

Non-linear approach to Random Walk Test in selected African countries

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Abstract

Purpose – The purpose of this paper is to re-examine the weak form efficiency of five African stock markets (South Africa, Nigeria, Egypt, Ghana and Mauritius) using various tests to assess the impact of non-linearity effect and thin trading which are prevalent in African markets on market efficiency.

Design/methodology/approach – The weekly returns of S&P/IFC return indices for five African countries over the period 2000-2013 were obtained from DataStream and analyzed. The study adopted the newly developed Non-Linear Fourier unit root test advanced by Enders and Lee (2004, 2009) which allows for an unknown number of structural breaks with unknown functional forms and non-linearity in data generating process of stock prices series to test the Random Walk Hypothesis (RWH) for the five markets, and an augment regression model.

Findings – In light of the empirical evidence the author(s) using Non-linear Fourier Unit Root Test only fail to reject the RWH for South Africa, Nigeria and Egypt leading to the conclusion that these markets follow the RWH and weak-form efficient whilst Ghana and Mauritius are weak-form inefficient. Besides, evaluating non-linear models without adjusting for thin trading effect shows that, South Africa and Ghana markets are weak-form efficient while Nigeria, Egypt and Mauritius are not. However, after accounting for thin trading effect, the author(s) find that South Africa and Egypt markets follow the RWH. The findings imply that market efficiency results depend on the methodology used.

Originality/value – This paper provides further evidence on stock market efficiency in emerging markets. The finding suggests that thin trading and non-linearity effect influences markets efficiency tests in African stock markets. Thus, recent structural adjustment and liberalization policies have not enhanced stock market operations in Africa. This paper therefore has implications for policy makers and international investors.

Keywords Africa, Market efficiency, Non-linearity, Non-Linear-Fourier-Unit Root-Test, Thin trading

Paper type Research paper

1. Introduction

Stock market efficiency has been at the forefront of financial theory for over four decades since the publication of Fama's (1970) seminal work. This is partly because stock markets play crucial roles in economic growth and development by mobilizing national savings, financing new investment projects and channeling investment and financial resources into productive enterprises (Adjasi and Biekpe, 2006; Levine and Zervos, 1998). Furthermore, in both emerging and developed economies efficient capital markets implies positive real economic activities and national prospects (Green *et al.*, 2005; Mishra, 2011). Market

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efficiency is important because efficient stock prices allow agents to diversify their sources of investment capital and spread investment risk (Caprio and Demirgüç-Kunt, 1998). Thus, capital market plays a pivotal role in the allocation of economic resources into the productive activities of the economy for economic development (Oskooe, 2011). This allocation takes place through the appropriate pricing of securities traded in the market. Such a market in which stock prices fully reflect all the available information is called efficient (Fama, 1970, 1991).

During the past decades, structural changes, financial liberalization and globalization policies undertaken by many emerging countries promoted an accelerated growth of stock exchanges around the world (Dhankar and Chakraborty, 2007). This led to an increased interest in determining the opportunities of investing in emerging markets to enhance portfolio returns. However, since Hinich and Patterson (1985) published their seminal work containing irrefutable evidence of non-linear behavior of stocks traded on the New York Stock Exchange, studies of such behavior on stock markets have become a growing subfield within financial econometrics (Broock *et al.*, 1996; Tsay, 1986; White, 1989, 1990). In the stock market efficiency literature, several reasons have been advanced to explain the non-linear behavior of financial time series data (Abhyankar *et al.*, 1997; Chappel *et al.*, 1998; Hsieh, 1991; Lim *et al.*, 2009; Opong *et al.*, 1999). The most frequently cited reasons for non-linearity can be summarized as follows: structural changes, high amplitude shocks, technological innovation, changes in regulations, characteristics of market microstructure, unreliable and unqualified information, existence of bid-ask spread, existence of market imperfections (market frictions, e.g. transaction cost, tax, government intervention), informational asymmetries, lack of reliable information and noise traders (Oskooe, 2011). In the case of emerging stock markets, other factors such as low liquidity, thin trading, price limits and market volatility also lead to non-linear behavior in the data generating process (DGP) (Antoniou *et al.*, 1997). Investigation of non-linearity is closely linked to the concept of efficient financial markets, that is, if markets are efficient, abnormal returns cannot be obtained using information available. Hence, the returns of financial assets, given some additional basic assumptions, must obey a random-walk model and therefore, their future performances cannot be predicted.

Most studies on stock market efficiency in Africa have not utilized non-linear methodologies in determining the efficiency of African markets. The studies ignore the possible non-linear nature of stock movements, specifically, in conditional mean of stock price movements (Afego, 2012; Magnusson and Wydick, 2002; Okpara, 2010; Olowe, 1999; Simons and Laryea, 2006; Smith *et al.*, 2002). For instance, the non-parametric runs test which is mostly used to test African markets, only tests for the existence of a linear relationship, which is inadequate as efficiency test on African stock markets show that return generating processes are likely to be non-linear (Appiah-Kusi and Menyah, 2003; Mlambo and Biekpe, 2007). Since the DGP of many economic financial time series is non-linear (Enders, 2015; Franses and Van Dijk, 2000; Teräsvirta *et al.*, 2010; Tsay, 2010), this paper uses the non-linear approach to Random Walk Hypothesis (RWH) to re-examine market efficiency test of some selected African stock markets. To avoid spurious results and provide valid implications, non-linearity in the behavior of financial time series should explicitly be accounted for. We adopt the non-linear Fourier Unit Root Test (Becker *et al.*, 2006; Enders and Lee, 2012b) which allows for an unknown number of structural breaks and non-linearity in the DGP of stock prices series. The results from the non-linear Fourier Unit Root Test indicates that accounting for possible unknown structural breaks and non-linearity in the stock prices series, South Africa, Nigeria and Egypt follow RWH and are efficient in weak form while Ghana and Mauritius are weak-form inefficient. Besides, evaluating non-linear models without adjusting for thin trading effect shows that, South Africa and Ghana markets are weak-form efficient while Nigeria, Egypt and Mauritius are not.

However, after accounting for thin trading effect, only South Africa and Egypt markets follow the RWH. The structure of the rest of the paper is as follows: Section 2 reviews the literature on stock market efficiency and RWH. Section 3 presents the data and methodology and Section 4 is devoted to the empirical findings. Finally, Section 5 concludes the paper.

2. Relevant literature review

Most early works on RWH generally confirmed the theory. For instance, the test of Cowles (1933), Working (1934), Kendall and Hill (1953) and Osborne (1959) computed the serial correlation coefficients for daily, weekly and monthly price changes and found them to be extremely closer to zero indicating evidence against dependence in the price changes. Eugene F. Fama (1965) examined the RWH using the natural log of price for each of the 30 stocks on Dow Jones Industrial Average for time periods that varied from stock to stock between 1957 and 1962. Using intervals of 4 days, 9 days and 16 days, Fama (1965) concluded that the amount of dependence in the data seemed to be either extremely slight or non-existent. Fama (1965) noted that for the 4 days interval, the average serial correlation coefficient was -0.039 while the 9 days and 16 days recorded average serial correlation of 0.053 and -0.057 , respectively. Lo and MacKinlay (1988) also reported that RWH was strongly rejected using the variance ratio (VR) statistics for the period, 1962-1985 on different sample periods for a variety of aggregate US indices and for size sorted portfolios. They also examined the finite-sample properties of VR test and found out that, compared to Dicky-Fuller test and Box-pierce Q -statistics, VR was more reliable under a heteroscedastic RWH null which is generally the case with financial prices. The RWH has also been examined in a number of stock markets apart from the US markets. For instance Niarchos (1971) examined the Greece market, Praetz (1972) tested Australian market, Jennergren and Korsvold (1975) tested Swedish and Norway markets. But, none of these studies found evidence against the RWH.

Darrat and Zhong (2000) investigated whether prices in two Chinese Stock Exchanges follow a random walk process as required by the efficient market hypothesis (EMH). Using the Standard VR test and a model comparison test (ARIMA, GARCH, and Artificial Neural Network), results from the model comparison approach were quite decisive in rejecting the RWH in both Chinese Stock Markets. Charles and Darné (2009) also examined the RWH for Chinese Stock Markets using the new multiple VR test, Kim's wild bootstrap tests and the conventional multiple Chow-Denning test. They realized that, Class B (Shenzhen Stock Exchange) shares for Chinese Stock Exchanges did not follow RWH, and hence significantly inefficient while Class A (Shanghai Stock Exchange) shares seemed more efficient. Budd (2012) obtained time series data for each sector of Tadawul exchange between June 19, 2007 and September 12, 2011, and used parametric and non-parametric approaches to analyze the data. Using the Runs test, VR test and serial correlation, Budd (2002) rejected the RWH for all sectors which was attributed to serial correlations, an evidence of weak-form efficiency in the Tadawul exchange. JaihanZulqarnain and Amir Shah (2013) used data from KSE-100 Index from July, 2006-June, 2011. Runs test, Unit roots test (ADF, PP, etc.), correlation test were used to test the hypothesis with the assistance of E-VIEWS and SPSS. The results showed that, Karachi Stock Exchange (KSE) does not follow the RWH since the Runs test, Autocorrelation and Unit root test all rejected the RWH endorsing market inefficiency. Jarrett and Kyper (2006) monthly closing prices of 62 firms listed on organized exchanges (NYSE and NASDAQ) were obtained for the period, April, 1992-September, 2002. A predictive model for measuring the effect of changes in the day of the week on closing prices of securities was used. For Tuesdays, all the coefficients were significant at levels equal to 0.0001 or less. The same results occurred for Wednesdays, Thursdays and Fridays. However, the findings that closing prices of securities follow RWH is doubtful. Chaity and

Sharmin (2011) used both All Share Price Indices and DSE General Indices for the period January, 1993-June, 2011 and January, 2002-July, 2011, respectively. Using serial correlation, autocorrelation and ARIMA models to test market efficiency, the results revealed the presence of non-randomness in the series thereby rejecting the null hypothesis.

Kompa and Matuszewska-Janica (2009) also examined the efficiency of Warsaw Stock Exchange using Runs test and VR test. Data obtained were grouped into sub periods. The result from the Runs test shows that, some securities were random while others were not random. For the VR test, sub periods $P3$ and $P4$ were not random while $P1$ was random. Camelia (2012) again tested informational efficiency of UE and BRIC emergent markets using the RWH. Various stationary tests for instantaneous returns (logarithmic) of stock indices were employed: Unit root tests (Augmented Dickey-Fuller test, Phillips-Perron test, autocorrelation coefficients and Ljung-Box test), Runs test and VR test. From the Runs test, during the ten years period, all the other capital markets, in Hungary, Czech Republic, Brazil, China, Russia (at cut point = median) and India (at cut point = mode) were weak-form efficient. From the VR test, the individual statistics generally rejected the null hypothesis, indicating Random Walks in the series. Arshad Haroon (2012) tested the weak-form efficiency of KSE using Random Walk theory. KS test, Runs test and Autocorrelation test were some of the statistical tools used to test for the hypothesis. Runs test, KS test and Serial Correlation test all indicated that, KSE is not weak-form efficient and did not follow a Random Walk.

Majority of efficient market research has been undertaken on US and European securities market with little focus on emerging markets, including Africa. The scanty studies on emerging markets is largely because these markets are small and there is not enough information to draw robust conclusions. However, Osei (2002) showed that Ghana Stock Exchange (GSE) was inefficient when earnings announcement was incorporated into stock prices, semi-strong form inefficiency. Also, Adjasi (2004) after examining GSE found out that stock returns exhibited volatility clustering and persistence indicating a positive asymmetry in volatility of stock returns when he modeled the returns using EGARCH (1, 2) in mean. Smith *et al.* (2002) tested the hypothesis that stock market price index followed a Random Walk using the multiple VR test on eight African markets. They observed that seven of the markets – Botswana, Egypt, Kenya, Mauritius, Morocco, Nigeria and Zimbabwe – did not follow a Random Walk because returns were auto-correlated, except South Africa's All Share Index. Olowe (1999) provided further evidence on the weak-form efficiency of the Nigerian Stock Exchange (NSE). Using correlation analysis to test monthly stock return data over the period of January 1981 and December 1992, the results supported the works of Samuels *et al.* (1981) and Ayadi (1984) that NSE appeared efficient in the weak-form. Afego (2012) also examined the weak-form efficiency for NSE by testing for Random Walk in monthly index returns. Using the Runs test, results showed that stock price changes in the NSE were not random and that exploitation patterns existed. Simons and Laryea (2006) also employed various tests to investigate weak-form of EMH for four African Stock Markets. The results of parametric and non-parametric tests showed that South African market was weak-form efficient whereas Ghana, Mauritius and Egypt are weak form inefficient. Again, Alagidede (2008) studied the behavior of stock returns on Africa's biggest markets (South Africa, Egypt, Nigeria, Kenya, Tunisia and Morocco). Using GARCH models, weak-form efficiency was rejected for all the markets. Employing parametric and non-parametric tests on eight African markets and eight individual national daily shares, Ntim (2013) reported statistically significant weak-form informational inefficiency of the African continent share price index over the eight individual national share prices irrespective of the test used. Magnusson and Wydick (2002) also used recent data to examine eight African emerging markets and compared them with emerging markets of South Asia and Latin America. Using the RWH, correlation analysis indicated that weak-form efficiency in emerging African Stock Markets compared favorably

with those performed on other emerging stock markets. Okpara (2010) tested RWH on NSE using the Runs test and correlogram as alternative instrument. The results indicated that NSE was efficient in weak form and followed a Random Walk. Again, Alagidede and Panagiotidis (2009) investigated the behavior of stock returns to test the validity of RWH using a battery of tests (GARCH, GARCH-M and EGARCH-M). They rejected the RWH in all cases and observed that, empirical stylized facts of volatility clustering leptokurtosis and leverage effects were present in the African market. Furthermore, Mlambo and Biekpe (2007) used serial correlation and the Runs test to study the weak-form efficiency of ten African markets and with the exception of Namibia, a significant number of stocks rejected the RWH. Finally, using GARCH approach with time-varying parameter, Jefferis and Smith (2005) examined the efficiency of African markets. A test of evolving efficiency (TEE) was implemented for periods starting from 1990 to June 1, 2001. The TEE test showed that, Johannesburg Stock Exchange was weak-form throughout the period while three markets became more efficient getting to the end of the period – Egypt, Morocco and Nigeria – which contrasted with Kenya and Zimbabwe markets revealing no tendency to weak-form efficiency.

3. Data and methodology

This paper investigates the RWH of five selected African Stock Markets by considering the effect of thin trading and non-linearity in the price series of the selected markets. To achieve the objectives of the study, weekly All Share Price Index (ASPI) from June, 2000 to November, 2013 was obtained from DataStream. ASPI was used because it represents the price variations of all the listed companies in the market. Closing index returns (R_t) was calculated using the first log difference for weekly price index for each of the five selected stock markets: $R_t = \ln(P_t/P_{t-1})$, where P_t and P_{t-1} represent the current and previous weekly market price index.

The Fourier series is used since the trigonometric terms are defined to capture the unknown non-linearity in the equilibrium level. More precisely, the Fourier Unit Root Test relies on Fourier approximation for the transition function which captures the structural changes. We also used Dickey-Fuller test in which the deterministic function is a time-dependent function denoted by $\alpha(t)$:

$$P_t = \alpha(t) + \rho P_{t-1} + \gamma t + \varepsilon_t \quad (1)$$

where ε_t is the stationary disturbance with variance σ_ε^2 and $\alpha(t)$ is a deterministic function of t . Becker *et al.* (2006) were interested in testing the null hypothesis that $\rho = 1$. When the form of $\alpha(t)$ is unknown, any test for $\rho = 1$ is problematic if $\alpha(t)$ is mis-specified. As an approximation of the unknown functional form of $\alpha(t)$, the study considers the Fourier expansion:

$$\alpha(t) = \alpha_0 + \sum_{k=1}^n \alpha_k \sin\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^n \beta_k \cos\left(\frac{2\pi kt}{T}\right), n \leq T/2 \quad (2)$$

where n represents the number of frequencies contained in the approximation; k the particular frequency; t the trend term; and T the number of observations. If $\alpha_1 = \beta_1 = \dots = \alpha_n = \beta_n = 0$, the process is linear, and the traditional Unit Root Testing methodologies are appropriate. However, if there is a break or non-linear trend, at least one Fourier frequency must be present in the DGP. As proposed by Gallant (1981), most other approximations, such as Taylor Series (including power series and Translog approximations), are valid at a particular point in the sample space. That is, they have local validity. An important advantage of Fourier approximation is that it is global, rather than local approximation. As a practical matter, it is not possible to use a large value of n in a regression framework. Using many frequency components uses degrees of freedom and can lead to an over fitting problem. Thus, instead of

positing the specific form of $\alpha(t)$, it is better to select the proper frequencies to include in Equation (2) above. Hence, suppose we use only a single frequency k , and consider the testing regression:

$$\Delta P_t = \rho P_{t-1} + c_1 + c_2 \sin\left(\frac{2\pi kt}{T}\right) + c_3 \cos\left(\frac{2\pi kt}{T}\right) + e_t. \quad (3)$$

It may take varying periods of time for an economic or financial series to display the effects of the realized events (Becker *et al.*, 2006; Enders and Lee, 2012a, b). To account for these features of structural breaks and control for the effect of unknown form of non-linear deterministic terms, Enders and Lee (2005, 2012a, b) a new Fourier Unit Root Test that replicates the pattern of structural breaks using a single-frequency Fourier function was proposed (Becker *et al.*, 2006; Enders and Lee, 2012a, b). In this method, the trigonometric terms are defined to capture unknown non-linearities in the equilibrium level.

Following others (Becker *et al.*, 2006; Christopoulos and León-Ledesma, 2010; Enders and Lee, 2012a, b), the study adopted the Non-Linear Fourier Unit Root Test as follows:

$$\Delta P_t = \rho P_{t-1} + c_1 + c_2 \sin\left(\frac{2\pi kt}{T}\right) + c_3 \cos\left(\frac{2\pi kt}{T}\right) + e_t \quad (4)$$

In theory, through selecting an appropriate single frequency k , the corresponding Fourier function [$c_2 \sin(2\pi kt/T) + c_3 \cos(2\pi kt/T)$] captures structural changes in the form of the sequence $\{P_t\}$. In practice, in order to determine the unknown value of k the equation is estimated for each integer value of k and the appropriate value of k is selected based on the smallest residual sum of squares (RSSs). After that, a formal test for the presence of unknown breaks in the DGP, $\{P_t\}$ is carried out by testing the hypothesis:

$$H_0. C_2 = C_3 = 0.$$

$$H_1. C_2 \neq 0, C_3 \neq 0.$$

If the null hypothesis is rejected, the Fourier function captures non-linearity. Also test for non-stationarity is carried on by applying OLS to estimate Equation (4) and testing the null hypothesis $H_0: \rho = 0$. If it is significantly different from zero (using the τ_{LM} statistics, Becker *et al.*, 2006), we reject the null hypothesis of Unit Root test taking into account non-linearity and possible structural breaks and therefore the sequence, P_t is said to be non-stationary and does not follow RWM.

However, it should be noted that, if error terms in Equation (4) reveal serial correlation, the augmented form of the test will be estimated using the lagged values of ΔP_t as follows:

$$\Delta P_t = \rho P_{t-1} + c_1 + c_2 \sin\left(\frac{2\pi kt}{T}\right) + c_3 \cos\left(\frac{2\pi kt}{T}\right) + \sum_{i=1}^p \beta_i \Delta P_{t-i} + e_t \quad (5)$$

To account for the possibility of non-linearity, most papers (Harrison and Moore, 2012) consider estimating an equation of the form:

$$R_{mt} = \varpi_0 + \varpi_1 R_{mt-1} + \varpi_2 (R_{mt-1})^2 + \varpi_3 (R_{mt-1})^3 + \varepsilon_t \quad (6)$$

R_{mt} is returns on the market. For the efficient market to hold, all the coefficients should be equal to zero ($\varpi_1 = \varpi_2 = \varpi_3 = 0$). While this test can account for quadratic non-linearity, it is also possible that non-linearity could be driven by an exponential DGP that nests other types of non-linearity (e.g. Threshold models, Markov switching models, etc.). Therefore, the equations are augmented with: $e^{-[R_{mt-1}]R_{mt-1}}$ to account for the possible exponential

non-linearity (Castle and Hendry, 2010):

$$R_{m_t} = \varpi_0 + \varpi_1 R_{m_{t-1}} + \varpi_2 (R_{m_{t-1}})^2 + \varpi_3 (R_{m_{t-1}})^3 + \varpi_4 e^{-[R_{m_{t-1}}]} R_{m_{t-1}} + \varepsilon_t. \tag{7}$$

In order to account for the presence of autocorrelation occurring because of relatively thin trading, Miller *et al.* (1994) suggest fitting an AR(1) model to obtain an adjustment for infrequent trading. However, the assumption of a fixed autoregressive coefficient is particularly unlikely to hold in emerging markets as these markets are now maturing. As an alternative, this study uses a state space model of the form:

$$R_{m_t} = a_o + \beta_t R_{m_{t-1}} + u_t \tag{8}$$

where $\beta_t = \gamma \beta_{t-1} + v_t$

R_{m_t} is the stock market return calculated at the period (t) (β_t) is an unknown parameter that is assumed to be a first order autoregressive process, while u_t and v_t are independent white noise processes that are assumed to have a zero mean and a constant variance. The fitted residuals ($\hat{u}_t$) from Equation (8) were then employed to estimate the adjusted return series ($R_{m_t}^A$), $R_{m_t}^A = u_t / 1 - \beta_t$.

We estimate Equation (7) for the adjusted return series as follows:

$$R_{m_t}^A = \varpi_0 + \varpi_1 R_{m_{t-1}}^A + \varpi_2 (R_{m_{t-1}}^A)^2 + \varpi_3 (R_{m_{t-1}}^A)^3 + \varpi_4 e^{-[R_{m_{t-1}}^A]} R_{m_{t-1}}^A + \varepsilon_t \tag{9}$$

4. Empirical results

For the purposes of testing weak-form efficiency and the presence of non-linearity in the DGP, weekly market index of five selected African countries were examined over the period, 2000-2013. The weekly returns series of the selected countries were computed as the logarithm price ratio of today to price yesterday. Table I summarizes the descriptive statistics of the return series. The highest weekly mean returns were recorded by Nigeria (0.0245), with Ghana having the lowest returns (0.0073). Interestingly, the lowest mean return recorded by Ghana coincides with the lowest standard deviation while Egypt recorded the highest standard deviation (0.0435).

Sample	South Africa January 6, 2000 to November 21, 2013	Nigeria January 6, 2000 to November 21, 2013	Egypt January 6, 2000 to November 21, 2013	Ghana January 7, 2000 to November 21, 2013	Mauritius January 6, 2000 to November 21, 2013
Period					
Mean	0.0161	0.0245	0.0117	0.0074	0.0185
Max	0.24976	0.13359	0.14594	0.21903	0.19425
Min	-0.1902	-0.1661	-0.2193	-0.1712	-0.2258
SD	0.03942	0.03692	0.04354	0.02548	0.02632
Skewness	-0.2392	-0.3919	-0.6461	-0.0671	0.19354
Kurtosis	6.8637	5.8432	6.0444	19.5373	19.0881
Jarque-Bera (JB)	0.000	0.000	0.000	0.000	0.000
Observation	723	723	723	723	723
Breusch- Godfrey LM Test(12)	176.45*** [0.000]	146.12*** [0.000]	77.65*** [0.000]	32.87*** [0.000]	43.21*** [0.000]

Table I.
Descriptive statistics
(logarithmic returns)

Notes: Breusch-Godfrey is the test statistics for autocorrelation. JB is the p -values of Jarque-Bera statistic for normality. ***Statistical significant at 1 percent level

The risk-return trade-off is the balance between the desire for the lowest possible risk and the highest possible return: if there are expectations of higher level of risk associated with investment then greater returns are required to compensate for that higher expected return. However, Egypt with the highest mean return did not record the highest risk. Alternatively, an investment with relatively lower levels of expected risk would require investors settle for relatively lower returns as depicted by Ghana which recorded both the least mean return and standard deviation of the series. Table I indicates that, generally, the standard deviation for the other countries is high implying that investors in Africa's emerging markets must be ready to accept risk in exchange for possible higher returns. The distributional properties of returns appear to show extreme observations. The highest kurtosis in the sample occurs in Ghana and Mauritius, with Egypt, Nigeria and South Africa having the lowest. However, the kurtosis of all countries exceeds the threshold of 3, implying that the returns have flatter tails that would be expected from a normally distributed variable. With the exception of Mauritius, all the return series are negatively skewed. The Jarque-Bera test rejects the normality assumption for all countries. Deviations from normality could be induced in part by the temporal dependencies in return, especially second moment temporal dependence, an indication that assuming a linear process for returns may leave important features of the data unexplained. The second moment dependence is reinforced by Breusch-Godfrey LM test calculated for 12 lags. The hypothesis that all autocorrelation up to the 12th lag is zero is rejected. A possible reason for autocorrelation in the returns is non-synchronous trading and thin trading which are common features of African markets. Moreover, majority of stocks scarcely trade and even the most active stocks only trade for a few hours in the working week.

The study reports the results of applying the non-linear Fourier Unit Root test to detect stochastic or deterministic trend of stock prices series for the five selected stock markets in Africa. First, a grid-search was performed to find the best frequency, as there is no a priori knowledge concerning the shape of the breaks in the data. Following recommendation of Enders and Lee (2012a,b) that a single frequency can capture a wide variety of breaks, Equation (2) was estimated in order to determine the unknown value of k . The second column in Table II displays the RSSs and indicates the single frequency that works best for each series. With the exception of Ghana and Mauritius, for which the results showed that frequency 2 is the best choice, all the other markets showed $k=1$ to be the best frequency.

The significant $F(\hat{k})$ statistics shown in column 4 of Table II also indicates whether both sine and cosine terms should be included in the estimated model and both are used to test for linearity in the DGP. The fifth column shows the number of lags of ΔP_t needed to remove serial correlation in residuals. The sixth column reports the results of the Unit Root Test with a non-linear Fourier Function based on the estimated frequency. Following Enders and

Countries	RSSs	$\hat{k}$	$F(\hat{k})$	The number of lags of ΔP_t	$\tau LM(\hat{k})$	Bootstrapping critical values		
						10%	5%	1%
South Africa	0.248	1	8.516**	1	1.78	2.63	2.33	3.21
Nigeria	0.980	1	16.211***	8	5.27	8.12	7.98	9.82
Egypt	0.01	1	12.541***	5	7.60	10.01	10.51	10.91
Ghana	0.055	2	10.539***	4	11.15***	4.88	5.02	5.78
Mauritius	0.681	2	11.210**	7	9.82***	4.13	4.41	5.42

Notes: The critical values of the $\tau LM(\hat{k})$ statistic t test for 10, 5 and 1 percent levels of significance are obtained by 10,000 bootstrapping replications. **,***Significant at 5 and 1 percent levels, respectively

Table II.
Unit root test with a
non-linear Fourier
function

Lee (2012a, b), the critical values of each country was obtained tailored to the sample size. Hence, the critical values of the $\tau LM(\hat{k})$ statistic test for 10, 5 and 1 percent are obtained by bootstrapping, replicating 10,000 times (see last columns of Table II). In the next step, the null hypothesis of linearity ($H_0: C2 = C3 = 0$) was tested with the F -statistics. After that, the augmented equation (Equation (5)) was estimated and appropriate lags based on randomness of the error terms recognized. The null hypothesis of unit root ($H_0: \rho = 0$) was also tested using the $\tau LM(\hat{k})$ from Equation (5).

As shown in Table II, the null hypothesis of linearity was rejected for all the five markets. Hence stock prices on these markets exhibit non-linearity in the DGP. These results are in consonance with earlier works of Darrat and Zhong (2000) and Lim, Hinich, and Brooks (2006) who posited that, stock price series follow a non-linear process and can be well captured by a more robust technique. From the analysis, all the markets exhibit non-linearity. This implies that findings can lead to bias conclusions if the non-linearity effect is not considered when measuring market efficiency.

To determine whether the markets are weak-form efficient or not, the null hypothesis of unit root was tested using the $\tau LM(\hat{k})$ statistic for ρ in equation five. From the $\tau LM(\hat{k})$ statistics, the null hypothesis of unit root was not rejected for three markets comprising of South Africa, Nigeria and Egypt. In line with the findings, it can be concluded that the series for the three markets is not stationary. In other words, it has a unit root. This means that, South Africa, Nigeria and Egypt Stock Market indices are non-stationary and as a result follow the Random Walk Theory leading to the conclusion that these markets are weak-form efficient. This further implies that investors cannot beat the market to obtain abnormal profits. The findings are consistent with previous studies for Nigerian market (Samuels *et al.*, 1981; Smith *et al.*, 2002), for South African market (Mlambo and Biekpe, 2007; Smith *et al.*, 2002) and for Egyptian market (Abdmoulah, 2010; Appiah-Kusi and Menyah, 2003; Harrison and Moore, 2012). The efficiency of the Egyptian market may be due to its openness to foreign investors, with ownership restriction only imposed on a few industries. For Ghana and Mauritius, the null hypothesis of unit root was rejected indicating, the series is stationary. In other words, they have no unit root and do not follow the Random Walk Theory making these markets inefficient. For Ghana, the result is consistent with the findings of Appiah-Kusi and Menyah (2003), Ntim *et al.* (2007) and Smith (2008). For the Mauritius market, the finding is consistent with the studies of Smith (2008), Appiah-Kusi and Menyah (2003) and Mlambo and Beikpe (2007). The results, nevertheless, are inconsistent with those of Afego (2012) and Smith (2008) for Nigeria Market, South Africa and Egypt Markets.

In order to evaluate the efficiency in the selected African Stock Exchanges, Table III provides estimates of the linear model, unadjusted and adjusted, for thin trading. Without adjusting for relatively thin trading in the five exchanges, the null hypothesis of market efficiency could be rejected in four of the markets under study. Only South Africa had its returns uncorrelated with previous values. The significance of the Ljung and Box Q -test in

Table III.
Linear test for stock
market efficiency

	Unadjusted for thin trading		Adjusted for thin trading	
	ϖ_1	Q -stat	ϖ_1	Q -stat
South Africa	-0.056 (-1.55)	44.86 [0.28]	0.0003 (0.09)	40.74 [0.44]
Nigeria	0.096 (2.61)***	90.92 [0.00]	-0.113 (-2.56)**	72.76 [0.00]
Egypt	0.074 (1.99)**	56.30 [0.04]	-0.08 (-1.96)	60.06 [0.02]
Ghana	0.161 (4.38)***	127.22 [0.00]	-0.134 (-0.38)	191.25 [0.00]
Mauritius	0.108 (2.93)**	152.44 [0.00]	-0.005 (-0.13)	97.03 [0.00]

Notes: t -Statistics are provided in parentheses below coefficient estimates and p -value in square bracket. Q -stat is test for autocorrelation. **, ***Significant at 5 and 1 percent levels of testing

the majority of the countries also suggests that there was serial correlation between contemporaneous and lagged returns for up to 52 lags. Based on the significance of the Q -test, Stock Exchanges used for the study appear to be weak-form inefficient except South Africa. One of the key characteristics of exchanges in developing countries is thin trading and if this phenomenon is not accounted for, it could lead to leptokurtic distortions in the measurement of portfolio returns. The last three columns of Table III also present the results for adjusting the returns for thin trading. After adjusting for thin trading, the lagged returns were significant in only one exchange compared to the unadjusted return results, though the Q -statistics still remained significant in four out of the five markets. These results confirm the works of Afego (2012) on Nigerian Stock Exchange. In addition, previous works such as Magnusson and Wydick (2002) and Smith *et al.* (2002) confirm results of Johannesburg Stock Exchange. The efficiency of GSE and MSE contradicts the findings of Mlambo and Biekpe (2007). The results certainly suggest that not accounting for the relatively thin trading on the markets can bias the determination of Stock Market efficiency.

Benartzi and Thaler (1995) noted that if investors are loss averse, that is, they are more sensitive to losses than gains, their behavior may appear to be risk-neutral or even risk-loving, thus violating one of the assumptions of efficient market model of rational or risk averse, investors. Such behavior might result in non-linear stock price behavior. A number of factors may induce non-linearity in stock returns. For instance, where investors and markets overreact to bad news and under react to good news (Bondt and Thaler, 1985), the feedback mechanism for returning the asset price to its equilibrium level may be non-linear. Hence, efficiency tests such as the Runs test that do not account for such non-linearity are likely to draw inappropriate conclusions from the results. The non-linear results for Stock Market efficiency that does not account for thin trading and one that accounts for thin trading are provided in Tables IV and V, respectively. In this instance, the market is weak-form efficient if we only fail to reject null hypothesis of ($\varpi_1 = \varpi_2 = \varpi_3 = 0$) at normal levels of testing and the residual term follows a white noise process. The F-statistics is provided in the last column of the tables.

Table IV.
Non-linear test for
stock market
efficiency (without
accounting for thin
trading)

	ϖ_1	ϖ_2	ϖ_3	ϖ_4	Q-stat	F-stat
South Africa	2.891 (1.17)	-0.671 (-1.43)	-9.711 (-1.05)	-3.097 (-1.20)	83.40 [0.15]	2.24 [0.61]
Nigeria	-15.363 (-3.90)***	-0.781 (-1.48)	73.643 (3.34)***	18.075 (4.12)***	121.20 [0.00]	7.86 [0.000]
Egypt	-7.534 (-2.87)***	0.647 (1.28)*	21.437 (1.89)*	8.086 (2.94)***	76.82 [0.00]	6.70 [0.000]
Ghana	0.137 (1.93)*	-0.006 (-0.48)	-0.596 (-1.61)	-0.144 (-1.95)*	102.56 [0.28]	1.40 [0.23]
Mauritius	1.097 (-15.17)***	-0.595 (-67.49)**	-1.193 (-4.43)**	-1.12 (16.02)***	167.00 [0.00]	17.57 [0.00]

Notes: t -Statistics are provided in parentheses below coefficient estimates and p -values in square bracket. *, **, ***Significant at 10, 5 and 1 percent levels of testing-Stat is test for autocorrelation

Table V.
Non-linear test for
stock market
efficiency (accounting
for thin trading)

	ϖ_1	ϖ_2	ϖ_3	ϖ_4	Q-stat	F-stat
South Africa	1.745 (0.64)	-0.886 (-1.81)	-5.381 (-0.49)	-1.855 (-0.81)	81.14 [0.43]	1.56 [0.18]
Nigeria	-13.258 (-3.78)***	-1.168 (-2.50)*	55.590 (3.14)	15.831 (4.25)***	96.45 [0.00]	6.68 [0.00]
Egypt	-1.602 (-2.53)	0.144 (1.12)	0.873 (1.42)	2.055 (2.91)***	67.11 [0.31]	4.91 [0.21]
Ghana	-0.357 (-7.93)***	0.143 (1.80)	0.759 (3.86)***	0.205 (4.38)***	97.56 [0.00]	1.90 [0.00]
Mauritius	-0.443 (-1.90)	-0.452 (-1.36)	-5.941 (-0.77)	0.595 (0.25)	137.01 [0.00]	3.91 [0.003]

Notes: t -Statistics are provided in parentheses below coefficient estimates and p -values in square bracket. *, **, ***Significant at 10 and 1 percent levels of testing-Stat is test for autocorrelation

From Table IV, the result reveals that, without accounting for thin trading but rather non-linearity, the introduction of the non-linear terms does not make changes to the earlier findings since the coefficient of R_{t-1} is still statistically significant in four markets with the exception of South Africa. However, the coefficients of the non-linear terms for some countries were significant. For instance, all the non-linear terms were not significant for South Africa. In Ghana and Nigeria, some of the non-linear terms were also not significant. Nigeria had its lagged returns being significant in both cases. The Q statistics results depict that, after the introduction of non-linear terms, serial correlation still existed in four markets as shown earlier in Table IV.

Thus, the inclusion of non-linear terms has not removed the error term having white noise properties for some countries. Since the coefficient of lagged returns for most countries and non-linear terms are significant and the error terms does not follow white noise properties, provisionally, the study rejects the null hypothesis that the market is efficient for four markets in Table IV using the F -statistics. Table V depicts the predictable behavior of stock returns after introducing non-linear terms and accounting for thin trading in the various markets. The lagged return series and non-linear terms for South Africa and Egypt are not statistically significant and error terms do not show any significant serial correlation. Hence, the market inefficiency of Egypt found earlier was due to thin trading, since after adjusting for thin trading and non-linearity, the market follows the Random Walk model just like South Africa. The efficiency of Egypt Stock Market may be attributed to the least restrictions in relation to foreign investment regulations among the MENA countries. Ghana, after adjusting for thin trading and non-linearity, proved to be weak-form inefficient and the lagged returns proved to be significant. In summary, three markets after accounting for thin trading and non-linearity proved that, we only fail to reject the null hypothesis of market efficiency.

5. Conclusions

It is well documented that stock prices or stock returns specifically in emerging Stock Markets behave in a non-linear manner. In this sense, the unknown structural breaks and non-linearity in the DGP should be taken into account in testing for the RWH in Stock Markets. Hence, the study set out to achieve two objectives. The first was to ascertain the validity of the Random Walk Theory. The second was to verify whether non-linearity exists in stock markets as stipulated by Antoniou *et al.* (1997) that, due to some reasons such as market microstructure, transaction cost, etc. stock prices are non-linear in nature. We adopted three different methodologies which had the ability to capture structural breaks and non-linearity in the DGP of stock price series. From the analysis, we conclude that the market index of the five markets under study are all non-linear (structural breaks exist) in nature. In other words, they exhibit non-linear characteristics in the DGP using the Non-linear Fourier Unit Root test. On the validity of the RWH for these markets, the Non-linear Fourier Unit Root test revealed that, South Africa, Nigeria and Egypt markets are efficient while Ghana and Mauritius are weak-form inefficient. Using the augmented regression equation where thin trading was accounted for in the return series and the addition of non-linear terms to ascertain the efficiency of the markets, the result depicts that South Africa and Egypt are efficient while Nigeria, Ghana and Mauritius are inefficient. Interestingly, without accounting for thin trading, Ghana market was efficient but after adjusting for thin trading, the market proved to be inefficient. Lastly, as to whether the non-linear nature of the returns affect market efficiency, the study revealed mixed results. This is because, even though some markets exhibited features of non-linearity, the study proved them to be efficient and vice versa. Hence, for the inefficient markets, smart investors can analyze the trend and make abnormal profits on their investments. The presence of non-linearity in stock returns of these five markets reveals that there are market imperfections. Therefore, it is

recommended that, steps should be taken to encourage more investors to participate in share trading. In this regard, the introduction of risk hedging instruments and allowance of short sales would be much effective. Furthermore, the findings that markets in some countries are inefficient means that, policy makers in these countries should strengthen the institutional structure of price-forming information. Thus, technologies that enhance the speed with which information is disseminated, the development of business journals and market regulations, are key policy recommendations that will enhance the efficiencies of these markets. Moreover, structural changes and reforms in these markets should be embarked on to enhance the efficiency of these markets. Further studies should explore in detail the microstructure and other factors that account for the inefficiency in some of the markets.

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