

UNIVERSITY OF GHANA

**MULTI-DIRECTIONAL PRODUCTIVITY CHANGE INDEX: BIENNIAL MEA
MALMQUIST INDEX**

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OPERATIONS

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DECLARATION

I Mark Cleur, hereby affirm that the contents of this work are the culmination of my independent research efforts and have not been previously submitted, either partially or in its entirety, for any academic recognition at this institution or elsewhere. All sources and references utilized in this work have been appropriately cited and acknowledged. Any deficiencies or limitations within this work are solely my responsibility.



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CERTIFICATION

I certify that this thesis was supervised in strict adherence to the established protocols outlined by the University of Ghana.



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DEDICATION

This thesis is dedicated to God Almighty for his Grace and Mercies.



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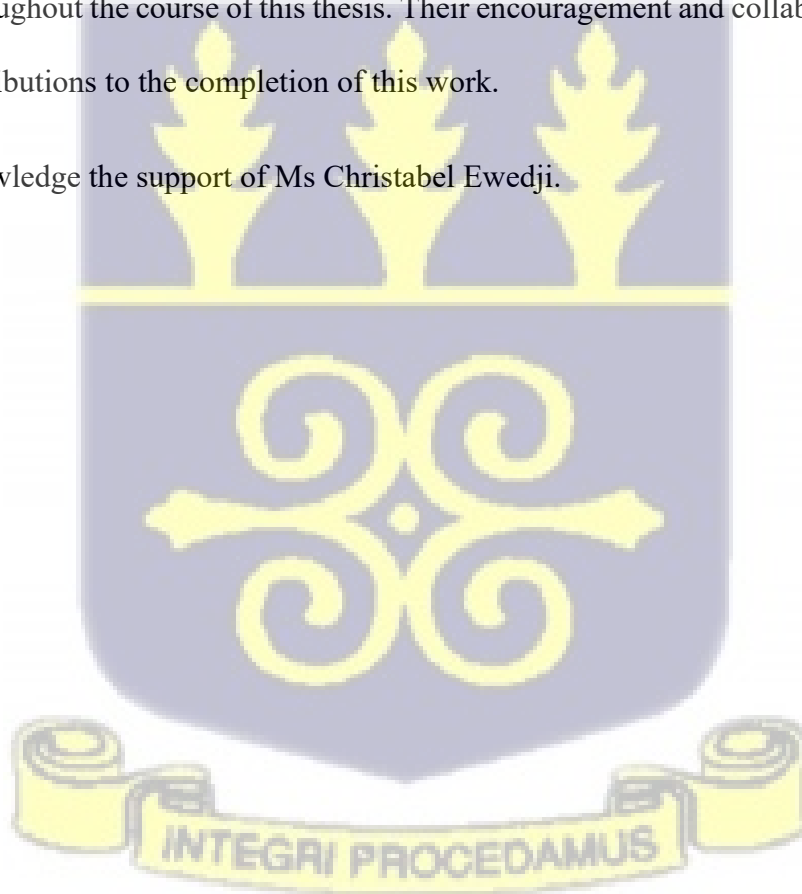


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TABLE OF ABBREVIATIONS

BMEAM Biennial Multi Directional Efficiency Analysis Malmquist

BMI Biennial Malmquist Index

BPEC Biennial Pure Efficiency Change

BPTC Biennial Pure Technological Change

BSEC Biennial Scale Efficiency Change

BSTC Biennial Scale Technological Change

CRS Constant Returns to Scale

DEA Data Envelopment Analysis

DMU Decision making Unit

EC Efficiency Change

EDA Exploratory Data Analysis

EIA Energy Information Administration

EU European Union

FCU Frontier catch-up

GDP Gross Domestic Product

LP Linear programming

MEA Multi- Directional Efficiency Analysis

MEAM Multi- Directional Efficiency Analysis Malmquist

MPI Malmquist productivity change index

OECD Organisation for economic Co-operation and Development

PI Potential Improvement

PPS Production possibility Set

PPP Purchasing Power Parity

PTCU Pure technological catch-up

PWT Penn World Tables

RDM Range Directional Model



RTS Returns to Scale

SDG Sustainable Development Goals

SFA Stochastic frontier analysis

TC Technical change

VRS Variable returns to scale



ABSTRACT

The study extends MEA-Malmquist to handle LP infeasibilities and scale effect in infinite mixed-period efficiency scores, develop a proposed productivity index (MEA Biennial Malmquist-BMEAM), decompose the productivity index into Pure Technical Change, Pure Efficiency Change, Scale Technical Change and Scale Efficiency Change and empirically apply the proposed productivity index to determine the energy productivity change of African states. A quantitative research methodology that includes extending multi-directional productivity change (MEA-Malmquist) to provide a comprehensive assessment of the energy sector of African states' performance and identify patterns, trends, and factors influencing their development over the examined period.

This study through MEA-Malmquist investigates the dynamic productivity of to 32 African states across the five sub-regions countries based on availability of data covering the periods from 2000 to 2019. In the first phase, the study develops and illustrates a proposed productivity index (MEA Biennial Malmquist-BMEAM) by formulating the MEA efficiency model, incorporating the variable specific efficiencies into Malmquist productivity index to measure productivity changes over time then solving the infeasibilities using a biennial frontier to include the two time periods. The results showed that after using the biennial frontier the infeasibilities in estimating productivity were resolved. The study further decomposed MEA Biennial Malmquist-BMEAM into 4 factors, the results revealed that the effect of managerial expertise as the main driver of the improvement observed in the industry. Also, the productivity change across the years shows that African energy industry, on average, experienced about 0.41% productivity improvement for capital services, 0.09% improvement productivity for labour, 0.02% decline for energy used.

CHAPTER ONE

INTRODUCTION

1.1 Background of Study

In recent decades, performance assessment and productivity measurement have become central to strategic decision-making in both public and private sectors (Asmild et al., 2007; Zhu, 2009). This focus reflects the need for organizations and economies to benchmark efficiency, allocate resources optimally, and improve service delivery in the face of increasing global competition and resource constraints.

Among the most prominent tools for assessing efficiency and productivity is Data Envelopment Analysis (DEA), a non-parametric, linear programming-based technique introduced by Charnes, Cooper, and Rhodes (1978). DEA evaluates the relative efficiency of decision-making units (DMUs) by comparing multiple inputs and outputs, without assuming a predefined functional form. It has been widely adopted for productivity analysis, particularly when integrated with the Malmquist Productivity Index (MPI) to assess productivity change over time (Färe et al., 1992, 1994).

The MPI, originally developed by Caves et al. (1982) and later adapted for DEA by Färe et al. (1994), decomposes productivity change into technical change (shifts in the production frontier) and efficiency change (movements towards or away from the frontier). Further extensions attempted to account for scale efficiency and pure technical efficiency, yet many of these decompositions suffer from internal inconsistencies and linear programming (LP) infeasibility issues (Ray & Desli, 1997; Simar & Wilson, 1998; Tone, 2001, 2002).

To address some of these limitations, the Multi-directional Efficiency Analysis (MEA) framework was introduced by Bogetoft and Hougaard (1999) and later extended by Asmild et al. (2003, 2016). Unlike traditional DEA, MEA estimates input- and output-specific efficiency scores and accommodates variable-specific improvement patterns. MEA is particularly useful when improvements are not uniformly distributed across inputs and outputs and when the analysis requires recognition of undesirable outputs or non-radial slacks, limitations poorly handled by radial DEA models. MEA has since been applied in diverse sectors, including agriculture, banking, and energy (Asmild & Matthews, 2012; Wang et al., 2013; Asmild et al., 2016).

However, MEA-based Malmquist indices (MEA-Malmquist or MEAM) also encounter LP infeasibilities and do not adequately address scale effects when applied to cross-period productivity measurement. This methodological shortcoming is critical in evaluating productivity change, especially in sectors or regions where data irregularities or structural heterogeneities prevail. Asmild et al. (2016a) acknowledge these limitations, creating a compelling need for a more robust productivity index.

Despite the extensive theoretical development of DEA and its variants, their application to energy productivity in Africa remains limited. Africa faces growing challenges in balancing energy access, economic growth, and environmental sustainability. Yet, existing studies rarely explore input-specific productivity changes or the scale-adjusted dynamics of energy use, capital deployment, and labour efficiency in African states. Much of the African context appears belatedly in the literature and is often treated peripherally, leaving a gap in actionable regional analysis. There is, therefore, a pressing need for methodological innovation that aligns with Africa's diverse and evolving development challenges.

This study proposes a novel extension to the MEA-Malmquist framework—the Biennial MEA Malmquist (BMEAM) index—which is designed to resolve LP infeasibilities and capture scale effects in productivity analysis. The BMEAM index offers a variable-specific, decomposition-rich approach to tracking productivity change over biennial periods, with empirical application to 32 African countries between 2000 and 2019. The study bridges a critical gap in both the methodological literature and applied energy economics by providing a tool that is both theoretically sound and practically useful for informing energy and development policy in Africa.

In conclusion, while DEA, MPI, and MEA have each advanced the field of productivity analysis, they fall short in addressing key technical limitations, especially when applied to cross-period, regionally heterogeneous data. This study addresses that gap through the development and application of the BMEAM index. It contributes both a methodological advancement and an empirically grounded framework for understanding productivity dynamics in Africa's energy sector, where such insights are urgently needed to guide sustainable and inclusive development.

1.2 Research Problem

Despite the increasing adoption of advanced performance measurement techniques such as Data Envelopment Analysis (DEA) and Multi-directional Efficiency Analysis (MEA), significant methodological gaps persist, particularly in their dynamic extensions. While DEA, introduced by Charnes et al. (1978), evaluates the relative technical efficiency of decision-making units (DMUs) using multiple inputs and outputs, MEA, developed by Bogetoft and Hougaard (1999) and operationalized by Asmild et al. (2003), offers variable-specific efficiency insights that enhance the diagnosis of inefficiencies.

A key extension of MEA is the Multi-directional Efficiency Analysis Malmquist index (MEA-Malmquist or MEAM), proposed by Asmild, Baležentis et al. (2016a), which facilitates variable-specific productivity analysis over time. However, MEAM presents two critical limitations. First, it is prone to linear programming (LP) infeasibilities when mixed-period efficiency scores become infinite, an issue acknowledged but not addressed in the original model. Second, the MEAM framework fails to capture scale effects, which are particularly relevant in real-world systems characterized by variable returns to scale.

These methodological shortcomings undermine the robustness and policy relevance of MEAM-based analyses, especially in empirical settings such as African energy economies, where structural diversity, scale heterogeneity, and data irregularities are prevalent. The inability to decompose scale effects or resolve LP infeasibilities limits the practical utility of MEAM in contexts requiring precision, interpretability, and adaptability.

This study seeks to bridge this methodological gap by developing a variable-specific productivity index, the Biennial MEA-Malmquist (BMEAM), which incorporates a four-way decomposition: Biennial Pure Efficiency Change (BPEC), Biennial Pure Technical Change (BPTC), Biennial Scale Technical Change (BSTC), and Biennial Scale Efficiency Change (BSEC). By addressing both LP infeasibilities and scale decomposition, the proposed BMEAM aims to provide a more accurate and policy-relevant tool for analyzing dynamic productivity change in scale-sensitive, data-constrained environments such as those found in many African states.

1.3 Research Objectives

The objective of this study is;

1. To develop a proposed variable specific productivity index (Biennial MEA Malmquist-BMEAM) that handles the LP infeasibilities and Scale effect and illustrate the proposed productivity index using a hypothetical example.
2. To decompose the productivity index into Pure Technical Change, Pure Efficiency Change, Scale Technical Change and Scale Efficiency Change
3. To empirically apply the proposed productivity index to determine the energy productivity change of African states.

1.4 Research Questions

Essentially, the study seeks to answer the following questions.

1. To what extent does the productivity index tend to solve the LP infeasibilities and scale effect.
2. What is the effect on efficiencies of African States when analyzed via MEA than DEA?
3. Which variables drive dynamic productivity?

1.5 Significance of the Study

In the world of academia, this would be the first extension of MEAM to handle LP infeasibilities leading to BMEAM.

An important contribution to the existing literature on efficiency studies is the premier decomposition of the productivity index into four factor components, which addresses the scale effect in the context of Multi-directional Efficiency Analysis (MEA). This decomposition allows for a more comprehensive understanding of productivity dynamics by capturing the impact of scale changes on efficiency assessments.

By decomposing the productivity index into four factor components, this approach provides insights into the specific factors driving productivity improvement or decline, including changes in technical efficiency, technological change, pure scale efficiency change, and scale efficiency change. This enables researchers to disentangle the effects of scale from other sources of efficiency change, leading to a more accurate and detailed analysis of productivity patterns.

The incorporation of this four-factor decomposition in MEA enhances the accuracy and reliability of efficiency studies. It contributes to the existing literature by offering a robust framework that accounts for scale effects and provides a more comprehensive understanding of the drivers of productivity change. This contribution adds depth and insight to the field of efficiency studies, advancing our understanding of performance evaluation and optimization in various contexts.

The proposed study on the energy productivity of African states, using the unique application of the methodology discussed, holds significant potential for generating valuable insights. By applying the proposed methodology to assess energy productivity, the study aims to identify areas within the operations of industry players that are inefficient and require continuous improvement. This analysis can help industry stakeholders gain a better understanding of their energy utilization patterns and identify specific areas for optimization.

The findings of this study will contribute to the body of knowledge on energy productivity in the African context, providing industry players with valuable information on where they can focus their efforts to enhance efficiency and productivity. By pinpointing inefficient areas, organizations can take targeted actions to improve their energy performance, reduce waste, and optimize resource utilization. This can lead to cost savings, environmental benefits, and overall sustainability in the energy sector.

The study's unique application of the proposed methodology to the energy productivity of African states adds novelty and relevance to the field. It has the potential to empower industry players with actionable insights and contribute to the broader goal of promoting efficient energy utilization in Africa.

The study's findings on energy productivity in the African energy sector would indeed serve as a valuable prescription for managerial and policy reviews.

From a managerial perspective, the study would provide insights into suitable structures and sector governance that can enhance the management and operation of energy organizations. By analyzing the efficiency and productivity patterns, management boards can identify areas where investments should be made to strengthen the sector, improving overall performance and competitiveness. This can guide strategic decision-making, resource allocation, and operational improvements within energy companies.

In terms of policy direction, the productivity evaluation across the African continent would offer regulators and governments detailed information on efficiency trends and major drivers within the sector. This knowledge would enable policymakers to develop realistic and effective solutions to address any inefficiencies or productivity gaps identified. By understanding the dynamics of efficiency across different time periods, policymakers can design targeted policies and create an enabling environment that fosters improved performance in lagging firms, aligning them with international standards.

Overall, the study's contribution to managerial and policy reviews in the African energy sector would empower both industry leaders and policymakers with the necessary information to make informed decisions, optimize resource allocation, and facilitate the sustainable growth of the sector. It enhances the understanding of sector trends, highlights productivity drivers, and supports

the development of strategies and policies that can lead to improved efficiency, competitiveness, and overall performance.

1.7 Scope and Limitation of the Study

This study investigates productivity of African states on an unbalanced sample of 32 countries based on availability of data covering the periods from 2000 to 2019.

Data is sourced from Penn World Tables (PWT 10.0) and US Energy Information Administration (EIA) in obtaining the input and output variables. The study is limited to energy sector of Africa states. It investigates the productivity of these African states under the period of study in relation to energy sector utilizing the proposed productivity index.

Two challenges presented with the data being utilized are the incomplete country coverage and limited time. These are common obstacles in research projects, especially when dealing with extensive geographic regions and historical data.

The absence of data for all countries may limit the generalizability of the findings. However, it is important to acknowledge that data availability is often beyond the control of researchers.

Transparent reporting of the limitations regarding country coverage is crucial for the interpretation and applicability of the study's results.

Similarly, the restricted time can affect the ability to capture long-term trends or cyclical patterns in energy productivity. Researchers should be mindful of this limitation and clearly communicate the timeframe under investigation. Despite the limited period, the study can still provide valuable insights into the efficiency and productivity dynamics during the available years, which can inform decision-making and policy direction.

Researchers can also acknowledge the limitations of data availability and emphasize the need for future studies to expand the coverage and extend the period. This can help guide future research

efforts and data collection initiatives to fill the gaps in knowledge and provide a more comprehensive understanding of energy productivity across the African continent.

In summary, while challenges related to data availability, such as incomplete country coverage and limited time, are common in research, transparent reporting of these limitations and their potential impact on the study's findings are essential. Despite these challenges, the study can still offer valuable insights and contribute to the existing knowledge on energy productivity in the African energy sector. The nature (quality and quantity) of the data acquired will be altered since the research will be tested using data from numerous African states and most of these energy companies in these countries are frequently reticent to disclose essential information about their operations.

1.8 Chapter Outline

The thesis is broken down into six chapters. The first chapter discusses the study's background, problem statement, contributions to policy, practice, and current literature, research goals and questions, and study scope. Chapter two (2) sets the context for the study by delving into the energy sector. It provides an overview of the energy sector, highlighting its significance and relevance in the current landscape. The chapter also explores some of the current trends in the energy sector, focusing specifically on the efficient utilization of energy resources. This includes discussions on energy efficiency initiatives, technological advancements, and best practices aimed at optimizing energy consumption.

Furthermore, the chapter delves into the sustainable development goals (SDGs) set forth by the United Nations. Specifically, it examines the 7th and 13th SDGs, which pertain to ensuring access to affordable and clean energy, as well as taking urgent action to combat climate change and its

impacts. These SDGs are critical in guiding the global community towards a sustainable energy future.

By discussing the current trends in the energy sector and incorporating the relevant SDGs, this chapter sets the foundation for the subsequent analysis in the study. It provides the necessary context and highlights the significance of energy efficiency and sustainability goals in shaping the research and its implications.

Chapter three (3) of the study focuses on providing the context for the research by conducting a comprehensive review of the current literature in the field. The chapter analyzes existing research related to performance assessment and efficiency studies, with a particular emphasis on Multi-directional Efficiency Analysis (MEA).

The literature review aims to identify and evaluate the key studies and methodologies employed in the field of performance assessment and efficiency studies. It examines various approaches used in assessing performance, such as traditional financial ratio analysis, Data Envelopment Analysis (DEA), and MEA. Special attention is given to MEA, as it offers a more comprehensive and nuanced perspective on efficiency analysis.

The chapter critically evaluates the strengths and limitations of the reviewed studies, highlighting the gaps and research opportunities that exist in the field. It also identifies the key findings and contributions of previous research, providing a foundation for the subsequent analysis in the study.

By conducting a thorough literature review, Chapter 3 aims to establish the theoretical and conceptual framework for the study. It helps situate the research within the broader scholarly discourse, highlighting the importance of MEA and performance assessment in the context of efficiency studies. This analysis informs the subsequent chapters and provides a basis for the methodology and findings presented in the study. During the evaluation process, emphasis is

placed on studies that focus on efficiency and concepts, as well as the methodologies employed. MEA in terms of productivity is then addressed further.

The fourth chapter touches on the methodology used. The research is described in this section. In this chapter input-orientation and output-orientation orientation models in data envelopment analysis (DEA) is explained. Moreover input-orientation and input-output orientation(non-orientation) models are also explained. Then finally, the multi-directional efficiency and productivity analysis (MEA) also known as potential improvement, is discussed.

Chapter five (5) of the study focuses on data analysis and discussion. This chapter is dedicated to conducting various analyses that align with the objectives outlined earlier in the study. Each analysis aims to achieve a specific objective, which was established in previous chapters.

In this chapter, the precise impacts that are evaluated in each analysis are described in detail. The data analysis includes the application of appropriate statistical techniques, models, or frameworks to analyze the collected data and derive meaningful insights. The results of each analysis are presented and discussed, providing a comprehensive understanding of the research findings.

Chapter Six (6) serves as a culmination of the study, covering several key aspects. It begins with a summary of the findings obtained from the data analysis conducted in Chapter 5. The chapter then highlights the contribution of the study to the existing body of knowledge in the field, emphasizing the novel insights and advancements made.

Following that, the chapter presents the conclusion of the study, summarizing the key findings and their implications. It provides a comprehensive overview of the research outcomes and their significance in addressing the research objectives and answering the research questions.

Additionally, the chapter includes suggestions and recommendations based on the study's findings. These suggestions may pertain to managerial practices, policy implications, or avenues for future research. The chapter concludes by outlining potential directions for future research, identifying gaps in the current study that could be explored further.

In summary, Chapter 5 focuses on data analysis and discussion, where each analysis aligns with the research objectives. Chapter 6 provides a summary of findings, discusses the study's contribution, presents the conclusion, and offers suggestions for future research. These chapters are crucial in presenting the key outcomes and implications of the study in a structured and comprehensive manner.



CHAPTER TWO

CONTEXT OF STUDY

2.1 Introduction

An overview of the distinctive features of the global energy sector is given by the study's setting.

This seeks to give enough context for understanding the empirical claims made in this study.

In this Chapter, there is a division into two parts, allowing for a clear and organized presentation of the information. This division helps to structure the chapter and facilitate the reader's understanding of the content.

The first part of Chapter 2 provides the contextual background of the study. It establishes the broader context and significance of the research topic, setting the stage for the subsequent analysis. This section may include discussions on the energy sector, its importance in the overall economy, and its role in sustainable development. It may also highlight key challenges, trends, and policy frameworks relevant to the energy sector. This part aims to provide a comprehensive understanding of the context in which the study is conducted. Given that the majority of fuel sources for power generation facilities are obtained from this sub-sector of the energy industry, the structure of the global oil and gas industry is first given. The operations in the oil and gas value chain as well as several other general value chains in the energy sector are clearly displayed. In addition, the section provides information on significant industry inputs and potential environmental hazards.

The second part focuses on current trends and sustainable goals related to energy efficiency and sustainability. It examines the recent developments and advancements in the energy sector, particularly in terms of efficient energy utilization. This section may discuss initiatives, technologies, and best practices aimed at improving energy efficiency and achieving sustainable energy goals. Additionally, it explores the alignment of these trends and goals with international

frameworks, such as the Sustainable Development Goals (SDGs). This aims to highlight the ongoing efforts and global commitments towards a more sustainable energy future.

By dividing this chapter into two parts, the study can provide a comprehensive overview of the contextual background and current trends related to the research topic. This division helps to organize the content and present the information in a logical and structured manner.

2.2 Overview of the Energy industry

Energy continues to be a key component of economic growth and prosperity (Asafu-Adjaye 2000; Zhang and Cheng 2009; Apergis, Aye et al. 2015; Ohene-Asare, Tetteh et al. 2020). Saving energy has been a key concern for sustainable development. Global economic growth during the past 300 years has been dependent on finite fossil fuels like Petro (Agency 2020). The interconnected economic, social, and environmental goals of sustainable human growth all heavily rely on the energy system (Ohene-Asare, Gakpey et al. 2018).

Energy is the source of practically all important economic activities; hence it is also the activity that keeps life on earth going (Mahmood and Kanwal 2017).

Africa's energy consumption growth rate was 2.5% between 2009 and 2019, compared to the global average of 1.9% over the same period (BP 2021).

Primary and secondary energy are alternative method for categorizing energy. Primary energy is a type of natural energy that has not undergone any engineering-based conversion procedures. It is the energy that is present in unprocessed fuels and other energy sources, such as trash, that are introduced into a system.

Renewable or non-renewable sources can be used for primary energy. Resources that have undergone conversion or storage are considered secondary energy (Øvergaard 2008).

Oil and petroleum-based goods, as well as gasoline, natural gas, diesel fuel, and nuclear power, are examples of non-renewable energy. Hydropower, solar power, and wind power are examples of renewable energy. Based on the sources, these two groups can also be divided into Primary and Secondary.

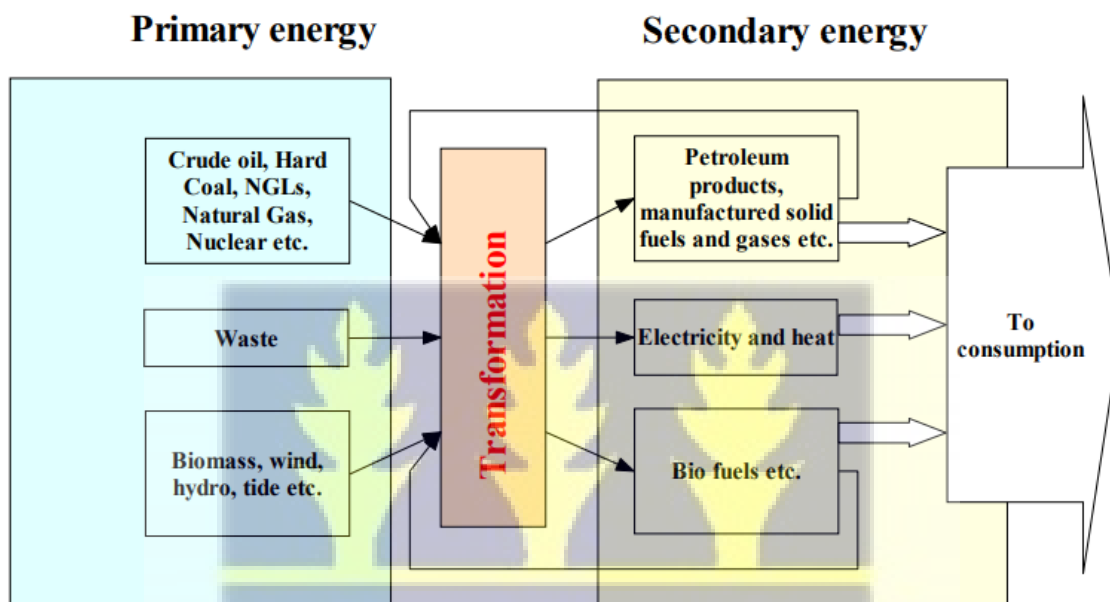


Figure 1 Primary and Secondary

According to the information provided, in 2021, there was a significant increase in primary energy compared to the previous year. The primary energy increased by 31 exajoules (EJ), which is the highest increase ever recorded. This increase compensated for the substantial decline observed in 2020. Furthermore, in 2021, there was an additional 8 EJ of primary energy available compared to the levels in 2019.

The rise in primary energy in 2021 can be attributed to the growth of economies, with China leading the way. China's primary energy increased by 10 EJ, contributing to the overall increase in global primary energy. This indicates the strong energy demand and consumption in emerging economies, particularly in China.

These figures highlight the resilience and recovery of the energy sector after the downturn experienced in 2020. The substantial increase in primary energy in 2021 reflects the rebound in economic activities and energy consumption, indicating a positive trend in global energy demand. It is important to note that the information provided is based on the BP Statistical Review of World Energy 2021. Energy is the primary source of value since it is necessary for the production of other components of production like labour and capital (Yang 2000; Ghali and El-Sakka 2004). To determine whether energy inputs serve as a complement or a replacement for other inputs, the energy efficiency indicator can be used in conjunction with labour and capital (Patterson 1996).

Renewable energy sources were solely responsible for the rise in primary energy demand between 2019 and 2021. Between 2019 and 2021, the amount of fossil fuel energy consumed remained constant, with increasing natural gas (5 EJ) and coal (3 EJ) consumption offsetting decreasing oil demand (-8 EJ) (BP 2021). Oil, gas, and coal continue to be the primary sources of energy consumed throughout the African continent. As of 2020, oil, natural gas, and coal accounted for 90.4% of primary energy consumption, while hydroelectricity contributed for 6.8% and nuclear and renewable energy sources made up just 2.8% (BP 2021).

Therefore, energy conservation is essential to enable economic expansion before new and alternative fuels are made available (Hu and Kao 2007).

The depletion of fossil resources is exacerbated by rapidly expanding energy use and rapidly evolving economies. However, a lot of people are concerned that significant energy reductions may stifle economic expansion.

Sustainable development requires integrating energy concerns with environmental management, particularly for countries that are growing quickly. The secret to managing energy in a sustainable manner is to increase energy efficiency. Reducing overall domestic energy usage without losing

economic development is a crucial challenge for economies all over the world given the limited availability of commercially effective alternative energy sources (De Nooij, Van der Kruk et al. 2003). Goals for energy conservation are crucial for all economies.

Energy efficiency is driven by environmental concerns in addition to economic growth considerations, according to research (Wang, Wei et al. 2013; Bi, Wang et al. 2014; Lei, Zhang et al. 2021).

In order to establish standards for the underperformers, it is crucial to look at the success so far experienced by the IGOs in assuring energy efficiency. Additionally, it is critical for policymakers to recognize the areas with the most potential for energy efficiency improvement as well as how energy efficiency relates to economic development.

Emerging markets and developing countries must take center stage in any evaluation of the future of global energy and emissions. There are hundreds of millions of people who lack the most basic access to modern electricity, and there are billions of people who do not yet have the housing stock and appliances that are taken for granted in the majority of developed nations.

The bulk of the two billion people that are expected to be added to the world's population by 2050 will live in emerging market and developing economies, excluding China, which now house two-thirds of the world's population. Sub-Saharan Africa's population is expected to increase particularly quickly and double by 2050 (Agency 2021).

According to the Global Industry Classification Standard (GICS), the energy sector encompasses companies involved in various aspects of the exploration, production, refining, sale, storage, and transportation of oil and gas, coal, and other consumable fuels (Barra 2009). This includes firms engaged in activities such as drilling for oil and gas reserves, operating refineries to process crude

oil, marketing and selling petroleum products, and managing infrastructure for the transportation and storage of energy resources.

In addition to the core activities directly related to energy production and distribution, the energy sector also includes businesses that provide equipment, services, and technology to support the oil and gas industry. These companies may manufacture and supply drilling equipment, pipelines, and other infrastructure components necessary for energy operations.

Given the capital-intensive nature of the energy industry, energy-related enterprises typically invest significant amounts of capital and possess substantial fixed assets. These assets can include land and properties used for oil reserves, refineries, and storage facilities. Moreover, energy companies may have extensive infrastructure networks for transportation, including pipelines, terminals, and logistical systems.

The energy sector plays a vital role in powering economies and supporting various industries worldwide. Its operations and investments contribute to the exploration and production of fossil fuels, the refining and processing of raw energy resources, and the overall infrastructure required for energy supply chains.

Energy corporations invest a lot of money on R&D to upgrade drilling and processing technologies, increase efficiency, and adjust to shifting environmental regulations.

Currently, 770 million people worldwide mostly in developing nations in Asia and Africa do not have access to electricity and preliminary data indicate that development internationally halted between 2019 and 2021 after declining 9 percent yearly on average between 2015 and 2019, and that the number of people without access to electricity increased in sub-Saharan Africa (Agency 2021).

Up to 90 million individuals with electrical connections in developing nations in Asia and Africa will no longer be able to buy a full package of services in 2020 due to the pandemic's impact on household earnings (Agency 2021). It's possible that households are choosing less powerful, inexpensive systems that offer less energy services than they would have in the absence of the epidemic. Access to clean cooking has made comparable progress backwards (Agency 2021).

An estimate shows that using biomass, coal, or kerosene for traditional cooking results in 2.5 million preventable deaths every year, delaying development and entrenching gender inequality (Agency 2021).

2.2.1 The Energy sector value chain

The energy industry includes businesses that are engaged in the production, delivery, storage, and exploitation of energy.

The energy industry is made up of a sizable collection of businesses with strong connections and a broad range of energy products. The Global Industry Classification Standard (GICS) further separates the energy sector into the oil, gas, and consumable fuels industry and the energy equipment & services industry.

a. Energy Equipment & Services Industry

Oil and gas drilling contractors, manufacturers of drilling equipment, and companies that offer services associated with the drilling and completion of oil or gas wells make up the energy equipment & services industry. A few of the participants in the US energy equipment & services market include Helmerich & Payne Inc., Unit Corporation, and Exterran Corporation.

b. Oil, Gas & Consumable Fuels Industry

Upstream, middle, and downstream companies in the oil and gas sector can all be grouped together. They are in various places along the supply chain.

The upstream businesses investigate out possible natural gas and crude oil reserves. Through the drilling and operation of wells, they also utilize these energy resources. In the upstream, mergers, acquisitions, and divestitures occur often.

The midstream sector of the energy industry primarily focuses on the storage, transportation, and wholesale marketing of oil, gas, and petroleum products. This sector plays a crucial role in connecting the upstream (exploration and production) and downstream (refining and marketing) components of the energy supply chain. Midstream companies are involved in the transportation of energy resources from production sites to refineries or distribution centers. They utilize various modes of transportation, including pipelines, barges, trains, and other logistics firms. These transportation services enable the movement of crude oil, natural gas, and petroleum products over long distances to meet demand in different regions (Fei, Chen et al. 2020).

In addition to transportation, midstream companies also provide storage facilities for oil, gas, and petroleum products. These storage facilities ensure a steady supply of energy resources, enable inventory management, and facilitate distribution to downstream markets. Storage terminals and tanks are strategically located near key demand centers to enhance operational efficiency and meet market requirements.

It is worth noting that while the midstream sector is often considered a part of the downstream sector, there can be some overlap and interdependencies between the two. The downstream sector primarily includes activities related to the refining of crude oil and unprocessed natural gas to produce consumable fuels like jet fuel, diesel, gasoline, and other refined petroleum products. Downstream businesses also engage in the wholesale marketing and distribution of these products.

Moreover, downstream operations may involve the production and sale of derivative goods derived from crude oil and natural gas, including waxes, lubricants, and various petrochemicals used in diverse industries. This sector focuses on transforming raw energy resources into finished products ready for consumption by end-users.

While the midstream sector primarily handles transportation and storage, its activities are closely linked to the downstream sector in terms of ensuring the efficient flow of energy resources and products to meet market demand. Together, the midstream and downstream sectors form an essential part of the energy industry's overall supply chain.

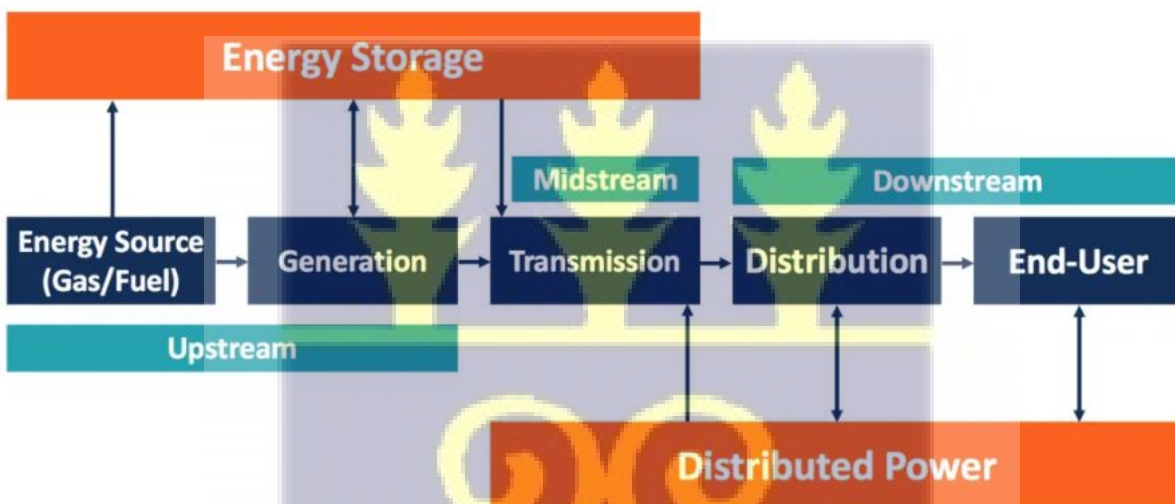


Figure 2 Energy sector value chain

The energy industry is extremely cyclical and macroeconomic conditions can affect it greatly. The degree of economic activity has a big influence on how much oil and gas people want. Among the macroeconomic factors are the industrial output index, GDP, disposable income and employment. Oil demand and price are inflated during expansionary periods by an increase in business activity and output. Reduced output during a recession decreases demand and prices, weakening the industry.

The weather and seasons have an influence on the energy industry in addition to the state of the economy. Typically, summertime gas costs are greater than wintertime ones. It is caused by both increased production costs for summer-grade fuels and increased travel throughout the summer. Natural catastrophes and extreme weather may damage infrastructure and interrupt the flow of electricity. Storms, Hurricanes, Fire outbreaks are only example.

Economic health is typically indicated by high and steady oil prices. As a result, certain groups coordinate energy providers and maintain market stability. One of them is the Organization of Petroleum Exporting Countries (OPEC). More than 40% of the world's oil output is accounted for by this international body. It works in concert with its 13 member nations to control oil supply and production in order to affect oil prices.

2.2.2 Environmental impacts of the energy sector

Considering the massive global need for electricity There is a push to lessen emissions related to energy generation as well.

Reducing fossil fuel usage has been a priority of environmental policy in many economies, both developed and developing ones, since the Kyoto Protocol came into force in February 2005(De Nooij, Van der Kruk et al. 2003). Furthermore, there has been a significant surge in nations' aspirations to decarbonize following the Paris Conference of Parties (COP21) in 2015. Net zero objectives and commitments now cover over 70% of global carbon emissions.

It is important to note that energy production and consumption have been connected to the release of dangerous air gasses in response to environmental concerns (Zhang and Cheng 2009). Therefore, economies must choose between increasing their usage of renewable energy and maintaining energy efficiency, which uses less energy to create the greatest amount of economic production (Ottinger and Williams 2002; Gerrard 2011; Scherman and Fleischer 2015).

Furthermore, according to the International Energy Agency (2017), with the right efficiency policies, the world could achieve more than 40% of the emissions reductions required to meet its climate goals without developing new technology. Energy efficiency can also promote economic growth, lower emissions, and enhance energy security. This has resulted in some African nations having to establish regional intergovernmental organizations (IGOs) to design energy efficiency laws and regulations to guarantee efficient energy use.

The Regional Center for Renewable Energy and Energy Efficiency (RCEEE), the ECOWAS Centre for Renewable Energy and Energy Efficiency (ECREEE), the East African Centre for Renewable Energy and Energy Efficiency (EACREEE), and the SADC Centre for Renewable Energy and Energy Efficiency (SACREEE) are some of the organizations that have been established in this regard.

As a result, according to the Sustainable Development Goals, renewable energy and breakthroughs in energy efficiency are presently among the top global problems for achieving sustainable development (Stuggins, Sharabaroff et al. 2013).

This is a challenge for Africa in particular since the continent is seeing an increase in energy consumption, primarily from non-renewable sources, and that demand is expected to continue to rise (Arouri, Ben Youssef et al. 2014).

2.3 Major trends in the energy industry

Investments in renewable resources are on the rise as a result of the risks associated with shifting environmental regulations that the energy sector is exposed to. The degree of economic activity, the climate and the seasons, as well as governmental rules and organizational policies, are some of the external factors that affect the energy industry.

According to estimates, the world's energy demand decreased by 4.5 percent in 2020 (BP 2021).Renewable energy sources like wind and solar PV continued to develop quickly in 2020,

even as economies sagged under the weight of Covid-19 lockdowns, and electric vehicles broke sales records. Renewable energy continued to grow significantly in 2020, with wind and solar energy leading the way. Surprisingly, wind and solar capacity rose by a staggering 238 GW last year, which is 50% more than any prior growth (BP 2021).

The future energy economy will be cleaner, more connected, efficient, and electrified. Its development is the result of a positive feedback loop between policymaking and technological advancement, and its momentum is currently being supported by decreased costs. The least expensive form of new power generation currently available in most markets is solar PV or wind. The field of clean energy technology is quickly emerging as a significant new source of employment and investment, as well as a vibrant setting for cross-border cooperation and rivalry (Agency 2021).

Illustrating examples of the decisions that must be made by leaders throughout the world before and after the pivotal 26th Conference of the Parties (COP26) in November 2021.

Governmental action plays a pivotal role in shaping the future of the energy sector. While other stakeholders such as individuals, communities, civic society, businesses, and investors can also contribute to the transition to a sustainable energy future, governments have a unique capacity to influence and steer the direction of the energy industry. Here are some key reasons why governments have a significant impact:

Establishing Regulatory Frameworks: Governments have the authority to establish and enforce regulations and policies that govern the energy sector. By setting clear and consistent framework conditions, governments can provide stability and guidance for energy investments and operations. This includes establishing standards for environmental protection, promoting renewable energy sources, and implementing measures to reduce greenhouse gas emissions.

Encouraging Innovation: Governments can support and incentivize research, development, and innovation in the energy sector. By providing funding, grants, and tax incentives, governments can spur technological advancements, promote clean energy technologies, and drive innovation in energy efficiency and renewable energy sources. This support can accelerate the deployment of new technologies and contribute to the overall transformation of the energy sector.

Signalling Long-Term Goals: Governments can send strong signals to the market and industry about their long-term energy goals. By setting ambitious targets for renewable energy adoption, emissions reductions, and sustainability, governments create a sense of direction and provide certainty for businesses and investors. These signals guide investment decisions, encourage long-term planning, and drive the transition to cleaner and more sustainable energy systems.

Taking Action and Implementing Policies: Governments have the power to implement policies and initiatives that drive the necessary actions to achieve their energy goals. This can include measures such as renewable energy subsidies, carbon pricing mechanisms, energy efficiency programs, and regulations that promote sustainable practices. Governments can also invest in infrastructure development, promote energy access, and support the deployment of clean energy technologies.

In summary, while multiple stakeholders can contribute to the energy transition, governments have a unique and influential role to play. By establishing the right framework conditions, encouraging innovation, signalling long-term goals, and implementing effective policies, governments can shape the energy landscape and drive the necessary changes to achieve a sustainable and resilient energy future.

The main scenarios in this Outlook are:

- The Net Zero Emissions by 2050 Scenario (NZE) lays out a constrained but doable course for the world's energy industry to reach net zero CO₂ emissions by 2050.
- Nationally Determined Contributions (NDCs) and longer-term net zero objectives are included in the Announced Pledges Scenario (APS), which assumes that all climate pledges announced by governments worldwide will be fully and on time met.
- Stated Policies Scenario (STEPS), which represents current policy settings based on a sector-by-sector evaluation of the particular policies that are in force as well as those that have been declared by governments throughout the world.

The academic literature has concentrated on the link between energy consumption and economic development, and there hasn't been much analysis of the macroeconomic effects of energy efficiency legislation, particularly in Africa (Rajbhandari and Zhang 2018). Accounting for the costs and benefits of energy efficiency initiatives will be made easier for countries if the relationship between energy efficiency and economic development is understood (Vivid Economics 2013).

Mahmood and Kanwal (2017) underscore the importance of investigating the causal connection between energy efficiency and economic advancement in both the short and long run. By doing so, we can gain insight into the sustainability of economic growth in Africa. The environmental impacts resulting from energy production and consumption, such as global warming and environmental degradation, serve as the underlying factors that generate a research-worthy argument regarding the correlation between economic development and energy efficiency (Mirza and Kanwal 2017). Despite the undeniable significance of energy efficiency in establishing a sustainable energy provision and promoting sustainable development, research on energy efficiency in Africa remains scarce (Ohene-Asare and Turkson 2019).

The few studies, for example, only take into account a few sub-regions, like ECOWAS; however, taking into account all the sub-regions will help to provide both country-specific benchmarks and regional benchmarks for the various regional organizations responsible for energy efficiency policies.

On the world perspective Energy efficiency improvements reduce the demand for electricity and all types of fuels. Global energy consumption is projected to increase in the Stated Policies Scenario (STEPS), plateau in the Announced Pledges Scenario (APS), and be 15% lower in the Net Zero Emissions Scenario (NZE) by 2030 than in the APS.

Therefore, the global economy's energy intensity declines by 4% annually between 2020 and 2030 under the Net Zero Emissions (NZE), which is more than twice as fast as it did on average during the preceding 10 years. Without this increase in energy efficiency, overall final consumption in the NZE would be nearly a third higher in 2030, which would make decarbonizing the electricity supply substantially more expensive and challenging (Agency 2021).

2.4 Conclusion

The chapter's goal is to give readers an understanding of the industry's operations and key developments. As a result, this chapter includes information on the value chain, significant industrial participants, significant environmental concerns, and developments over the research period.



CHAPTER THREE

LITERATURE REVIEW

3.1 Chapter Introduction

In this chapter, the theoretical foundations that underpin this study and review previous research conducted on efficiencies and productivity in different industries that employ MEA techniques. Available literature shows that while there have been several valuable studies on efficiency utilizing MEA and other extensions whiles focusing on developed countries, (Asmild and Matthews 2012; Wang, Wei et al. 2013; Bi, Wang et al. 2014; Wang, Yu et al. 2014; Asmild, Baležentis et al. 2016; Zhu, Wang et al. 2019).

This study develops a new MEA productivity change index incorporating the use of biennial Malmquist to resolve LP infeasibilities and scale effect. The study further uses the developed productivity change index to investigate the efficiency and productivity of the African energy sector. The index is then decomposed into a four-factor decomposition (Wheellock & Wilson) which consist of Pure Efficiency Change, Pure Technical Change, Scale Efficiency change and scale technical change.

3.2 Efficiency and Productivity

Daraio and Simar (2007) refer to various authorities, including Debreu (1951), Pareto-Koopmans (1951), Fare and Lovell (1978), and Russell (1985, 1988, 1990), among others. These scholars have extensively studied and defined efficiency from multiple perspectives (Daraio and Simar 2007). These dimensions of efficiency mentioned, including technological, allocative, revenue, scale, and structural, have been extensively discussed in the literature. Farrell (1957) argues that overall productive efficiency can be seen as the combination of technical and allocative efficiency. Technical efficiency refers to a firm's ability to generate the maximum output from a given set of

inputs (output orientation) or to minimizing all inputs for a given set of outputs (input orientation). The notion of structural efficiency pertains to the industry level and evaluates the extent to which an industry aligns with the performance benchmarks set by its top-performing firms (Farrell 1957). In essence, it gauges how effectively an industry distributes its output among firms, thereby assessing the optimal allocation of resources and output levels within the industry in the short term. Efficiency is the measure of performance at a time while Productivity on the other hand is the measure of performance over time, thus between two time periods.

Over the last decades, there has been much research on measuring productivity change over time (also known as dynamic productivity) for a set of comparable production units, both theoretically and practically. Typically, researchers have commonly integrated traditional Data Envelopment Analysis (DEA) methods with the Malmquist index in a non-parametric framework. This approach allows for the evaluation of productivity change. The DEA-based Malmquist index can be divided into two parts: efficiency change (EC) and technical change (TC). The application of the Malmquist index, along with its different decompositions, has yielded fruitful results, as demonstrated by numerous subsequent empirical studies that have highlighted its economic significance. These techniques mentioned, including the use of the general Malmquist index and the Multi-directional Efficiency Analysis (MEA) approach proposed by Bogetoft and Hougaard (1999) and Asmild et al. (2003), can be employed when conducting a productivity change study using the MEA methodology within a non-parametric framework. Additionally, they serve as an alternative to Data Envelopment Analysis (DEA) for measuring efficiency and productivity. The main distinction lies in the fact that by utilizing the MEA approach, a set of input-specific (and/or output-specific) efficiency scores for each unit is obtained. This, in combination with the standard Malmquist index, enables us to analyze the progress of productivity in each individual input

dimension (and/or output dimension) over time. In essence, MEA provides a more detailed and comprehensive understanding of how efficiency and productivity evolve in specific dimensions of inputs and outputs.

3.2.1 Resource Dependence Theory

Resource Dependence Theory (RDT), developed by Pfeffer and Salancik (1978), posits that organizations operate within open systems and are dependent on acquiring and maintaining access to critical external resources to achieve survival, growth, and operational effectiveness. These resources include financial capital, human expertise, technical infrastructure, regulatory frameworks, and institutional support, all of which are essential for enhancing performance and sustaining productivity.

The theory emphasizes that external constraints, such as limited access to inputs or unstable policy environments, are significant determinants of firm-level decisions and performance outcomes. In the context of this study, RDT provides a relevant lens for understanding the differences in efficiency and productivity among energy-based Decision-Making Units (DMUs) across African states. Specifically, resource access influences how efficiently inputs are converted into outputs and shapes the productivity dynamics of firms operating under varying structural and institutional conditions (Pfeffer and Salancik, 1978; Daily et al., 2003).

Although RDT has traditionally been applied to analyze the role of corporate boards in acquiring external support, its broader principle, that organizational performance is shaped by environmental interdependencies, remains highly applicable to national energy systems. In this regard, RDT complements the empirical outputs of DEA and MEA-based models by offering a contextual explanation for performance variation across DMUs that may not stem purely from internal inefficiencies but from systemic input deprivation (Zona et al., 2018).

Daily, Dalton et al. (2003) further argue that organizations require constant access to financial, technical, and informational resources to operate effectively and meet their stated objectives. This reinforces the idea that efficiency outcomes, particularly in the energy sector, are strongly contingent upon resource flow. Where such resources are limited or unstable, overall productivity suffers, regardless of managerial effort.

The theoretical implication is that RDT enriches the interpretation of the BMEAM results by embedding efficiency scores within broader realities of resource dependence, infrastructural gaps, and institutional asymmetries. In energy-constrained and developmentally diverse regions like sub-Saharan Africa, this perspective is essential for understanding why units with similar goals may exhibit divergent performance trajectories.

3.2.2 Agency Theory

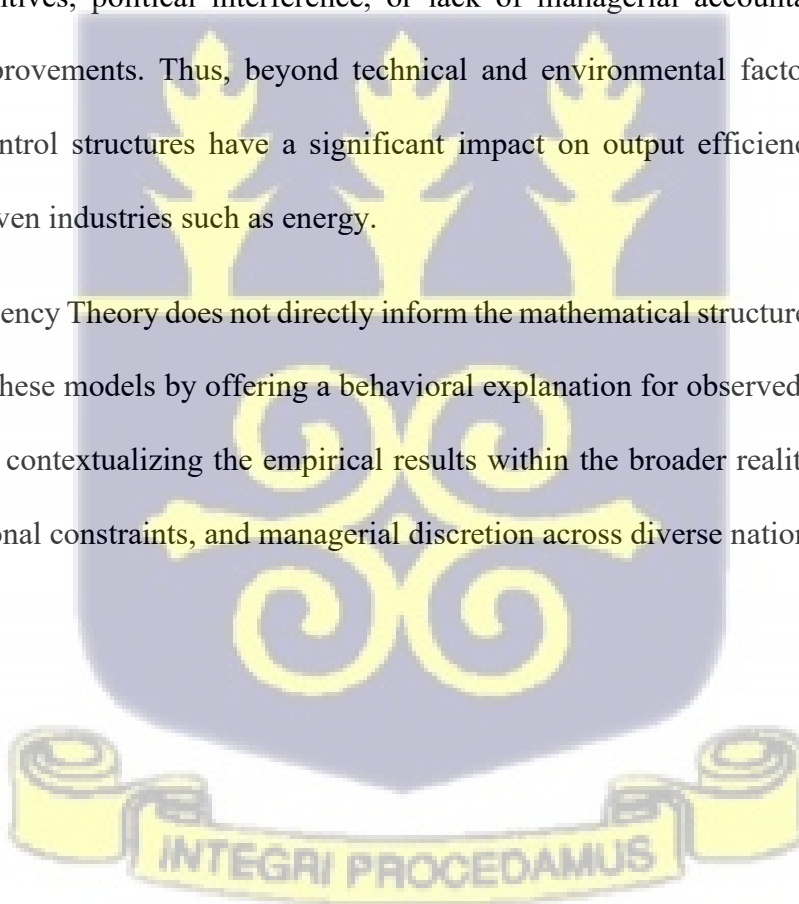
Agency Theory, originally formalized by Jensen and Meckling (1976), addresses the inherent conflicts that arise from the separation of ownership and control in organizations. It centers on the relationship between principals (owners or shareholders) and agents (managers), wherein agents are entrusted to operate the organization on behalf of the principals. However, due to information asymmetry and self-interest, agents may act opportunistically, pursuing personal objectives that diverge from the wealth-maximizing goals of the owners (Ross, 1973; Padilla, 2002).

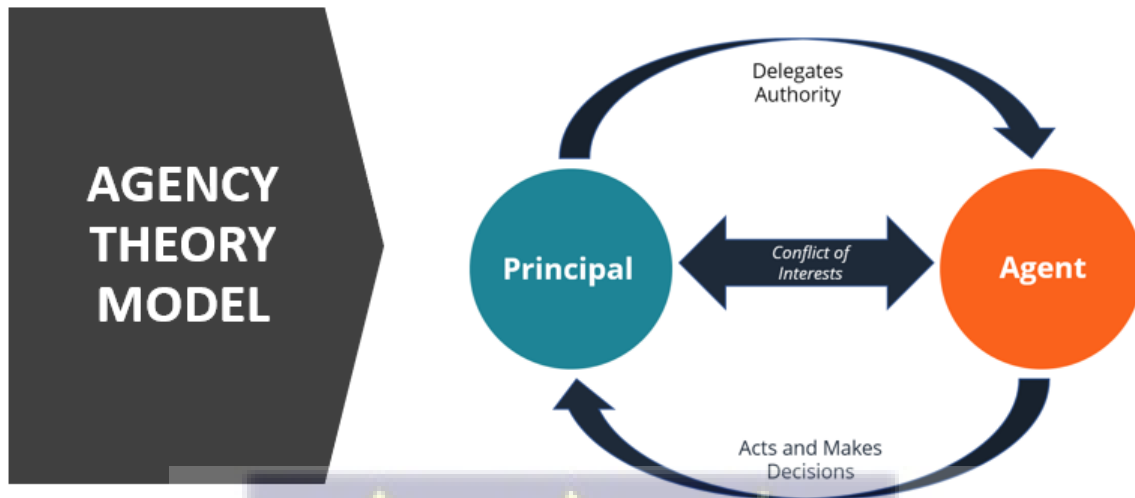
This divergence leads to what is commonly referred to as the “agency problem.” Agency Theory seeks to understand and resolve this misalignment by introducing monitoring mechanisms, incentive structures, and governance frameworks that align managerial actions with shareholder interests (Clarke, 2004). Jensen and Meckling (1976) argue that agency costs, expenses incurred to control managerial behavior, are unavoidable but can be minimized through effective oversight.

In the context of this study, Agency Theory provides a useful conceptual lens for understanding how governance quality and institutional control influence the efficiency and productivity of energy sector Decision-Making Units (DMUs) across African states. While DEA, MEA, and Malmquist Index techniques quantify technical and scale efficiency, Agency Theory explains the behavioral dynamics behind why certain DMUs may underperform. For instance, weak institutional accountability or lack of transparency in state-owned enterprises may result in persistent inefficiencies, even in the presence of adequate resources.

Agency Theory also supports the interpretation of performance variations by acknowledging that misaligned incentives, political interference, or lack of managerial accountability can hinder productivity improvements. Thus, beyond technical and environmental factors, organizational behavior and control structures have a significant impact on output efficiency, particularly in public sector-driven industries such as energy.

In sum, while Agency Theory does not directly inform the mathematical structure of DEA or MEA, it complements these models by offering a behavioral explanation for observed inefficiencies. Its relevance lies in contextualizing the empirical results within the broader realities of governance quality, institutional constraints, and managerial discretion across diverse national settings.





(CFI, 2022)

Figure 3 Agency theory model

3.2.3 Production Theory

Production theory provides a foundational framework for analyzing how firms and sectors transform inputs into outputs, and how they optimize production decisions based on resource availability, technological constraints, and market conditions (Farrell, 1957; Färe and Lovell, 1978; Debreu, 1951). Rooted in microeconomic principles, production theory models the production process as a function of input combinations (such as capital, labor, energy, and technology) that yield outputs. This conceptual framework is critical to the study of efficiency and productivity, particularly when applying frontier-based methods such as Data Envelopment

Analysis (DEA), Multi-directional Efficiency Analysis (MEA), and their extensions (Daraio and Simar, 2007).

At its core, production theory centers around the production possibility set, which encompasses all feasible input-output combinations. The boundary of this set, the production frontier, represents technically efficient performance. Any deviation from this frontier signals inefficiency. A Decision-Making Unit (DMU) that lies on the frontier is achieving maximum output for a given set of inputs, while those below the frontier are operating inefficiently. This provides the theoretical basis for calculating technical, scale, and allocative efficiency measures (Farrell, 1957).

In dynamic contexts, the theory is further extended through tools like the Malmquist Productivity Index (MPI), which decomposes productivity change into efficiency change and technical change (Färe et al., 1994). The MEA-Malmquist and the proposed Biennial MEA-Malmquist (BMEAM) index used in this study build on this principle by introducing variable-specific assessments of input and output productivity. These models track not only how DMUs perform relative to the frontier, but also how the frontier itself shifts due to technological progress or structural changes.

Production theory also informs decisions regarding capital structure and long-term sustainability. It guides firms in aligning output goals with capital allocation strategies, ensuring efficient resource utilization and sustainable growth (Juma et al., 2024). Efficient input management is central to optimizing output and profitability, particularly in resource-constrained environments like those found in many African energy systems (Hutauruk, 2023).

Additionally, modern production theory incorporates environmental dimensions, acknowledging the interactions between production processes and ecological sustainability. This is particularly relevant in energy-related sectors, where production choices often have significant environmental

implications (Anton, 2022). Thus, production theory supports not only economic optimization but also the adoption of sustainable and responsible production practices.

Historical contributions, such as those by Ibn Khaldun and Abdul Manan, further enrich production theory by emphasizing the importance of labor specialization and organizational structure in promoting efficiency and societal well-being (Hidayatullah, 2023).

By grounding this study within the broader context of production theory, the research ensures methodological coherence, economic relevance, and interpretive clarity. The theory provides a robust and flexible lens through which the productivity of African energy sector DMUs can be evaluated over time, accounting for both technical performance and environmental or institutional constraints.

3.3 Empirical Literature

The existing literature indicates that there is a scarcity of published papers that specifically focus on the development of MEA (Multi-directional Efficiency Analysis), its extensions, and empirical applications of the technique. However, it is worth noting that MEA has gained recognition as an alternative approach to traditional methods such as Data Envelopment Analysis (DEA) for measuring efficiency and productivity. While there may be a limited number of studies dedicated to MEA, researchers have acknowledged its potential and have started exploring its various applications in different fields, this includes (Asmild, Hougaard et al. 2003; Asmild and Matthews 2012; Wang, Wei et al. 2013; Bi, Wang et al. 2014; Wang, Yu et al. 2014; Asmild, Baležentis et al. 2016a; Asmild, Kronborg et al. 2016b; Zhu, Wang et al. 2019). These papers that have utilized the MEA technique have employed it in various application areas. The motivation for this study was inspired by a paper by Asmild, Baležentis, and Hougaard (2016) that extended the MEA

approach by incorporating the Malmquist total factor productivity index. This extension allowed for a more comprehensive analysis of productivity using MEA methodology. Building upon their work, the current study aimed to further explore and apply the MEA technique in different contexts, potentially contributing to the existing body of knowledge in this field.

Asmild et al 2016a recognized that LP infeasibilities and scale effect can occur when mixed-period efficiency scores are infinite and (MEA-Malmquist or MEAM) could not handle it, hence the need for this study.

In the study conducted by Asmild et al. 2009 examined railway operations in 23 European nations from 1995 to 2001, when the European Commission began a series of reform measures and examined whether these reform initiatives enhanced the efficiency of the railway networks. Multi-directional Efficiency Analysis is used to assess efficiency, allowing researchers to investigate how railway reforms influence the inefficiencies of various cost drivers. The major conclusions are that reform measures enhance technical efficiency in general, but in various ways for distinct cost drivers.

The research specifically presents empirical evidence that accounting separation is critical for improving the efficiency of both material and labor costs, whereas other changes only affected one of these elements.

Asmild and Matthews (2012) showcased the capabilities of the modern MEA technique in exploring changes in efficiency patterns among different subgroups within a dataset. Specifically, they focused on examining Joint Stock Banks and State-Owned Banks in China. By applying the MEA methodology, they were able to empirically test hypotheses regarding the differences in both the levels and patterns of inefficiencies between these two types of banks. Their research provided empirical evidence supporting most of the hypothesized patterns, indicating variations in

objectives and constraints that contribute to the observed differences in efficiency between the two bank types.

In line with the Charnes et al. (1981) framework, Asmild et al. (2016b) conducted a study on Lithuanian family farms to assess both managerial and program efficiencies. However, they departed from the traditional approach of data envelopment analysis and instead employed the multi-directional efficiency analysis methodology to estimate efficiency. This methodology allowed for the consideration of input-specific efficiencies.

The study revealed significant variations in efficiency scores across different inputs as well as among different types of farms, including crop farms, livestock farms, and mixed farms. Notably, crop farms exhibited the highest program efficiency but the lowest managerial efficiency. On the other hand, mixed farms demonstrated lower program efficiency but not necessarily the highest managerial efficiency.

These findings highlight the importance of considering both managerial and technical efficiencies separately and the unique efficiency characteristics exhibited by different types of farms. The multi-directional efficiency analysis approach provided valuable insights into the specific areas of efficiency improvement that can be targeted for each farm type. The study demonstrates that, as compared to standard DEA, the MEA approach yields more insights. The disaggregation into input-specific efficiency scores provides a more comprehensive view of management and program inefficiencies. They present one example, when comparing the management efficiency of livestock and crop farms using DEA, one would infer that the managerial efficiencies in these two categories are not considerably different.

However, using MEA, they demonstrate that this is only true for labor efficiency, with considerable disparities in managerial efficiencies on intermediate consumption and depreciation.

Benchmarking within farm types is a potentially helpful activity for all farms since it can eliminate some of the managerial inefficiencies caused by farming techniques and management abilities.

In Zhu's study, the author examines the trajectory of improvement, potential for improvement, and dynamic evolution of regional energy efficiency in China from 2005 to 2016. To conduct the analysis, an improved multidirectional efficiency analysis and the Markov model are utilized.

The findings of the study indicate that China's provincial energy efficiency follows an "olive-shaped" pattern, displaying a significant geographical imbalance. The central provinces, along with a few provinces from both the eastern and western regions, face the challenge of addressing weak links in their energy efficiency. For these provinces, a one-way breakthrough approach is necessary to enhance their energy efficiency.

On the other hand, most western provinces can benefit from adopting a step-by-step progressive or leap-forward approach to improve their energy efficiency. These strategies offer a more suitable path for enhancing energy efficiency in these regions.

Overall, the study highlights the diverse improvement paths and potential for energy efficiency in different regions of China, providing valuable insights for policymakers and stakeholders in implementing targeted measures to enhance energy efficiency across the country (Zhu, Wang et al. 2019)

The eastern and western areas have a relatively big potential for energy savings, and the central region has a large potential for CO₂ emission reduction. In the short run, the overall energy efficiency of different provinces is not extremely fluid across levels. In the long run, China's energy efficiency equilibrium will be in the mid-low range (Zhu, Wang et al. 2019). As a result, it is proposed that exchanges between various areas be strengthened, resource advantages of each

region be fully used, and differentiated and targeted energy efficiency programs be implemented in different provinces.

The study conducted by Wang et al. (2013), focused on examining the energy and emission efficiency patterns of different regions in China. To assess efficiency, they employed the multi-directional efficiency analysis (MEA) approach.

The empirical findings of the study revealed an overall increasing trend in MEA efficiency across China during the period of analysis, which spanned from 1997 to 2010. In terms of regional variations, the east area of China demonstrated higher MEA efficiency compared to the central and west areas throughout the study period.

The higher efficiency of the east area can be attributed to both its higher energy-specific efficiency and higher emissions-specific efficiency compared to the other two regions. This suggests that the east area was more effective in utilizing energy resources and reducing emissions relative to the central and west areas.

The study also identified several provinces, including Hebei, Shanxi, Inner Mongolia, Shandong, Henan, and Hubei, as having significant potential for energy savings and emissions reduction. These provinces were highlighted as playing crucial roles in China's efforts to conserve energy and mitigate CO₂ emissions.

Overall, the research by Wang et al. provides insights into the efficiency patterns and regional disparities in energy and emission performance in China. The findings can inform policymakers and stakeholders in identifying priority areas for energy conservation and climate change mitigation initiatives (Wang, Wei et al. 2013).

In the study conducted by Suhaimi et al. (2017), the authors aimed to assess the technical inefficiency of dairy farms in the Malaysian dairy industry. They further investigated the factors that contribute to technical inefficiency in this sector.

To analyze technical inefficiency, the study employed multi-directional efficiency analysis on a sample consisting of 200 farm observations. This analysis allowed for the calculation of technical inefficiency scores, providing insights into the performance and productivity of the dairy farms.

To identify the factors influencing technical inefficiency, the authors employed a single-bootstrap truncated regression model. This statistical approach enabled them to examine the relationship between various factors and the degree of technical inefficiency observed in the dairy farms.

By utilizing these methodologies, the study shed light on the technical inefficiency levels in the Malaysian dairy industry and identified the key factors influencing inefficiency. The findings can be valuable for policymakers, industry stakeholders, and farmers in developing strategies to improve productivity and address the inefficiency challenges within the dairy sector. Scores for managerial and program inefficiency are provided for intensive and semi-intensive manufacturing systems (Suhaimi, de Mey et al. 2017). The results showed significant variability in inefficiency ratings across inputs and production methods.

In the Malaysian dairy farming business, intensive systems have the lowest management and program inefficiency ratings (Suhaimi, de Mey et al. 2017). Policymakers might use this information to urge dairy producers to switch to an intensive farming strategy. In order to enhance the performance of the dairy industry, they recommend the Malaysian government to rethink its farm finance policy and target young farmers when establishing training and extension programs.

The existing literature on Southeast Asian dairy production has not investigated input-specific

efficiency or compared management and program efficiency. Hence Suhaimi et al, (2017) seeks to fill that gap utilizing MEA to analyse inefficiency.

The MEA approach has undergone various extensions of which includes works from (Asmild and Pastor 2010; Kapelko and Lansink 2017; Tziogkidis, Philippas et al. 2020).

Asmild and Pastor (2010) have made significant contributions to the field of multi-directional efficiency analysis (MEA). Their research expands upon two directional distance function models, namely the Multi-directional Efficiency Analysis (MEA) Model and the Range Directional Model (RDM), to incorporate various types of technological inefficiencies, including both directional and non-directional inefficiencies.

In their study, the authors focused on the variable returns to scale (VRS) scenario, as both VRS-MEA and RDM are translation invariant models capable of handling negative data. One of the key achievements of their work is the development of a comprehensive efficiency measure that is both unit and translation invariant, applicable to both the MEA and RDM models.

Furthermore, Asmild and Pastor introduced the RDM model with constant returns to scale (CRS) and presented a novel comprehensive efficiency metric in this context. By extending the models and introducing new efficiency measures, their research offers valuable tools for analyzing efficiency and performance in various settings.

Overall, Asmild and Pastor's work has advanced the field of MEA by providing innovative models and efficiency metrics that can be applied to different scenarios, contributing to a deeper understanding of efficiency analysis and measurement.

The study undertaken by Kapelko et al. (2017), extends the method of multi-directional inefficiency analysis to account for dynamics of firms' production decisions. The resulting technique, dynamic multi-directional inefficiency analysis, employs Data Envelopment Analysis

to assess variable input- and investment-specific inefficiency. The empirical application is based on panel data from big dairy manufacturing enterprises in three European regions from 2005 to 2012. These regions are Eastern, Southern and Western. As per the findings, investments are the most inefficient element for enterprises across all European areas, followed by labor and materials. However, the data indicate a wide range of investment-specific inefficiency scores within the sample. The article also discovers that the median dynamic inefficiency is quickly changing in the years after the start of the financial crisis (Kapelko and Lansink 2017).

The study conducted by Tziogkidis et al. (2020), suggested a unique two-step technique for assessing nations' innovation efficiency and response to increases in innovation inputs, while resolving inadequacies associated with composite measures. Based on the findings, they suggest innovation policies that are suited to the various economic contexts of the numerous nations in our research. An independent efficiency rating is estimated for each innovation input and output using multidirectional efficiency analysis on data from the Global Innovation Index (Tziogkidis, Philippas et al. 2020).

Sensitivities are then estimated for each country, by applying partial least squares on explanatory and response matrices which are determined by the nearest neighbors of the country under consideration (Tziogkidis, Philippas et al. 2020). The research findings demonstrate significant variations in innovation efficiencies and sensitivities across different national innovation systems. These asymmetries highlight the diversity and uniqueness of each system.

In light of these findings, the study suggested three policy directions that can guide decision-making to enhance innovation performance. These policy directions are outlined to provide a platform for more informed and effective decision-making in the realm of innovation.

By considering both innovation efficiencies and sensitivities, policymakers can better understand the strengths and weaknesses of their national innovation systems. This knowledge can inform targeted policies and interventions to foster innovation, address bottlenecks, and capitalize on the specific advantages of each system.

Overall, the study contributes to the understanding of national innovation systems and offers practical insights for policymakers to promote innovation and drive economic growth. The identified policy directions serve as a valuable framework for making informed decisions to improve the innovation ecosystem within a country or region (Tziogkidis, Philippas et al. 2020).



Table 1 A taxonomy of some available studies utilising MEA

	Authors	Industry	Method	Country Sample Year	Inputs and Outputs	Findings
1	Wang et al., 2013	Energy	MEA approach	China 1997-2010	Input: Number of employees, fixed assets, Total Deposits Outputs: Net interest earnings, non-interest earnings	1. China's average MEA efficiency increased. with certain sectors being efficient than others.
2	Zhu et al., 2019	Energy	Improved MEA Markov Model	China's 30 Provinces 2005-2016	Input: Total energy consumption, the capital stock, labor force Outputs: desirable output variable GDP, CO2 emissions (undesirable)	1.To improve energy efficiency, most western provinces had to employ a step-by-step progressive approach. 2. The energy saving potential of the eastern and western regions was relatively large, and the potential value of CO2 emission reduction in the central region is relatively large.
3	Wang et al., 2014	Environment	MEA	China 2006-2010	Input: energy: total energy consumption of industrial enterprises, labor: number of employed person capital: net value of fixed assets Desirable output: value added Undesirable outputs: industrial emissions, waste water discharge, solid waste discharge.	1.Environmental efficiency increased in the less developed cities than the cities in the well-developed region. 2. Different Chinese cities had to employ different strategies in dealing with pollutants. Where East China cities pay more attention to their industrial waste gas emissions and industrial wastewater discharges, West China cities should focus on their industrial soot and dust emissions, and solid waste discharges.
4	Wang et al., 2017	Energy	Non-parametric frontier approach, MEA	China 2006-2010	Inputs: labor, Capital ,Energy Outputs: GDP (desirable), Solid and liquid waste (Undesirable), CO2 emissions (Undesirable)	1. There was significant sectoral heterogeneity in the country's energy performances. 2. The transport sector showed the most substantial contribution to economy-wide energy productivity growth, while the agriculture sector and industry sector had seen a decline in energy productivity.
5	Bi et al., 2014	Transportation	MEA	China 2006-2011	Input: labor and capital stocks (two nonenergy input, the volume of energy consumed in the transportation sector(energy input)	The energy saving potential was identified and CO2 emissions reduction potential for each province and area in China.

					Outputs: value-added amount(desirable output), volume of CO2 emissions related to fuel used (undesirable output).	
6	Asmild et al., 2009	Transportation-Railway	MEA	23 European nations 1995 to 2001	Discretionary inputs – Staff costs (in million euros) – Material purchases and external charges (in million euros) Non-discretionary input – Network length Outputs – Passenger train-kilometres (in thousands), Freight train-kilometres (in thousands)	Major conclusions are that reform measures enhance technical efficiency in general, but in various ways for distinct cost drivers.
7	Suhaimi et al., (2017)	Agriculture	MEA	Malaysia	Inputs: - Land size (number of hectares) - Labor (total labor used for dairy activities) - Herd size (number of cows that a farmer owned), Feed (total cost of purchased feed) Output: (total revenue)	Scores for managerial and program inefficiency are provided for intensive and semi-intensive manufacturing systems. The results showed significant variability in inefficiency ratings across inputs and production methods.
8	Asmild M & Matthews K., 2012	Banking	MEA	China 1997-2008	Inputs: Number of employees, fixed assets, Total Deposits Outputs: net interest earnings, non-interest earnings	State owned banks (SOBs) are less efficient than Joint Stock Banks(JSBs) on utilization of labour & production of non-performing loans.
9	Asmild et al., 2016	Agriculture	MEA	Lithuania 3,308 different Lithuanian family farms 2004–2011	Inputs: land, labour, capital, and materials employed Outputs: total revenue	There exist differences between the efficiency scores on the different inputs as well as between the farm types of crop, livestock and mixed farms respectively. Moreover, crop farms have the highest program efficiency, but the lowest managerial efficiency and that the mixed farms have the lowest program efficiency (yet not the highest managerial efficiency).
10	Baležentiš & De Witte, 2015	Agriculture	MEA	Netherlands 1200 observations 2004–2009	Input - Land, Labour - Intermediate consumption, Assets Output - Crop Production subsidies	Combining the traditional one-directional and multi-directional efficiency framework yielded valuable insights. It allows one to identify what factors matter in terms of output production and input consumption

The taxonomy above contains a taxonomy of some available efficiency studies, along with some information.

The standard Data Envelopment Analysis (DEA) technique often assumes that the decision-making units (DMUs) can only contract or expand radially on all input or output variables simultaneously. However, this assumption may not always capture the realistic and specific efficiency metrics of different input-output combinations.

To address this limitation, researchers have developed alternative approaches, such as the Multi-directional Efficiency Analysis (MEA) technique, which allows for input-specific or output-specific efficiency evaluations. MEA considers the possibility of different inputs or outputs changing in different proportions, providing a more nuanced and realistic assessment of efficiency.

By considering input-specific or output-specific efficiency measures, the MEA approach enables a more accurate evaluation of the relative performance of DMUs, as it recognizes the heterogeneity in resource allocation and output production patterns. This allows for a better understanding of the factors driving inefficiencies and offers more targeted recommendations for improving performance in specific areas.

The use of MEA and similar techniques expands the scope of efficiency evaluation beyond the constraints of radial contractions or expansions, leading to more comprehensive and insightful analyses in various industries, including insurance.

The Multi-directional Efficiency Analysis (MEA) approach selects benchmarks in a manner that considers the potential for improvement in each input or output variable individually. This means that the contractions in inputs or expansions in outputs are proportional to the improvement

potential identified for each specific variable. MEA takes into account not only the efficiency levels but also the efficiency patterns within the data.

By considering the improvement potential separately for each input or output, MEA aims to provide a more accurate and comprehensive assessment of efficiency. This approach acknowledges that different variables may have varying degrees of potential for improvement and adjusts the benchmarks accordingly.

While it can be assumed that the MEA approach may yield better results compared to the traditional DEA approach, it ultimately depends on the specific context and objectives of the analysis. MEA's ability to account for input-specific or output-specific efficiency can provide a more nuanced understanding of performance and inform targeted improvement strategies.

3.4 Conclusion

In this chapter, various aspects of efficiency was discussed in the context of business and its application worldwide. This discussion was started by examining the concept of efficiency, its historical development, and different forms it can take.

The existing literature and research were delved into to show how various studies utilized the Multi-directional Efficiency Analysis (MEA) approach. This was followed by how the MEA concept has been applied in different studies and examined the specific applications and findings of some MEA efficiency studies. This presented insights into the practical implications and relevance of the MEA approach in various domains.

Furthermore, a taxonomy of MEA efficiency studies, categorizing and summarizing their key characteristics and contributions is provided. This taxonomy serves as a useful reference for understanding the diversity of research conducted using the MEA framework.

Finally, a review of the more traditional approach of Data Envelopment Analysis (DEA), providing a general overview of its use and characteristics, was undertaken. Comparing DEA with the MEA approach, distinctive characteristics and potential advantages of MEA were highlighted in capturing slacks and making realistic efficiency metrics.

Overall, this chapter provides a comprehensive overview of efficiency research, specifically focusing on the MEA approach and its applications. By examining the existing literature and summarizing key findings, we contribute to the understanding and advancement of efficiency analysis in various fields of study.



CHAPTER FOUR

RESEARCH METHODOLOGY

4.1 Chapter Introduction

This chapter focuses on various aspects related to the design of the study, population data collection techniques, intervention methods, and the planned data analysis. To determine the main methods for analysing the collected data, we conducted exploratory data analysis (EDA) on the gathered information.

Exploratory data analysis is a crucial step in understanding the dataset and identifying basic patterns exhibited by the variables under study. It involves techniques such as summarizing the data, examining distributions, identifying outliers, and exploring relationships between variables. By performing EDA, we gain insights into the characteristics of the data and identify initial trends or patterns.

The observations and findings from the exploratory data analysis informs decisions regarding the most appropriate methods for analysing the data. These methods may include statistical techniques, hypothesis testing, or any other relevant analytical approaches based on the research objectives and the nature of the data.

Overall, this chapter emphasizes the importance of the study design, population data collection techniques, and intervention methods. Additionally, it highlights the role of exploratory data analysis in guiding the selection of appropriate data analysis methods by providing a preliminary understanding of the data's patterns and characteristics.

Various types of charts, such as line graphs and curves, are used to aid in the explanation and visualization of the results. Some of these charts can be found in this chapter, the following chapter, and the appendix section of the dissertation.

To conduct the analysis, the R-language statistical software employed. R is a widely used programming language and software environment for statistical computing and graphics. It offers a wide range of functions and packages that enable advanced statistical analysis, including the computation of MEA and DEA technical efficiency scores.

The choice of R as the statistical software was based on its capability to perform multi-directional efficiency analysis (MEA), which is essential for our research. As far as my knowledge extends, R is one of the software programs that can handle the complexities of MEA, by utilizing R, we were able to leverage its computational power and extensive statistical functionalities to analyze the data and generate the MEA and DEA efficiency scores required for the studies.

Overall, the use of R as the statistical software in this study enabled is essential to conduct the necessary calculations and analysis, making it an essential tool for our research.

4.2 Research Design

The research design outlines the systematic procedures for collecting, analyzing, and interpreting data to ensure alignment with the study's objectives. This study adopts a non-experimental quantitative approach, grounded in a post-positivist philosophical paradigm, which views knowledge as measurable, testable, and derived through observation and empirical evidence (Creswell, 2014). The post-positivist stance justifies the application of Data Envelopment Analysis (DEA) and its extensions, as these models rely on observable input-output relationships and deterministic efficiency frontiers to quantify performance.

DEA aligns with post-positivism by emphasizing objective benchmarking based on empirical data rather than subjective interpretation. It assumes that real-world phenomena, such as the transformation of energy, capital, and labor into economic output, can be modeled and assessed without imposing prior statistical distributions. However, it is important to acknowledge that DEA's deterministic nature does not account for random noise or measurement error, which could influence results. While this study does not implement Stochastic DEA (SDEA), it recognizes its relevance in addressing such stochastic influences and suggests it as a potential avenue for future research.

The design is especially relevant for analyzing dynamic productivity change across African states using the newly developed Biennial MEA-Malmquist (BMEAM) index, which incorporates variable-specific decomposition. This non-parametric benchmarking framework supports detailed cross-country and temporal analysis without relying on distributional assumptions, making it ideal for the heterogeneous and data-limited environments typical of many African countries.

Although DEA is primarily diagnostic, measuring relative efficiency rather than predicting future outcomes, the results can inform predictive or policy-oriented models. For instance, efficiency scores generated through DEA or MEAM can serve as inputs or explanatory variables in regression models, panel analysis, or clustering algorithms to explore determinants of performance, identify best-practice frontiers, or simulate future productivity scenarios.

This design allows the study to not only measure productivity but also evaluate systemic inefficiencies and scale effects that constrain energy performance in African states. The methodological choice is therefore both appropriate and innovative, offering a structured and data-driven pathway to assess and compare energy productivity over time and across regions.

4.3 Data sample and sources

This study conducts a cross-country evaluation of energy productivity using a sample of 32 African states over the period 2000–2019. The selection of countries was guided primarily by data availability across key variables of interest. Although the original design intended a balanced panel, limitations in the availability of reliable and complete data resulted in an unbalanced panel dataset. This characteristic was explicitly addressed by excluding years and countries with significant missing values. Where appropriate, simple interpolation techniques were used for minor data gaps, while countries with systematic data absence were excluded from the final sample.

The inclusion criteria for country selection were based on: (i) the availability of continuous energy and economic data from 2000 to 2019; (ii) regional representativeness across North, East, West, Central, and Southern Africa; and (iii) political and macroeconomic stability sufficient to ensure data quality. Exclusion criteria included: persistent missing data, extreme outliers with no feasible correction, and inconsistencies in national reporting standards.

Data were sourced from three reputable databases: the Penn World Table (PWT 10.0), the U.S. Energy Information Administration (EIA), and the World Bank's World Development Indicators (WDI). The PWT provides purchasing power parity (PPP)-adjusted macroeconomic data, enabling cross-country comparisons of output and capital stock. The EIA offers annual energy data including energy use (kg of oil equivalent per capita) and production. The WDI provides supplementary socioeconomic indicators relevant for understanding the broader development context. These sources are widely recognized for their accuracy, standardization, and frequent updates (Heston, Summers et al., 1998).

4.3.1 Selection of Input and Output Variables

The selection of input and output variables is a critical component of the Data Envelopment Analysis (DEA) framework, as it determines the validity and interpretability of the efficiency results. In this study, the inputs selected are capital services, labour, and energy used, while the outputs include real GDP as a desirable output and CO₂ emissions as an undesirable output.

These choices are grounded in established economic production theory and supported by empirical applications in the literature. Capital and labour are traditional inputs in growth accounting and efficiency analysis, while energy use reflects the intensity of energy resources consumed to support economic activities. Real GDP is widely accepted as a comprehensive measure of economic output, and CO₂ emissions capture the environmental cost associated with productive processes. Including both desired and undesired outputs enables the study to assess not only economic efficiency but also eco-efficiency, consistent with recent advances in productivity measurement (Färe et al., 1994; Zhou et al., 2008).

The variables were selected based on theoretical relevance, data availability, and their importance in capturing the input-output dynamics of national energy systems. Ramanathan (2003) emphasizes that variable selection in DEA should be context-specific, guided by the nature of the decision-making units (DMUs) and the objectives of the analysis. In this context, the DMUs are African states with diverse but comparable energy and economic structures, justifying the use of macro-level indicators.

This study adopts a constant-returns to scale (CRS) assumption in constructing the global reference frontier used in the Biennial MEA Malmquist (BMEAM) index. The CRS assumption ensures consistency in evaluating performance across countries and over time, enabling the decomposition

of productivity change into its component effects. Although variable returns to scale (VRS) could reveal insights into scale inefficiencies, CRS was selected due to its suitability for panel data structures and to maintain comparability across the heterogeneous DMUs under study. Furthermore, data constraints in certain countries make VRS estimation less stable in this context.

While the DEA literature acknowledges the existence of flexible measures, variables that may function as either inputs or outputs depending on context (Cook and Zhu, 2007), this study employs a fixed classification to maintain conceptual clarity and model robustness. Each variable's designation as an input or output is based on established production logic and the intended analytical purpose.

The selected inputs and outputs are conceptually sound, empirically supported, and well-suited to the objectives of this study. They allow for a meaningful and multidimensional assessment of both technical and scale efficiency, as well as the environmental implications of energy use across African economies.

4.4 Frontier Efficiency Analysis

Productivity and efficiency measurement have become critical tools in evaluating the performance of organizations, sectors, and national economies. Frontier efficiency analysis aims to benchmark decision-making units (DMUs) against a constructed efficiency frontier, identifying best-performing entities and diagnosing inefficiencies. The foundation of this approach dates back to Farrell (1957), with significant advancement made by Charnes, Cooper, and Rhodes (1978), who formalized Data Envelopment Analysis (DEA). DEA is a non-parametric linear programming method that constructs a production frontier from observed input-output combinations and evaluates the relative efficiency of each unit against this frontier (Cooper, Seiford, & Tone, 2011).

In this study, a more advanced methodology, Multi-Directional Efficiency Analysis (MEA) and its extension, the Biennial MEA Malmquist (BMEAM) index, is employed to assess productivity changes across African energy systems. MEA, originally proposed by Bogetoft and Hougaard (1999) and further developed by Asmild, Hougaard, Kronborg, and Thorsen (2003), offers a granular approach by allowing input- or output-specific efficiency assessments. This is particularly relevant in multi-input systems like national energy sectors, where improvements may not be uniform across all resource dimensions. The MEA-Malmquist framework facilitates variable-specific productivity change analysis, overcoming the aggregation bias inherent in traditional DEA-Malmquist models (Asmild, Baležentis, & Hougaard, 2016a).

The BMEAM index developed in this study addresses two critical methodological limitations often observed in conventional MEAM approaches: linear programming infeasibilities and the inability to capture scale effects. By introducing a biennial window and decomposing productivity change into four components, BPEC (Pure Efficiency Change), BPTC (Pure Technical Change), BSTC (Scale Technical Change), and BSEC (Scale Efficiency Change), the model enables a more robust, detailed assessment of productivity drivers. This decomposition enhances interpretability and aligns with dynamic production environments where scale and technology evolve over time (Førsund, Lovell, & Schmidt, 1980; Pastor & Lovell, 2005).

Although DEA is widely favored for its flexibility and minimal assumptions, it is important to consider its alternatives, particularly Stochastic Frontier Analysis (SFA). SFA is a parametric approach that estimates efficiency relative to a statistically defined frontier, accounting for random noise (Aigner, Lovell, & Schmidt, 1977). However, SFA requires the specification of a functional form and a distributional assumption for the inefficiency term, which can introduce model bias, especially in cross-country studies where input-output relationships are complex and

heterogeneous (Greene, 2008). DEA, in contrast, is non-parametric, does not impose a predefined production structure, and can simultaneously handle multiple inputs and outputs without requiring price or cost data. These advantages make DEA more suitable for this study's macro-level assessment of energy productivity across diverse African states (Zhou, Ang, & Poh, 2008; Daraio & Simar, 2007).

To enhance the transparency and robustness of the results, a model validation step was conducted. A sensitivity analysis was performed by re-estimating the BMEAM model under alternative input-output combinations and excluding selected DMUs with incomplete data histories. The resulting efficiency trends remained stable across these specifications, confirming the model's reliability. This step provides additional confidence in the interpretability and policy relevance of the findings, addressing potential concerns about the influence of outliers or data imbalances (Simar & Wilson, 2007).

The BMEAM model applied in this study not only extends the MEA-Malmquist framework to account for practical challenges like infeasibilities and scale effects, but also offers a robust, contextually grounded method for measuring energy productivity change. Its integration into a DEA-based frontier approach ensures that the analysis remains methodologically sound and empirically insightful for guiding policy and performance improvement across African energy systems.

4.5 Formalising the basic DEA model

Let x_{ij} ($i = 1, \dots, m$) be m different inputs and y_{rj} ($r = 1, \dots, s$) be s different outputs for DMU $_j$ ($j = 1, \dots, n$).

$$\theta^* = \min \theta_{\lambda_j \theta}$$

s.t

$$\sum_{j=1}^n x_{ij} \lambda_j \leq \theta x_{io}, \quad i = 1, \dots, m$$

$$\sum_{j=1}^n y_{rj} \lambda_j \geq y_{ro}, \quad r = 1, \dots, s$$

$$\sum_{j=1}^n \lambda_j = 1 (vrs); \quad \lambda_j > 0; j = 1, \dots, n$$

where DMU represents 1 of the n DMUs under evaluation x_{io} & y_{ro} are i th input & r th output of DMU.

θ is the (input-oriented) TE score of DMU, ϕ is the (output-oriented) TE score of DMU.

If $\theta = 1$, $\phi = 1$ then the current input levels can't be reduced (radially), indicating that DMU is on frontier/efficient.

If $\theta < 1$, $\phi > 1$ DMU is dominated by frontier & inefficient. The efficiency of a Decision Making Unit (DMU) can be estimated using either the envelopment model or the multiplier model.

Table 2 Envelopment and Multiplier Models

Input orientation	
Envelopment Model	Multiplier Model



$\min \theta$ s.t $\sum_{j=1}^n x_{ij} \lambda_j \leq \theta x_{io}, i = 1, 2, \dots, m$ $\sum_{j=1}^n y_{rj} \lambda_j \geq y_{ro}, r = 1, 2, \dots, s$ $\lambda_j \geq 0, j = 1, 2, \dots, n$	$\max z = \sum_{j=1}^n \mu_r y_{ro}$ s.t $\sum_{j=1}^s \mu_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0$ $\sum_{i=1}^m v_i x_{io} = 1$ $\mu_r, v_i \geq 0$
Output orientation	
Envelopment Model	Multiplier Model
$\min \phi$ s.t $\sum_{j=1}^n x_{ij} \lambda_j \leq \phi x_{io}, i = 1, 2, \dots, m$ $\sum_{j=1}^n y_{rj} \lambda_j \geq \phi y_{ro}, r = 1, 2, \dots, s$ $\lambda_j \geq 0, j = 1, 2, \dots, n$	$\min q = \sum_{j=1}^n v_i x_{io}$ s.t $\sum_{i=1}^m v_i x_{ij} - \sum_{j=1}^s \mu_r y_{rj} \geq 0$ $\sum_{i=1}^m \mu_r y_{ro} = 1$ $\mu_r, v_i \geq 0$

Table 2 above provides information on each of these models. In these models, λ , μ and v are weights and θ , ϕ , z and q are the efficiency scores to be estimated.

4.6 Input and Output Orientations

In DEA models, the choice between input-orientation and output-orientation depends on the focus of the analysis. The input-orientation framework, as described by Lovell and Pastor (1999), aims to minimize the input quantities required to produce a given level of outputs. It focuses on identifying the most efficient way to utilize inputs while keeping the outputs constant or at least not decreasing them. This approach helps to assess how efficiently inputs are used and how input reductions can be achieved without sacrificing the current output levels. By comparing the input requirements of each Decision Making Unit (DMU) to the efficient boundary, the input-orientation framework provides insights into improving input utilization and reducing inefficiencies. Daraio and Simar (2007) use the term "input saving" to emphasize that the output level remains constant while input amounts are lowered proportionally until the frontier is achieved.

This framework is widely utilized when the decision maker has the ability to control and manipulate the input variables but has limited control over the output variables. It allows the decision maker to assess and improve efficiency by identifying the most effective utilization of available inputs to achieve desired output levels. By focusing on input-oriented analysis, this framework offers valuable insights and guidance for optimizing resource allocation and operational strategies.

To provide a visual representation, let's consider the the hypothetical data provided in in Table 3 and illustrative example shown in Figure 4 below. We have seven Decision Making Units (DMUs) denoted as A, B, ..., G. Each DMU consumes a single input to produce a single output (Cooper, Seiford et al. 2011).

Table 3 Data for an $Input(x)$ and $Output(y)$

DMU	Input (x)	Output (y)
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A	2.4	4
B	7	12
C	4	10
D	1	3
E	1	2
F	6	12
G	5.6	9

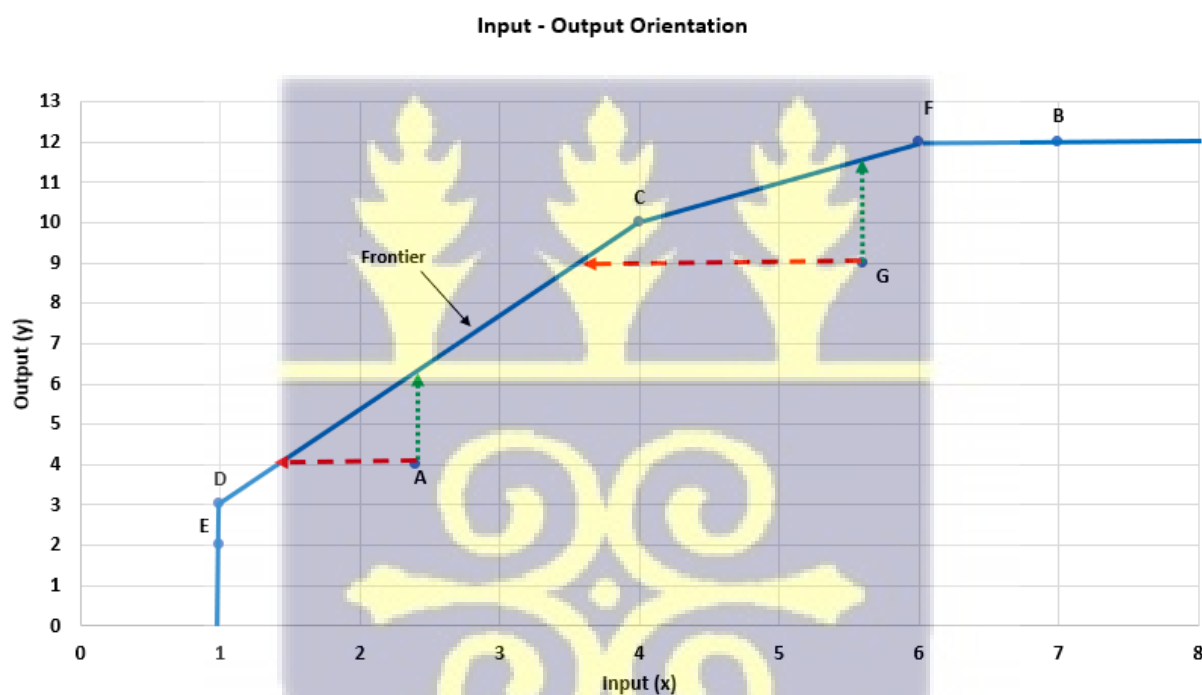


Figure 4 Projection to frontier for the input-oriented and output-oriented model

Figures 4 display the efficient frontier and DEA projections for both the input-oriented and output-oriented models. The red dash lines represent the input-oriented model, while the green dash lines represent the output-oriented model. In both cases, the efficient frontier obtained from the VRS model is indicated by arrowheads. It is worth noting that an inefficient DMU (Decision Making

Unit) could be projected to various points on the frontier depending on the orientation. (Cooper, Seiford et al. 2011). In an input orientation, efficiency is enhanced by reducing inputs while keeping the inputs constant. On the other hand, in an output orientation, efficiency is achieved by maximizing outputs while keeping inputs constant. It is important to differentiate between a boundary point and an efficient boundary point.

Furthermore, the efficiency of a boundary point can be influenced by the chosen model orientation. In other words, whether a boundary point is considered efficient or not may vary depending on the specific orientation of the model being used. (Cooper, Seiford et al. 2011).

Figure 4 illustrates that DMUs B, C, D, E, and F are positioned on the frontier, indicating that they are efficient. However, DMUs A and G are inefficient because they are not located on the frontier. To evaluate the efficiency of DMU G we can solve the following input-oriented envelopment linear programming (LP) model:

min θ

subject to

$$2.4\lambda_1 + 7\lambda_2 + 4\lambda_3 + 1\lambda_4 + 1\lambda_5 + 6\lambda_6 + 5.6\lambda_7 \leq 5.6\theta \text{ (input)}$$

$$4\lambda_1 + 12\lambda_2 + 10\lambda_3 + 3\lambda_4 + 2\lambda_5 + 12\lambda_6 + 9\lambda_7 \geq 9 \text{ (output)}$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 = 1 \text{ (VRS axiom)}$$

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7 \geq 0 \text{ (non-negativity)}$$

The data used in the above example contain a single input and a single output. This model can be solved to find the values of the efficiency score (θ) and the weights (λ_j)

Utilizing figure 4, the efficiency scores for the two inefficient DMUs i.e DMU A and DMU G can be calculated as follows:

$$Efficiency_{DMU A} = \frac{1.4}{2.4} = 0.5833 = 58.33\%$$

$$Efficiency_{DMU G} = \frac{3.6}{5.6} = 0.6429 = 64.29\%$$

The results indicate that DMU A, should reduce inputs x by 58% of their current levels without changing the output level and similarly DMU G should reduce inputs x by 64% of their levels.

4.7 Merits and Demerits of DEA

The benefits and drawbacks of DEA are summarized here ((Coelli, Rao et al. 2005; Daraio and Simar 2007).

Merits

1. Able to handle multiple inputs & outputs simultaneously.
2. Researcher needs not specify restrictive functional forms, unlike stochastic frontier analysis (Coelli, Rao et al. 2005; Fried, Lovell et al. 2008).
3. Able to decompose efficiency into several component (profit, cost, revenue, technical, allocative, scale etc)
4. DEA is unit invariant, i.e., works with variables of different units (metrics) without the need for standardization (Tone 2001)
5. DEA generates relative prices/weights if desired, but they are not required (Allen, Athanassopoulos et al. 1997).
6. It is useful for target-setting, identification of “suitable” efficient peers for inefficient DMUs to emulate.

Demerits

1. DEA is deterministic making it difficult in making statistical inference. It assumes all deviations from frontier is due to inefficiencies without noise (measurement error) (Daraio and Simar 2007)
2. Statistical inferences are not easy in DEA. No parameters for economic interpretation
3. Sensitive to extremes/outliers, though partial frontiers can help.
4. Curse of dimensionality, sample size bias can also occur

4.8 Multi-directional Efficiency Analysis (MEA)

The multi-directional efficiency analysis (MEA) approach is an enhancement of the conventional DEA (Data Envelopment Analysis) approach, as discussed by Asmild et al. (2003) in their research. The MEA methodology was pioneered by Bogetoft and Hougaard (1999) and further refined and utilized by Asmild et al. (2003) (Asmild, Hougaard et al. 2003). The decision to employ MEA instead of the widely recognized DEA approach developed by (Charnes, Cooper et al. 1978) is rooted in a fundamental difference: whereas DEA employs radial contraction of all inputs (in its input-oriented form) or expansion of outputs (in its output-oriented form), MEA offers a distinct approach. MEA (Multi-directional Efficiency Analysis) adopts a benchmark selection method that ensures the reductions in inputs or expansions in outputs align with the potential efficiency improvements identified for each input or output individually. Consequently, this approach is particularly well-suited for examining the efficiency patterns of each Decision Making Unit (DMU) separately, as it allows for a detailed analysis of their performance in relation to inputs and outputs (Asmild, Baležentis et al. 2014; Bi, Wang et al. 2014; Baležentis and De Witte 2015; Asmild, Kronborg et al. 2019; Zhu, Baležentis et al. 2020).

In addition, MEA is particularly suitable for situations where the objective of each DMU is to simultaneously decrease certain inputs while increasing certain outputs. Since MEA assesses the potential for improvement in each variable independently, it is well-equipped to analyze and evaluate such circumstances effectively (Wang, Wei et al. 2013; Bi, Wang et al. 2014; Zhu, Baležentis et al. 2020).

Asmild and Pastor (2010), highlighted that the Multi-directional Efficiency Analysis (MEA) Model, conceptualized by Bogetoft and Hougaard, fundamentally operates as a linear directional distance function efficiency model. However, the measurement of directional efficiency needs to be derived retrospectively and relies on the selection of benchmarks that may include non-directional slack. MEA can be defined under various assumptions regarding returns to scale. Asmild and Pastor (2010) also suggest that the Variable Return to Scale (VRS) MEA is a translation-invariant model, meaning that it can handle negative data effectively.

Within the framework of the Multi-directional Efficiency Analysis, every unit undergoing assessment is designated an ideal point, signifying the direction leading toward the efficient frontier, where the benchmark position is ideally located. MEA selects a unique ideal point for each unit individually, primarily focusing on benchmark selection. Additionally, Asmild and Pastor (2010) noted that in MEA, deriving an efficiency measure is straightforward but only possible after gaining experience and knowledge about the inefficient input (a posteriori). However, this process requires the imposition of specific normalization conditions. It is worth mentioning that MEA can be defined under any assumption regarding returns to scale, allowing for flexibility in its application.

MEA, outlines an ideal point specific each unit under analysis. In an input-oriented analysis, the largest reduction potentials in each input are identified independently and produce the smallest

feasible input consumption in each dimension, which are then combined to produce the ideal point. MEA is explicitly intended to produce input-specific (or output-specific) efficiency scores, making it a logical choice for measuring input-specific productivity change (Wang, Wei et al. 2013; Bi, Wang et al. 2014; Zhu, Baležentis et al. 2020).

Yet, there are other non-radial efficiency indices that indirectly can be used to obtain input-specific scores. Common for these indices is that they either build on an aggregation of input specific scores or are determined by minimizing some distance to the efficient frontier of the production possibility set. One of the key drawbacks of non-radial measures, as highlighted, is that they require the establishment of an objective function that involves an arbitrary combination of input-specific efficiency scores. This means that determining the appropriate weighting scheme for the aggregation of these scores can be subjective and lack a clear basis. This subjectivity in defining the objective function is considered a fundamental disadvantage of non-radial measures. (Asmild, Baležentis et al. 2016a). The use of MEA to estimate the efficiency measures used in the Malmquist index makes it easier to evaluate variable-specific productivity increases.

In order to define the MEA, a production technology that uses a vector of input x to produce vectors of output y (desirable) and c (undesirable). The technology set is given by:

$$\psi = \{(x, y, c) \in \mathbb{R}_+^{m+s_1+s_2} \mid x \text{ can produce } y, c\} \quad (1)$$

The production technology ψ considers undesirable outputs as by-products since they are generated alongside desired outputs (Färe & Grosskopf, 2004); (Bi et al., 2014).

There are three assumptions that reinforce the joint production technology for the asymmetric treatment of the desired and undesired outputs (Färe et al., 2005; Färe & Grosskopf, 2004; Reyna & Fuentes, 2018).

Strong or free disposability of desirable outputs:

If $(x, y, c) \in \psi$ and $y^* \leq y$, then $(x, y, c) \in \psi$. This assumption is like the classical assumption handling the disposability of desirable outputs. The axiom states that any output vector with a smaller desirable output is feasible if the observed desirable and bad outputs vectors are feasible. This assumption suggests that we can always dispose of certain desired outputs freely and without incurring any costs. (Färe et al., 2005; Färe & Grosskopf, 2004).

Weak disposability of undesirable outputs (Shephard, 1970) :

If $\{(x, y, x) \in \varphi \text{ and } 0 \leq \varphi \leq 1, \text{ then } (x, \varphi y, \varphi c) \in T\}$

Weak disposability means that the proportional reduction of desirable and undesirable outputs is possible, hence for any given input, bad outputs can be reduced if and only if good inputs are also reduced in proportion. The assumption is to introduce the idea that it is not feasible to reduce undesirable outputs solely, hence the sole reduction of undesirable outputs is impossible, due to their costly disposal which affects desirable outputs (Färe et al., 2005; Färe & Grosskopf, 2004; Reyna & Fuentes, 2018).

Desirable and undesirable outputs being null-joint (Shephard, Ronald & Fare, 1944):

If $\{(x, y, c) \in \omega \text{ and } c = 0, \text{ then } y = 0\}$. This axiom suggests that undesirable outputs are by-products of desirable outputs, hence the production of desirable outputs and the production of undesirable outputs are inseparable. Relating the assumption to the energy environment, the emission of carbon dioxide is linked with the production of energy from fossil fuels - as desirable outputs are being produced, undesirable outputs will be produced alongside (Färe et al., 2005; Reyna & Fuentes, 2018).

Referring to the first and second assumption, strong disposability of good output vector suggests weak disposability of both good and bad outputs. Consistent with Bi et al. (2014) and others, the production technology ψ is modelled for a DEA framework as:

$$\psi = \left\{ (x, y, c) \mid \begin{aligned} & x \geq \sum_{j=1}^n \lambda_j x_{ij}, i = 1, 2, \dots, m, y \leq \sum_{j=1}^n \lambda_j y_{rj}, r = 1, 2, \dots, s_1, c \geq \sum_{j=1}^n \lambda_j c_{kj}, 1 \\ & = \sum_{j=1}^n \lambda_j, k = 1, 2, \dots, s_2, \lambda_j \geq 0 \end{aligned} \right\}, \quad (2)$$

Lambda (λ_j) in equations (2) shows the weights or the intensity factor of the variables whilst $(\sum_{j=1}^n [\lambda_j = 1])$ shows the convexity constraint. The strong or free disposability of the good outputs and the weak disposability of the bad outputs are impossible because of the equality and inequality on the bad outputs and good outputs respectively.

Following (Asmild & Matthews, 2012) and (L. Zhu et al., 2019) an MEA model is formalization is performed in 2 stages. First, we identify the ideal reference point (IRP). In order to find the ideal reference point for the specific DMU₀ (x_{i0}, y_{ro}, c_{ko}), the LPP (3)-(5) is first solved, by considering the subvector efficiencies for the input dimension, desirable output dimension and undesirable output dimensions. The non-oriented MEA model is used since we are primarily interested in the reduction potentials for the inputs and undesired output as well as the augmentation potential of desirable output.

$$d_{i0}^* = \min_{\lambda_j, d_{i0}} d_{i0}$$

$$\text{s. t. } \begin{cases} \sum_{j=1}^n \lambda_j x_{ij} \leq d_{i0}, \\ \sum_{j=1}^n \lambda_j x_{-ij} \leq x_{-i0} : -i = 1, \dots, i-1, i+1, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj} \geq y_{r0} : r = 1, \dots, s_1 \\ \sum_{j=1}^n \lambda_j c_{kj} = c_{k0} \quad k = 1, \dots, s_2 \\ \sum_{j=1}^n \lambda_j = 1 \quad (\text{if VRS}), \\ \lambda_j \geq 0, \quad j = 1, \dots, n \end{cases} \quad (3)$$

$$\delta_{r0}^* = \max_{\lambda_j, \delta_r} \delta_{r0}$$

$$\text{s. t. } \begin{cases} \sum_{j=1}^n \lambda_j y_{rj} \geq \delta_{r0}, \\ \sum_{j=1}^n \lambda_j y_{-rj} \geq y_{-r0} : -r = 1, \dots, r-1, r+1, \dots, s_1 \\ \sum_{j=1}^n \lambda_j x_{ij} \leq x_{i0} : i = 1, \dots, m \\ \sum_{j=1}^n \lambda_j c_{kj} = c_{k0} \quad k = 1, \dots, s_2 \\ \sum_{j=1}^n \lambda_j = 1 \quad (\text{if VRS}), \lambda_j \geq 0, \quad \forall j = 1, \dots, n \end{cases} \quad (4)$$

$$\begin{aligned}
 \varphi_{i0}^* &= \min_{\lambda_j, \varphi_k} \varphi_{k0} \\
 \text{s. t. } &\begin{cases} \sum_{j=1}^n \lambda_j c_{kj} = \varphi_{k0} \\ \sum_{j=1}^n \lambda_j c_{-kj} = c_{-k0} : \quad -k = 1, \dots, k-1, k+1, \dots, s_2 \\ \sum_{j=1}^n \lambda_j x_{ij} \leq x_{i0} : \quad i = 1, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj} \geq y_{r0} : \quad r = 1, \dots, s_1 \\ \sum_{j=1}^n \lambda_j = 1 \quad (\text{if VRS}), \lambda_j \geq 0, \quad \forall j = 1, \dots, n \end{cases} \quad (5)
 \end{aligned}$$

In model (3)-(5), we consider a set of n DMU's ($j=1, \dots, n$) that produce s_1 of y desirable outputs ($r=1, \dots, s_1$) and s_2 of c undesirable outputs ($k=1, \dots, s_2$) using m of x inputs ($i=1, \dots, m$). The IRP ($d_{i0}^*, \delta_{r0}^*, \varphi_{i0}^*$) of the evaluated DMU ($d_{i0}, \delta_{r0}, \varphi_{i0}$) is then the optimal solutions of the estimated models (3)-(5). By estimating the IRP of each variable, the LPP, eqn (6), can be employed to ascertain the MEA efficiency of each variable of the assessed DMU.

$$\beta_0^* = \max_{\lambda_j, \beta_0} (\beta_{i0} + \beta_{r0} + \beta_{k0})$$

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$$\text{s. t. } \begin{cases} \sum_{j=1}^n \lambda_j x_{ij} \leq x_{io} - \beta_{io}(x_{io} - d_{io}^*), & i = 1, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj} \geq y_{ro} + \beta_{ro}(\delta_{ro}^* - y_{ro}), & r = 1, \dots, s1 \\ \sum_{j=1}^n \lambda_j c_{kj} = c_{ko} - \beta_{ko}(c_{ko} - \varphi_{ko}^*), & k = 1, \dots, s2 \\ \sum_{j=1}^n \lambda_j = 1 & (\text{if VRS}), \\ \lambda_j \geq 0, & \forall j = 1, \dots, n \end{cases} \quad (6)$$

Model (6) will then be used to finally determine the benchmark selection β_o^* for the specific DMU which can be used to determine the MEA efficiency of each variable of the evaluated unit. $\beta_{io}, \beta_{ro}, \beta_{ko}$ measures the proportion by which the desirable outputs are added while the undesirable outputs and inputs are contracted in the same proportion (Bi et al., 2014; Bogetoft & Hougaard, 1999; Kapelko & Lansink, 2017; Tziogkidis, Philippas, & Tsionas, 2020). β_{ij}, β_{rj} and β_{kj} always falls within the interval, $[0,1]$ hence a DMU is said to have reached the frontier of the best practice firms if $\beta_{ij} = \beta_{rj} = \beta_{kj} = 0$ otherwise, the DMU is farther away from the frontier of the best practice firms (Asmild et al., 2003; Bi et al., 2014; Bogetoft & Hougaard, 1999; Kapelko & Lansink, 2017). The optimal solution of model (6) is $(\lambda^*, \beta_i^*, \beta_r^*, \beta_k^*)$ and can be used to define the *relative variable-specific MEA efficiency* for each variable separately.

Hence for the input variable x_{io} we can define specific input MEA efficiency as follows:

$$e_i = \frac{x_{io} - \beta_{io}^*(x_{io} - d_{io}^*)}{x_{io}} \quad (7)$$

For the desirable output y_{ro} we can define specific desirable output MEA efficiency as follows:

$$e_r = \frac{y_{ro}}{y_{ro} + \beta_{r0}^*(\delta_{r0}^* - y_{ro})} \quad (8)$$

For the undesirable output $(c)_{ko}$, we can define specific undesirable output MEA efficiency as follows:

$$e_k = \frac{c_{ko} - \beta_{k0}^*(c_{ko} - \varphi_{k0}^*)}{c_{ko}} \quad (9)$$

where β_o^* is the value of the estimated benchmark from equation (6), X_{io} represent the specific input to be reduced, Y_{ro} represent the specific desirable output to be increased, the C_{ko} represent the specific undesirable output to be reduced and $d_i^* \dots$ is the value of the ideal reference point that correspond with each of the observations respectively.

Using the variable-specific MEA efficiency values defined in equations (7) – (9) and based on the slack-based measure (SBM) model aggregation idea of (Tone, 2001), an integrated MEA efficiency is established consisting of all the input and output (desirable and undesirable) variables. Following Asmild and Matthews (2012) and (L. Zhu et al., 2019), the overall MEA efficiency score is given as:

$$\theta_0 = \frac{1 - \frac{1}{m} \sum_{i=1}^m \frac{\beta_{i0}^*(x_{i0} - d_{i0}^*)}{x_{i0}}}{1 + \frac{1}{s_1 + s_2} \left[\sum_{r=1}^{s_1} \frac{\beta_{r0}^*(\delta_{r0}^* - y_{r0})}{y_{r0}} + \sum_{k=1}^{s_2} \frac{\beta_{k0}^*(c_{k0} - \varphi_{k0}^*)}{c_{k0}} \right]} \quad (10)$$

4.8.1 An illustrative example

This section utilizes a numerical example to explain the information that can be gleaned using the comparison of the classical DEA and MEA. Considering the first 7 hypothetical DMUs each using one input (x) and one output (y) in a two time periods as shown in table 4.

Table 4 Hypothetical Data Set for two time periods

DMU	Year	Input (x)	Output(y)
A	1	2.4	4
B	1	7	12
C	1	4	10
D	1	1	3
E	1	1	2
F	1	6	12
G	1	5.6	9
A	2	4.8	13
B	2	6.2	17
C	2	5	17
D	2	3	6
E	2	5	10
F	2	7.4	15
G	2	5.6	9

The dataset for period 1 is plotted in Fig. 5 with piece-wise linear frontier (VRS) associated with convexity and monotonicity property well-known in DEA. The convex VRS frontier is depicted by the frontier EDCFB and the free disposability lines showing monotonicity axiom.



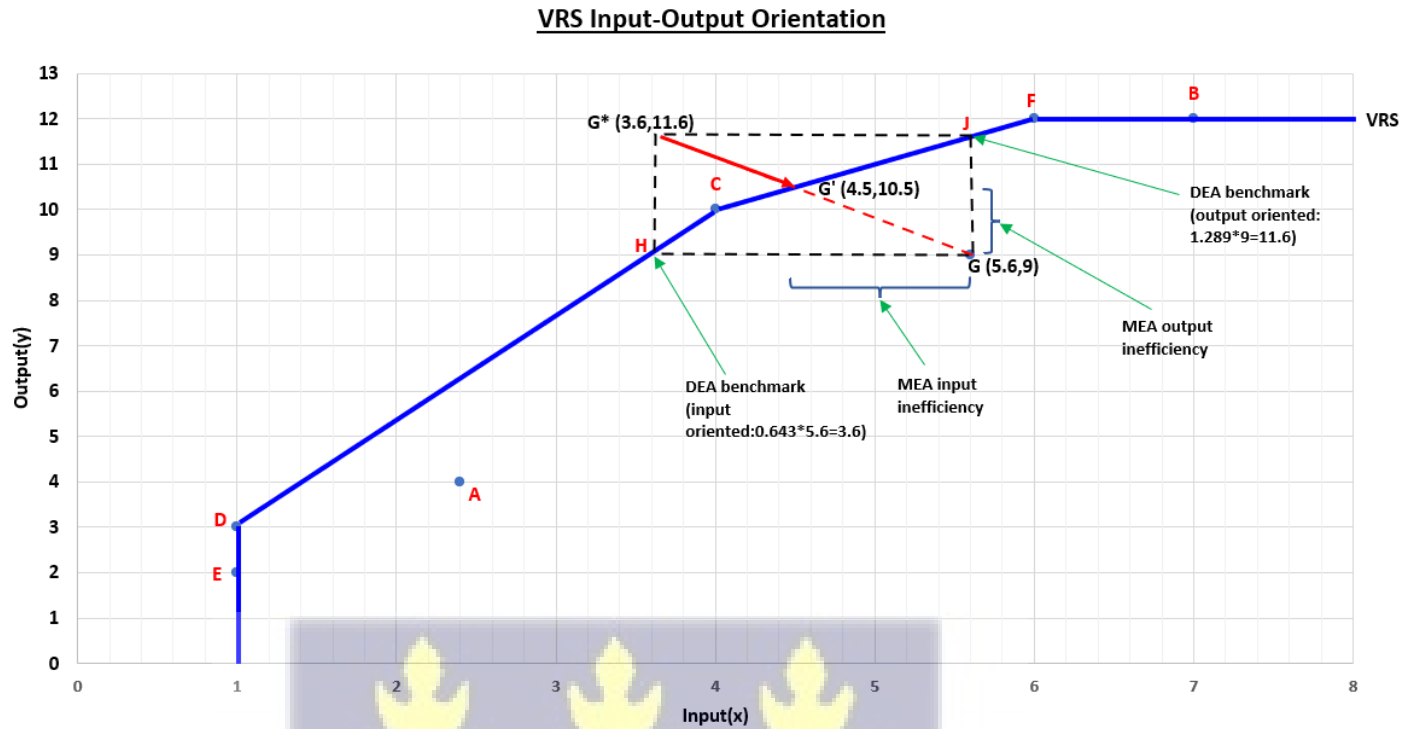


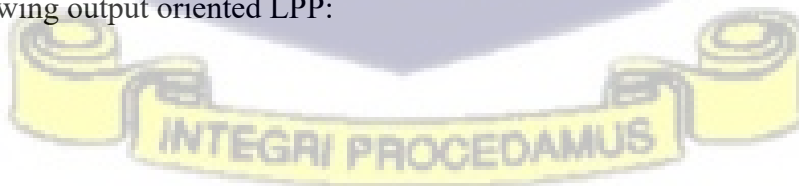
Figure 5 Illustrating MEA

Generally, the CCR (1978) DEA technical efficiency (TE) of a target DMU is defined as,

$$TE(x, y) = \frac{\text{Distance from the origin to the VRS production frontier}}{\text{Distance from the origin to the unit}}$$

As an example, the output-oriented DEA TE of DMU G is $TE_G(x, y) = \frac{11.6}{9} = 1.28$ and the input-oriented TE of this same DMU is $TE_G(x, y) = \frac{3.6}{9} = 0.64$.

The classical DEA efficiency score of DMU G is obtained by mathematically formulating and solving the following output oriented LPP:



$$\Phi_{rG}^* = \max_{\lambda_{A-I}, \phi} \phi_{rG}$$

$$\text{s. t. } \begin{cases} 2.4\lambda_A + 7\lambda_B + 4\lambda_C + \lambda_D + \lambda_E + 6\lambda_F + 5.6\lambda_G \leq 5.6 & (\text{input x constraint}) \\ 4\lambda_A + 12\lambda_B + 10\lambda_C + 3\lambda_D + 2\lambda_E + 12\lambda_F + 9\lambda_G \geq 9\phi & (\text{output y constraint}) \\ \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G = 1 & (\text{if VRS}), \\ \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G \geq 0, & j = 1, \dots, 7 \end{cases}$$

The input oriented LPP can be similarly formulated. The output-oriented scores for DMU G are

$$\phi_F=2.24, \lambda_A=0.14, \lambda_C=0.86 \text{ with } \lambda_{j=A-I, j \neq A, C}=0,$$

Using similar definitions, the DEA input (Ei) and output (Eo) orientations of all DMUs are computed and presented in Table 2. This was programmed in R using the *Benchmarking* and *dear* packages. The R codes are shown in appendix xx. In the table, we also show the slacks and targets or benchmarks under input-oriented efficiency measure. We find that DMU G is 64.3% efficient under input orientation and 128.9% under output orientation. The output-oriented DEA score means that DMU G is inefficient and hence, to be efficient, it should be able to raise all the outputs radially to 128.9% of the current level ($1.289 \times 9 = 11.6$) denoted as point J in the graph, without the need for extra inputs, whilst remaining in the production possibility set. Its input efficiency score of 64.3% means that, to be efficient, it should cut its input of 5.6 units to a target value of 3.6 (0.643×5.6) as in point H in Figure 5.

DMU G is dominated by the frontier and hence, is inefficient. Two DMUs under input orientation identified are inefficient A and G). It is also observed that the efficiency scores can change depending on orientation used. One drawback of this DEA radial approach cannot simultaneously contract inputs and augment outputs.

Table 5 DEA Input and output oriented technical efficiency results for time period 1

DMU	X	y	Ei	Eo
A	2.4	4	0.5	1.567
B	7	12	0.8571	1
C	4	10	1	1
D	1	3	1	1
E	1	2	1	1.5
F	6	12	1	1
G	5.6	9	0.6378	1.289

Table 6 Output-oriented lambda values across DMUs

DMU	L1	L2	L3	L4	L5	L6	L7
A	0	0	0.466667	0.533333	0	0	0
B	0	0	0	0	0	1	0
C	0	0	1	0	0	0	0
D	0	0	0	1	0	0	0
E	0	0	0	1	0	0	0
F	0	0	0	0	0	1	0
G	0	0	0.2	0	0	0.8	0

Table 7 Input-oriented lambda values across DMUs

DMU	L1	L2	L3	L4	L5	L6	L7
A	0	0	0.142857	0.857143	0	0	0
B	0	0	0	0	0	1	0
C	0	0	1	0	0	0	0
D	0	0	0	1	0	0	0
E	0	0	0	1	0	0	0
F	0	0	0	0	0	1	0
G	0	0	0.857143	0.142857	0	0	0

The capacity of the MEA approach to evaluate efficiency under non-orientation or both input and output orientation simultaneously, among other benefits, makes it beneficial. To illustrate the MEA, we use the original data set in Table 4 and the associated Figure 5.

MEA for DMU G is executed in two stages. The ideal reference point (IRP) is first identified by drawing a horizontal line (signifying input radial contraction potential) in a DEA fashion and a vertical line (signifying output radial augmentation potential) from the G coordinates until they hit the boundary of the convex production possibility set. Merging the improvement potentials in all of the dimensions (GH, GJ and HG* and JG*), as in this graph, provides (typically unobtainable, optimal possible point) the IRP, F'' (3.6, 11.6). Points H and J, where the DEA input contraction and output expansion intersect the frontier are the *DEA input and output benchmarks* or targets respectively. The intersection creates a diagonal distance G* which is an MEA non-radial line. Because the IRP, G*, is not within the PPS, the *MEA benchmark* for G in the direction of the ideal point is G' (4.5,10.5).

Consequently, unlike DEA, MEA considers the shape of the part of the frontier that dominates the point and selects a benchmark that is proportional to the improvement potentials. In effect the MEA benchmark therefore has some desirable properties espoused by Bogetoft and Hougaard (1999) and Asmild et al. (2003) as explained earlier. For instance, whilst DEA selects weakly efficient benchmarks (E or B), MEA selects strongly efficient benchmarks (F or D).

Given the MEA equations (3) – (5), the MEA input-and-output specific efficiency scores for unit G are computed using the linear programs below while assuming a variable returns to scale technology.



Stage 1: Find the ideal reference point for DMU F (7,5)

$$d_{iG}^* = \min_{\lambda_{j=A-I}, d_{iF}} d_{iG}$$

$$\text{s. t. } \begin{cases} 2.4\lambda_A + 7\lambda_B + 4\lambda_C + \lambda_D + \lambda_E + 6\lambda_F + 5.6\lambda_G \leq d_{1G} : & i = 1 \\ 4\lambda_A + 12\lambda_B + 10\lambda_C + 3\lambda_D + 2\lambda_E + 12\lambda_F + 9\lambda_G \geq 9 : & r = 1 \\ \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G = 1 \quad (\text{if VRS}), \\ \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G \geq 0, \quad j = 1, \dots, 7 \end{cases}$$

$$\delta_{rF}^* = \min_{\lambda_{j=A-I}, d_{rF}} \delta_{rF}$$

$$\text{s. t. } \begin{cases} 2.4\lambda_A + 7\lambda_B + 4\lambda_C + \lambda_D + \lambda_E + 6\lambda_F + 5.6\lambda_G \leq 5.6 : & i = 1 \\ 4\lambda_A + 12\lambda_B + 10\lambda_C + 3\lambda_D + 2\lambda_E + 12\lambda_F + 9\lambda_G \geq \delta_{rG} : & r = 1 \\ \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G = 1 \quad (\text{if VRS}), \\ \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G \geq 0, \quad j = 1, \dots, 7 \end{cases}$$

If there had been an undesirable output in this hypothetical data, we would have formulated the LPP for that. The LPP can be solved using excel solver or any LP software. Using R software with the Benchmarking package. The commands for the estimations are shown below:

```
k=read.delim("clipboard")
require(Benchmarking)
y=cbind(k$y[1:7])#column bind y vectors into matrix y
x=cbind(k$x[1:7])#column bind x vectors into matrix x
de=dea(x,y,RTS="vrs", ORIENTATION = "graph",XREF = x,YREF = y)#run TE only if the DEA
is for input output is when you use the hyperbolic graph orientation
me=mea(x,y,RTS="vrs", ORIENTATION = "in-out",XREF = x,YREF = y)#run non-oriented
MEA
dsl=slack(x,y,de) #run slack analysis under DEA
msl=slack(x,y,me) #run slack analysis under MEA
round(data.frame("d"=de$eff,"m"=me$eff,de$lambda,me$lambda,peers(de),peers(me),dsl$sx,dsl
$sy,msl$sx,msl$sy,me$direct),2)#create data frame of results
```

From the R output, the vertical and horizontal distance (along the diagonal path) toward the IRP is given by the `me$direct` command. This provides the excess inputs (output shortfalls) to subtract from (add to) actual inputs (outputs) to obtain the IRP. The IRP $(d_{io}^*, \delta_{r0}^*, \varphi_{io}^*)$ for DMU F is thus computed as follows:

IRPx = actual input - excess input (input-orientation)

IRPy = actual output + output shortfall (output-orientation). In effect, we have:

$$d_{1G}^* = 5.6 - 2 = 3.6$$

$$\delta_{1G}^* = 9 + 2.6 = 11.6$$

Consequently, the ideal reference point for DMU G is $(d_{1G}^*, \delta_{1G}^*) = (3.6, 11.6)$. This implies that the largest possible contraction (augmentation) of input x (output y) is 3.6 (11.2) units. The weights of the peers of DMU G are 0.86 from peer unit C and 0.14 from peer unit A

The IRP is computed in the R programming language as;
 IRPx=(x-ecx) # ecx= excess input from `me$direct`, x= input
 IRPy=(y+ecy) # ecy= output shortfall, from `me$direct`, y= input

Still, the IRP is ideal and hence infeasible as it lies outside the boundary of the PPS.

The step two in the MEA involves a second LP to move the IRP back to the frontier to determine the true MEA benchmark or the potential improvement point (PIP). This obviously explains the reason behind the different benchmark points selected by DEA as against MEA for an evaluated unit. The second LPP maximizes β_0^* , the proportion by which the inputs are contracted and/or the outputs are augmented to make the IRP possible and establish the MEA benchmark or the PIP.

$$\beta_{io}^* = \min_{\lambda_{j=A-1}, \beta_{TG}} (\beta_{io})$$

$$\text{s. t.} \begin{cases} 2.4\lambda_A + 7\lambda_B + 4\lambda_C + \lambda_D + \lambda_E + 6\lambda_F + 5.6\lambda_G \leq 5.6 - \beta_{io}(5.6 - 3.6) \\ 4\lambda_A + 12\lambda_B + 10\lambda_C + 3\lambda_D + 2\lambda_E + 12\lambda_F + 9\lambda_G \geq 9, \\ \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G = 1 \quad (\text{if VRS}) \\ \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G \geq 0, \quad j = 1, \dots, 7 \end{cases}$$

$$\beta_{ro}^* = \max_{\lambda_{j=A-1}, \beta_{rG}} (\beta_{ro})$$

$$\text{s. t.} \begin{cases} 2.4\lambda_A + 7\lambda_B + 4\lambda_C + \lambda_D + \lambda_E + 6\lambda_F + 5.6\lambda_G \leq 5.6 \\ 4\lambda_A + 12\lambda_B + 10\lambda_C + 3\lambda_D + 2\lambda_E + 12\lambda_F + 9\lambda_G \geq 9 + \beta(11.6 - 9) \\ \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G + \lambda_H + \lambda_I = 1 \quad (\text{if VRS}) \\ \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G + \lambda_H + \lambda_I \geq 0, \quad j = 1, \dots, 7 \end{cases}$$

The optimal solution to the above LPP are the lambdas and beta values (λ^*, β_0^*) . In R software, the beta values, β_0^* , are the MEA input and output-specific inefficiencies. Hence, the MEA benchmark or PIP is given by,

$$PIP_x = x_{io} - \beta_{io}(x_{io} - d_{io}^*)$$

$$PIP_y = y_{ro} + \beta_{ro}(\delta_{ro}^* - y_{ro})$$

where d_{io}^* and δ_{ro}^* are the input and output coordinates of IRP respectively. For DMU G, the PIP is given by,

$$PIP_{x,G} = 5.6 - 0.56(5.6 - 3.6) = 4.5$$

$$PIP_{y,G} = 9 + 0.56(11.6 - 9) = 10.5$$

The MEA benchmark or potential improvement point for DMU G can easily be observed at point G'(4.5, 10.5) on the DEA frontier which is feasible unlike the IRP. This means that DMU G will be MEA efficient when it uses 4.5 units of input x to produce 10.5 units of output y.

The PIP is computed in the R programming language as;
 PIPx=x-me\$eff*(x-IRPx) #MEA benchmark for x for all DMUs
 PIPx=x-me\$eff*ecx #same answer for PIP
 PIPy=y+me\$eff*(IRPy-y) #MEA benchmark for y for all DMUs
 PIPy=y+me\$eff*ecy #same answer for PIP

The relative input-specific and output-specific (x, y) MEA efficiency scores for DMU G are computed as:

$$e_{iG} = \frac{x_{io} - \beta_{i0}^*(x_{io} - d_{i0}^*)}{x_{io}} = \frac{PIP_x}{x_{io}} = \frac{4.5}{5.6} = 0.8036$$

$$e_{rG} = \frac{y_{ro}}{y_{ro} + \beta_{r0}^*(\delta_{r0}^* - y_{ro})} = \frac{y_{ro}}{PIP_y} = \frac{9}{10.5} = 0.8571$$

DMU G is simultaneously 80.4% efficient in the utilization of the input variable and 85.7% efficient in output augmentation whilst accounting for all slacks. These are termed the patterns of efficiency as stated by Asmild and Matthews, (2012). One minus these efficiency scores should deliver inefficiency scores.

Finally, the integrated, comprehensive or overall, MEA non-oriented efficiency score for unit F is given as

$$\theta_0 = \frac{1 - \frac{1}{m} \sum_{i=1}^m \frac{\beta_{i0}^*(x_{i0} - d_{i0}^*)}{x_{i0}}}{1 + \frac{1}{s_1 + s_2} \left[\sum_{r=1}^{s_1} \frac{\beta_{r0}^*(\delta_{r0}^* - y_{r0})}{y_{r0}} + \sum_{k=1}^{s_2} \frac{\beta_{k0}^*(c_{k0} - \varphi_{k0}^*)}{c_{k0}} \right]}$$

$$\theta_G = \frac{1 - \frac{1}{1} (0.1964 + 0)}{1 + \frac{1}{1+0} [0.1429 + 0]}$$

$$\theta_G = 0.731$$

This implies that much of the low 73% efficiency of DMU G emanates from the low efficiency on the output than on the input side. Tables 8,9,10 and 11 below show the complete results of ideal reference points, potential improvement points, MEA integrated efficiency scores, MEA input- and-output-specific efficiency and inefficiency scores for all the hypothetical DMUs.

It is observed that the outputs of some of the DMUs exhibit LP infeasibilities because the ideal reference point projection and MEA benchmark of the observed variable is located outside the boundary of the PPS. This observation applies to both periods 1 and 2 from the data set in tables 10 and 11 below.

Table 8 Results of hypothetical example (year 1)

DMU	Input (x1)	Output(y1)	IRPx1	IRPy1	PIPx1	PIPy1	MEA Variable Specific inefficiency (x1)	MEA Variable Specific inefficiency (y1)	MEA Variable Specific efficiency (x1)	MEA Variable Specific efficiency (y1)	Over Efficiency Score OE1
A	2.40	4.00	1.43	6.27	1.91	5.13	0.20	3.53	0.80	0.78	0.62
B	7.00	12.00	6.00	12.00	6.00	12.00	0.14	Inf	0.86	1.00	0.86
C	4.00	10.00	4.00	10.00	4.00	10.00	0.00	Inf	1.00	1.00	1.00
D	1.00	3.00	1.00	3.00	1.00	3.00	0.00	Inf	1.00	1.00	1.00
E	1.00	2.00	1.00	3.00	1.00	3.00	0.00	2.00	1.00	0.67	0.67
F	6.00	12.00	6.00	12.00	6.00	12.00	0.00	Inf	1.00	1.00	1.00
G	5.60	9.00	3.57	11.60	4.46	10.46	0.20	6.16	0.80	0.86	0.69

Table 9 Results of hypothetical example (year 2)

DMU	Input (x2)	Output(y2)	IRPx2	IRPy2	PIPx2	PIPy2	MEA Variable Specific inefficiency (x2)	MEA Variable Specific inefficiency (y2)	MEA Variable Specific efficiency (x2)	MEA Variable Specific efficiency (y2)	Over Efficiency Score OE2
A	4.80	13.00	4.27	15.90	4.54	14.45	0.05	8.97	0.95	0.90	0.85
B	6.20	17.00	5.00	17.00	5.00	17.00	0.19	Inf	0.81	1.00	0.81
C	5.00	17.00	5.00	17.00	5.00	17.00	0.00	Inf	1.00	1.00	1.00
D	3.00	6.00	3.00	6.00	3.00	6.00	0.00	Inf	1.00	1.00	1.00
E	5.00	10.00	3.73	17.00	4.36	13.50	0.13	2.86	0.87	0.74	0.65
F	7.40	15.00	4.64	17.00	4.96	16.77	0.33	8.49	0.67	0.89	0.60
G	5.60	9.00	3.55	17.00	4.40	13.68	0.21	1.92	0.79	0.66	0.52

Table 10 Results of hypothetical example year 1 relative to year 2 frontier

DMU	Input (x1)	Output(y1)	IRPx12	IRPy12	PIPx12	PIPy12	MEA Variable Specific inefficiency (x12)	MEA Variable Specific inefficiency (y12)	MEA Variable Specific efficiency (x12)	MEA Variable Specific efficiency (y12)	Over Efficiency Score OE12
A	2.40	4.00	3.00	-1E+30	NA	NA	NA	NA	NA	NA	NA
B	7.00	12.00	4.09	17.00	4.59	17.00	0.32	3.15	0.66	0.71	0.52
C	4.00	10.00	3.73	11.50	4.36	14.25	0.03	13.33	1.09	0.70	0.90
D	1.00	3.00	3.00	-1E+30	NA	NA	NA	NA	NA	NA	NA
E	1.00	2.00	3.00	-1E+30	NA	NA	NA	NA	NA	NA	NA
F	6.00	12.00	4.09	17.00	5.16	16.35	0.22	3.54	0.86	0.73	0.61
G	5.60	9.00	3.55	17.00	4.40	13.68	0.21	1.92	0.79	0.66	0.52

Table 11 Results of hypothetical example year 2 relative to year 1 frontier

DMU	Input (x2)	Output(y2)	IRPx21	IRPy21	PIPx21	PIPy21	MEA Variable Specific inefficiency (x21)	MEA Variable Specific inefficiency (y21)	MEA Variable Specific efficiency (x21)	MEA Variable Specific efficiency (y21)	Over Efficiency Score OE21
A	4.8	13	-1E+30	10.8	-Inf	Inf	Inf	0	-Inf	0	NaN
B	6.2	17	-1E+30	12	-Inf	Inf	Inf	0	-Inf	0	NaN
C	5	17	-1E+30	11	-Inf	Inf	Inf	0	-Inf	0	NaN
D	3	6	2E+00	7.67	2.64	6.83	0.12	7.2	0.88	0.88	0.77
E	5	10	4E+00	11	4.5	10.5	0.1	20.00	0.90	0.95	0.86
F	7.4	15	-1E+30	12	NA	NA	NA	NA	NA	NA	NA
G	5.6	9	4E+00	11.6	4.46	10.46	0.2	6.16	0.80	0.86	0.69

Table 12 Results of hypothetical example year 1 relative to biennial frontier

DMU	Input (x1)	Output(y1)	BIRPx12	BIRPy12	BIPx12	BIPy12	MEA Variable Specific inefficiency (x1b)	MEA Variable Specific inefficiency (y1b)	MEA Variable Specific efficiency (x1b)	MEA Variable Specific efficiency (y1b)	Over Efficiency Score OE1b
A	2.4	4	1.29	7.90	1.84	5.95	0.23	2.05	0.77	0.67	0.52
B	7	12	3.57	17.00	4.58	15.53	0.35	3.40	0.65	0.77	0.51
C	4	10	3.00	13.50	3.50	11.75	0.13	5.71	0.88	0.85	0.74
D	1	3	1.00	3.00	1.00	3.00	0.00	Inf	1.00	1.00	1.00
E	1	2	1.00	3.00	1.00	3.00	0.00	2.00	1.00	0.67	0.67
F	6	12	3.57	17.00	4.47	15.15	0.25	3.81	0.75	0.79	0.59
G	5.6	9	2.71	17.00	3.99	13.46	0.29	2.02	0.71	0.67	0.48

Table 13 Results of hypothetical example year 2 relative to biennial frontier

DMU	Input (x2)	Output (y2)	BIRPx2b	BIRPy2b	BIPx2b	BIPy2b	MEA Variable Specific inefficiency (x2b)	MEA Variable Specific inefficiency (y2b)	MEA Variable Specific efficiency (x2b)	MEA Variable Specific efficiency (y2b)	Over Efficiency Score OE2b
A	4.8	13	3.86	16.30	3.04	14.65	0.10	7.88	0.38	0.89	0.80
B	6.2	17	5.00	17.00	3.57	17.00	0.19	Inf	0.74	1.00	0.81
C	5	17	5.00	17.00	5.00	17.00	0.00	Inf	0.70	1.00	1.00
D	3	6	1.86	10.00	2.00	8.00	0.19	3.00	0.33	0.75	0.61
E	5	10	3.00	17.00	3.00	13.50	0.20	2.86	0.20	0.74	0.59
F	7.4	15	4.43	17.00	4.19	16.68	0.34	8.94	0.60	0.90	0.60
G	5.6	9	2.71	17.00	3.99	13.46	0.29	2.02	0.71	0.67	0.48

From tables 12 and 13 it is observed that the variable specific cross period infeasibilities are resolved by introducing a biennial frontier as the base reference.

Moreover, since the efficiency gains in this study result from a combination of reducing inputs consumed, increasing the production of desirable outputs, and mitigating the emission of unwanted outputs, we strongly believe that utilizing the MEA methodology is a more suitable approach for achieving efficiency improvements. MEA, with its ability to analyse multiple directions and consider different variables independently, is well-suited for capturing and evaluating the multidimensional nature of efficiency enhancements in such cases.

In R, the special values NaN, -Inf, Inf, and NA are used to represent many concepts:

NaN is an abbreviation meaning "Not a Number." It is a value that reflects the outcomes of undefined or unrepresentable mathematical processes. NaN is commonly encountered when dividing zero by zero or computing the square root of a negative number. Any arithmetic procedure that involves NaN will yield NaN.

-Inf: -Inf stands for negative infinity. It denotes a value that is less than any other numeric value.

It occurs when dividing a negative integer by zero or computing the logarithm of zero. Any mathematical operation that involves -Inf will yield -Inf.

Inf: Inf stands for positive infinity. It denotes a value that is greater than any other numeric value. It occurs when dividing a positive number by zero or computing the logarithm of a positive number greater than or equal to Inf. Any mathematical procedure that involves Inf will yield Inf.

NA: NA is an abbreviation for "Not Available" or "Missing Value." It symbolizes data that is missing or unavailable. When a value is not present or cannot be ascertained, NA is used. It is frequently used to identify missing values in data frames or vectors. Because NA values propagate via calculations, every operation that involves NA will almost always result in NA.

These unusual values are critical in dealing with missing data, conducting calculations with undefined or infinite outcomes, and communicating extraordinary situations in R computations. It is critical to manage these numbers correctly based on the unique needs of your analysis or calculations.

4.9 Malmquist productivity change index (MPI)

Shephard and Malmquist (1953) were the first to propose the productivity index function, although they did so without allowing for changes in the production function over time.

(Caves, Christensen et al. 1982) then proposed the first kind of the Malmquist index, based on the prior research of (Sten 1953) and Shephard (1953) and demonstrated how a firm's production may be assessed over time using the distance function. Nishimizu and Page (1982) incorporated the initial MPI type into the SFA parametric framework (Nishimizu and Page 1982). Malmquist productivity change indices, as established by (Caves, Christensen et al. 1982) have been the most often used indices to measure productivity changes.

In their work, Färe, Grosskopf, et al. (1992) recommended the calculation of Malmquist indices using DEA efficiency scores. They proposed a two-factor decomposition of the constant returns to scale Malmquist index, separating it into efficiency change and technical change components.

This decomposition approach has gained significant acceptance in the field, although it is not universally embraced by all researchers.(Färe, Grosskopf et al. 1992).

The subsequent three-factor decomposition, which breaks down the efficiency change component into a scale change and a pure technical efficiency change component, has faced significant criticism due to its internal inconsistencies and has been questioned (Ray and Desli 1997) though this decomposition has in turn been criticised in (Balk 2001)for failing to distinguish between scale efficiency change and the input mix or output mix effects. Fare (Färe, Grosskopf et al. 1994) extended the MPI by decomposing it into pure efficiency change, scale change and technological change under both CRS&VRS.

Based on the CRS assumption, the Malmquist index can handle many inputs and outputs with few assumptions and can be split into efficiency change (catching up) and technological change (frontier shift) components. The difference in productivity attributed to managerial abilities or experience is measured by the efficiency change component. That is, changes in DMU performance caused by managers' decisions on input-output mix. The component of technical change assesses dynamic productivity as a result of external influences or frontier shifts such as process and product developments, laws, and trend changes. These are industry-wide reforms that benefit some enterprises while hurting others.

In the decomposition in the scale change effect is obtained by considering both constant returns to scale and variable returns to scale technologies (Ray and Desli 1997).

Utilizing mixed-period efficiency scores in estimation against the latter can result in linear programming infeasibilities, which is a primary argument against employing this decomposition method. The similar infeasibilities problem exists, though it is not explicitly addressed, in (Balk 2001) where mixed-period efficiency scores are calculated relative to a variable return to scale

technology and used to determine input or output mix effects. In any situation, the data set will obviously reveal how much information is lost through infeasibilities.

Assuming the production technology is defined at each period t , $t = 1, \dots, T$, to be the set of all possible input and output vectors. If $x^t \in R_+^N$ denotes an input vector at period t and $y^t \in R_+^M$ an output vector in period t , then the technology is the set N^t , where can produce.

The value of $\emptyset$ attained is the technical efficiency score for the DMU. A DMU is said to be technically efficient if $\emptyset=1$ meaning that the current output level cannot be increased radially, meaning that DMU is on the frontier on the other hand a DMU is technically inefficient if $\emptyset>1$.

The CRS assumption is appropriate when all DMUs are operating at an optimal scale (Charnes et al., 1978a). However, some factors such as policies, regulations, financial constraints, Environmental and Societal causes may cause a firm not to operate at an optimal (Coelli, Rao et al. 2005). A VRS model can then be formulated by adding the constraint.

The Malmquist Productivity Index (MPI) is a measure used to evaluate changes in productivity over time. It requires defining efficiency scores in relation to two different time periods. The efficiency of a DMU in time $t+1$ (x_j^{t+1}, y_j^{t+1}) with respect to frontier in time t . The $\emptyset_o^t(x_j^{t+1}, y_j^{t+1})$ result may be greater than, equal to, or less than 1. This is because (x_j^{t+1}, y_j^{t+1}) does not belong to the technology t . It is also necessary to define the efficiency score of the DMU in time $t+1$ (x_j^{t+1}, y_j^{t+1}) with respect to frontier $\emptyset_o^{t+1}(x_j^{t+1}, y_j^{t+1})$

Caves et al. (1982) defined a period t based Malmquist as:

$$MPI_o^t = \frac{\emptyset_o^t(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})}{\emptyset_o^t(x_j^t, y_j^t, y_j^{bt})}$$

Alternatively, one could define the MPI for time $t+1$ as:

$$MPI_o^{t+1} = \frac{\phi_o^{t+1}(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})}{\phi_o^{t+1}(x_j^t, y_j^t, y_j^{bt})}$$

The above equations represent own-period output-oriented efficiency scores which may be greater than or equal to 1

To eliminate the need for selecting an arbitrary benchmark or technology, the MPI (Malmquist Productivity Index) can be defined as the geometric mean of the two indices. This approach ensures a balanced representation of productivity changes over time. (Färe, Grosskopf et al. 1992). This formula necessitates the computation of four separate efficiency scores, which are as follows:

$$MPI_o^{t+1} = \left[\frac{\phi_o^t(x_j^t, y_j^t)}{\phi_o^t(x_j^{t+1}, y_j^{t+1})} \times \frac{\phi_o^{t+1}(x_j^t, y_j^t)}{\phi_o^{t+1}(x_j^{t+1}, y_j^{t+1})} \right]^{1/2}$$

The above represent cross-period efficiency scores which can be less than, greater than, or equal to 1. The cross-period super-efficiency occurs when the efficiency score is less than one.

It should be noted that only cross-period efficiency scores can be super-efficient. Färe et al. (1992) decomposed the output oriented MPI into efficiency change and technological change:

$$MPI_o^{t+1} = \underbrace{\frac{\phi_o^t(x_j^t, y_j^t)}{\phi_o^{t+1}(x_j^{t+1}, y_j^{t+1})}}_{\text{Efficiency change}} \left[\underbrace{\frac{\phi_o^t(x_j^t, y_j^t)}{\phi_o^t(x_j^{t+1}, y_j^{t+1})} \times \frac{\phi_o^{t+1}(x_j^t, y_j^t)}{\phi_o^{t+1}(x_j^{t+1}, y_j^{t+1})}}_{\text{[Technological change]}} \right]^{1/2}$$

The component of efficiency change quantifies the productivity change that can be attributed to managerial skills or experience. On the other hand, the component of technological change measures the dynamic productivity change resulting from external factors or shifts in the frontier, such as process or product innovations, regulations, and changes in trends. (Färe, Grosskopf et al. 1994).

Four-factor decomposition of the MPI

Following Wheelock and Wilson (1999) four factor decompositions

$$MPI(x_j^t, x_j^{t+1}, y_j^t, y_j^{t+1}, y_j^{bt}, y_j^{bt+1}) = PEC \times PTC \times SEC \times STC$$

$$\begin{aligned}
 &MPI(x_j^t, x_j^{t+1}, y_j^t, y_j^{t+1}, y_j^{bt}, y_j^{bt+1}) \\
 &= \underbrace{\left[\frac{\phi_v^t(x_j^t, y_j^t, y_j^{bt})}{\phi_v^{t+1}(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})} \right]}_{PEC} \times \underbrace{\left[\frac{\phi_v^{t+1}(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})}{\phi_v^t(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})} \times \frac{\phi_v^{t+1}(x_j^t, y_j^t, y_j^{bt})}{\phi_v^t(x_j^t, y_j^t, y_j^{bt})} \right]}_{PTC} \\
 &\times \underbrace{\left[\frac{\phi_c^{t+1}(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})}{\phi_v^{t+1}(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})} \times \frac{\phi_v^t(x_j^t, y_j^t, y_j^{bt})}{\phi_c^t(x_j^t, y_j^t, y_j^{bt})} \right]}_{SEC} \\
 &\times \underbrace{\left[\frac{\phi_c^t(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})/\phi_v^t(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})}{\phi_c^{t+1}(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})/\phi_v^{t+1}(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})} \right.} \\
 &\quad \left. \times \frac{\phi_c^t(x_j^t, y_j^t, y_j^{bt})/\phi_v^t(x_j^t, y_j^t, y_j^{bt})}{\phi_c^{t+1}(x_j^t, y_j^t, y_j^{bt})/\phi_v^{t+1}(x_j^t, y_j^t, y_j^{bt})} \right]}_{STC}
 \end{aligned}$$

Table 14 Variable Specific Malmquist Productivity Scores (Hypothetical Data) showing infeasibilities.

DMU	Input Variable (x)			Output Variable (y)		
	MPI _x	EC _x	TC _x	MPI _y	EC _y	TC _y
A	NA	0.84	NA	NA	0.87	NA
B	NaN	1.06	NaN	0	1	0
C	NaN	1.00	NaN	0	1	0
D	NA	1.00	NA	NA	1	NA
E	NA	1.15	NA	NA	0.9	NA
F	NA	1.49	NA	NA	1.12	NA
G	1.01	1.01	1	1.31	1.31	1

Table 14 above illustrates the MPI and its numerous components using the hypothetical data provided before. The MPI scores exhibits some infeasibilities due to infeasibilities in the cross-period efficiency scores, which are used to calculate the Technological Change.

4.10 Biennial Malmquist Index (BMI)

Despite its numerous advantages, the MPI (Malmquist Productivity Index) suffers from a significant drawback. When cross-period efficiencies are calculated assuming Variable Returns to Scale (VRS), there is a possibility of encountering infeasibilities in the linear programming (LP) solution. This occurs when the observed input-output combination exceeds the frontier of the previous period. Consequently, this issue also impacts the assessment of the scale change component proposed by Ray and Desli (1997b), which necessitates both Constant Returns to Scale (CRS) and VRS technologies.

The biennial Malmquist index is a methodology that addresses the issue of infeasibilities in the Malmquist index calculation. It not only evaluates both technical regress and progress but also preserves previously computed productivity changes when new time periods are included in the dataset. This approach ensures a more robust and consistent assessment of productivity changes over time. The biennial technology corresponds to a DEA window analysis with a window width of two (Pastor and Lovell 2007). In the case of panel data comprising of two time periods, the biennial technology is identical to a pooled or meta-frontier technology also used to construct a global Malmquist index (Pastor and Lovell 2007). In the general scenario of having more than two time periods, a set of overlapping biennial technologies is established for each pairwise comparison between adjacent time periods. This means that for each combination of consecutive periods, a separate biennial technology is defined, allowing for a more detailed analysis of productivity changes over the entire time span.

The biennial Malmquist index is defined precisely for the adjacent time periods t and $t+1$ since two adjacent time periods are enough to establish the necessary properties of avoiding infeasibility, allowing technical regress, and maintaining previous productivity calculations. Although having two time periods is sufficient to achieve the desirable properties, it does not prevent the construction of a triennial Malmquist index. However, it's important to note that the biennial Malmquist index is not transitive due to its construction from a series of overlapping two-period technologies, which can vary. As a result, when comparing productivity changes over longer time spans, the biennial Malmquist index may not exhibit transitive properties. The biennial Malmquist approach addresses the issue of LP infeasibilities by generating a distinct biennial Variable Returns to Scale (VRS) frontier that encompasses all observations from two consecutive time periods. This methodology effectively avoids the problems associated with infeasible linear programming solutions. By using this approach, the biennial Malmquist index provides a comprehensive framework for assessing productivity changes over time without encountering infeasibility concerns. The biennial CRS Malmquist index can be calculated with reference to a biennial technology set that combines both time periods $(x_j^t, y_j^t, x_j^{t+1}, y_j^{t+1}, y_j^{bt}, y_j^{bt+1}) \in \mu^B$ as:

$$BMI_c^B(x_j^t, y_j^t, x_j^{t+1}, y_j^{t+1}) = \frac{\phi_c^B(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})}{\phi_c^B(x_j^t, y_j^t, y_j^{bt})}$$

It is important to mention that the biennial frontier, which captures both time periods similar to the traditional Malmquist index, eliminates the need for calculating the geometric mean. This is because the biennial frontier acts as the reference frontier for both periods simultaneously. Therefore, the biennial Malmquist index inherently incorporates the reference frontier for each pair of consecutive time periods without requiring additional calculations (Pastor, Asmild et al.

2011). The biennial technology in CRS can be reformulated in reference to a VRS frontier by changing the subscript c to v. The biennial VRS Malmquist index is defined by:

$$BMI_v^B(x_j^t, y_j^t, x_j^{t+1}, y_j^{t+1}) = \frac{\phi_v^B(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})}{\phi_v^B(x_j^t, y_j^t, y_j^{bt})}$$

In line with the decomposition proposed by Ray and Desli (1997a) for the adjacent Malmquist Productivity Index (MPI), the Biennial Malmquist Index (BMI) can be further decomposed into three components: Biennial Pure Efficiency Change (BPEC), Biennial Pure Technological Change (BPTC), and Biennial Scale Efficiency Change (BSEC). These components help analyze and understand the specific sources of productivity changes between consecutive biennial periods. The BPEC is defined similarly to the pure efficiency change component under the adjacent MPI as:

BMI scores above 1 indicate improvements in productivity, suggesting that productivity has increased between the biennial periods. On the other hand, BMI scores below 1 signify a deterioration in productivity, indicating a decline in productivity over the given time span. BMI scores equal to 1 represent productivity stagnation, suggesting that there has been no significant change in productivity between the two biennial periods.

4.11 Biennial MEA - Malmquist Index (BMEAM)

This study seeks to develop a BMEAM for which we use the Biennial MEA Malmquist index in the world of multi-directional efficiency analysis. This combines the ability for a biennial Malmquist to solve infeasibilities and handle scale effect together with multi-directional efficiency analysis which estimates and selects benchmarks in such a way that the input reductions or output expansions are proportional to the potential efficiency improvements identified by considering the improvement potential in each input or output separately.

The biennial Malmquist index (BMI) proposed by Pastor et al. (2011) offers three significant advantages. Firstly, it addresses the issue of linear programming (LP) infeasibilities when calculating mixed-period efficiency scores under Variable Returns to Scale (VRS). This ensures a more robust and accurate assessment of efficiency. Secondly, the BMI accounts for technical progress, enabling the analysis of changes in productivity over time due to technological advancements. Lastly, the BMI does not require recomputation when new time periods are added to the dataset, allowing for seamless integration of additional data without disrupting previously obtained results. These advantages make the BMI a valuable tool for productivity analysis and monitoring. Indeed, the Biennial Malmquist Index (BMI) is not the sole approach to addressing infeasibility challenges in dynamic productivity estimations.

Two alternative methodologies, namely the Sequential Malmquist Index proposed by Shestalova (2003) and the Global Malmquist Index presented by Pastor and Lovell (2005), also provide immunity against infeasible observations.

The Sequential Malmquist Index allows for a sequential analysis of productivity changes by incorporating intermediate time periods. This approach helps avoid infeasible observations and provides a comprehensive understanding of productivity dynamics over time.

Similarly, the Global Malmquist Index proposed by Pastor and Lovell (2005) offers a holistic perspective on productivity changes across multiple time periods. By considering a global frontier encompassing all observations, this method avoids infeasible observations and provides a robust estimation of productivity changes over an extended time horizon.

These alternative methodologies, along with the BMI, contribute to a range of solutions available to address infeasibility concerns and enhance the accuracy of dynamic productivity estimations.

Pastor et al. (2011) argue that the Sequential Malmquist Index has limitations in capturing technical regress, which may be present in certain real-life market systems. This index may not effectively account for situations where there is a decline in technical efficiency over time.

On the other hand, the Global Malmquist Index needs to be recomputed when a new time period is added to the dataset that includes all time periods. This can lead to the estimated dynamic productivity between two specific periods being influenced by observations from other periods. This property is referred to as the "relevance of irrelevant alternatives," as the inclusion of additional periods can impact the productivity estimation between the two target periods.

These considerations highlight the trade-offs and potential limitations associated with different approaches to addressing infeasible observations and estimating dynamic productivity. Researchers need to carefully consider the specific characteristics of the market system and the goals of their analysis when selecting an appropriate methodology.

Indeed, one advantage of the Biennial Malmquist Index (BMI) is that it avoids the issue of non-existent technology setbacks that can arise when adopting sequence DEA (Data Envelopment Analysis). This makes the BMI more suitable for capturing changes in productivity in real-life market systems.

However, it is important to note that the BMI, along with the Sequential Malmquist Index and the classical Adjacent Malmquist Index, suffers from a loss of transitivity or circularity. Transitivity refers to the property of maintaining consistent rankings when comparing productivity changes across multiple time periods. This weakness means that the relative rankings of productivity changes between different periods may not always be preserved when comparing them in a cyclical manner.

It is essential for researchers to consider this limitation when interpreting and comparing productivity changes using these indices. Depending on the specific research objectives and context, alternative approaches may need to be explored to address the issue of transitivity in productivity analysis. Another significant drawback of the Biennial Malmquist Index (BMI) is its inability to incorporate non-orientation, non-radial slacks, and undesirable outputs. These are important considerations in productivity analysis that allow for a more comprehensive assessment of efficiency and productivity changes.

Table 16 below illustrates how the infeasibilities of the MPI scores and its numerous components gets resolved when a biennial frontier is introduced and using the same hypothetical data. It can also be observed that the infeasibilities in PTC scores is resolved as the reference frontier now becomes the biennial frontier.

Four-factor Decompositions of the BMEAM

The proposed biennial MEA Malmquist (BMEAM) productivity index along the lines of Wheelock and Wilson (1999) as follows:

$$BMEAM_c^t(x_j^t, x_j^{t+1}, y_j^t, y_j^{t+1}, y_j^{bt}, y_j^{bt+1}) = BPEC \times BPTC \times BSEC \times BSTC$$

$$BMEAM_c^t(x_j^t, x_j^{t+1}, y_j^t, y_j^{t+1}, y_j^{bt}, y_j^{bt+1})$$

$$= \underbrace{\left[\frac{\phi_v^{t+1}(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})}{\phi_v^t(x_j^t, y_j^t, y_j^{bt})} \right]}_{BPEC} \times \underbrace{\left[\frac{\phi_v^B(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})}{\phi_v^{t+1}(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})} \times \frac{\phi_v^t(x_j^t, y_j^t, y_j^{bt})}{\phi_v^B(x_j^t, y_j^t, y_j^{bt})} \right]}_{BPTC}$$

$$\times \left[\frac{\phi_c^{t+1}(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})}{\phi_v^{t+1}(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})} \times \frac{\phi_v^t(x_j^t, y_j^t, y_j^{bt})}{\phi_c^t(x_j^t, y_j^t, y_j^{bt})} \right]$$

BSEC

$$\times \left[\frac{\phi_c^B(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1}) / \phi_v^B(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})}{\phi_c^{t+1}(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1}) / \phi_v^{t+1}(x_j^{t+1}, y_j^{t+1}, y_j^{bt+1})} \times \frac{\phi_c^t(x_j^t, y_j^t, y_j^{bt}) / \phi_v^t(x_j^t, y_j^t, y_j^{bt})}{\phi_c^B(x_j^t, y_j^t, y_j^{bt}) / \phi_v^B(x_j^t, y_j^t, y_j^{bt})} \right]$$

BSTC

Using the hypothetical data shown below for illustration purposes.

Table 15 Hypothetical Data to Illustrate BMEAM

DMU	Year	Input (x)	Output(y)
A1	1	2.4	4
B1	1	7	12
C1	1	4	10
D1	1	1	3
E1	1	1	2
F1	1	6	12
G1	1	5.6	9
A2	2	4.8	13
B2	2	6.2	17
C2	2	5	17
D2	2	3	6
E2	2	5	10
F2	2	7.4	15
G2	2	5.6	9

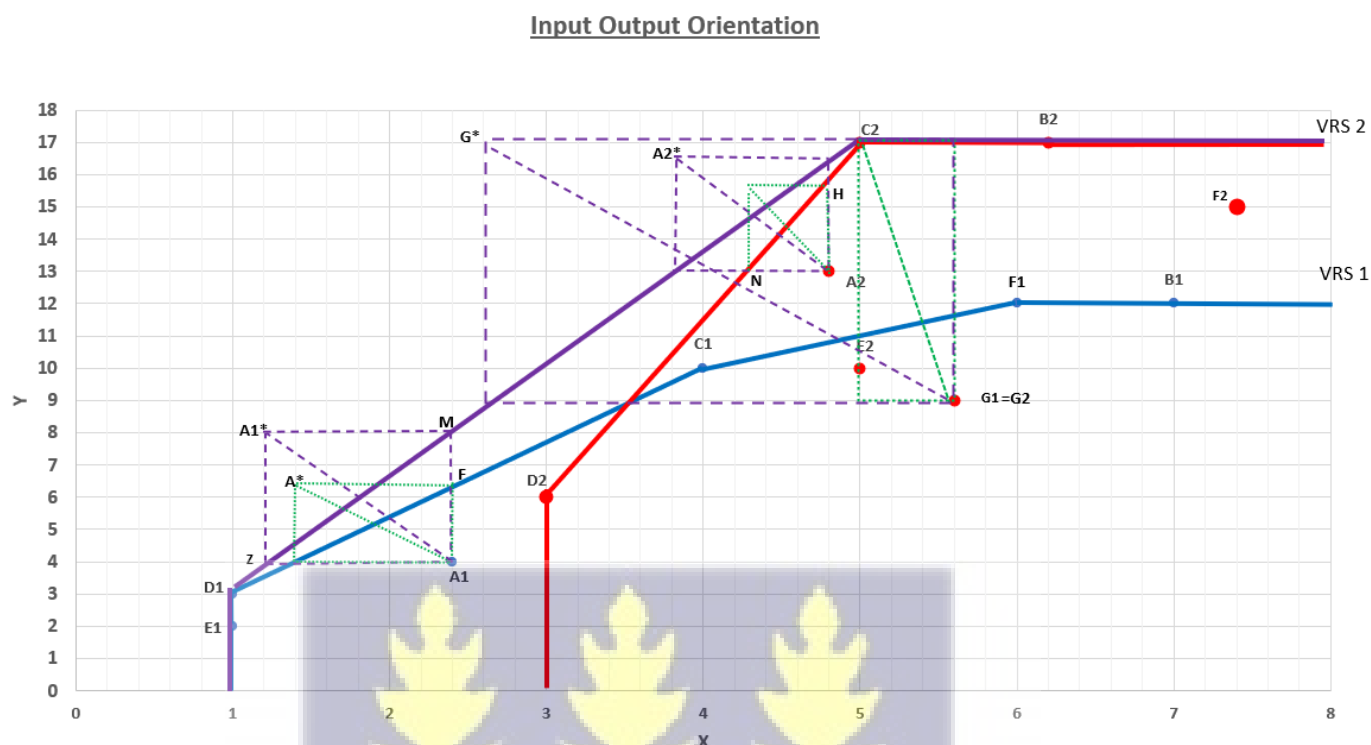


Figure 6 MEA Biennial Malmquist illustration

Using the same hypothetical data, the biennial frontier can be illustrated in the Figure 6 above.

The solution to this problem of VRS infeasibilities is to construct a biennial VRS frontier that envelopes both period one and period two's frontiers, as illustrated. The biennial VRS frontier is the purple convex frontier that envelopes both frontiers of periods one and two.

By this, the cross-period frontier for DMU A in period one can be computed with reference to the biennial frontier which envelopes its own-period frontier too.

Input Productivity Decline ($MPI_{io} > 1$) is computed by $1 - (1/MPI_{io})$

Table 16 Results of Variable Specific Biennial Malmquist Productivity Scores (Hypothetical Data) without infeasibilities.

DMU	Input Variable (x)				Output Variable (y)			
	MPI _x	BMI _x	BPTC _x	BPEC _x	MPI _y	BMI _y	BPTC _y	BPEC _y
A	NA	2.00	2.37	0.84	NA	0.76	0.87	0.87
B	NaN	0.89	0.83	1.06	0	0.77	0.77	1.00
C	NaN	1.25	1.25	1.00	0	0.85	0.85	1.00
D	NA	3.00	3.00	1.00	NA	1.33	1.33	1.00
E	NA	5.00	4.36	1.15	NA	0.90	1.00	0.90
F	NA	1.23	0.83	1.49	NA	0.88	0.79	1.12
G	1.01	1.00	0.99	1.01	1.31	1.00	0.76	1.31

As indicated in table 16 it can be observed that the introduction of the biennial also assists in resolving the infeasibilities of the MPI that were as a result of variable specific cross period efficiencies being infeasible.

Table 17 Results showing Variable Specific four factor decomposition using the hypothetical data for input (x)

DMU	MPI _x	BMI _{cx}	BMI _{vx}	BPEC _x	BPTC _x	BSEC _x	BSTC _x
A	NA	2.00	2.00	0.84	2.37	1.03	0.97
B	NaN	0.89	0.89	1.06	0.83	0.82	1.22
C	NaN	1.25	1.25	1.00	1.25	0.92	1.09
D	NA	3.00	3.00	1.00	3.00	1.26	0.79
E	NA	5.00	5.00	1.15	4.36	0.92	1.09
F	NA	1.23	1.23	1.49	0.83	0.70	1.43
G	1.01	1.00	1.00	1.01	0.99	1.03	0.97

Table 18 Results showing Variable Specific four factor decomposition using the hypothetical data for input (y)

DMU	MPI _y	BMI _{cy}	BMP _{vy}	BPEC _y	BPTC _y	BSEC _y	BSTC _y
A	NA	0.31	0.76	0.87	0.87	0.93	0.44
B	0	0.71	0.77	1.00	0.77	0.81	1.12
C	0	0.59	0.85	1.00	0.85	0.91	0.76
D	NA	0.50	1.33	1.00	1.33	1.35	0.28
E	NA	0.20	0.90	0.90	1.00	1.20	0.19
F	NA	0.80	0.88	1.12	0.79	0.96	0.95

G	1.31	1.00	1.00	1.31	0.76	0.83	1.20
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In addition to the infeasibilities being resolved by the introduction of the biennial frontier, a four factor decomposition based on Wheelock and Wilson four factor decomposition to be undertaken in order to identify factors that drive productivity, as shown in tables 17 and 18.

4.12 Infeasibility of super-efficiency data envelopment analysis models

Data envelopment analysis (DEA) has had one of the fastest increases in interest among management science subfields during the past 20 years (Seiford and Zhu 1999). One common use of DEA is to determine the relative efficiency of a collection of decision-making units (DMUs) that consume several inputs and generate numerous outputs.

A DMU is regarded as efficient if its performance in contrast to another DMU in the sample cannot be improved. According to Banker, Charnes, and Cooper (BCC) (1984), the original DEA model developed by Charnes, Cooper, and Rhodes (CCR) (1978) may be viewed as combining a technical efficiency measure with a scale efficiency measure.

The measurement mentioned is closely related to the concept of returns to scale (RTS) in economics. The Constant Returns to Scale (CRS) and the Variable Returns to Scale (VRS) models, such as the CCR (Charnes, Cooper, and Rhodes) and BCC (Banker, Charnes, and Cooper) models, can be utilized to determine the RTS categories of rising, constant, and declining returns to scale (IRS, CRS, and DRS) for each Decision-Making Unit (DMU).

By applying these models, it is possible to assess the scale efficiency of DMUs and classify them into categories based on their RTS characteristics. This analysis helps identify whether the production process exhibits increasing, constant, or decreasing returns to scale, providing valuable insights into the efficiency and scalability of the entities under evaluation. In recent times, literature has reported on super-efficiency DEA models, a modification on the basic DEA model.

Super-efficiency DEA models, in which the DMU being evaluated is not a part of the reference set, can be highly beneficial in a number of circumstances. The efficiency categories were utilized by Charnes et al. (1992) and Zhu (1996) to test their sensitivity.

In 1993, Andersen and Petersen suggested incorporating them into a plan for identifying the top DMUs. They were used by Fare, Grosskopf, and Lovell (1994) to assess productivity and technical advancement.

Numerous studies have addressed the problem of the VRS super-efficiency model's impossibility. According to Lovell and Rouse (2003), a user-defined scaling factor can be used to create the VRS super-efficiency model. Cook et al. (2009) note that for any DMUs with impractical solutions, Cook et al. (2009) go on to construct a super-efficiency score in terms of both input and output super-efficiencies.

The super-efficiency CCR model might not be feasible, according to Thrall (1996). Until and unless a specific set of zero patterns were available in the data domain, Zhu (1996) showed that the super-efficiency CCR model was impracticable. Several super-efficiency DEA models could also be impractical when the input or output data don't include any zero patterns. The practicality of super-efficiency DEA models has become more important even if they are already employed in many different scenarios.

Chen (2005) stated that one needs use the input- and output-oriented super-efficiency models in order to properly describe super-efficiency when infeasibility arises. According to Chen (2005), to integrate the input and output super-efficiency scores, one must solve the input-oriented and output-oriented Variable Returns to Scale (VRS) super-efficiency models. These models are used to determine the super-efficiency scores by considering both input and output factors.

By solving these models, it becomes possible to evaluate the overall efficiency of Decision-Making Units (DMUs) and obtain a comprehensive assessment of their performance. The input-oriented VRS super-efficiency model focuses on minimizing the input quantities while maintaining the same level of outputs, while the output-oriented VRS super-efficiency model aims to maximize the output quantities while keeping the inputs constant.

Integrating these scores provides a more holistic view of efficiency, considering both input and output factors simultaneously, and allows for a more accurate and comprehensive evaluation of DMUs' performance (Chen and Liang 2011).

An efficient DMU that is being assessed may only show super-efficiency performance in outputs if the input-oriented super-efficiency is not feasible, according to Seiford and Zhu (1999) and Chen (2005).

If the output-oriented super-efficiency is not feasible, a certain efficient DMU under investigation may only exhibit super efficiency performance in inputs. Chen (2005) asserts that super-efficiency may be seen as input savings and output surpluses obtained by a successful DMU.



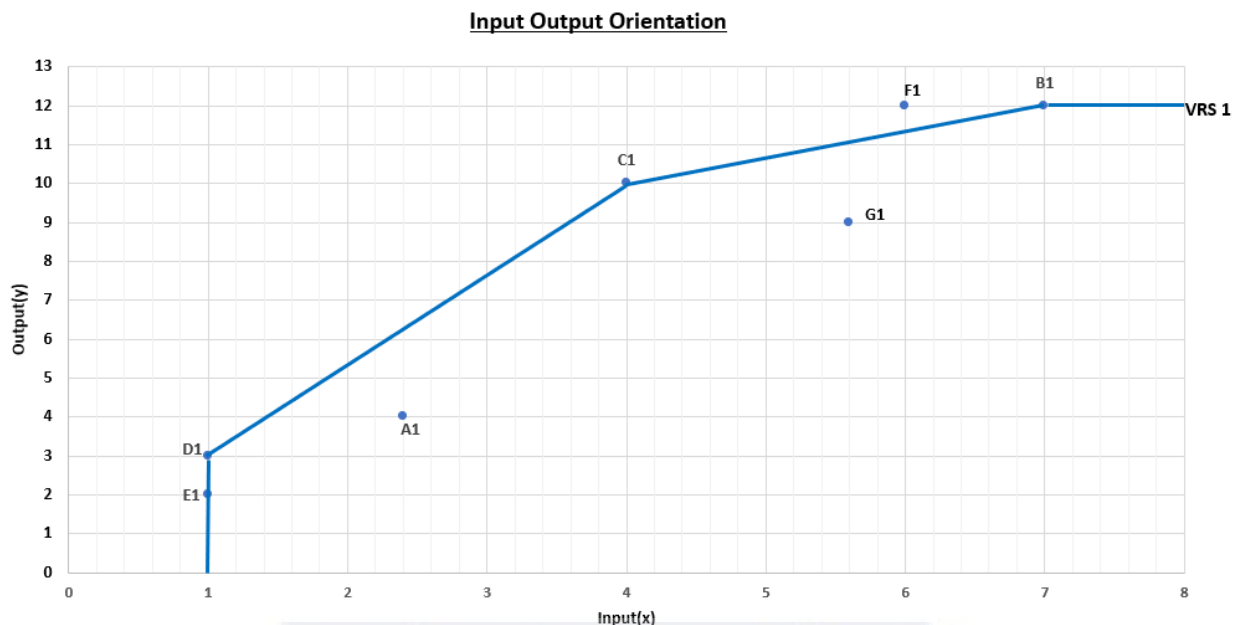


Figure 7 Illustration of Super Efficiency

To illustrate the discussion, let us consider the example presented above in figure 7, consisting of seven DMUs, labelled A1, G1 each consuming a single input to produce a single output. It can be observed that DMUs F1 exhibits super efficiency as it falls outside the efficiency frontier.

4.13 Conclusion

This chapter provided a comprehensive overview of the research design, data sources, target population, sample size, and data gathering procedure. It also discusses the important considerations in selecting inputs and outputs, as well as the different orientations of input and output in the analysis. The mathematical formulations of both DEA and MEA models are explained in detail, accompanied by illustrative examples and graphical representations to enhance understanding.

CHAPTER FIVE

DATA ANALYSIS AND DISCUSSION

5.1 Chapter Introduction

This chapter is divided into two parts and focuses on the data analysis aspect of the study. The first part of the chapter involves an examination of various data statistics. This includes descriptive statistics such as means, standard deviations, or correlations, as well as any other relevant statistical analyses to gain insights into the data.

The second section of the chapter delves into the assessment of efficiency and productivity levels of the countries under study. This analysis involves applying the developed productivity index to evaluate productivity performance of the countries. This section provides insights into the relative performance of the countries in terms of their efficiency and productivity levels.

By dividing the chapter into these two parts, the study aims to provide a comprehensive understanding of the data, including its characteristics and the efficiency/productivity landscape of the countries under investigation.

The descriptive statistics of the inputs and outputs, as well as their correlations, are provided at the beginning of the chapter to evaluate the data's isotonicity characteristic. To determine the actual technology guiding the African energy market, a return to scale test is conducted following Simar and Wilson (2002) non parametric returns to scale test.

The MEA efficiency and productivity of all the countries are also evaluated over the time periods of 2000-2019 and decomposed in line with Wheelock and Wilson (1999) in order to identify the real causes of productivity changes. The productivity patterns of the study's involved countries are calculated with an emphasis on the outcomes of an examination of the MEA scores under the new Biennial Malmquist MEA.

5.2 Data Description

In the last chapter, it was noted that data used is from a cross-country assessment of African states on an unbalanced sample of 32 nations based on the availability of data for the years 2000 to 2019. The sample includes 5 countries from the Arab Maghreb Union (AMU), 5 countries from the East African Community (EAC), 6 countries from the Economic Community of Central African States (ECCAS), 9 countries from the Economic Community of West African States (ECOWAS), and 7 countries from the Southern African Development Community (SADC). This sample composition allows for a comprehensive analysis of efficiency and productivity levels across different regional organizations in Africa.

Table 19 Contingency Table

	Region Economic Groups					Count of Observation
	AMU	EAC	ECCAS	ECOWAS	SADC	
2000	5	5	6	9	7	32
2001	5	5	6	9	7	32
2002	5	5	6	9	7	32
2003	5	5	6	9	7	32
2004	5	5	6	9	7	32
2005	5	5	6	9	7	32
2006	5	5	6	9	7	32
2007	5	5	6	9	7	32
2008	5	5	6	9	7	32
2009	5	5	6	9	7	32
2010	5	5	6	9	7	32
2011	5	5	6	9	7	32
2012	5	5	6	9	7	32
2013	5	5	6	9	7	32
2014	5	5	6	9	7	32
2015	5	5	6	9	7	32
2016	5	5	6	9	7	32
2017	5	5	6	9	7	32
2018	5	5	6	9	7	32
2019	5	5	6	9	7	32
Total	100	100	120	180	140	640

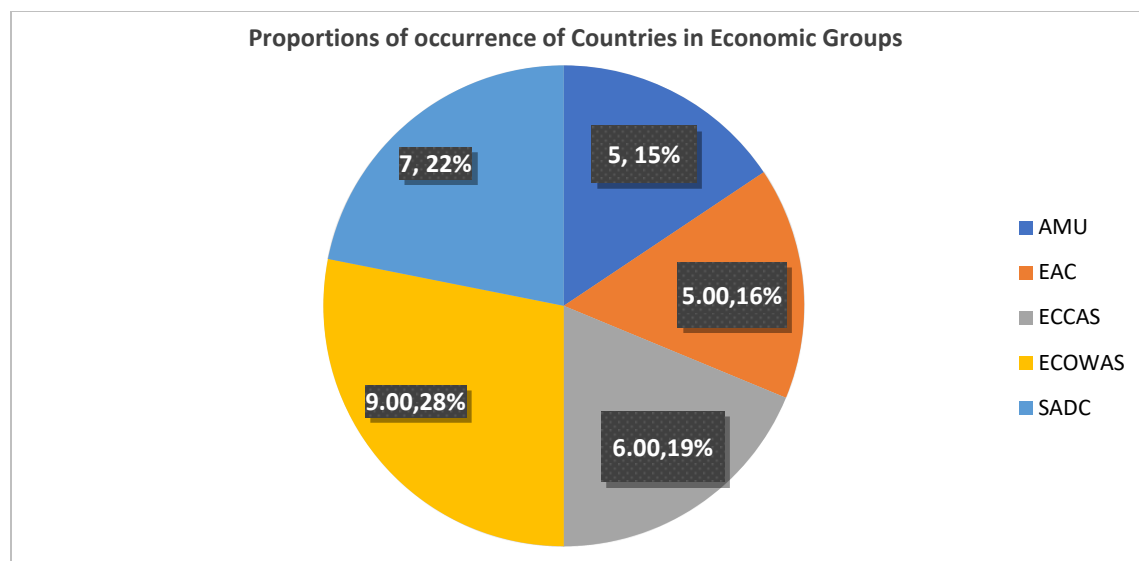


Figure 8 Proportions of Occurrence

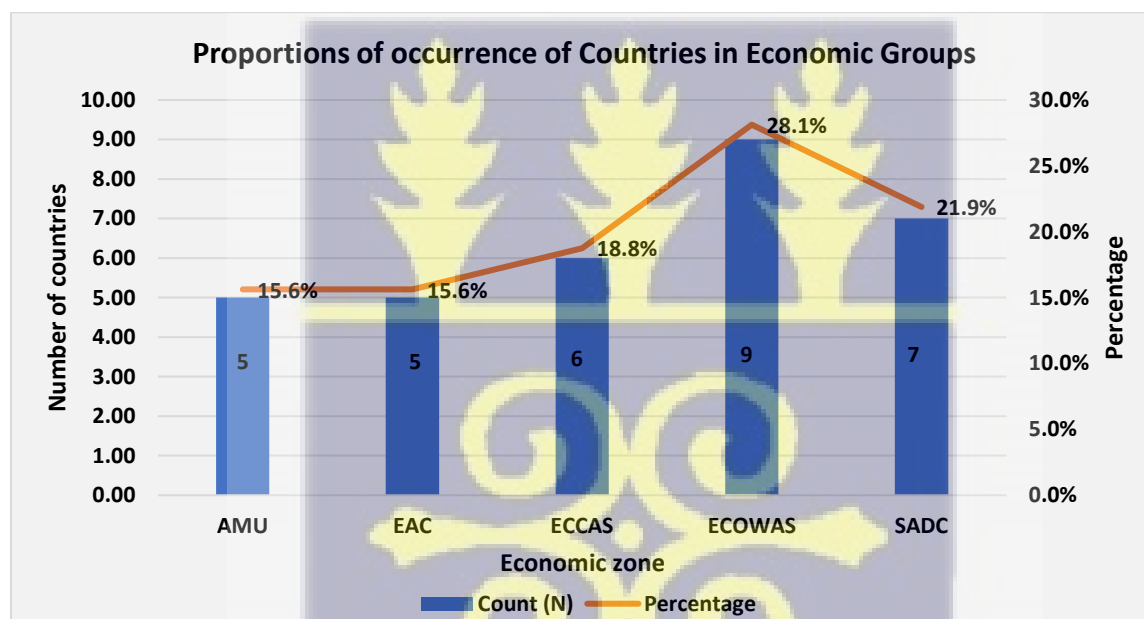


Figure 9 Summary of Data Distribution

Penn World Tables (PWT 10.0), US Energy Information Administration and World Development Indicators are the sources of the data. Both the input and the output data are included in this. More information on the variables utilized as inputs and outputs in the study is provided in Table 20 below.

Table 20 Definitions of Inputs and Outputs

Variable	Description
Input 1	Capital Service at constant 2017 national prices (in mil. 2017US\$) (xa)
Input 2	Labour (in millions) (xb)
Input 3	Energy Used (quadrillion btu) (xc)
Output 1	Real Gross Domestic Product (ya) (desirable)
Output 2	Carbon Dioxide -CO ₂ , undesirable output (yb)

Since our focus is on resource allocation given the level of output achieved by the various African countries under study, we use a non-orientation (input-output) framework for the analysis because the multi-directional analysis (MEA) technique aims at minimization of inputs and expansion of outputs. The countries covered represent about 60% of countries of the African continent. The African continent which is the second largest and second-most populated continent in the world after Asia, is represented by the countries examined in this study. There by assuring that the DEA convention, according to which there should be at least three times as many DMUs as inputs plus output, was followed.

In the study, the selection of inputs and output variables for modelling purposes was based on the intermediation approach. The intermediation approach focuses on identifying and analysing the inputs and outputs that play a crucial role in the intermediation process of a specific context or industry.

By adopting the intermediation approach, the study aims to capture the key factors that contribute to the efficiency and productivity of the system under investigation. These factors are typically identified based on their significance in the intermediation process, where inputs are transformed into outputs through various operational activities.

The selection of inputs and outputs based on the intermediation approach allows for a more targeted and context-specific analysis, enabling researchers to focus on the factors that are most relevant and influential in the intermediation process of the particular industry or system being studied. The input variables consist of Capital Service at constant 2017 national prices (in mil. 2017US\$) (xa), Labour (in millions) (xb), Energy Used (Quadrillion btu) (xc) whilst output variables consist of one (1) desirable output Real Gross Domestic Product (RGDP)(ya) and an undesirable outputs carbon dioxide (CO2)(yb).

Table 21 shows the descriptive statistics of inputs and outputs variables used. The mean, standard deviation, minimum and maximum of the pooled data for each component from 2000 to 2019 are presented below.

Table 21 Pooled Descriptive statistics for variables (2000-2019)

		Capital Service (million US\$)-xa	Labour (millions US\$)-xb	Energy Used (quadrillion btu)-xc	Real GDP (millions US\$)-ya	CO2 emissions (kt)-yb
Pooled	Count(N)	640	640	640	640	640
	Mean	2835.82	7.26	0.40	109292.98	29712.12
	SD	6086.90	10.25	1.08	220757.83	84765.14
	Min	3.14	0.04	0.00	374.08	47.67
	Max	32363.46	73.02	5.73	1287589.00	503112.40
Time F-Statistic ^a		16.6***	7.617***	1.991	9.963***	1.235

Count=number of observations, SD =standard deviation, Min=minimum value, max=Maximum value.

****, ***, **, * denote significance at 0.1%, 1%, 5% and 10% respectively

^a Test for differences of variables across time.

The range can be determined by calculating the difference between the maximum and minimum values for each variable, as shown in Table 21 above. The range of values differs for both the input and output variables, indicating that the countries under study in Africa exhibit variations in size. This observation is supported by the higher standard deviation values, which are compared to the mean values of both the input and output variables. The higher standard deviations indicate greater dispersion or variability in the data, further reinforcing the notion of differences in the sizes of the countries being examined.

The mean values of the inputs show that, on average 2835.82million US dollars is used in terms of capital services compared to a minimum of 3.14million US dollars and maximum of 32363.46million US dollars.

Labour usage also is represented on average 7.26million US dollars compared to a minimum of 0.04 million US dollars and maximum of 73.02 million US dollars.

Energy usage, on the average is 0.4 quadrillion btu compared to a minimum of 0.0005 quadrillion btu and maximum of 5.7322 quadrillion btu.

Considering the outputs, the real GDP produced represents an average 109292.98million US dollars compared to a minimum of 374.08million US dollars and maximum of 1287589.00million US dollars.

Carbon dioxide emission which is a bad output of the energy generation processes of these countries has an average 23626.6100kilo tonnes compared to a minimum of 33.00 kilo tonnes and maximum of 503112.40 kilo tonnes.

Table 22 Correlation matrix of inputs and outputs (Isotonicity Test)

	Capital Service (xa)	Labour (xb)	Energy Used (xc)	Real GDP (ya)	CO2 emissions (yb)
Capital Service (xa)	1				
Labour (xb)	0.8339***	1			
Energy Used (xc)	0.7033***	0.4486***	1		
Real GDP (ya)	0.8657***	0.7886***	0.8186***	1	
CO2 emissions (yb)	0.7183***	0.4381***	0.9879***	0.7725***	1

***p<0.1%, **p<1% and *p<5% and p<10%

An isotonicity test (Avkiran 1999) which involves the calculation of all inter-correlations between inputs and outputs, is carried out to determine if increasing amounts of inputs result in increasing outputs in order to further confirm the validity of DEA calculations.

The isotonicity property requires that requires that all inputs and outputs have a positive correlation (Thanassoulis and Allen 1998; Cooper, Seiford et al. 2011). This was done using the input-output

correlation test. It is recommended to do this before measuring productivity under DEA in order to meet the isotonicity criterion. At a significance level of 0.1%, Table 22's inputs all exhibit a significant and positive connection with real GDP. The relevance of all three research inputs is demonstrated by the strong positive correlations between capital service and GDP (87%), labour and GDP (79%) and energy consumption and GDP (82%). As a result, the isotonicity property—which states that an increase in input should not result in a decrease in the associated output—is met. Emissions of CO₂ are viewed as an unwanted output, there is no need to contrast their positive correlation with inputs.

Based on the findings, there is a significant positive correlation between the input and output variables. This indicates that increasing inputs would lead to higher outputs, aligning with the assumptions of Data Envelopment Analysis (DEA) regarding isotonic relationships. The observed positive link supports the notion that increasing resource inputs can result in improved output levels, reflecting the productive efficiency of the entities being analysed.

The observed positive correlation between the input and output variables aligns with the non-parametric nature of the MEA (Multi Directional Efficiency Analysis) technique. Table 22 presents the correlation matrix results for the pooled meta-data set, revealing a generally high positive correlation coefficient between these variables. These strong positive correlations further support the notion that changes in inputs tend to correspond with changes in outputs, emphasizing the interdependence and relationship between these variables in the context of the study. The highest correlation of (0.9879) was recorded between CO₂ used and Energy consumption, thus there is more CO₂ emission when we consume more energy which is a known fact.

During the study period, the lowest correlation coefficient of 0.4381 is observed between the variables of Energy used and Labour. As anticipated, all the p-values associated with the

correlation coefficients were found to be statistically significant. This confirms that there is a substantial correlation among all the variables, indicating a high level of interdependence. The fact that the output variables are largely dependent on the input variables supports the significant correlations observed.

5.3 Nonparametric test of global returns to scale

If an industry's genuine RTS technology is not evaluated before its performance assessments, results may be generated that are skewed and deceptive (Simar and Wilson 1998; Daraio and Simar 2007). In order to find the proper RTS describing the energy business in Africa, this study conducts three distinct tests of the return to scale of Léopold Simar & Wilson, 2002, & 2011. Results of the tests, which comprise the mean of ratios (S1), ratio of means (S2), and mean of ratios minus 1 (S3), are provided in Table 23.

One of the presumptions driving the employment of DEA is that either VRS or CRS technology may be used to support the industry under evaluation. The choice between CRS and VRS affects productivity and efficiency results. To do this, the suitable scale elasticity attribute in the African energy industry was tested. The study made use of Simar and Wilson's test approach (2002). The alternative hypothesis, VRS, is compared to the null hypothesis, CRS. When the p-value is less than the level of significance, which is 1%, the null hypothesis is rejected.

Test of returns to scale

Table 23 Test of Returns to Scale

Ho: μ is CRS	Significance level	Mean of ratios (S1)	Ratio of means (S2)	Mean of ratios minus 1 (S3)	Conclusion
Test Static		0.9298467***	0.4620527*	-0.06705516***	
Critical Level	5%	0.9664829	0.6381129	-0.02658735	Reject Ho
Critical Level	1%	0.9626069	0.3377091	-0.02647794	

***p<0.1%, **p<1% and *p<5% and p<10%

5.4 Efficiency Analysis

Throughout the study, both the constant return to scale (CRS) and variable return to scale (CRS) technologies were employed. The reason being that the variable-return-to-scale (VRS) score solely measures technical efficiency and in computing the decompositions, the constant return to scale (CRS) scores would be used to account for the productivity metrics in the decomposition of the MPI and BMI.

It is important to note that MEA efficiency score has a range of 0 to 1, unlike the DEA efficiency score. In the MEA framework, DMUs that are efficient receive a score of 0, while inefficient DMUs receive progressively higher scores indicating their level of inefficiency, with higher scores representing greater inefficiency (Bogetoft and Hougaard 1999).

To demonstrate how different MEA is from DEA and the steps in which MEA estimates efficiency scores of DMUs, using an empirical example with data from 2018 to estimate DEA scores, MEA technical efficiency, Ideal reference point, Potential improvement Point, MEA inefficiency, MEA efficiency and Over MEA Efficiency the following tables show for each variable how MEA scores are estimated. These variable specific efficiencies are shown in tables 24 to 28. In the appendix part of this study, a detailed variable specific MEA efficiency score for the countries represented in the sample have been outlined with from 2000 to 2019.

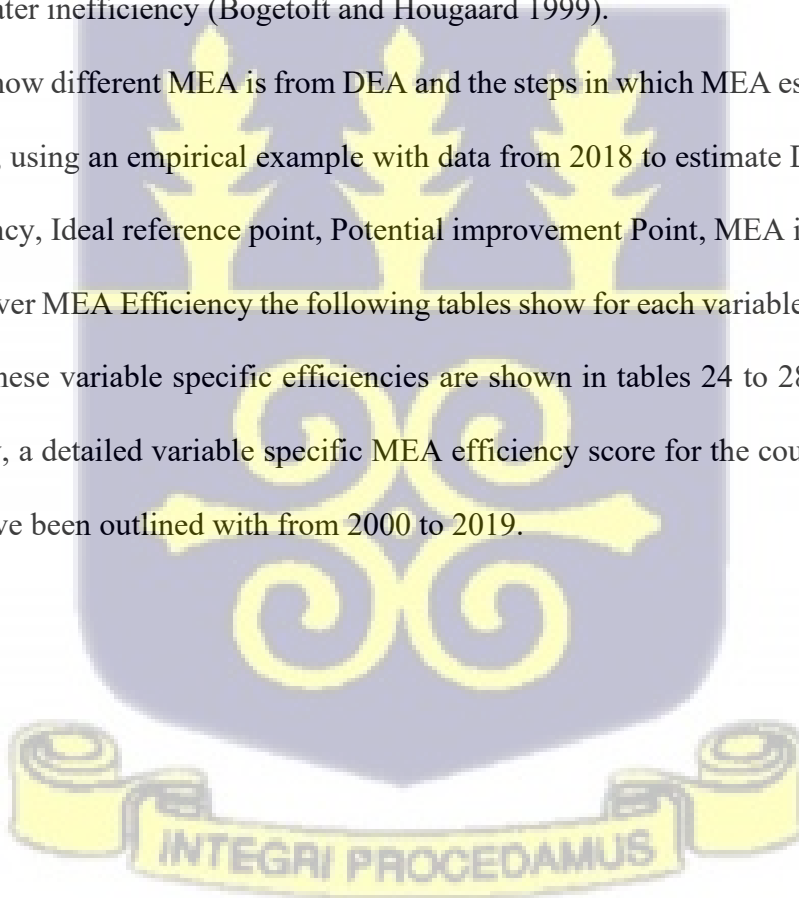


Table 24 MEA Variable efficiency scores of all the 32 states for Capital Service (xa)-2018

Countries	Capital services	DEA Efficiency (oev1.eff)	MEA Efficiency (mv1.eff)	Diagonal distance to IRP (mv1.direct)	Ideal reference point (IRPx1xa)	Potential Improvement point (PIPx1xa)	MEA inefficiency (xa)	MEA efficiency (efx1xa)	Overall MEA Efficiency (ovse1m)
Angola	13642.12	1	0	5.11E-08	13642.12	13642.12	0	1	1
Benin	887.2405	0.671595	0.377185	3.71E+02	516.126	747.2615	0.157769	0.842231	0.629567
Botswana	1850.156	0.982541	0.333333	5.14E+02	1336.47	1678.927	0.092548	0.907452	0.944538
Burkina Faso	871.8946	0.800836	0.363988	2.24E+02	648.3906	790.5418	0.093306	0.906694	0.763546
Burundi	193.3141	0.803403	0.361282	7.10E+01	122.3285	167.6683	0.132664	0.867336	0.761389
Cabo Verde	191.9693	0.676795	0.379703	1.08E+02	84.27132	151.0761	0.21302	0.78698	0.657314
Cameroon	2486.112	0.83771	0.387262	8.79E+02	1607.344	2145.799	0.136886	0.863114	0.802544
Central African Republic	324.595	1	0	0.00E+00	324.5951	324.595	0	1	1
Chad	477.6592	0.889049	0.41505	6.33E+01	414.4078	451.4067	0.054961	0.945039	0.834653
Côte d'Ivoire	2559.19	1	0	0.00E+00	2559.19	2559.19	0	1	1
Djibouti	113.2305	0.948472	0.222097	1.05E+01	102.7235	110.8969	0.020609	0.979391	0.938584
Egypt	18320.14	1	0	0.00E+00	18320.14	18320.14	0	1	1
Eswatini	329.7085	1	0	0.00E+00	329.7085	329.7085	0	1	1
Gabon	1279.221	1	0	0.00E+00	1279.221	1279.221	0	1	1
Guinea	573.3508	0.901222	0.309213	7.49E+01	498.4572	550.1927	0.040391	0.959609	0.879393
Kenya	5372.316	1	0	4.06E-09	5372.316	5372.316	0	1	1
Lesotho	277.1008	0.715829	0.330538	1.65E+02	112.4367	222.673	0.196419	0.803581	0.702361
Mauritania	976.59	0.725316	0.327341	6.30E+02	346.47	770.3259	0.211209	0.788792	0.713374
Mauritius	970.1594	1	0	0.00E+00	970.1594	970.1594	0	1	1
Morocco	15392.91	0.885106	0.334136	1.11E+04	4298.334	11685.82	0.240831	0.759169	0.814248
Mozambique	1225.147	0.504705	0.425041	6.95E+02	529.9514	929.6604	0.241185	0.758815	0.481018
Niger	1311.49	0.625713	0.479595	8.84E+02	427.7117	887.6342	0.323186	0.676814	0.628598
Nigeria	30957.93	1	0	0.00E+00	30957.93	30957.93	0	1	1
Rwanda	502.3325	1	0	0.00E+00	502.3325	502.3325	0	1	1
Sao Tome and Principe	37.3574	1	0	0.00E+00	37.3574	37.3574	0	1	1
Sierra Leone	167.1296	1	0	0.00E+00	167.1296	167.1296	0	1	1
South Africa	28973.59	0.833144	0.5	1.80E+04	10975.98	19974.79	0.310586	0.689414	0.674861
Sudan	3153.801	1	0	0.00E+00	3153.801	3153.801	0	1	1
Togo	485.0028	0.649278	0.34681	2.71E+02	214.4584	391.1754	0.193458	0.806542	0.62552
Tunisia	3549.378	1	0	0.00E+00	3549.378	3549.378	0	1	1
U.R. of Tanzania: Mainland	3910.371	1	0	0.00E+00	3910.371	3910.371	0	1	1
Zimbabwe	707.2451	0.939113	0.5	4.31E+01	664.1828	685.7139	0.030444	0.969556	0.768743

Table 25 MEA Variable efficiency scores of all the 32 states for Labour (xb)-2018

Countries	Labour	DEA Efficiency (oev1.eff)	MEA Efficiency (mv1.eff)	Diagonal distance to IRP (mv1.direct)	Ideal reference point (IRPx1xb)	Potential Improvement point (PIPx1xb)	MEA inefficiency (xb)	MEA efficiency (efx1xb)	Overall MEA Efficiency (ovse1m)
	16.0783								
Angola	3	1	0	0	16.07833	16.07833	0	1	1
Benin	4.15568	0.671595	0.377185	3.371276	0.784405	2.884084	0.30599	0.69401	0.629567
	0.91396								
Botswana	6	0.982541	0.333333	0.057409	0.856558	0.89483	0.020938	0.979063	0.944538
Burkina Faso	6.75672	0.800836	0.363988	4.350987	2.405733	5.173012	0.23439	0.76561	0.763546
	4.84106								
Burundi	8	0.803403	0.361282	3.643193	1.197875	3.524849	0.271886	0.728114	0.761389
	0.20823								
Cabo Verde	7	0.676795	0.379703	0.090579	0.117658	0.173844	0.165163	0.834837	0.657314
	10.0732								
Cameroon	1	0.83771	0.387262	5.270314	4.802897	8.03222	0.202616	0.797384	0.802544
Central African Republic	1.80255	1	0	0	1.80255	1.80255	0	1	1
	5.76620								
Chad	2	0.889049	0.41505	2.033227	3.732976	4.922312	0.146351	0.853649	0.834653
	7.28638								
Côte d'Ivoire	2	1	0	0	7.286382	7.286382	0	1	1
	0.36498								
Djibouti	2	0.948472	0.222097	0.09845	0.266532	0.343117	0.059908	0.940092	0.938584
Egypt	25.9848	1	0	0	25.9848	25.9848	0	1	1
	0.30542								
Eswatini	2	1	0	0	0.305422	0.305422	0	1	1
	0.64187								
Gabon	8	1	0	0	0.641878	0.641878	0	1	1
	4.51553								
Guinea	5	0.901222	0.309213	1.535581	2.979954	4.040713	0.105153	0.894847	0.879393
Kenya	24.2133	1	0	0	24.2133	24.2133	0	1	1
	0.65409								
Lesotho	3	0.715829	0.330538	0.468436	0.185657	0.499257	0.236719	0.763281	0.702361
	1.09561								
Mauritania	7	0.725316	0.327341	0.647647	0.44797	0.883616	0.1935	0.8065	0.713374
Mauritius	0.57499	1	0	0	0.57499	0.57499	0	1	1

Morocco	11.37258	0.885106	0.334136	4.259521	7.113061	9.949325	0.125148	0.874852	0.814248
Mozambique	10.66711	0.504705	0.425041	9.914533	0.752582	6.453029	0.395054	0.604946	0.481018
	8.53264								
Niger	9	0.625713	0.479595	5.674381	2.858268	5.811243	0.31894	0.68106	0.628598
Nigeria	71.11517	1	0	0	71.11517	71.11517	0	1	1
	5.16152								
Rwanda	3	1	0	0	5.161523	5.161523	0	1	1
	0.06344								
Sao Tome and Principe	6	1	0	0	0.063446	0.063446	0	1	1
	2.47238								
Sierra Leone	8	1	0	0	2.472388	2.472388	0	1	1
	18.6863								
South Africa	6	0.833144	0.5	3.11794	15.56842	17.12739	0.083428	0.916572	0.674861
	10.0248								
Sudan	3	1	0	0	10.02483	10.02483	0	1	1
	3.08190								
Togo	4	0.649278	0.34681	2.43525	0.646654	2.237335	0.274041	0.725959	0.62552
	3.64583								
Tunisia	4	1	0	0	3.645834	3.645834	0	1	1
U.R. of Tanzania: Mainland	22.91124	1	0	0	22.91124	22.91124	0	1	1
	6.71495								
Zimbabwe	2	0.939113	0.5	5.173028	1.541924	4.128438	0.385187	0.614813	0.768743

Table 26 MEA Variable efficiency scores of all the 32 states for Energy used (xc)-2018.

Countries	Energy Used	DEA Efficiency (oev1.eff)	MEA Efficiency (mv1.eff)	Diagonal distance to IRP (mv1.direct)	Ideal reference point (IRP1xc)	Potential Improvement point (PIP1xc)	MEA inefficiency (xc)	MEA efficiency (efx1xc)	Overall MEA Efficiency (ovse1m)
Angola	0.338	1	0	0	0.338	0.338	0	1	1
Benin	0.112	0.671595	0.377185	0.065637	0.046363	0.087243	0.221047	0.778953	0.629567
Botswana	0.079	0.982541	0.333333	0.00191	0.07709	0.078363	0.00806	0.99194	0.944538
Burkina Faso	0.065	0.800836	0.363988	0.024739	0.040261	0.055995	0.138532	0.861468	0.763546
Burundi	0.011	0.803403	0.361282	0.003631	0.007369	0.009688	0.11924	0.88076	0.761389
Cabo Verde	0.015	0.676795	0.379703	0.007922	0.007078	0.011992	0.200532	0.799468	0.657314
Cameroon	0.161	0.83771	0.387262	0.034109	0.126891	0.147791	0.082043	0.917957	0.802544
Central African Republic	0.005	1	0	0	0.005	0.005	0	1	1
Chad	0.024	0.889049	0.41505	0.006723	0.017277	0.021209	0.116274	0.883726	0.834653
Côte d'Ivoire	0.195	1	0	0	0.195	0.195	0	1	1
Djibouti	0.01	0.948472	0.222097	0.002435	0.007565	0.009459	0.05407	0.94593	0.938584

Egypt	4.031	1	0	0	4.031	4.031	0	1	1
Eswatini	0.023	1	0	0	0.023	0.023	0	1	1
Gabon	0.06	1	0	0	0.06	0.06	0	1	1
Guinea	0.048	0.901222	0.309213	0.013344	0.034656	0.043874	0.085958	0.914042	0.879393
Kenya	0.352	1	0	0	0.352	0.352	0	1	1
Lesotho	0.016	0.715829	0.330538	0.007183	0.008817	0.013626	0.148397	0.851603	0.702361
Mauritania	0.059	0.725316	0.327341	0.022048	0.036952	0.051783	0.122327	0.877673	0.713374
Mauritius	0.097	1	0	0	0.097	0.097	0	1	1
Morocco	0.855	0.885106	0.334136	0.141073	0.713927	0.807863	0.055131	0.944869	0.814248
Mozambique	0.245	0.504705	0.425041	0.208919	0.036081	0.156201	0.362446	0.637554	0.481018
Niger	0.034	0.625713	0.479595	0.015534	0.018466	0.02655	0.219122	0.780878	0.628598
Nigeria	1.655	1	0	0	1.655	1.655	0	1	1
Rwanda	0.019	1	0	0	0.019	0.019	0	1	1
Sao Tome and Principe	0.002	1	0	0	0.002	0.002	0	1	1
Sierra Leone	0.015	1	0	0	0.015	0.015	0	1	1
South Africa	5.492	0.833144	0.5	3.193973	2.298027	3.895013	0.290784	0.709216	0.674861
Sudan	0.366	1	0	0	0.366	0.366	0	1	1
Togo	0.034	0.649278	0.34681	0.021362	0.012638	0.026591	0.217899	0.782101	0.62552
Tunisia	0.416	1	0	0	0.416	0.416	0	1	1
U.R. of Tanzania: Mainland	0.192	1	0	0	0.192	0.192	0	1	1
Zimbabwe	0.162	0.939113	0.5	0.055439	0.106561	0.134281	0.171107	0.828893	0.768743

Table 27 MEA Variable efficiency scores of all the 32 states for CO2 emissions (yb)-2018

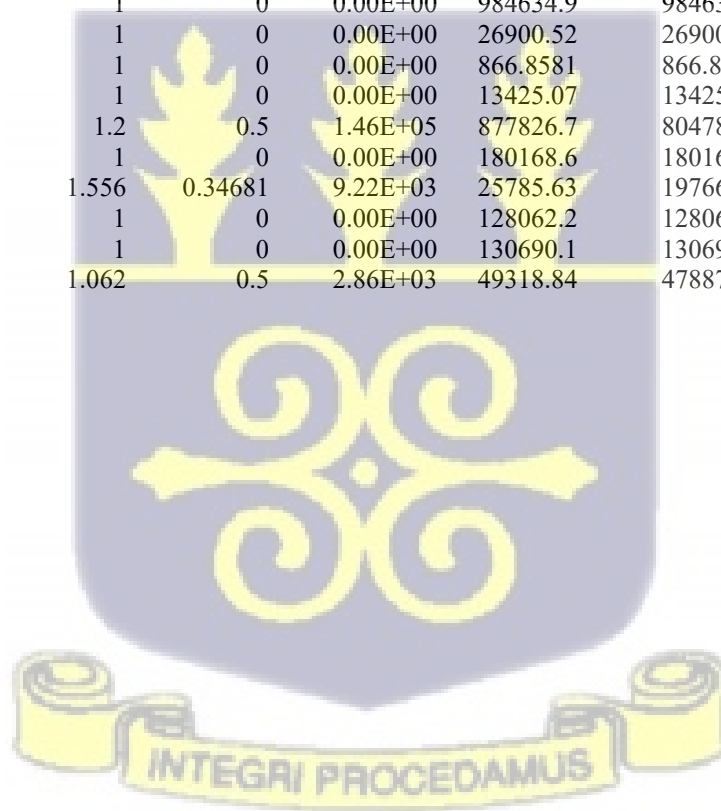
Countries	CO2 emissions	DEA Efficiency (oev1.eff)	MEA Efficiency (mv1.eff)	Diagonal distance to IRP (mv1.direct)	Ideal reference point (IRP1yb)	Potential Improvement point (PIP1yb)	MEA inefficiency (yb)	MEA efficiency (efx1yb)	Overall MEA Efficiency (ovse1m)
Angola	23960	1	0	1.94E-08	23960	23960	0	1	1
Benin	7420	0.671595	0.377185	6.35E+03	1069.836	5024.811	0.322802	0.677198	0.629567
Botswana	7310	0.982541	0.333333	1.85E+03	5459.281	6693.094	0.084392	0.915608	0.944538
Burkina Faso	4670	0.800836	0.363988	3.16E+03	1511.349	3520.288	0.246191	0.753809	0.763546
Burundi	690	0.803403	0.361282	2.16E+02	474.0284	611.9734	0.113082	0.886918	0.761389
Cabo Verde	610	0.676795	0.379703	4.19E+02	191.2551	451.0014	0.260653	0.739347	0.657314
Cameroon	9590	0.83771	0.387262	3.97E+03	5620.416	8052.732	0.160299	0.839701	0.802544
Central African Republic	230	1	0	0.00E+00	230	230	0	1	1

Chad	2190	0.889049	0.41505	1.08E+03	1108.574	1741.154	0.204952	0.795048	0.834653
Côte d'Ivoire	10190	1	0	0.00E+00	10190	10190	0	1	1
Djibouti	400	0.948472	0.222097	1.06E+02	293.8451	376.4234	0.058942	0.941058	0.938584
Egypt	247910	1	0	0.00E+00	247910	247910	0	1	1
Eswatini	910	1	0	0.00E+00	910	910	0	1	1
Gabon	5120	1	0	0.00E+00	5120	5120	0	1	1
Guinea	3750	0.901222	0.309213	1.82E+03	1930.003	3187.233	0.150071	0.849929	0.879393
Kenya	17490	1	0	3.56E-08	17490	17490	0	1	1
Lesotho	730	0.715829	0.330538	4.69E+02	260.9932	574.9752	0.212363	0.787637	0.702361
Mauritania	3690	0.725316	0.327341	2.93E+03	762.6284	2731.751	0.259688	0.740312	0.713374
Mauritius	4130	1	0	0.00E+00	4130	4130	0	1	1
Morocco	64960	0.885106	0.334136	3.62E+04	28721.56	52851.45	0.1864	0.8136	0.814248
Mozambique	6950	0.504705	0.425041	6.18E+03	765.6002	4321.375	0.378219	0.621781	0.481018
Niger	1950	0.625713	0.479595	8.73E+02	1077.199	1531.409	0.214662	0.785338	0.628598
Nigeria	109890	1	0	0.00E+00	109890	109890	0	1	1
Rwanda	1290	1	0	0.00E+00	1290	1290	0	1	1
Sao Tome and Principe	140	1	0	0.00E+00	140	140	0	1	1
Sierra Leone	860	1	0	0.00E+00	860	860	0	1	1
South Africa	434350	0.833144	0.5	3.01E+05	133532.6	283941.3	0.346285	0.653715	0.674861
Sudan	20860	1	0	0.00E+00	20860	20860	0	1	1
Togo	12000	0.649278	0.34681	1.14E+04	583.7375	8040.727	0.329939	0.670061	0.62552
Tunisia	2310	1	0	0.00E+00	2310	2310	0	1	1
U.R. of Tanzania; Mainland	29890	1	0	0.00E+00	29890	29890	0	1	1
Zimbabwe	12380	0.939113	0.5	6.03E+03	6348.946	9364.473	0.243581	0.75642	0.768743

Table 28 MEA Variable efficiency scores of all the 32 states for real GDP (ya)-2018

Countries	GDP	DEA Efficiency (oev1.eff)	MEA Technical Efficiency (mv1.eff)	Diagonal distance to IRP (mv1.direct)	Ideal reference point (IRP1ya)	Potential Improvement point (PIP1ya)	MEA inefficiency (ya)	MEA efficiency (efx1ya)	Overall MEA Efficiency (ovse1m)
Angola	225540	1	0	2.76E-07	225540	225540	Inf	1	1
Benin	36617.88	1.499	0.377185	1.83E+04	54895.9	43512.08	5.311401	0.841557	0.629567
Botswana	37341.26	1.013	0.333333	4.72E+02	37812.99	37498.5	237.4746	0.995807	0.944538
Burkina Faso	39322.71	1.21	0.363988	8.26E+03	47578.45	42327.71	13.0858	0.929007	0.763546
Burundi	9089.612	1.289	0.361282	2.62E+03	11713.08	10037.42	9.590106	0.905572	0.761389
Cabo Verde	3871.192	1.532	0.379703	2.06E+03	5931.687	4653.568	4.948	0.831876	0.657314
Cameroon	90019.03	1.167	0.387262	1.51E+04	105079.1	95851.22	15.43486	0.939154	0.802544
Central African Republic	4426.902	1	0	0.00E+00	4426.902	4426.902	Inf	1	1

Chad	24261.21	1.1	0.41505	2.43E+03	26692.26	25270.22	24.04467	0.960071	0.834653
Côte d'Ivoire	118822.5	1	0	0.00E+00	118822.5	118822.5	Inf	1	1
Djibouti	4436.217	1.063	0.222097	2.77E+02	4713.6	4497.823	72.00968	0.986303	0.938584
Egypt	1219797	1	0	0.00E+00	1219797	1219797	Inf	1	1
Eswatini	9557.596	1	0	0.00E+00	9557.596	9557.596	Inf	1	1
Gabon	30885.04	1	0	0.00E+00	30885.04	30885.04	Inf	1	1
Guinea	31043.89	1.093	0.309213	2.88E+03	33922.4	31933.96	34.87791	0.972128	0.879393
Kenya	215975.4	1	0	3.03E-08	215975.4	215975.4	Inf	1	1
Lesotho	6315.904	1.427	0.330538	2.70E+03	9013.708	7207.632	7.08277	0.87628	0.702361
Mauritania	21582.33	1.385	0.327341	8.31E+03	29895.31	24303.51	7.931235	0.888033	0.713374
Mauritius	29215.56	1	0	0.00E+00	29215.56	29215.56	Inf	1	1
Morocco	280390.1	1.125	0.334136	3.49E+04	315300.5	292054.9	24.03728	0.96006	0.814248
Mozambique	37536.65	1.855	0.425041	3.21E+04	69621.15	51173.89	2.752512	0.733512	0.481018
Niger	26082.36	1.34	0.479595	8.86E+03	34943.79	30332.26	6.137175	0.859889	0.628598
Nigeria	984634.9	1	0	0.00E+00	984634.9	984634.9	Inf	1	1
Rwanda	26900.52	1	0	0.00E+00	26900.52	26900.52	Inf	1	1
Sao Tome and Principe	866.8581	1	0	0.00E+00	866.8581	866.8581	Inf	1	1
Sierra Leone	13425.07	1	0	0.00E+00	13425.07	13425.07	Inf	1	1
South Africa	731735.1	1.2	0.5	1.46E+05	877826.7	804780.9	10.01748	0.909235	0.674861
Sudan	180168.6	1	0	0.00E+00	180168.6	180168.6	Inf	1	1
Togo	16570.34	1.556	0.34681	9.22E+03	25785.63	19766.29	5.18478	0.838313	0.62552
Tunisia	128062.2	1	0	0.00E+00	128062.2	128062.2	Inf	1	1
U.R. of Tanzania: Mainland	130690.1	1	0	0.00E+00	130690.1	130690.1	Inf	1	1
Zimbabwe	46457.1	1.062	0.5	2.86E+03	49318.84	47887.97	32.46776	0.970121	0.768743



To demonstrate infeasibilities across time the table 29 to 33 below shows variable specific efficiency estimated own period, across time periods and relative to biennial for periods 2018 and 2019.

Table 29 Efficiency estimates with infeasibilities: 2018-2019 for Capital Services

MEA Variable Specific Efficiency (Capital Services)						
Country	Efficiency 2018	Efficiency 2019	Efficiency 2018r2019	Efficiency 2019r2018	Efficiency 2018r Biennial	Efficiency 2019r Biennial
Angola	1	0.804831	NA	0.7836804	1	0.7814537
Benin	0.842231	0.833618	0.9584816	0.8288396	0.8399134	0.823957
Botswana	0.907452	0.876117	0.9397521	NA	0.8694261	0.8758368
Burkina Faso	0.906694	0.869042	1.0356909	0.865543	0.8946376	0.8629912
Burundi	0.867336	0.833321	0.9027292	0.8442699	0.8521573	0.8343845
Cabo Verde	0.78698	0.784522	0.8297898	0.7863258	0.7837674	0.782825
Cameroon	0.863114	0.870513	0.9453259	0.8752308	0.8633681	0.866385
Central African Republic	1	1	NA	NA	1	1
Chad	0.945039	0.958302	1.0091657	0.934062	0.9347278	0.9249499
Côte d'Ivoire	1	1	NA	NA	1	1
Djibouti	0.979391	1	1.1119823	0.9701511	0.9747284	0.9671127
Egypt	1	1	NA	NA	1	1
Eswatini	1	1	0.9796036	NA	0.9869698	1
Gabon	1	1	1.010772	NA	0.9842796	1
Guinea	0.959609	0.943652	0.9613213	0.961937	0.9403406	0.943752
Kenya	1	1	NA	NA	1	1
Lesotho	0.803581	0.781613	0.8748753	0.7836096	0.7998629	0.7799529
Mauritania	0.788792	0.792171	0.8142514	0.7961047	0.7851484	0.7920574
Mauritius	1	1	1.0591483	NA	0.9982025	1
Morocco	0.759169	0.746285	0.8024638	0.7554624	0.7529482	0.7447989
Mozambique	0.758815	0.719844	0.9126289	0.7155997	0.7558411	0.7131454
Niger	0.676814	0.654212	0.6927781	0.680391	0.647388	0.6504963
Nigeria	1	1	NA	NA	1	1
Rwanda	1	1	NA	NA	1	1
Sao Tome and Principe	1	1	NA	NA	1	1
Sierra Leone	1	1	0.9842081	NA	0.9846355	1
South Africa	0.689414	0.692377	0.7157777	0.6834235	0.6893349	0.6833477
Sudan	1	1	1.008043	NA	0.998731	1
Togo	0.806542	0.809817	0.8569331	0.8071683	0.7995029	0.8033417
Tunisia	1	1	NA	NA	1	1
U.R. of Tanzania:						
Mainland	1	1	1.0265217	NA	0.8969095	1
Zimbabwe	0.969556	0.925255	0.9927544	0.9233055	0.9618169	0.9162348

Table 30 Efficiency estimates with infeasibilities: 2018-2019 for Labour

MEA Variable Specific Efficiency (Labour)						
Country	Efficiency 2018	Efficiency 2019	Efficiency 2018r2019	Efficiency 2019r2018	Efficiency 2018r Biennial	Efficiency 2019r Biennial
Angola	1	0.968819	NA	0.9545358	1	0.9525645
Benin	0.69401	0.713214	0.713952	0.722185	0.6968989	0.7067097
Botswana	0.979063	0.950487	0.9405241	NA	0.9354939	0.9504867
Burkina Faso	0.76561	0.794658	0.8139611	0.7660737	0.7681358	0.7739653
Burundi	0.728114	0.702391	0.7348829	0.6968786	0.7176597	0.6946371
Cabo Verde	0.834837	0.840366	0.84699	0.8407142	0.832567	0.837968
Cameroon	0.797384	0.82677	0.8389721	0.8219201	0.8027406	0.8255915
Central African Republic	1	1	NA	NA	1	1
Chad	0.853649	0.872507	0.9035757	0.8454747	0.8565913	0.8471822
Côte d'Ivoire	1	1	NA	NA	1	1
Djibouti	0.940092	1	0.9882989	0.9712172	0.932734	0.9674304
Egypt	1	1	NA	NA	1	1
Eswatini	1	1	1.0124269	NA	0.9893239	1
Gabon	1	1	1.014135	NA	0.9960527	1
Guinea	0.894847	0.867377	0.9068536	0.8835394	0.8721031	0.8563458
Kenya	1	1	NA	NA	1	1
Lesotho	0.763281	0.756074	0.7794988	0.7534498	0.7629581	0.7528172
Mauritania	0.8065	0.808654	0.8234674	0.8127269	0.8030284	0.8088841
Mauritius	1	1	1.0099482	NA	0.9988725	1
Morocco	0.874852	0.833034	0.8407455	0.8711634	0.8329551	0.833821
Mozambique	0.604946	0.593796	0.6281235	0.5913632	0.6056093	0.5916173
Niger	0.68106	0.666021	0.6674735	0.6907793	0.6522188	0.6624123
Nigeria	1	1	NA	NA	1	1
Rwanda	1	1	NA	NA	1	1
Sao Tome and Principe	1	1	NA	NA	1	1
Sierra Leone	1	1	0.995373	NA	0.9781682	1
South Africa	0.916572	0.90847	0.9057251	0.9181862	0.906893	0.9084699
Sudan	1	1	1.0185152	NA	0.9964128	1
Togo	0.725959	0.746097	0.74739	0.738638	0.7203677	0.736353
Tunisia	1	1	NA	NA	1	1
U.R. of Tanzania:						
Mainland	1	1	0.8117327	NA	0.7783207	1
Zimbabwe	0.614813	0.567913	0.6810975	0.5966733	0.6064221	0.5956762

Table 31 Efficiency estimates with infeasibilities: 2018-2019 for Energy used

MEA Variable Specific Efficiency (Energy Used)						
Country	Efficiency 2018	Efficiency 2019	Efficiency 2018r2019	Efficiency 2019r2018	Efficiency 2018r Biennial	Efficiency 2019r Biennial
Angola	1	0.992637	NA	0.9603064	1	0.9609099
Benin	0.778953	0.817281	0.7110278	0.8195679	0.777648	0.8124293
Botswana	0.99194	0.980116	0.9575941	NA	0.9575941	0.9801156
Burkina Faso	0.861468	0.862641	0.8647095	0.8639959	0.8493826	0.8585178
Burundi	0.88076	0.870621	1.0278501	0.8398692	0.8765369	0.8397746
Cabo Verde	0.799468	0.828418	0.8162822	0.8134311	0.7924865	0.8054043
Cameroon	0.917957	0.93384	0.9177683	0.9373261	0.910887	0.9283959
Central African Republic	1	1	NA	NA	1	1
Chad	0.883726	0.920722	0.9585467	0.8767387	0.8863088	0.8799574
Côte d'Ivoire	1	1	NA	NA	1	1
Djibouti	0.94593	1	0.8980561	0.9849059	0.9435509	0.9815063
Egypt	1	1	NA	NA	1	1
Eswatini	1	1	0.9629421	NA	0.9937333	1
Gabon	1	1	0.95716	NA	0.9809714	1
Guinea	0.914042	0.913695	0.9779087	0.9104548	0.9003738	0.8941038
Kenya	1	1	NA	NA	1	1
Lesotho	0.851603	0.849738	0.9057821	0.837541	0.8461291	0.8327838
Mauritania	0.877673	0.874342	0.890057	0.8857309	0.8656874	0.8731012
Mauritius	1	1	1.0314453	NA	0.9980848	1
Morocco	0.944869	0.929278	0.9626713	0.9451745	0.9300511	0.928908
Mozambique	0.637554	0.6276	0.6417001	0.6275242	0.6361646	0.6256026
Niger	0.780878	0.786741	0.8055554	0.7944472	0.7643902	0.7744603
Nigeria	1	1	NA	NA	1	1
Rwanda	1	1	NA	NA	1	1
Sao Tome and Principe	1	1	NA	NA	1	1
Sierra Leone	1	1	0.9167149	NA	0.9563393	1
South Africa	0.709216	0.692438	0.7113656	0.7040356	0.6972011	0.6923056
Sudan	1	1	0.943369	NA	0.9861746	1
Togo	0.782101	0.811641	0.7752924	0.8010492	0.7812908	0.7999211
Tunisia	1	1	NA	NA	1	1
U.R. of Tanzania:						
Mainland	1	1	0.9718231	NA	0.9691434	1
Zimbabwe	0.828893	0.691199	0.828683	0.7188764	0.79748	0.7018845

Table 32 Efficiency estimates with infeasibilities: 2018-2019 for Carbon Dioxide Emissions

MEA Variable Specific Efficiency (CO2 emissions)						
Country	Efficiency 2018	Efficiency 2019	Efficiency 2018r2019	Efficiency 2019r2018	Efficiency 2018r Biennial	Efficiency 2019r Biennial
Angola	1	0.953531	NA	0.9396082	1	0.941154
Benin	0.677198	0.693727	0.6693419	0.689021	0.6813247	0.6873104
Botswana	0.915608	0.898659	0.8563976	NA	0.8613434	0.8986592
Burkina Faso	0.753809	0.755348	0.8335429	0.7455779	0.7568031	0.7465883
Burundi	0.886918	0.869022	0.9166931	0.8596576	0.8824461	0.8586692
Cabo Verde	0.739347	0.745176	0.788001	0.7387416	0.7389746	0.7377004
Cameroon	0.839701	0.897638	0.8633194	0.8771355	0.8437714	0.8768426
Central African Republic	1	1	NA	NA	1	1
Chad	0.795048	0.826342	0.8547703	0.7878057	0.7995033	0.7921911
Côte d'Ivoire	1	1	NA	NA	1	1
Djibouti	0.941058	1	1.0099053	0.981627	0.9329771	0.9762347
Egypt	1	1	NA	NA	1	1
Eswatini	1	1	1.0331897	NA	0.9872689	1
Gabon	1	1	1.0057359	NA	0.9873671	1
Guinea	0.849929	0.859748	0.8841537	0.8571936	0.8351785	0.8435439
Kenya	1	1	NA	NA	1	1
Lesotho	0.787637	0.780073	0.8335369	0.7712573	0.7871656	0.7711438
Mauritania	0.740312	0.748341	0.7839753	0.7518919	0.7340695	0.7459526
Mauritius	1	1	1.0058423	NA	0.9976504	1
Morocco	0.8136	0.797675	0.869711	0.8112203	0.8034241	0.7973377
Mozambique	0.621781	0.610408	0.6733524	0.6063304	0.623097	0.6072785
Niger	0.785338	0.752611	0.8007497	0.7803561	0.758137	0.7529813
Nigeria	1	1	NA	NA	1	1
Rwanda	1	1	NA	NA	1	1
Sao Tome and Principe	1	1	NA	NA	1	1
Sierra Leone	1	1	1.0235099	NA	0.9924696	1
South Africa	0.653715	0.64432	0.6517843	0.6522804	0.6454978	0.6441293
Sudan	1	1	0.9887013	NA	0.9971894	1
Togo	0.670061	0.689574	0.7029585	0.6759705	0.671437	0.6813556
Tunisia	1	1	NA	NA	1	1
U.R. of Tanzania:						
Mainland	1	1	0.7063119	NA	0.6963401	1
Zimbabwe	0.75642	0.664004	0.7227399	0.6882898	0.7381395	0.6817057

Table 33 Efficiency estimates with infeasibilities: 2018-2019 for GDP

Country	MEA Variable Specific Efficiency (GDP)					
	Efficiency 2018	Efficiency 2019	Efficiency 2018r2019	Efficiency 2019r2018	Efficiency 2018r Biennial	Efficiency 2019r Biennial
Angola	1	0.994121	NA	0.9740374	1	0.9742829
Benin	0.841557	0.853432	0.8057744	0.8577284	0.8353584	0.8469299
Botswana	0.995807	0.989272	0.9599833	NA	0.9767418	0.9892724
Burkina Faso	0.929007	0.918256	0.9042809	0.9123118	0.922702	0.9108756
Burundi	0.905572	0.880393	0.8977852	0.8781997	0.8967747	0.8743624
Cabo Verde	0.831876	0.83933	0.8072741	0.8425793	0.8246763	0.8345731
Cameroon	0.939154	0.948714	0.9206862	0.9522645	0.9312911	0.9472127
Central African Republic	1	1	NA	NA	1	1
Chad	0.960071	0.976152	0.9634873	0.9509758	0.9567247	0.9490057
Côte d'Ivoire	1	1	NA	NA	1	1
Djibouti	0.986303	1	0.9354623	0.9927851	0.9834194	0.9911409
Egypt	1	1	NA	NA	1	1
Eswatini	1	1	0.9895104	NA	0.9967901	1
Gabon	1	1	0.9708499	NA	0.9962073	1
Guinea	0.972128	0.966943	0.9330152	0.9742185	0.9617938	0.9628497
Kenya	1	1	NA	NA	1	1
Lesotho	0.87628	0.865827	0.8746715	0.8595389	0.8746781	0.8580912
Mauritania	0.888033	0.890591	0.8533931	0.8959426	0.8819044	0.8895095
Mauritius	1	1	0.972043	NA	0.9995516	1
Morocco	0.96006	0.944783	0.9255971	0.9597889	0.9463195	0.944837
Mozambique	0.733512	0.670566	0.7275552	0.6681568	0.7305263	0.6656704
Niger	0.859889	0.845334	0.8143852	0.8676517	0.8312584	0.8424223
Nigeria	1	1	NA	NA	1	1
Rwanda	1	1	NA	NA	1	1
Sao Tome and Principe	1	1	NA	NA	1	1
Sierra Leone	1	1	0.959823	NA	0.9939337	1
South Africa	0.909235	0.899532	0.897009	0.9111482	0.8976233	0.899532
Sudan	1	1	0.989926	NA	0.9992707	1
Togo	0.838313	0.862615	0.8176896	0.852764	0.8324282	0.8526787
Tunisia	1	1	NA	NA	1	1
U.R. of Tanzania:						
Mainland	1	1	0.9392356	NA	0.9724141	1
Zimbabwe	0.970121	0.925523	1.0207054	0.9201732	0.9628292	0.9134997

5.5 Productivity change assessments from 2000 to 2019

The second and third objectives are to empirically apply the proposed productivity index to determine the energy productivity change of African states of this study and to decompose the developed productivity index into Pure Technical Change, Pure Efficiency Change, Scale Technical Change and Scale Efficiency Change. This captures the dynamic changes in energy use and to apply it to the case of African countries. This is crucial for understanding the energy performance of African countries over time and for designing effective policies to improve it. The proposed productivity index decomposes the total productivity change into four components: pure technical change, pure efficiency change, scale technical change and scale efficiency change. By doing so, sources of productivity change and the factors that affect them can be identified. These components reflect the changes in technology, efficiency and scale of production that influence the energy use of a country. An estimate and decomposition of the dynamic variable specific productivity of African countries using a balanced panel data set covering the period from 2000 to 2019.

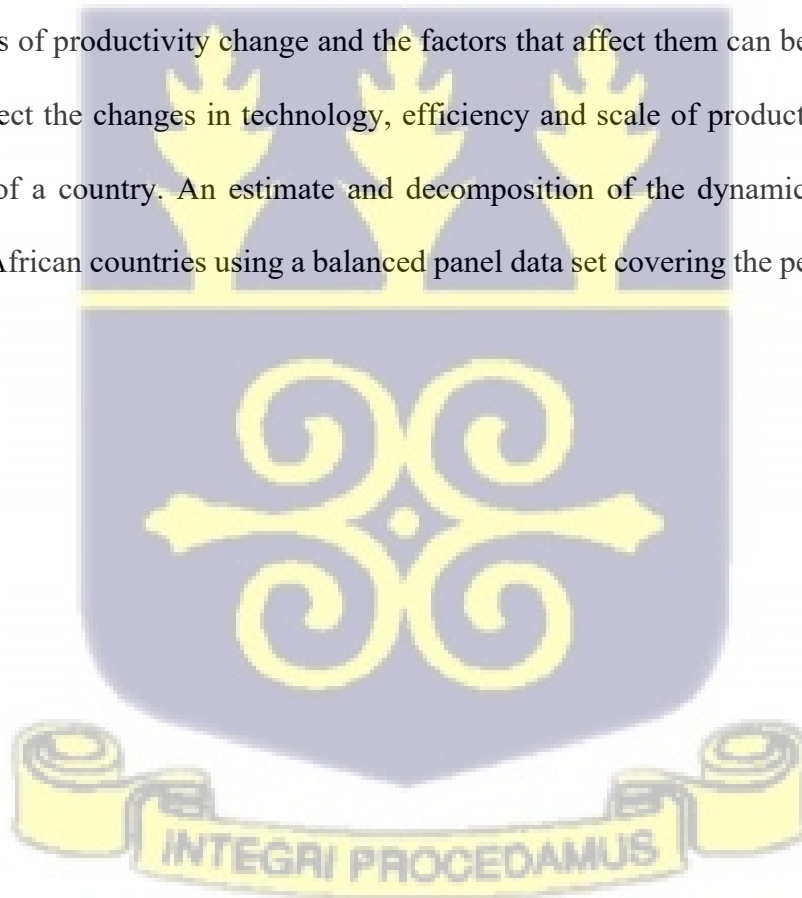


Table 34 Efficiency and Productivity estimates with infeasibilities: 2018-2019 for capital service (xa)

Country	Eff 2018	Eff 2019	Eff 2018r2019	Eff 2018r2019	MPIxa	ECxa	TCxa
Angola	1.0000	0.8048	NA	0.7837	NA	1.2425	NA
Benin	0.8422	0.8336	0.9585	0.8288	0.9347	1.0103	0.9251
Botswana	0.9075	0.8761	0.9398	NA	NA	1.0358	NA
Burkina Faso	0.9067	0.8690	1.0357	0.8655	0.9338	1.0433	0.8950
Burundi	0.8673	0.8333	0.9027	0.8443	0.9866	1.0408	0.9479
Cabo Verde	0.7870	0.7845	0.8298	0.7863	0.9750	1.0031	0.9719
Cameroon	0.8631	0.8705	0.9453	0.8752	0.9581	0.9915	0.9663
Central African Republic	1.0000	1.0000	NA	NA	NA	1.0000	NA
Chad	0.9450	0.9583	1.0092	0.9341	0.9554	0.9862	0.9688
Côte d'Ivoire	1.0000	1.0000	NA	NA	NA	1.0000	NA
Djibouti	0.9794	1.0000	1.1120	0.9702	0.9244	0.9794	0.9438
Egypt	1.0000	1.0000	NA	NA	NA	1.0000	NA
Eswatini	1.0000	1.0000	0.9796	NA	NA	1.0000	NA
Gabon	1.0000	1.0000	1.0108	NA	NA	1.0000	NA
Guinea	0.9596	0.9437	0.9613	0.9619	1.0087	1.0169	0.9920
Kenya	1.0000	1.0000	NA	NA	NA	1.0000	NA
Lesotho	0.8036	0.7816	0.8749	0.7836	0.9596	1.0281	0.9334
Mauritania	0.7888	0.7922	0.8143	0.7961	0.9867	0.9957	0.9909
Mauritius	1.0000	1.0000	1.0591	NA	NA	1.0000	NA
Morocco	0.7592	0.7463	0.8025	0.7555	0.9786	1.0173	0.9620
Mozambique	0.7588	0.7198	0.9126	0.7156	0.9092	1.0541	0.8625
Niger	0.6768	0.6542	0.6928	0.6804	1.0080	1.0345	0.9743
Nigeria	1.0000	1.0000	NA	NA	NA	1.0000	NA
Rwanda	1.0000	1.0000	NA	NA	NA	1.0000	NA
Sao Tome and Principe	1.0000	1.0000	NA	NA	NA	1.0000	NA
Sierra Leone	1.0000	1.0000	0.9842	NA	NA	1.0000	NA
South Africa	0.6894	0.6924	0.7158	0.6834	0.9750	0.9957	0.9792
Sudan	1.0000	1.0000	1.0080	NA	NA	1.0000	NA
Togo	0.8065	0.8098	0.8569	0.8072	0.9686	0.9960	0.9725
Tunisia	1.0000	1.0000	NA	NA	NA	1.0000	NA
U.R. of Tanzania: Mainland	1.0000	1.0000	1.0265	NA	NA	1.0000	NA
Zimbabwe	0.9696	0.9253	0.9928	0.9233	0.9872	1.0479	0.9421

Table 35 Efficiency and Productivity estimates with infeasibilities: 2018-2019 for Labour (xb)

Country	Eff 2018	Eff 2019	Eff 2018r2019	Eff 2018r2019	MPIxb	ECxb	TCxb
Angola	1.0000	0.9688	NA	0.9545	NA	1.0322	NA
Benin	0.6940	0.7132	0.7140	0.7222	0.9921	0.9731	1.0196
Botswana	0.9791	0.9505	0.9405	NA	NA	1.0301	NA
Burkina Faso	0.7656	0.7947	0.8140	0.7661	0.9522	0.9634	0.9884
Burundi	0.7281	0.7024	0.7349	0.6969	0.9915	1.0366	0.9564
Cabo Verde	0.8348	0.8404	0.8470	0.8407	0.9930	0.9934	0.9996
Cameroon	0.7974	0.8268	0.8390	0.8219	0.9720	0.9645	1.0079
Central African Republic	1.0000	1.0000	NA	NA	NA	1.0000	NA
Chad	0.8536	0.8725	0.9036	0.8455	0.9568	0.9784	0.9779
Côte d'Ivoire	1.0000	1.0000	NA	NA	NA	1.0000	NA
Djibouti	0.9401	1.0000	0.9883	0.9712	0.9612	0.9401	1.0224
Egypt	1.0000	1.0000	NA	NA	NA	1.0000	NA
Eswatini	1.0000	1.0000	1.0124	NA	NA	1.0000	NA
Gabon	1.0000	1.0000	1.0141	NA	NA	1.0000	NA
Guinea	0.8948	0.8674	0.9069	0.8835	1.0026	1.0317	0.9718
Kenya	1.0000	1.0000	NA	NA	NA	1.0000	NA
Lesotho	0.7633	0.7561	0.7795	0.7534	0.9878	1.0095	0.9785
Mauritania	0.8065	0.8087	0.8235	0.8127	0.9921	0.9973	0.9948
Mauritius	1.0000	1.0000	1.0099	NA	NA	1.0000	NA
Morocco	0.8749	0.8330	0.8407	0.8712	1.0432	1.0502	0.9933
Mozambique	0.6049	0.5938	0.6281	0.5914	0.9794	1.0188	0.9613
Niger	0.6811	0.6660	0.6675	0.6908	1.0287	1.0226	1.0060
Nigeria	1.0000	1.0000	NA	NA	NA	1.0000	NA
Rwanda	1.0000	1.0000	NA	NA	NA	1.0000	NA
Sao Tome and Principe	1.0000	1.0000	NA	NA	NA	1.0000	NA
Sierra Leone	1.0000	1.0000	0.9954	NA	NA	1.0000	NA
South Africa	0.9166	0.9085	0.9057	0.9182	1.0113	1.0089	1.0024
Sudan	1.0000	1.0000	1.0185	NA	NA	1.0000	NA
Togo	0.7260	0.7461	0.7474	0.7386	0.9806	0.9730	1.0078
Tunisia	1.0000	1.0000	NA	NA	NA	1.0000	NA
U.R. of Tanzania: Mainland	1.0000	1.0000	0.8117	NA	NA	1.0000	NA
Zimbabwe	0.6148	0.5679	0.6811	0.5967	0.9739	1.0826	0.8996

Table 36 Efficiency and Productivity estimates with infeasibilities: 2018-2019 for Energy Used (xc)

Country	Eff 2018	Eff 2019	Eff 2018r2019	Eff 2018r2019	MPIxc	ECxc	TCxc
Angola	1.0000	0.9926	NA	0.9603	NA	1.0074	NA
Benin	0.7790	0.8173	0.7110	0.8196	1.0481	0.9531	1.0997
Botswana	0.9919	0.9801	0.9576	NA	NA	1.0121	NA
Burkina Faso	0.8615	0.8626	0.8647	0.8640	0.9989	0.9986	1.0003
Burundi	0.8808	0.8706	1.0279	0.8399	0.9092	1.0116	0.8987
Cabo Verde	0.7995	0.8284	0.8163	0.8134	0.9807	0.9651	1.0162
Cameroon	0.9180	0.9338	0.9178	0.9373	1.0020	0.9830	1.0193
Central African Republic	1.0000	1.0000	NA	NA	NA	1.0000	NA
Chad	0.8837	0.9207	0.9585	0.8767	0.9370	0.9598	0.9762
Côte d'Ivoire	1.0000	1.0000	NA	NA	NA	1.0000	NA
Djibouti	0.9459	1.0000	0.8981	0.9849	1.0185	0.9459	1.0768
Egypt	1.0000	1.0000	NA	NA	NA	1.0000	NA
Eswatini	1.0000	1.0000	0.9629	NA	NaN	1.0000	NaN
Gabon	1.0000	1.0000	0.9572	NA	NA	1.0000	NA
Guinea	0.9140	0.9137	0.9779	0.9105	0.9651	1.0004	0.9647
Kenya	1.0000	1.0000	NA	NA	NA	1.0000	NA
Lesotho	0.8516	0.8497	0.9058	0.8375	0.9626	1.0022	0.9605
Mauritania	0.8777	0.8743	0.8901	0.8857	0.9995	1.0038	0.9957
Mauritius	1.0000	1.0000	1.0314	NA	NA	1.0000	NA
Morocco	0.9449	0.9293	0.9627	0.9452	0.9991	1.0168	0.9827
Mozambique	0.6376	0.6276	0.6417	0.6275	0.9967	1.0159	0.9811
Niger	0.7809	0.7867	0.8056	0.7944	0.9894	0.9925	0.9968
Nigeria	1.0000	1.0000	NA	NA	NA	1.0000	NA
Rwanda	1.0000	1.0000	NA	NA	NA	1.0000	NA
Sao Tome and Principe	1.0000	1.0000	NA	NA	NA	1.0000	NA
Sierra Leone	1.0000	1.0000	0.9167	NA	NA	1.0000	NA
South Africa	0.7092	0.6924	0.7114	0.7040	1.0068	1.0242	0.9830
Sudan	1.0000	1.0000	0.9434	NA	NA	1.0000	NA
Togo	0.7821	0.8116	0.7753	0.8010	0.9978	0.9636	1.0355
Tunisia	1.0000	1.0000	NA	NA	NA	1.0000	NA
U.R. of Tanzania: Mainland	1.0000	1.0000	0.9718	NA	NA	1.0000	NA
Zimbabwe	0.8289	0.6912	0.8287	0.7189	1.0200	1.1992	0.8505

Table 37 Efficiency and Productivity estimates with infeasibilities: 2018-2019 for Carbon Dioxide Emission

Country	Eff 2018	Eff 2019	Eff 2018r2019	Eff 2018r2019	MPIyb	ECyb	TCyb
Angola	1.0000	0.9535	NA	0.9396	NA	1.0487	NA
Benin	0.6772	0.6937	0.6693	0.6890	1.0024	0.9762	1.0269
Botswana	0.9156	0.8987	0.8564	NA	NA	1.0189	NA
Burkina Faso	0.7538	0.7553	0.8335	0.7456	0.9448	0.9980	0.9467
Burundi	0.8869	0.8690	0.9167	0.8597	0.9783	1.0206	0.9586
Cabo Verde	0.7393	0.7452	0.7880	0.7387	0.9644	0.9922	0.9720
Cameroon	0.8397	0.8976	0.8633	0.8771	0.9749	0.9355	1.0422
Central African Republic	1.0000	1.0000	NA	NA	NA	1.0000	NA
Chad	0.7950	0.8263	0.8548	0.7878	0.9417	0.9621	0.9787
Côte d'Ivoire	1.0000	1.0000	NA	NA	NA	1.0000	NA
Djibouti	0.9411	1.0000	1.0099	0.9816	0.9564	0.9411	1.0163
Egypt	1.0000	1.0000	NA	NA	NA	1.0000	NA
Eswatini	1.0000	1.0000	1.0332	NA	NaN	1.0000	NaN
Gabon	1.0000	1.0000	1.0057	NA	NA	1.0000	NA
Guinea	0.8499	0.8597	0.8842	0.8572	0.9790	0.9886	0.9903
Kenya	1.0000	1.0000	NA	NA	NA	1.0000	NA
Lesotho	0.7876	0.7801	0.8335	0.7713	0.9666	1.0097	0.9573
Mauritania	0.7403	0.7483	0.7840	0.7519	0.9741	0.9893	0.9846
Mauritius	1.0000	1.0000	1.0058	NA	NA	1.0000	NA
Morocco	0.8136	0.7977	0.8697	0.8112	0.9754	1.0200	0.9563
Mozambique	0.6218	0.6104	0.6734	0.6063	0.9577	1.0186	0.9402
Niger	0.7853	0.7526	0.8007	0.7804	1.0084	1.0435	0.9664
Nigeria	1.0000	1.0000	NA	NA	NA	1.0000	NA
Rwanda	1.0000	1.0000	NA	NA	NA	1.0000	NA
Sao Tome and Principe	1.0000	1.0000	NA	NA	NA	1.0000	NA
Sierra Leone	1.0000	1.0000	1.0235	NA	NA	1.0000	NA
South Africa	0.6537	0.6443	0.6518	0.6523	1.0076	1.0146	0.9932
Sudan	1.0000	1.0000	0.9887	NA	NA	1.0000	NA
Togo	0.6701	0.6896	0.7030	0.6760	0.9666	0.9717	0.9948
Tunisia	1.0000	1.0000	NA	NA	NA	1.0000	NA
U.R. of Tanzania: Mainland	1.0000	1.0000	0.7063	NA	NA	1.0000	NA
Zimbabwe	0.7564	0.6640	0.7227	0.6883	1.0416	1.1392	0.9143

Table 38 Efficiency and Productivity estimates with infeasibilities: 2018-2019 for Real GDP

Country	Eff 2018	Eff 2019	Eff 2018r2019	Eff 2018r2019	MPI _{ya}	EC _{ya}	TC _{ya}
Angola	1.0000	0.9941	NA	0.9740	NA	1.0059	NA
Benin	0.8416	0.8534	0.8058	0.8577	1.0245	0.9861	1.0390
Botswana	0.9958	0.9893	0.9600	NA	NA	1.0066	NA
Burkina Faso	0.9290	0.9183	0.9043	0.9123	1.0103	1.0117	0.9986
Burundi	0.9056	0.8804	0.8978	0.8782	1.0031	1.0286	0.9752
Cabo Verde	0.8319	0.8393	0.8073	0.8426	1.0171	0.9911	1.0262
Cameroon	0.9392	0.9487	0.9207	0.9523	1.0119	0.9899	1.0222
Central African Republic	1.0000	1.0000	NA	NA	NA	1.0000	NA
Chad	0.9601	0.9762	0.9635	0.9510	0.9853	0.9835	1.0018
Côte d'Ivoire	1.0000	1.0000	NA	NA	NA	1.0000	NA
Djibouti	0.9863	1.0000	0.9355	0.9928	1.0231	0.9863	1.0373
Egypt	1.0000	1.0000	NA	NA	NA	1.0000	NA
Eswatini	1.0000	1.0000	0.9895	NA	NA	1.0000	NA
Gabon	1.0000	1.0000	0.9708	NA	NA	1.0000	NA
Guinea	0.9721	0.9669	0.9330	0.9742	1.0246	1.0054	1.0191
Kenya	1.0000	1.0000	NA	NA	NA	1.0000	NA
Lesotho	0.8763	0.8658	0.8747	0.8595	0.9973	1.0121	0.9854
Mauritania	0.8880	0.8906	0.8534	0.8959	1.0232	0.9971	1.0261
Mauritius	1.0000	1.0000	0.9720	NA	NA	1.0000	NA
Morocco	0.9601	0.9448	0.9256	0.9598	1.0265	1.0162	1.0102
Mozambique	0.7335	0.6706	0.7276	0.6682	1.0023	1.0939	0.9163
Niger	0.8599	0.8453	0.8144	0.8677	1.0410	1.0172	1.0234
Nigeria	1.0000	1.0000	NA	NA	NA	1.0000	NA
Rwanda	1.0000	1.0000	NA	NA	NA	1.0000	NA
Sao Tome and Principe	1.0000	1.0000	NA	NA	NA	1.0000	NA
Sierra Leone	1.0000	1.0000	0.9598	NA	NA	1.0000	NA
South Africa	0.9092	0.8995	0.8970	0.9111	1.0133	1.0108	1.0025
Sudan	1.0000	1.0000	0.9899	NA	NA	1.0000	NA
Togo	0.8383	0.8626	0.8177	0.8528	1.0067	0.9718	1.0359
Tunisia	1.0000	1.0000	NA	NA	NA	1.0000	NA
U.R. of Tanzania:							
Mainland	1.0000	1.0000	0.9392	NA	NA	1.0000	NA
Zimbabwe	0.9701	0.9255	1.0207	0.9202	0.9721	1.0482	0.9274

Tables 34 to 38 displays the average Malmquist indices and their breakdowns for the periods 2018 to 2019. It can be observed that the presence of LP infeasibilities results in some MEA variable specific scores of the Malmquist index (MPI) yielding scores with LP infeasibilities.

In these tables are also listed the country name, VRS-MEA variable specific own-period and cross-period efficiency scores in years 2018 and 2019, the productivity estimates (MPI), which is split into the efficiency change (EC) and technological change (TC) respectively from columns 1 to 8. Clearly, from the MPI column, 16 out of the 32 countries were affected by LP mixed-period infeasibilities, representing 50% "Not Available" (NA) which would lead to significantly inaccurate conclusions in the adjacent Malmquist index. They have not been fully separated because these numerous infeasibilities generate average values based on fewer observations than for the MEA-based biennial Malmquist index. This illustration highlights the potential risks of being forced to use a subset of observations, which is the likely effect of breaking down the VMPI.

Table 39 Efficiency and Productivity estimates after handling infeasibilities: 2018-2019 for Capital Services

Country	Eff 2018r2019	Eff 2018r2019	MPI _{xa}	EC _{xa}	TC _{xa}	BMI _{xa}	BPEC _{xa}	BPTC _{xa}
Angola	NA	0.7837	NA	1.2425	NA	1.2797	1.2425	1.0299
Benin	0.9585	0.8288	0.9347	1.0103	0.9251	1.0194	1.0103	1.0089
Botswana	0.9398	NA	NA	1.0358	NA	0.9927	1.0358	0.9584
Burkina Faso	1.0357	0.8655	0.9338	1.0433	0.8950	1.0367	1.0433	0.9936
Burundi	0.9027	0.8443	0.9866	1.0408	0.9479	1.0213	1.0408	0.9812
Cabo Verde	0.8298	0.7863	0.9750	1.0031	0.9719	1.0012	1.0031	0.9981
Cameroon	0.9453	0.8752	0.9581	0.9915	0.9663	0.9965	0.9915	1.0051
Central African Republic	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Chad	1.0092	0.9341	0.9554	0.9862	0.9688	1.0106	0.9862	1.0248
Côte d'Ivoire	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Djibouti	1.1120	0.9702	0.9244	0.9794	0.9438	1.0079	0.9794	1.0291
Egypt	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Eswatini	0.9796	NA	NaN	1.0000	NaN	0.9870	1.0000	0.9870
Gabon	1.0108	NA	NA	1.0000	NA	0.9843	1.0000	0.9843

Guinea	0.9613	0.9619	1.0087	1.0169	0.9920	0.9964	1.0169	0.9798
Kenya	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Lesotho	0.8749	0.7836	0.9596	1.0281	0.9334	1.0255	1.0281	0.9975
Mauritania	0.8143	0.7961	0.9867	0.9957	0.9909	0.9913	0.9957	0.9955
Mauritius	1.0591	NA	NA	1.0000	NA	0.9982	1.0000	0.9982
Morocco	0.8025	0.7555	0.9786	1.0173	0.9620	1.0109	1.0173	0.9938
Mozambique	0.9126	0.7156	0.9092	1.0541	0.8625	1.0599	1.0541	1.0054
Niger	0.6928	0.6804	1.0080	1.0345	0.9743	0.9952	1.0345	0.9620
Nigeria	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Rwanda	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Sao Tome and Principe	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Sierra Leone	0.9842	NA	NA	1.0000	NA	0.9846	1.0000	0.9846
South Africa	0.7158	0.6834	0.9750	0.9957	0.9792	1.0088	0.9957	1.0131
Sudan	1.0080	NA	NA	1.0000	NA	0.9987	1.0000	0.9987
Togo	0.8569	0.8072	0.9686	0.9960	0.9725	0.9952	0.9960	0.9993
Tunisia	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
U.R. of Tanzania: Mainland	1.0265	NA	NA	1.0000	NA	0.8969	1.0000	0.8969
Zimbabwe	0.9928	0.9233	0.9872	1.0479	0.9421	1.0497	1.0479	1.0018
Geomean	0.9253	0.8186	0.9652	1.0153	0.9511	1.0096	1.0153	0.9943
Min	0.6928	0.6804	0.9092	0.9794	0.8625	0.8969	0.9794	0.8969
Max	1.1120	0.9702	1.0087	1.2425	0.9920	1.2797	1.2425	1.0299

Table 40 Efficiency and Productivity estimates after handling infeasibilities: 2018-2019 for Labour

Country	Eff 2018r2019	Eff 2019r2018	MPIxb	ECxb	TCxb	BMIxb	PECxb	PTCxb
Angola	NA	0.9545	NA	1.0322	NA	1.0498	1.0322	1.0171
Benin	0.7140	0.7222	0.9921	0.9731	1.0196	0.9861	0.9731	1.0134
Botswana	0.9405	NA	NA	1.0301	NA	0.9842	1.0301	0.9555
Burkina Faso	0.8140	0.7661	0.9522	0.9634	0.9884	0.9925	0.9634	1.0301
Burundi	0.7349	0.6969	0.9915	1.0366	0.9564	1.0331	1.0366	0.9966
Cabo Verde	0.8470	0.8407	0.9930	0.9934	0.9996	0.9936	0.9934	1.0001
Cameroon	0.8390	0.8219	0.9720	0.9645	1.0079	0.9723	0.9645	1.0082
Central African Republic	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Chad	0.9036	0.8455	0.9568	0.9784	0.9779	1.0111	0.9784	1.0334
Côte d'Ivoire	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Djibouti	0.9883	0.9712	0.9612	0.9401	1.0224	0.9641	0.9401	1.0256
Egypt	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Eswatini	1.0124	NA	NaN	1.0000	NaN	0.9893	1.0000	0.9893
Gabon	1.0141	NA	NA	1.0000	NA	0.9961	1.0000	0.9961

Guinea	0.9069	0.8835	1.0026	1.0317	0.9718	1.0184	1.0317	0.9871
Kenya	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Lesotho	0.7795	0.7534	0.9878	1.0095	0.9785	1.0135	1.0095	1.0039
Mauritania	0.8235	0.8127	0.9921	0.9973	0.9948	0.9928	0.9973	0.9954
Mauritius	1.0099	NA	NA	1.0000	NA	0.9989	1.0000	0.9989
Morocco	0.8407	0.8712	1.0432	1.0502	0.9933	0.9990	1.0502	0.9512
Mozambique	0.6281	0.5914	0.9794	1.0188	0.9613	1.0237	1.0188	1.0048
Niger	0.6675	0.6908	1.0287	1.0226	1.0060	0.9846	1.0226	0.9629
Nigeria	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Rwanda	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Sao Tome and Principe	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Sierra Leone	0.9954	NA	NA	1.0000	NA	0.9782	1.0000	0.9782
South Africa	0.9057	0.9182	1.0113	1.0089	1.0024	0.9983	1.0089	0.9894
Sudan	1.0185	NA	NA	1.0000	NA	0.9964	1.0000	0.9964
Togo	0.7474	0.7386	0.9806	0.9730	1.0078	0.9783	0.9730	1.0054
Tunisia	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
U.R. of Tanzania: Mainland	0.8117	NA	NA	1.0000	NA	0.7783	1.0000	0.7783
Zimbabwe	0.6811	0.5967	0.9739	1.0826	0.8996	1.0180	1.0826	0.9404
Geomean	0.8446	0.7849	0.9884	1.0030	0.9863	0.9912	1.0030	0.9883
Min	0.6281	0.5914	0.9522	0.9401	0.8996	0.7783	0.9401	0.7783
Max	1.0185	0.9712	1.0432	1.0826	1.0224	1.0498	1.0826	1.0334

Table 41 Efficiency and Productivity estimates after handling infeasibilities: 2018-2019 for Energy Used

Country	Eff 2018r2019	Eff 2019r2018	MPIxc	ECxc	TCxc	BMIxc	BPECxc	BPTCxc
Angola	NA	0.9603	NA	1.0074	NA	1.0407	1.0074	1.0330
Benin	0.7110	0.8196	1.0481	0.9531	1.0997	0.9572	0.9531	1.0043
Botswana	0.9576	NA	NA	1.0121	NA	0.9770	1.0121	0.9654
Burkina Faso	0.8647	0.8640	0.9989	0.9986	1.0003	0.9894	0.9986	0.9907
Burundi	1.0279	0.8399	0.9092	1.0116	0.8987	1.0438	1.0116	1.0318
Cabo Verde	0.8163	0.8134	0.9807	0.9651	1.0162	0.9840	0.9651	1.0196
Cameroon	0.9178	0.9373	1.0020	0.9830	1.0193	0.9811	0.9830	0.9981
Central African Republic	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Chad	0.9585	0.8767	0.9370	0.9598	0.9762	1.0072	0.9598	1.0494
Côte d'Ivoire	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Djibouti	0.8981	0.9849	1.0185	0.9459	1.0768	0.9613	0.9459	1.0163
Egypt	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Eswatini	0.9629	NA	NaN	1.0000	NaN	0.9937	1.0000	0.9937
Gabon	0.9572	NA	NA	1.0000	NA	0.9810	1.0000	0.9810

Guinea	0.9779	0.9105	0.9651	1.0004	0.9647	1.0070	1.0004	1.0066
Kenya	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Lesotho	0.9058	0.8375	0.9626	1.0022	0.9605	1.0160	1.0022	1.0138
Mauritania	0.8901	0.8857	0.9995	1.0038	0.9957	0.9915	1.0038	0.9877
Mauritius	1.0314	NA	NA	1.0000	NA	0.9981	1.0000	0.9981
Morocco	0.9627	0.9452	0.9991	1.0168	0.9827	1.0012	1.0168	0.9847
Mozambique	0.6417	0.6275	0.9967	1.0159	0.9811	1.0169	1.0159	1.0010
Niger	0.8056	0.7944	0.9894	0.9925	0.9968	0.9870	0.9925	0.9944
Nigeria	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Rwanda	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Sao Tome and Principe	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Sierra Leone	0.9167	NA	NA	1.0000	NA	0.9563	1.0000	0.9563
South Africa	0.7114	0.7040	1.0068	1.0242	0.9830	1.0071	1.0242	0.9832
Sudan	0.9434	NA	NA	1.0000	NA	0.9862	1.0000	0.9862
Togo	0.7753	0.8010	0.9978	0.9636	1.0355	0.9767	0.9636	1.0136
Tunisia	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
U.R. of Tanzania: Mainland	0.9718	NA	NA	1.0000	NA	0.9691	1.0000	0.9691
Zimbabwe	0.8287	0.7189	1.0200	1.1992	0.8505	1.1362	1.1992	0.9475
Geomean	0.8822	0.8369	0.9889	1.0010	0.9881	0.9985	1.0010	0.9975
Min	0.6417	0.6275	0.9092	0.9459	0.8505	0.9563	0.9459	0.9475
Max	1.0314	0.9849	1.0481	1.1992	1.0997	1.1362	1.1992	1.0494

Table 42 Efficiency and Productivity estimates after handling infeasibilities: 2018-2019 for Carbon Dioxide Emissions

Country	Eff 2018r2019	Eff 2018r2019	MPIyb	ECyb	TCyb	BMIyb	PECyb	PTCyb
Angola	NA	0.9396	NA	1.0487	NA	1.0625	1.0487	1.0132
Benin	0.6693	0.6890	1.0024	0.9762	1.0269	0.9913	0.9762	1.0155
Botswana	0.8564	NA	NA	1.0189	NA	0.9585	1.0189	0.9407
Burkina Faso	0.8335	0.7456	0.9448	0.9980	0.9467	1.0137	0.9980	1.0158
Burundi	0.9167	0.8597	0.9783	1.0206	0.9586	1.0277	1.0206	1.0070
Cabo Verde	0.7880	0.7387	0.9644	0.9922	0.9720	1.0017	0.9922	1.0096
Cameroon	0.8633	0.8771	0.9749	0.9355	1.0422	0.9623	0.9355	1.0287
Central African Republic	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Chad	0.8548	0.7878	0.9417	0.9621	0.9787	1.0092	0.9621	1.0490
Côte d'Ivoire	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Djibouti	1.0099	0.9816	0.9564	0.9411	1.0163	0.9557	0.9411	1.0155
Egypt	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Eswatini	1.0332	NA	NaN	1.0000	NaN	0.9873	1.0000	0.9873
Gabon	1.0057	NA	NA	1.0000	NA	0.9874	1.0000	0.9874

Guinea	0.8842	0.8572	0.9790	0.9886	0.9903	0.9901	0.9886	1.0015
Kenya	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Lesotho	0.8335	0.7713	0.9666	1.0097	0.9573	1.0208	1.0097	1.0110
Mauritania	0.7840	0.7519	0.9741	0.9893	0.9846	0.9841	0.9893	0.9947
Mauritius	1.0058	NA	NA	1.0000	NA	0.9977	1.0000	0.9977
Morocco	0.8697	0.8112	0.9754	1.0200	0.9563	1.0076	1.0200	0.9879
Mozambique	0.6734	0.6063	0.9577	1.0186	0.9402	1.0260	1.0186	1.0073
Niger	0.8007	0.7804	1.0084	1.0435	0.9664	1.0068	1.0435	0.9649
Nigeria	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Rwanda	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Sao Tome and Principe	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Sierra Leone	1.0235	NA	NA	1.0000	NA	0.9925	1.0000	0.9925
South Africa	0.6518	0.6523	1.0076	1.0146	0.9932	1.0021	1.0146	0.9877
Sudan	0.9887	NA	NA	1.0000	NA	0.9972	1.0000	0.9972
Togo	0.7030	0.6760	0.9666	0.9717	0.9948	0.9854	0.9717	1.0141
Tunisia	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
U.R. of Tanzania:								
Mainland	0.7063	NA	NA	1.0000	NA	0.6963	1.0000	0.6963
Zimbabwe	0.7227	0.6883	1.0416	1.1392	0.9143	1.0828	1.1392	0.9505
Geomean	0.8382	0.7711	0.9772	1.0022	0.9769	0.9900	1.0022	0.9878
Min	0.6518	0.6063	0.9417	0.9355	0.9143	0.6963	0.9355	0.6963
Max	1.0332	0.9816	1.0416	1.1392	1.0422	1.0828	1.1392	1.0490

Table 43 Efficiency and Productivity estimates after handling infeasibilities: 2018-2019 for Real GDP

Country	Eff 2018r2019	Eff 2018r2019	MPI ₁ _{ya}	EC _{ya}	TC _{ya}	BMI ₁ _{ya}	PEC _{ya}	PTC _{ya}
Angola	NA	0.9740	NA	1.0059	NA	1.0264	1.0059	1.0204
Benin	0.8058	0.8577	1.0245	0.9861	1.0390	0.9863	0.9861	1.0003
Botswana	0.9600	NA	NA	1.0066	NA	0.9873	1.0066	0.9809
Burkina Faso	0.9043	0.9123	1.0103	1.0117	0.9986	1.0130	1.0117	1.0013
Burundi	0.8978	0.8782	1.0031	1.0286	0.9752	1.0256	1.0286	0.9971
Cabo Verde	0.8073	0.8426	1.0171	0.9911	1.0262	0.9881	0.9911	0.9970
Cameroon	0.9207	0.9523	1.0119	0.9899	1.0222	0.9832	0.9899	0.9932
Central African Republic	NA	0.0000	NA	1.0000	NA	1.0000	1.0000	1.0000
Chad	0.9635	0.9510	0.9853	0.9835	1.0018	1.0081	0.9835	1.0250
Côte d'Ivoire	NA	0.0000	NA	1.0000	NA	1.0000	1.0000	1.0000
Djibouti	0.9355	0.9928	1.0231	0.9863	1.0373	0.9922	0.9863	1.0060
Egypt	NA	0.0000	NA	1.0000	NA	1.0000	1.0000	1.0000
Eswatini	0.9895	0.0000	0.0000	1.0000	0.0000	0.9968	1.0000	0.9968

Gabon	0.9708	NA	NA	1.0000	NA	0.9962	1.0000	0.9962
Guinea	0.9330	0.9742	1.0246	1.0054	1.0191	0.9989	1.0054	0.9936
Kenya	NA	0.0000	NA	1.0000	NA	1.0000	1.0000	1.0000
Lesotho	0.8747	0.8595	0.9973	1.0121	0.9854	1.0193	1.0121	1.0072
Mauritania	0.8534	0.8959	1.0232	0.9971	1.0261	0.9915	0.9971	0.9943
Mauritius	0.9720	NA	NA	1.0000	NA	0.9996	1.0000	0.9996
Morocco	0.9256	0.9598	1.0265	1.0162	1.0102	1.0016	1.0162	0.9856
Mozambique	0.7276	0.6682	1.0023	1.0939	0.9163	1.0974	1.0939	1.0033
Niger	0.8144	0.8677	1.0410	1.0172	1.0234	0.9867	1.0172	0.9700
Nigeria	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Rwanda	NA	NA	NA	1.0000	NA	1.0000	1.0000	1.0000
Sao Tome and Principe	NA	0.0000	NA	1.0000	NA	1.0000	1.0000	1.0000
Sierra Leone	0.9598	NA	NA	1.0000	NA	0.9939	1.0000	0.9939
South Africa	0.8970	0.9111	1.0133	1.0108	1.0025	0.9979	1.0108	0.9872
Sudan	0.9899	NA	NA	1.0000	NA	0.9993	1.0000	0.9993
Togo	0.8177	0.8528	1.0067	0.9718	1.0359	0.9763	0.9718	1.0046
Tunisia	NA	0.0000	NA	1.0000	NA	1.0000	1.0000	1.0000
U.R. of Tanzania: Mainland	0.9392	NA	NA	1.0000	NA	0.9724	1.0000	0.9724
Zimbabwe	1.0207	0.9202	0.9721	1.0482	0.9274	1.0540	1.0482	1.0055
Geomean	0.9048	0.8949	1.0112	1.0049	1.0023	1.0026	1.0049	0.9978
Min	0.7276	0.6682	0.9721	0.9718	0.9163	0.9724	0.9718	0.9700
Max	1.0207	0.9928	1.0410	1.0939	1.0390	1.0974	1.0939	1.0250

After using the proposed BMEAM index on the various variables, the results (BMI) of capital services (x_a) is shown in table 39 and applied to other variables shown in tables 40 to 43. These show that, all the 50% LP infeasibilities handled, and extensive sources of the productivity changes are identified. The geometric mean of the productivity of capital services from table 39 indicated $BMI_{xa} = 1.0096$ which would have been $VMPI = 0.9652$, which is much lower and misleading. It should be noted that in situations where a particular observation appears in one year but not the other year, the productivity change score and its component is not calculated but the observation is included in the construction of the biennial frontier.

Table 44 Results of variable specific MEA biennial Malmquist productivity change estimates and its components: 2018-2019 for Capital Service

Country	BMIcxa	BMIvxa	BPECxa	BPTCxa	BSECxa	BSTCxa
Angola	1.261837	1.279666	1.242497	1.029915	0.997677	0.988363
Benin	1.017945	1.019366	1.010332	1.008941	1.003916	0.99471
Botswana	1.012523	0.992681	1.035766	0.958403	0.983094	1.037529
Burkina Faso	1.028676	1.036671	1.043326	0.993621	0.994875	0.997399
Burundi	1.019811	1.0213	1.040818	0.981248	0.987514	1.011167
Cabo Verde	0.996136	1.001204	1.003134	0.998076	0.99187	1.003093
Cameroon	1.000829	0.996518	0.991501	1.00506	1.00417	1.000156
Central African Republic	1.010653	1	1	1	1.041539	0.970346
Chad	1.013612	1.010571	0.98616	1.024754	1.069494	0.937835
Côte d'Ivoire	1	1	1	1	1	1
Djibouti	1.007348	1.007875	0.979391	1.029083	1.022536	0.977449
Egypt	1	1	1	1	1	1
Eswatini	0.983112	0.98697	1	0.98697	0.988385	1.007797
Gabon	0.982131	0.98428	1	0.98428	1	0.997817
Guinea	0.995167	0.996385	1.01691	0.979817	0.999685	0.999092
Kenya	1.021837	1	1	1	1.007881	1.013847
Lesotho	1.022894	1.025527	1.028106	0.997492	0.996536	1.000899
Mauritania	0.991142	0.991277	0.995734	0.995524	0.996909	1.002964
Mauritius	0.993358	0.998203	1	0.998203	1	0.995147
Morocco	1.010732	1.010942	1.017264	0.993785	0.991849	1.008009
Mozambique	1.05288	1.05987	1.054139	1.005436	1.008741	0.984796
Niger	0.986098	0.995222	1.034548	0.961987	0.976452	1.014728
Nigeria	1.007142	1	1	1	0.982645	1.02493
Rwanda	1	1	1	1	1	1
Sao Tome and Principe	1.016206	1	1	1	1.011308	1.004844
Sierra Leone	0.980997	0.984636	1	0.984636	1	0.996304
South Africa	1.007451	1.008762	0.99572	1.013098	1.004769	0.993961
Sudan	0.99868	0.998731	1	0.998731	1	0.999949
Togo	0.99723	0.995222	0.995956	0.999263	1.001165	1.000852
Tunisia	1	1	1	1	1	1
U.R. of Tanzania:						
Mainland	1.020998	0.89691	1	0.89691	1.021845	1.114016
Zimbabwe	1.05463	1.049749	1.04788	1.001784	1.0111	0.99362
Geomean	1.014395	1.009572	1.015343	0.994316	1.002857	1.001916

Table 45 Results of variable specific MEA biennial Malmquist productivity change estimates and its components: 2018-2019 for Labour

Country	BMIcxb	BMIvxb	BPECxb	BPTCxb	BSECxb	BSTCxb
Angola	1.109311	1.049798	1.032185	1.017064	1.062475	0.994555
Benin	0.985751	0.986118	0.973074	1.013405	1.002368	0.997267
Botswana	0.992571	0.984226	1.030064	0.9555	0.959914	1.050593
Burkina Faso	0.990034	0.992468	0.963446	1.030124	1.016641	0.981219
Burundi	1.033685	1.033143	1.036622	0.996645	0.981389	1.019498
Cabo Verde	0.988968	0.993555	0.993422	1.000134	0.997848	0.99753
Cameroon	0.976519	0.972322	0.964458	1.008154	1.000425	1.00389
Central African Republic	1.003344	1	1	1	1.041481	0.963382
Chad	1.011387	1.011106	0.978386	1.033443	1.016016	0.984509
Côte d'Ivoire	1	1	1	1	1	1
Djibouti	0.953876	0.964136	0.940092	1.025576	1.01002	0.979544
Egypt	1	1	1	1	1	1
Eswatini	0.975902	0.989324	1	0.989324	0.99859	0.987826
Gabon	0.995342	0.996053	1	0.996053	1	0.999287
Guinea	1.009702	1.018401	1.03167	0.987138	0.99528	0.99616
Kenya	0.972708	1	1	1	0.953501	1.020143
Lesotho	1.012482	1.013471	1.009532	1.003901	1.001919	0.997111
Mauritania	0.991948	0.992761	0.997337	0.995412	0.996898	1.002291
Mauritius	0.99712	0.998873	1	0.998873	1	0.998245
Morocco	0.999479	0.998962	1.0502	0.951211	0.957571	1.044851
Mozambique	1.021143	1.02365	1.018779	1.004782	1.000995	0.996559
Niger	0.981751	0.984612	1.02258	0.96287	0.974915	1.022751
Nigeria	1.009531	1	1	1	1.009858	0.999676
Rwanda	1	1	1	1	1	1
Sao Tome and Principe	1.024243	1	1	1	1.021186	1.002994
Sierra Leone	0.972924	0.978168	1	0.978168	1	0.994639
South Africa	0.998556	0.998264	1.008918	0.98944	0.998905	1.001389
Sudan	0.996131	0.996413	1	0.996413	1	0.999717
Togo	0.977947	0.978291	0.973008	1.00543	1.000882	0.998767
Tunisia	1	1	1	1	1	1
U.R. of Tanzania:						
Mainland	0.974232	0.778321	1	0.778321	0.98261	1.273862
Zimbabwe	1.067116	1.01804	1.082582	0.940381	0.957035	1.095264
Geomean	1.00035	0.991196	1.002976	0.988255	0.997844	1.011416

Table 46 Results of variable specific MEA biennial Malmquist productivity change estimates and its components: 2018-2019 for Energy used.

Country	BMIcxc	BMIvxc	BPECxc	BPTCxc	BSECxc	BSTCxc
Angola	1.039129	1.04068	1.007418	1.033017	1.027831	0.971472
Benin	0.957207	0.957189	0.953104	1.004286	0.999551	1.000468
Botswana	0.989656	0.977022	1.012065	0.965375	0.995913	1.017089
Burkina Faso	0.989834	0.989359	0.998641	0.990706	1.001038	0.999442
Burundi	1.039905	1.043776	1.011645	1.031762	1.007916	0.988466
Cabo Verde	0.983481	0.983961	0.965054	1.019592	1.026607	0.973608
Cameroon	0.9869	0.981141	0.982992	0.998117	1.006125	0.999746
Central African Republic	0.994191	1	1	1	0.988038	1.006228
Chad	1.00702	1.007218	0.959819	1.049383	1.014035	0.985966
Côte d'Ivoire	1	1	1	1	1	1
Djibouti	0.953488	0.96133	0.94593	1.016279	1.003345	0.988537
Egypt	1	1	1	1	1	1
Eswatini	0.992325	0.993733	1	0.993733	1.0002	0.998383
Gabon	0.980508	0.980971	1	0.980971	1	0.999527
Guinea	1.004302	1.007013	1.00038	1.00663	1.012531	0.984966
Kenya	0.979573	1	1	1	0.975932	1.003731
Lesotho	1.015148	1.016025	1.002195	1.0138	1.017935	0.981533
Mauritania	0.991334	0.991509	1.003809	0.987746	0.998317	1.00151
Mauritius	0.994208	0.998085	1	0.998085	1	0.996116
Morocco	1.005486	1.001231	1.016777	0.98471	1.003086	1.001161
Mozambique	1.017463	1.016883	1.015861	1.001006	0.995316	1.005279
Niger	0.990153	0.986997	0.992547	0.994409	0.988479	1.01489
Nigeria	1.003175	1	1	1	1.006518	0.996679
Rwanda	1	1	1	1	1	1
Sao Tome and Principe	1.103194	1	1	1	1.111637	0.992404
Sierra Leone	0.949868	0.956339	1	0.956339	1	0.993233
South Africa	1.00492	1.007071	1.02423	0.983247	0.99645	1.001419
Sudan	0.986055	0.986175	1	0.986175	1	0.999879
Togo	0.976723	0.97671	0.963604	1.013601	1.005675	0.99437
Tunisia	1	1	1	1	1	1
U.R. of Tanzania:						
Mainland	0.976277	0.969143	1	0.969143	0.981509	1.026339
Zimbabwe	1.086231	1.136198	1.19921	0.947456	0.897303	1.065439
Geomean	0.999453	0.998459	1.001002	0.99746	1.001502	0.999494

Table 47 Results of variable specific MEA biennial Malmquist productivity change estimates and its components: 2018-2019 for Carbon Dioxide Emission

Country	BMlcyb	BMlvyb	BPECyb	BPTCyb	BSECyb	BSTCyb
Angola	1.11755	1.062525	1.048734	1.013151	1.059247	0.992957
Benin	0.991195	0.991291	0.976175	1.015486	1.00264	0.99727
Botswana	0.974913	0.958476	1.01886	0.940734	0.963891	1.055254
Burkina Faso	1.009303	1.013682	0.997962	1.015752	1.010863	0.98498
Burundi	1.027097	1.02769	1.020593	1.006954	0.999851	0.999572
Cabo Verde	0.99307	1.001727	0.992177	1.009625	0.997695	0.993648
Cameroon	0.96429	0.962284	0.935457	1.028678	1.034704	0.968475
Central African Republic	1.003479	1	1	1	1.026374	0.977694
Chad	1.010044	1.00923	0.962129	1.048956	1.021881	0.979377
Côte d'Ivoire	1	1	1	1	1	1
Djibouti	0.902758	0.955689	0.941058	1.015547	0.963064	0.980843
Egypt	1	1	1	1	1	1
Eswatini	0.981085	0.987269	1	0.987269	0.998499	0.99523
Gabon	0.985282	0.987367	1	0.987367	1	0.997888
Guinea	0.999363	0.990083	0.988578	1.001522	1.031106	0.978923
Kenya	1.011053	1	1	1	1.010561	1.000487
Lesotho	1.018417	1.020777	1.009697	1.010973	1.008501	0.989278
Mauritania	0.983359	0.98407	0.989271	0.994742	0.998739	1.000539
Mauritius	0.993478	0.99765	1	0.99765	1	0.995818
Morocco	1.012188	1.007633	1.019964	0.987911	0.998333	1.006197
Mozambique	1.022847	1.026048	1.018631	1.007282	1.001177	0.995707
Niger	1.000908	1.006847	1.043485	0.964889	0.979999	1.01439
Nigeria	1.008062	1	1	1	0.992647	1.01553
Rwanda	1	1	1	1	1	1
Sao Tome and Principe	1.086265	1	1	1	1.089631	0.996911
Sierra Leone	0.986532	0.99247	1	0.99247	1	0.994017
South Africa	0.999244	1.002125	1.014583	0.987721	0.994307	1.002835
Sudan	0.99704	0.997189	1	0.997189	1	0.99985
Togo	0.985507	0.985443	0.971702	1.014141	0.996311	1.003769
Tunisia	1	1	1	1	1	1
U.R. of Tanzania:						
Mainland	0.9925	0.69634	1	0.69634	0.983515	1.4492
Zimbabwe	1.069999	1.082783	1.13918	0.950494	0.928334	1.064481
Geomean	1.003341	0.99002	1.002215	0.987832	1.002507	1.01092

Table 48 Results of variable specific MEA biennial Malmquist productivity change estimates and its components: 2018-2019 for Real GDP

Country	BMlcy	BMlvy	BPECy	BPTCy	BSECy	BSTCy
Angola	1.027723	1.010973	1.005914	1.005029	1.021736	0.994942
Benin	0.986544	1.054056	0.986085	1.06893	0.999493	0.936425
Botswana	0.988512	1.016619	1.006605	1.009948	0.997748	0.974548
Burkina Faso	1.000768	1.070718	1.011708	1.058327	1.008377	0.926905
Burundi	1.023416	1.043806	1.0286	1.014783	0.994658	0.985731
Cabo Verde	0.985293	1.044147	0.991119	1.053503	1.002038	0.941715
Cameroon	0.988005	1.021094	0.989923	1.031489	0.998465	0.969082
Central African Republic	1.003927	1.029703	1	1.029703	0.941195	1.035883
Chad	1.011079	1.037975	0.983526	1.055361	1.015874	0.958867
Côte d'Ivoire	1	1.062318	1	1.062318	1	0.941338
Djibouti	0.977612	1.06871	0.986303	1.083551	0.956694	0.956167
Egypt	1	1.055577	1	1.055577	1	0.947349
Eswatini	0.995405	1.008211	1	1.008211	0.997157	0.990114
Gabon	0.995762	1.032567	1	1.032567	1	0.964355
Guinea	1.002446	1.055288	1.005362	1.049659	1.017458	0.933628
Kenya	0.99006	1.053657	1	1.053657	0.964648	0.974077
Lesotho	1.018849	1.034783	1.012073	1.02244	1.002972	0.981684
Mauritania	0.991372	1.050248	0.997128	1.053274	0.998939	0.944943
Mauritius	0.998758	1.035044	1	1.035044	1	0.964943
Morocco	1.013064	1.037611	1.016169	1.021101	1.001158	0.975213
Mozambique	1.086783	1.12249	1.09387	1.026164	1.014223	0.954612
Niger	1.010428	1.044261	1.017217	1.026586	1.063188	0.910094
Nigeria	1.002301	1.021939	1	1.021939	0.979638	1.00117
Rwanda	1	1.094326	1	1.094326	1	0.913804
Sao Tome and Principe	1.08541	1.013016	1	1.013016	0.886011	1.209312
Sierra Leone	0.991204	1.048677	1	1.048677	1	0.945195
South Africa	1.000162	0.999401	1.010787	0.988735	1.011799	0.989092
Sudan	0.999226	1.011994	1	1.011994	1	0.987384
Togo	0.977362	1.027992	0.971827	1.057794	0.998312	0.952356
Tunisia	1	1.010433	1	1.010433	1	0.989675
U.R. of Tanzania:						
Mainland	0.986925	1.040269	1	1.040269	0.906454	1.046629
Zimbabwe	1.057224	0.968626	1.048187	0.924097	1.042958	1.046512
Geomean	1.005795	1.037946	1.004867	1.032919	0.993841	0.975029

Table 44 to 48 shows individual countries and summary results of the proposed Biennial MEA Productivity Index (BMIv) for the sample, with its four-factor variable specific decompositions ($BMIc = BPEC \times BPTC \times BSEC \times BSTC$). The table displays countries names, Biennial MEA variable specific productivity under constant and variable returns to scale (BMIc and BMIv).

Following the four-factor decomposition of Wheelock and Wilson (1999). The proposed Biennial MEA Malmquist index can be decomposed into four parts. These being the Biennial Pure Efficiency Change (BPEC), Biennial Pure Technical Change (BPTC), Biennial Scale Efficiency Change (BSEC) and Biennial Scale Technical Change (BSTC).

Tables 49 to 53 and figures 10 to 19 present the variable specific observations for variables under this study.

Table 49 Results of BMEAM and its decomposition (2000-2019) for variable xa(Capital Service)

Period	Inf	MPIvxa	BMIcxa	BMIvxa	BPECxa	BPTCxa	BSECxa	BSTCxa
2000-2001	19/32	0.9786	0.9952	0.9797	0.9632	1.0172	1.0488	0.9685
2001-2002	18/23	0.9972	0.9850	0.9881	1.0170	0.9716	0.9581	1.0405
2002-2003	22/32	0.9658	0.9914	0.9956	0.9896	1.0061	1.0116	0.9844
2003-2004	20/32	0.9681	1.0014	0.9897	0.9868	1.0030	1.0112	1.0006
2004-2005	21/32	0.9789	0.9873	0.9849	1.0076	0.9774	0.9857	1.0170
2005-2006	19/32	0.9679	0.998	1.004	1.000	1.004	0.996	0.998
2006-2007	21/32	0.9728	0.9997	0.9929	0.9873	1.0057	1.0049	1.0020
2007-2008	16/32	0.9486	1.0107	1.0083	1.0020	1.0064	0.9873	1.0152
2008-2009	17/32	0.9656	0.9975	0.9975	0.9972	1.0002	0.9964	1.0036
2009-2010	16/32	0.9717	1.0078	1.0105	1.0021	1.0084	0.9980	0.9993
2010-2011	20/32	0.9645	1.0033	1.0051	0.9996	1.0055	1.0015	0.9967
2011-2012	17/32	0.9548	0.9956	0.9971	0.9951	1.0020	1.0006	0.9979
2012-2013	18/32	0.9461	1.0073	1.0084	0.9969	1.0116	0.9982	1.0007
2013-2014	14/32	0.9506	1.0192	1.0101	0.9967	1.0135	1.0147	0.9944
2014-2015	16/32	0.9782	1.0174	1.0177	1.0213	0.9966	0.9838	1.0161
2015-2016	15/32	0.9547	1.0118	1.0136	0.9995	1.0141	0.9977	1.0006
2016-2017	14/32	0.9508	1.0155	1.0121	1.0001	1.0120	1.0029	1.0005
2017-2018	17/32	0.9601	0.9997	0.9927	0.9927	1.0000	1.0195	0.9878
2018-2019	16/32	0.9652	1.0144	1.0096	1.0153	0.9943	1.0029	1.0019
Geomean		0.9652	1.0030	1.0008	0.9983	1.0025	1.0009	1.0013
SD		0.0129	0.0101	0.0109	0.0127	0.0116	0.0178	0.0146
Min		0.9461	0.9850	0.9797	0.9632	0.9716	0.9581	0.9685
Max		0.9972	1.0192	1.0177	1.0213	1.0172	1.0488	1.0405

Table 49 above shows geometric mean productivity increased over time, but with some fluctuations and with the highest increases occurring in the periods 2013-2014 with a value of 1.0192 for the CRS measure and 2014-2015 with a value of 1.0177 for the VRS measure. This is graphically represented in figure 10 and its decomposition in figure 11.

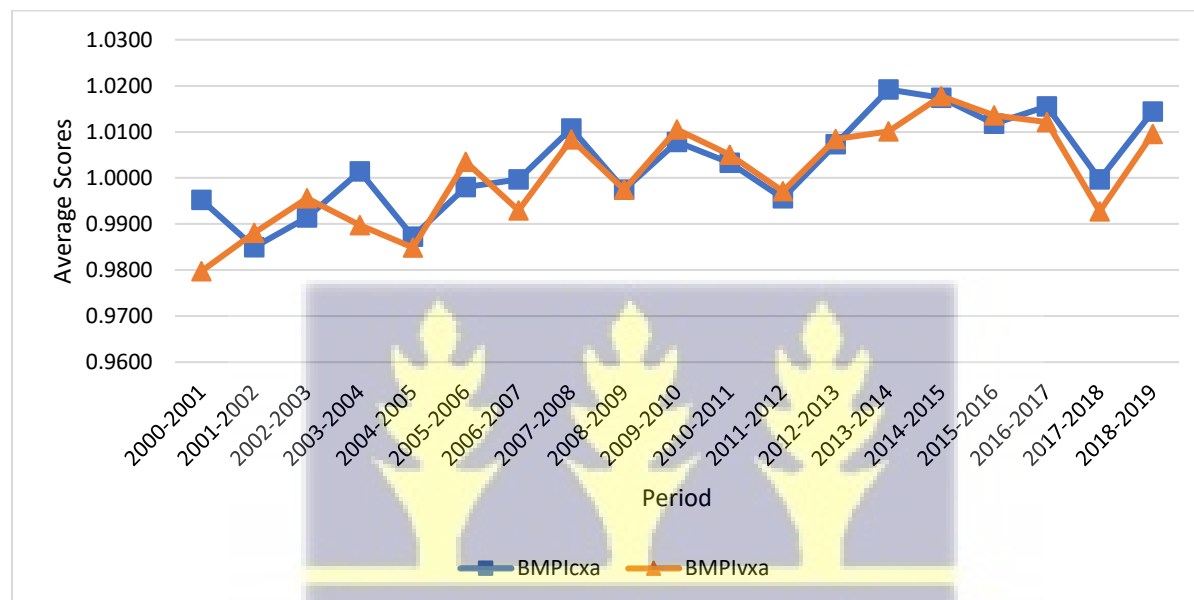


Figure 10 Capital Service average productivity trends (2000-2019)

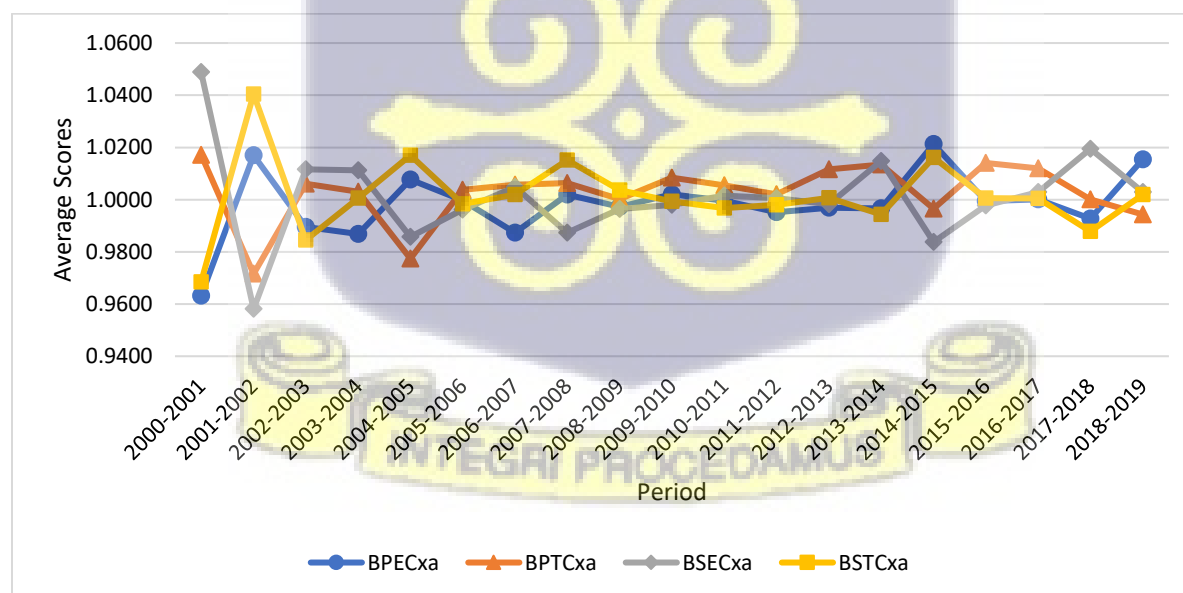


Figure 11 Components of average productivity trends for Capital Service (2000-2019)

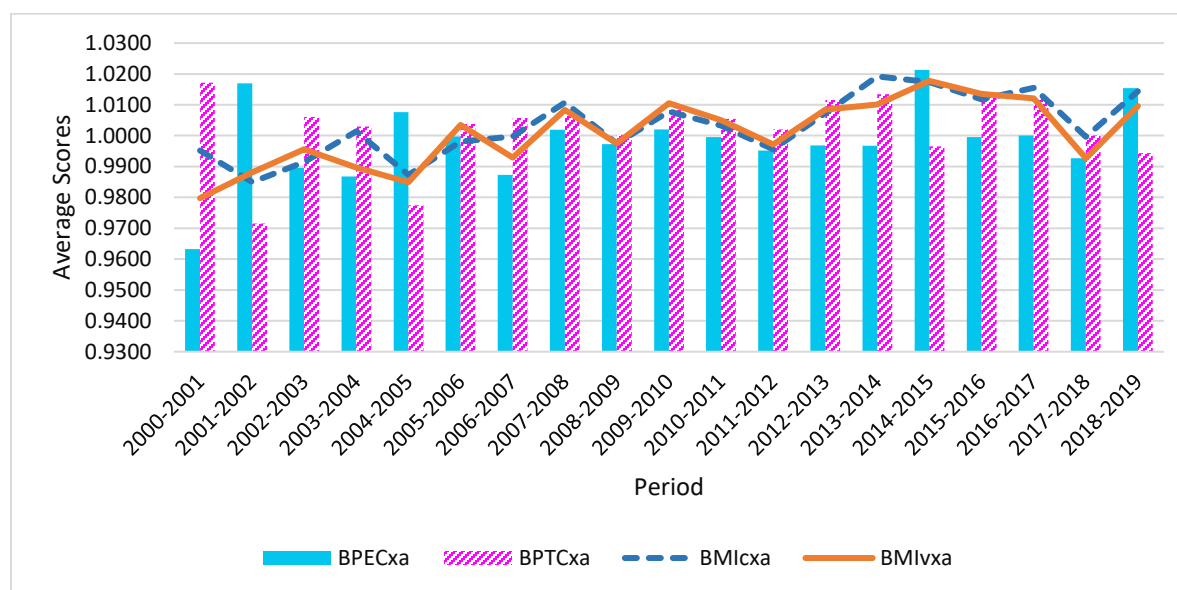


Figure 12 Capital Service Average Productivity (2000-2019)

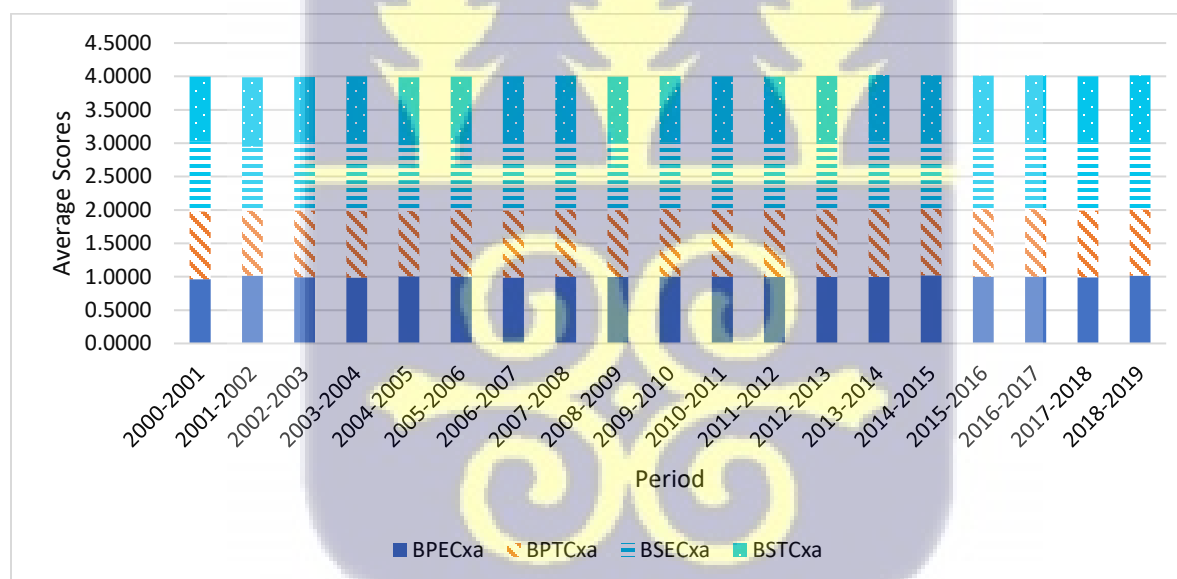


Figure 13 Capital service productivity change decomposition (2000-2019)

Table 50 Results of BMEAM and its decomposition (2000-2019) for Labour

Period	Inf	MPIvxb	BMPIcxb	BMPIvxb	BPECxb	BPTCxb	BSECxb	BSTCxb
2000-2001	19/32	1.0026	0.9913	0.9825	0.9883	0.9942	1.0108	0.9981
2001-2002	18/32	0.9810	0.9892	0.9911	0.9936	0.9975	0.9886	1.0096
2002-2003	22/32	0.9855	0.9956	0.9940	0.9974	0.9966	1.0031	0.9985
2003-2004	20/32	0.9873	0.9993	0.9998	1.0034	0.9965	0.9951	1.0044

2004-2005	21/32	0.9969	0.9948	0.9910	0.9999	0.9911	1.0043	0.9996
2005-2006	19/32	0.9897	0.9997	0.9941	0.9930	1.0011	1.0048	1.0009
2006-2007	21/32	0.9938	1.0053	1.0035	1.0021	1.0014	1.0026	0.9992
2007-2008	16/32	0.9810	0.9972	1.0004	0.9904	1.0101	0.9943	1.0026
2008-2009	17/32	0.9996	0.9986	1.0018	0.9990	1.0029	1.0037	0.9931
2009-2010	16/32	1.0060	1.0063	1.0142	1.0052	1.0089	1.0021	0.9901
2010-2011	20/32	1.0174	0.9983	1.0093	1.0194	0.9901	0.9868	1.0023
2011-2012	17/32	1.0061	0.9889	0.9976	0.9983	0.9993	0.9966	0.9947
2012-2013	18/32	0.9947	1.0098	1.0131	1.0137	0.9994	1.0014	0.9954
2013-2014	14/32	1.0141	1.0112	1.0067	1.0054	1.0013	1.0075	0.9970
2014-2015	16/32	1.0062	1.0068	1.0191	1.0115	1.0074	0.9971	0.9908
2015-2016	15/32	0.9972	1.0073	1.0127	1.0095	1.0032	0.9899	1.0048
2016-2017	14/32	0.9877	1.0067	1.0031	0.9945	1.0086	1.0077	0.9959
2017-2018	17/32	0.9965	0.9969	0.9971	0.9988	0.9983	1.0052	0.9945
2018-2019	16/32	0.9884	1.0003	0.9912	1.0030	0.9883	0.9978	1.0114
Geomean		0.9964	1.0002	1.0011	1.0014	0.9998	1.0000	0.9991
SD		0.0104	0.0067	0.0096	0.0082	0.0063	0.0068	0.0058
Min		0.9810	0.9889	0.9825	0.9883	0.9883	0.9868	0.9901
Max		1.0174	1.0112	1.0191	1.0194	1.0101	1.0108	1.0114

Table 50 presents the results of the Biennial Multi-directional Efficiency Analysis Malmquist (BMEAM) index and its decomposition for labour across 32 African states from 2000 to 2019. The analysis reflects variations in labour productivity over time using different perspectives: the traditional Malmquist Productivity Index under Variable Returns to Scale (MPIvxb), the Constant Returns to Scale BMEAM index (BMPIcxb), and the Variable Returns to Scale BMEAM index (BMPIvxb). To enhance analytical depth, the results are decomposed into Pure Efficiency Change (BPECxb), Pure Technical Change (BPTCxb), Scale Efficiency Change (BSECxb), and Scale Technical Change (BSTCxb).

Over the observed period, labour productivity exhibited modest but fluctuating improvement. The geometric mean of the traditional MPIvxb is slightly below unity, indicating a general decline in productivity when assessed under conventional assumptions. However, the BMEAM results provide a more optimistic outlook. Under CRS, the BMPIcxb recorded a geometric mean of 1.0002, while under VRS, the BMPIvxb recorded 1.0011. These values suggest that labour

productivity remained largely stable, with slight gains over the two decades, particularly when accounting for variable-specific and scale-sensitive dynamics.

The most significant growth in labour productivity occurred between 2013 and 2015. The year 2014–2015 recorded the highest CRS-based productivity score ($BMPI_{cxb} = 1.0191$), while 2013–2014 had the highest VRS-based score ($MPI_{vxb} = 1.0141$). These results likely reflect targeted policy reforms, improved labour utilization, or the impact of region-specific economic activities that elevated efficiency during that period. However, from 2015 to 2019, a consistent downward trend emerged, with both CRS and VRS scores showing mild decline. This trend may be attributed to macroeconomic slowdowns, structural bottlenecks, or inefficiencies in labour management across various economies.

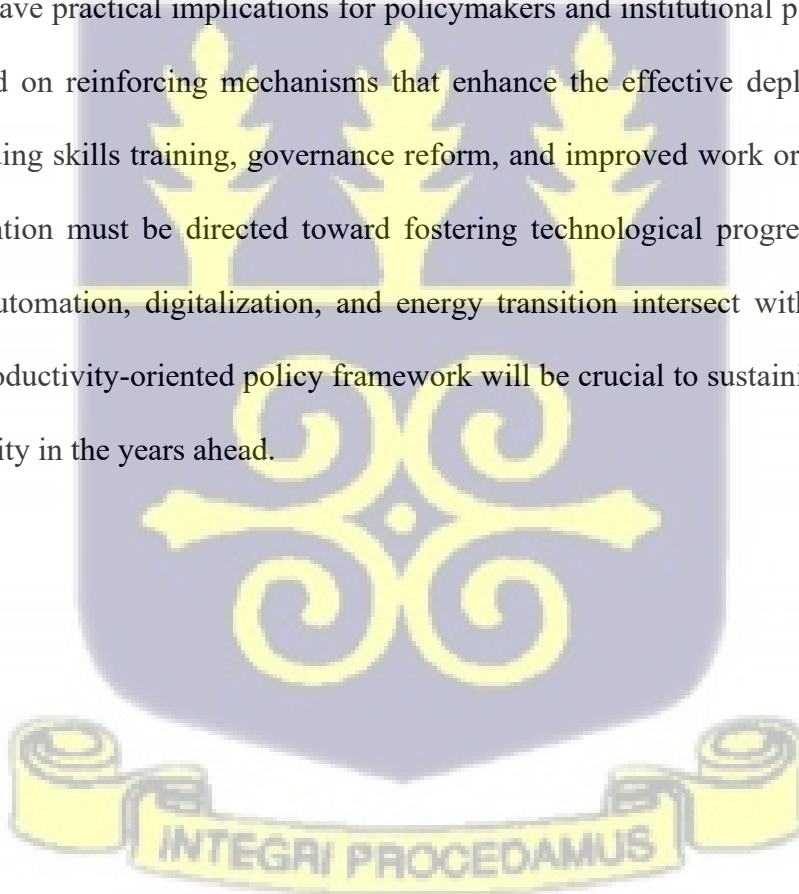
The decomposition of the productivity index provides greater insight into the drivers of change. Pure Efficiency Change (BPE_{cxb}) emerged as the dominant contributor to labour productivity improvement, with a geometric mean of 1.0014. This indicates that the productivity gains were largely the result of more effective use of existing labour inputs rather than technological advancement. In contrast, Pure Technical Change (BPT_{cxb}) averaged around 0.9998, suggesting limited technological progress throughout the region during the period. The average values for Scale Efficiency Change (BSE_{cxb}) and Scale Technical Change (BST_{cxb}) hovered around unity, reflecting the minimal role that scale economies played in influencing labour productivity over time.

There were noticeable periods of inefficiency and volatility. Labour productivity deteriorated significantly in 2001–2002 and 2007–2008, with MPI_{vxb} values dropping to 0.9810, coinciding with global economic disturbances and regional governance challenges. Infeasibility rates, recorded as the number of DMUs for which valid BMEAM scores could not be computed, varied

from 14 to 22 out of 32, indicating issues related to data quality or model limitations in certain years.

The results suggest that internal labour efficiency, rather than innovation or expansion, has been the main driver of productivity change in African countries. The lack of substantial technical progress highlights the need for stronger innovation ecosystems, increased investment in labour-enhancing technologies, and improved institutional capacity. Furthermore, the weak influence of scale efficiency suggests that most countries may not be operating at optimal scale, pointing to opportunities for regional integration and structural realignment to achieve scale-related gains.

These findings have practical implications for policymakers and institutional planners. Emphasis should be placed on reinforcing mechanisms that enhance the effective deployment of labour resources, including skills training, governance reform, and improved work organization. At the same time, attention must be directed toward fostering technological progress, particularly in sectors where automation, digitalization, and energy transition intersect with labour. A more inclusive and productivity-oriented policy framework will be crucial to sustaining and enhancing labour productivity in the years ahead.



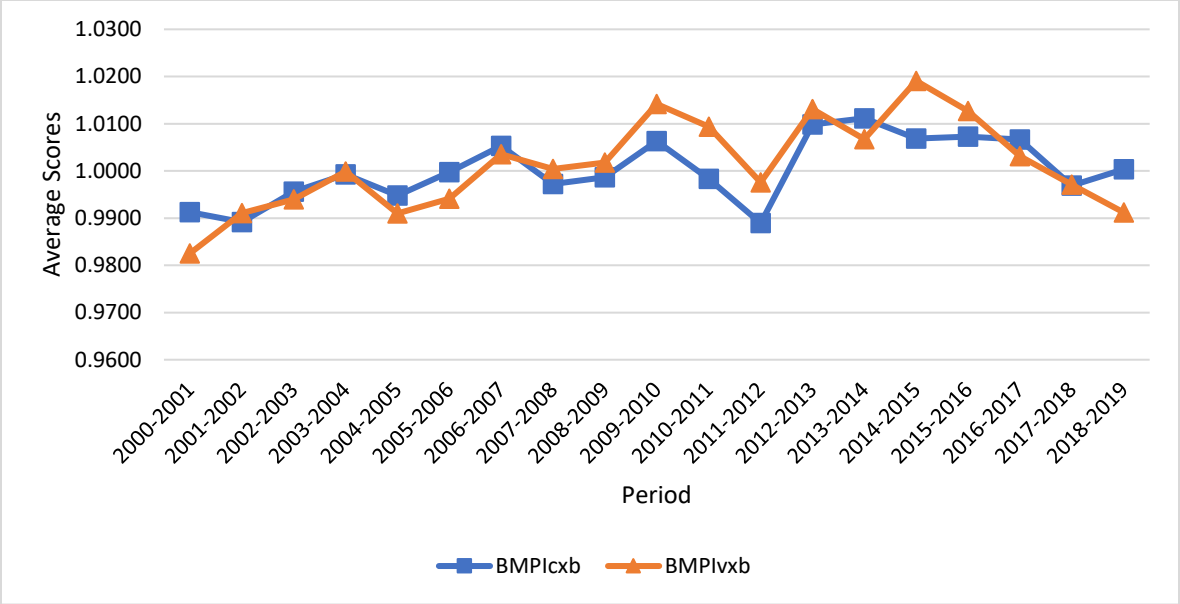


Figure 14 Labour average productivity trends (2000-2019)

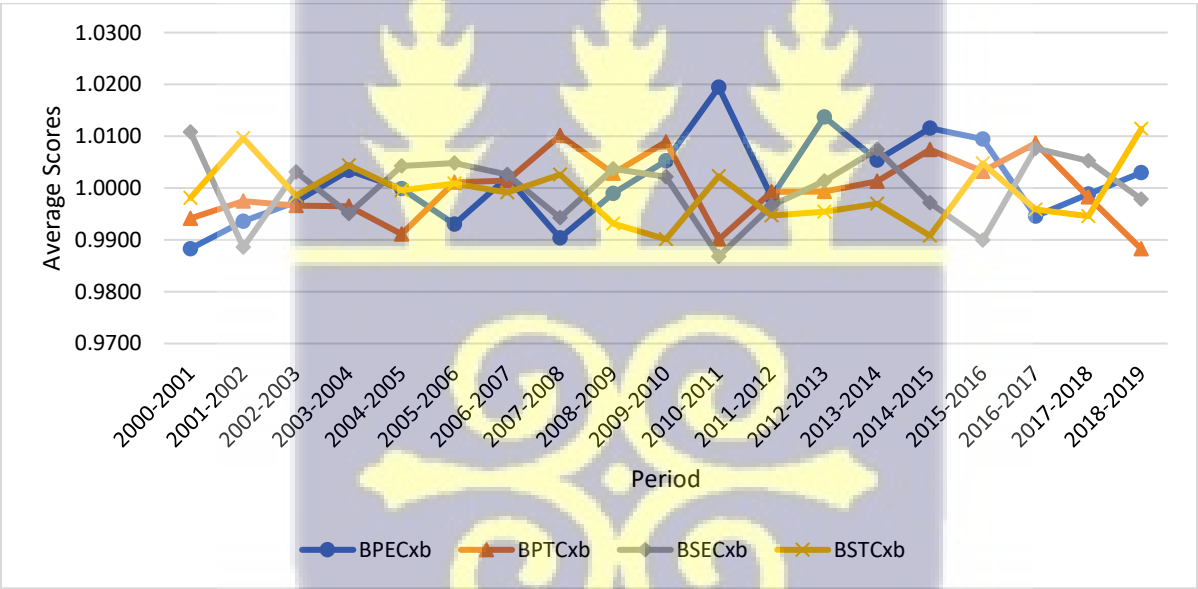


Figure 15 Components of average productivity trends for Labour (2000-2019)

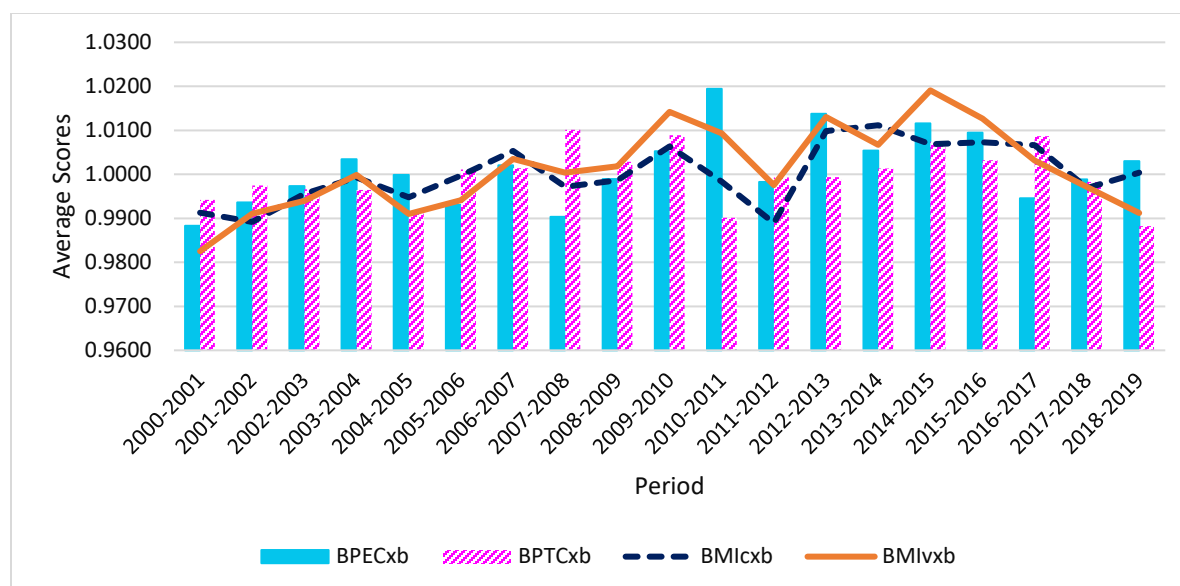


Figure 16 Labour average Productivity change (2000-2019)

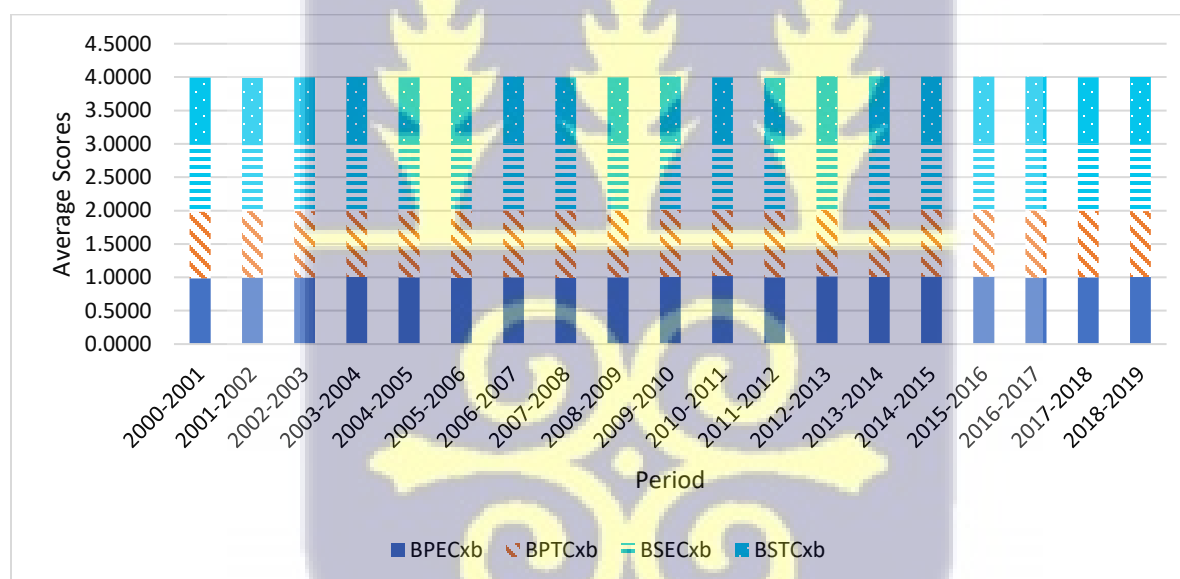


Figure 17 Labour productivity change decomposition (2000-2019)

Table 51 Results of BMEAM and its decomposition (2000-2019) for Energy Used

Period	Inf	MPIvxc	BMPlcxc	BMPIvxc	BPECxc	BPTCxc	BSECxc	BSTCxc
2000-2001	19/32	0.9879	0.990	0.976	0.979	0.997	1.020	0.994
2001-2002	18/32	1.0113	0.989	0.994	1.008	0.986	0.985	1.010
2002-2003	22/32	0.9865	0.993	0.988	0.990	0.998	1.014	0.991
2003-2004	20/32	1.0267	0.997	0.990	1.014	0.977	1.014	0.993
2004-2005	21/32	0.9725	0.995	0.990	1.011	0.979	0.996	1.009
2005-2006	19/32	0.9846	0.991	0.997	0.994	1.003	0.989	1.004
2006-2007	21/32	0.9952	1.002	0.995	1.007	0.989	1.007	0.999

2007-2008	16/32	0.9761	1.000	1.006	1.000	1.006	0.989	1.005
2008-2009	17/32	0.9674	0.992	0.995	0.996	0.999	1.014	0.983
2009-2010	16/32	0.9686	1.005	1.021	1.004	1.017	1.000	0.985
2010-2011	20/32	1.0043	0.998	1.022	1.019	1.003	0.983	0.994
2011-2012	17/32	1.0014	0.988	0.999	1.002	0.998	0.996	0.993
2012-2013	18/32	0.9606	1.000	1.001	0.990	1.011	1.008	0.990
2013-2014	14/32	0.9894	1.002	1.001	0.992	1.008	1.006	0.996
2014-2015	16/32	1.0064	1.008	1.019	1.020	0.999	0.989	0.999
2015-2016	15/32	0.9804	1.006	1.014	0.999	1.016	0.988	1.004
2016-2017	14/32	0.9498	1.005	1.008	0.975	1.034	0.997	1.000
2017-2018	17/32	1.0091	0.996	0.994	0.996	0.998	1.001	1.001
2018-2019	16/32	0.9889	0.999	0.998	1.001	0.997	1.002	0.999
Geomean		0.9876	0.998	1.000	1.000	1.001	1.000	0.997
SD		0.0194	0.006	0.012	0.012	0.013	0.011	0.008
Min		0.9498	0.988	0.976	0.975	0.977	0.983	0.983
Max		1.0267	1.008	1.022	1.020	1.034	1.020	1.010

Table 51 presents the results of the Biennial Multi-directional Efficiency Analysis Malmquist (BMEAM) index and its decomposition for energy use across 32 African countries from 2000 to 2019. The analysis captures the dynamics of energy-related productivity using three performance indicators: the traditional Malmquist Productivity Index under Variable Returns to Scale (MPI_{vxc}), the BMEAM index under Constant Returns to Scale (BMPI_{cxc}), and the BMEAM index under VRS (BMPI_{vxc}). The decomposition into four components, Pure Efficiency Change (BPEC_{cxc}), Pure Technical Change (BPTC_{cxc}), Scale Efficiency Change (BSEC_{cxc}), and Scale Technical Change (BSTC_{cxc}), provides further granularity to understand the drivers behind energy productivity fluctuations.

Across the observed period, energy productivity remained largely stable, as evidenced by the geometric mean scores. The CRS-based BMEAM index (BMPI_{cxc}) recorded a geometric mean of 0.998, while the VRS-based BMEAM index (BMPI_{vxc}) slightly exceeded unity at 1.000. The traditional MPI_{vxc}, however, recorded a lower mean of 0.9876, which suggests that conventional analysis underrepresents the nuanced patterns of energy efficiency gains and losses captured by the BMEAM framework.

Significant increases in energy productivity were recorded during the periods 2010–2011 and 2014–2015. The year 2010–2011 produced the highest BMEAM score under CRS ($BMPI_{cxc} = 1.022$), while 2014–2015 showed the strongest VRS-based performance ($BMPI_{vxc} = 1.008$). These periods likely reflect coordinated investment in energy infrastructure, policy interventions supporting efficient energy use, or favourable macroeconomic conditions. However, from 2015 to 2019, a gradual decline is evident in both CRS and VRS-based productivity scores, suggesting potential regressions in energy efficiency or systemic inefficiencies in energy management during the latter part of the decade.

The decomposition of the BMEAM results reveals the underlying sources of change. Pure Efficiency Change ($BPEC_{xc}$) averaged 1.000, indicating that energy was being utilized with relatively consistent efficiency over the two-decade span. Pure Technical Change ($BPTC_{xc}$) recorded a slightly higher geometric mean of 1.001, pointing to modest technological improvements in energy production or utilization processes. These suggest that any productivity gains were likely driven by incremental advances in energy-related technology, rather than radical efficiency enhancements.

Scale Efficiency Change ($BSEC_{xc}$) and Scale Technical Change ($BSTC_{xc}$) showed more variation. While their geometric means were near unity, 1.000 and 0.997 respectively, the standard deviations indicate modest volatility. The period 2016–2017, for instance, recorded the lowest Pure Efficiency Change (0.975) and the highest Pure Technical Change (1.034), suggesting uneven productivity patterns in that year, possibly influenced by external shocks, policy inconsistencies, or uneven scale economies across countries.

Notably, the scale-related components showed mixed effects. The fluctuations in Scale Technical Change ($BSTC_{xc}$) suggest that not all countries benefited uniformly from scale-related

technological advances. In some cases, smaller or less integrated energy systems may have struggled to realize economies of scale, thereby weakening their productivity scores.

Infeasibility counts remained persistent, ranging from 14 to 22 out of 32 countries, underscoring the data and structural limitations often encountered in cross-country DEA analyses. These infeasibilities may reflect gaps in energy data, reporting inconsistencies, or extreme variations in energy usage patterns, all of which may affect the reliability of scores in certain periods.

Overall, the energy productivity landscape across African countries during the period under review was characterized by stability, intermittent technological gains, and modest efficiency improvements. The results reflect a region where energy systems are evolving, but where gains are not uniformly distributed and where external vulnerabilities and internal inefficiencies continue to influence outcomes. The modest rise in technical change indicators points to opportunities for strengthening innovation, particularly through investment in cleaner, more efficient energy technologies and regional integration of energy markets.

The declining trend observed in the latter part of the period calls for renewed policy focus on energy efficiency, sustainable energy deployment, and supportive regulatory environments. Emphasis should be placed on enhancing the technological capabilities of energy sectors, improving institutional frameworks for energy governance, and addressing data gaps that undermine effective monitoring and benchmarking. As the region moves toward sustainable energy transitions, future improvements in energy productivity will increasingly depend on the successful alignment of technical progress, policy innovation, and efficiency-focused investment.

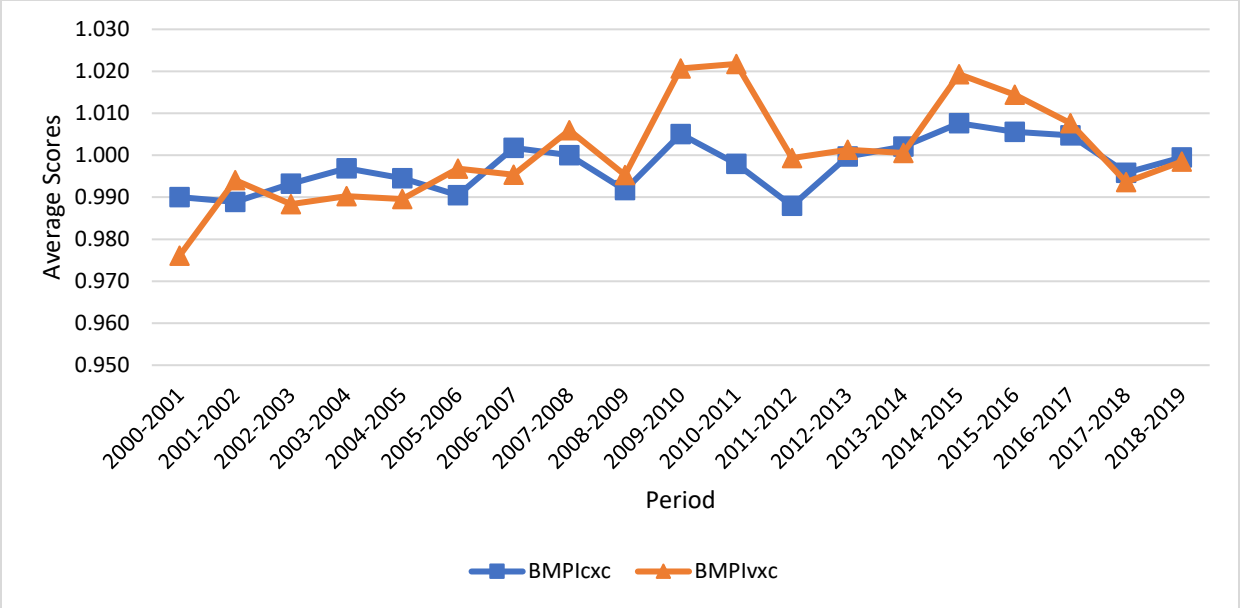


Figure 18 Energy Used average productivity trends (2000-2019)

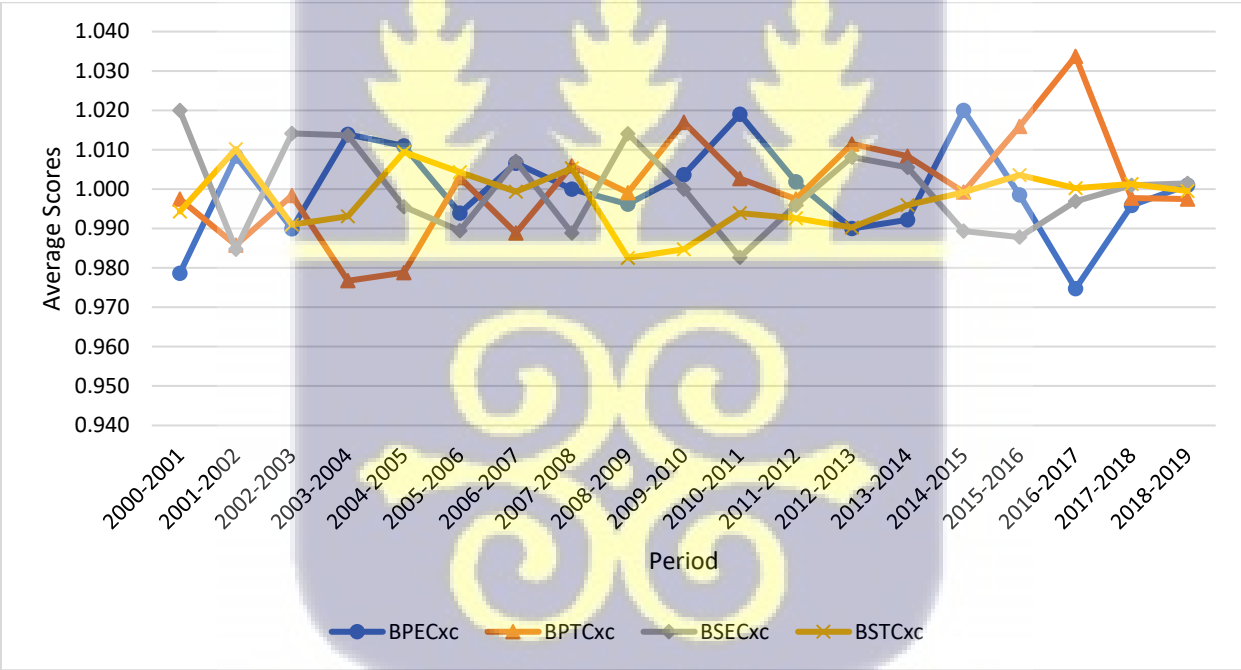


Figure 19 Components of average productivity trends (2000-2019) for Energy used

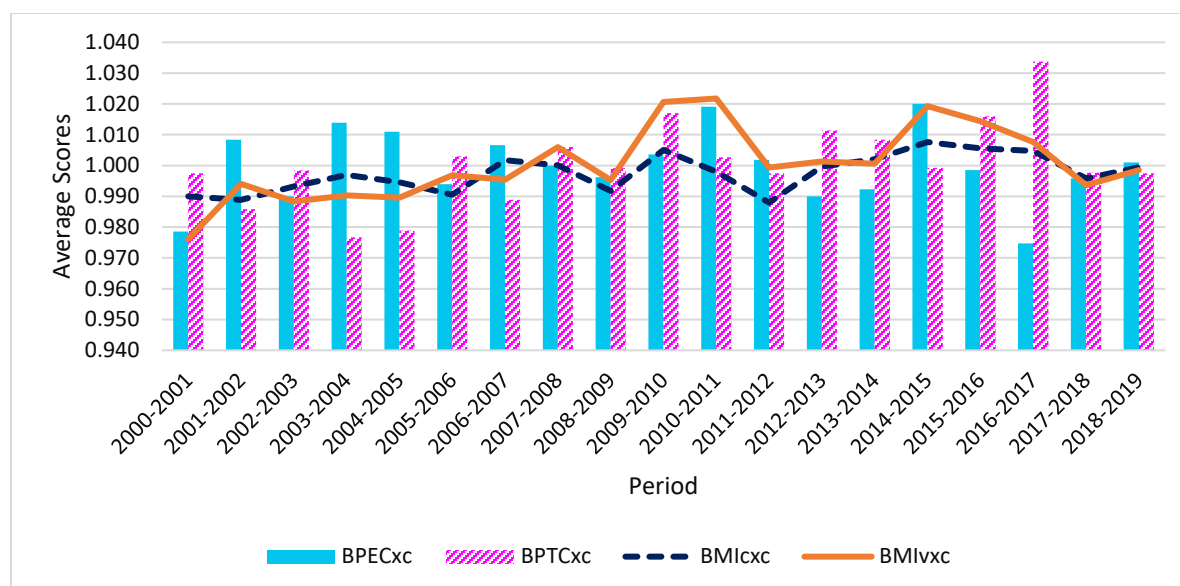


Figure 20 Energy consumption average Productivity change (2000-2019)

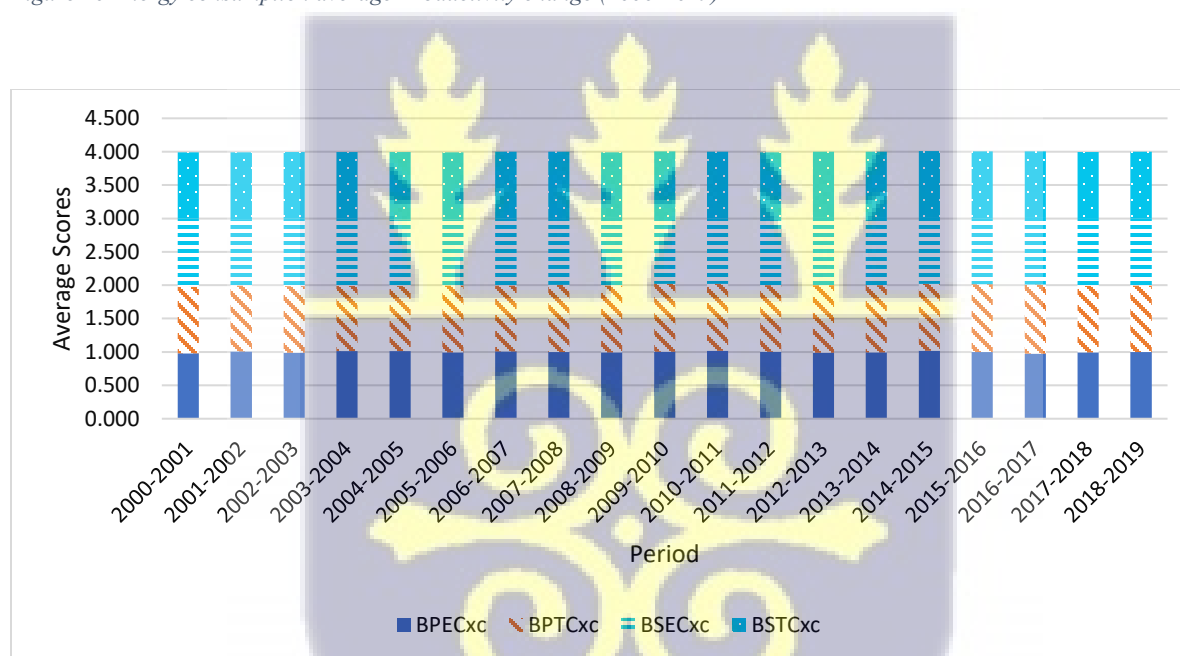


Figure 21 Energy consumption productivity change decomposition (2000-2019)

Table 52 Results of BMEAM and its decomposition (2000-2019) for Carbon Dioxide Emission

Period	Inf	MPIIvby	BMPIIcyb	BMPIIvby	BPECyb	BPTCyb	BSECyb	BSTCyb
2000-2001	19/32	0.9963	0.9961	0.9799	0.9877	0.9921	1.0177	0.9988
2001-2002	18/32	1.0176	0.9917	0.9968	1.0087	0.9882	0.9792	1.0160
2002-2003	22/32	0.9917	0.9954	0.9848	0.9836	1.0013	1.0192	0.9917
2003-2004	20/32	0.9671	1.0007	0.9934	0.9992	0.9942	0.9980	1.0094

2004-2005	21/32	1.0010	0.9926	0.9858	1.0011	0.9847	1.0055	1.0014
2005-2006	19/32	0.9737	0.9973	1.0067	1.0087	0.9980	0.9933	0.9974
2006-2007	21/32	0.9755	0.9999	0.9942	0.9886	1.0056	1.0085	0.9973
2007-2008	16/32	0.9961	0.9968	1.0001	1.0067	0.9934	0.9842	1.0127
2008-2009	17/32	0.9901	0.9951	1.0020	1.0020	1.0000	1.0008	0.9924
2009-2010	16/32	0.9718	1.0053	1.0107	1.0022	1.0085	0.9985	0.9962
2010-2011	20/32	0.9924	1.0034	1.0102	1.0063	1.0039	1.0014	0.9919
2011-2012	17/32	1.0110	0.9827	0.9963	1.0005	0.9958	0.9925	0.9938
2012-2013	18/32	0.9573	1.0079	1.0108	1.0031	1.0077	1.0032	0.9940
2013-2014	14/32	0.9793	1.0072	1.0089	1.0023	1.0066	1.0116	0.9869
2014-2015	16/32	1.0165	1.0038	1.0096	1.0121	0.9975	0.9932	1.0011
2015-2016	15/32	0.9753	1.0058	1.0106	1.0011	1.0094	0.9931	1.0022
2016-2017	14/32	1.0700	1.0025	1.0120	1.0637	0.9514	1.0291	0.9625
2017-2018	17/32	0.9271	0.9870	0.9813	0.9723	1.0093	0.9944	1.0115
2018-2019	16/32	0.9772	1.0033	0.9900	1.0022	0.9878	1.0025	1.0109
Geomean		0.9884	0.9986	0.9991	1.0026	0.9965	1.0013	0.9983
SD		0.0292	0.0069	0.0110	0.0177	0.0134	0.0122	0.0120
Min		0.9271	0.9827	0.9799	0.9723	0.9514	0.9792	0.9625
Max		1.0700	1.0079	1.0120	1.0637	1.0094	1.0291	1.0160

Table 52 presents the results of the Biennial Multi-directional Efficiency Analysis Malmquist

(BMEAM) index and its decomposition for carbon dioxide emissions across 32 African countries from 2000 to 2019. The analysis includes traditional Malmquist productivity under Variable Returns to Scale (MPI_{vyb}), BMEAM under Constant Returns to Scale (BMPI_{cyb}), and BMEAM under Variable Returns to Scale (BMPI_{vyb}). In addition to these indices, the analysis disaggregates the drivers of productivity change into four dimensions: Pure Efficiency Change (BPEC_{cyb}), Pure Technical Change (BPTC_{cyb}), Scale Efficiency Change (BSEC_{cyb}), and Scale Technical Change (BSTC_{cyb}).

Across the entire period, carbon emissions productivity remained generally stable with intermittent fluctuations. The geometric mean of the CRS-based BMEAM index (BMPI_{cyb}) was 0.9986, while the VRS-based BMEAM index (BMPI_{vyb}) averaged 0.9991, suggesting a near-stationary trend in emissions productivity. The conventional Malmquist index (MPI_{vyb}) recorded a slightly lower geometric mean of 0.9884, underlining once again that the BMEAM methodology captures nuances that standard indices may overlook.

Two peak periods stand out. The highest productivity growth under the VRS-based measure occurred in 2016–2017 ($BMPI_{vyb} = 1.0120$), which coincided with strong gains in pure efficiency ($BPEC_{yb} = 1.0637$), despite a sharp drop in technological change ($BPTC_{yb} = 0.9514$). Under the CRS-based measure, the most notable increase was in 2012–2013 ($BMPI_{cyb} = 1.0079$), suggesting modest gains in environmental productivity during that year. These improvements may reflect effective environmental regulations, shifts toward cleaner technologies, or macroeconomic dynamics that influenced energy use and emissions intensity.

Despite these gains, there were substantial periods of decline. Notable downturns occurred between 2011–2012 and 2017–2018, where both CRS and VRS-based scores dropped significantly. The period 2017–2018 saw the lowest value for the traditional MPI_{vyb} (0.9271), while $BMPI_{cyb}$ and $BMPI_{vyb}$ also dipped, suggesting widespread inefficiencies or a reversal in environmental productivity progress across many countries. Such declines may have resulted from economic slowdowns, energy mix shifts back toward fossil fuels, institutional weaknesses in environmental governance, or data anomalies.

The decomposition analysis reveals the drivers behind these productivity shifts. Pure Efficiency Change ($BPEC_{yb}$) recorded a geometric mean of 1.0026, suggesting a consistent improvement in how emissions were managed relative to inputs. This implies that African countries made slight but positive strides in improving the efficiency with which CO_2 emissions were linked to productive output. Pure Technical Change ($BPTC_{yb}$), however, recorded a geometric mean of 0.9965, indicating a marginal regression in emissions-related technology over the period. This reflects either a stagnation in clean technology adoption or limited diffusion of emissions-reducing innovations.

Scale-related factors played a relatively neutral role. Scale Efficiency Change (BSECyb) and Scale Technical Change (BSTCyb) recorded geometric means of 1.0013 and 0.9983, respectively. These figures imply that scale effects neither significantly improved nor worsened emissions productivity in most periods. However, substantial volatility was observed in select years. For instance, Scale Technical Change reached a minimum of 0.9625 in 2016–2017, while Scale Efficiency Change peaked at 1.0291 in the same year. These swings suggest varying capacity across countries to capitalize on scale-related innovations and infrastructure optimization.

Infeasibility remained a recurring feature throughout the period, with between 14 and 22 countries encountering issues in specific years. This reinforces the complexity and heterogeneity of emissions performance across African economies and may indicate uneven data quality, particularly where smaller or resource-constrained countries report inconsistently on environmental metrics.

The overall findings suggest that carbon emissions productivity in African countries remained relatively unchanged over the period, with temporary gains being offset by structural and technological limitations. While slight efficiency improvements were evident, technical and scale-related drivers were inconsistent. The decline in emissions productivity during key periods points to the vulnerability of progress in the absence of sustained policy intervention, innovation, and investment in environmental infrastructure.

Moving forward, there is a need to build institutional frameworks that prioritize climate-conscious productivity, especially in energy-intensive sectors. Policy tools should aim to decouple economic growth from emissions intensity by incentivizing renewable energy adoption, strengthening environmental regulations, and fostering regional cooperation. Furthermore, consistent and

reliable environmental data collection remains critical for performance measurement and effective decision-making.

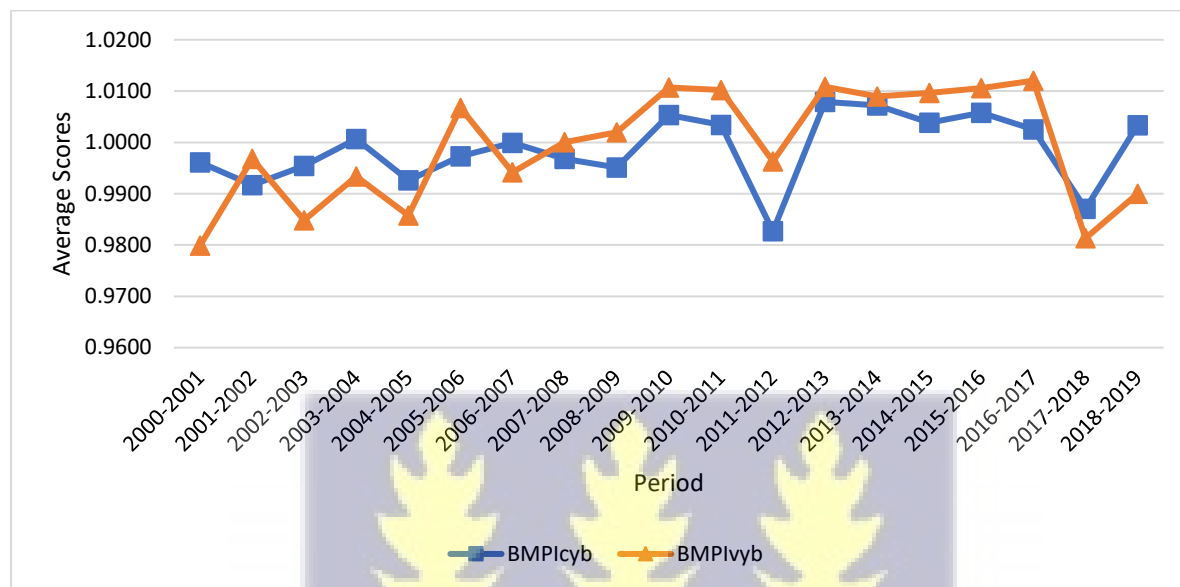


Figure 22 Average productivity trends (2000-2019) for Carbon Dioxide Emission

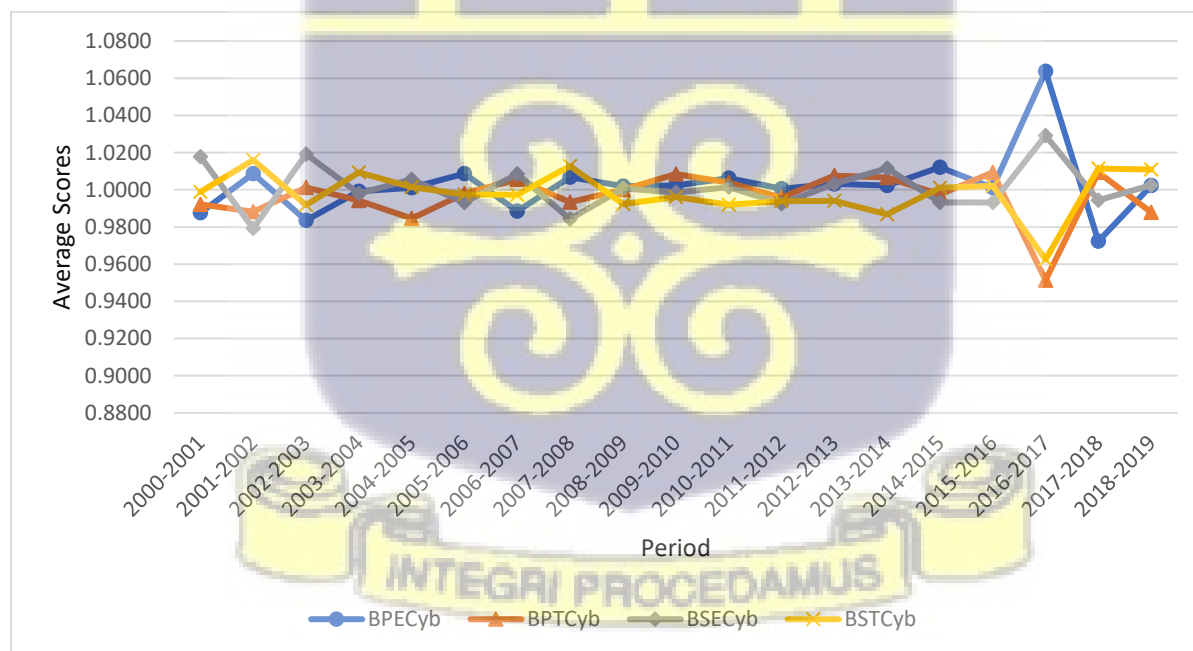


Figure 23 Components of average productivity trends (2000-2019) for Carbon Dioxide Emission

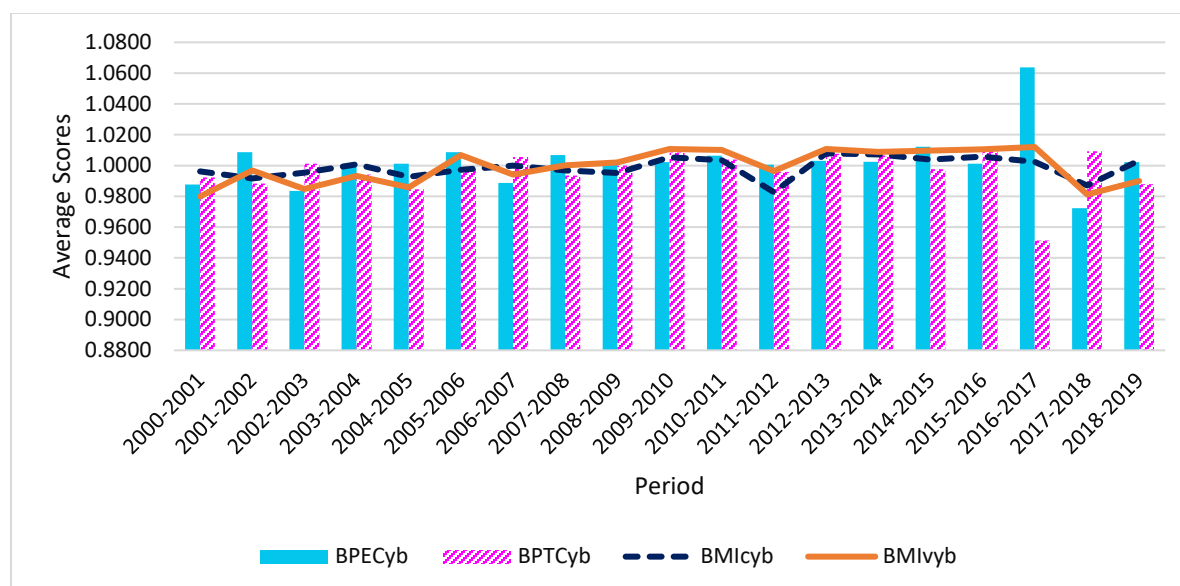


Figure 24 Carbon dioxide average Productivity change (2000-2019)

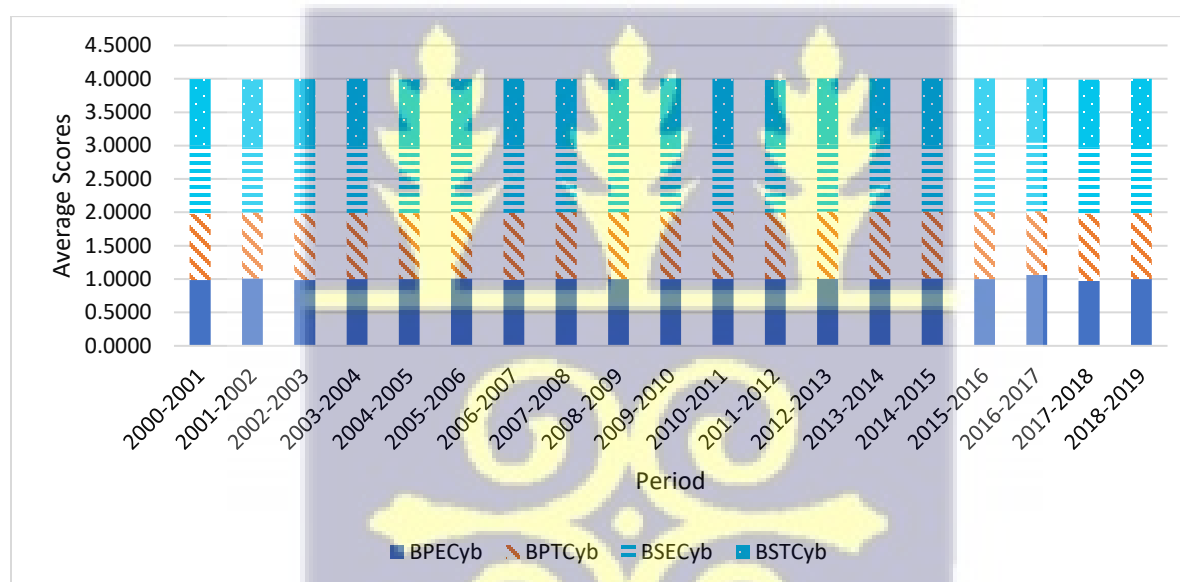


Figure 25 Carbon dioxide emission productivity change decomposition (2000-2019)

Table 53 Results of BMEAM and its decomposition (2000-2019) for Real GDP

Period	Inf	MPIIvya	BMPIIcya	BMPIIvya	BPECya	BPTCya	BSECya	BSTCya
2000-2001	20/32	1.0238	0.9952	1.0378	0.9964	1.0415	0.9872	0.9714
2001-2002	21/32	1.0016	0.9968	1.0480	0.9960	1.0522	0.9876	0.9630
2002-2003	20/32	1.0036	1.0008	1.0408	1.0001	1.0407	0.9903	0.9710
2003-2004	19/32	0.9943	1.0010	1.0500	0.9980	1.0521	0.9882	0.9646
2004-2005	17/32	1.0136	0.9990	1.0442	1.0036	1.0405	0.9907	0.9656

2005-2006	20/32	0.9979	0.9971	1.0509	0.9907	1.0608	0.9897	0.9587
2006-2007	21/32	1.0061	1.0048	1.0581	1.0015	1.0565	0.9824	0.9666
2007-2008	19/32	1.0074	0.9975	1.0423	0.9926	1.0500	0.9841	0.9726
2008-2009	23/32	1.0166	0.9936	1.0420	0.9969	1.0452	0.9854	0.9677
2009-2010	20/32	1.0186	1.0039	1.0687	1.0009	1.0677	0.9901	0.9488
2010-2011	21/32	1.0184	1.0032	1.0581	1.0017	1.0564	0.9975	0.9505
2011-2012	17/32	1.0201	0.9918	1.0608	0.9944	1.0668	0.9937	0.9408
2012-2013	19/32	1.0089	1.0082	1.0501	1.0027	1.0473	0.9969	0.9631
2013-2014	20/32	1.0226	1.0055	1.0579	1.0068	1.0508	0.9919	0.9581
2014-2015	17/32	1.0112	1.0048	1.0404	0.9993	1.0411	0.9923	0.9732
2015-2016	17/32	0.9961	1.0093	1.0484	0.9977	1.0509	0.9960	0.9666
2016-2017	13/32	1.0121	1.0081	1.0524	1.0140	1.0378	0.9894	0.9682
2017-2018	13/32	1.0137	0.9944	1.0312	0.9923	1.0392	0.9952	0.9690
2018-2019	15/32	1.0035	1.0058	1.0379	1.0049	1.0329	0.9938	0.9750
Geomean		1.0100	1.0011	1.0484	0.9995	1.0489	0.9906	0.9639
SD		0.0089	0.0054	0.0095	0.0056	0.0097	0.0043	0.0090
Min		0.9943	0.9918	1.0312	0.9907	1.0329	0.9824	0.9408
Max		1.0238	1.0093	1.0687	1.0140	1.0677	0.9975	0.9750

Table 53 presents the results of the Biennial Multi-directional Efficiency Analysis Malmquist (BMEAM) index and its decomposition for Real Gross Domestic Product (Real GDP) across 32 African states between 2000 and 2019. The analysis includes the conventional Malmquist Productivity Index under Variable Returns to Scale (MPI_{vy}), the BMEAM index under Constant Returns to Scale (BMPI_{cya}), and the BMEAM index under VRS (BMPI_{vy}). These indices are further decomposed into Pure Efficiency Change (BPEC_{ya}), Pure Technical Change (BPTC_{ya}), Scale Efficiency Change (BSEC_{ya}), and Scale Technical Change (BSTC_{ya}) to provide a more granular assessment of productivity dynamics.

The geometric mean values indicate that Real GDP productivity increased consistently over the study period. The CRS-based BMEAM index (BMPI_{cya}) has a geometric mean of 1.0011, while the VRS-based BMEAM index (BMPI_{vy}) registers a higher value of 1.0484. These figures demonstrate that Real GDP grew not only in absolute terms but also in productivity-adjusted performance, particularly under the VRS framework. The conventional MPI_{vy} similarly supports this trend, with a geometric mean of 1.0100, indicating a steady upward trajectory in GDP productivity across most countries.

Productivity peaked under CRS in the period 2009–2010, where $BMPI_{Icy}$ reached 1.0687, reflecting strong economic growth likely influenced by global recovery dynamics following the 2008 financial crisis. Under VRS, the most notable productivity gain occurred in 2015–2016 ($BMPI_{Icy} = 1.0093$), possibly reflecting regional resilience and improved internal efficiencies in managing economic resources. These peaks suggest strategic policy improvements, better macroeconomic management, or investment flows that temporarily boosted national output performance.

Despite the overall growth, several fluctuations and temporary declines were observed. A notable downturn occurred between 2011 and 2012, as well as between 2017 and 2018, especially under the VRS framework. The period 2017–2018 recorded the lowest CRS-based productivity score ($BMPI_{Icy} = 0.9944$), indicating inefficiencies or structural rigidities that may have hindered economic performance in that period. These dips may be attributed to external economic pressures, political instability, or declining commodity prices, which tend to heavily impact many African economies.

The decomposition of productivity change provides further clarity on the drivers of performance. Pure Technical Change ($BPTC_{Icy}$) recorded a geometric mean of 1.0489, making it the primary driver of productivity improvement across the period. This indicates that technological progress, innovation, or improvements in production processes substantially contributed to the observed GDP growth. However, Pure Efficiency Change ($BPEC_{Icy}$) remained relatively neutral, with a geometric mean of 0.9995, suggesting that the ability of countries to effectively utilize their existing inputs did not improve significantly over time.

Scale-related factors exhibited a more complex pattern. Scale Efficiency Change ($BSEC_{Icy}$) had a geometric mean of 0.9906, pointing to slight inefficiencies in aligning production scale with

optimal capacity. This suggests that while output was increasing, it was not always aligned with the most efficient operational scale, possibly due to overcapacity, underutilization of resources, or fragmented markets. Scale Technical Change (BSTC_{ya}) also recorded a low geometric mean of 0.9639, reinforcing the idea that scale-related technological adjustments were relatively weak across the observed countries.

Notably, the period 2011–2012 recorded the lowest scale technical score (BSTC_{ya} = 0.9408), indicating significant scale-related inefficiencies, possibly resulting from rapid but uneven growth, inadequate infrastructure, or policy misalignment. Conversely, the highest technical progress was noted during 2009–2010, when BPTC_{ya} reached 1.0677, aligning with a post-crisis rebound in investment and productivity-enhancing innovations.

Persistent infeasibility rates across many periods, with up to 23 out of 32 DMUs encountering computational issues in 2008–2009, reflect data variability and structural disparities between countries. These infeasibilities may be linked to inconsistencies in GDP reporting, rapid economic transitions, or mismatches between output and input data availability.

In summary, Real GDP productivity in African states demonstrated a sustained positive trend between 2000 and 2019, primarily driven by technological advancement rather than improvements in efficiency or scale optimization. While this reflects progress in innovation and economic transformation, the findings also suggest the need to improve institutional efficiency and better align economic operations with optimal scale. Future policy interventions should aim to strengthen resource utilization, deepen regional integration, and enhance technological diffusion across sectors to ensure balanced and inclusive productivity growth.

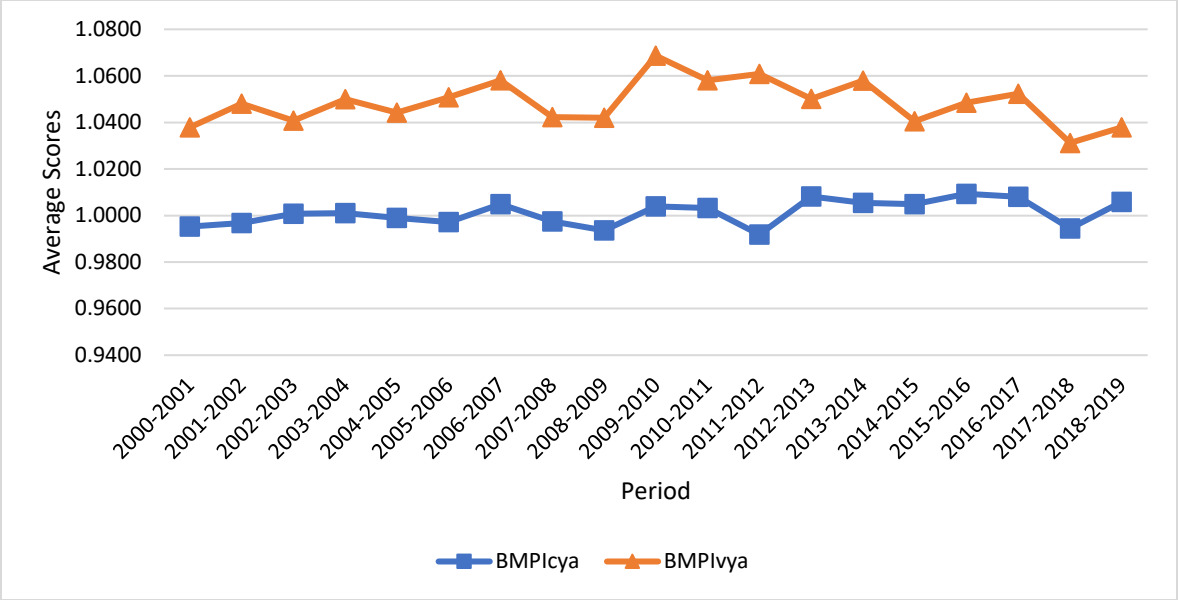


Figure 26 Real GDP average productivity trends (2000-2019)

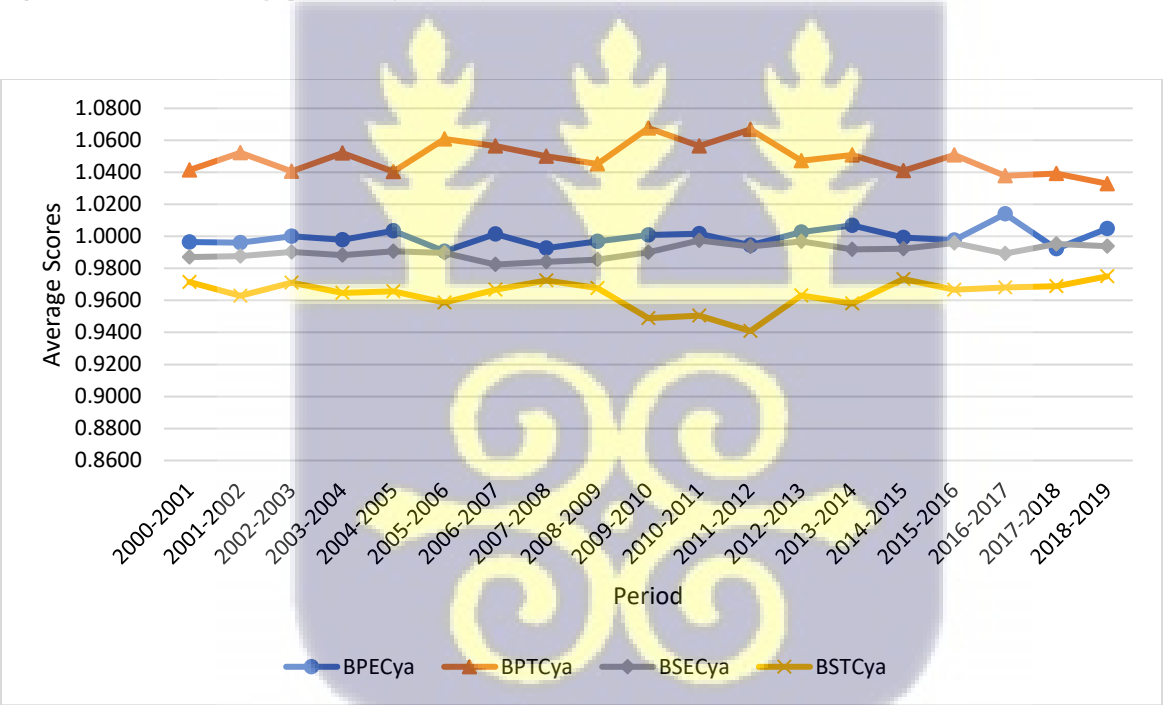
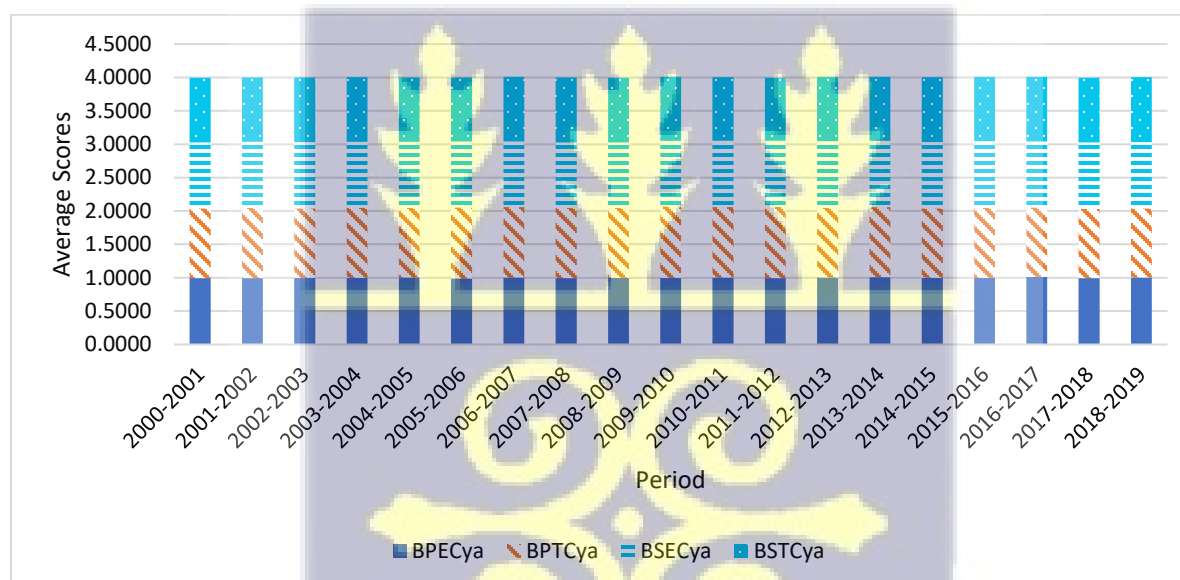
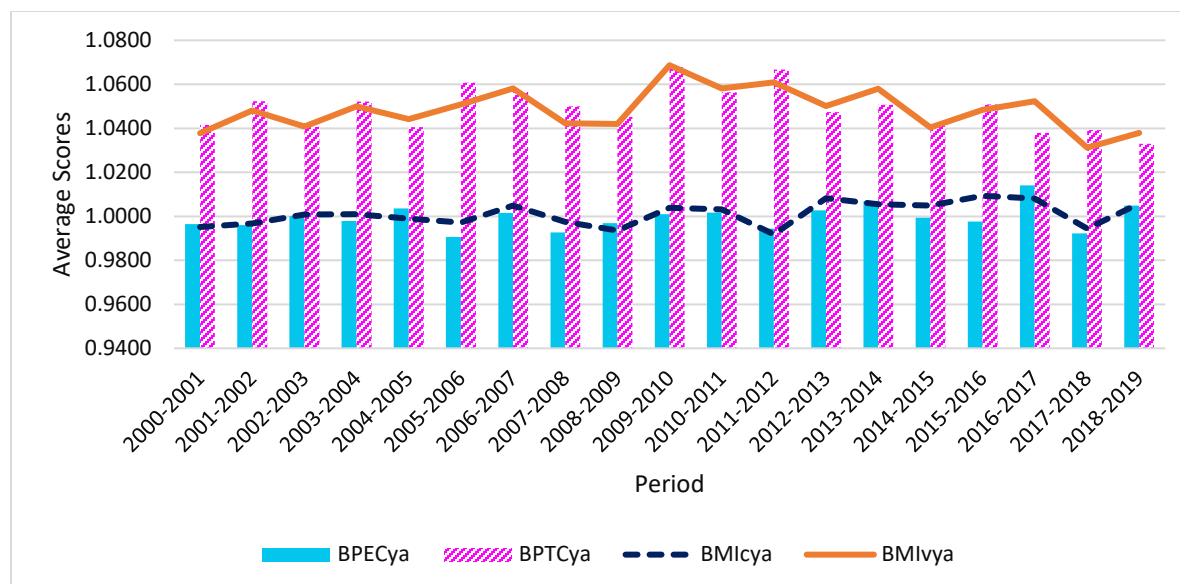


Figure 27 Components of average productivity trends (2000-2019) for Real GDP



5.6 Conclusion

The present chapter provides a comprehensive summary of the study's findings and discusses their implications. It highlights how the study successfully addressed its objectives and provided answers to the research questions that were posed. By consolidating and synthesizing the key findings, this chapter offers a clear understanding of the study's outcomes and their significance.

in relation to the broader research context. The study used a sample size of 32 African countries for all first stage MEA analysis because of data limitations for the rest of the countries. The data covered the period from 2000 to 2019.

The first objective of the study was to develop a proposed variable specific productivity index (MEA Biennial Malmquist-BMEAM) that handles the LP infeasibilities, Scale effect and illustrate the proposed productivity index using a hypothetical example. This was achieved by formulating the MEA efficiency model, incorporating the variable specific efficiencies into Malmquist productivity index to measure productivity changes over time then solving the infeasibilities using a biennial frontier to include the two time periods. The results showed that after using the biennial frontier the infeasibilities in estimating productivity were resolved.

The second objective aimed to decompose the productivity index into Biennial Pure Technical Change, Biennial Pure Efficiency Change, Biennial Scale Technical Change and Biennial Scale Efficiency Change. While the third objective aim at empirically applying the proposed productivity index to determine the energy productivity change of African states. Results showed that African energy industry, on average, experienced about 0.41% productivity improvement for capital services, 0.09% improvement productivity for labour, 0.02% decline for energy used. The study's four-factor decomposition revealed the effect of managerial expertise as the main driver of the improvement observed in the industry.



CHAPTER SIX

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

The concluding chapter comprises three parts. The initial section outlines the study's objectives and presents the significant findings. The subsequent section draws conclusions based on the identified key findings and their alignment with the objectives. Lastly, policy and practice recommendations, along with suggestions for future research directions, are presented.

6.2 Summary of the study

The primary objective of this study was to create a variable-specific productivity index called Biennial MEA Malmquist-BMEAM that effectively addresses issues related to linear programming (LP) infeasibilities and the scale effect. Additionally, the study aimed to demonstrate the application of this productivity index using a hypothetical example.

In addition to developing the productivity index, the study also aimed to empirically apply the productivity index to evaluate the variable-specific productivity changes in African states. This empirical analysis involved utilizing data from the years 2000 to 2019. By applying the productivity index to this dataset, the study aimed to assess and quantify the changes in productivity for each specific variable in the African states over the specified time period. Furthermore, the study intended to decompose the productivity index into four components: Pure Technical Change, Pure Efficiency Change, Scale Technical Change, and Scale Efficiency Change.

The study focused on evaluating the MEA (Malmquist Efficiency Analysis) efficiency and dynamic productivity of African states by analyzing the changes in the utilization of capital

services, labor, and energy in relation to economic productivity within these states. It was noted that despite a positive correlation between the performance of these variables and economic growth, Africa has exhibited low levels of energy consumption compared to the global average. Furthermore, this energy consumption has been inefficient, contributing to lower levels of economic (GDP) growth over time in the region.

Numerous studies have investigated the dynamics of capital service, labor, and energy efficiency in Africa as the United Nations (UN) aims to promote sustainable consumption and production resources, along with economic, social, and technological growth. However, these studies have often overlooked variable-specific efficiencies that optimize these finite resources and consumption. Such optimizations are crucial for achieving affordable energy for all by 2030, in line with the 7th Sustainable Development Goal (SDG). Additionally, they align with other SDG goals such as the 13th SDG, which focuses on combating climate change and its impacts. This current study makes a unique contribution by addressing these gaps, providing valuable insights for policy, practice, and academic research in this context.

Information regarding inputs such as capital services and labor, as well as the output variable of real GDP, were obtained from the Penn World Tables (PWT version 10.0). Primary energy consumption data was sourced from the US Energy Information Administration (EIA). Furthermore, data related to CO2 emissions variables were collected from the World Development Indicators (WDI). These diverse data sources were utilized to ensure comprehensive coverage and facilitate a thorough analysis of the variables involved in the study.

The study employed a two-stage analysis approach. In the first stage, a non-parametric Malmquist Efficiency Analysis (MEA) was conducted to assess variable-specific efficiencies and dynamic

productivities. This analysis aimed to measure the performance and changes in efficiency over time for each specific variable.

Following the MEA analysis, the second stage involved decomposing the results of dynamic productivity using the developed biennial MEA productivity index. This decomposition allowed for a more detailed understanding of the underlying factors contributing to changes in productivity.

Before delving into the analysis, the study presented descriptive statistics of the pooled data and also provided separate statistics for the regional blocs in Africa. This allowed for a comprehensive overview of the data and an understanding of any regional variations.

Furthermore, tests were conducted to determine the returns to scale and to examine the correlations between the inputs and outputs used in the study. These tests helped to assess the relationships between the variables and provided insights into their interdependencies.

Overall, this two-stage analysis approach provided a thorough assessment of variable-specific efficiencies, dynamic productivities, and the relationships between inputs and outputs in the African context.

In the analysis, the estimation of variable-specific MEA efficiencies and dynamic productivity was conducted using the Bogetoft and Asmild methodology developed for MEA. The efficiencies of selected states were estimated over a 20-year period, ranging from 2000 to 2019.

Next, the Malmquist Productivity Index (MPI) was calculated to assess changes in productivity over time. However, it was observed that a significant portion, around 50-60%, of the estimated MPIs exhibited infeasibilities. Ignoring these infeasibilities would lead to skewed results, making it necessary to address them.

To resolve these infeasibilities and ensure accurate productivity measurement, the developed productivity index (BMEAM) was employed. This index was specifically designed to handle infeasibilities and provide a more robust assessment of dynamic productivity.

By utilizing the BMEAM productivity index, the analysis aimed to overcome the challenges posed by infeasibilities in the estimation of the Malmquist Productivity Index, thus ensuring more accurate and reliable results for evaluating productivity changes in the African states over the given time period.

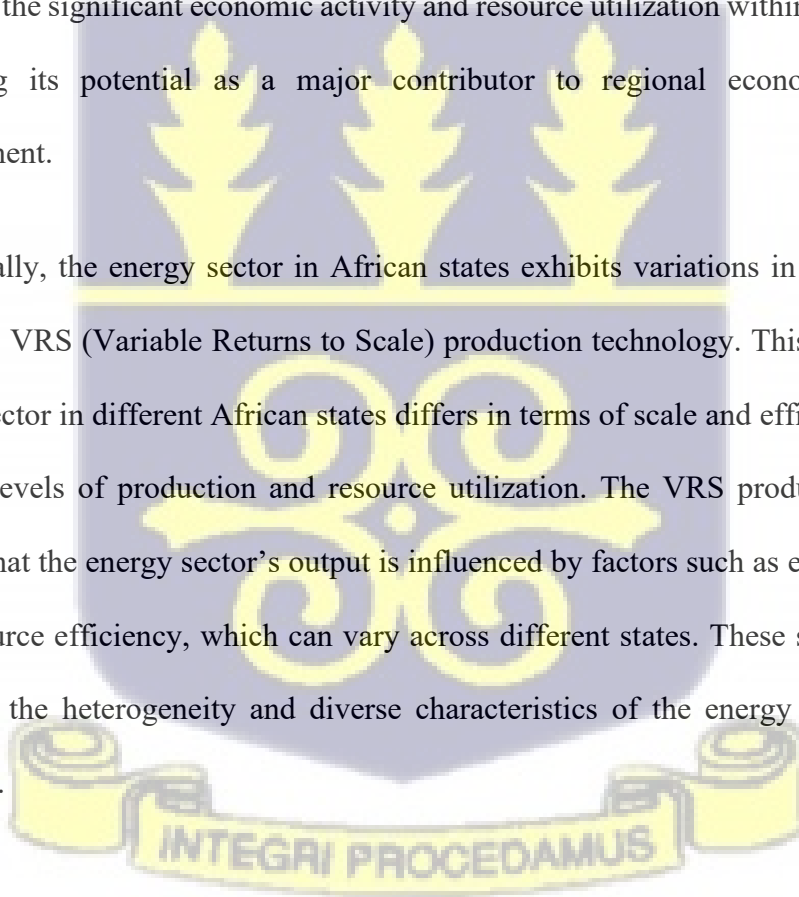
The study expanded upon the BMEAM productivity index by introducing a four-factor decomposition to uncover the primary factors influencing productivity changes in the energy industry. The obtained results were further broken down into four distinct components: Biennial Pure Efficiency Change (BPEC), Biennial Pure Technical Change (BPTC), Biennial Scale Efficiency Change (BSEC), and Biennial Scale Technical Change (BSTC). This decomposition analysis aimed to provide a comprehensive understanding of the underlying drivers behind productivity changes in the energy sector. By examining efficiency improvements, technological advancements, and changes in scale, the study sought to identify the specific factors contributing to overall productivity changes within the industry over the biennial period.

All estimations in the study were conducted using R version 4.1.2, along with its associated statistical packages. The key findings identified in the study are as follows:

- a.** Based on the pooled statistics, it was found that, on average, African states utilized approximately 2,835.82 million units of capital services, 7.26 million units of labor, and 0.4 quadrillion British thermal units (Btu) of energy to generate around 109,292.98 million US dollars of real GDP. Additionally, the average CO₂ emissions were measured at 29,712.12 kilotons. These statistics highlight significant variations in the variables across

different states in Africa. It indicates that the levels of capital services, labor, energy consumption, GDP, and CO₂ emissions vary considerably among the African states, reflecting the diverse economic and developmental conditions present in the region.

- b.** At the regional level, the AMU (Arab Maghreb Union) region stood out by employing the highest amount of capital services, totaling 4,702.98 million US dollars. Additionally, the AMU region also had the highest energy consumption, reaching 0.91 quadrillion British thermal units (Btu). This substantial energy usage might have contributed to the region's ability to achieve the highest real GDP of 266,239.84 million US dollars. These figures highlight the significant economic activity and resource utilization within the AMU region, indicating its potential as a major contributor to regional economic growth and development.
- c.** Statistically, the energy sector in African states exhibits variations in size and operates under the VRS (Variable Returns to Scale) production technology. This suggests that the energy sector in different African states differs in terms of scale and efficiency, indicating varying levels of production and resource utilization. The VRS production technology implies that the energy sector's output is influenced by factors such as economies of scale and resource efficiency, which can vary across different states. These statistical findings highlight the heterogeneity and diverse characteristics of the energy sector in African countries.



6.3 Conclusion

The findings of this study have raised important considerations that should be taken into account in cross-country studies of variable-specific efficiency and dynamic productivity assessments in the energy sector.

Firstly, the significant differences observed in all variables across regional blocs and over time provide clear evidence that the size of the energy sector matters in Africa. This suggests that the scale of operations and resource utilization vary among different blocs and time periods, indicating the importance of specific characteristics and dynamics of each region when analyzing energy sector efficiency and productivity.

Furthermore, the study revealed that the selected states in the energy sector operate under the VRS production technology. This implies that these states are not operating at their optimal scales, indicating the presence of either increasing returns to scale, decreasing returns to scale, or a combination of both in the industry. This finding suggests that there is room for improving efficiency and productivity by optimizing the scale of operations in the energy sector of these states.

These insights highlight the need for further research and policy considerations to address the disparities in size and scale efficiency within the energy sector across African countries. By focusing on optimizing scales of operation and addressing specific regional characteristics, efforts can be made to enhance efficiency and productivity in the energy sector, ultimately contributing to sustainable development and economic growth in the region.

According to Ohene-Asare et al. (2019), firms operating under increasing or decreasing returns to scale are also experiencing economies or diseconomies of scales respectively. In this context, such

states have the potential to improve by either increasing or decreasing their scales of operations to achieve optimal levels of outputs while reducing inputs.

The International Energy Agency (IEA) acknowledges that Africa has the potential to achieve significant economic and industrial growth, surpassing previous economies, provided that appropriate policies are implemented. This growth can be supported by promoting the expansion of clean technologies and placing emphasis on enhancing energy efficiency.

To ensure a reliable, affordable, and clean energy supply, Africa is making efforts to enhance its power generation capacity and upgrade transmission grids. One strategy being pursued is the establishment of cross-country transmission interconnection systems in the region. These interconnections aim to create a sub-regional balance between energy supply and demand across Africa, contributing to a more stable and efficient energy sector.

By implementing these strategies and focusing on clean energy technologies and energy efficiency enhancements, Africa aims to achieve its goal of providing reliable and affordable energy to its population. These initiatives align with the broader global objectives of sustainable development and mitigate the environmental impact of energy production and consumption.

6.4 Recommendations

Based on the study's findings and conclusions, it is possible to offer recommendations for policy implementation and practical measures, as well as suggest avenues for future academic research.

These recommendations are well defined below:

6.4.1 For policy:

The MEA variable-specific dynamic productivity analysis demonstrated a better ability to capture multiple sources of productivity change. These findings reveal that while some African states

achieved modest productivity gains, others experienced stagnation or decline, often correlating with disparities in institutional capacity, infrastructure, and regulatory coherence.

To enhance clarity and traceability, a policy matrix has been developed below, linking observed productivity challenges to actionable strategies by region, input/output, and BMEAM component:

Regionalized Policy Recommendations Based on BMEAM Findings

This section draws on the decomposition of the BMEAM index to provide region-specific, actionable recommendations. Each regional discussion aligns with key productivity components, such as pure efficiency, technical change, or scale effects, identified as dominant drivers or constraints within the empirical results. By contextualizing the findings through regional patterns, the study ensures that the policy interventions are both grounded and targeted.

In **North Africa (AMU)**, the analysis revealed strong contributions to productivity growth from scale efficiency in capital services, suggesting that the region is effectively leveraging economies of scale in capital investment. To sustain and build on this trend, it is recommended that countries in this region expand financing mechanisms for large-scale, capital-intensive projects, particularly in clean technology sectors where economies of scale are critical.

In **West Africa (ECOWAS)**, labour inefficiency emerged as a persistent challenge, especially in energy production and transformation. The weak performance in pure efficiency change (BPEC) suggests that human capital is not being fully optimized. Policymakers should prioritize workforce development by investing in technical and vocational training, particularly in energy sector operations and infrastructure maintenance, to improve labour productivity.

In **Central Africa (ECCAS)**, low scores in pure technical change (BPTC) for energy inputs indicate technological stagnation and limited innovation in energy systems. To address this, the

region should focus on upgrading grid infrastructure, supporting research and development in renewable energy, and encouraging the adoption of clean energy technologies through fiscal incentives and regional cooperation.

In **Southern Africa (SADC)**, scale technical change (BSTC) was identified as a limiting factor, especially in electricity distribution. The region displays variability in performance, indicating that electricity infrastructure is not consistently scaled or optimized. Investment in grid harmonization, expansion of cross-border electricity trading, and infrastructure optimization, particularly in underserved rural areas, can improve productivity.

In **East Africa (EAC)**, mixed performance in GDP output was associated with low scores in both pure efficiency and technical change. This suggests challenges in institutional performance and technological upgrading. To address these, countries in East Africa should enhance regulatory frameworks, promote SME-driven innovation, and incentivize energy-efficient production practices in key industries to achieve more consistent and sustainable productivity growth.

6.4.2 For practice:

To enhance performance, management should implement competent managerial policies focused on optimizing the input mix, investing in efficient cost techniques, and making adjustments to operational size. These measures can contribute to improved productivity and sustainability in the energy sector.

For instance, the observed highest performance improvement in the AMU region indicates its potential for further progress. To capitalize on this achievement, the AMU should consider expanding its investments in cleaner energy resource management technologies and expertise. These factors were identified as key drivers of their performance and align with the objectives of Sustainable Development Goal 7 (SDG7), which emphasizes affordable and clean energy for all.

By prioritizing optimal resource management, investing in cleaner energy technologies, and leveraging their expertise, the AMU region can enhance its overall performance and contribute significantly to the attainment of SDG7 targets. Effective managerial policies and strategic investments will be instrumental in realizing the full potential of the AMU region and fostering sustainable energy practices.

Overall, it is crucial for management to recognize the importance of adopting competent policies that optimize inputs, implement efficient cost techniques, and adapt operations to enhance performance. This will not only improve productivity but also contribute to the broader goal of achieving sustainable and clean energy systems.

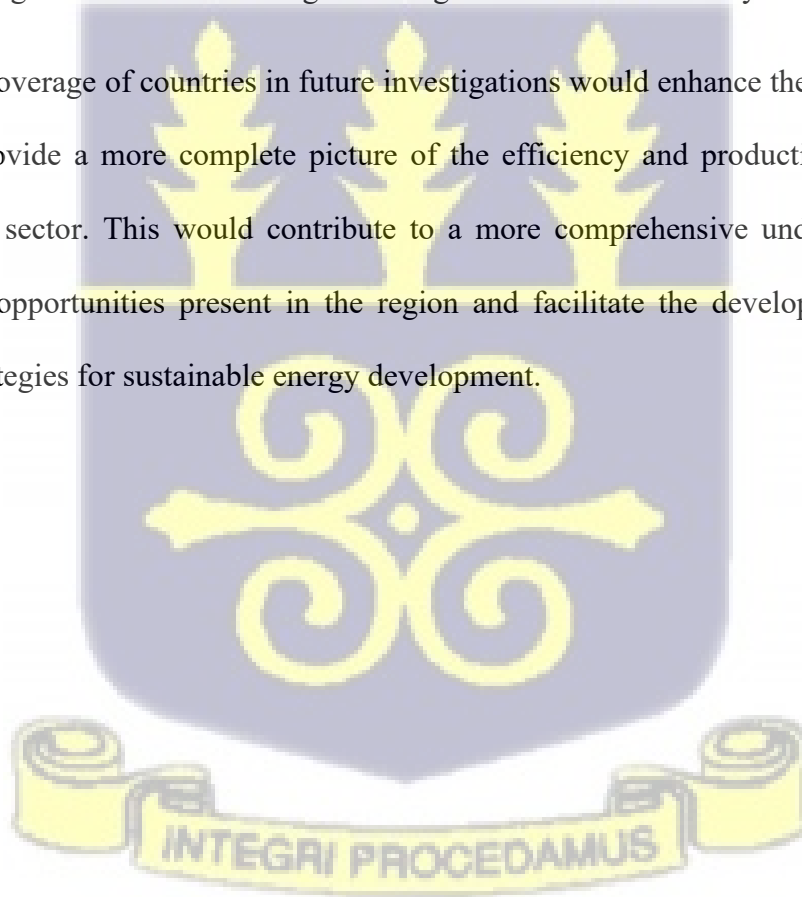
6.4.3 For further research:

1. In the estimation of the BMEAM the issue of transitivity was not handled. Further research can be extended to the developed BMEAM to resolve transitivity by drawing inspirations from the Global Malmquist index of Pastor and Lovell (2005).
2. Comparative studies across different African regions and countries can identify regional variations in energy sector performance, efficiency, and productivity, leading to the sharing and implementation of best practices. These studies can be conducted from a variable specific efficiency and dynamic productivity point of view utilizing MEA and its extensions
3. The MEA efficiency and dynamic productivity measures in the study were based on a sample of 32 African states, representing a subset of the total 54 African states. However, it is important to note that the analysis was limited to the available data, and further investigation could be undertaken to include the remaining countries.

The study acknowledges that the unavailability of data for certain countries resulted in the exclusion of potential observations, which could have provided valuable insights and intriguing findings. The limited sample size may have constrained the generalizability of the results and the comprehensive understanding of efficiency and productivity dynamics across the entire African continent.

To address this limitation, future research can aim to expand the scope of analysis to encompass a broader range of African countries. By incorporating additional data and increasing the sample size, a more comprehensive assessment of efficiency and productivity in the energy sector can be achieved, enabling a better understanding of the regional and continental dynamics.

Expanding the coverage of countries in future investigations would enhance the robustness of the findings and provide a more complete picture of the efficiency and productivity landscape in Africa's energy sector. This would contribute to a more comprehensive understanding of the challenges and opportunities present in the region and facilitate the development of targeted policies and strategies for sustainable energy development.



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APPENDIX A**Countries sampled for this study.**

Country	ID	Code	Group (Economic Zone)	Times Sampled
Angola	1	AGO	ECCAS	10
Benin	2	BEN	ECOWAS	10
Botswana	3	BWA	SADC	10
Burkina Faso	4	BFA	ECOWAS	10
Burundi	5	BDI	EAC	10
Cabo Verde	6	CPV	ECOWAS	10
Cameroon	7	CMR	ECCAS	10
Central African Republic	8	CAF	ECCAS	10
Chad	9	TCD	ECCAS	10
Côte d'Ivoire	10	CIV	ECOWAS	10
Djibouti	11	DJI	EAC	10
Egypt	12	EGY	AMU	10
Eswatini	13	SWZ	SADC	10
Gabon	14	GAB	ECCAS	10
Guinea	15	GIN	ECOWAS	10
Kenya	16	KEN	EAC	10
Lesotho	17	LSO	SADC	10
Mauritania	18	MRT	AMU	10
Mauritius	19	MUS	SADC	10
Morocco	20	MAR	AMU	10
Mozambique	21	MOZ	SADC	10
Niger	22	NER	ECOWAS	10
Nigeria	23	NGA	ECOWAS	10
Rwanda	24	RWA	EAC	10
Sao Tome and Principe	25	STP	ECCAS	10
Sierra Leone	26	SLE	ECOWAS	10
South Africa	27	ZAF	SADC	10
Sudan	28	SDN	AMU	10
Togo	29	TGO	ECOWAS	10
Tunisia	30	TUN	AMU	10
U.R. of Tanzania: Mainland	31	TZA	EAC	10
Zimbabwe	32	ZWE	SADC	10

APPENDIX B

Taxonomy Table of MEA related Studies

	Authors	Industry	Method	Country Sample Year	Inputs and Outputs	Objectives	Findings
1	Wang et al., 2013	Energy	MEA approach	China 1997-2010	Input: Number of employees, fixed assets, Total Deposits Outputs: net interest earnings, non-interest earnings	1. To evaluate Chinese regional energy and emissions efficiency, using multi-directional efficiency analysis (MEA) technique rather than the usual radial DEA.	1. Over the course of the investigation, China's average MEA efficiency increased. 2. During the research period, certain sectors were more efficient than others.
2	Zhu et al., 2019	Energy	Improved MEA Markov Model	China's 30 Provinces 2005-2016	Input: total energy consumption, the capital stock, labor force Outputs: desirable output(GDP), undesirable output CO2 emissions	1. To examine the regional energy efficiency's dynamic evolution, improvement trends, and improvement potential.	1. Center provinces, as well as a few eastern and western provinces, were to use a one-way breakthrough path for their weak linkages. To enhance energy efficiency, most western provinces were to use a step-by-step progressive. 2. The energy saving potential of the eastern and western regions was relatively large, and the potential value of CO2 emission reduction in the central region are relatively large.
3	Wang et al., 2014	Environment	MEA	China 2006-2010	Input: energy: total energy consumption of industrial enterprises , labor: number of employed person	1. To Evaluate the environmental efficiency of 30 Chinese large cities' industrial sectors, as well as finding efficiency patterns and predicting pollution reduction potential	1. Environmental efficiency increased in the less developed cities were faster than the cities in the well-developed region. 2. Although cities showed similar

					capital: net value of fixed assets Desirable output: value added Undesirable outputs: total volume of industrial waste gas emission , soot emission, dust emission, waste water discharge, solid waste discharge.		environmental efficiency levels, the undesirable output variable-specific efficiency patterns of these cities were diversified, and according to the variable-specific efficiency. 3. Different cities were to have different industrial pollutant reduction priorities: East China cities was to pay more attention to their industrial waste gas emissions and industrial wastewater discharges, while western cities were to focus on their industrial soot and dust emissions, and solid waste discharges.
4	Wang et al., 2017	Energy	Non-parametric frontier approach	China 2006-2010	Inputs: labor, Capital, Energy Outputs: GDP (desirable), Solid and liquid waste (Undesirable),CO2 emissions (Undesirable)	1. To Study economy-wide energy efficiency and productivity performances by accounting for sectoral heterogeneity. 2. To Obtain an aggregation of the sectoral energy efficiency performances of regions an economy-wide energy efficiency index. 3. To assess the change in the aggregate energy performance over time.	1.The study revealed significant sectoral heterogeneities in the country’s energy performance. 2. The study found that the transport sector showed the most substantial contribution to the economy-wide energy productivity growth, while the agriculture sector and industry sector saw a decline in energy productivity.
5	Asmild & Pastor, 2010	Health Care	MEA RDM	World wide 27 countries 1998	Input Expenditure Life expectancy Infant survival Output comprehensive efficiency measure	To utilise second phase models that ensure slack free benchmarks to provide new comprehensive efficiency measures for VRS-MEA and RDM	The proposed model by the study benchmarked selections in both the original models and the slack free models. MEA selected benchmarks with lower expenditures and higher infant survival, but lower life expectancy than those of RDM where slacks existed. An example is USA exhibited slacks in RDM but not in MEA and,

							as a consequence of the differences in benchmark selections
6	Hansson et al, 2018	Agriculture	MEA	Sweden 421 Swedish dairy farms 2013	Input; Variable costs Fixed cost Asset Labour Output Milk & Meat revenue	To evaluate and identify possibilities of waste reduction in the utilisation of production factors.	The findings had clear implications for policy schemes aiming at pushing farms towards the efficient production frontier
7	Bi et al., 2014	Transportation	MEA	China 2006-2011	Input: labor and capital stocks (two nonenergy input, the volume of energy consumed in transportation sector(energy input) Outputs: value-added amount(desirable output), volume of CO2 emissions related to fuel used in the sector (undesirable output).	To evaluate the energy and environmental efficiency level and trend of China's transportation sector. whiles investigating the efficiency patterns of 30 regions and three major areas of China.	The study identified the energy saving potential and CO2 emissions reduction potential for each province and area in China.
8	Asmild et al. ,2009	Transportation-Railway	MEA	23 European nations 1995 to 2001	Discretionary inputs – Staff costs (in million euros) – Material purchases and external charges (in million euros) Non-discretionary input	To examine whether railway reforms initiatives launched by the European Commission enhanced the efficiency of the railway networks	Major conclusions are that reform measures enhanced technical efficiency in general, but in various ways for distinct cost drivers.

					– Network length Outputs – Length of Passenger and Freight trains		
9	Suhaimi et al., (2017)	Agriculture	MEA	Malaysia	Inputs: Land size (number of hectares) Labor (total labor used for dairy activities) Herd size (number of cows that a farmer owned) Feed (total cost of purchased feed) Output: (total revenue)	To study the technical inefficiency of dairy farms and subsequently investigate the factors affecting technical inefficiency in the Malaysian dairy industry.	The results suggested that the scores for managerial and program inefficiency were provided for intensive and semi-intensive manufacturing systems. This showed significant variability in inefficiency ratings across inputs and production methods.
10	Asmild M & Matthews K., 2012	Banking	MEA	China 1997-2008	Input: Number of employees, fixed assets, Total Deposits Outputs: net interest earnings, non-interest earnings	To evaluate differences in efficiency patterns between different banking subgroups using the more recent MEA methodology.	The results showed that State owned banks (SOBs) were less efficient than Joint Stock Banks(JSBs) on utilization of labour & production of non-performing loans.
11	Asmild et al., 2016	Agriculture	MEA	Lithuania 3,308 different Lithuanian family farms 2004–2011	Inputs: land, labour, capital, and materials employed Outputs: Total revenue	To analyse both managerial and technical efficiencies of Lithuanian family farms using multi-directional efficiency analysis envelopment analysis approach which considers input-specific efficiencies.	The study showed that there existed differences between the efficiency scores on the different inputs as well as between the farm types of crop, livestock and mixed farms respectively. Additionally, crop farms had the highest technical efficiency, but the lowest managerial efficiency and the mixed farms have the lowest technical efficiency (yet

							not the highest managerial efficiency).
12	Holvad et al, 2004	Transportation	DEA MEA	Norway 154 companies	Input Fuel costs Driver costs Output Seat (kilometers)	To study the cost the Norwegian bus industry using data previously used by Jørgensen et al. (1995), which studied the cost structure of the Norwegian bus companies using a modified Cobb-Douglas function	The results showed how it becomes possible to disaggregate inefficiency into different components using MEA and corresponding to different types of cost generating variables. Thereby providing both managers of the bus companies and policy makers with more detailed information on possible improvements in performance
13	Asmild, Kronborg & Matthews, 2016	Banking	MEA	China 64 Observations 5 year period	Input Labour Fixed Assets Output net interest revenue non-interest revenue	To propose a model that can be used to analyse the variable specific inefficiency contributions.	The proposed model by the study revealed that the SOCBs that had the largest concentration, i.e. the least variance, in all dimensions. The reason is that state owned banks in China were created in 1979 from the commercial arm of the Peoples Bank of China, which had a monopoly on all banking activities. Nonetheless, despite modernization, listing and reforms, the policy oriented activity of the state-owned banks, its branches and organizational structure remained much the same as part of its heritage
14	Baležentis & De Witte, 2015	Agriculture	MEA	Netherlands 1200 observations 2004–2009	Input Land Labour Intermediate consumption Assets Output Crops Production subsidies	To combine the traditional measures to estimate relative efficiency with multi-directional efficiency.	The study showed that combining the traditional one-directional and multi-directional efficiency framework yielded valuable insights. There by allowing one to identify what factors mattered in terms of output production and input consumption
15	Tasevska, -Hansson,	Agriculture	MEA	Sweden	Input	To analyse the regional TE of agricultural resources	The results indicated the largest improvements in the efficiency of

	Asmild & Surry, 2018			21 observations 1998-2013	Variable costs Fixed costs Labour Assets Output Agricultural output Diversified output social		diversified output and labour, especially when concerns about environmental conditions and rurality were included in CAP, 2008-2013. Additional improvements in regional efficiency could be achieved by creating possibilities for diversified output and structural changes in assets.
16	Ohene-Asare, Gakpey & Turkson, 2018	Energy	DEA	46 unique OPCs that belong to the four IGOs 2000 to 2013	Input Labor force Gas reserves Oil reserves Output Gas Produced Oil produced	This study was to compare the production efficiencies and frontiers differences of oil-producing countries (OPCs) in four inter-governmental organizations (IGOs) in the international petroleum industry with the aim of providing such countries understanding of group characteristics that help maximize their supply interests	Results indicated that groups with high inter and intra-group collaborations which ensure exchange of information, organizational learning and innovation tend to do better than groups with even higher hydro-carbon endowment. Additionally, hydro-carbon resource endowment may not be the solution to group inefficiency without higher endowment in human capital, economic stability, technology and infrastructure
17	Ohene-Asare, Tetteh & Asuah, 2020	Energy	DEA	African countries 46 countries 1975–2015	Input Electricity Coal Gasoline Output real GDP CO2	1.To contribute to the energy efficiency literature by evaluating the total factor energy efficiency (TFEE) of African countries and sub-regions while considering carbon dioxide emissions; second. 2. To contribute to the economic development literature by investigating the impact of economic development on TFEE in Africa. 3. To contribute to the second stage regression in efficiency literature by investigating the	The results showed African countries to be on average, 56% energy efficient within the study period. Other African sub-regions could adopt the energy efficiency policies of North Africa as benchmark to improve energy efficiency. Economic development and technological progress are found to have significant positive effects on energy efficiency of African countries, while higher energy prices lead to higher inefficiency. Also, a bi-causal relationship is found to exist between total factor energy efficiency and economic

						possible bidirectional link between energy efficiency and economic development	development, giving support to the concept of sustainable development and confirming the International Energy Agency's assertion on the positive macroeconomic impacts of energy efficiency
18	Sueyoshi & Goto, 2012	Energy	DEA	China 2005 to 2009	Input Oil reserve Gas Reserve Operating Cost No. of Employee Output Oil Production Gas Production CO2 Emission	The purpose of this study is to compare the performance of petroleum firms under public with that of firms under private ownership by fully utilizing the proposed DEA models for environmental assessment	The result showed that national oil companies under public ownership outperform international oil companies under private ownership in terms of unified (operational and environmental) efficiency and unified efficiency under natural disposability.
19	Jebali, Essid & Khraief, 2017	Energy	DEA	Mediterranean countries 1000 observations 2009-2012	Input Labor force Energy consumption gross fixed capital Output GDP	To evaluate the economy-wide energy efficiency in Mediterranean countries for the period 2009-2012 and specify its determinants by using a two-stage double bootstrap method	In the first stage, the study explored trends in energy efficiency of Mediterranean countries. The DEA approach offered the advantage of a substantial increase in the sample size by pooling the data over time. Such a practice is important for obtaining reliable estimates of efficiency
20	Manevska Tasevska, Hansson, Asmild & Surry, 2021	Agriculture	Farm Accounting Data Network (FADN) MEA	Sweden	Input variable costs fixed costs labour assets value Output agricultural output diversified output	To analyse the regional efficiency of agricultural resources in Sweden and, to explore how efficiency patterns of the Swedish regions evolved during the last three CAP programmes: CAP 1 (for 1998–2002); CAP 2 (for 2003–2007); and CAP 3 (for 2008–2013).	The studied showed that the efficiency of labour, assets and diversified output vary the most. Over the CAP periods, the efficiency gap across the counties decreased, indicating that changes in the policy helped for regions cohesion.



APPENDIX C

MEA Variable efficiency scores of all the 32 states from 2000 -2009 for Capital Service (xa)

Country	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Angola	1	1	1	1	1	1	1	1	1	0.88140
Benin	0.6899	0.696273	0.71099	0.69404	0.72716	0.73347	0.77137	0.78957	0.80320	0.80115
Botswana	1	1	1	1	1	1	1	1	1	1
Burkina Faso	0.93981	1	1	1	1	1	1	0.9883	0.99545	0.99395
Burundi	1	1	1	1	1	1	1	1	1	1
Cabo Verde	1	1	1	1	1	0.92938	0.94175	0.94074	0.99954	0.92101
Cameroon	0.80682	1	1	1	1	1	1	0.91461	0.94404	0.91341
Central African Republic	1	1	1	0.72660	1	0.71896	0.70390	0.71016	0.66995	0.70325
Chad	1	1	1	1	1	1	1	1	1	1
Côte d'Ivoire	0.78968	0.789179	0.79298	0.8414	0.84580	0.84540	0.90915	0.93027	0.96626	1
Djibouti	1	1	1	1	1	1	1	1	1	1
Egypt	1	1	1	1	1	1	1	1	1	1
Eswatini	0.71762	0.718968	0.73294	0.82035	1	1	1	1	1	1
Gabon	1	1	1	1	1	1	1	1	1	1
Guinea	1	1	1	1	1	1	1	1	1	1
Kenya	1	1	1	1	1	1	1	1	1	1
Lesotho	0.66283	0.655213	0.67999	0.69381	0.73623	0.73271	0.75858	0.75209	0.82132	0.80119
Mauritania	1	1	1	1	1	1	1	1	1	0.94764
Mauritius	1	1	1	1	1	1	0.9135	0.94056	0.96950	1
Morocco	0.71450	0.728402	0.74275	0.75120	0.75294	0.74934	0.76006	0.75862	0.76012	0.76024
Mozambique	0.76854	0.779363	0.77949	0.78777	0.81631	0.84992	0.92380	0.95636	0.98867	1
Niger	0.6925	1	0.72212	0.69452	0.54871	0.58521	0.66543	0.56471	0.63162	0.58469
Nigeria	1	1	1	1	1	1	1	1	1	1
Rwanda	1	1	1	1	1	1	1	1	1	1
Sao Tome and Principe	1	1	1	1	1	1	1	1	1	1
Sierra Leone	0.80544	0.845557	0.96333	0.99372	0.95126	0.99109	1	1	1	1
South Africa	0.61434	1	0.63633	1	1	1	0.67068	1	0.69016	0.68896
Sudan	1	1	1	1	1	1	1	1	1	1
Togo	0.65714	0.663296	0.67490	0.6708	0.67291	0.66439	0.69438	0.69434	0.71010	0.69539
Tunisia	0.70769	0.692394	0.69133	0.71877	0.73569	0.75534	0.79536	0.93749	0.84909	1
U.R. of Tanzania: Mainland	1	1	1	1	1	1	1	1	1	1
Zimbabwe	0.62227	0.622327	0.61669	0.60925	0.61657	0.61761	0.62841	0.63064	0.64533	0.80688
GEO MEAN	0.8678	0.9009	0.8859	0.8952	0.9072	0.9003	0.9006	0.9122	0.9104	0.9129

MEA Variable efficiency scores of all the 32 states from 2010 -2019 for Capital Service (xa)

Country	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	Average
Angola	0.914466	1	1	1	1	0.824909	0.843988	1	1	0.804831	0.963
Benin	0.793831	0.82249	0.860153	0.865837	0.876973	0.891322	0.889462	0.87115	0.842231	0.833618	0.798
Botswana	1	1	1	1	1	1	0.908383	0.907655	0.907452	0.876117	0.980
Burkina Faso	0.988463	1	0.953422	0.937707	0.941387	0.925156	0.952433	0.940839	0.906694	0.869042	0.967
Burundi	1	0.885202	0.890145	0.908883	1	0.953508	0.954873	1	0.867336	0.833321	0.965
Cabo Verde	0.895627	0.848153	0.876763	0.861504	0.823343	0.809142	0.793928	0.777005	0.78698	0.784522	0.899
Cameroon	0.896976	0.909612	0.915165	1	0.921056	0.909664	0.906544	0.888786	0.863114	0.870513	0.933
Central African Republic	0.597191	0.607925	0.6348	0.606407	0.623276	0.625569	0.635937	0.674318	1	1	0.762
Chad	1	1	1	1	1	1	1	1	0.945039	0.958302	0.995
Côte d'Ivoire	0.996128	1	1	1	1	1	1	1	1	1	0.935
Djibouti	1	1	1	1	1	1	0.990793	0.975776	0.979391	1	0.997
Egypt	1	1	1	1	1	1	1	1	1	1	1.000
Eswatini	1	1	1	0.980128	1	1	1	1	1	1	0.949
Gabon	1	1	1	1	1	1	1	1	1	1	1.000
Guinea	1	1	1	1	1	1	0.938595	0.956841	0.959609	0.943652	0.990
Kenya	1	1	1	1	1	1	1	1	1	1	1.000
Lesotho	0.818842	0.830959	0.839781	0.817682	0.801244	0.808567	0.818947	0.779168	0.803581	0.781613	0.770
Mauritania	1	1	0.908023	1	0.877151	0.871312	1	0.807889	0.788792	0.792171	0.950
Mauritius	1	1	1	1	1	1	1	1	1	1	0.991
Morocco	0.759599	0.764614	0.759384	0.761442	0.763492	0.760164	0.758708	0.761646	0.759169	0.746285	0.754
Mozambique	1	0.993739	0.932994	0.874236	0.811442	0.807261	0.805185	0.805486	0.758815	0.719844	0.858
Niger	0.57194	0.547024	0.609819	0.642358	0.664311	0.664373	0.665857	0.682735	0.676814	0.654212	0.653
Nigeria	1	1	1	1	1	1	1	1	1	1	1.000
Rwanda	1	1	1	1	1	1	1	1	1	1	1.000
Sao Tome and Principe	1	1	1	1	1	1	1	1	1	1	1.000
Sierra Leone	1	0.973394	0.995264	1	1	1	1	1	1	1	0.976
South Africa	0.698511	0.70604	0.707687	0.70798	1	0.691508	0.688102	0.687851	0.689414	0.692377	0.778
Sudan	1	1	1	1	1	1	1	1	1	1	1.000
Togo	0.712069	0.725993	0.751534	0.770148	0.775407	0.807247	0.811633	0.806667	0.806542	0.809817	0.729
Tunisia	1	1	1	1	0.928224	0.861481	0.857994	0.839347	1	1	0.869
U.R. of Tanzania: Mainland	1	1	1	0.995434	0.974416	0.96337	0.952707	1	1	1	0.994
Zimbabwe	0.83583	0.873557	0.926429	0.925826	0.932796	0.947636	0.953626	0.961378	0.969556	0.925255	0.783
GEO MEAN	0.9110	0.9114	0.9159	0.9187	0.9218	0.9026	0.9031	0.9062	0.9097	0.8959	

MEA Variable efficiency scores of all the 32 states from 2000 -2009 for Labour (xb)

Country	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Angola	1	1	1	1	1	1	1	1	1	0.951989

Benin	0.746027	0.734906	0.755011	0.713118	0.737829	0.739011	0.732039	0.741718	0.743822	0.720476
Botswana	1	1	1	1	1	1	1	1	1	1
Burkina Faso	0.924458	1	1	1	1	1	1	0.982652	0.995088	0.993974
Burundi	1	1	1	1	1	1	1	1	1	1
Cabo Verde	1	1	1	1	1	0.913763	0.902045	0.887021	0.999503	0.888491
Cameroon	0.886964	1	1	1	1	1	1	0.932439	0.972664	0.920772
Central African Republic	1	1	1	0.978772	1	0.992202	0.981217	0.977248	0.928357	0.937863
Chad	1	1	1	1	1	1	1	1	1	1
Côte d'Ivoire	0.823155	0.769525	0.774818	0.842472	0.807596	0.788553	0.843131	0.906217	0.964239	1
Djibouti	1	1	1	1	1	1	1	1	1	1
Egypt	1	1	1	1	1	1	1	1	1	1
Eswatini	0.853943	0.88934	0.920248	0.989424	1	1	1	1	1	1
Gabon	1	1	1	1	1	1	1	1	1	1
Guinea	1	1	1	1	1	1	1	1	1	1
Kenya	1	1	1	1	1	1	1	1	1	1
Lesotho	0.655185	0.644395	0.67146	0.676122	0.706732	0.699303	0.715519	0.713061	0.767113	0.757984
Mauritania	1	1	1	1	1	1	1	1	1	0.973304
Mauritius	1	1	1	1	1	1	0.967962	0.966471	0.986364	1
Morocco	0.762994	0.788818	0.792338	0.828554	0.809981	0.792287	0.82032	0.80789	0.811496	0.825343
Mozambique	0.683138	0.689506	0.709232	0.708642	0.747176	0.768049	0.791695	0.804321	0.84084	1
Niger	0.87742	1	0.912598	0.868943	0.717044	0.801297	0.869515	0.789389	0.826221	0.741589
Nigeria	1	1	1	1	1	1	1	1	1	1
Rwanda	1	1	1	1	1	1	1	1	1	1
Sao Tome and Principe	1	1	1	1	1	1	1	1	1	1
Sierra Leone	0.808493	0.833843	0.978482	0.996606	0.961486	0.99291	1	1	1	1
South Africa	0.984435	1	0.990416	1	1	1	0.9938	1	0.989726	0.991013
Sudan	1	1	1	1	1	1	1	1	1	1
Togo	0.649834	0.668405	0.67742	0.656666	0.655557	0.645342	0.670677	0.650075	0.651947	0.630801
Tunisia	0.859795	0.85055	0.846603	0.863106	0.877037	0.885636	0.89797	0.990106	0.923154	1
U.R. of Tanzania: Mainland	1	1	1	1	1	1	1	1	1	1
Zimbabwe	0.594318	0.59384	0.591824	0.585431	0.589383	0.584566	0.584908	0.576321	0.583321	0.6687
GEO MEAN	0.8998	0.9104	0.9163	0.9187	0.9156	0.9157	0.9222	0.9203	0.9292	0.9302

MEA Variable efficiency scores of all the 32 states from 2010 -2019 for Labour (xb)

Country	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	Average
Angola	0.996042	1	1	1	1	0.979133	0.995097	1	1	0.968819	0.995
Benin	0.706035	0.717673	0.779008	0.78008	0.780888	0.767884	0.724317	0.730334	0.69401	0.713214	0.738
Botswana	1	1	1	1	1	1	0.934236	0.943215	0.979063	0.950487	0.990

Burkina Faso	0.960519	1	0.915162	0.892941	0.88278	0.799348	0.879105	0.82033	0.76561	0.794658	0.930
Burundi	1	0.763208	0.694483	0.734978	1	0.791315	0.760092	1	0.728114	0.702391	0.909
Cabo Verde	0.886509	0.85163	0.88854	0.932721	0.851843	0.845117	0.836984	0.826878	0.834837	0.840366	0.909
Cameroon	0.88559	0.913295	0.921441	1	0.918196	0.951574	0.894181	0.822942	0.797384	0.82677	0.932
Central African Republic	0.897573	0.818531	0.809605	0.658939	0.656441	0.65855	0.679182	0.664459	1	1	0.882
Chad	1	1	1	1	1	1	1	1	0.853649	0.872507	0.986
Côte d'Ivoire	0.993088	1	1	1	1	1	1	1	1	1	0.926
Djibouti	1	1	1	1	1	1	0.955124	0.92257	0.940092	1	0.991
Egypt	1	1	1	1	1	1	1	1	1	1	1.000
Eswatini	1	1	1	0.979607	1	1	1	1	1	1	0.982
Gabon	1	1	1	1	1	1	1	1	1	1	1.000
Guinea	1	1	1	1	1	1	0.804122	0.838688	0.894847	0.867377	0.970
Kenya	1	1	1	1	1	1	1	1	1	1	1.000
Lesotho	0.765135	0.778118	0.789235	0.773352	0.763427	0.752624	0.761172	0.736373	0.763281	0.756074	0.732
Mauritania	1	1	0.964373	1	0.956947	0.969496	1	0.869243	0.8065	0.808654	0.967
Mauritius	1	1	1	1	1	1	1	1	1	1	0.996
Morocco	0.810774	0.82183	0.830351	0.845694	0.85705	0.856157	0.869868	0.894143	0.874852	0.833034	0.827
Mozambique	1	0.795439	0.762338	0.667064	0.554472	0.566887	0.578264	0.601699	0.604946	0.593796	0.723
Niger	0.688091	0.624127	0.628311	0.645235	0.664225	0.672133	0.67232	0.686575	0.68106	0.666021	0.752
Nigeria	1	1	1	1	1	1	1	1	1	1	1.000
Rwanda	1	1	1	1	1	1	1	1	1	1	1.000
Sao Tome and Principe	1	1	1	1	1	1	1	1	1	1	1.000
Sierra Leone	1	0.98213	0.996212	1	1	1	1	1	1	1	0.978
South Africa	0.996577	0.988515	0.995434	0.999899	1	0.962742	0.951176	0.933574	0.916572	0.90847	0.980
Sudan	1	1	1	1	1	1	1	1	1	1	1.000
Togo	0.632814	0.645123	0.65967	0.667861	0.67027	0.677915	0.696648	0.693773	0.725959	0.746097	0.669
Tunisia	1	1	1	1	0.990784	0.934289	0.92094	0.907796	1	1	0.937
U.R. of Tanzania: Mainland	1	1	1	0.839681	0.797159	0.788834	0.792222	1	1	1	0.961
Zimbabwe	0.656495	0.650557	0.733511	0.633482	0.600639	0.623226	0.608174	0.576379	0.614813	0.567913	0.611
GEO MEAN	0.9254	0.9077	0.9093	0.8970	0.8922	0.8820	0.8737	0.8785	0.8795	0.8769	

MEA Variable efficiency scores of all the 32 states from 2000 -2009 for Energy Used (xc)

Country	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Angola	1	1	1	1	1	1	1	1	1	0.892731
Benin	0.850985	0.853531	0.863061	0.811586	0.777752	0.759909	0.764258	0.763514	0.774853	0.72445
Botswana	1	1	1	1	1	1	1	1	1	1
Burkina Faso	0.965922	1	1	1	1	1	1	0.957364	0.990131	0.979136
Burundi	1	1	1	1	1	1	1	1	1	1

Cabo Verde	1	1	1	1	1	0.86861	0.888758	0.871912	0.921528	0.884231
Cameroon	0.901965	1	1	1	1	1	1	0.933423	0.951521	0.904609
Central African Republic	1	1	1	0.952808	1	0.800819	0.855718	0.839736	0.803792	0.822186
Chad	1	1	1	1	1	1	1	1	1	1
Côte d'Ivoire	0.816482	0.814218	0.828642	0.846276	0.845122	0.824667	0.894401	0.898434	0.933682	1
Djibouti	1	1	1	1	1	1	1	1	1	1
Egypt	1	1	1	1	1	1	1	1	1	1
Eswatini	0.816841	0.826287	0.838327	0.888021	1	1	1	1	1	1
Gabon	1	1	1	1	1	1	1	1	1	1
Guinea	1	1	1	1	1	1	1	1	1	1
Kenya	1	1	1	1	1	1	1	1	1	1
Lesotho	0.784043	0.792965	0.799521	0.806642	0.806377	0.76632	0.777039	0.745856	0.831551	0.786426
Mauritania	1	1	1	1	1	1	1	1	1	0.95465
Mauritius	1	1	1	1	1	1	0.898138	0.892252	0.919606	1
Morocco	0.860501	0.877128	0.875716	0.879596	0.869669	0.835345	0.849538	0.82262	0.8162	0.815568
Mozambique	0.639758	0.63691	0.648578	0.645592	0.648815	0.669161	0.678169	0.702615	0.728574	1
Niger	0.910959	1	0.94538	0.909144	0.613013	0.648862	0.879356	0.626559	0.68414	0.609448
Nigeria	1	1	1	1	1	1	1	1	1	1
Rwanda	1	1	1	1	1	1	1	1	1	1
Sao Tome and Principe	1	1	1	1	1	1	1	1	1	1
Sierra Leone	0.842975	0.848998	0.970583	0.993484	0.872642	0.974495	1	1	1	1
South Africa	0.669812	1	0.683483	1	1	1	0.716938	1	0.710613	0.710514
Sudan	1	1	1	1	1	1	1	1	1	1
Togo	0.73871	0.755074	0.739937	0.707485	0.670722	0.664578	0.696226	0.67065	0.665405	0.625555
Tunisia	0.863831	0.851898	0.850445	0.887944	0.917402	0.904103	0.942269	0.993838	0.955741	1
U.R. of Tanzania: Mainland	1	1	1	1	1	1	1	1	1	1
Zimbabwe	0.608099	0.612701	0.606209	0.595883	0.588336	0.57528	0.573755	0.557408	0.566528	0.658067
GEO MEAN	0.9061	0.9259	0.9182	0.9275	0.9148	0.9049	0.9104	0.9044	0.9045	0.9079

MEA Variable efficiency scores of all the 32 states from 2010 -2019 for Energy Used (xc)

Country	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	Average
Angola	0.979523	1	1	1	1	0.986213	0.997184	1	1	0.992637	0.992
Benin	0.694187	0.718211	0.750456	0.790526	0.809752	0.805233	0.796715	0.807365	0.778953	0.817281	0.786
Botswana	1	1	1	1	1	1	0.965453	0.976627	0.99194	0.980116	0.996
Burkina Faso	0.929315	1	0.809925	0.87951	0.872341	0.778719	0.863744	0.883526	0.861468	0.862641	0.932
Burundi	1	0.820517	0.791899	0.801943	1	0.809966	0.787301	1	0.88076	0.870621	0.938
Cabo Verde	0.876731	0.821997	0.846767	0.848045	0.820963	0.818277	0.807803	0.810548	0.799468	0.828418	0.886
Cameroon	0.857432	0.857759	0.84789	1	0.888674	0.89922	0.930648	0.91511	0.917957	0.93384	0.937

Central African Republic	0.811168	0.688666	0.687466	0.677761	0.676481	0.673633	0.680807	0.80903	1	1	0.839
Chad	1	1	1	1	1	1	1	1	0.883726	0.920722	0.990
Côte d'Ivoire	0.961494	1	1	1	1	1	1	1	1	1	0.933
Djibouti	1	1	1	1	1	1	0.966745	0.954281	0.94593	1	0.993
Egypt	1	1	1	1	1	1	1	1	1	1	1.000
Eswatini	1	1	1	0.990378	1	1	1	1	1	1	0.968
Gabon	1	1	1	1	1	1	1	1	1	1	1.000
Guinea	1	1	1	1	1	1	0.816235	0.866796	0.914042	0.913695	0.976
Kenya	1	1	1	1	1	1	1	1	1	1	1.000
Lesotho	0.797292	0.84346	0.841424	0.858239	0.856993	0.854605	0.869194	0.812722	0.851603	0.849738	0.817
Mauritania	1	1	0.976229	1	0.978822	0.987122	1	0.913233	0.877673	0.874342	0.978
Mauritius	1	1	1	1	1	1	1	1	1	1	0.985
Morocco	0.819281	0.848908	0.851975	0.878509	0.893062	0.902387	0.929202	0.947666	0.944869	0.929278	0.872
Mozambique	1	0.691709	0.639489	0.610904	0.547379	0.561857	0.581041	0.620146	0.637554	0.6276	0.676
Niger	0.574002	0.548823	0.576577	0.595089	0.614436	0.601234	0.592728	0.740961	0.780878	0.786741	0.712
Nigeria	1	1	1	1	1	1	1	1	1	1	1.000
Rwanda	1	1	1	1	1	1	1	1	1	1	1.000
Sao Tome and Principe	1	1	1	1	1	1	1	1	1	1	1.000
Sierra Leone	1	0.933129	0.98603	1	1	1	1	1	1	1	0.971
South Africa	0.711832	0.722893	0.731303	0.729721	1	0.716346	0.711018	0.709883	0.709216	0.692438	0.796
Sudan	1	1	1	1	1	1	1	1	1	1	1.000
Togo	0.630053	0.644709	0.660404	0.666456	0.663008	0.66495	0.712097	0.750117	0.782101	0.811641	0.696
Tunisia	1	1	1	1	0.995279	0.970027	0.970965	0.962146	1	1	0.953
U.R. of Tanzania: Mainland	1	1	1	0.992988	0.967529	0.978794	0.955525	1	1	1	0.995
Zimbabwe	0.649809	0.654235	0.714401	0.68425	0.657856	0.677511	0.745237	0.812596	0.828893	0.691199	0.653
GEO MEAN	0.9046	0.8877	0.8861	0.8950	0.9020	0.8843	0.8856	0.9086	0.9124	0.9115	

MEA Variable efficiency scores of all the 32 states from 2000 -2009 for CO2 emissions (yb)

Country	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Angola	1	1	1	1	1	1	1	1	1	0.898376
Benin	0.833481	0.808617	0.804215	0.773591	0.798751	0.795027	0.767357	0.772786	0.782457	0.763505
Botswana	1	1	1	1	1	1	1	1	1	1
Burkina Faso	0.973207	1	1	1	1	1	1	0.977897	0.989317	0.987139
Burundi	1	1	1	1	1	1	1	1	1	1
Cabo Verde	1	1	1	1	1	0.93792	0.941314	0.953419	0.999852	0.949519
Cameroon	0.980141	1	1	1	1	1	1	0.942884	0.973935	0.951296
Central African Republic	1	1	1	0.9843	1	0.99412	0.989201	0.987688	0.951584	0.966815
Chad	1	1	1	1	1	1	1	1	1	1

Côte d'Ivoire	0.817364	0.798589	0.81361	0.880784	0.845234	0.828316	0.880548	0.925584	0.972116	1
Djibouti	1	1	1	1	1	1	1	1	1	1
Egypt	1	1	1	1	1	1	1	1	1	1
Eswatini	0.912305	0.938687	0.960513	0.994374	1	1	1	1	1	1
Gabon	1	1	1	1	1	1	1	1	1	1
Guinea	1	1	1	1	1	1	1	1	1	1
Kenya	1	1	1	1	1	1	1	1	1	1
Lesotho	0.653201	0.638745	0.664168	0.668204	0.702618	0.689268	0.702766	0.70591	0.761161	0.75123
Mauritania	1	1	1	1	1	1	1	1	1	0.980554
Mauritius	1	1	1	1	1	1	0.967649	0.96922	0.987184	1
Morocco	0.832252	0.807822	0.820888	0.842397	0.845701	0.825641	0.837167	0.838782	0.838807	0.83379
Mozambique	0.81371	0.809886	0.818056	0.789893	0.811228	0.829884	0.859882	0.875701	0.944114	1
Niger	0.968753	1	0.989386	0.947417	0.900716	0.959542	0.98474	0.955313	0.914034	0.81458
Nigeria	1	1	1	1	1	1	1	1	1	1
Rwanda	1	1	1	1	1	1	1	1	1	1
Sao Tome and Principe	1	1	1	1	1	1	1	1	1	1
Sierra Leone	0.874289	0.865051	0.935506	0.991904	0.93846	0.988764	1	1	1	1
South Africa	0.645764	1	0.63954	1	1	1	0.655616	1	0.647544	0.642519
Sudan	1	1	1	1	1	1	1	1	1	1
Togo	0.730823	0.741502	0.746336	0.722665	0.727569	0.708764	0.736025	0.705422	0.68685	0.642454
Tunisia	0.905933	0.858815	0.856689	0.902305	0.913362	0.928654	0.940359	0.956905	0.950826	1
U.R. of Tanzania: Mainland	1	1	1	1	1	1	1	1	1	1
Zimbabwe	0.60028	0.607753	0.605652	0.600683	0.612369	0.59595	0.594304	0.586465	0.594112	0.732319
GEO MEAN	0.9144	0.9258	0.9178	0.9331	0.9339	0.9328	0.9247	0.9354	0.9292	0.9273

MEA Variable efficiency scores of all the 32 states from 2010 -2019 for CO emissions (yb)

Country	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	Average
Angola	0.995105	1	1	1	1	0.935351	0.943005	1	1	0.953531	0.986
Benin	0.734156	0.743956	0.771283	0.781305	0.788329	0.791249	0.778646	0.662586	0.677198	0.693727	0.766
Botswana	1	1	1	1	1	1	0.970352	0.945601	0.915608	0.898659	0.987
Burkina Faso	0.952571	1	0.860851	0.873145	0.878313	0.845972	0.861985	0.672306	0.753809	0.755348	0.919
Burundi	1	0.917474	0.951231	0.976953	1	0.989321	0.945223	1	0.886918	0.869022	0.977
Cabo Verde	0.926455	0.892794	0.954361	0.961063	0.892385	0.900138	0.904356	0.724555	0.739347	0.745176	0.921
Cameroon	0.922984	0.949159	0.945859	1	0.967652	0.975233	0.956442	0.674086	0.839701	0.897638	0.949
Central African Republic	0.977077	0.98357	0.990974	0.897414	0.883543	0.888659	0.913418	0.741304	1	1	0.957
Chad	1	1	1	1	1	1	1	1	0.795048	0.826342	0.981
Côte d'Ivoire	0.995968	1	1	1	1	1	1	1	1	1	0.938
Djibouti	1	1	1	1	1	1	0.968114	0.796164	0.941058	1	0.985

Egypt	1	1	1	1	1	1	1	1	1	1	1.000
Eswatini	1	1	1	0.991769	1	1	1	1	1	1	0.990
Gabon	1	1	1	1	1	1	1	1	1	1	1.000
Guinea	1	1	1	1	1	1	0.842257	0.761309	0.849929	0.859748	0.966
Kenya	1	1	1	1	1	1	1	1	1	1	1.000
Lesotho	0.75932	0.762552	0.761957	0.752144	0.732306	0.733491	0.744714	0.702421	0.787637	0.780073	0.723
Mauritania	1	1	0.972211	1	0.909194	0.931597	1	0.828607	0.740312	0.748341	0.956
Mauritius	1	1	1	1	1	1	1	1	1	1	0.996
Morocco	0.842284	0.857986	0.84662	0.853253	0.857838	0.873816	0.884886	0.865672	0.8136	0.797675	0.841
Mozambique	1	0.9186	0.795393	0.735225	0.61208	0.666248	0.662771	0.593381	0.621781	0.610408	0.788
Niger	0.762025	0.719873	0.711603	0.739527	0.743931	0.744376	0.753445	0.626603	0.785338	0.752611	0.839
Nigeria	1	1	1	1	1	1	1	1	1	1	1.000
Rwanda	1	1	1	1	1	1	1	1	1	1	1.000
Sao Tome and Principe	1	1	1	1	1	1	1	1	1	1	1.000
Sierra Leone	1	0.92514	0.939681	1	1	1	1	1	1	1	0.973
South Africa	0.647379	0.656797	0.663887	0.662587	1	0.658191	0.655961	0.661512	0.653715	0.64432	0.757
Sudan	1	1	1	1	1	1	1	1	1	1	1.000
Togo	0.652291	0.672448	0.691316	0.701387	0.707125	0.711623	0.731672	0.668118	0.670061	0.689574	0.702
Tunisia	1	1	1	1	0.944052	0.912519	0.919779	0.884939	1	1	0.944
U.R. of Tanzania: Mainland	1	1	1	0.955405	0.929268	0.92119	0.943294	1	1	1	0.987
Zimbabwe	0.698423	0.67704	0.774603	0.686686	0.660284	0.679145	0.719977	0.754207	0.75642	0.664004	0.660
GEO MEAN	0.9253	0.9195	0.9190	0.9161	0.9140	0.9030	0.9020	0.8480	0.8722	0.8702	

MEA Variable efficiency scores of all the 32 states from 2010 -2019 for GDP (ya)

Country	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Angola	1	1	1	1	1	1	1	1	1	0.963887
Benin	0.865036	0.864675	0.883566	0.827178	0.834133	0.816771	0.847761	0.862023	0.870992	0.829291
Botswana	1	1	1	1	1	1	1	1	1	1
Burkina Faso	0.982045	1	1	1	1	1	1	0.99527	0.998638	0.998121
Burundi	1	1	1	1	1	1	1	1	1	1
Cabo Verde	1	1	1	1	1	0.961384	0.966224	0.966757	0.999902	0.962694
Cameroon	0.981776	1	1	1	1	1	1	0.970624	0.986914	0.971072
Central African Republic	1	1	1	0.989923	1	0.996221	0.991973	0.990592	0.955585	0.970201
Chad	1	1	1	1	1	1	1	1	1	1
Côte d'Ivoire	0.868803	0.837058	0.839788	0.908724	0.898155	0.894396	0.949964	0.969779	0.989049	1
Djibouti	1	1	1	1	1	1	1	1	1	1
Egypt	1	1	1	1	1	1	1	1	1	1
Eswatini	0.935024	0.953894	0.969893	0.996133	1	1	1	1	1	1

Gabon	1	1	1	1	1	1	1	1	1	1	1
Guinea	1	1	1	1	1	1	1	1	1	1	1
Kenya	1	1	1	1	1	1	1	1	1	1	1
Lesotho	0.763986	0.769024	0.793368	0.800751	0.825301	0.787887	0.808951	0.780459	0.892505	0.86359	
Mauritania	1	1	1	1	1	1	1	1	1	0.989532	
Mauritius	1	1	1	1	1	1	0.978326	0.981376	0.992246	1	
Morocco	0.891222	0.913913	0.913047	0.923203	0.914152	0.891542	0.904362	0.888414	0.888045	0.8948	
Mozambique	0.868617	0.877663	0.890352	0.879315	0.908518	0.93526	0.964318	0.978055	0.994265	1	
Niger	0.977031	1	0.992923	0.969785	0.915679	0.968939	0.989991	0.970799	0.949859	0.884	
Nigeria	1	1	1	1	1	1	1	1	1	1	
Rwanda	1	1	1	1	1	1	1	1	1	1	
Sao Tome and Principe	1	1	1	1	1	1	1	1	1	1	
Sierra Leone	0.893349	0.906044	0.989117	0.998445	0.976758	0.996222	1	1	1	1	
South Africa	0.984476	1	0.99047	1	1	1	0.993818	1	0.989692	0.99099	
Sudan	1	1	1	1	1	1	1	1	1	1	
Togo	0.736009	0.76519	0.762805	0.716589	0.707919	0.66365	0.724197	0.683972	0.686855	0.60968	
Tunisia	0.924491	0.877082	0.873648	0.920087	0.93984	0.945791	0.962968	0.99654	0.97247	1	
U.R. of Tanzania: Mainland	1	1	1	1	1	1	1	1	1	1	
Zimbabwe	0.470423	0.477211	0.466818	0.45527	0.4893	0.479131	0.49257	0.50354	0.54556	0.81213	
GEO MEAN	0.933404	0.936749	0.940506	0.940417	0.942343	0.938997	0.947832	0.946391	0.953417	0.956384	

Country	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	AVG
Angola	0.99805	1	1	1	1	0.991258	0.998213	1	1	0.994121	0.997
Benin	0.801263	0.840351	0.882003	0.879863	0.883555	0.9028	0.890402	0.885227	0.841557	0.853432	0.858
Botswana	1	1	1	1	1	1	0.982534	0.987089	0.995807	0.989272	0.998
Burkina Faso	0.993665	1	0.972379	0.969401	0.968255	0.934961	0.967399	0.955586	0.929007	0.918256	0.979
Burundi	1	0.877202	0.933204	0.970084	1	0.988866	0.958561	1	0.905572	0.880393	0.976
Cabo Verde	0.951852	0.904178	0.961223	0.971759	0.880653	0.888234	0.909454	0.820856	0.831876	0.83933	0.941
Cameroon	0.94822	0.967142	0.966721	1	0.976712	0.983834	0.969945	0.944273	0.939154	0.948714	0.978
Central African Republic	0.966203	0.976329	0.986731	0.826789	0.806037	0.813966	0.858069	0.684256	1	1	0.941
Chad	1	1	1	1	1	1	1	1	0.960071	0.976152	0.997
Côte d'Ivoire	0.998561	1	1	1	1	1	1	1	1	1	0.958
Djibouti	1	1	1	1	1	1	0.991448	0.982352	0.986303	1	0.998
Egypt	1	1	1	1	1	1	1	1	1	1	1.000
Eswatini	1	1	0.994331	1	1	1	1	1	1	1	0.992
Gabon	1	1	1	1	1	1	1	1	1	1	1.000

Guinea	1	1	1	1	1	1	0.946524	0.965572	0.972128	0.966943	0.993
Kenya	1	1	1	1	1	1	1	1	1	1	1.000
Lesotho	0.8757	0.897601	0.899845	0.892644	0.885883	0.877473	0.895528	0.810425	0.87628	0.865827	0.843
Mauritania	1	1	0.989356	1	0.985913	0.990569	1	0.934433	0.888033	0.890591	0.983
Mauritius	1	1	1	1	1	1	1	1	1	1	0.998
Morocco	0.889582	0.908304	0.910177	0.925423	0.932438	0.936362	0.951182	0.963618	0.96006	0.944783	0.917
Mozambique	1	0.995802	0.949194	0.885988	0.790832	0.788539	0.77905	0.788532	0.733512	0.670566	0.884
Niger	0.83397	0.747293	0.711052	0.745478	0.766151	0.785741	0.813904	0.822357	0.859889	0.845334	0.878
Nigeria	1	1	1	1	1	1	1	1	1	1	1.000
Rwanda	1	1	1	1	1	1	1	1	1	1	1.000
Sao Tome and Principe	1	1	1	1	1	1	1	1	1	1	1.000
Sierra Leone	1	0.990078	0.998053	1	1	1	1	1	1	1	0.987
South Africa	0.99659	0.988473	0.99545	0.9999	1	0.961554	0.948944	0.929044	0.909235	0.899532	0.979
Sudan	1	1	1	1	1	1	1	1	1	1	1.000
Togo	0.625643	0.674901	0.711935	0.726701	0.726151	0.762988	0.800497	0.79925	0.838313	0.862615	0.729
Tunisia	1	1	1	1	0.997068	0.980046	0.979392	0.972261	1	1	0.967
U.R. of Tanzania: Mainland	1	1	1	0.997548	0.98708	0.988353	0.977805	1	1	1	0.998
Zimbabwe	0.832084	0.876823	0.931892	0.929661	0.934919	0.947177	0.952128	0.960858	0.970121	0.925523	0.723
GEO MEAN	0.955528	0.953939	0.959296	0.956758	0.950281	0.950913	0.953127	0.939962	0.94727	0.942682	



APPENDIX D

R Commands for Analysis

```
##maininputoutputefficiency##
```

```
k=read.delim("clipboard") # import full dataset with clipboard command
```

```
require(Benchmarking)
```

```
x1=cbind(k$xa[1:32],k$xb[1:32],k$xc[1:32],k$yb[1:32]) #column bind vectors x into matrix x in  
year 1 matrix
```

```
y1=cbind(k$ya[1:32]) #column bind vectors y into matrix y in year 1 matrix
```

```
x2=cbind(k$xa[33:64],k$xb[33:64],k$xc[33:64],k$yb[33:64]) #column bind vectors x into matrix  
x in year 2 matrix
```

```
y2=cbind(k$ya[33:64]) #column bind vectors y into matrix y in year 2 matrix
```

```
x1b=cbind(k$xa[1:64],k$xb[1:64],k$xc[1:64],k$yb[1:64]) #column bind vectors x into matrix  
(biennial)
```

```
y1b=cbind(k$ya[1:64]) #column bind vectors y into matrix (biennial)
```

```
x2b=cbind(k$xa[1:64],k$xb[1:64],k$xc[1:64],k$yb[1:64]) #column bind vectors x into matrix  
(biennial)
```

```
y2b=cbind(k$ya[1:64]) #column bind vectors y into matrix (biennial)
```

```
oev1=dea(x1,y1,RTS = "vrs",ORIENTATION = "in",XREF = x1,YREF = y1) #own period 1,  
RTS=vrs
```

```
oev2=dea(x2,y2,RTS = "vrs",ORIENTATION = "in",XREF = x2,YREF = y2) #own period 2,  
RTS=vrs
```

```
cev12=dea(x1,y1,RTS = "vrs",ORIENTATION = "in",XREF = x2,YREF = y2) #cross period of  
yr 1 rel to F2, RTS=vrs
```

```
cev21=dea(x2,y2,RTS = "vrs",ORIENTATION = "in",XREF = x1,YREF = y1) #cross period of  
yr 2 rel to F1, RTS=vrs
```

```
ev1b=dea(x1,y1,RTS = "vrs",ORIENTATION = "in",XREF = x1b,YREF = y1b) #yr 1 rel biennial  
F, RTS=vrs
```

ev2b=dea(x2,y2,RTS = "vrs",ORIENTATION = "in",XREF = x2b,YREF = y2b) #yr 1 rel biennial F, RTS=vrs

mv1=mea(x1,y1,RTS = "vrs",ORIENTATION = "in-out",XREF = x1,YREF = y1)#own period

mv2=mea(x2,y2,RTS = "vrs",ORIENTATION = "in-out",XREF = x2,YREF = y2)#own period

mv12=mea(x1,y1,RTS = "vrs",ORIENTATION = "in-out",XREF = x2,YREF = y2)#cross period

mv21=mea(x2,y2,RTS = "vrs",ORIENTATION = "in-out",XREF = x1,YREF = y1)#cross period

mev1b=mea(x1b,y1b,RTS = "vrs",ORIENTATION = "in-out",XREF = x1b,YREF = y1b) #yr 1 rel biennial F, RTS=vrs

mev2b=mea(x2b,y2b,RTS = "vrs",ORIENTATION = "in-out",XREF = x2b,YREF = y2b) #yr 2 rel biennial F, RTS=vrs

mv1\$eff # gives the beta value, gives the MEA technical eff score

mv2\$eff # gives the beta value, gives the MEA technical eff score

mv12\$eff # gives the beta value, gives the MEA technical eff score of yr 1 relative to yr 2

mv21\$eff # gives the beta value, gives the MEA technical eff score of yr 1 relative to yr 2

mev1b\$eff # gives the beta value, gives the MEA technical eff score of yr 1 relative to the biennial yr

mev2b\$eff # gives the beta value, gives the MEA technical eff score of yr 2 relative to the biennial yr

data.frame(mv1\$eff,mv2\$eff,mv12\$eff,mv21\$eff,mev1b\$eff,mev2b\$eff)

mv1\$lambda # gives the weight

mv2\$lambda # gives the weight

mv12\$lambda # gives the weight

mv21\$lambda # gives the weight

mev1b\$lambda

mev2b\$lambda

peers(mv1) # gives peers in MEA

peers(mv2)# gives peers in MEA

peers(mv12)# gives peers in MEA

peers(mv21)# gives peers in MEA

peers(mev1b)

peers(mev2b)

mv1\$direct # gives distance (diagonal) which to travel to the ideal point and also gives the vertical& horizontal distance to the DEA benchmark

mv2\$direct

mv12\$direct

mv21\$direct

mev1b\$direct

mev2b\$direct

#isolate required column from results and calculates the excess inputs to add to the actual input to give IRP for yr 1

ecx1xa=mv1\$direct[,1]

ecx1xb=mv1\$direct[,2]

ecx1xc=mv1\$direct[,3]

ecx1yb=mv1\$direct[,4]

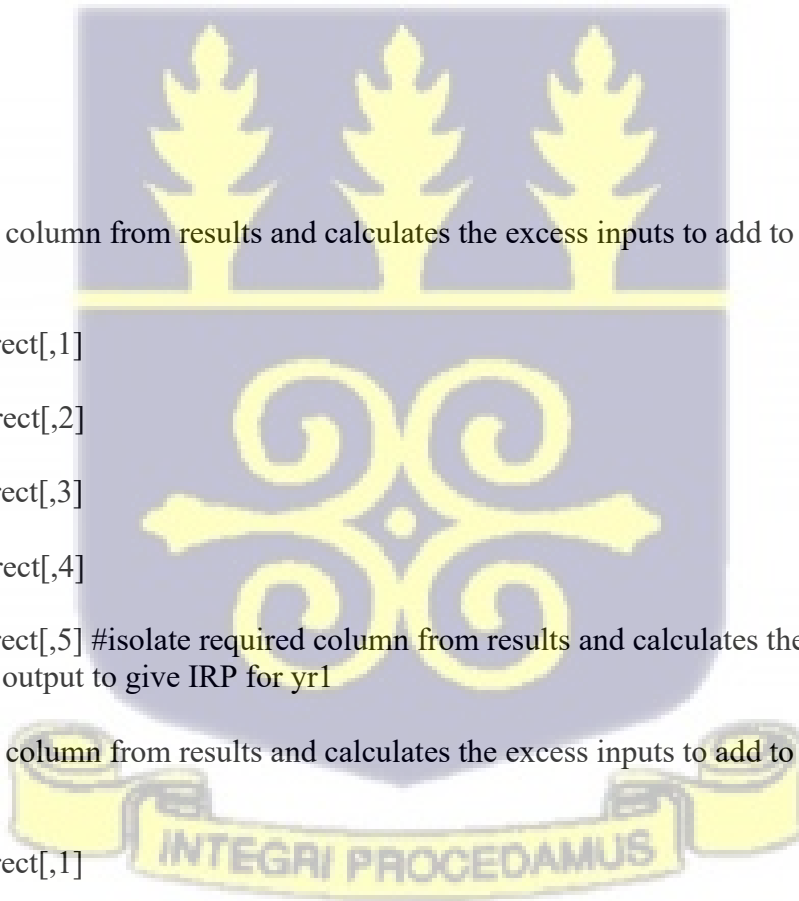
ecy1ya=mv1\$direct[,5] #isolate required column from results and calculates the excess outputs to add to the actual output to give IRP for yr1

#isolate required column from results and calculates the excess inputs to add to the actual input to give IRP for yr 1

ecx2xa=mv2\$direct[,1]

ecx2xb=mv2\$direct[,2]

ecx2xc=mv2\$direct[,3]



```
ecx2yb=mv2$direct[,4]
```

```
ecy2ya=mv2$direct[,5]#isolate required column from results and calculates the excess outputs to  
add to the actual output to give IRP for yr2
```

```
#isolate required column from results and calculates the excess inputs to add to the actual input to  
give IRP for year 1 relative to yr2
```

```
ecx12xa=mv12$direct[,1]
```

```
ecx12xb=mv12$direct[,2]
```

```
ecx12xc=mv12$direct[,3]
```

```
ecx12yb=mv12$direct[,4]
```

```
ecy12ya=mv12$direct[,5]#isolate required column from results and calculates the excess outputs  
to add to the actual output to give IRP for yr 1 relative to yr2
```

```
#isolate required column from results and calculates the excess inputs to add to the actual input to  
give IRP for year 2 relative to yr1
```

```
ecx21xa=mv21$direct[,1]
```

```
ecx21xb=mv21$direct[,2]
```

```
ecx21xc=mv21$direct[,3]
```

```
ecx21yb=mv21$direct[,4]
```

```
ecy21ya=mv21$direct[,5]#isolate required column from results and calculates the excess outputs  
to add to the actual output to give IRP for yr 2 relative to yr1
```

```
#isolate required column from results and calculates the excess inputs to add to the actual input to  
give IRP for yr 1 relative to biennial yr
```

```
ecx1bxa=mev1b$direct[,1]
```

```
ecx1bxb=mev1b$direct[,2]
```

```
ecx1bxc=mev1b$direct[,3]
```

```
ecx1byb=mev1b$direct[,4]
```

```
#isolate required column from results and calculates the excess outputs to add to the actual output  
to give IRP for year 1 relative to biennial yr
```

```
ecy1bya=mev1b$direct[,5]
```

#isolate required column from results and calculates the excess inputs to add to the actual input to give IRP for year 2 relative to biennial yr

```
ecx2bxa=mev2b$direct[,1]
```

```
ecx2bxb=mev2b$direct[,2]
```

```
ecx2bxc=mev2b$direct[,3]
```

```
ecx2byb=mev2b$direct[,4]
```

#isolate required column from results and calculates the excess outputs to add to the actual output to give IRP for year 2 relative to biennial yr

```
ecy2bya=mev2b$direct[,5]
```

ideal ref point = actual input - excess input (and this excess if found in medirect results input-orientation)

ideal ref point = actual output + surplus output (and this excess if found in medirect results output-orientation)

#Step 1; Identify ideal reference point and solve LPP for the ideal point, 1 for each input and/or output direction

#IRP= actual input - excess input; the excess input is found in the me\$direct results. if it is in output orientation, the IRP= actual output + Surplus output

##Step 2; Find the potential Improvement point(PIP) or MEA benchmark using the Ideal reference by moving the input/output from the IRP to the target point(MEA benchmark)

```
#PIP = Xi0 - Beta(Xi0-d*)
```

```
5.6-0.56*(5.6-3.6)#PIP(Gx)=4.48
```

```
#PIP = Yi0 + Beta(d*-Yi0)
```

```
9+0.56*(11.6-9)#PIP(Gy)= 10.45
```

```
#thus K=(4.48,10.45)
```

```
#Relative variable specific inefficiency = Beta(Xi0-d*)/Xi0
```

```
#Relative variable specific efficiency = Xi0 - Beta(Xi0-d*)/Xi0
```

#calculates for IRP for inputs xa,xb,xc,yb(undesirable output) for year 1

$$IRPx1xa=(x1[,1]-ecx1xa)$$

$$IRPx1xb=(x1[,2]-ecx1xb)$$

$$IRPx1xc=(x1[,3]-ecx1xc)$$

$$IRPx1yb=(x1[,4]-ecx1yb)$$

#calculates for IRP for output ya for year 1

$$IRPy1ya=(y1[,1]+ecy1ya)$$

#calculates for IRP for inputs xa,xb,xc,yb(undesirable output) for year 2

$$IRPx2xa=(x2[,1]-ecx2xa)$$

$$IRPx2xb=(x2[,2]-ecx2xb)$$

$$IRPx2xc=(x2[,3]-ecx2xc)$$

$$IRPx2yb=(x2[,4]-ecx2yb)$$

#calculates for IRP for output ya for year 2

$$IRPy2ya=(y2[,1]+ecy2ya)$$

#calculates for IRP for inputs xa,xb,xc,yb(undesirable output) for year 1 relative to yr 2

$$IRPx12xa=(x1[,1]-ecx12xa)$$

$$IRPx12xb=(x1[,2]-ecx12xb)$$

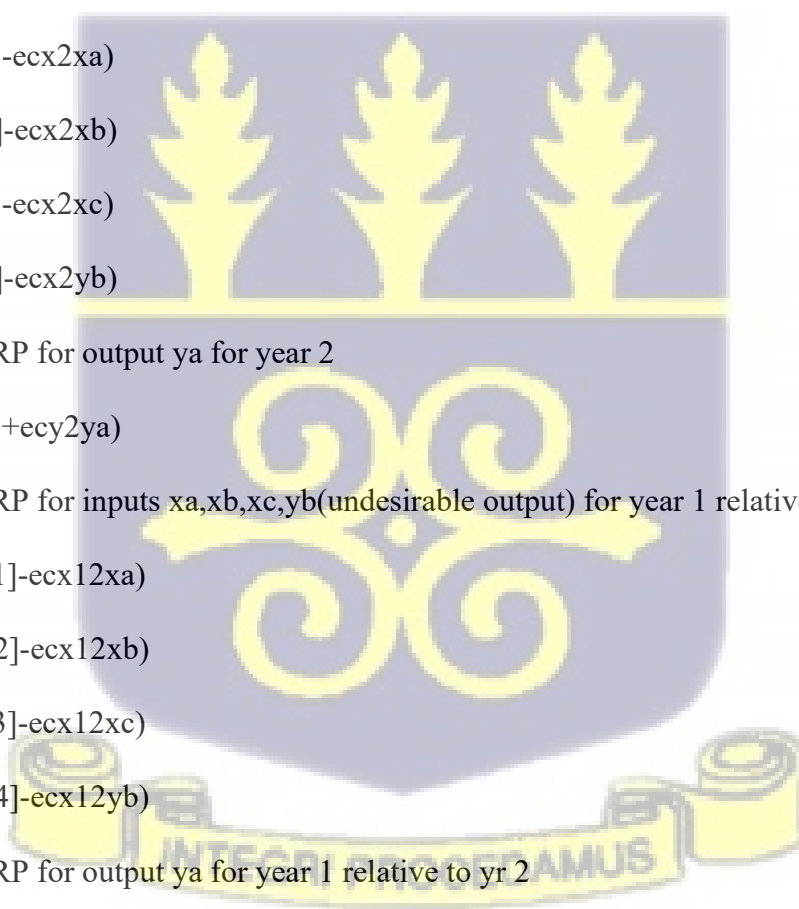
$$IRPx12xc=(x1[,3]-ecx12xc)$$

$$IRPx12yb=(x1[,4]-ecx12yb)$$

#calculates for IRP for output ya for year 1 relative to yr 2

$$IRPy12ya=(y1[,1]+ecy12ya)$$

#calculates for IRP for inputs xa,xb,xc,yb(undesirable output) for year 2 relative to yr 1



$$IRPx21xa=(x2[,1]-ecx21xa)$$

$$IRPx21xb=(x2[,2]-ecx21xb)$$

$$IRPx21xc=(x2[,3]-ecx21xc)$$

$$IRPx21yb=(x2[,4]-ecx21yb)$$

#calculates for IRP for output ya for year 2 relative to yr 1

$$IRPy21ya=(y2[,1]+ecy21ya)$$

#calculates for IRP for inputs xa,xb,xc,yb(undesirable output) for year 1 relative to biennial yr

$$BIRPx1bxa=(x1b[,1]-ecx1bxa)$$

$$BIRPx1bxb=(x1b[,2]-ecx1bxb)$$

$$BIRPx1bxc=(x1b[,3]-ecx1bxc)$$

$$BIRPx1byb=(x1b[,4]-ecx1byb)$$

#calculates for IRP for output ya for year 1 relative to biennial yr

$$BIRPy1bya=(y1b[,1]+ecy1bya)$$

#calculates for IRP for inputs xa,xb,xc,yb(undesirable output) for year 2 relative to biennial yr

$$BIRPx2bxa=(x2b[,1]-ecx2bxa)$$

$$BIRPx2bxb=(x2b[,2]-ecx2bxb)$$

$$BIRPx2bxc=(x2b[,3]-ecx2bxc)$$

$$BIRPx2byb=(x2b[,4]-ecx2byb)$$

#calculates for IRP for output ya for year 2 relative to biennial yr

$$BIRPy2bya=(y2b[,1]+ecy2bya)$$

#calculates for PIP for inputs xa,xb,xc,yb(undesirable output) this is the MEA benchmark for year 1

$$PIPx1xa=x1[,1]-mv1\$eff*(x1[,1]-IRPx1xa)$$

$$PIPx1xb=x1[,2]-mv1\$eff*(x1[,2]-IRPx1xb)$$

$$PIP_{x1xc} = x1[,3] - mv1\$eff*(x1[,3] - IRP_{x1xc})$$

$$PIP_{x1yb} = x1[,4] - mv1\$eff*(x1[,4] - IRP_{x1yb})$$

$PIP_{y1ya} = y1[,1] + mv1\$eff*(IRP_{y1ya} - y1[,1])$ #calculates for PIP for output ya, this is the MEA benchmark for year 1

#calculates for PIP for inputs x1, this is the MEA benchmark for year 2

$$PIP_{x2xa} = x2[,1] - mv2\$eff*(x2[,1] - IRP_{x2xa})$$

$$PIP_{x2xb} = x2[,2] - mv2\$eff*(x2[,2] - IRP_{x2xb})$$

$$PIP_{x2xc} = x2[,3] - mv2\$eff*(x2[,3] - IRP_{x2xc})$$

$$PIP_{x2yb} = x2[,4] - mv2\$eff*(x2[,4] - IRP_{x2yb})$$

$PIP_{y2ya} = y2[,1] + mv2\$eff*(IRP_{y2ya} - y2[,1])$ #calculates for PIP for output ya, this is the MEA benchmark for year 2

#calculates for PIP for inputs xa,xb,xc,yb(undesirable output) this is the MEA benchmark for year 1 relative to yr 2

$$PIP_{x12xa} = x2[,1] - mv12\$eff*(x2[,1] - IRP_{x12xa})$$

$$PIP_{x12xb} = x2[,2] - mv12\$eff*(x2[,2] - IRP_{x12xb})$$

$$PIP_{x12xc} = x2[,3] - mv12\$eff*(x2[,3] - IRP_{x12xc})$$

$$PIP_{x12yb} = x2[,4] - mv12\$eff*(x2[,4] - IRP_{x12yb})$$

$PIP_{y12ya} = y2[,1] + mv12\$eff*(IRP_{y12ya} - y2[,1])$ #calculates for PIP for output y1, this is the MEA benchmark for year 1 relative to yr 2

#calculates for PIP for inputs x1, this is the MEA benchmark for year 2 relative to yr 1

$$PIP_{x21xa} = x2[,1] - mv21\$eff*(x2[,1] - IRP_{x21xa})$$

$$PIP_{x21xb} = x2[,2] - mv21\$eff*(x2[,2] - IRP_{x21xb})$$

$$PIP_{x21xc} = x2[,3] - mv21\$eff*(x2[,3] - IRP_{x21xc})$$

$$PIP_{x21yb} = x2[,4] - mv21\$eff*(x2[,4] - IRP_{x21yb})$$

$PIP_{y21ya} = y2[,1] + mv21\$eff*(IRP_{y21ya} - y2[,1])$ #calculates for PIP for output y1, this is the MEA benchmark for year 2 relative to yr 1

#calculates for PIP for inputs bx12, this is the MEA benchmark for year 1 relative to biennial yr

$BPIP_{x1bxa} = x1b[,1] - mev1b\$eff * (x1b[,1] - BIRP_{x1bxa})$

$BPIP_{x1bxb} = x1b[,2] - mev1b\$eff * (x1b[,2] - BIRP_{x1bxb})$

$BPIP_{x1bxc} = x1b[,3] - mev1b\$eff * (x1b[,3] - BIRP_{x1bxc})$

$BPIP_{x1byb} = x1b[,4] - mev1b\$eff * (x1b[,4] - BIRP_{x1byb})$

$BPIP_{y1bya} = y1b[,1] + mev1b\$eff * (BIRP_{y1bya} - y1b[,1])$ #calculates for PIP for output by12, this is the MEA benchmark for year 1 relative to biennial yr

#calculates for PIP for inputs bx12, this is the MEA benchmark for year 2 relative to biennial yr

$BPIP_{x2bxa} = x2b[,1] - mev2b\$eff * (x2b[,1] - BIRP_{x2bxa})$

$BPIP_{x2bxb} = x2b[,2] - mev2b\$eff * (x2b[,2] - BIRP_{x2bxb})$

$BPIP_{x2bxc} = x2b[,3] - mev2b\$eff * (x2b[,3] - BIRP_{x2bxc})$

$BPIP_{x2byb} = x2b[,4] - mev2b\$eff * (x2b[,4] - BIRP_{x2byb})$

$BPIP_{y2bya} = y2b[,1] + mev2b\$eff * (BIRP_{y2bya} - y2b[,1])$ #calculates for PRP for output by12, this is the MEA benchmark for year 2 relative to biennial yr

FOR INPUTS VARIABLES RTS = VRS

#MEA inefficiency score for xa,xb,xc,yb(undesirable output) for year 1, NB:
 $ecx1xa = mv2\$direct[, -2]$

$inx1xa = (mv1\$eff * ecx1xa) / x1[,1]$

$inx1xb = (mv1\$eff * ecx1xb) / x1[,2]$

$inx1xc = (mv1\$eff * ecx1xc) / x1[,3]$

$inx1yb = (mv1\$eff * ecx1yb) / x1[,4]$

#MEA inefficiency score for xa,xb,xc,yb(undesirable output) for year 2

$inx2xa = (mv2\$eff * ecx2xa) / x2[,1]$

$inx2xb = (mv2\$eff * ecx2xb) / x2[,2]$

$inx2xc = (mv2\$eff * ecx2xc) / x2[,3]$

$inx2yb=(mv2\$eff*ecx2yb)/x2[,4]$

#MEA ineff score for xa,xb,xc,yb(undesirable output) for year 1 relative to yr 2

$inx12xa=(mv12\$eff*ecx12xa)/x1[,1]$

$inx12xb=(mv12\$eff*ecx12xb)/x1[,2]$

$inx12xc=(mv12\$eff*ecx12xc)/x1[,3]$

$inx12yb=(mv12\$eff*ecx12yb)/x1[,4]$

#MEA ineff score for xa,xb,xc,yb(undesirable output) for year 2 relative to yr 1

$inx21xa=(mv21\$eff*ecx21xa)/x2[,1]$

$inx21xb=(mv21\$eff*ecx21xb)/x2[,2]$

$inx21xc=(mv21\$eff*ecx21xc)/x2[,3]$

$inx21yb=(mv21\$eff*ecx21yb)/x2[,4]$

#MEA ineff score for x for year 1 relative to biennial yr

$inx1bxa=(mev1b\$eff*ecx1bxa)[1:32]/x1b[,1][1:32]$

$inx1bxb=(mev1b\$eff*ecx1bxb)[1:32]/x1b[,2][1:32]$

$inx1bxc=(mev1b\$eff*ecx1bxc)[1:32]/x1b[,3][1:32]$

$inx1byb=(mev1b\$eff*ecx1byb)[1:32]/x1b[,4][1:32]$

#MEA ineff score for x for year 2 relative to biennial yr

$inx2bxa=(mev2b\$eff*ecx2bxa)[33:64]/x2b[,1][33:64]$

$inx2bxb=(mev2b\$eff*ecx2bxb)[33:64]/x2b[,2][33:64]$

$inx2bxc=(mev2b\$eff*ecx2bxc)[33:64]/x2b[,3][33:64]$

$inx2byb=(mev2b\$eff*ecx2byb)[33:64]/x2b[,4][33:64]$

FOR INPUT EFFCICINECY SCORE VRS

$\#nPIP_x1=x1-(mv1\$eff*mv1\$direct[, -2])$

#eff score for inputs xa,xb,xc,yb(undesirable output) yr 1

$efx1xa = PIPx1xa/x1[,1]$

$efx1xb = PIPx1xb/x1[,2]$

$efx1xc = PIPx1xc/x1[,3]$

$efx1yb = PIPx1yb/x1[,4]$

#eff score for inputs xa,xb,xc,yb(undesirable output) yr 2

$efx2xa = PIPx2xa/x2[,1]$

$efx2xb = PIPx2xb/x2[,2]$

$efx2xc = PIPx2xc/x2[,3]$

$efx2yb = PIPx2yb/x2[,4]$

#eff score for inputs xa,xb,xc,yb(undesirable output) yr 1 rel to yr 2

$efx12xa = PIPx12xa/x1[,1]$

$efx12xb = PIPx12xb/x1[,2]$

$efx12xc = PIPx12xc/x1[,3]$

$efx12yb = PIPx12yb/x1[,4]$

#eff score for inputs xa,xb,xc,yb(undesirable output) yr 2 rel to yr 1

$efx21xa = PIPx21xa/x2[,1]$

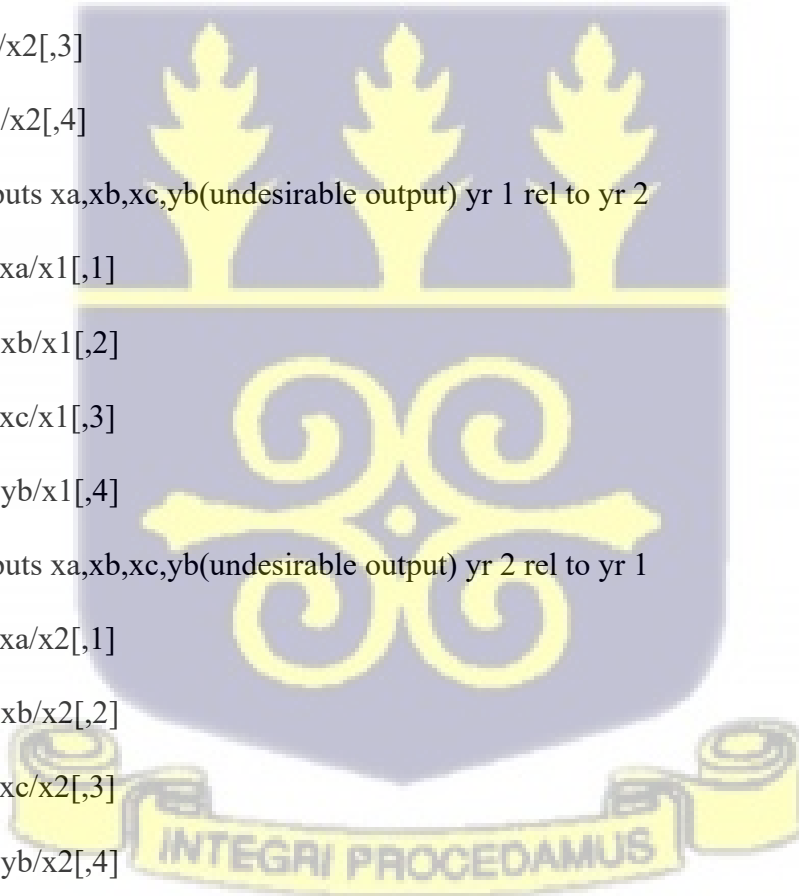
$efx21xb = PIPx21xb/x2[,2]$

$efx21xc = PIPx21xc/x2[,3]$

$efx21yb = PIPx21yb/x2[,4]$

#eff score for inputs xa,xb,xc,yb(undesirable output) yr 1 rel to biennial

$efx1bxa = BPIPx1bxa[1:32]/x1b[,1][1:32]$



```

efx1bxb=BPIPx1bxb[1:32]/x1b[,2][1:32]
efx1bxc=BPIPx1bxc[1:32]/x1b[,3][1:32]
efx1byb=BPIPx1byb[1:32]/x1b[,4][1:32]
#eff score for inputs xa,xb,xc,yb(undesirable output) yr 2 rel to biennial
efx2bxa=BPIPx2bxa[33:64]/x2b[,1][33:64]
efx2bxb=BPIPx2bxb[33:64]/x2b[,2][33:64]
efx2bxc=BPIPx2bxc[33:64]/x2b[,3][33:64]
efx2byb=BPIPx2byb[33:64]/x2b[,4][33:64]

# for input xa
data.frame(inx1xa,efx1xa,inx2xa,efx2xa,inx12xa,efx12xa,inx21xa,efx21xa,inx1bxa,efx1bxa,inx
2bxa,efx2bxa)
data.frame(efx1xa,efx2xa)
data.frame(efx1xa,efx2xa,efx12xa,efx21xa,efx1bxa,efx2bxa)

# for input xb
data.frame(inx1xb,efx1xb,inx2xb,efx2xb,inx12xb,efx12xb,inx21xb,efx21xb,inx1bxb,efx1bxb,in
x2bxb,efx2bxb)
data.frame(efx1xb,efx2xb)
data.frame(efx1xb,efx2xb,efx12xb,efx21xb,efx1bxb,efx2bxb)

# for input xc
data.frame(inx1xc,efx1xc,inx2xc,efx2xc,inx12xc,efx12xc,inx21xc,efx21xc,inx1bxc,efx1bxc,in
x2bxc,efx2bxc)
data.frame(efx1xc,efx2xc)
data.frame(efx1xc,efx2xc,efx12xc,efx21xc,efx1bxc,efx2bxc)

# for output yb

```

```
data.frame(inx1yb,efx1yb,inx2yb,efx2yb,inx12yb,efx12yb,inx21yb,efx21yb,inx1byb,efx1byb,inx2byb,efx2byb)
```

```
data.frame(efx1yb,efx2yb)
```

```
data.frame(efx1yb,efx2yb,efx12yb,efx21yb,efx1byb,efx2byb)
```

```
#results to show first table from variable to eff showing PIP for 1 year
```

```
data.frame(k$xa[1:32],mv1$eff,ecx1xa,IRPx1xa,PIPx1xa,inx1xa,efx1xa)
```

```
data.frame(k$xb[1:32],mv1$eff,ecx1xb,IRPx1xb,PIPx1xb,inx1xb,efx1xb)
```

```
data.frame(k$xc[1:32],mv1$eff,ecx1xc,IRPx1xc,PIPx1xc,inx1xc,efx1xc)
```

```
data.frame(k$yb[1:32],mv1$eff,ecx1yb,IRPx1yb,PIPx1yb,inx1yb,efx1yb)
```

```
data.frame(efx1xa,efx1xb,efx1xc,efx1yb) #year 1
```

```
data.frame(efx2xa,efx2xb,efx2xc,efx2yb) #year 2
```

```
# VARIABLE SPECIFIC MALMQUIST FOR INPUT RTS=VRS
```

```
VMPIxa=((efx1xa/efx2xa))*(((efx2xa/efx12xa)*(efx21xa/efx1xa))^0.5)
```

```
VMPIxb=((efx1xb/efx2xb))*(((efx2xb/efx12xb)*(efx21xb/efx1xb))^0.5)
```

```
VMPIxc=((efx1xc/efx2xc))*(((efx2xc/efx12xc)*(efx21xc/efx1xc))^0.5)
```

```
VMPIyb=((efx1yb/efx2yb))*(((efx2yb/efx12yb)*(efx21yb/efx1yb))^0.5)
```

```
# input variable specific eff change for x
```

```
VPECxa=efx1xa/efx2xa
```

```
VPECxb=efx1xb/efx2xb
```

```
VPECxc=efx1xc/efx2xc
```

```
VPECyb=efx1yb/efx2yb
```

```
#pure technical change
```

```
VPTCxa=((efx2xa/efx12xa)*(efx21xa/efx1xa))^0.5
```

```
VPTCxb=((efx2xb/efx12xb)*(efx21xb/efx1xb))^0.5
```

$$VPTC_{xc} = ((efx_{2xc}/efx_{12xc}) * (efx_{21xc}/efx_{1xc}))^{0.5}$$

$$VPTC_{yb} = ((efx_{2yb}/efx_{12yb}) * (efx_{21yb}/efx_{1yb}))^{0.5}$$

#calculates biennial malquist MEA input variable specific eff change for x

$$VBMI_{xa} = efx_{1bxa}/efx_{2bxa}$$

$$VBMI_{xb} = efx_{1bxb}/efx_{2bxb}$$

$$VBMI_{xc} = efx_{1bxc}/efx_{2bxc}$$

$$VBMI_{yb} = efx_{1byb}/efx_{2byb}$$

input variable specific biennial eff change for x

$$VBPEC_{xa} = efx_{1xa}/efx_{2xa}$$

$$VBPEC_{xb} = efx_{1xb}/efx_{2xb}$$

$$VBPEC_{xc} = efx_{1xc}/efx_{2xc}$$

$$VBPEC_{yb} = efx_{1yb}/efx_{2yb}$$

biennial pure technical change

$$VBPTC_{xa} = (efx_{2xa}/efx_{2bxa}) * (efx_{1bxa}/efx_{1xa})$$

$$VBPTC_{xb} = (efx_{2xb}/efx_{2bxb}) * (efx_{1bxb}/efx_{1xb})$$

$$VBPTC_{xc} = (efx_{2xc}/efx_{2bxc}) * (efx_{1bxc}/efx_{1xc})$$

$$VBPTC_{yb} = (efx_{2yb}/efx_{2byb}) * (efx_{1byb}/efx_{1yb})$$

#FOR OUTPUT INEFFCICINECY VARIABLES RTS =VRS

iny1ya=y1[,1]/(mv1\$eff*ecy1ya) #MEA ineff score for output y in year 1 , NB:
ecy1=mv2\$direct[, -1]

iny2ya=y2[,1]/(mv2\$eff*ecy2ya) #MEA ineff score for output y in year 2

iny12ya=y1[,1]/(mv12\$eff*ecy12ya) #MEA ineff score for output y for year 1 relative to yr 2

iny21ya=y2[,1]/(mv21\$eff*ecy21ya) #MEA ineff score for output y for year 2 relative to yr 1

$iny1bya = y1b[,1]/(mev1b\$eff*ecy1bya)$ #MEA ineff score for output y for year 1 relative to biennial yr

$iny2bya = y2b[,1]/(mev2b\$eff*ecy2bya)$ #MEA ineff score for output y for year 2 relative to biennial yr

FOR OUTPUT EFFCICINECY SCORE RTS=VRS

Given $efy1 = y1/(y1 + (me\$eff*(IRPy1 - y1)))$ MEA eff score for output y in year 1, $IRPy = (y1 + ecy)$

$PIPy1 = y1 + (mv1\$eff*(IRPy1 - y1))$

$efy1ya = y1[,1]/PIPy1ya$ #eff score

$efy2ya = y2[,1]/PIPy2ya$ #eff score

$efy12ya = y1[,1]/PIPy12ya$ #eff score

$efy21ya = y2[,1]/PIPy21ya$ #eff score

$efy1bya = y1b[,1][1:32]/BPIPy1bya[1:32]$ #eff score

$efy2bya = y2b[,1][1:32]/BPIPy2bya[33:64]$ #eff score

for output ya

`data.frame(iny1ya,efy1ya,iny2ya,efy2ya,iny12ya,efy12ya,iny21ya,efy21ya,iny1bya,efy1bya,iny2bya,efy2bya)`

`data.frame(efy1ya,efy2ya)`

`data.frame(efy1ya,efy2ya,efy12ya,efy21ya,efy1bya,efy2bya)`

`data.frame(efx1xa,efx2xa,efx1xb,efx2xb,efx1xc,efx2xc,efx1yb,efx2yb,efy1ya,efy2ya)` #DATA FRAME

$riny1ya = 1/iny1ya$ #reciprocals of MEA ineff score for output y in year 1

$riny2ya = 1/iny2ya$ #reciprocals of MEA ineff score for output y in year 1

$riny12ya = 1/iny12ya$ #reciprocals of MEA ineff score for output y for year 1 relative to yr 2

$riny21ya = 1/iny21ya$ #reciprocals of MEA ineff score for output y for year 2 relative to yr 1

$riny1bya = 1/iny1bya[1:32]$ #reciprocals of MEA ineff score for output y for year 1 relative to biennial yr

$riny2bya = 1 / iny2bya[33:64]$ #reciprocals of MEA ineff score for output y for year 2 relative to biennial yr

Variable specific Malmquist for output

$VMPIya = ((efy1ya / efy12ya) * (efy21ya / efy2ya))^{0.5}$ # output variable Malmquist productivity index

$VPECya = efy1ya / efy2ya$ # output variable specific eff change for y ,Pure eff change

$VPTCya = ((efy2ya / efy12ya) * (efy21ya / efy1ya))^{0.5}$ # output variable specific technological change for y

#calculates biennial Malmquist MEA input variable specific eff change for y

$VBMIya = efy1bya / efy2bya$

input variable specific biennial eff change for x

$VBPECya = efy1ya / efy2ya$

biennial pure technical change

$VBPTCya = (efy2ya / efy2bya) * (efy1bya / efy1ya)$

$data.frame(VMPIxa, VPECxa, VPTCxa, VBMIxa, VBPECxa, VBPTCxa)$

$data.frame(VMPIxb, VPECxb, VPTCxb, VBMIxb, VBPECxb, VBPTCxb)$

$data.frame(VMPIxc, VPECxc, VPTCxc, VBMIxc, VBPECxc, VBPTCxc)$

$data.frame(VMPIyb, VPECyb, VPTCyb, VBMIyb, VBPECyb, VBPTCyb)$

$data.frame(VMPIya, VPECya, VPTCya, VBMIya, VBPECya, VBPTCya)$

OVERALL VARIABLE SPECIFIC EFFICIENCY RTS=VRS

$ovse1m = (1 - ((1/4) * (inx1xa + inx1xb + inx1xc + inx1yb))) / (1 + ((1/1) * (riny1ya)))$ #MEA overall eff score for multiple inputs and multiple outputs for year 1

$ovse2m = (1 - ((1/4) * (inx2xa + inx2xb + inx2xc + inx2yb))) / (1 + ((1/1) * (riny2ya)))$ #MEA overall eff score for multiple inputs and multiple outputs for year 2

$ovse12m = (1 - ((1/4) * (inx12xa + inx12xb + inx12xc + inx12yb))) / (1 + ((1/1) * (riny12ya)))$ #MEA
overall eff score for multiple inputs and multiple outputs for year 1 relative to yr 2

$ovse21m = (1 - ((1/4) * (inx21xa + inx21xb + inx21xc + inx21yb))) / (1 + ((1/1) * (riny21ya)))$ #MEA
overall eff score for multiple inputs and multiple outputs for year 2 relative to yr 1

$ovse1bm = (1 - ((1/4) * (inx1bxa + inx1bxb + inx1bxc + inx1byb))) / (1 + ((1/1) * (riny1bya)))$ #MEA
overall eff score for multiple inputs and multiple outputs for year 1 relative to yr 2

$ovse2bm = (1 - ((1/4) * (inx2bxa + inx2bxb + inx2bxc + inx2byb))) / (1 + ((1/1) * (riny2bya)))$ #MEA
overall eff score for multiple inputs and multiple outputs for year 2 relative to yr 1

`data.frame(k$xa[1:32], oev1$eff, mv1$eff, peers(mv1), mv1$direct[,1], IRPx1xa, PIPx1xa, inx1xa, ef
x1xa, ovse1m) #data year 1`

`data.frame(k$xb[1:32], oev1$eff, mv1$eff, peers(mv1), mv1$direct[,2], IRPx1xb, PIPx1xb, inx1xb, e
fx1xb, ovse1m) #data year 1`

`data.frame(k$xc[1:32], oev1$eff, mv1$eff, peers(mv1), mv1$direct[,3], IRPx1xc, PIPx1xc, inx1xc, ef
x1xc, ovse1m) #data year 1`

`data.frame(k$yb[1:32], oev1$eff, mv1$eff, peers(mv1), mv1$direct[,4], IRPx1yb, PIPx1yb, inx1yb, e
fx1yb, ovse1m) #data year 1`

`data.frame(k$ya[1:32], oev1$eff, mv1$eff, peers(mv1), mv1$direct[,5], IRPy1ya, PIPy1ya, iny1ya, ef
y1ya, ovse1m) #data year 1`

`data.frame(k$xa[33:64], oev2$eff, mv2$eff, peers(mv2), mv2$direct[,1], IRPx2xa, PIPx2xa, inx2xa, e
fx2xa, ovse2m) #data year 2`

`data.frame(k$xb[33:64], oev2$eff, mv2$eff, peers(mv2), mv2$direct[,2], IRPx2xb, PIPx2xb, inx2xb, e
fx2xb, ovse2m) #data year 2`

`data.frame(k$xc[33:64], oev2$eff, mv2$eff, peers(mv2), mv2$direct[,3], IRPx2xc, PIPx2xc, inx2xc, e
fx2xc, ovse2m) #data year 2`

`data.frame(k$yb[33:64], oev2$eff, mv2$eff, peers(mv2), mv2$direct[,4], IRPx2yb, PIPx2yb, inx2yb, e
fx2yb, ovse2m) #data year 2`

`data.frame(k$ya[33:64], oev2$eff, mv2$eff, peers(mv2), mv2$direct[,5], IRPy2ya, PIPy2ya, iny2ya, e
fy2ya, ovse2m) #data year 2`

```

data.frame(ovsel1m,ovse2m,ovsel12m,ovse21m,ovsel1bm,ovse2bm) #overall eff

data.frame(VMPIx,VPECx,VPTCx,VBMIx,VMPIya,VPECya,VPTCy,VBMIya,VBPECya,VB
PTCy) # data frame for MEA MPI and BMI results

# FOR RTS = CRS (CONSTANT RETURNS TO SCALE)

oev1c=dea(x1,y1,RTS = "crs",ORIENTATION = "in",XREF = x1,YREF = y1) #own period 1,
RTS=crs

oev2c=dea(x2,y2,RTS = "crs",ORIENTATION = "in",XREF = x2,YREF = y2) #own period 2,
RTS=crs

cev12c=dea(x1,y1,RTS = "crs",ORIENTATION = "in",XREF = x2,YREF = y2) #cross period of
yr 1 rel to F2, RTS=crs

cev21c=dea(x2,y2,RTS = "crs",ORIENTATION = "in",XREF = x1,YREF = y1) #cross period of
yr 2 rel to F1, RTS=crs

ev1bc=dea(x1,y1,RTS = "crs",ORIENTATION = "in",XREF = x1b,YREF = y1b) #yr 1 rel
biennial F, RTS=vrs

ev2bc=dea(x2,y2,RTS = "crs",ORIENTATION = "in",XREF = x2b,YREF = y2b) #yr 1 rel
biennial F, RTS=vrs

#dea plott#

dea.plot(x1, y1, RTS="crs", txt=TRUE,add=TRUE) # plot for RTS = CRS

dea.plot(x2, y2, RTS="crs", txt=TRUE, col="red",add=TRUE) #run this command before running
above command

dea.plot(bx12, by12, RTS="crs", txt=TRUE,add=TRUE, col="green") # this draws the biennial
graph

points(x2, y2, col="red", pch=16)

text(x2, y2, 1:dim(x2)[1], col="red", adj=-1)

mc1=mea(x1,y1,RTS = "crs",ORIENTATION = "in-out",XREF = x1,YREF = y1)#own period
RTS=crs

mc2=mea(x2,y2,RTS = "crs",ORIENTATION = "in-out",XREF = x2,YREF = y2)#own period
RTS=crs

```

mc12=mea(x1,y1,RTS = "crs",ORIENTATION = "in-out",XREF = x2,YREF = y2)#cross period
RTS=crs

mc21=mea(x2,y2,RTS = "crs",ORIENTATION = "in-out",XREF = x1,YREF = y1)#cross period
RTS=crs

mec1b=mea(x1b,y1b,RTS = "crs",ORIENTATION = "in-out",XREF = x1b,YREF = y1b) #yr 1
rel biennial F, RTS=crs

mec2b=mea(x2b,y2b,RTS = "crs",ORIENTATION = "in-out",XREF = x2b,YREF = y2b) #yr 2
rel biennial F, RTS=crs

mc1\$eff # gives the beta value, gives the MEA technical eff score, RTS = crs

mc2\$eff # gives the beta value, gives the MEA technical eff score RTS = crs

mc12\$eff # gives the beta value, gives the MEA technical eff score of yr 1 relative to yr 2 RTS =
crs

mc21\$eff # gives the beta value, gives the MEA technical eff score of yr 2 relative to yr 1 RTS =
crs

mec1b\$eff # gives the beta value, gives the MEA technical eff score of yr 1 relative to the biennial
yr RTS = crs

mec2b\$eff # gives the beta value, gives the MEA technical eff score of yr 2 relative to the biennial
yr RTS = crs

mc1\$lambda # gives the weight RTS = crs

mc2\$lambda # gives the weight RTS = crs

mc12\$lambda # gives the weight RTS = crs

mc21\$lambda # gives the weight RTS = crs

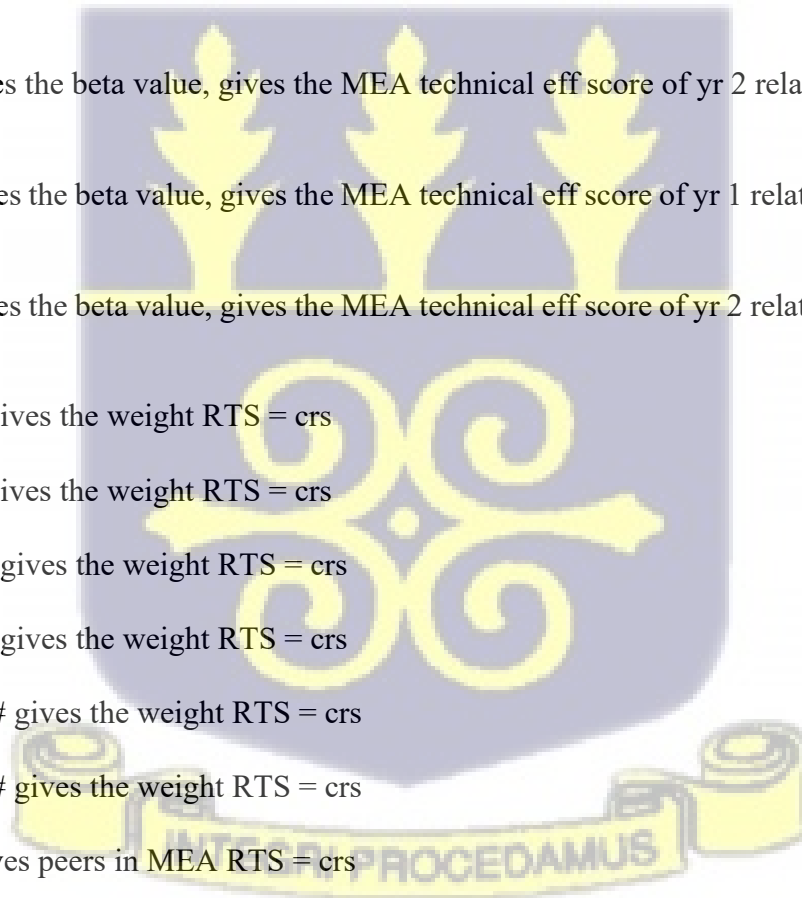
mec1b\$lambda # gives the weight RTS = crs

mec2b\$lambda # gives the weight RTS = crs

peers(mc1) # gives peers in MEA RTS = crs

peers(mc2) # gives peers in MEA RTS = crs

peers(mc12)# gives peers in MEA RTS = crs



peers(mc21)# gives peers in MEA RTS = crs

peers(bmec1b) # gives peers in MEA RTS = crs

peers(mec2b) # gives peers in MEA RTS = crs

mc1\$direct # gives distance (diagonal) which to travel to the ideal point and also gives the vertical& horizontal distance to the DEA benchmark RTS = crs

mc2\$direct # gives distance (diagonal) which to travel to the ideal point and also gives the vertical& horizontal distance to the DEA benchmark RTS = crs

mc12\$direct # gives distance (diagonal) which to travel to the ideal point and also gives the vertical& horizontal distance to the DEA benchmark RTS = crs

mc21\$direct # gives distance (diagonal) which to travel to the ideal point and also gives the vertical& horizontal distance to the DEA benchmark RTS = crs

mec1b\$direct # gives distance (diagonal) which to travel to the ideal point and also gives the vertical& horizontal distance to the DEA benchmark RTS = crs

mec2b\$direct # gives distance (diagonal) which to travel to the ideal point and also gives the vertical& horizontal distance to the DEA benchmark RTS = crs

#isolate required column from results and calculates the excess inputs to add to the actual input to give IRP for yr 1 RTF=CRS

ecx1cxa=mc1\$direct[,1]

ecx1cxb=mc1\$direct[,2]

ecx1cxc=mc1\$direct[,3]

ecx1cyb=mc1\$direct[,4]

ecy1cya=mc1\$direct[,5] #isolate required column from results and calculates the excess outputs to add to the actual output to give IRP for yr1

#isolate required column from results and calculates the excess inputs to add to the actual input to give IRP for yr 2 RTF=CRS

ecx2cxa=mc2\$direct[,1]

ecx2cxb=mc2\$direct[,2]

ecx2cxc=mc2\$direct[,3]

```
ecx2cyb=mc2$direct[,4]
```

```
ecy2cya=mc2$direct[,5] #isolate required column from results and calculates the excess outputs  
to add to the actual output to give IRP for yr2
```

```
#isolate required column from results and calculates the excess inputs to add to the actual input to  
give IRP for yr 1 rel to yr 2 RTF=CRS
```

```
ecx12cxa=mc12$direct[,1]
```

```
ecx12cxb=mc12$direct[,2]
```

```
ecx12cxc=mc12$direct[,3]
```

```
ecx12cyb=mc12$direct[,4]
```

```
ecy12cya=mc12$direct[,5] #isolate required column from results and calculates the excess outputs  
to add to the actual output to give IRP for yr1 rel to yr 2
```

```
#isolate required column from results and calculates the excess inputs to add to the actual input to  
give IRP for yr 2 rel to yr 1 RTF=CRS
```

```
ecx21cxa=mc21$direct[,1]
```

```
ecx21cxb=mc21$direct[,2]
```

```
ecx21cxc=mc21$direct[,3]
```

```
ecx21cyb=mc21$direct[,4]
```

```
ecy21cya=mc21$direct[,5] #isolate required column from results and calculates the excess outputs  
to add to the actual output to give IRP for yr2 rel to yr 1 RTS=CRS
```

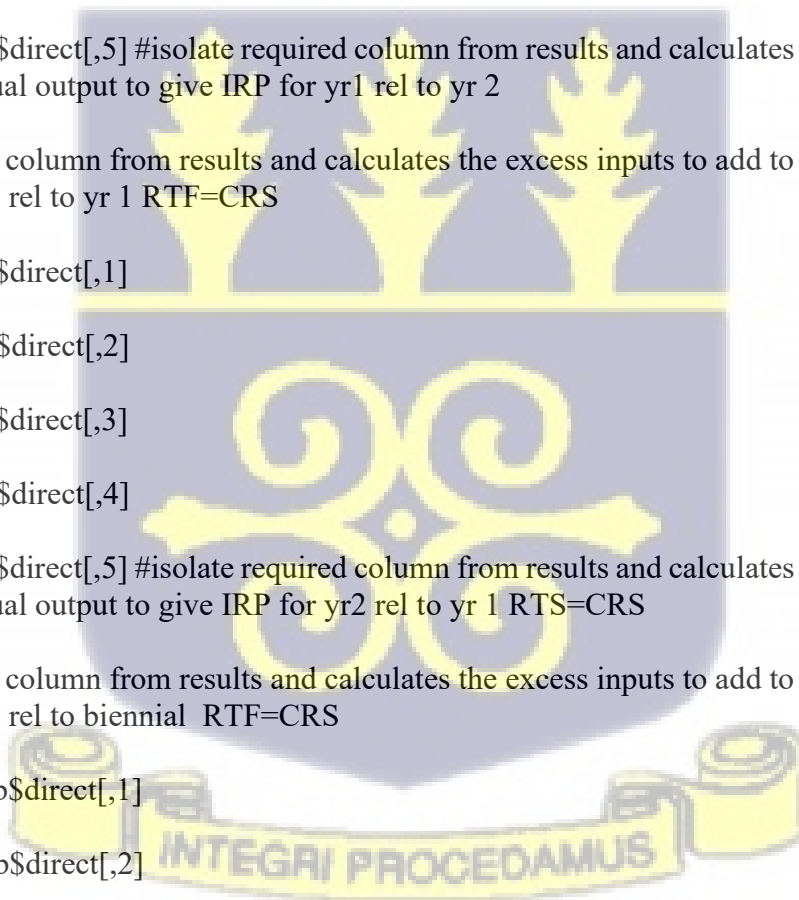
```
#isolate required column from results and calculates the excess inputs to add to the actual input to  
give IRP for yr 1 rel to biennial RTF=CRS
```

```
ecx1bcxa=mec1b$direct[,1]
```

```
ecx1bcxb=mec1b$direct[,2]
```

```
ecx1bcxc=mec1b$direct[,3]
```

```
ecx1bcyb=mec1b$direct[,4]
```



ecy1bcya=mec1b\$direct[,5] #isolate required column from results and calculates the excess outputs to add to the actual output to give IRP for yr1 rel to biennial

#isolate required column from results and calculates the excess inputs to add to the actual input to give IRP for yr 2 rel to biennial RTF=CRS

ecx2bcxa=mec2b\$direct[,1]

ecx2bcxb=mec2b\$direct[,2]

ecx2bcxc=mec2b\$direct[,3]

ecx2bcyb=mec2b\$direct[,4]

ecy2bcya=mec2b\$direct[,5] #isolate required column from results and calculates the excess outputs to add to the actual output to give IRP for yr2 rel to biennial

ideal ref point = actual input - excess input (and this excess if found in medirect results input-orientation)

ideal ref point = actual output + surplus output (and this excess if found in medirect results output-orientation)

#Step 1; Identify ideal reference point and solve LPP for the ideal point, 1 for each input and/or output direction

#IRP= actual input - excess input; the excess input is found in the me\$direct results. if it is in output orientation, the IRP= actual output + Surplus output

##Step 2; Find the potential Improvement point(PIP) or MEA benchmark using the Ideal reference by moving the input/output from the IRP to the target point(MEA benchmark)

#PIP = $X_{i0} - \text{Beta}(X_{i0} - d^*)$

$5.6 - 0.56 * (5.6 - 3.6)$ #PIP(Gx)=4.48

#PIP = $Y_{i0} + \text{Beta}(d^* - Y_{i0})$

$9 + 0.56 * (11.6 - 9)$ #PIP(Gy)= 10.45

#thus K=(4.48,10.45)

#Relative variable specific inefficiency = $\text{Beta}(X_{i0} - d^*) / X_{i0}$

#Relative variable specific efficiency = $X_{i0} - \text{Beta}(X_{i0}-d^*)/X_{i0}$

#calculates for IRP for inputs xa,xb,xc,yb(undesirable output) for year 1 RTS =CRS

$IRPx1cxa=(x1[,1]-ecx1cxa)$

$IRPx1cxb=(x1[,2]-ecx1cxb)$

$IRPx1cxc=(x1[,3]-ecx1cxc)$

$IRPx1cyb=(x1[,4]-ecx1cyb)$

#calculates for IRP for output ya for year 1

$IRPy1cya=(y1[,1]+ecy1cya)$

#calculates for IRP for inputs xa,xb,xc,yb(undesirable output) for year 2 RTS=CRS

$IRPx2cxa=(x2[,1]-ecx2cxa)$

$IRPx2cxb=(x2[,2]-ecx2cxb)$

$IRPx2cxc=(x2[,3]-ecx2cxc)$

$IRPx2cyb=(x2[,4]-ecx2cyb)$

#calculates for IRP for output ya for year 2

$IRPy2cya=(y2[,1]+ecy2cya)$

#calculates for IRP for inputs xa,xb,xc,yb(undesirable output) for year 1 relative to yr 2 RTS=CRS

$IRPx12cxa=(x1[,1]-ecx12cxa)$

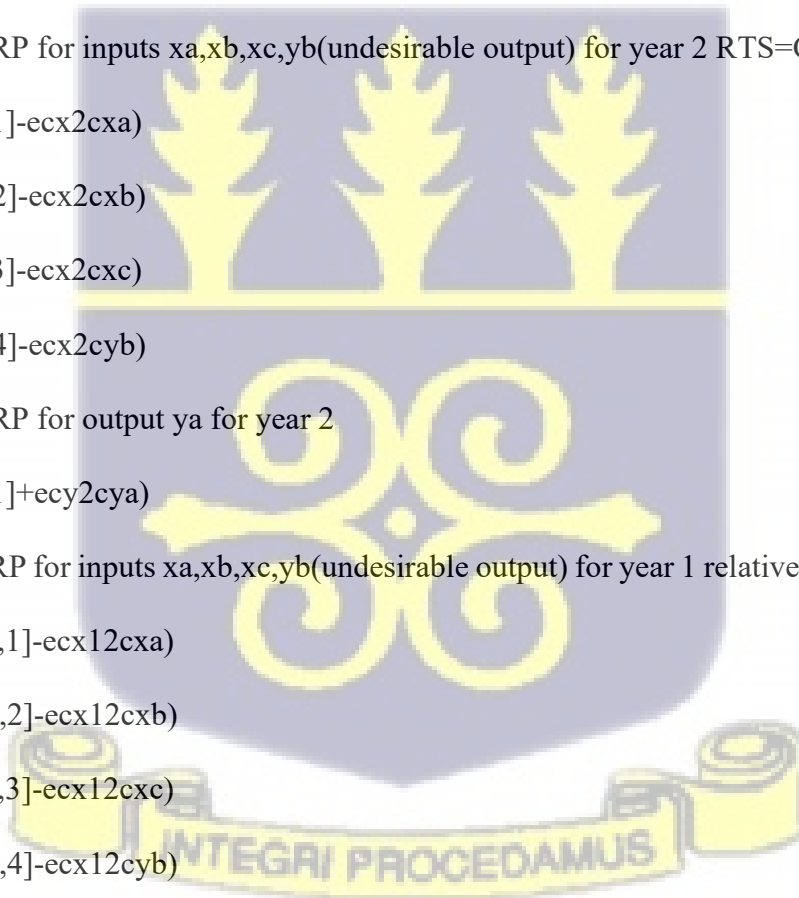
$IRPx12cxb=(x1[,2]-ecx12cxb)$

$IRPx12cxc=(x1[,3]-ecx12cxc)$

$IRPx12cyb=(x1[,4]-ecx12cyb)$

#calculates for IRP for output ya for year 1 relative to yr 2

$IRPy12cya=(y1[,1]+ecy12cya)$



#calculates for IRP for inputs xa,xb,xc,yb(undesirable output) for year 2 relative to yr 1 RTS=CRS

$$IRPx21cxa=(x2[,1]-ecx21cxa)$$

$$IRPx21cxb=(x2[,2]-ecx21cxb)$$

$$IRPx21cxc=(x2[,3]-ecx21cxc)$$

$$IRPx21cyb=(x2[,4]-ecx21cyb)$$

#calculates for IRP for output ya for year 2 relative to yr 1

$$IRPy21cya=(y2[,1]+ecy21cya)$$

#calculates for IRP for inputs xa,xb,xc,yb(undesirable output) for year 1 relative to biennial yr
RTS = CRS

$$BIRPx1bcxa=(x1b[,1]-ecx1bcxa)$$

$$BIRPx1bcxb=(x1b[,2]-ecx1bcxb)$$

$$BIRPx1bcxc=(x1b[,3]-ecx1bcxc)$$

$$BIRPx1bcyb=(x1b[,4]-ecx1bcyb)$$

#calculates for IRP for output ya for year 1 relative to biennial yr

$$BIRPy1bcya=(y1b[,1]+ecy1bcya)$$

#calculates for IRP for inputs xa,xb,xc,yb(undesirable output) for year 2 relative to biennial yr
RTS =CRS

$$BIRPx2bcxa=(x2b[,1]-ecx2bcxa)$$

$$BIRPx2bcxb=(x2b[,2]-ecx2bcxb)$$

$$BIRPx2bcxc=(x2b[,3]-ecx2bcxc)$$

$$BIRPx2bcyb=(x2b[,4]-ecx2bcyb)$$

#calculates for IRP for output ya for year 2 relative to biennial yr RTS =CRS

$$BIRPy2bcya=(y2b[,1]+ecy2bcya)$$

#calculates for PIP for inputs xa,xb,xc,yb(undesirable output) this is the MEA benchmark for year
1 RTS =CRS

$$PIP_{x1cxa} = x1[1] - mc1\$eff * (x1[1] - IRP_{x1cxa})$$

$$PIP_{x1cxb} = x1[2] - mc1\$eff * (x1[2] - IRP_{x1cxb})$$

$$PIP_{x1cxc} = x1[3] - mc1\$eff * (x1[3] - IRP_{x1cxc})$$

$$PIP_{x1cyb} = x1[4] - mc1\$eff * (x1[4] - IRP_{x1cyb})$$

$PIP_{y1cya} = y1[1] + (mc1\$eff * (IRP_{y1cya} - y1[1]))$ #calculates for PIP for output ya, this is the MEA benchmark for year 1 RTS=CRS

#calculates for PIP for inputs x1, this is the MEA benchmark for year 2

$$PIP_{x2cxa} = x2[1] - mc2\$eff * (x2[1] - IRP_{x2cxa})$$

$$PIP_{x2cxb} = x2[2] - mc2\$eff * (x2[2] - IRP_{x2cxb})$$

$$PIP_{x2cxc} = x2[3] - mc2\$eff * (x2[3] - IRP_{x2cxc})$$

$$PIP_{x2cyb} = x2[4] - mc2\$eff * (x2[4] - IRP_{x2cyb})$$

$PIP_{y2cya} = y2[1] + mv2\$eff * (IRP_{y2cya} - y2[1])$ #calculates for PIP for output ya, this is the MEA benchmark for year 2

#calculates for PIP for inputs xa,xb,xc,yb(undesirable output) this is the MEA benchmark for year 1 relative to yr 2

$$PIP_{x12cxa} = x2[1] - mc12\$eff * (x2[1] - IRP_{x12cxa})$$

$$PIP_{x12cxb} = x2[2] - mc12\$eff * (x2[2] - IRP_{x12cxb})$$

$$PIP_{x12cxc} = x2[3] - mc12\$eff * (x2[3] - IRP_{x12cxc})$$

$$PIP_{x12cyb} = x2[4] - mc12\$eff * (x2[4] - IRP_{x12cyb})$$

$PIP_{y12cya} = y2[1] + mv12\$eff * (IRP_{y12cya} - y2[1])$ #calculates for PIP for output y1, this is the MEA benchmark for year 1 relative to yr 2

#calculates for PIP for inputs x1, this is the MEA benchmark for year 2 relative to yr 1

$$PIP_{x21cxa} = x2[1] - mc21\$eff * (x2[1] - IRP_{x21cxa})$$

$$PIP_{x21cxb} = x2[2] - mc21\$eff * (x2[2] - IRP_{x21cxb})$$

$$PIP_{x21cxc} = x2[3] - mc21\$eff * (x2[3] - IRP_{x21cxc})$$

PIP_{x21cyb}=x₂[,4]-mc₂₁\$eff*(x₂[,4]-IRP_{x21cyb})

PIP_{y21cya}=y₂[,1]+mc₂₁\$eff*(IRP_{y21cya}-y₂[,1]) #calculates for PIP for output y1, this is the MEA benchmark for year 2 relative to yr 1

#calculates for PIP for inputs x1b, this is the MEA benchmark for year 1 relative to biennial yr

BPIP_{x1bcxa}=x_{1b}[,1]-mec_{1b}\$eff*(x_{1b}[,1]-BIRP_{x1bcxa})

BPIP_{x1bcxb}=x_{1b}[,2]-mec_{1b}\$eff*(x_{1b}[,2]-BIRP_{x1bcxb})

BPIP_{x1bcxc}=x_{1b}[,3]-mec_{1b}\$eff*(x_{1b}[,3]-BIRP_{x1bcxc})

BPIP_{x1bcyb}=x_{1b}[,4]-mec_{1b}\$eff*(x_{1b}[,4]-BIRP_{x1bcyb})

BPIP_{y1bcya}=y_{1b}[,1]+mec_{1b}\$eff*(BIRP_{y1bcya}-y_{1b}[,1]) #calculates for PIP for output y1b, this is the MEA benchmark for year 1 relative to biennial yr

#calculates for PIP for inputs bx12, this is the MEA benchmark for year 2 relative to biennial yr

BPIP_{x2bcxa}=x_{2b}[,1]-mec_{2b}\$eff*(x_{2b}[,1]-BIRP_{x2bcxa})

BPIP_{x2bcxb}=x_{2b}[,2]-mec_{2b}\$eff*(x_{2b}[,2]-BIRP_{x2bcxb})

BPIP_{x2bcxc}=x_{2b}[,3]-mec_{2b}\$eff*(x_{2b}[,3]-BIRP_{x2bcxc})

BPIP_{x2bcyb}=x_{2b}[,4]-mec_{2b}\$eff*(x_{2b}[,4]-BIRP_{x2bcyb})

BPIP_{y2bcya}=y_{2b}[,1]+mec_{2b}\$eff*(BIRP_{y2bcya}-y_{2b}[,1]) #calculates for PRP for output y1b, this is the MEA benchmark for year 2 relative to biennial yr

FOR INPUTS VARIABLES RTS = CRS

#MEA inefficiency score for xa,xb,xc,yb(undesirable output) for year 1, NB:
excl_{xa}=mv1\$direct[,-2]

inx_{1cxa}=(mc₁\$eff*excl_{1cxa})/x₁[,1]

inx_{1cxb}=(mc₁\$eff*excl_{1cxb})/x₁[,2]

inx_{1cxc}=(mc₁\$eff*excl_{1cxc})/x₁[,3]

inx_{1cyb}=(mc₁\$eff*excl_{1cyb})/x₁[,4]

#MEA inefficiency score for xa,xb,xc,yb(undesirable output) for year 2

$$\text{inx2cxa}=(\text{mc2\$eff}*\text{ecx2cxa})/\text{x2[,1]}$$

$$\text{inx2cxb}=(\text{mc2\$eff}*\text{ecx2cxb})/\text{x2[,2]}$$

$$\text{inx2cxc}=(\text{mc2\$eff}*\text{ecx2cxc})/\text{x2[,3]}$$

$$\text{inx2cyb}=(\text{mc2\$eff}*\text{ecx2cyb})/\text{x2[,4]}$$

#MEA ineff score for xa,xb,xc,yb(undesirable output) for year 1 relative to yr 2

$$\text{inx12cxa}=(\text{mc12\$eff}*\text{ecx12cxa})/\text{x1[,1]}$$

$$\text{inx12cxb}=(\text{mc12\$eff}*\text{ecx12cxb})/\text{x1[,2]}$$

$$\text{inx12cxc}=(\text{mc12\$eff}*\text{ecx12cxc})/\text{x1[,3]}$$

$$\text{inx12cyb}=(\text{mc12\$eff}*\text{ecx12cyb})/\text{x1[,4]}$$

#MEA ineff score for xa,xb,xc,yb(undesirable output) for year 2 relative to yr 1

$$\text{inx21cxa}=(\text{mc21\$eff}*\text{ecx21cxa})/\text{x2[,1]}$$

$$\text{inx21cxb}=(\text{mc21\$eff}*\text{ecx21cxb})/\text{x2[,2]}$$

$$\text{inx21cxc}=(\text{mc21\$eff}*\text{ecx21cxc})/\text{x2[,3]}$$

$$\text{inx21cyb}=(\text{mc21\$eff}*\text{ecx21cyb})/\text{x2[,4]}$$

#MEA ineff score for x for year 1 relative to biennial yr

$$\text{inx1bcxa}=(\text{mec1b\$eff}*\text{ecx1bcxa})[1:32]/\text{x1b[,1]}[1:32]$$

$$\text{inx1bcxb}=(\text{mec1b\$eff}*\text{ecx1bcxb})[1:32]/\text{x1b[,2]}[1:32]$$

$$\text{inx1bcxc}=(\text{mec1b\$eff}*\text{ecx1bcxc})[1:32]/\text{x1b[,3]}[1:32]$$

$$\text{inx1bcyb}=(\text{mec1b\$eff}*\text{ecx1bcyb})[1:32]/\text{x1b[,4]}[1:32]$$

#MEA ineff score for x for year 2 relative to biennial yr

$$\text{inx2bcxa}=(\text{mec2b\$eff}*\text{ecx2bcxa})[33:64]/\text{x2b[,1]}[33:64]$$

$$\text{inx2bcxb}=(\text{mec2b\$eff}*\text{ecx2bcxb})[33:64]/\text{x2b[,2]}[33:64]$$

```
inx2bcxc=(mec2b$eff*ecx2bcxc)[33:64]/x2b[,3][33:64]
```

```
inx2bcyb=(mec2b$eff*ecx2bcyb)[33:64]/x2b[,4][33:64]
```

FOR INPUT EFFCICINECY SCORE CRS

```
#nPIPx1=x1-(mv1$eff*mv1$direct[,-2])
```

```
#eff score for inputs xa,xb,xc,yb(undesirable output) yr 1
```

```
efx1cxa=PIPx1cxa/x1[,1]
```

```
efx1cxb=PIPx1cxb/x1[,2]
```

```
efx1cxc=PIPx1cxc/x1[,3]
```

```
efx1cyb=PIPx1cyb/x1[,4]
```

```
#eff score for inputs xa,xb,xc,yb(undesirable output) yr 2
```

```
efx2cxa=PIPx2cxa/x2[,1]
```

```
efx2cxb=PIPx2cxb/x2[,2]
```

```
efx2cxc=PIPx2cxc/x2[,3]
```

```
efx2cyb=PIPx2cyb/x2[,4]
```

```
#eff score for inputs xa,xb,xc,yb(undesirable output) yr 1 rel to yr 2
```

```
efx12cxa=PIPx12cxa/x1[,1]
```

```
efx12cxb=PIPx12cxb/x1[,2]
```

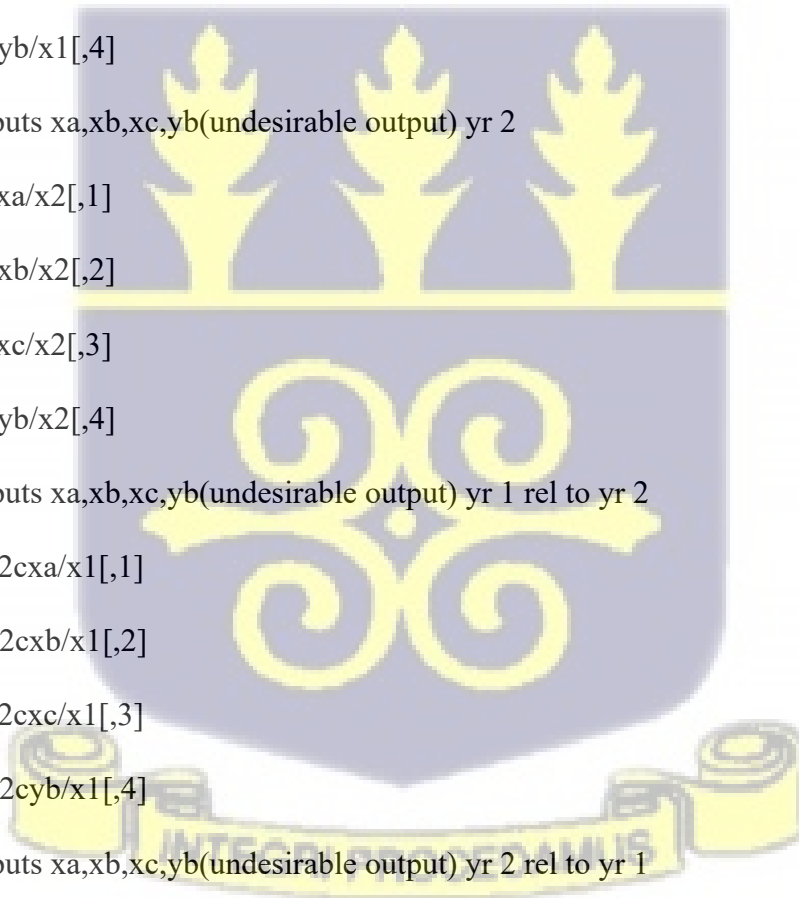
```
efx12cxc=PIPx12cxc/x1[,3]
```

```
efx12cyb=PIPx12cyb/x1[,4]
```

```
#eff score for inputs xa,xb,xc,yb(undesirable output) yr 2 rel to yr 1
```

```
efx21cxa=PIPx21cxa/x2[,1]
```

```
efx21cxb=PIPx21cxb/x2[,2]
```



```
efx21cxc=PIPx21cxc/x2[,3]
```

```
efx21cyb=PIPx21cyb/x2[,4]
```

```
#eff score for inputs xa,xb,xc,yb(undesirable output) yr 1 rel to biennial
```

```
efx1bcxa=BPIPx1bcxa[1:32]/x1b[,1][1:32]
```

```
efx1bcxb=BPIPx1bcxb[1:32]/x1b[,2][1:32]
```

```
efx1bcxc=BPIPx1bcxc[1:32]/x1b[,3][1:32]
```

```
efx1bcyb=BPIPx1bcyb[1:32]/x1b[,4][1:32]
```

```
#eff score for inputs xa,xb,xc,yb(undesirable output) yr 2 rel to biennial
```

```
efx2bcxa=BPIPx2bcxa[33:64]/x2b[,1][33:64]
```

```
efx2bcxb=BPIPx2bcxb[33:64]/x2b[,2][33:64]
```

```
efx2bcxc=BPIPx2bcxc[33:64]/x2b[,3][33:64]
```

```
efx2bcyb=BPIPx2bcyb[33:64]/x2b[,4][33:64]
```

```
# for input xa crs
```

```
data.frame(inx1cxa,efx1cxa,inx2cxa,efx2cxa,inx12cxa,efx12cxa,inx21cxa,efx21cxa,inx1bcxa,efx1bcxa,inx2bcxa,efx2bcxa)
```

```
data.frame(efx1cxa,efx2cxa)
```

```
data.frame(efx1cxa,efx2cxa,efx12cxa,efx21cxa,efx1bcxa,efx2bcxa)
```

```
# for input xb crs
```

```
data.frame(inx1cxb,efx1cxb,inx2cxb,efx2cxb,inx12cxb,efx12cxb,inx21cxb,efx21cxb,inx1bcxb,efx1bcxb,inx2bcxb,efx2bcxb)
```

```
data.frame(efx1cxb,efx2cxb)
```

```
data.frame(efx1cxb,efx2cxb,efx12cxb,efx21cxb,efx1bcxb,efx2bcxb)
```

```
# for input xc crs
```

```
data.frame(inx1cxc,efx1cxc,inx2cxc,efx2cxc,inx12cxc,efx12cxc,inx21cxc,efx21cxc,inx1bcxc,efx1bcxc,inx2bcxc,efx2bcxc)
```

```
data.frame(efx1cxc,efx2cxc)
```

```
data.frame(efx1cxc,efx2cxc,efx12cxc,efx21cxc,efx1bcxc,efx2bcxc)
```

```
# for output yb crs
```

```
data.frame(inx1cyb,efx1cyb,inx2cyb,efx2cyb,inx12cyb,efx12cyb,inx21cyb,efx21cyb,inx1bcyb,efx1bcyb,inx2bcyb,efx2bcyb)
```

```
data.frame(efx1cyb,efx2cyb)
```

```
data.frame(efx1cyb,efx2cyb,efx12cyb,efx21cyb,efx1bcyb,efx2bcyb)
```

```
# VARIABLE SPECIFIC MALMQUIST FOR INPUT RTS=CRS
```

```
CMPIcxa=((efx1cxa/efx2cxa)*(((efx2cxa/efx12cxa)*(efx21cxa/efx1cxa))^0.5)
```

```
CMPIcxb=((efx1cxb/efx2cxb)*(((efx2cxb/efx12cxb)*(efx21cxb/efx1cxb))^0.5)
```

```
CMPIcxc=((efx1cxc/efx2cxc)*(((efx2cxc/efx12cxc)*(efx21cxc/efx1cxc))^0.5)
```

```
CMPIcyb=((efx1cyb/efx2cyb)*(((efx2cyb/efx12cyb)*(efx21cyb/efx1cyb))^0.5)
```

```
# input variable specific eff change for x
```

```
CPECcxa=efx1cxa/efx2cxa
```

```
CPECcxb=efx1cxb/efx2cxb
```

```
CPECcxc=efx1cxc/efx2cxc
```

```
CPECcyb=efx1cyb/efx2cyb
```

```
#pure technical change
```

```
CPTCcxa=((efx2cxa/efx12cxa)*(efx21cxa/efx1cxa))^0.5
```

```
CPTCcxb=((efx2cxb/efx12cxb)*(efx21cxb/efx1cxb))^0.5
```

```
CPTCcxc=((efx2cxc/efx12cxc)*(efx21cxc/efx1cxc))^0.5
```

```
CPTCcyb=((efx2cyb/efx12cyb)*(efx21cyb/efx1cyb))^0.5
```

#calculates biennial malquist MEA input variable specific eff change for x

$CBMI_{cxa} = efx1bcxa / efx2bcxa$

$CBMI_{cxb} = efx12cxb / efx2bcxb$

$CBMI_{cxc} = efx12cxc / efx2bcxc$

$CBMI_{cyb} = efx12cyb / efx2bcyb$

input variable specific biennial eff change for x

$CBPEC_{cxa} = efx1cxa / efx2cxa$

$CBPEC_{cxb} = efx1cxb / efx2cxb$

$CBPEC_{cxc} = efx1cxc / efx2cxc$

$CBPEC_{cyb} = efx1cyb / efx2cyb$

biennial pure technical change

$CBPTC_{cxa} = (efx2cxa / efx2bcxa) * (efx1bcxa / efx1cxa)$

$CBPTC_{cxb} = (efx2cxb / efx2bcxb) * (efx1bcxb / efx1cxb)$

$CBPTC_{cxc} = (efx2cxc / efx2bcxc) * (efx1bcxc / efx1cxc)$

$CBPTC_{cyb} = (efx2cyb / efx2bcyb) * (efx1bcyb / efx1cyb)$

#FOR OUTPUT VARIABLES RTS=CRS

$iny1cya = y1 / (mc1\$eff * ecy1cya)$ #MEA ineff score for output y in year 1 ,
NB: $ecy1c = mc1\$direct[-1]$, RTS= CRS

$iny2cya = y2 / (mc2\$eff * ecy2cya)$ #MEA ineff score for output y in year 2 ,
NB: $ecx2c = mc2\$direct[-2]$

$iny12cya = y1 / (mc12\$eff * ecy12cya)$ #MEA ineff score for y for year 1 relative to yr 2 RTS = crs

$iny21cya = y2 / (mc21\$eff * ecy21cya)$ #MEA ineff score for y for year 2 relative to yr 2 RTS = crs

$iny1bcya = y1b / (mec1b\$eff * ecy1bcya)$ #MEA ineff score for y for year 1 relative to biennial yr
RTS = crs

$iny2bcya = y2b / (mec2b\$eff * ecy2bcya)$ #MEA ineff score for y for year 2 relative to biennial yr
RTS = crs

FOR OUTPUT EFFCICINECY SCORE RTS=CRS

Given $efy1 = y1 / (y1 + (me\$eff * (IRPy1 - y1)))$ MEA eff score for output y in year 1, $IRPy = (y1 + ecy)$

$PIPy1 = y1 + (mv1\$eff * (IRPy1 - y1))$

$efy1cya = y1 / PIPy1cya$ #eff score

$efy2cya = y2 / PIPy2cya$ #eff score

$efy12cya = y1 / PIPy12cya$ #eff score

$efy21cya = y2 / PIPy21cya$ #eff score

$efy1bcya = y1b[1:32] / BPIPy1bcya[1:32]$ #eff score

$efy2bcya = y2b[33:64] / BPIPy1bcya[33:64]$ #eff score

$riny1cya = 1 / iny1cya$ #reciprocals of MEA ineff score for output y in year 1

$riny2cya = 1 / iny2cya$ #reciprocals of MEA ineff score for output y in year 1

$riny12cya = 1 / iny12cya$ #reciprocals of MEA ineff score for output y for year 1 relative to yr 2

$riny21cya = 1 / iny21cya$ #reciprocals of MEA ineff score for output y for year 2 relative to yr 1

$riny1bcya = 1 / iny1bcya[1:32]$ #reciprocals of MEA ineff score for output y for year 1 relative to biennial yr

$riny2bcya = 1 / iny2bcya[33:64]$ #reciprocals of MEA ineff score for output y for year 2 relative to biennial yr

Variable specific Malmquist for output RTS = CRS

CMPIcya=((efy1cya/efy12cya)*(efy21cya/efy2cya))^0.5 # output variable Malmquist productivity index

CPECcya=efy1cya/efy2cya # output variable specific eff change for y ,Pure eff change

CPTCcya=((efy2cya/efy12cya)*(efy21cya/efy1cya))^0.5 # output variable specific technological change for y

#calculates biennial Malmquist MEA input variable specific eff change for y

CBMIcya=efy1bcya/efy2bcya

input variable specific biennial eff change for x

CBPECcya=efy1cya/efy2cya

biennial pure technical change

CBPTCcya=(efy2cya/efy2bcya)*(efy1bcya/efy1cya)

data.frame(CMPIcya,CPECcya,CPTCcya,CBMIcya,CBPECcya,CBPTCcya)

#OVERALL VARIABLE SPECIFIC EFFICIENCY RTS=CRS

ovse1mc=(1-((1/4)*(inx1cxa+inx1cxb+inx1cxc+inx1cyb)))/(1+((1/1)*(riny1cya))) #MEA
overall eff score for multiple inputs and multiple outputs for year 1

ovse2mc=(1-((1/4)*(inx2cxa+inx2cxb+inx2cxc+inx2cyb)))/(1+((1/1)*(riny2cya))) #MEA
overall eff score for multiple inputs and multiple outputs for year 2

ovse12mc=(1-((1/4)*(inx12cxa+inx12cxb+inx12cxc+inx12cyb)))/(1+((1/1)*(riny12cya)))
#MEA overall eff score for multiple inputs and multiple outputs for year 1 relative to yr 2

ovse21mc=(1-((1/4)*(inx21cxa+inx21cxb+inx21cxc+inx21cyb)))/(1+((1/1)*(riny21cya)))
#MEA overall eff score for multiple inputs and multiple outputs for year 2 relative to yr 1

ovse1bmc=(1-((1/4)*(inx1bcxa+inx1bcxb+inx1bcxc+inx1bcyb)))/(1+((1/1)*(riny1bcya)))
#MEA overall eff score for multiple inputs and multiple outputs for year 1 relative to yr 2

ovse2bmc=(1-((1/4)*(inx2bcxa+inx2bcxb+inx2bcxc+inx2bcyb)))/(1+((1/1)*(riny2bcya)))
#MEA overall eff score for multiple inputs and multiple outputs for year 2 relative to yr 1

#WHEELOCK & WILSON 4 FACTOR #BMPI# FOR INPUTS and UNDESIRABLE OUTPUT

BPECxa <- (efx1xa)/(efx2xa) #pure efficiency change (same as classical)

BPECxb <- (efx1xb)/(efx2xb)

BPECxc <- (efx1xc)/(efx2xc)

BPECyb <- (efx1yb)/(efx2yb)

BSECxa <- (efx2xa/efx2cxa)*(efx1cxa/efx1xa)#scale efficiency change (same as classical, no cross period)

BSECxb <- (efx2xb/efx2cxb)*(efx1cxb/efx1xb)

BSECxc <- (efx2xc/efx2cxc)*(efx1cxc/efx1xc)

BSECyb <- (efx2yb/efx2cyb)*(efx1cyb/efx1yb)

BPTCxa <- ((efx2xa)/efx2bxa)*((efx1bxa/efx1xa)#biennial pure technical change (no geomean)

BPTCxb <- ((efx2xb)/efx2bxb)*((efx1bxb/efx1xb))

BPTCxc <- ((efx2xc)/efx2bxc)*((efx1bxc/efx1xc))

BPTCyb <- ((efx2yb)/efx2byb)*((efx1byb/efx1yb))

BSTCxa <- ((efx2bxa/efx2bcxa)/(efx2xa/efx2cxa))*((efx1xa/efx1cxa)/(efx1bxa/efx1bcxa))

BSTCxb <- ((efx2bxb/efx2bcxb)/(efx2xb/efx2cxb))*((efx1xb/efx1cxb)/(efx1bxb/efx1bcxb))

BSTCxc <- ((efx2bxc/efx2bcxc)/(efx2xc/efx2cxc))*((efx1xc/efx1cxc)/(efx1bxc/efx1bcxc))

BSTCyb <- ((efx2byb/efx2bcyb)/(efx2yb/efx2cyb))*((efx1yb/efx1cyb)/(efx1byb/efx1bcyb))

BMPIxa <- BPECxa*BPTCxa*BSECxa*BSTCxa

BMPIxb <- BPECxb*BPTCxb*BSECxb*BSTCxb

BMPIxc <- BPECxc*BPTCxc*BSECxc*BSTCxc

BMPIyb <- BPECyb*BPTCyb*BSECyb*BSTCyb

data.frame(BPECxa,BPTCxa,BSECxa,BSTCxa)

data.frame(BPECxb,BPTCxb,BSECxb,BSTCxb)

data.frame(BPECxc,BPTCxc,BSECxc,BSTCxc)

data.frame(BPECyb,BPTCyb,BSECyb,BSTCyb)

#For variable xa

BMPIcxa

BMPIvxa <- BPECxa*BPTCxa

BMPIvxa

results2xa <- data.frame(VMPIxa,BMPIcxa,BMPIvxa,BPECxa,BPTCxa,BSECxa,BSTCxa)

#For variable xb

BMPIcxb

BMPIvxb <- BPECxb*BPTCxb

BMPIvxb

results2xb <- data.frame(VMPIxb,BMPIcxb,BMPIvxb,BPECxb,BPTCxb,BSECxb,BSTCxb)

#For variable xc

BMPIcxc

BMPIvxc <- BPECxc*BPTCxc

BMPIvxc

results2xc <- data.frame(VMPIxc,BMPIcxc,BMPIvxc,BPECxc,BPTCxc,BSECxc,BSTCxc)

#For variable yb

BMPIcyb

BMPIvyb <- BPECyb*BPTCyb

BMPIvyb

results2yb <- data.frame(VMPIyb,BMPIcyb,BMPIvyb,BPECyb,BPTCyb,BSECyb,BSTCyb)

#WHEELOCK & WILSON 4 FACTOR #BMPI# FOR OUTPUTS

BPECya <- (efy1ya)/(efy2ya) #pure efficiency change (same as classical)

BSECya <- (efy2ya/efy2cya)*(efy1cya/efy1ya) #scale efficiency change (same as classical, no cross period)

```
BPTCya <- ((efy2ya)/efy2bya)*((efy1bya/efy1ya))#biennial pure technical change (no geomean)
```

```
BSTCya <- ((efy2bya/efy2bcya)/(efy2ya/efy2cya))*((efy1ya/efy1cya)/(efy1bya/efy1bcya))
```

```
#For variable ya
```

```
BMPIcya <- BPECya*BPTCya*BSECya*BSTCya
```

```
BMPIcya
```

```
BMPIvya <- BPECya*BPTCya
```

```
BMPIvya
```

```
results2ya <- data.frame(VMPIya,BMPIcya,BMPIvya,BPECya,BPTCya,BSECya,BSTCya)
```

