# MODELLING GHANA STOCK EXCHANGE INDICES AND EXCHANGE RATES WITH STABLE DISTRIBUTIONS

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# DECLARATION

**Candidate's Declaration**

This is to certify that, this thesis is the result of my own research work and that no part of it has been presented for another degree in this University or elsewhere.

SIGNATURE: …………………………. DATE…………………………..….

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**Supervisors' Declaration**

We hereby certify that this thesis was prepared from the candidate's own work and supervised in accordance with guidelines on supervision of thesis laid down by the University of Ghana.

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# ABSTRACT

Most of the concepts in theoretical and empirical finance that have been developed over the last 50 years rest upon the assumption that the return or price distribution for financial data follows a normal distribution. But this assumption is not justified by empirical data. Rather, the empirical observations (financial returns) exhibit excess kurtosis, more colloquially known as fat tails or heavy tails. This research first described the stable distribution family - $\alpha-$stable, Levy stable, Cauchy and Gaussian or Normal distributions. The study presented three methods of estimating parameters of $\alpha-$stable distributions, namely Maximum Likelihood estimation, Empirical Characteristic function and Sample Quantile methods, and goodness of fit tests- K-S and Chi-square, were used to quantitatively assess the quality performance of their respective estimates. A sample of weekly financial data (GSE All-Shares index, USD/GHC, GBP/GHC and EUR/GHC exchange rates) covering the period of 02/01/2000 − 31/12/2011 was analysed, and fitted to $\alpha-$stable, Cauchy and Normal distributions. Diagnostic tests such as P-P and Q-Q plots and goodness of fit tests (K-S, Chi-square, Anderson-Darling and Shapiro-Wilk) were graphically and quantitatively used to assess fitness to the returns of the data respectively. The study concludes that the weekly return distributions of Ghana financial data are heavy tailed and asymmetry and the maximum likelihood estimation method produce the most accurate and efficient estimates for the $\alpha-$stable fit to the data. The weekly financial data considered were modelled with $\alpha-$stable distribution and recommends that for efficient risk and assets returns management, analysts should explore and discover actual return distributions of financial data and not desist from speculative assumptions.

# DEDICATION

I dedicate this thesis to my guardian Madam Jessica Reath, Philipine Mordzinu and my beloved late parents; Mr. Samuel Maama K. Dagadu and Mrs. Margret A. Adjei Dagadu.

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I thank the Almighty God who has given me the care, knowledge and the opportunity to pursue education up to this level.

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Finally, to the management of Ghana Stock Exchange, Bank of Ghana and all my friends especially 2013 batch of MPhil students of Statistics Department, and to all of you including those not mentioned here, I ask for Gods guidance and mercies.

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# LIST OF ABBREVIATIONS

APT-------------------Arbitrage Pricing Theory

CAPM ---------------Capital Asset Pricing Model

CF--------------------Characteristic Function

ECF-------------------Empirical Characteristic Function

EUR-------------------European Euro

EUR/GHC-----------Euro to Ghana Cedi exchange rate

FFT -------------------Fast Fourier Transform

GBP-------------------Great Britain Pound

GBP/GHC-----------British pound to Ghana Cedi exchange rate

GCLT-----------------General Central Limit Theorem

GSE-------------------Ghana Stock Exchange

GSEI------------------Ghana Stock Exchange all-shares index

i.i.d. -------------------independent identically distributed

K-S--------------------Kolmogorov-Smirnov test

MCMC---------------Markov Chain Monte Carlo

MLE------------------Maximum Likelihood Estimation

P-P--------------------Probability-Probability plot

Q-Q-------------------Quantile-Quantile plot

S. Quantile-----------Sample Quantile

USD-------------------United State of America Dollar

USD/GHC-----------US dollar to Ghana Cedi exchange rate

# Chapter One

# INTRODUCTION

## 1.0 Background of the Study

Most of the concepts in theoretical and empirical finance that have been developed over the last 50 years rest upon the assumption that the return or price distribution for financial assets follows a normal distribution. Yet, with rare exception, studies that have investigated the validity of this assumption since the 1960s fail to find support for the normal distribution or Gaussian distribution as it is also called. Moreover, there is ample empirical evidence that many, if not most, financial return series are heavy-tailed and possibly skewed (Rachev, Menn & Fabozzi, 2005).

The “tails” of the distribution are where the extreme values occur. Empirical distributions for stock prices and returns have found that the extreme values are more likely than would be predicted by the normal distribution. This means that, between periods where the market exhibits relatively modest changes in prices and returns, there will be periods where there are changes that are much higher (i.e., crashes and booms) than predicted by the normal distribution. This is not only of concern to financial theorists, but also to practitioners who are, in view of the frequency of sharp market down turns in the equity markets, troubled by the compelling evidence that something is wrong in the foundation of the statistical edifice used, for example, to produce probability estimates for financial risk assessment by Hope’s study (Rachev et al., 2005).

Stable distributions have been widely used for fitting data in which extreme values are frequent due to the fact that it accommodates heavy-tailed financial series and therefore produces more reliable measures of tail risk such as value at risk (Garcia, Renault & Veredas, 2010).

Today it is widely acknowledged that the proper management of assets and prices or other related investment risks, requires the proper modelling of the return distribution of financial assets. For instance, the answer to whether it is possible to beat the market except by chance depends on whether stock market prices display long memory and how probable are very large price fluctuations (Alfonso, Mansilla & Terrero-Escalante, 2011). The crucial difficulty, however, is that the financial market is a very complex system; it has a large number of non-linearly interacting internal elements, and is highly sensible to the action of external forces. Even more, the real challenge here is that the number of the system constituents and the details of their interactions and of the external factors acting upon it is actually barely known (Alfonso et al., 2011).

It is often argued that financial asset returns are the cumulative outcome of a vast number of pieces of information and individual decisions arriving almost continuously in time. Hence, in the presence of heavy-tails it is natural to assume that they are approximately governed by a stable non-Gaussian distribution. Stable distributions have been proposed as a model for many types of physical and economic systems. There are several reasons for using a stable distribution to describe a system. The first is where there are solid theoretical reasons for expecting a non-Gaussian stable model, for example, reflection of rotating mirror yielding Cauchy distribution, hitting times from a Brownian motion yielding Levy distribution, the gravitational field of stars yielding the Holtsmark distribution (Feller, 1971; Uchaikin & Zolotarev, 1999).

The second reason is the Generalized Central Limit Theorem, which states that the only possible non-trivial limit of normalized sums of independent identically distributed terms is stable. It is argued that some observed quantities are the sum of many small terms, for example, the price of a stock, and hence a stable model should be used to describe such systems. The third argument for modelling with stable distributions is empirical: many large data sets exhibit heavy tails and skewness. The strong empirical evidence for these features combined with the Generalized Central Limit Theorem are used to justify the use of stable models.

Stable distributions have been successfully fit to stock returns, excess bond returns, foreign exchange rates, commodity price returns and real estate returns (McCulloch, 1996; Rachev & Mittnik, 2000). For a developing country like Ghana, there is a need for the proper management of assets and prices (and the related investment risks) and that requires the proper modelling of the return distribution of financial assets.

## 1.1 Problem Statement

The behaviour of extreme variations of economic indices, stock prices or even currencies, has been a topic of interest in finance and economics, and its study has become relevant in the context of risk management and financial risk theory. However, the analyses of stock prices, asset returns and exchange rates are usually difficult to perform due to the small number of extreme values in the tails of the distributions of financial time series variations.

Many techniques in modern finance rely heavily on the assumption that the random variables under investigation follow a Gaussian distribution. However, time series data observed in finance, and in other applications, often deviate from the normal distribution, in that their marginal distributions are heavy-tailed and, possibly asymmetric. In such situations, the appropriateness of the commonly adopted normal assumption is highly questionable (Borak, Härdle, Wolfgang & Weron, 2005).

Many of the concepts in theoretical and empirical finance developed over the past decades rest upon the assumption that asset returns follow a normal distribution. However, it has been long known that asset returns practically are not normally distributed. Rather, the empirical observations of financial asset returns and stock price indices exhibit fat tails. In response to the empirical evidence, Mandelbrot (1963) and Fama (1965) proposed the stable distribution as an alternative model.

Frain (2009) states that, the use of α-stable distribution in Finance was originally proposed by Mandelbrot to model various goods and asset prices and it became popular in the sixties and seventies but interest waned thereafter. This decline in interest was due not only to its mathematical complexity and the considerable computation resources required but to the considerable success of the Merton-Black-Scholes Gaussian approach to finance theory which was developed at the same time (Frain, 2009). It is widely recognized that the key to develop successful strategies for risk management and asset pricing is to parsimoniously describe the stochastic process governing asset dynamics (Xu, Xiao, Wu & Dong, 2011).

The financial market of Ghana remains under developed as compared to emerging markets of developed countries, due to lack of ability to manage risk and pricing of assets. Managing risk of an asset depends on knowing riskiness of the asset and the distribution of the returns of asset. Tsay (2005) states emphatically that, proper managing of risky assets depends on both the positive and negative returns of the asset or stock. An investor may be able to hold a short or long position of an asset, or put short or put long of an asset if he/she knows the return distribution of the asset/stock. Investors take positions in currency swaps, options, futures or forward because they know the return distributions of that currency exchange rate. Studying the return distribution of assets and stocks make the exchange market efficient and vibrant, and prevent arbitrageurs from taking advantage of the market.

The study anticipates finding out the distribution of the financial data of Ghana stock exchange. It is observed that due to mathematical complexity of computing parameters of stable distributions, financial, statistical analysts and economists resort to the assumptions that financial data follows the Gaussian (normal) distribution.

## 1.2 Objectives of the Study

The main objective of this study is to investigate the empirical performance of $\alpha$-stable distribution in fitting the behaviour of asset returns and exchange rates and also to explore which of the existing methods for estimating the parameters of $\alpha$-stable distribution is much better in terms of fitting stable models.

This study specifically seeks to;

1. Determine the return distribution of financial assets (currency exchange rates and GSE all-shares index).
2. Investigate which of the following methods (Maximum Likelihood Estimation, Sample Quantile and Empirical Characteristic Function) of estimating the four parameters of stable distribution produces the best fit.
3. Fit an $\alpha$-stable model to the Ghana Stock Exchange All-Shares Index and Currency Exchange rates.

## 1.3 Scope of the Study

The analysts have rarely studied the modelling of financial data of Ghana using stable laws. The study will consider a sample of data covering a period of eleven years (2000-2011) and it will consist of daily stock indices from Ghana Stock Exchange and daily or monthly exchange rates from Bank of Ghana. The study will consider the Ghana Stock Exchange (GSE) All-Shares index and the three major exchange rates; US Dollar to Ghana Cedi, Euro to Ghana Cedi and British Pound to Ghana Cedi.

## 1.4 Significance of the Study

Although there are several studies that have examined the performance of stock prices and assets returns, there are a few or no studies that have investigated the performance of returns in the Ghana stock market using stable distributions. Findings from this study will go a long way in assisting economists, financial analysts and policy makers to make decent government economic or financial policies that will yield impressive gains in all the sectors of economic. The results from this study will be very important in assisting investors to develop successful strategies for risk management and asset pricing, especially for investors who adopt chasing index returns and exchange rates investment strategy in the Ghanaian stock market.

It will also help the economists and financial analysts to stabilise the Ghanaian currency (Cedi) and also enable them to make accurate future forecast and predictions, since empirical results have shown that, assets returns with heavy tails give a much better stable model fit than  Gaussian model fit (Nolan, 2010). On the whole, it will add knowledge to the academic field, since there is little or no work that has been carried out in Ghana.

## 1.5 Limitations of the Study

The research intended to model the daily logarithm returns of Ghanaian financial data, but due to the constant nature and low volatility of the data, the study couldn't modelled the daily logarithm returns of the data. The returns of the financial data could not satisfied some of the assumptions of stable laws, therefore the fractional moment method of estimating stable parameters was not applicable because the returns of the financial data contains zero. The Ghana Stock Exchange change the calculation of the indices from All-Shares index to composite index and financial index and that took effect from second January, 2011 and so that year data was not considered which reduces the sample size of GSE all-shares index.

## 1.6 Organization of the Study

The rest of the thesis is organized into four chapters, chapter two comprised of reviews of related literature on the topic and chapter three looks at the family of stable distributions, methods of estimating the parameters of stable distribution and goodness of fit tests of stable model. Chapter four analyzes and discusses the findings of the study and finally chapter five presents the summary, conclusion and recommendations of the study.

# Chapter Two

## REVIEW OF RELATED LITERATURE

It is often argued that financial asset returns are the cumulative outcome of a vast number of pieces of information and individual decisions arriving almost continuously in time. Hence, in the presence of heavy-tails it is natural to assume that they are approximately governed by a stable non-Gaussian distribution. McCulloch (1997) states that other leptokurtic distributions, including Student's $t$, Weibull and Hyperbolic, do not obey the central limit theorem.

Tsay (2005) disputes the traditional assumption that simple financial returns are independently and identically distributed as normal with mean and variance. He explained that the simple returns have a lower bound of -1 but the normal distribution has a lower bound of -∞. Also, he said if the simple returns are normally distributed, then the multi-period simple return is not normally distributed because it is a product of one-period returns. Tsay concluded that, the normality assumption is not supported by many empirical asset returns (Tsay, 2005).

The most common assumption in some literature is that the logarithm returns of an asset are independent and identically distributed as normal with mean $\mu$ and variance $\sigma^2$ but Tsay (2005) dispute that the lognormal assumption is not consistent with all the properties of stock returns, which lean towards positive excess kurtosis (Tsay, 2005).

Stable distributions have been successfully fitted to stock returns, excess bond returns, foreign exchange rates, commodity price returns and real estate returns (McCulloch, 1996; Rachev & Mittnik, 2000). In recent years, however, several studies have found what appears to be strong evidence against the stable model (Gopikrishnan et al., 1999; McCulloch, 1997).

The implication that returns of financial assets have a heavy-tailed distribution may be profound to a risk manager in a financial institution. Bradley and Taqqu (2003) argue that, a 3σ events may occur with a much larger probability when the return distribution is heavy-tailed than when it is normal. Quantile based measures of risk, such as value at risk, may also be drastically different if calculated for a heavy-tailed distribution. This is especially true for the highest quantiles of the distribution associated with very rare but very damaging adverse market movements (Bradley & Taqqu, 2003).

DuMouchel (1973) estimated the Paretian tail index directly from the tail observations and using either the Pareto distribution or a generalization of the Pareto distribution proposed by DuMouchel, found a tail index that appears to be significantly greater than 2; the maximum permissible value for a stable distribution (McCulloch, 1997).

An issue here is whether the underlying distributions are actually stable. Stability only holds for $0 < \alpha \leq 2$ and some authors have found that the tails of some financial time series have to be modelled with $\alpha > 2$ (Gopikrishnan, et al., 1999; Pagan, 1996; Coronel-Brizio & Hernandez-Montoya, 2005).

Alfonso et al. (2011) and Cont (2001), argue that, in order for a parametric distributional model to reproduce the properties of the empirical distribution it must have at least four parameters: a location parameter, a scale parameter, a parameter describing the decay of the tails and an asymmetry parameter. There are other heavy-tailed distributions such as

Student's $t$, hyperbolic or normal inverse Gaussian which fulfilled this condition. Therefore, in order to grasp the universal laws behind markets dynamics, it is important to keep accumulating empirical facts about the statistics of different financial indices around the world (Alfonso et al., 2011).

A rigorous statistical analysis of logarithm returns of daily fluctuations of the IPC; the leading Mexican Stock Market Index, shows that the stable Levy distribution best fits the data after considering the Gaussian, normal inverse Gaussian and Levy distributions and the corresponding $\alpha$ was indeed in the range for a stable Levy distribution (Alfonso et al., 2011).

Bradley and Taqqu (2001) argue that despite Markowitz's mean–variance portfolio theory, as well as the CAPM (Capital Asset Pricing Model) and APT (Arbitrage Pricing Theory) models, relies either explicitly or implicitly on the assumption of normally distributed asset returns. Today, with long histories of price or return data available for many financial assets, it is easy to see that this assumption is inadequate. Empirical evidence using NASDAQ composite index, suggests that asset returns have distributions which are heavier-tailed than the normal distribution (Bradley & Taqqu, 2003).

Xu et al. (2011) fitted the Shanghai Composite Index and Shenzhen Component Index returns with $\alpha$ - stable distribution, and the empirical results show that the asymmetric leptokurtic features present in the Shanghai Composite index and Shenzhen Component index returns can be captured by $\alpha$-stable law. Their findings of empirical result shows that $\alpha$ - stable distribution is better fitted to Chinese stock returns than the Black–Scholes model.

Khindanova, Rachev and Schwartz (2001), argue that one of the most important tasks of financial institutions is evaluating the exposure to market risks, which arise from variations in prices of equities, commodities, exchange rates, and interest rates. The dependence on market risks can be measured by changes in the portfolio value, or profits and losses. The empirical observations of financial data exhibit “fat” tails and excess kurtosis. The historical methods (delta method, historical simulation, Monte Carlo simulation, and stress-testing) do not impose distributional assumptions but it is not reliable in estimating low quantiles of changes in prices with a small number of observations in the tails.

The value-at-risk (VAR) measurements are widely applied to estimate the exposure to market risks. Khindanova et al. (2001) shows that the traditional approaches to VAR computations-the Delta method, Historical simulation, Monte Carlo simulation, and Stress-testing, do not provide satisfactory evaluation of possible losses. The Delta-normal methods do not describe well financial data with heavy tails. Hence, they underestimate VAR measurements in the tails. The Historical simulation does not produce robust VAR estimates since it is not reliable in approximating low quantiles with a small number of observations in the tails. The stable Paretian model, while sharing the main properties of the normal distribution leading to the CLT (central limit theorem), outperforms the normal modelling for high values of the VAR confidence level 99% and provides superior fit in modelling VAR (Khindanova et al., 2001).

Many of the concepts in theoretical and empirical finance developed over the past decades such as the classical portfolio theory, the Black-Scholes-Merton option pricing model or the Risk Metrics variance-covariance approach to VaR, rest upon the assumption that asset returns follow a normal distribution. But this assumption is not justified by empirical data, but rather, the empirical observations exhibit excess kurtosis and was justified by Guillaume et al. (1997) and Rachev and Mittnik (2000) and Borak, Misiorek & Weron (2010).

The Basle Committee on Banking Supervision's study (as cited in Borak et al., 2010) suggested that for the purpose of determining minimum capital reserves, financial institutions should use a 10-day VaR at the 99% confidence level multiplied by a safety factor $s \in [3, 4]$. Stahl (1997) and Danielsson, Hartmann and De Vries' study (as cited in Borak et al., 2010) also argue convincingly that the range of $s$ is as a result of the heavy-tailed nature of asset returns (Borak, Misiorek & Weron , 2010).

Borak et al. (2010) fitted two samples of financial data (Dow Jones Industrial Average (DJIA) index and the Polish WIG20 index) to Gaussian, Hyperbolic, Normal-Inverse Gaussian ( NIG) and Stable distributions and found that for both datasets, Kolmogorov and Anderson-Darling goodness-of-fit statistics suggest the Hyperbolic and NIG distributions as the best models for filtered and standardized returns, respectively.

Peters, Sisson and Fan (2009) states that models constructed with α-stable distributions possess several useful properties, including infinite variance, skewness and heavy tails and also justified by Zolotarev (1986), Alder, Feldman and Taqqu (1998), Samorodnitsky and Taqqu (1994) and Nolan (2010).

Peters et al. (2009) argue convincingly that, statistical inference for α-stable models is challenging due to the computational intractability of the density function. In practice this limits the range of models fitted, to univariate and bivariate cases. By adopting likelihood-free Bayesian methods they were able to circumvent this difficulty, and provide approximate, but credible posterior inference in the general multivariate case, at a moderate computational cost. They show that multivariate projections of data onto the unit hypersphere, in combination with sample quantile estimators, are adequate for this task and their method shows greater sampler consistency than alternative samplers, such as the auxiliary Gibbs or Markov Chain Monte Carlo (MCMC) inversion plus series expansion samplers (Peters, Sisson & Fan, 2009).

Alfonso et al. (2011) found out that, despite the simplifications of the normal distribution provides in analytical calculation are very valuable, empirical studies by Mandelbrot (1963), Mantegna and Stanley (1995), Gopikrishnan et al. (1999) and Cont (2001) show that the distribution of returns has a tail heavier than that of a Gaussian. They illustrated this fact, with the histogram plot of daily logarithm differences of the Mexican IPC index from April 9th, 2000 to April 9th, 2010.

The chi-square goodness of fit test, the Anderson-Darling test and the Kolmogorov-Smirnov (K-S) test rejected the Mexican financial index, IPC, being distributed normally or as a normal inverse Gaussian. On the other hand, the three tests show that IPC data comes from $\alpha$-stable levy distribution (Alfonso et al., 2011). Alfonso et al. (2011) illustrate that the sample size have impact on estimating the tail index and that high frequency data is needed in order to determine whether or not a given distribution is stable.

Cartea and Howison (2009) claim that based on the Generalised Central Limit Theorem (GCLT); there are two ways of modelling stock prices or stock returns, in general terms. If it is believed that stock returns are at least approximately governed by a Levy-Stable distribution then the accumulation of the random events is additive. On the other hand, if it is believed that the logarithms of stock prices are approximately governed by a Levy-stable distribution then the accumulation is multiplicative.

McCulloch (1996) assumes that assets returns are log Levy-Stable and prices options using a utility maximisation argument also follow Levy-Stable process. Also, Carr and Wu (2003) states that priced European options of the log-stock price returns follow a maximally skewed Levy-Stable process (Cartea & Howison, 2009).

Cartea and Howison, use the Black-Scholes model in finance for pricing assets as a benchmark to compare the option prices obtained when the returns follow Levy-Stable process. Their findings were consistent with the findings of Hull and White (1987) where the Black-Scholes model under prices in- and out of-the-money call option prices and overprices at-the-money options (Cartea & Howison, 2009).

Until recently, it has been difficult to use stable laws in practical problems because of computational difficulties. Nolan (1997) developed a software program, known as STABLE which can compute stable densities, cumulative distribution functions, parameters and quantiles. Nolan (1997) described the basic method used in the program –Fourier transform and simulation. Later improvements to the program was made by incorporating the Chambers, Mallows and Stuck (1976) method of simulating stable random variables, which improved accuracy in the calculations, and estimation of stable parameters from data sets (Nolan, 2003).

The basic estimation problem for stable laws is to estimate the four parameters $(\alpha,\ \beta,\ \gamma,\ \delta)$ from an i.i.d. random sample $X_1, X_2, ..., X_n$. There are several methods available for this basic estimation problem: a Quantile method of McCulloch (1986), a Fractional Moment method of Nikias and Shao (1995), Empirical Characteristic Function (ECF) method of Kogon and Williams (1998) based on ideas of Koutrouvelis, and Maximum Likelihood (ML) estimation of DuMouchel (1971) and Nolan (2001). Ojeda (2001), compared these methods in a simulation study and found that the Maximum Likelihood estimates are almost always more accurate, with the Empirical Characteristic Function estimates next best, followed by the Sample Quantile method, and finally the Moment method. The ML method has the added advantage that one can give large sample confidence intervals for the parameters, based on numerical computations of the Fisher information matrix (Nolan, 2003).

Standard exploratory data analysis of graphical techniques can be adapted to informally evaluate the closeness of a stable fit. Nolan observed that comparing smoothed data density plots to a proposed fit gives a good sense of how good the fit is near the centre of the data. Nolan argues that the P–P plots allow a comparison over the range of the data whiles Q–Q plots not as satisfactory for comparing heavy tailed data to the proposed fit and for technical reasons he recommend the “variance stabilized” P–P plot of Michael (1983). He claim that heavy tailed data set have many more extreme values than a typical sample from finite variance population and that it forces a Q–Q plot to be visually compressed, with a few extreme values dominating the plot. Also, the heavy tails imply that the extreme order statistics will have a lot of variability, and hence deviations from an ideal straight line Q–Q plot are hard to assess (Nolan, 2003).

The above discussion has mainly highlighted the return distribution of financial data. It reviews a range of financial data that have been modelled with different heavy-tail distributions. The chapter also reviews different methods of estimating the parameters of $\alpha-$stable distribution and concluded with diagnostic tests for fitting the models. The next chapter elaborates further on this statistical technique and provides a detailed account of their procedures.

# Chapter Three

# METHODOLOGY

## 3.0 Introduction

This chapter discusses the various stable distribution families which will best fit the GSE All-Shares index and the three currency exchange rates under study. These involve α-stable, Levy-stable, Cauchy and Gaussian (Normal) distributions. Specifically, this chapter examines the α-stable distribution and the normal distribution.

Techniques for estimating the parameters of the distributions as well as Quantile-Quantile (Q-Q) and Probability-Probability (P-P) plots used in assessing goodness of fit to the data set are described. The Kolmogorov-Smirnov, Chi-square, Shapiro-Wilk and Anderson-Darling goodness of fit tests applied in this research work are also discussed.

### 3.1.0 The Stable Distribution Family

This section gives detailed description of the four stable distributions that this study explores to model GSE all-shares index and exchange rates. The study considers the stable families of distributions due to their stability property and nature of the financial data. This research considers the α-stable distribution, Levy stable distribution, Cauchy distribution and Gaussian or normal distribution. This family of distributions has a very interesting pattern of shapes, allowing for asymmetry and thick tails, that makes them suitable for the modelling of several phenomena, ranging from the engineering (noise of degraded audio sources) to the financial; asset returns (Lombardi, 2007).

### 3.1.1 $\alpha-$Stable Distribution

Stable distributions are a class of probability laws that have intriguing theoretical and practical properties. The $\alpha-$stable family of distributions stems from a more general version of the central limit theorem which replaces the assumption of the finiteness of the variance with a much less restrictive one concerning the regular behaviour of the tails (Gnedenko & Kolmogorov, 1954).

Their applications to financial modelling comes from the fact that they generalize the normal (Gaussian) distribution and allow heavy tails and skewness, which are frequently seen in financial data. In general, there are no closed form formulas for α-stable density functions $f$ and cumulative distribution functions $F$, but there are now reliable computer programs for working with these laws (Nolan, 2005) and α-stable distributions are represented by characteristic functions.

**Definition 1**

The complex-valued function

$$\phi_X(t) = E\left[e^{itx}\right] \tag{3.01}$$

is called the characteristic function (c.f.) of a real random variable. Here $t$ is some real-valued variable and if the density $f(x)$ exists, then the Fourier transform of that density is given as;

$$\phi_X(t) = \int_{-\infty}^{\infty} e^{itx} f(x)\,dx \tag{3.02}$$

The density function $f(x)$ is derived using inverse Fourier transform which allows us to reconstruct the density of a distribution from a known c.f. by the uniqueness theorem and is given as;

$$f(x) = \tfrac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx}\phi_X(t)\,dx \tag{3.03}$$

**Definition 2**

A random variable $X$ is α-stable distributed; denoted by $S(\alpha,\beta,\gamma,\delta_0;0)$ , if it has the characteristic function

$$E\left[\exp(iuX)\right] = \begin{cases} \exp\left(-\gamma^{\alpha}|u|^{\alpha}\left[1+i\beta\left(\tan\dfrac{\pi\alpha}{2}\right)(sign\ u)\left(|\gamma u|^{1-\alpha}-1\right)\right]+i\delta_0 u\right) & \alpha \neq 1, \\ \exp\left(-\gamma^{\alpha}|u|^{\alpha}\left[1+i\beta\dfrac{2}{\pi}(sign\ u)\ln|u|\right]+i\delta_0 u\right) & \alpha = 1, \end{cases} \tag{3.04}$$

where sign($u$) is 1 if $u$ > 0, 0 if $u$ = 0, and –1 if $u$ < 0.

**Definition 3**

A random variable $X$ is α-stable distributed; denoted by $S(\alpha,\beta,\gamma,\delta_1;1)$ , if it has characteristic function as given below;

$$E\left[\exp(iuX)\right] = \begin{cases} \exp\left(-\gamma^{\alpha}|u|^{\alpha}\left[1-i\beta\left(\tan\dfrac{\pi\alpha}{2}\right)(sign\ u)\right]+i\delta_1 u\right) & \alpha \neq 1, \\ \exp\left(-\gamma^{\alpha}|u|\left[1+i\beta\dfrac{2}{\pi}(sign\ u)\ln|u|\right]+i\delta_1 u\right) & \alpha = 1, \end{cases} \tag{3.05}$$

where sign($u$) is 1 if $u$ > 0, 0 if $u$ = 0, and –1 if $u$ < 0.

The parameter $\alpha$ is called the index of the law or the index of stability or characteristic exponent and must be in the range $0 < \alpha \le 2$ or $(0,2]$. The parameter $\beta$ is called the skewness of the law, and must be in the range $-1 \le \beta \le 1$. If $\beta = 0$, the distribution is symmetric, if $\beta > 0$ it is skewed towards the right and if $\beta < 0$, it is skewed towards the left. The parameters $\alpha$ and $\beta$ determines the shape of the distribution. The parameter $\gamma$ is a scale parameter and it can be any positive number, i.e. $\gamma \ge 0$. The parameter $\delta$ is a location parameter and range $-\infty < \delta < \infty$, it shifts the distribution right if $\delta > 0$, and left if $\delta < 0$.

The two different definitions of α-stable distribution above, according to Zolotarev (1986) and Nolan (2003) are the two common different parameterizations, which this study will consider and are denoted by $S(\alpha,\beta,\gamma,\delta_0;0)$ and $S(\alpha,\beta,\gamma,\delta_1;1)$. The first is use in applications, because it has better numerical behaviour and intuitive meaning but the formulation of the characteristic function is, in this case, quite more cumbersome, and the analytic properties have a less intuitive meaning. The second parameterization has a quite manageable expression of its characteristic function and can straightforwardly produce several interesting analytic results (Zolotarev, 1986), but has a major drawback for what concerns estimation and inferential purposes: it is not continuous with respect to the parameters, having a pole at $\alpha = 1$, but is more commonly used in the literature.

### 3.1.2 Stable Density and Distribution Functions

The lack of closed form formulas for most α-stable densities and distribution functions has negative consequences. For example, during maximum likelihood estimation computationally burdensome numerical approximations have to be used. There generally are two approaches to this problem. Either the Fast Fourier transform (FFT) has to be applied to the characteristic function (Mittnik, Rachev, Doganoglu & Chenyao, 1999) or direct numerical integration has to be utilized (Nolan, 1997). The FFT based approach is faster for large samples, whereas the direct integration method favours small data sets since it can be computed at any arbitrarily chosen point (Borak et al., 2005).

The probability density function $f(x;\alpha,\beta)$, of a standard α-stable random variable is given below as:

Let $\zeta = -\beta \tan \frac{\pi\alpha}{2}$,

- When $\alpha \neq 1,\ x > \zeta$:

$$f(x;\alpha,\beta) = \frac{\alpha(x-\zeta)^{\frac{1}{\alpha-1}}}{\pi|\alpha-1|} \int_{\xi}^{\frac{\pi}{2}} V(\theta;\alpha,\beta)\exp\left\{-(x-\zeta)^{\frac{\alpha}{\alpha-1}} V(\theta;\alpha,\beta)\right\} d\theta, \qquad (3.06)$$

- When $\alpha \neq 1,\ x = \zeta$:

$$f(x;\alpha,\beta) = \frac{\Gamma\left(1+\frac{1}{\alpha}\right)\cos(\xi)}{\pi\left(1+\zeta^2\right)^{\frac{1}{2\alpha}}}, \qquad (3.07)$$

- When $\alpha \neq 1$ and $x < \zeta$:

$$f(x;\alpha,\beta) = f(-x;\alpha,-\beta), \qquad (3.08)$$

- When $\alpha = 1,\ x \in \mathbb{R}$:

$$f(x;1,\beta) = \begin{cases} \dfrac{1}{2|\beta|} e^{-\frac{\pi x}{2\beta}} \displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} V(\theta;1,\beta)\exp\left\{-e^{-\frac{\pi x}{2\beta}} V(\theta;1,\beta)\right\} d\theta, & \beta \neq 0 \\[2ex] \dfrac{1}{\pi\left(1+x^2\right)}, & \beta = 0 \end{cases} \qquad (3.09)$$

where

$$\xi = \begin{cases} \frac{1}{\alpha}\arctan(-\zeta), & \alpha \neq 1 \\ \frac{\pi}{2}, & \alpha = 1 \end{cases} \tag{3.10}$$

and

$$V(\theta;\alpha,\beta) = \begin{cases} (\cos\alpha\xi)^{\frac{1}{\alpha-1}}\left(\frac{\cos\theta}{\sin\alpha(\xi+\theta)}\right)^{\frac{\alpha}{\alpha-1}}\frac{\cos\{\alpha\xi+(\alpha-1)\theta\}}{\cos\theta}, & \alpha \neq 1 \\ \frac{2}{\pi}\left(\frac{\frac{\pi}{2}+\beta\theta}{\cos\ \theta}\right)\exp\left\{\frac{1}{\beta}\left(\frac{\pi}{2}+\beta\theta\right)\tan\theta\right\}, & \alpha = 1,\ \beta \neq 0 \end{cases} \tag{3.11}$$

The cumulative distribution function (cdf) $F(x;\alpha,\beta)$ of a standard $\alpha$-stable random variable can be expressed as:

- When $\alpha \neq 1$ and $x > \zeta$:

$$F(x;\alpha,\beta) = c_1(\alpha,\beta) + \frac{sign(1-\alpha)}{\pi}\int_{-\xi}^{\frac{\pi}{2}}\exp\left\{-(x-\zeta)^{\frac{\alpha}{\alpha-1}}V(\theta;\alpha,\beta)\right\}d\theta, \tag{3.12}$$

where

$$c_1(\alpha,\beta) = \begin{cases} \frac{1}{\pi}\left(\frac{\pi}{2}-\xi\right), & \alpha < 1 \\ 1, & \alpha > 1 \end{cases} \tag{3.13}$$

- When $\alpha \neq 1,\ x = \zeta$ :

$$F(x;\alpha,\beta) = \tfrac{1}{\pi}\left(\tfrac{\pi}{2} - \xi\right), \tag{3.14}$$

- When $\alpha \neq 1,\ x < \zeta$ :

$$F(x;\alpha,\beta) = 1 - F(-x;\alpha,-\beta), \tag{3.15}$$

- When $\alpha = 1,\ x \in \mathbb{R}$:

$$F(x;\alpha,\beta) = \begin{cases} \dfrac{1}{\pi}\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp\left\{-e^{-\frac{\pi x}{2\beta}}\, V(\theta;1,\beta)\right\} d\theta, & \beta > 0, \\ \dfrac{1}{2} + \dfrac{1}{\pi}\arctan x, & \beta = 0, \\ 1 - F(x;1,-\beta), & \beta < 0. \end{cases} \tag{3.16}$$

The density formula (eqn. 3.06) above requires numerical integration of the function $p(.)\exp\{-p(.)\}$, where $p(\theta;x,\alpha,\beta) = (x-\zeta)^{\frac{\alpha}{\alpha-1}} V(\theta;\alpha,\beta)$. The integrand is 0 at $-\xi$, and increases monotonically to a maximum of $\frac{1}{e}$ at point $\theta^*$ for which $p(\theta^*;x,\alpha,\beta) = 1$, and then decreases monotonically to 0 at $\dfrac{\pi}{2}$. However, in some cases the integrand becomes very peaked and numerical algorithms can miss the spike and underestimate the integral. To avoid this problem we need to find the argument $\theta^*$ of the peak numerically and compute the integral as a sum of two integrals: one from $-\xi$ to $\theta^*$ and the other from $\theta^*$ to $\frac{\pi}{2}$ (Nolan, 1997).

The defining characteristic, and reason for the term stable, is that they retain their shape (up to scale and shift) under addition. If $X_1, X_2, ..., X_n$ are independent and identically distributed stable random variables, with distribution function $F$, then for every $n$;

$$X_1 + X_2 + ... + X_n \underset{=}{d} c_n X + d_n \tag{3.17}$$

for some constants $c_n > 0$ and $d_n$. The symbol $\underset{=}{d}$ means equality in distribution, i.e., the right and left hand sides have the same distribution.

The law is called strictly stable if $\underset{=}{d} = 0$ for all $n$. In terms of financial returns, one could say that the sum of daily returns is up to scale and location equally distributed as the monthly return or yearly return.

## 3.2 Special Cases of the Stable Distribution

There are three special cases of the stable distributions; Levy stable, Cauchy and Gaussian distributions that have closed-form expression of density functions. The case where $\alpha = 2$ (and $\beta = 0$, which plays no role in this case) and with the reparameterization in scale, where $\widehat{\gamma} = \sqrt{2\gamma}$ , yields the normal distribution. The case where $\alpha = 1$ and $\beta = 0$ yields the Cauchy distribution with much fatter tails than the normal distribution. The third and last special case is obtained for $\alpha = ½$ and $\beta = 1$, yields the Lévy distribution.

### 3.2.1 Levy Stable Distribution

A random variable $X$ is Levy stable with parameters $(\mu, \sigma; \mu = \delta \text{ and } \sigma = \gamma)$ has the probability density function (pdf) $f(x; \mu, \sigma)$ given as:

$$f(x;\mu,\sigma) = \left(\frac{\sigma}{2\pi}\right)^{\frac{1}{2}} \frac{1}{(x-\mu)^{3/2}} \exp\left\{-\frac{\sigma}{2(x-\mu)}\right\}, \quad x > \mu \qquad (3.18)$$

The probability density of the Lévy distribution is concentrated on the interval (μ, +∞). The cumulative distribution function (cdf) $F(x; \mu, \sigma)$ of a Lévy distribution random variable can be expressed as:

$$F(x;\mu,\sigma) = \left(\frac{\sigma}{2\pi}\right)^{\frac{1}{2}} \int_{\mu}^{\infty} \frac{1}{(x-\mu)^{3/2}} \exp\left\{-\frac{\sigma}{2(x-\mu)}\right\} dx \text{ , } x > \mu \qquad (3.19)$$

$$F(x;\mu,\sigma) = erfc\left(\sqrt{\frac{\sigma}{2(x-\mu)}}\right), \; x > \mu$$

where $erfc(x)$ is the complementary error function. Both the expected value (mean) and the variance of a Levy stable distribution does not exist, ($\infty$).

The characteristic function of a Levy stable distribution is given as

$$\phi_X(t) = \exp\left\{i\mu t - \sqrt{-2i\sigma t}\right\} \qquad (3.20)$$

### 3.2.2 The Cauchy Distribution

A random variable $X$ has the standard Cauchy distribution, if $X$ has probability density function (pdf) given as:

$$f(x) = \frac{1}{\pi\left(1+x^2\right)},\ x \in \mathbb{R} \tag{3.21}$$

The distribution function $F(x)$ of the standard Cauchy distribution is given by

$$F(x) = 1/2 + 1/\pi \arctan(x),\ x \in \mathbb{R} \tag{3.22}$$

The Cauchy distribution is generalized by adding location and scale parameters. Suppose that Z has the standard Cauchy distribution and that $\mu \in \mathbb{R}$ and $\sigma \in (0,\infty)$. Then $X = \mu + \sigma Z$ has the Cauchy distribution with location parameter $\mu$ and scale parameter $\sigma$ and the probability density function of $X$ is given as:

$$h(x) = \frac{\sigma}{\pi\left[\sigma^2 + (x-\mu)^2\right]},\ x \in \mathbb{R} \tag{3.23}$$

The distribution function $H(x)$ of the general Cauchy distribution is given by

$$H(x) = \frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{x-\mu}{\sigma}\right),\quad x \in \mathbb{R} \tag{3.24}$$

The characteristic function of a standard Cauchy distribution is given as

$$\phi_X(t) = \exp(-|t|),\quad t \in \mathbb{R} \tag{3.25}$$

The characteristic function of a general Cauchy distribution is given as

$$\phi_X(t) = \exp\left(\mu i t - \sigma|t|\right), \quad t \in \mathbb{R} \tag{3.26}$$

The Cauchy distribution is a simple family of distributions for which the expected value (and other moments) do not exist. Secondly, the family is closed under sums of independent variables, and hence is an infinitely divisible family of distributions. It is a symmetrical 'bell-shaped' function like the Gaussian distribution, but it differs from that in the behaviour of their tails: the tails of the Cauchy density decrease as $x^{-2}$.

### 3.2.3 The Gaussian (Normal) Distribution

The most popular distribution in various physical and engineering applications is the Gaussian or Normal distribution. A random variable $X$ has a Normal distribution with parameters $\mu$ $(mean)$ and $\sigma^2$ $(Variance)$ if $X$ has probability density function (pdf) given as:

$$f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, \qquad -\infty < x < \infty,\ \sigma > 0 \text{ and } \mu \in \mathbb{R} \tag{3.27}$$

The distribution function $F(x)$ of the normal distribution is given as

$$\begin{aligned} F(x) &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left\{-\frac{(t-\mu)^2}{2\sigma^2}\right\} dt \\ &= \Phi\left(\tfrac{x-\mu}{\sigma}\right) = \frac{1}{2}\left[1 - erf\left(\tfrac{x-\mu}{\sigma\sqrt{2}}\right)\right], \quad -\infty < x < \infty,\ \sigma > 0 \text{ and } \mu \in \mathbb{R} \end{aligned} \tag{3.28}$$

The random variable $x$ having a standard normal distribution is given as:

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{x^2}{2}\right\}, \ -\infty < x < \infty \tag{3.29}$$

The distribution function $F(x)$ of the standard normal distribution is given as:

$$\begin{aligned} F(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left\{-\frac{t^2}{2}\right\} dt, \ -\infty < x < \infty \\ &= \Phi(x) \end{aligned} \tag{3.30}$$

The characteristic functions of both the standard normal distribution and the normal distribution are given respectively as:

$$\phi(t) = \exp\left(-\frac{t^2}{2}\right),$$

$$\phi(t) = \exp\left\{i\mu t - \tfrac{1}{2}(\sigma t)^2\right\}$$

The expected value of X is denoted as μ and is called the location parameter and the variance of $X$ denoted as $\sigma^2$ is known as the shape parameter. The Gaussian distribution is symmetrical 'bell-shaped' about the mean $(\mu)$ and it has the mean, mode and median been equal.

### 3.3 Probability-Probability (P-P) Plot

The probability-probability plot is a graphical technique for assessing whether or not a data set follows a specified theoretical distribution. The data are plotted against a theoretical distribution in such a way that the points should form approximately a straight line. Departures from this straight line indicate departures from the specified distribution. P-P plots tend to magnify deviations from the proposed distribution in the middle.

The probability-probability plot is constructed using the theoretical cumulative distribution function, $F(x_i)$, where $i = 1, 2, 3, ..., n$, of the specified model. The values in the sample of the data are ordered from the smallest to the largest and are denoted as; $X_{(1)}, X_{(2)}, X_{(3)}, \ldots, X_{(n)}$. For $i = 1, 2, 3, ..., n$, $F(x_{(i)})$ is plotted against $\left[\frac{i-\frac{1}{2}}{n}\right]$. The P-P plot was used in this study to illustrate how the returns of financial data fit the Gaussian distribution and the $\alpha-$stable distribution.

### 3.4 Quantile-Quantile (Q-Q) Plot

The quantile-quantile (Q-Q) plot is a diagnostic graph of the input (observed) data values plotted against the theoretical (fitted) distribution quantiles. Both axes of this graph are in units of the input data set. The plotted points should be approximately linear if the specified theoretical distribution is the correct model. The quantile-quantile plot is more sensitive to the deviances from the theoretical distribution (normal) in the tails and tends to magnify deviation from the proposed distribution on the tails.

The Q-Q plot is constructed using the theoretical cumulative distribution function, $F(x)$, of the specified theoretical model or distribution. The values in the sample of the data are ordered from the smallest to the largest and are denoted as $X_{(1)}, X_{(2)}, X_{(3)}, \ldots, X_{(n)}$. For $i = 1, 2, 3, \ldots, n$, $X_{(i)}$'s are plotted against the inverse cumulative distribution function; $F^{-1}\left[\frac{i-\frac{1}{2}}{n}\right]$. The Q-Q plot was used in this study to graphically assess goodness of fit of the returns of financial data to the Gaussian distribution and the $\alpha-$stable distribution.

## 3.5 Goodness-of-Fit Tests

The goodness of fit (GOF) tests measures the compatibility of a random sample with a theoretical probability distribution function. In other words, a test for goodness of fit usually involves examining a random sample from some unknown distribution in order to test the null hypothesis that the unknown distribution function is in fact a known, specified distribution. The general procedure consists of defining a test statistic which is some function of the data measuring the distance between the hypothesised distribution and the data, and then calculating the probability of obtaining data which have a still larger value of this test statistic than the value observed. Assuming the hypothesis is true, this probability is called the confidence level.

### 3.5.1 Anderson-Darling Test

The Anderson-Darling procedure is a general test to compare the fit of an observed cumulative distribution function to an expected cumulative distribution function. The Anderson-Darling test is used to test if a sample of data came from a population with a specific distribution. It is a modification of the Kolmogorov-Smirnov (K-S) test and gives more weight to the tails than the K-S test does. The K-S test is distribution free in the sense that the critical values do not depend on the specific distribution being tested but the Anderson-Darling test makes use of the specific distribution in calculating the critical values. This has the advantage of allowing a more sensitive test and the disadvantage that critical values must be calculated for each distribution.

The Anderson-Darling tests the hypothesis:

$H_0$ : The data follow the specified distribution.

$H_1$ : The data do not follow the specified distribution.

The Anderson-Darling test statistic ( $A^2$ ) is defined as;

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\ln F(X_i) + \ln(1-F(X_{n-i+1}))\right] \tag{3.31}$$

where $F$ is the cumulative distribution function (cdf) of the specified distribution and $X_i$ is the ordered data. The hypothesis regarding the distributional form is rejected at the chosen significance level $(\alpha)$ if the test statistic, $A^2$, is greater than the critical value obtained from a table.

### 3.5.2 Kolmogorov-Smirnov Goodness-of-Fit Test

The Kolmogorov-Smirnov (K-S) goodness of fit test is a nonparametric test used to assess whether a sample comes from a population with a specified distribution. A hypothesis test involves calculation of a test statistic from the data and the probability of obtaining a value at least as large as a tail area if the correct distribution is chosen. The K-S test is based on the empirical cumulative distribution function (ECDF), it measures the Supremun distance between the cumulative distribution function of the theoretical distribution and the empirical distribution function, over all the sample points. The K-S test is distribution free since its critical values do not depend on the specific distribution being tested. The K-S test is relatively insensitive to differences in the tails but more sensitive to points near the median of the distribution.

**Definition 4**

Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample. The empirical distribution function $S(x)$ is a function of $X$, which equals the fraction of $X_i's$ that are less than or equal to $X$ for each $X_i, -\infty < X_i < \infty$,

$$S(x) = \frac{1}{n}\sum_{i=1}^{n} I_{\{x_i \le x\}} \tag{3.32}$$

Where $I_{\{x_i \le x\}}(x)$ is an indicator function and

$$I_{\{x_i \le x\}}(x) = \begin{cases} 0, & \text{if } x_i > x \\ 1, & \text{if } x_i \le x \end{cases} \tag{3.33}$$

The empirical distribution function $S(x)$ is useful as an estimator of $F(x)$, the unknown distribution function of the $X_i's$. We compare the empirical distribution function $S(x)$ with the hypothesized distribution function $F^*(x)$ to investigate if there is good fit. Conover (1999) states that, Kolmogorov in 1933, suggested the test statistic as be the greatest (denoted by "sup" for Supremum) vertical distance between $S(x)$ and $F^*(x)$, and is define as;

$$D = \sup_i \left\| F^*(x_i) - S(x_i) \right\| \tag{3.34}$$

For testing the hypothesis,

$H_0: \ F(x) = F^*(x), \ \text{for all} - \infty < x < \infty$

$H_1: \ F(x) \ne F^*(x), \ \text{for at least one value of } x$

The Kolmogorov-Smirnov goodness of fit test is used in this study to test the goodness of fit of the financial data to the Normal, Cauchy and $\alpha-$stable distributions.

### 3.5.3 Chi-square Goodness of Fit Test

The Chi-square goodness of fit test is the test which makes a statement or claim concerning the nature of the distribution for the whole population and it is a parametric test. The data in the sample is examined in order to see whether this distribution is consistent with the hypothesized distribution of the population or not. One way in which the Chi-square goodness of fit test can be used is to examine how closely a sample matches a population of known distribution.

The Chi-Squared test is used to determine if a sample comes from a population with a specific distribution. This test is applied to binned data, so the value of the test statistic depends on how the data is binned. Although there is no optimal choice for the number of bins $(k)$, there are several formulas which can be used to calculate this number based on the sample size $(N)$. The value of $k$ can be determined by the empirical formula;

$$k = 1 + \log_2 N \tag{3.35}$$

The data can be grouped into intervals of equal probability or equal width. The first approach is generally more acceptable since it handles peaked data much better. Each bin should contain at least 5 or more data points, so certain adjacent bins sometimes need to be joined together for this condition to be satisfied.

**Definition 5**

The Chi-Square statistic is defined as;

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}, \tag{3.36}$$

where $O_i$ is the observed frequency for bin $i$, and $E_i$ is the expected frequency for bin $i$ calculated by $E_i = N[F(x_2^i) - F(x_1^i)]$, where $F$ is the cumulative distribution function (cdf) of the probability distribution being tested, $N$ the total sample size and $x_1$, $x_2$ are the limits for bin $i$. The test statistic is used for testing the hypothesis;

$H_0$ : The data follow the specified distribution.

$H_1$ : The data do not follow the specified distribution.

The hypothesis regarding the distributional form is rejected at the chosen significance level $(\alpha)$ if the test statistic is greater than the critical value defined as $\chi^2_{\alpha,k-c}$, where $(k-c)$ is the degrees of freedom and $c = u+1$, where $u$ is the number of estimated parameters from the sample.

### 3.5.4 Shapiro-Wilk Test

The Shapiro-Wilk test was proposed in 1965 by Shapiro and Wilk, for testing a random sample, $X_1, X_2, ..., X_n$ comes from (specifically) a normal distribution. The test statistic is denoted as $W$ and small values of $W$ are evidence of departure from normality and percentage points for the $W$ statistic are obtained via Monte Carlo simulations and were reproduced by Pearson and Hartley (1972). The $W$ statistic is defined as follows:

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}, \tag{3.37}$$

where $x_{(i)}$ are the ordered sample values and $a_i$'s are constants generated from the means, variances and covariances of the order statistics of a sample of size $n$ from a normal distribution. Let $m' = \left(m_1,\ m_2,\ m_3,\ ...,\ m_n\right)$, where $m_i = E(X_{(i)})$ and $V$ is $n$ x $n$ covariance matrix of $\upsilon_{ij}$ where $\upsilon_{ij} = \text{Cov}(X_{(i)}, X_{(j)})$. The vector $a$ of the weights $a_i$'s from eqn. (3.37) is defined as:

$$a' = m'V^{-1}\left[\left(m'V^{-1}\right)\left(V^{-1}m\right)\right]^{1/2} \tag{3.38}$$

The null hypothesis that the random sample comes the normal distribution is rejected, if the test statistic $W$ is less than or equal to $w_\alpha$, the critical value for the significance level $\alpha$; the probability of type I error.

### 3.6 Estimation the Parameters of $\alpha$ - Stable Distribution

This section discusses the methods of estimating the parameters of $\alpha -$stable distribution by the use of sample information. It is natural to expect that if a sample is at all representative of the population, one can use it to make an estimate that is better than a sheer guess (Lindgren, 1993).

The lack of known closed-form density functions of $\alpha -$stable distribution complicates statistical inference estimation of stable distributions. For instance, maximum likelihood (ML) estimates have to be based on numerical approximations or direct numerical integration of the formulas presented in Section 3.1. Consequently, ML estimation is difficult to implement and time consuming for samples encountered in modern finance. However, there are also other numerical methods that have been found useful in practice are sample quantile and empirical characteristic function (Borak et al., 2010).

The entire purpose of estimation theory is to arrive at an estimator, and preferably an implementable one that could actually be used. The estimator takes the measured data as input and produces an estimate of the parameters. It is also preferable to derive an estimator that exhibits optimality – achieving minimum average error over some class of estimators, for example, a minimum variance unbiased estimator. In this case, the class is the set of unbiased estimators, and the average error measure is variance (average squared error between the value of the estimate and the parameter).

### 3.6.1 Maximum Likelihood (ML) Estimation Method

If a random sample is available from a population for which the probability distribution is known except for the value of one parameter $\theta$, where $\theta = (\theta_1, \theta_2, ..., \theta_k)$. Each specific value of $\theta$ determines one particular distribution from among the large number of theoretically possible distribution. The estimation of $\theta$ is therefore equivalent to selecting that particular distribution which applies to the population that produced the sample (Odoom, 2007).

The Maximum Likelihood Principle suggests that the criterion for making the selection of an estimate should be the probability (or likelihood) which a particular distribution can produce the given sample. Thus, the distribution with the highest probability of producing the sample must be taken as the appropriate distribution that produced the sample. The value of $\theta$ for that distribution is the Maximum Likelihood Estimate of the unknown $\theta$ (Odoom, 2007).

The Maximum Likelihood Estimation (MLE) technique is asymptotically consistent, that is, as the sample size gets larger, the estimates converge to the right values. It is asymptotically efficient, it produces the most precise estimates and asymptotically unbiased, that is, for large samples one expects to get the right value on average.

Let $X = \left(X_1, X_2, ..., X_n\right)$ be an i.i.d. $\alpha -$stable random sample of size $n$, then the maximum likelihood (ML) estimate of the parameter vector $\theta = \left(\alpha,\ \beta,\ \gamma,\ \delta\right)$ is obtained by maximizing the log-likelihood function:

$$L(x;\theta) = \sum_{i=1}^{n} \log \tilde{f}\left(x_i;\theta\right), \tag{3.39}$$

where $\tilde{f}\left(x_i;\theta\right)$ is the $\alpha -$stable pdf. By maximizing $L$, the maximum likelihood estimators (MLE) of $\theta = (\alpha,\ \beta,\ \gamma,\ \delta)$ are the simultaneous solutions of four (4) equations such that:

$$\frac{\partial L}{\partial \theta} = 0, \text{ where } \theta = (\alpha,\ \beta,\ \gamma,\ \delta) \tag{3.40}$$

In general, we do not know the explicit form of the $\alpha-$stable density and have to approximate it numerically. However, all of them have an appealing common feature – under certain regularity conditions that the ML estimator is asymptotically normal with the variance specified by the Fisher information matrix (DuMouchel, 1973). The latter can be approximated either by using the Hessian matrix arising in maximization or, as in Nolan (2001), by numerical integration. DuMouchel (1971) developed an approximate ML method, which was based on grouping the data set into bins and using a combination of means to compute the density (FFT for the central values of $x$ and series expansions for the tails) to compute an approximate log-likelihood function. This function was then numerically maximized.

### 3.6.2 Sample Quantile Method

Fama and Roll (1971), were the first to provide a very simple estimates for parameters of symmetric $(\beta=0,\ \delta=0)$ stable laws with $\alpha>1$. A decade later McCulloch (1986) generalized their method and provided consistent estimators of all four stable parameters (with the restriction $\alpha \geq 0.6$ ). McCulloch (1986) define the estimators as:

$$\upsilon_{\alpha}=\frac{x_{0.95}-x_{0.05}}{x_{0.75}-x_{0.25}} \tag{3.41}$$

$$\upsilon_{\beta}=\frac{x_{0.95}+x_{0.05}-2x_{0.50}}{x_{0.95}-x_{0.05}} \tag{3.42}$$

$$\upsilon_{\gamma}=\frac{x_{0.75}-x_{0.25}}{\gamma} \tag{3.43}$$

Where $x_f$ denotes the $f-$th population quantile and the $x_{(i)}$ are ordered in ascending order and the $x_f$ matched, so that $S(\alpha,\ \beta,\ \gamma,\ \delta)(x_f)=f(x;\ \theta)$. The statistics $\upsilon_{\alpha}$ and $\upsilon_{\beta}$

are functions of $\alpha$ and $\beta$ only, i.e. they are independent of both $\gamma$ and $\delta$. This relationships are inverted and the parameters $\alpha$ and $\beta$ is viewed as functions of $\upsilon_\alpha$ and $\upsilon_\beta$ :

$$\alpha = \psi_1(\upsilon_\alpha,\ \upsilon_\beta) \quad \text{and} \quad \beta = \psi_2(\upsilon_\alpha,\ \upsilon_\beta) \tag{3.44}$$

Substituting $\upsilon_\alpha$ and $\upsilon_\beta$ by their sample values and applying linear interpolation between values found in tables given by McCulloch (1986) yields estimators $\hat{\alpha}$ and $\hat{\beta}$. The scale parameters, $(\gamma)$, is also estimated in a similar way and finally, the location parameter, $(\delta)$, is estimated using the sample mean $\bar{x}$ $(\text{when } \alpha > 1)$. However, due to the discontinuity of the characteristic function (CF) for $\alpha = 1$ and $\beta \neq 0$, in the representation of stable distributions, this procedure is much more complicated (Borak et al., 2010).

### 3.6.3 Empirical Characteristic Function Method

Press (1972) was the first to use the empirical characteristic function in the context of statistical inference for stable laws. He proposed a simple estimation method for all four parameters, called the method of moments, based on transformations of the characteristic function (CF). However, the convergence of this method to the population values depends on the choice of four estimation points, whose selection is problematic.

Given an i.i.d. random sample $X = (X_1, X_2, \ldots, X_n)$ of size $n$, the empirical characteristic function is define:

$$\hat{\phi}(t) = \frac{1}{n}\sum_{i=1}^{n} \exp(itx_i) \tag{3.45}$$

Since $\left|\hat{\phi}(t)\right|$ is bounded by unity and all moments of $\hat{\phi}(t)$ are finite and, for any fixed $t$, it is the sample average of i.i.d. random variables $\exp\left(itx_i\right)$. Hence, by the law of large numbers, $\hat{\phi}(t)$ is a consistent estimator of the characteristic function $\phi(t)$.

Koutrouvelis (1980) presented a much more accurate regression-type method which starts with an initial estimate of the parameters and proceeds iteratively until some prespecified convergence criterion is satisfied. Each iterations, consists of two weighted regression runs. The number of points to be used in these regressions depends on the sample size and the starting values of $\alpha$. Typically not more than two or three iterations are needed and the speed of the convergence depends on the initial estimates and the convergence criterion. The regression method is based on the following observations concerning the characteristic function $\phi(t)$ (Borak et al., 2010).

Koutrouvelis (1980), derive the regression equation for estimating the parameters from the stable characteristic function as:

$$\log\left(-\log\left|\phi(t)\right|^2\right)=\log\left(2\gamma^\alpha\right)+\alpha\log\left|t\right| \tag{3.46}$$

The real and imaginary parts of $\phi(t)$ for $\alpha\neq 1$ are given by:

$$\Re\{\phi(t)\} = \exp\left(-\left|\gamma t\right|^\alpha\right)\cos\left[\delta t+\left|\gamma t\right|^\alpha \beta sign(t)\tan\frac{\pi\alpha}{2}\right], \tag{3.47}$$

$$\Im\{\phi(t)\} = \exp\left(-\left|\gamma t\right|^\alpha\right)\sin\left[\delta t+\left|\gamma t\right|^\alpha \beta \mathrm{sign}(t)\tan\frac{\pi\alpha}{2}\right], \tag{3.48}$$

Apart from considerations of principal values, we have:

$$\arctan\left(\frac{\Im\{\phi(t)\}}{\Re\{\phi(t)\}}\right) = \delta t + \beta\gamma^{\alpha}\tan\frac{\pi\alpha}{2} sign(t)|t|^{\alpha} \tag{3.49}$$

The equation (3.46) depends only on $\alpha$ and $\gamma$ and, these two parameters can be estimated by regressing $y = \log\left(-\log|\phi_n(t)|^2\right)$ on $\omega = \log|t|$ in the model:

$$y_k = m + \alpha\omega_k + \varepsilon_k, \tag{3.50}$$

where $t_k$ is an appropriate set of real numbers, $m = \log(2\gamma)$, and $\varepsilon_k$ denotes an error term. Koutrouvelis (1980) proposed to use $t_k = \frac{\pi k}{25}$, $k = 1,\ 2,\ ....,N$; with $N$ ranging between 9 and 134 for different values of $\alpha$ and sample sizes. Once $\hat{\alpha}$ and $\hat{\gamma}$ have been obtained and $\alpha$ and $\gamma$ fixed at these values, estimates of $\beta$ and $\delta$ are now obtained using (3.49). Next, the regressions are repeated with $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ and $\hat{\delta}$ as the initial parameters. The iterations continue until a prespecified convergence criterion is satisfied. Koutrouvelis suggested to use Fama and Roll's (1971) formula to estimate $\gamma$ and the 25% truncated mean ( $\bar{X}$ ) for initial estimates of $\gamma$ and $\delta$, respectively.

Koutrouvelis (1980) method was slower in estimating the parameters and therefore Kogon and Williams (1998) eliminated Koutrouvelis iteration procedure and simplified the regression method using McCulloch's (1986) method with the continuous representation of the Characteristic function instead of the classical one. They used a fixed set of only ten

(10) equally spaced frequency points $t_k$ and in terms of computational speed, their method was favourably faster than the original regression method (Borak et al., 2010).

Borak et al. (2010) stated that the sample quantile method is fast and simple than the other two methods but it is the least accurate of all. While the maximum likelihood estimation method is the slower and complex of all but much better, in terms of accuracy and computational time. The study used a program known as STABLE with R statistical software package to estimate all the parameters of $\alpha-$stable distribution, under the three different estimating methods discussed in this section. The next chapter present data analysis and discussions of the study.

### 3.7.1 Central Limit Theorem

Feller (1971) states that if the random variables $X_1, X_2, \ldots, X_n$ are independent and identically distributed with standard deviation $\sigma < \infty$, then normalized sum $S_n / \sqrt{n}$ is approximately normal with standard deviation $\sigma$, if $n$ is sufficiently large.

where

$$S_n = \sum_{j=1}^{n} x_j \tag{3.51}$$

Dependent variables do not satisfy the conditions for the simplest Central Limit Theorem, even if they are stationary and have finite variance. This is illustrated by the extreme case where the $X's$ are exact duplicates. In this situation, the normalized sums are identically zero, or grow to (plus or minus) infinity with the sample size. However, the independence assumption can be substantially relaxed while preserving the limiting normal distribution (Goldberg, Hayes, Barbieri, Dubikovsky & Gladkevich, 2009).

Mathematically, the variables $Y_1, Y_2, \ldots$ constitute a martingale if the expected value of $Y_t$ conditional on information at time $t-1$ is $Y_{t-1}$; $E_{t-1}(Y_t) = Y_{t-1}$ (Hall & Heyde,1980).

Let $X_t$ be the variable representing changes in value; i.e. $X_t$ is the difference of $Y_t - Y_{t-1}$, then Jacod and Protter (2003) states the Martingale Central Limit Theorem as follows:

If the random variables $X_1, X_2, \ldots, X_n$ satisfy

$$\begin{aligned} &(i) \quad E_{t-1}\left[X_t\right] = 0 \\ &(ii) \quad E_{t-1}\left[X_t^2\right] = \sigma^2 < \infty \\ &(iii) \quad E_{t-1}\left[\left|X_t\right|^3\right] < \infty, \end{aligned} \tag{3.52}$$

then the normalized sum $S_n/\sqrt{n}$ is approximately normal with standard deviation $\sigma$ if $n$ is sufficiently large.

### 3.7.2 Generalized Central Limit Theorem

Let $X_1, X_2, \ldots, X_n$ be independent identically distributed random variables with the distribution function $F_X(x)$ satisfying the conditions;

$$\begin{aligned} &1 - F_X(x) \sim cs^{-\mu}, \quad x \to \infty \\ &F_X(x) \sim d\left|x\right|^{-\mu}, \quad x \to \infty \end{aligned}$$

with $\mu > 0$ . Then there exist sequences $a_n$ and $b_n > 0$ such that the distribution of the centred and normalized sum

$$Z_n = \frac{1}{b_n}\left(\sum_{i=1}^{n} X_i - a_n\right), \tag{3.53}$$

weakly converges to the stable distribution with parameters

$$\alpha = \begin{cases} \mu, & \mu \le 2 \\ 2, & \mu > 2 \end{cases} \qquad \text{and} \qquad \beta = \frac{c-d}{c+d}$$

and as $n \to \infty$;

$$F_{Z_n}(x) \Rightarrow G^A(x;\alpha,\beta) \tag{3.54}$$

where $G^A(x;\alpha,\beta)$ is the distribution function of the stable distribution.

### 3.7.3 Infinite Variance

It is commonly assumed that financial returns have finite variance. Even if this assumption holds, the squares of financial returns may have infinite variance- or equivalently, financial returns may have an infinite fourth moment. An extension of the Central Limit Theorem addresses this situation, although its conclusion differs from the previous versions.

If the $X_i$'s are identical and independent with infinite variance, there may be a limiting distribution for appropriately scaled sums, and it is the $\alpha-$stable distribution (Goldberg et al., 2009).

### 3.7.4 Infinite Variance Central Limit Theorem

If the variables $X_1, X_2, ..., X_n$ are independent and identically distributed with tail index $\alpha \in (0,2)$, then the normalized sum $S_n / n^{1/\alpha}$ is, under technical conditions, approximately $\alpha-$stable if $n$ is sufficiently large (Embrechts, Kluppelberg & Mikosch, 1997).

# Chapter Four

# ANALYSIS AND DISCUSSIONS

## 4.0 Introduction

This chapter basically discusses the return distributions of Ghanaian financial data from 2000 to 2011. The chapter used time series plots to illustrate to the progression and the volatility of four variables considered over the eleven years period. The various diagnostics tests (P-P plot, Q-Q plot, Z-Z plot and density plot) were used to graphically demonstrate goodness of fit to the Normal distribution, the Cauchy distribution and the $\alpha-\text{Stable}$ distribution. The Goodness-of-Fit tests (Kolmogorov-Smirnov and Chi-square tests) were used to statistically test fitness of the Normal distribution, the Cauchy distribution and the $\alpha-\text{Stable}$ distribution to the financial data. The financial data considered was modelled with $\alpha-\text{Stable}$ distribution.

## 4.1 Ghana Stock Exchange All-Shares Index

The sample size of 571 of GSE All-Shares index was used to perform these analysis and the data span from 2nd January, 2000 to 31st December, 2010. Due to the constant nature of the daily GSE All-Shares index, a weekly GSE All-Shares index was used. The weekly logarithm returns was computed using the formula below;

$$X(t) = \log(Y_{t+1}) - \log(Y_t) \tag{4.01}$$

where $Y_t$ is the weekly all-shares index at week $t$ and $Y_{t+1}$ is the successive weekly all-shares index.

Time series plot of GSE All-Shares index in Figure 1 displays a constant growth between 2000 and 2002, and then increased up to 2004 before it started decreasing till almost 2005. The process begun again and attained its maximum index at mid-way of 2008 to 2009. The rise and fall in GSE All-Shares index was as a result of high inflation rates during the general elections in 2004 and 2008.

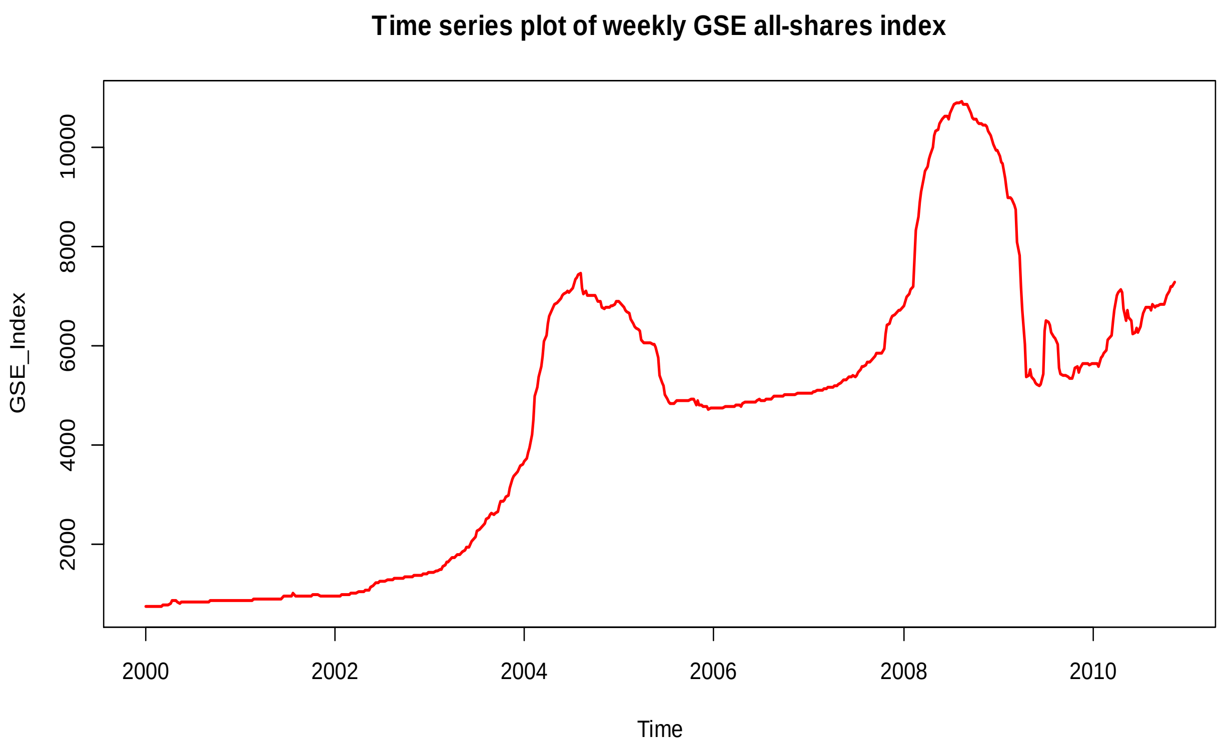


**Figure 1: The GSE All-Shares index evolution from 02/01/00 to 31/12/10**
Source: GSE (2000-2010)

Times series plot of logarithm returns and returns of weekly GSE All-Shares index in Figure 2 illustrates that the Ghana stock market indices are not volatile and dynamic as emerging markets of the developed countries. The plot shows that between 2008 and 2011, the market was a little volatile. The volatility of the actual returns and the logarithm returns is the same as displayed on Figure 2.

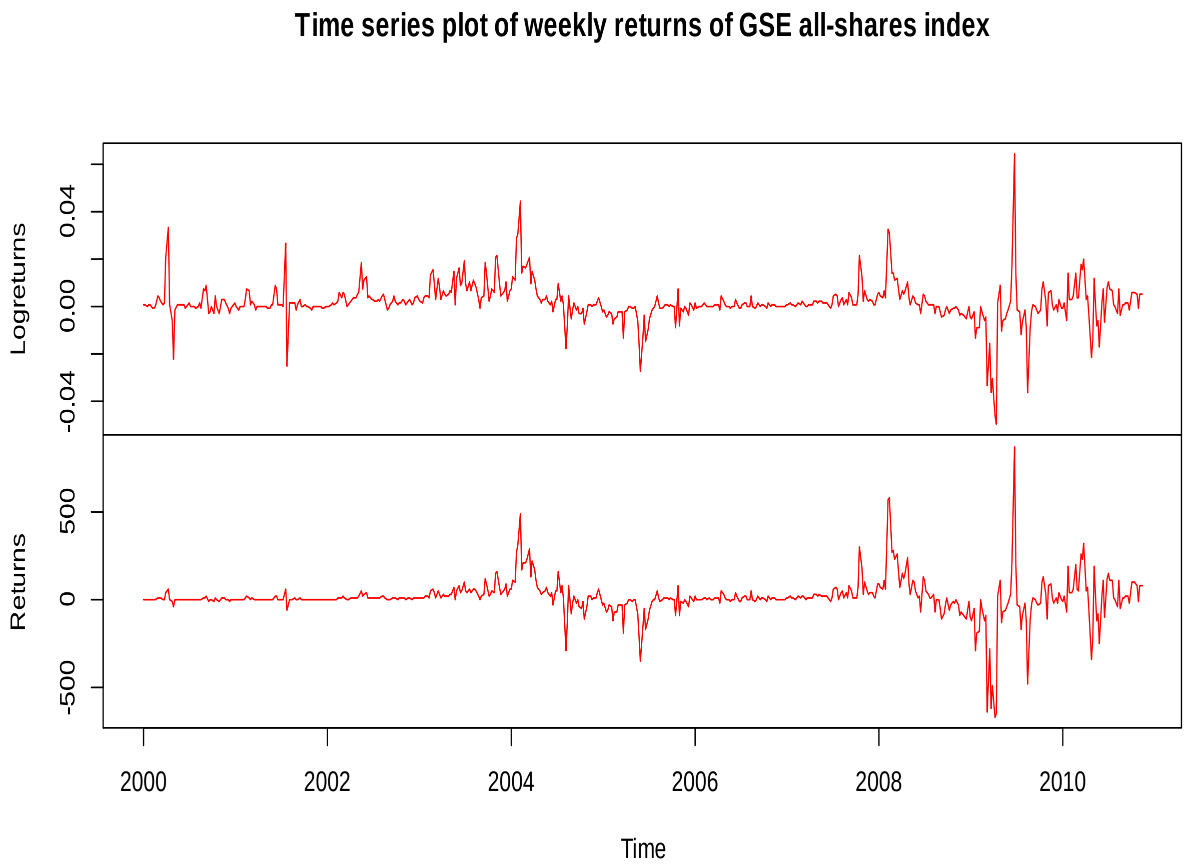


**Figure 2: Logarithm returns of GSE All-Shares index**

Source: GSE (2000-2010)

Figure 3 displays a histogram plot with fitted line of the data and Normal fit to the log returns of the weekly GSE All-Shares index. The plot illustrates that, the log returns is highly picked and heavy tail than the Normal fit which conformed to the studies in the literature that, assets returns are heavy-tailed. The Figure 3 graphically demonstrates that the logarithm returns of GSE All-Shares index are not normally distributed. It is observed that the logarithm returns are in the neighbourhood of zero.

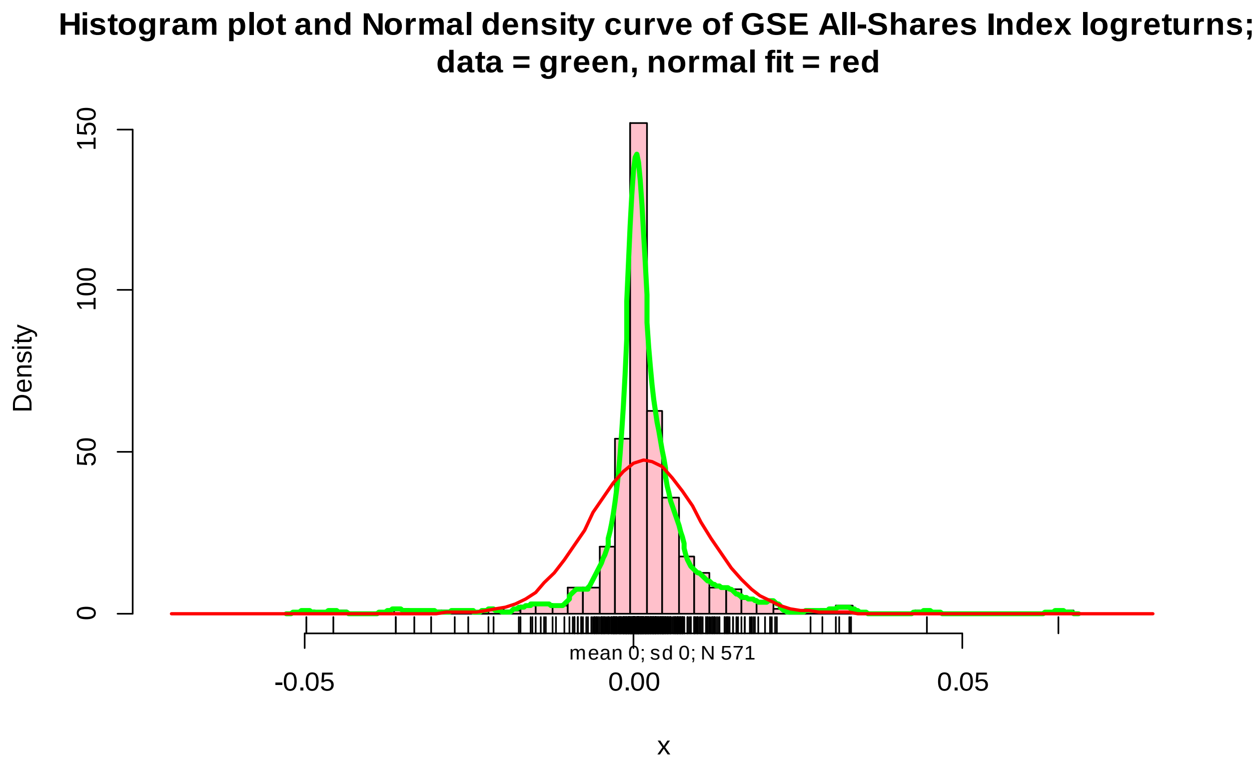


**Figure 3: GSE All-Shares index from 02/01/00- 31/12/10**

Source: GSE (2000-2010)

NB: X denote the weekly logarithm returns of GSE All-Shares index

The density plots in Figure 4 demonstrates that, the $\alpha$ -stable distribution produces the best fit to the logarithm returns of GSE All-Shares index than the normal and Cauchy fits graphically. The $\alpha$-stable fit with parameters $\alpha = 1.005$, $\beta = 0.31$, $\gamma = 0.002$ and $\delta$=0.001 fit graphically best to the data than the Normal and Cauchy distributions. This displays graphically that, the weekly logarithm returns of GSE All-Shares index comes from leptokurtic distribution and heavy-tailed as proposed in the literature by Mandelbrot (1963) that the returns can be modelled by stable distributions with four parameters.

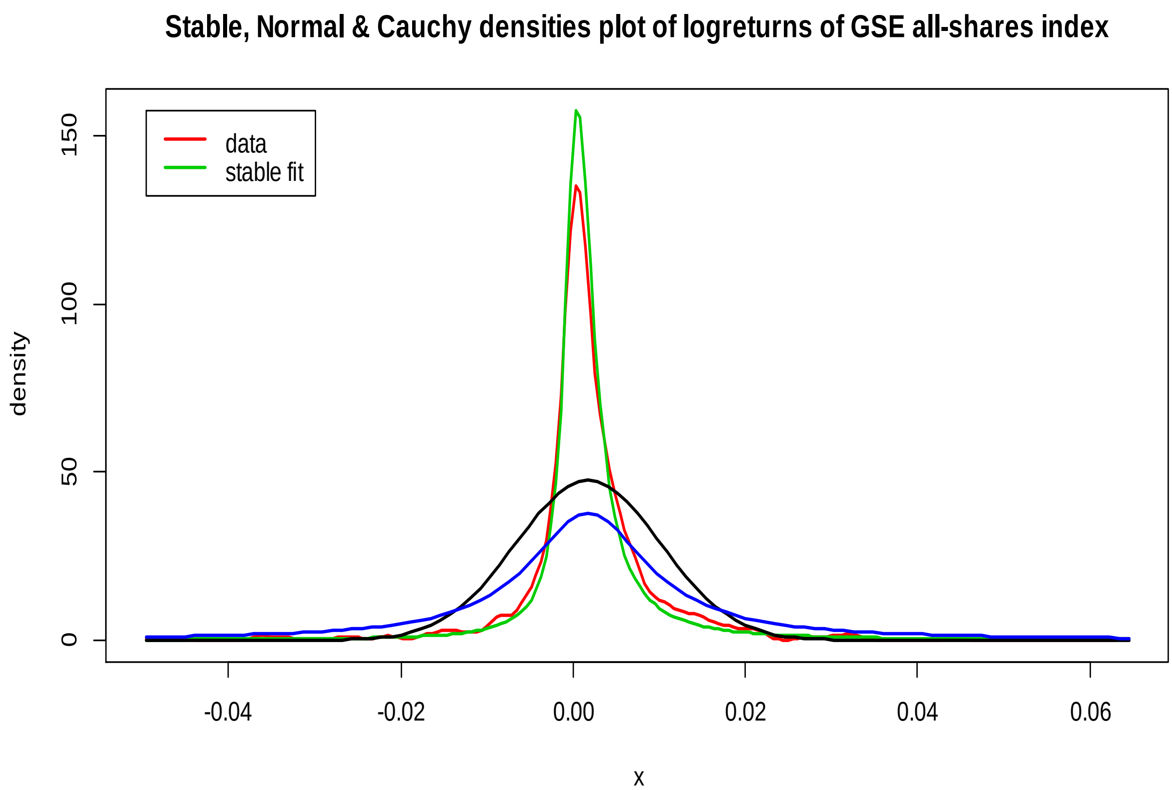


**Figure 4: The density plots of log returns of GSE All-Shares index**

Data = red line, Stable fit = green line, Normal fit = black line and Cauchy fit = blue line

Source: GSE (2000-2010)

NB: X denote the weekly logarithm returns of GSE All-Shares index

The figures (Fig. 5, Fig. 6 & Fig. 7) are Q-Q and P-P plots of the normal distribution and the stable distribution and confirm that the weekly logarithm returns of GSE All-Shares index are not normally distributed but stable distributed with parameters $\alpha = 1.005 \pm 0.104$, $\beta = 0.31 \pm 0.135$, $\gamma = 0.002 \pm 0.0002$ and $\delta = 0.001 \pm 0.0002$. Figure 5 shows deviation of the data from normal fit at the tails, signifying that the GSE All-Shares index returns are heavy tailed. Figure 6 and Figure 7 illustrate a perfect fit of the logarithm returns of GSE All-Shares index to the $\alpha$-stable fit with the stable Q-Q plot displaying few outliers at the tails. The P-P and Q-Q plots of $\alpha$-stable fit illustrate the argument by Nolan (2003) that P-P plot allows comparison over the range of the data while Q-Q plot is not as satisfactory for comparing heavy tailed data and this is revealed by the outliers observed in Figure 6.

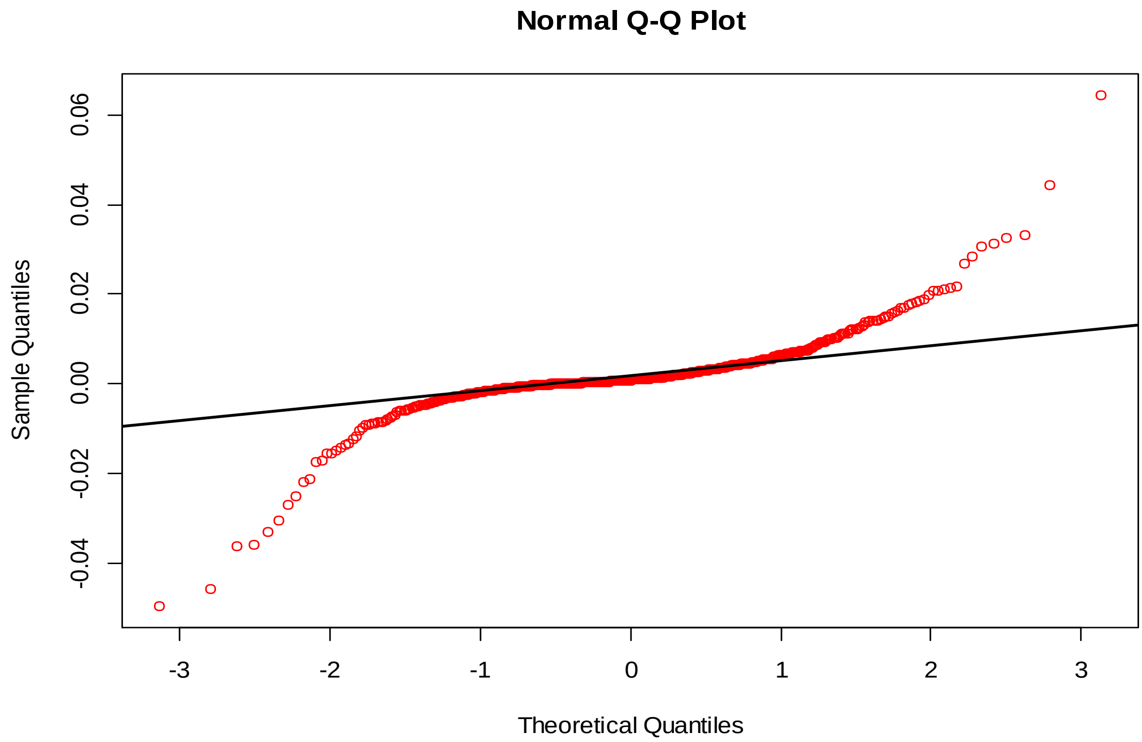


**Figure 5: The Normal Q-Q plot GSE All-Shares index**

Source: GSE (2000-2010)

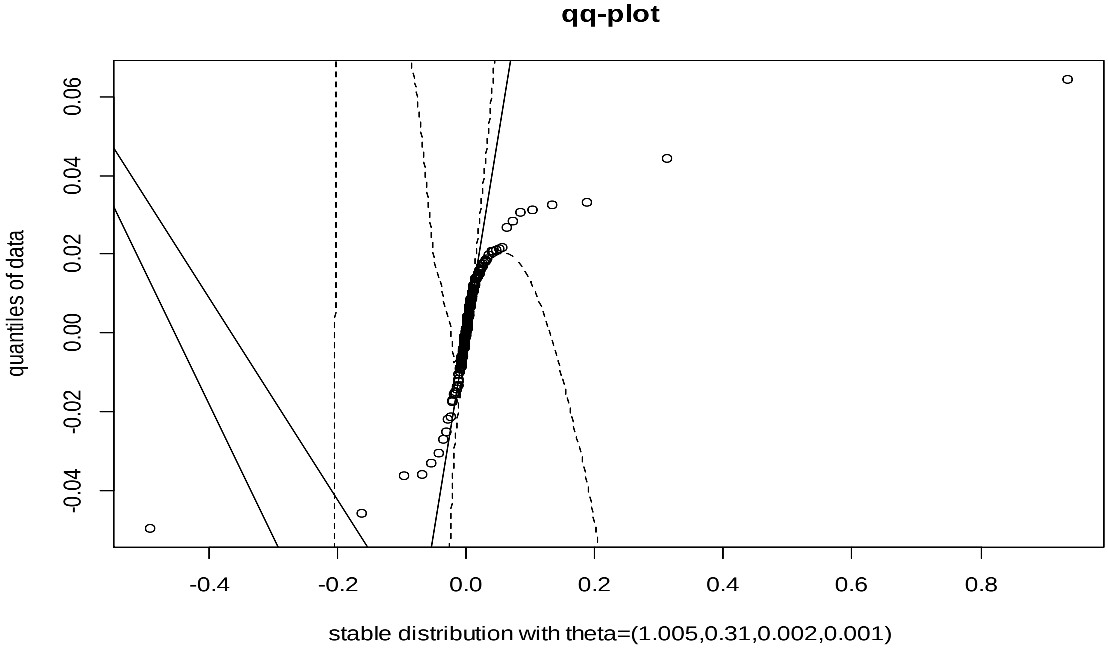


**Figure 6: Stable Q-Q plot of GSE All-Shares index**

Source: GSE (2000-2010)

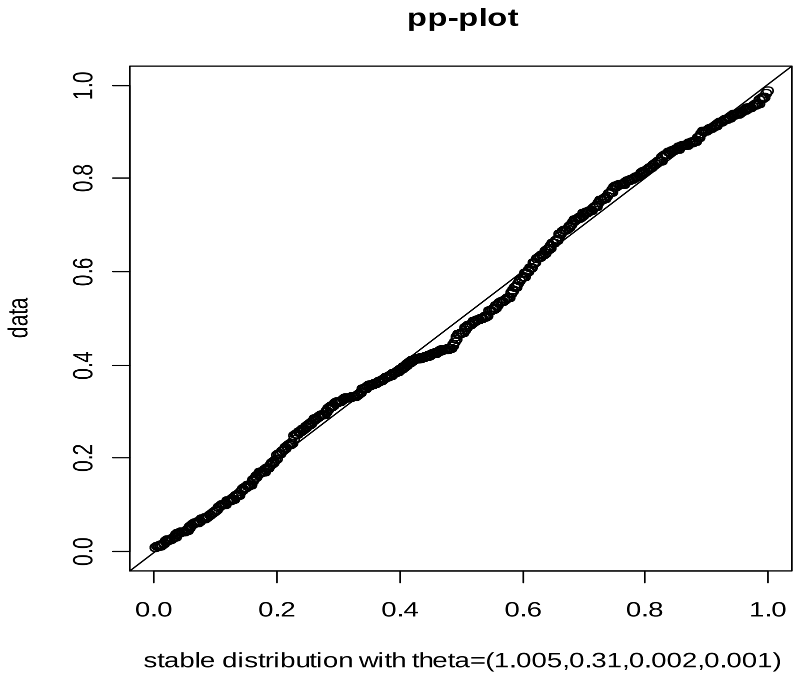


**Figure 7: The Stable P-P plot of GSE All-Shares index**

Source: GSE (2000-2010)

Table 1 displays estimates of the four parameters of the stable fit and 95% confidence interval of the estimates. The index of stability (α) is approximately 1 for the maximum likelihood estimation method while the sample quantile method value of index of stability is 1.124 and empirical characteristic function method produces α as 1.174. The skewness estimates from the three methods are positive and that explains that the returns of GSE index are positively skew. The three methods approximately estimate the same value of the scale parameter ($\gamma$) to be 0.002 and different location parameter ($\delta$).

**Table 1: Estimated parameters of the Stable distribution**

| | Stable Parameters | | | |
|---|---|---|---|---|
| **Methods** | $\hat{\alpha}$ | $\hat{\beta}$ | $\hat{\gamma}$ | $\hat{\delta}$ |
| **MLE** | $1.005 \pm 0.104$ | $0.31 \pm 0.135$ | $0.002 \pm 0.0002$ | $0.001 \pm 0.0002$ |
| **S. Quantile** | $1.124 \pm 0.115$ | $0.196 \pm 0.155$ | $0.0023 \pm 0.0002$ | $0.0007 \pm 0.0003$ |
| **ECF** | $1.174 \pm 0.116$ | $0.399 \pm 0.155$ | $0.0023 \pm 0.0002$ | $0.0009 \pm 0.0003$ |

Source: GSE (2000-2010)

Table 2 illustrates the Kolmogorov-Smirnov and Chi-Square goodness of fit tests of the maximum likelihood estimation, sample quantile and empirical characteristic function methods of estimating parameters of $\alpha$- stable distribution. The K-S test and Chi-square test shows that maximum likelihood estimates of the stable distribution fit perfectly well to the logarithm returns of GSE all-Shares index. The K-S test produces a p-value of 0.138 and Chi-square test p-value is 0.20, which means that, the hypothesis that logarithm returns of GSE All-Shares index is α-stable distributed with maximum likelihood estimation method is not rejected at 10% or 5% significance level. This implies that maximum likelihood estimation method produce more accurate estimates of the data.

**Table 2: Goodness of fit tests**

| | Statistical Tests | |
|---|---|---|
| **Methods** | **K-S** | **Chi-Square** |
| **MLE** | $D = 0.048$<br>$\mathrm{p} - value = 0.138$ | $\chi^2 = 3.28$<br>$\mathrm{p} - value > 0.20$ |
| **Sample Quantile** | $D = 0.084$<br>$\mathrm{p} - value = 0.0006$ | $\chi^2 = 21.36$<br>$\mathrm{p} - value < 0.005$ |
| **ECF** | $D = 0.059$<br>$\mathrm{p} - value = 0.035$ | $\chi^2 = 6.75$<br>$\mathrm{p} - value > 0.10$ |

Source: GSE (2000-2010)

The K-S and the Chi-square goodness of fit tests both rejected the hypothesis that the logarithm returns of GSE all-shares index is α-stable distributed with sample quantile estimates of stable parameters. Both goodness of fit test estimated an approximate p-value of 0.0, which means that α-stable fit with sample quantile estimates does not fit successfully to the GSE All-Shares index. The rejection of the sample quantile method implies that the estimates are not accurate for modelling log returns of GSE All-Shares index data. The K-S test rejected α-stable fit to the logarithm returns of GSE All-Shares index with empirical characteristic function estimation method at 5% significance level. But the Chi-square goodness of fit test says otherwise. The Chi-square test estimates a p-value of approximately 0.10, which means that the empirical characteristic function estimation method of fitting the log returns of GSE All-Shares index to α-stable fit is not significant at 5% significance level and the hypothesis is not rejected. The table 2 revealed that the MLE method produces the best and most accurate estimates and next to it is empirical characteristic function estimates and the sample quantile method produces the worst estimates as indicated in the methodology.

Table 3 demonstrates the goodness of fit of the logarithm returns of GSE All-Shares index to the three distributions under study. The four tests for normality considered in this study (Kolmogorov-Smirnov, Chi-square, Anderson-Darling and Shapiro-Wilk) all rejected the claim that logarithm returns of GSE All-Shares index is normally distributed at even 1% significance level. Since Anderson-Darling and Shapiro-Wilk tests cannot be used to test goodness of fit for the Cauchy and the α-stable distributions, only K-S and Chi-square tests are employed. The K-S test and Chi-square test estimated p-values of 0.0 and 0.017 respectively for the Cauchy fit. This implies that both tests (K-S and Chi-square) rejected the claim that logarithm returns of GSE All-Shares index is Cauchy distributed.

**Table 3: Goodness of fit tests for Normal, Cauchy and Stable distributions**

| **Cases** | **K-S** | **Chi-Square** | **Anderson-Darling** | **Shapiro-Wilk** |
|---|---|---|---|---|
| **Normal fit** | $D = 0.18$<br>p-$value = 0.0$ | $\chi^2 = 408.26$<br>p-$value < 0.005$ | $A^2 = 36.78$<br>p-$value < 0.005$ | $W = 0.78$<br>p-$value = 0.0$ |
| **Cauchy fit** | $D = 0.486$<br>p-$value = 0.0$ | $\chi^2 = 32432$<br>p-$value = 0.017$ | | |
| **Stable fit** | $D = 0.048$<br>p-$value = 0.138$ | $\chi^2 = 3.28$<br>p-$value > 0.20$ | | |

Source: GSE (2000-2010)

The K-S and Chi-square goodness of fit tests consistently fail to reject the claim that logarithm returns of GSE all-shares index is α-stable distributed at 5% or 10% significance level. Both tests estimated the p-values as 0.138 for K-S test and approximately 0.20 for Chi-square test. These results conformed to the results obtained by Alfonso et al. (2011) and Xu et al. (2011) that assets returns are α-stable distributed and the two goodness of fit tests agreed with the graphical tests in Figures 4, 6 and 7.

## 4.2 Currency Exchange Rates

This section investigates the return distribution of the three major exchange rates (US dollar to Ghana cedi, Britain pounds sterling to Ghana cedi and European Euro to Ghana cedi). The sample sizes of the three exchange rates are as follows; 02/01/01 to 31/12/11 of 524 weekly data points for both US dollar and British Pounds to the Ghana cedi and 02/01/02 to 31/12/11 of 474 data points for the European Euro to the Ghana cedi. The weekly exchange rates are used due to the fact that the daily exchange rates are not volatile enough to be modelled with $\alpha$-stable distributions. The weekly logarithm returns was calculated using equation (4.01) in section 4.1.

The figures (Fig.8 - Fig.13) display time series plots of the respective exchange rates. Figure 8 illustrates a progressive weekly USD/GHC exchange rate over the period of study and it shows that, there was steep increase between 2008 and 2009. This may be due to the 2008 general election held in December which led to high inflation indices.

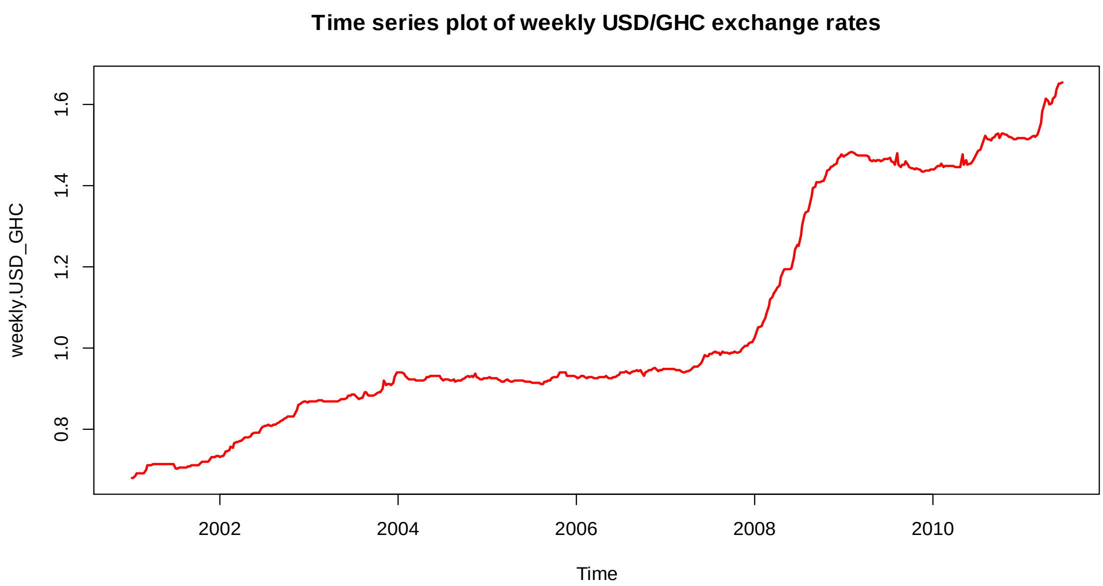


**Figure 8: USD/GHC progression for the 11 years period**

Source: Bank of Ghana (2001-2011)

Figure 9 also demonstrates that, there were greater positive returns than negative returns over the 11 years period of study and in risk management it is good to have equal percentage of positive and negative returns. It shows that the volatility of the weekly USD/GHC exchange rates exhibits large extreme returns as compared to the developed markets exchange rates in the literature. The simple returns and the logarithm returns of USD/GHC exchange rates are almost the same in terms of volatility displayed in Figure 9.

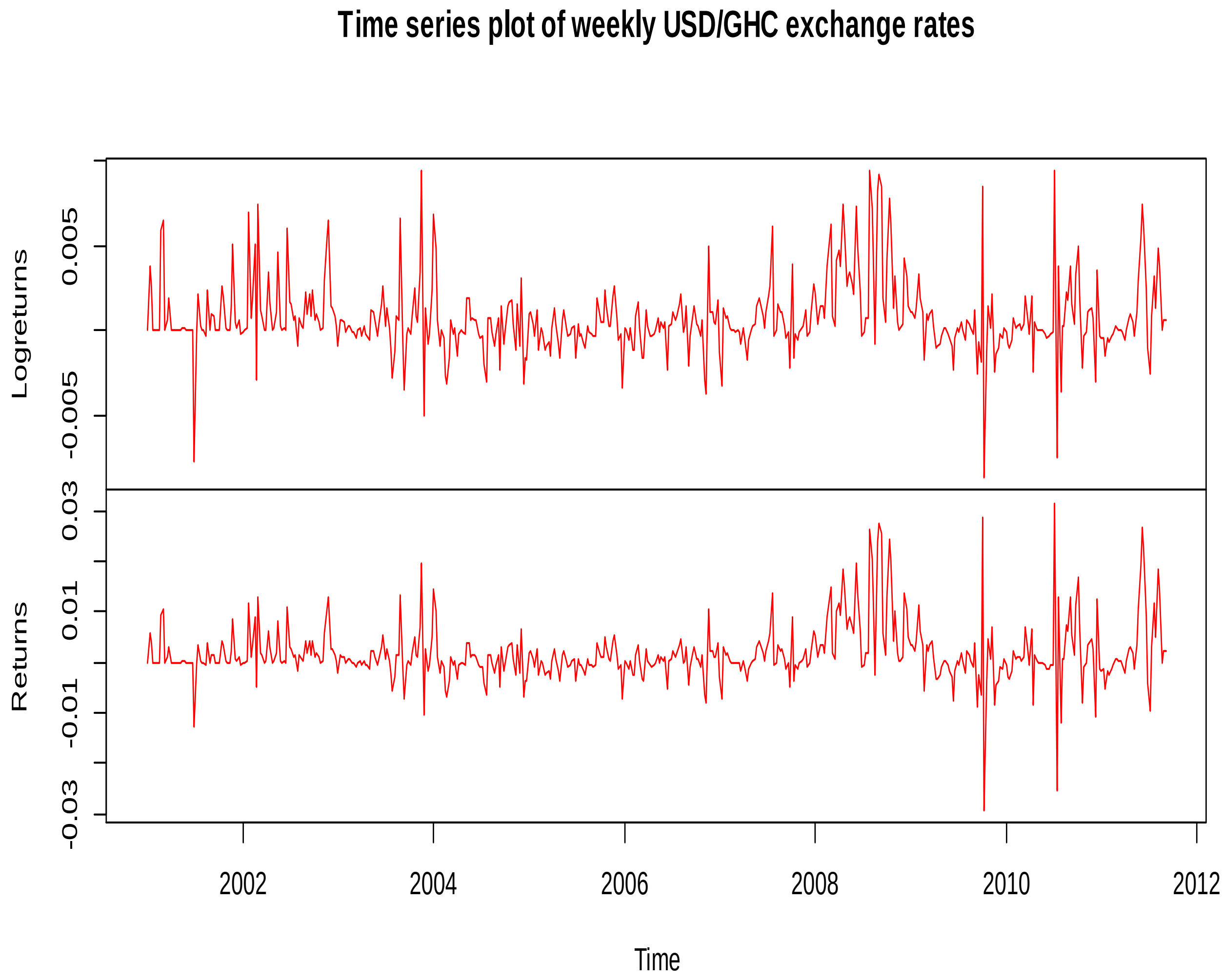


**Figure 9: USD/GHC of log returns of volatility over the 11 years period**

Source: Bank of Ghana (2001-2011)

Figure 10 displays the progressive growth of the 11 years weekly British pound sterling to Ghana cedi starting from 02/01/2001-31/12/2011. The graph shows a continuous increase over the period with fluctuations than the USD/GHC exchange rate.

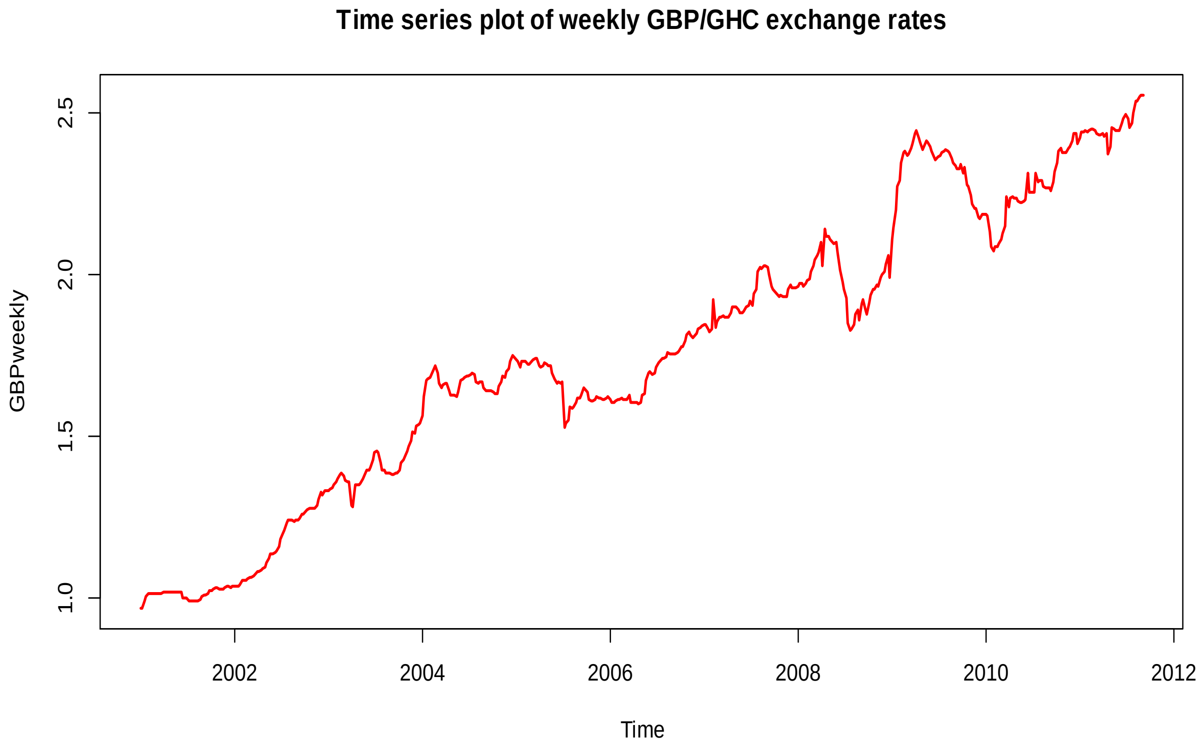


**Figure 10: GBP/GHC progression for the 11 years period**

Source: Bank of Ghana (2001-2011)

The log-returns and simple returns of the GBP/GHC weekly exchange rates are shown graphically on Figure 11 of the entire period of study; 2/01/01-31/12/11. It is observed that logarithm returns of GBP/GHC weekly exchange rates are uniformly distributed on average between -0.01 to 0.01 with exception of some few extreme returns which may be due to the inefficiency of the Ghanaian market.


**Figure 11: GBP/GHC of log returns of volatility over the 11 years period**

Source: Bank of Ghana (2001-2011)

Figure12 displays the evolution of the 10 years weekly European Euro to Ghana cedi. The graph shows a continuous increased with fluctuations over the period with approximately the same trend as the GBP/GHC exchange rate.

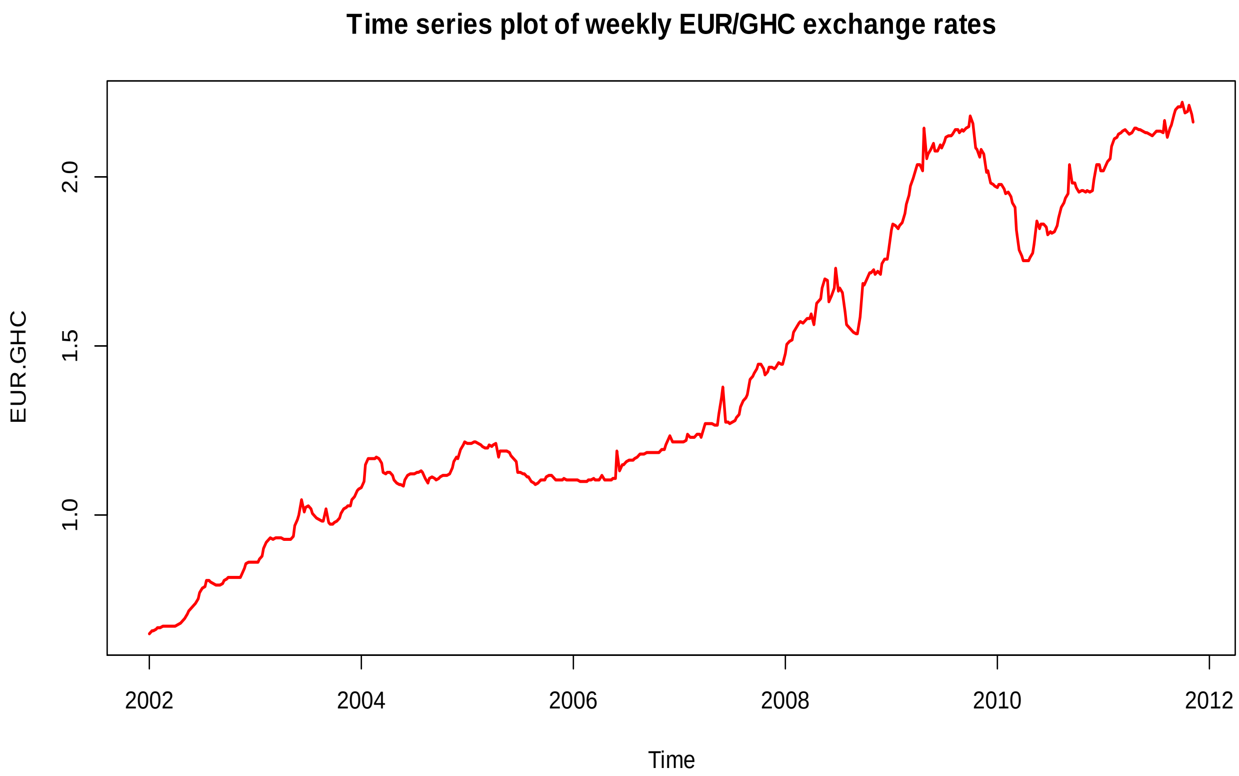


**Figure 12: EUR/GHC progression for the 10 years period**

Source: Bank of Ghana (2002-2011)

The weekly EUR/GHC exchange rate returns of the actual returns and log-returns are shown graphically on Figure 13 and demonstrating an average logarithm returns between -0.015 to 0.015 over the entire period of study; 2/01/02-31/12/11. The simple and logarithm returns of the euro exchange rate parades extreme returns equal to the returns of British pounds exchange rate.


**Figure 13: EUR/GHC log returns of volatility over the 10 years period**

Source: Bank of Ghana (2002-2011)

Figure 14 displays a histogram plot with fitted line of the data and Normal fit to the log returns of the weekly USD/GHC exchange rate. The plot illustrates that, the log returns is of USD/GHC exchange rate is more picked and heavy tail than the Normal fit and this parades that the weekly USD/GHC exchange rate log returns come from a leptokurtic distribution family.

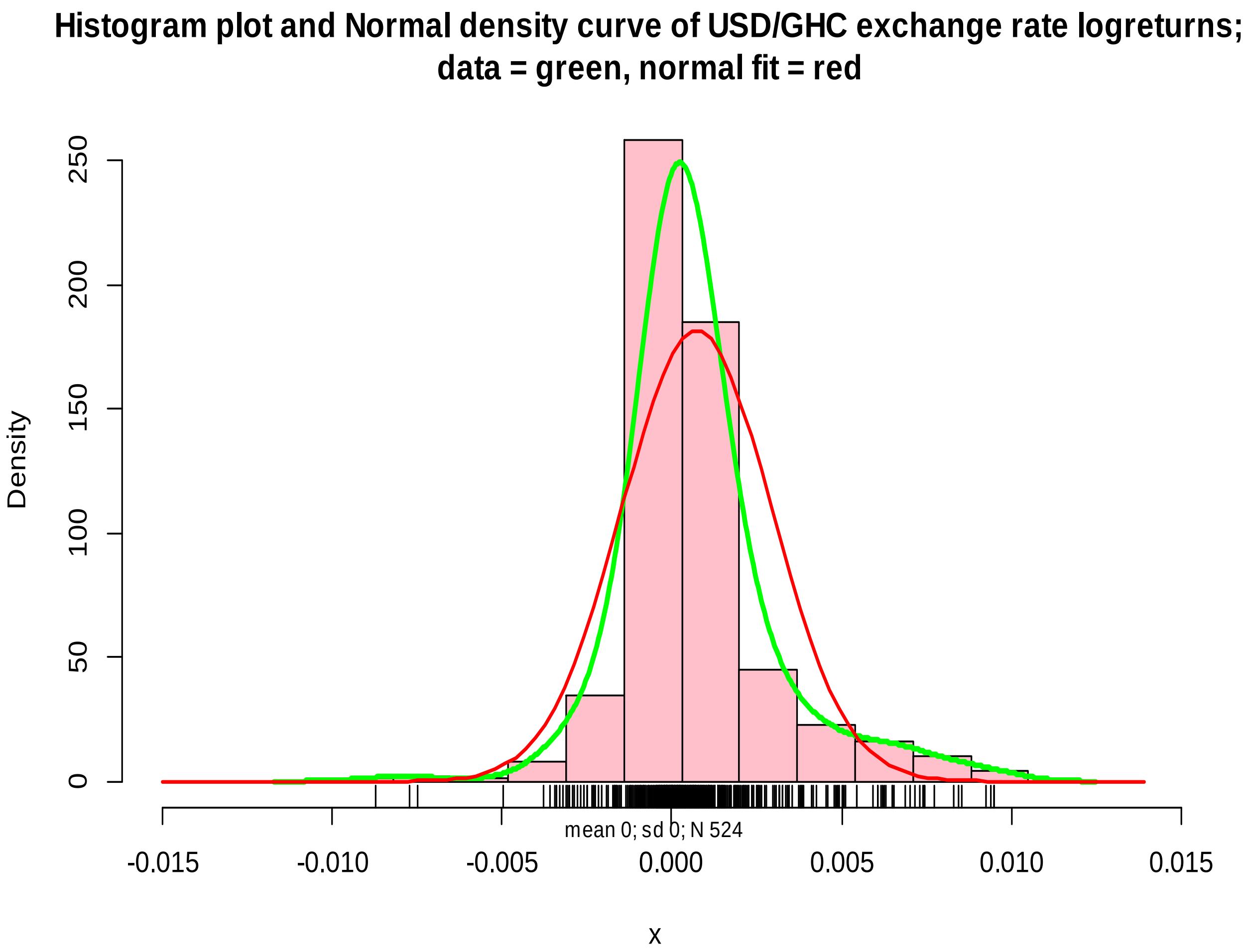


**Figure 14: Histogram plot of log returns of USD/GHC**

Source: Bank of Ghana (2001-2011)

NB: X denote the weekly logarithm returns of USD/GHC exchange rate

Figures 15 and 16 also demonstrate that, the weekly logarithm returns of GBP/GHC and EUR/GHC exchange rates do not fit to the normal distribution but come from a leptokurtic distribution family like the USD/GHC exchange rate logarithm returns.

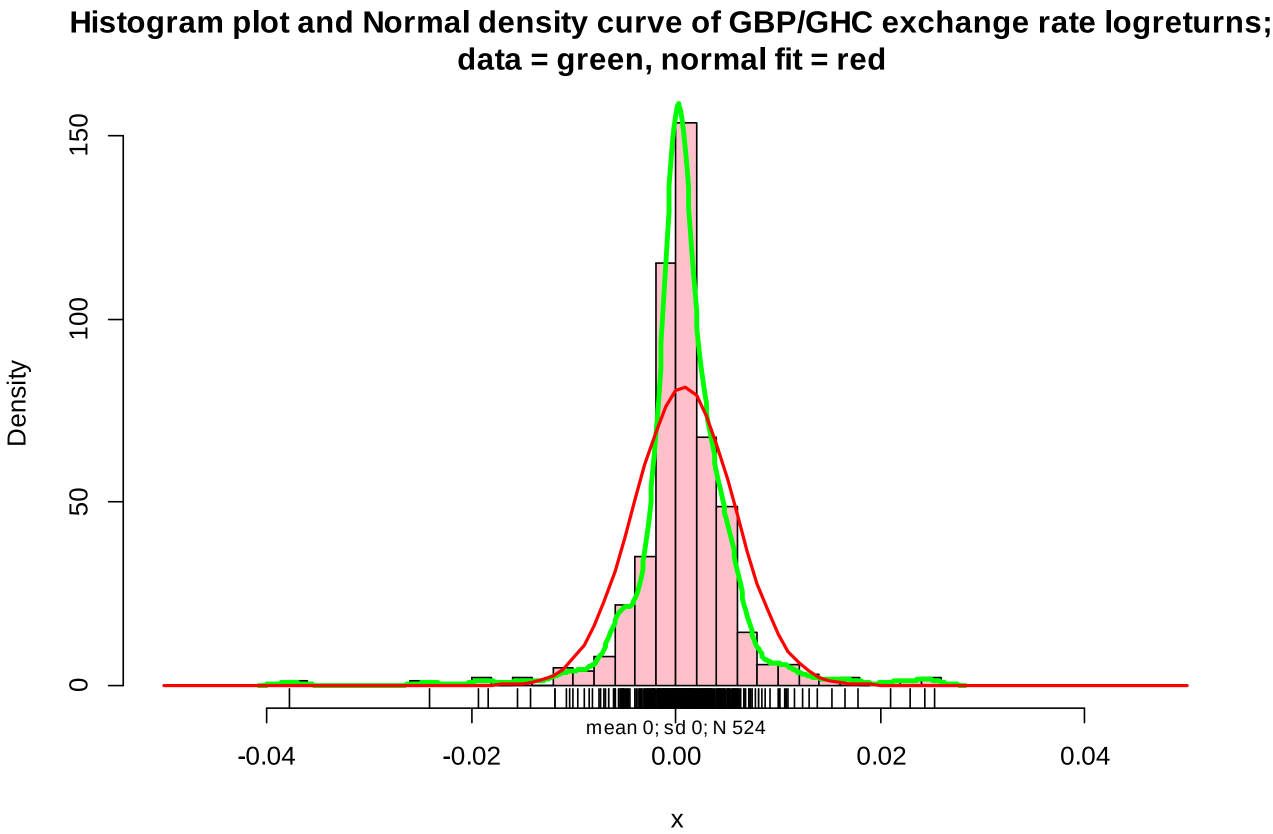


**Figure 15: Histogram plot of log returns of GBP/GHC**

Source: Bank of Ghana (2001-2011)

NB: X denote the weekly logarithm returns of GBP/GHC exchange rate

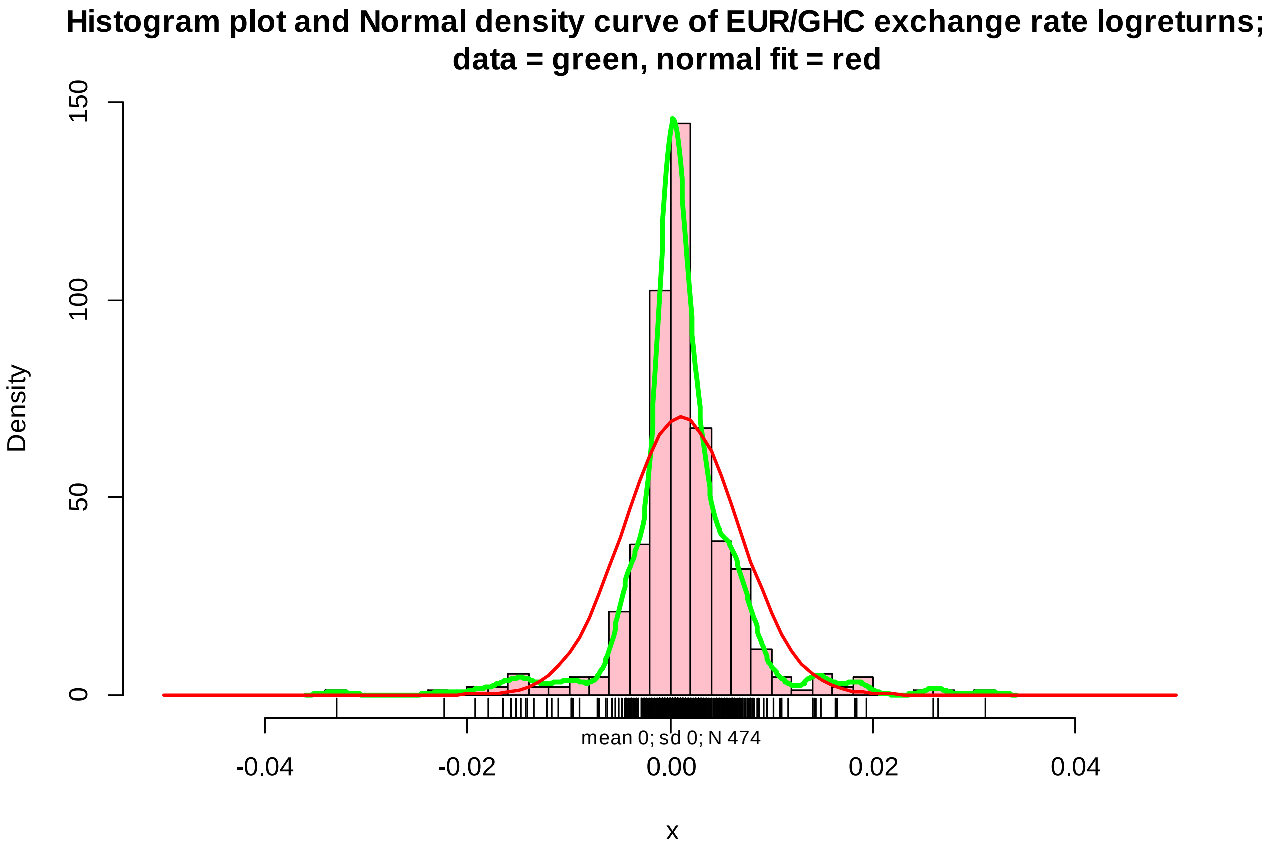


**Figure 16: Histogram plot logarithm returns of EUR/GHC**

Source: Bank of Ghana (2002-2011)

NB: X denote the weekly logarithm returns of EUR/GHC exchange rate

The figures 17, 18 and 19 illustrate the density plots of stable, normal and Cauchy fits to the weekly logarithm returns of the three exchange rates understudy. The three density plots clearly demonstrate graphically that, the return distribution of the three exchange rates are neither normally distributed nor Cauchy distributed but are $\alpha$-stable distributed. The $\alpha$-stable fitted well to the three exchange rates with the following parameters; USD/GHC exchange rate estimates are: $\alpha = 1.073$, $\beta = 0.394$, $\gamma = 0.001$ and $\delta$=0.0, GBP/GHC exchange rate estimates are: $\alpha = 1.289$, $\beta = 0.173$, $\gamma = 0.002$ and $\delta$=0.0 and EUR/GHC exchange rate estimates are: $\alpha = 1.236$, $\beta = 0.234$, $\gamma = 0.002$ and $\delta$=0.001.

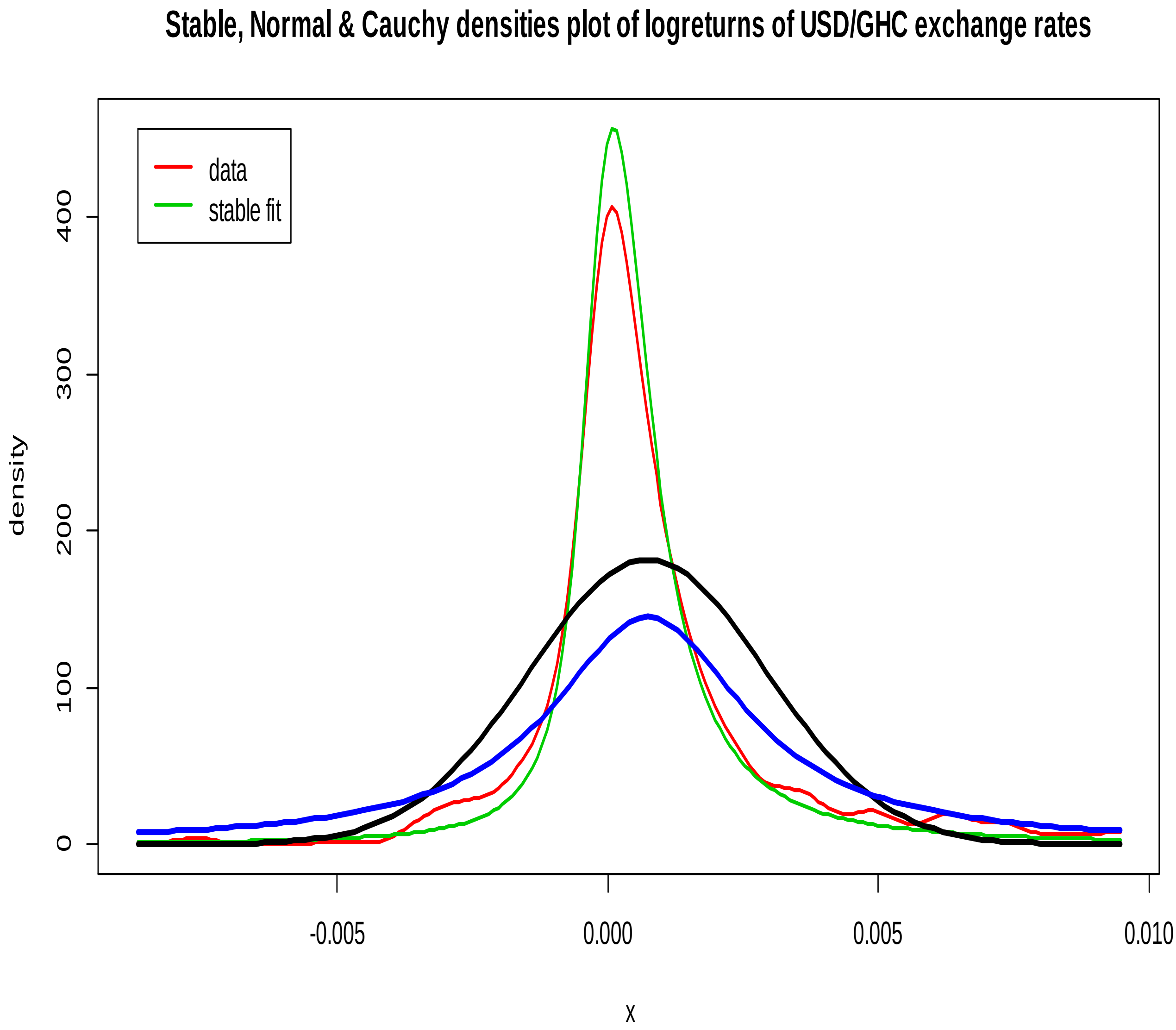


**Figure 17: The density plots of log returns of the data**
USD/GHC = red line, Stable fit = green line, Normal fit = black line and Cauchy fit = blue line;
Source: Bank of Ghana (2001-2011)

NB: X denote the weekly logarithm returns of USD/GHC exchange rate

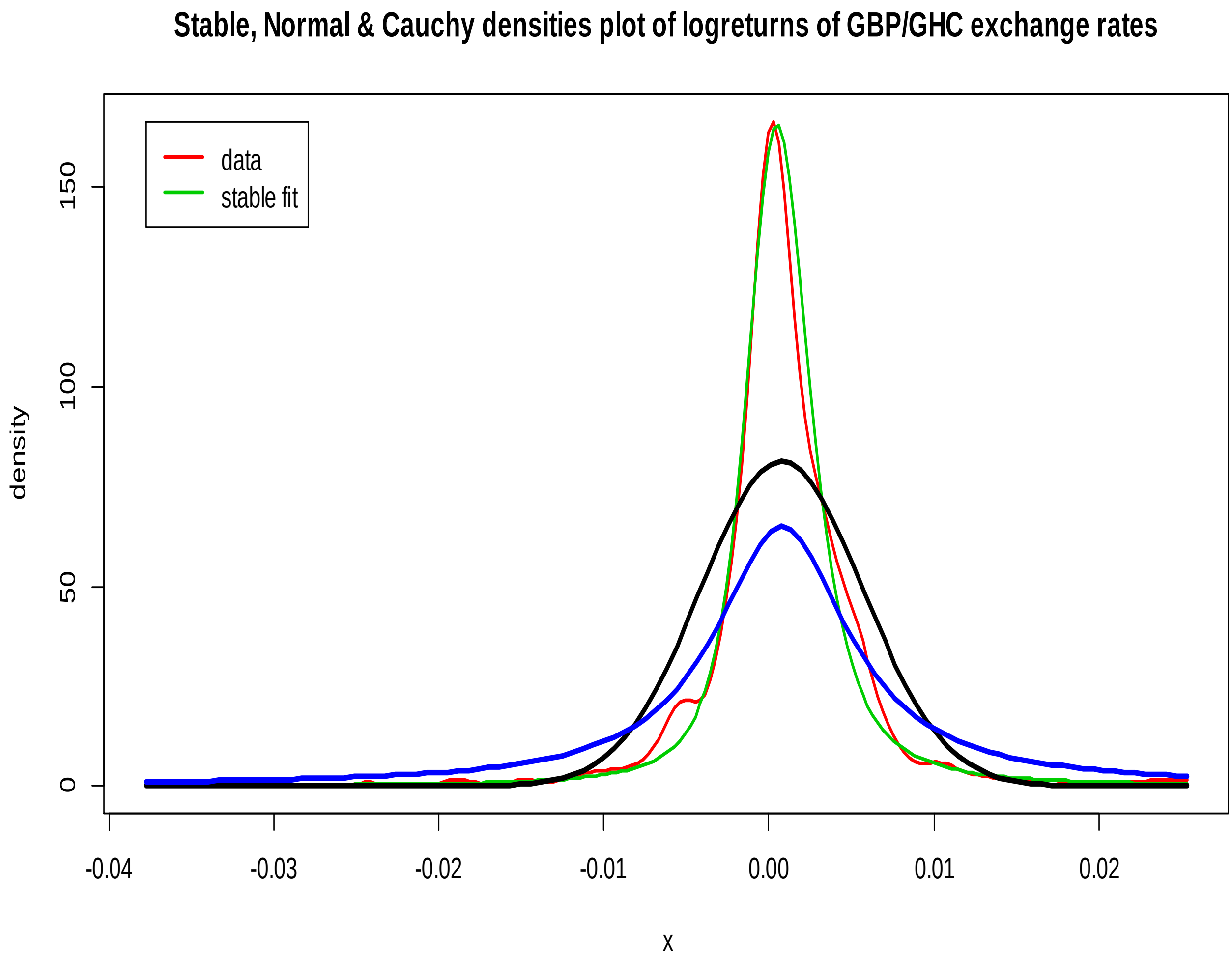


**Figure 18: The density plots of log returns of the data**
GBP/GHC = red line, Stable fit = green line, Normal fit = black line and Cauchy fit = blue line;
Source: Bank of Ghana (2001-2011)

NB: X denote the weekly logarithm returns of GBP/GHC exchange rate

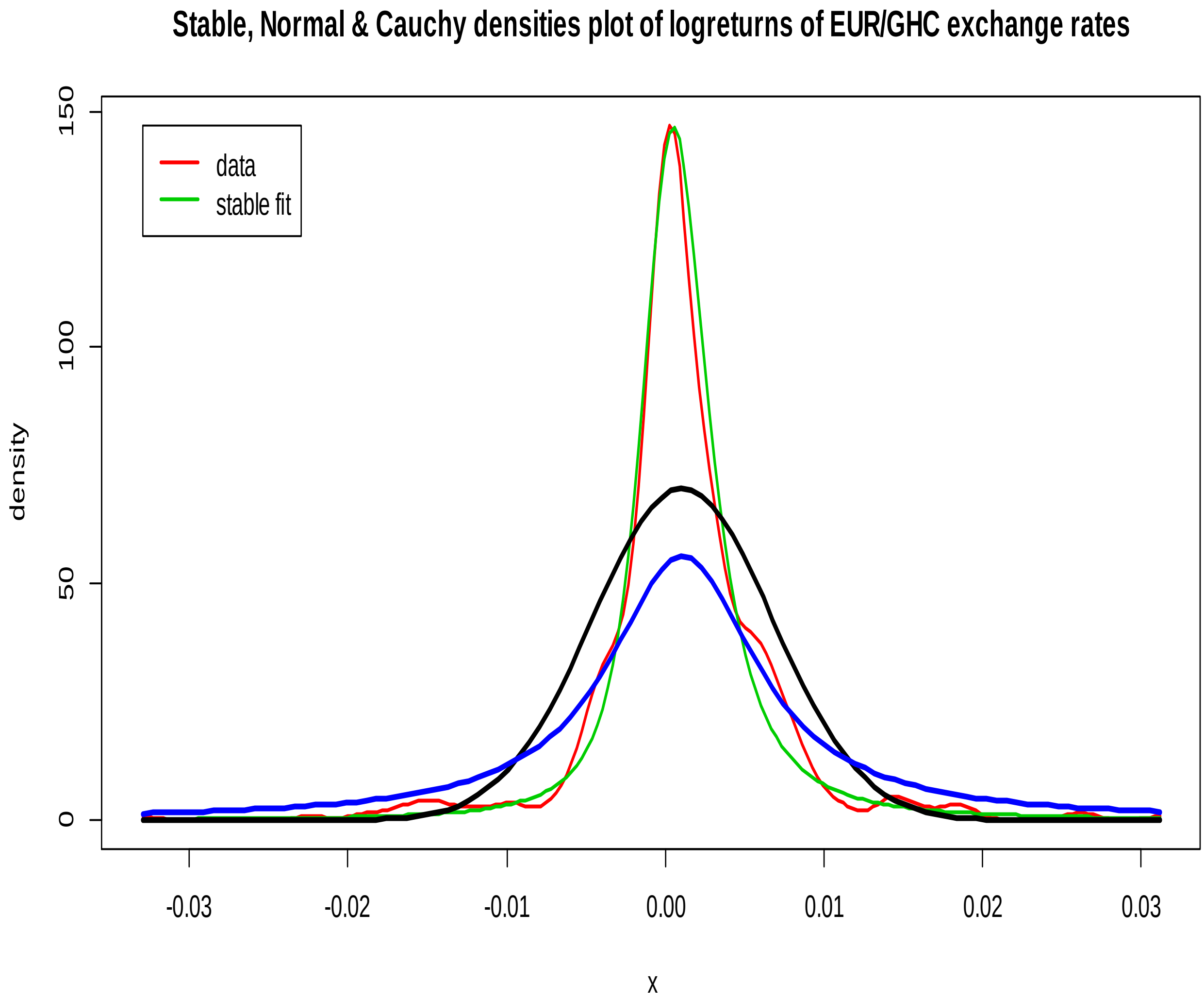


**Figure 19: The density plots of log returns of the data**
EUR/GHC = red line, Stable fit = green line, Normal fit = black line and Cauchy fit = blue line;
Source: Bank of Ghana (2002-2011)

NB: X denote the weekly logarithm returns of EUR/GHC exchange rate

Figure 20 displays the normal Q-Q plot of log-returns of USD/GHC exchange rate with large deviation away from the normal distribution. The normal q-q plot shows a significant deviation away from the tails of the theoretical quantiles of the normal fit and this revealed that the USD/GHC weekly log-returns are not normally distributed.

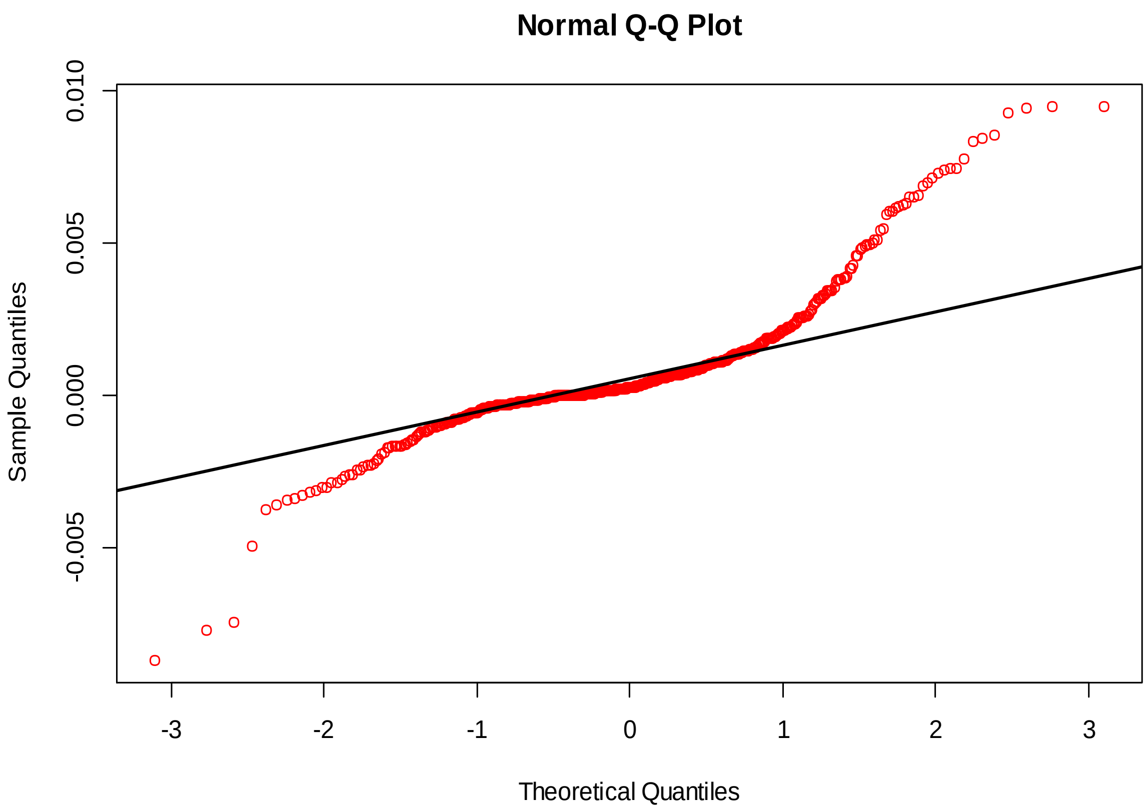


**Figure 20 : Normal Q-Q plot of USD/GHC exchange rate**
Source: Bank of Ghana (2001-2011)

Figure 21 illustrates the $\alpha$-stable fit diagnostic tests for log-returns of the weekly USD/GHC exchange rate. Figure 21 displays four graphs of $\alpha$-stable fit namely; density plot of $\alpha$-stable fit to the data, Q-Q plot, P-P plot and Z-Z plot. The α-stable Q-Q plot illustrates a perfect fit of the quantiles of the weekly USD/GHC log-returns to the quantiles of the theoretical α-stable fit but with few outliers at the tails. The $\alpha$-stable P-P plot displays a perfect fit of the probabilities of the USD/GHC weekly log returns to the theoretical probabilities of the α-stable distribution and finally the Z-Z plot also demonstrates a perfect fit of the inverse cumulative distribution of α-stable fit of the data to the theoretical inverse of normal cumulative distribution plotted on both axes of the P-P plot.

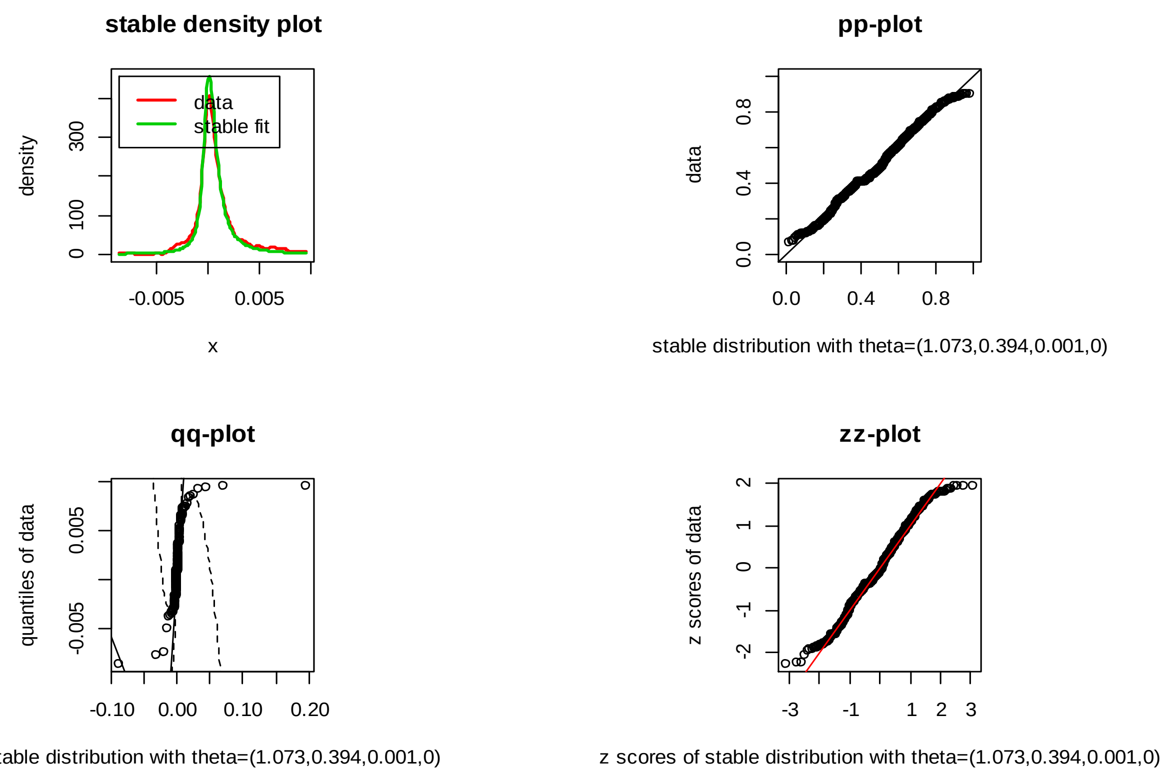


**Figure 21: The diagnostics tests of USD/GHC data**

Source: Bank of Ghana (2001-2011)

NB: X denote the weekly logarithm returns of USD/GHC exchange rate

Figure 22 displays the normal Q-Q-plot for the weekly log returns of GBP/GHC exchange rate showing deviation away from the normal distribution. The normal Q-Q plot shows a significant deviation away from the tails of the theoretical quantiles of the normal fit but forming a good fit at the centre of the theoretical quantiles. This revealed that the GBP/GHC weekly log-returns deviated away from the normally distribution but the average log-return will be equal to the normal distribution estimate.

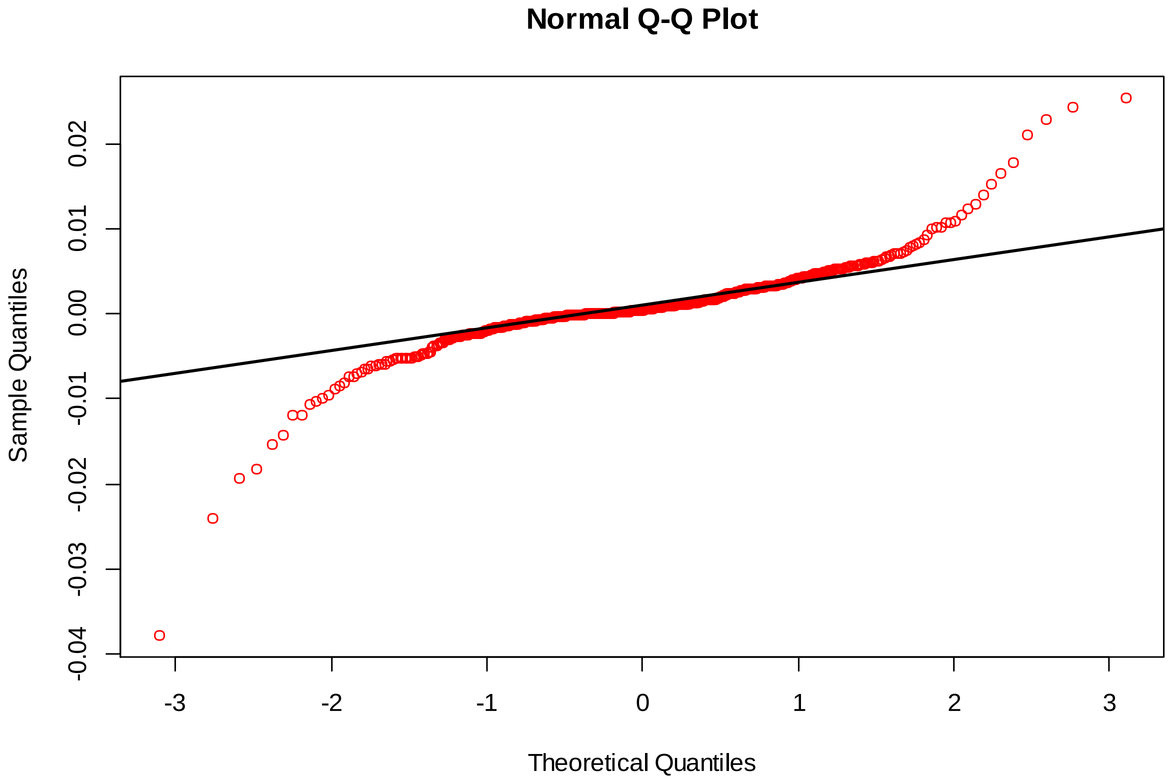


**Figure 22: Normal Q-Q plot of GBP/GHC exchange rate**
Source: Bank of Ghana (2001-2011)

Figure 23 displays the four $\alpha$-stable fit diagnostic tests for the weekly log returns of GBP/GHC exchange rate. The density plot of $\alpha$-stable fit to the data, Q-Q plot, P-P plot and Z-Z plot attest to the fact that weekly log-returns of GBP/GHC exchange rate perfectly fit to α-stable distribution but not normal distribution.

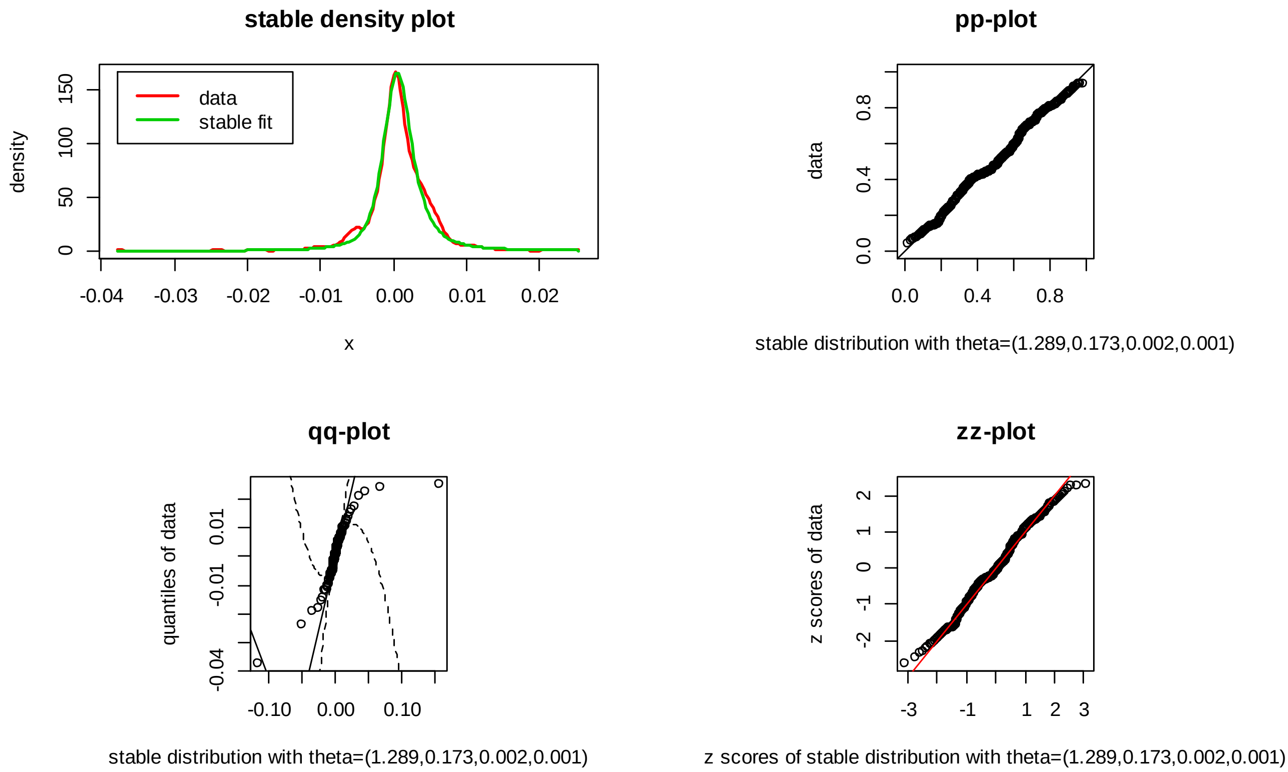


**Figure 23: The diagnostics tests of GBP/GHC data**

Source: Bank of Ghana (2001-2011)

NB: X denote the weekly logarithm returns of GBP/GHC exchange rate

Figure 24 parades the normal Q-Q plot for the weekly log returns of EUR/GHC exchange rate showing a significant deviation from the tails of the theoretical quantiles of the normal distribution. The normal Q-Q plot shows a good fit at the centre of theoretical quantiles of the normal fit and this means that the average log return will be the same as the mean estimate computed from the normal distribution. The Q-Q plot shows that the log-returns of EUR/GHC weekly exchange rates are not normally distributed.

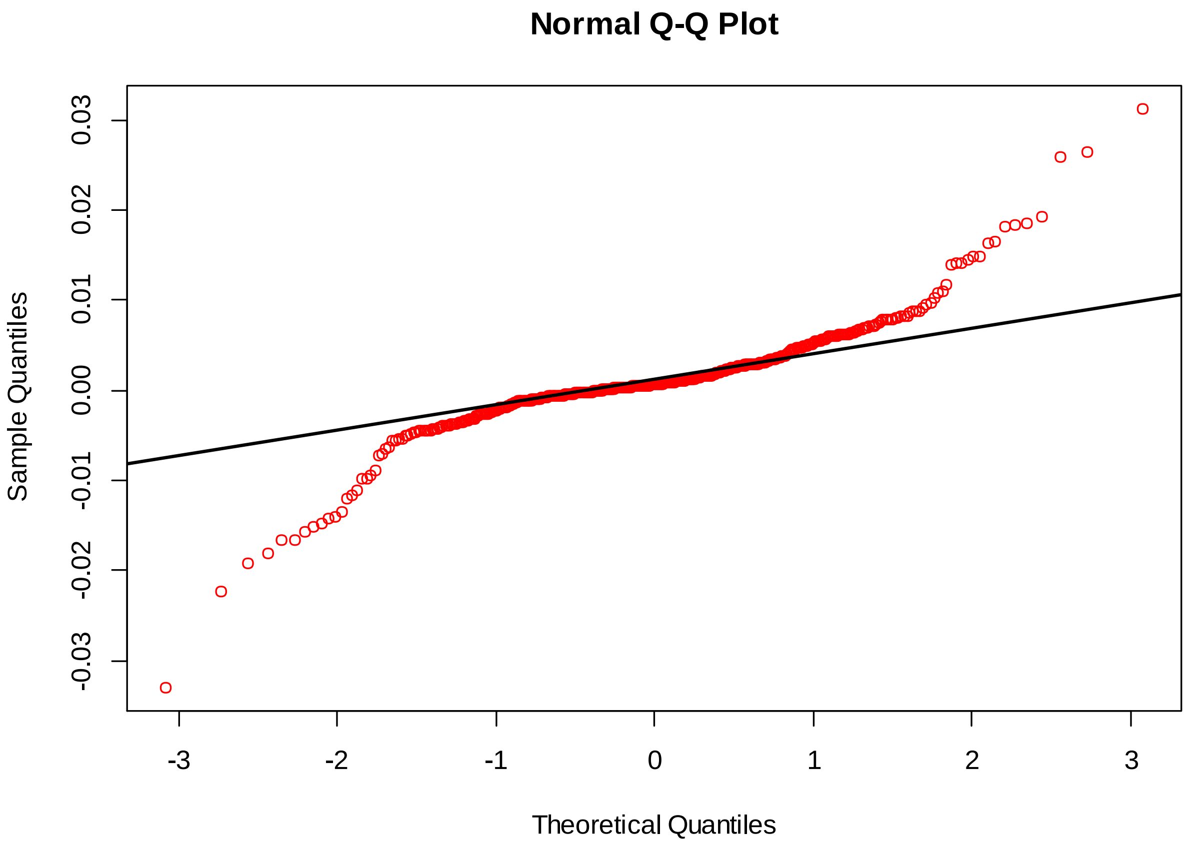


**Figure 24: Normal Q-Q plot of EUR/GHC exchange rate**
Source: Bank of Ghana (2002-2011)

Figure 25 displays the four $\alpha$-stable fit diagnostic tests for the weekly log-returns of EUR/GHC exchange rate. The density plot of $\alpha$-stable fit to the data, Q-Q plot, P-P plot and Z-Z plot attest to the fact that weekly log-returns of EUR/GHC exchange rate perfectly fit to α-stable distribution but not normal distribution. The Q-Q plot displays outliers at the tails of the theoretical quantiles of $\alpha$-stable distribution, showing the demerit of Q-Q plot indicated by Nolan (2003).

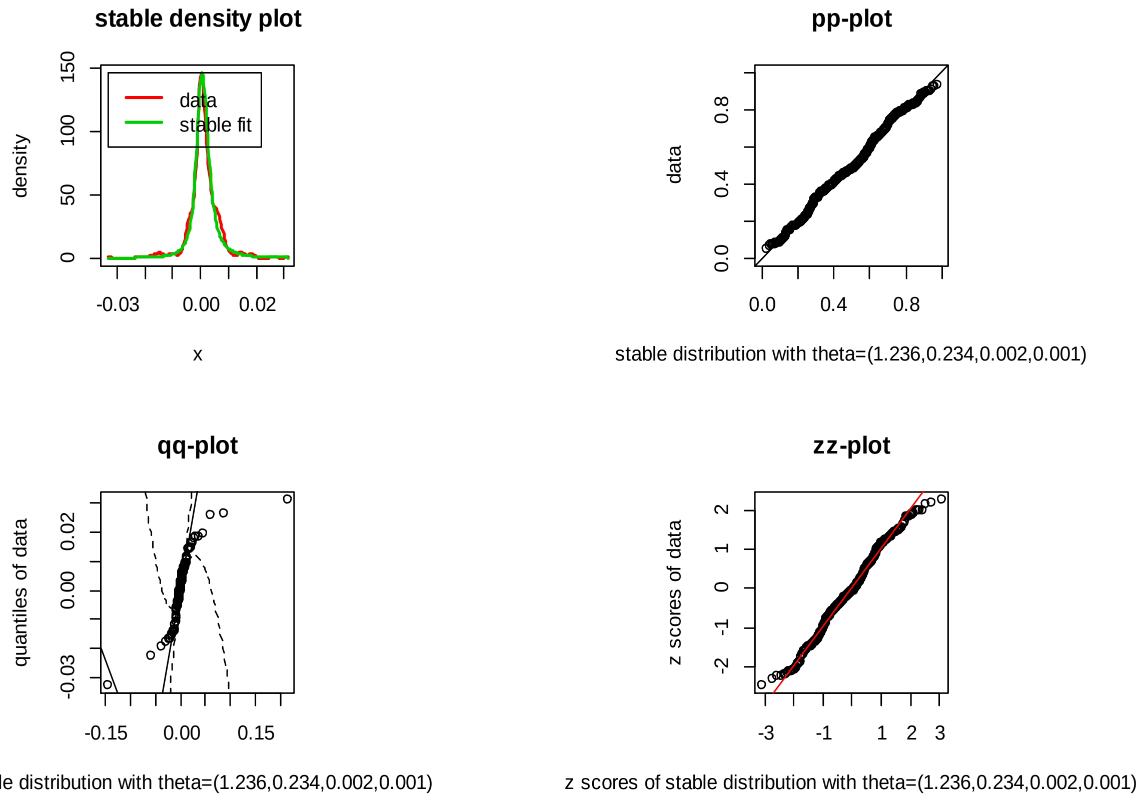


**Figure 25: The diagnostics tests of EUR/GHC data**

Source: Bank of Ghana (2002-2011)

NB: X denote the weekly logarithm returns of EUR/GHC exchange rate

Tables 4, 5 and 6 display estimates of the four parameters of stable fit and 95% confidence interval of the estimates using the three methods (Maximum likelihood estimation, Sample quantile and Empirical Characteristic function) methods of estimating the $\alpha$-stable parameters. Table 4 displays $\alpha$-stable fit estimates of USD/GHC exchange rate, Table 5 displays $\alpha$-stable fit estimates of USD/GHC exchange rate and Table 6 shows $\alpha$-stable fit estimates of EUR/GHC exchange rate.

**Table 4: Estimated parameters of the Stable distribution**

| USD/GHC Exchange Rate Stable Parameters | | | | |
|---|---|---|---|---|
| **Methods** | $\hat{\alpha}$ | $\hat{\beta}$ | $\hat{\gamma}$ | $\hat{\delta}$ |
| **MLE** | $1.073 \pm 0.114$ | $0.394 \pm 0.147$ | $6.6*10^{-4} \pm 7.62*10^{-5}$ | $2.3*10^{-4} \pm 8.9*10^{-5}$ |
| **S. Quantile** | $1.08 \pm 0.12$ | $0.396 \pm 0.147$ | $6.9*10^{-4} \pm 7.87*10^{-5}$ | $1.4*10^{-4} \pm 9.3*10^{-5}$ |
| **ECF** | $1.26 \pm 0.13$ | $0.205 \pm 0.186$ | $7.8*10^{-4} \pm 8.1*10^{-5}$ | $2.5*10^{-4} \pm 1.1*10^{-4}$ |

Source: Bank of Ghana (2001-2011)

**Table 5: Estimated parameters of the Stable distribution**

| GBP/GHC Exchange Rate Stable Parameters | | | | |
|---|---|---|---|---|
| **Methods** | $\hat{\alpha}$ | $\hat{\beta}$ | $\hat{\gamma}$ | $\hat{\delta}$ |
| **MLE** | $1.289 \pm 0.129$ | $0.173 \pm 0.193$ | $1.8*10^{-3} \pm 1.8*10^{-4}$ | $6.1*10^{-4} \pm 2.5*10^{-4}$ |
| **S. Quantile** | $1.38 \pm 0.13$ | $0.071 \pm 0.22$ | $1.9*10^{-3} \pm 1.8*10^{-4}$ | $4.3*10^{-4} \pm 2.7*10^{-4}$ |
| **ECF** | $1.48 \pm 0.14$ | $0.136 \pm 0.247$ | $2.0*10^{-3} \pm 1.8*10^{-4}$ | $7.0*10^{-4} \pm 2.9*10^{-4}$ |

Source: Bank of Ghana (2001-2011)

**Table 6: Estimated parameters of the Stable distribution**

| EUR/GHC Exchange Rate Stable Parameters | | | | |
|---|---|---|---|---|
| **Methods** | $\hat{\alpha}$ | $\hat{\beta}$ | $\hat{\gamma}$ | $\hat{\delta}$ |
| **MLE** | $1.236 \pm 0.126$ | $0.234 \pm 0.18$ | $2.02*10^{-3} \pm 2.1*10^{-4}$ | $6.9*10^{-4} \pm 2.8*10^{-4}$ |
| **S. Quantile** | $1.32 \pm 0.14$ | $0.18 \pm 0.21$ | $1.9*10^{-3} \pm 2.1*10^{-4}$ | $5.1*10^{-4} \pm 2.9*10^{-4}$ |
| **ECF** | $1.404 \pm 0.139$ | $0.425 \pm 0.218$ | $2.23*10^{-3} \pm 2.2*10^{-4}$ | $6.4*10^{-4} \pm 3.5*10^{-4}$ |

Source: Bank of Ghana (2002-2011)

Tables 7, 8 and 9 below illustrate Kolmogorov-Smirnov and Chi-Square goodness of fit tests of the Maximum likelihood estimation, Sample Quantile and Empirical Characteristic function methods of estimating parameters of stable distribution of the weekly logarithm returns of the three exchange rates understudy.

Table 7, illustrates that both the K-S test and Chi-square test fail to reject the claim that the Maximum likelihood estimates of the $\alpha$-stable distribution perfectly fit to the logarithm returns of USD/GHC exchange rate. The K-S goodness of fit test rejected the Sample quantile method estimates of $\alpha$-stable fit while the Chi-square goodness of fit test says otherwise. The Chi-square test shows a perfect fit of the Sample quantile method estimates of α-stable fit to the USD/GHC exchange rate data. The K-S and Chi-square goodness of fit tests illustrate a poor fit of $\alpha$-stable estimates of the Empirical Characteristic function method to the logarithm returns of USD/GHC exchange rate.

**Table 7: Goodness of fit tests for USD/GHC exchange rates**

| | Statistical Tests | |
|---|---|---|
| **Methods** | **K-S** | **Chi-Square** |
| **MLE** | $D = 0.037$<br>p-$value = 0.45$ | $\chi^2 = 7.85$<br>p-$value > 0.05$ |
| **Sample Quantile** | $D = 0.069$<br>p-$value = 0.014$ | $\chi^2 = 6.25$<br>p-$value > 0.10$ |
| **ECF** | $D = 0.08$<br>p-$value = 0.002$ | $\chi^2 = 18.06$<br>p-$value < 0.005$ |

Source: Bank of Ghana (2001-2011)

Table 8, demonstrates that the K-S goodness of fit test shows a good fit of Maximum likelihood estimates of the $\alpha$-stable distribution to the logarithm returns of GBP/GHC exchange rate at any significance level while the Chi-square goodness of fit test shows a poor fit of the Maximum likelihood estimates of the α-stable distribution to the logarithm returns of GBP/GHC exchange rate at 5% significance level. Both the K-S goodness of fit test and Chi-square goodness of fit test rejected the Sample quantile method estimates of $\alpha$-stable fit at any significance level. The K-S test for Empirical Characteristic function estimates of $\alpha$-stable fit shows a good fit at 1% significance level and a poor fit 5% significance level to the data while the Chi-square goodness of fit test shows a poor fit to the GBP/GHC exchange rate data at any significance level.

**Table 8: Goodness of fits test for GBP/GHC exchange rates**

| | Statistical Tests | |
|---|---|---|
| **Methods** | **K-S** | **Chi-Square** |
| **MLE** | $D = 0.046$<br>$\mathrm{p}-value = 0.21$ | $\chi^2 = 11.65$<br>$\mathrm{p}-value > 0.01$ |
| **Sample Quantile** | $D = 0.086$<br>$\mathrm{p}-value = 0.001$ | $\chi^2 = 38.60$<br>$\mathrm{p}-value < 0.005$ |
| **ECF** | $D = 0.06$<br>$\mathrm{p}-value = 0.042$ | $\chi^2 = 14.03$<br>$\mathrm{p}-value < 0.01$ |

Source: Bank of Ghana (2001-2011)

Table 9 demonstrates that K-S goodness of fit test fail to reject the hypothesis that the three methods (MLE, S. Quantile and ECF) of estimating parameters of $\alpha$ -stable distribution perfectly fit to the weekly logarithm returns of EUR/GHC exchange rate and this implies that the K-S test shows a good fit of the three estimating methods to the data at 5% significance level. The Chi-square goodness of fit test in the other hand shows a good fit to the Maximum likelihood estimates and a poor fit to the Sample Quantile and Empirical Characteristic function methods of α-stable fit.

**Table 9: Goodness of fit tests for EUR/GHC exchange rates**

| | Statistical Tests | |
|---|---|---|
| **Methods** | **K-S** | **Chi-Square** |
| **MLE** | $D = 0.044$<br>$\mathrm{p}-value = 0.32$ | $\chi^2 = 7.54$<br>$\mathrm{p}-value > 0.10$ |
| **Sample Quantile** | $D = 0.061$<br>$\mathrm{p}-value = 0.058$ | $\chi^2 = 13.38$<br>$\mathrm{p}-value = 0.01$ |
| **ECF** | $D = 0.058$<br>$\mathrm{p}-value = 0.082$ | $\chi^2 = 11.57$<br>$\mathrm{p}-value > 0.01$ |

Source: Bank of Ghana (2002-2011)

The maximum likelihood estimation method consistently demonstrate through the two goodness of fit tests that, it is superior and better estimator than the other two estimators (quantile and ECF) in modelling the returns of GSE All-Shares index and the three exchange rates. Therefore, the Maximum likelihood estimation produces the paramount estimating method for estimating the four parameters of α-stable distribution.

Tables 10-12 demonstrate the goodness of fit of the logarithm returns the three exchange rates to the three distributions under study. The four tests for normality considered in this study (Kolmogorov-Smirnov, Chi-square, Anderson-Darling and Shapiro-Wilk) all reject the claim that logarithm returns of the three exchange rates (USD/GHC, GBP/GHC and EUR/GHC) are normally distributed even at 1% significance level. This means that the return distributions of Ghana exchange rates are not normally distributed and this conformed to (McCulloch, 1996; Rachev & Mittnik, 2000) findings. The K-S and Chi-square goodness of fit tests both rejected the claim that the three exchange rates returns are Cauchy distributed. The K-S goodness of fit test parades impeccable fit of the weekly logarithm returns of all the three exchange rates to the α-stable fit and the Chi-square goodness of fit test displays a good fit of USD/GHC and EUR/GHC data to α-stable fit at 5% significance level and good fit of GBP/GHC data to $\alpha$-stable fit at 1% significance level. This implies that the Chi-square goodness of fit test is very sensitive of the deviation of the data to the theoretical distribution.

**Table 10: Goodness of fit tests for the distributions to USD/GHC data**

| Cases | K-S Test | Chi-Square | Anderson-Darling | Shapiro-Wilk |
|---|---|---|---|---|
| **Normal fit** | $D = 0.161$<br>p-$value \cong 0.0$ | $\chi^2 = 273.30$<br>p-$value < 0.005$ | $A^2$=25.99<br>p-$value < 0.005$ | W= 0.86<br>p-$value \cong 0.0$ |
| **Cauchy fit** | $D = 0.497$<br>p-$value \cong 0.0$ | $\chi^2 = 260428$<br>p-$value \cong 0.0$ | | |
| **Stable fit** | $D = 0.037$<br>p-$value = 0.45$ | $\chi^2 = 7.85$<br>p-$value > 0.05$ | | |

Source: Bank of Ghana (2001-2011)

**Table 11: Goodness of fit tests for the distributions to GBP/GHC data**

| Cases | K-S Test | Chi-Square | Anderson-Darling | Shapiro-Wilk |
|---|---|---|---|---|
| **Normal fit** | $D = 0.138$<br>p-$value \cong 0.0$ | $\chi^2 = 212.18$<br>p-$value < 0.005$ | $A^2$ = 20.09<br>p-$value < 0.005$ | W=0.834<br>p-$value \cong 0.0$ |
| **Cauchy fit** | $D = 0.492$<br>p-$value \cong 0.0$ | $\chi^2 = 269860$<br>p-$value \cong 0.0$ | | |
| **Stable fit** | $D = 0.0461$<br>p-$value = 0.21$ | $\chi^2 = 11.65$<br>p-$value > 0.01$ | | |

Source: Bank of Ghana (2001-2011)

**Table 12: Goodness of fit tests for distributions to the EUR/GHC Exchange data**

| Cases | K-S Test | Chi-Square | Anderson-Darling | Shapiro-Wilk |
|---|---|---|---|---|
| **Normal fit** | $D = 0.148$<br>p-$value \cong 0.0$ | $\chi^2 = 191.21$<br>p-$value < 0.005$ | $A^2$ =17.99<br>p-$value < 0.005$ | W=0.864<br>p-$value \cong 0.0$ |
| **Cauchy fit** | $D = 0.491$<br>p-$value \cong 0.0$ | $\chi^2 = 223254$<br>p-$value = 0.017$ | | |
| **Stable fit** | $D = 0.044$<br>p-$value = 0.32$ | $\chi^2 = 7.54$<br>p-$value > 0.10$ | | |

Source: Bank of Ghana (2002-2011)

Table 13 displays the estimated parameters (mean, standard error and standard deviation) of the normal distribution of GSE All-Shares index, USD/GHC, GBP/GHC and EUR/GHC exchange rates. It also shows the estimated parameters of skewness and kurtosis of the four variables considered under this study. It is observed that average logarithm returns of the GSE all-shares index and the three exchange rates is approximately zero. The return distribution of all the four financial data are asymmetry, and the GSE all-shares index and USD/GHC exchange rate are positively skewed while the other two exchange rates data (GBP/GHC and EUR/GHC) are negatively skewed. The kurtoses of the four financial data under study are higher than the normal distribution which is 3 and this conformed to the literature that asset returns are heavy tails and asymmetry.

**Table 13: Estimated parameters of the Normal distribution**

| Variable | Mean | SE Mean | Standard Dev. | Skewness | Kurtosis |
|---|---|---|---|---|---|
| **GSEI** | 0.00175 | 0.00035 | 0.0084 | 0.09 | 13.6 |
| **USD/GHC** | 0.00074 | 0.000096 | 0.0022 | 1.00 | 4.18 |
| **GBP/GHC** | 0.0008 | 0.000214 | 0.0049 | -0.61 | 12.65 |
| **EUR/GHC** | 0.00109 | 0.000261 | 0.0057 | -0.04 | 7.48 |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

This chapter presented the findings and discussions from the analysis of the study. The progression and the volatility of the financial time series data was displayed and the graphical density plots of the logarithm returns of the financial data illustrating the distribution of the returns of the four variables study was also presented. The diagnostics test was performed to demonstrate fitness to the Gaussian or normal, Cauchy and $\alpha$ -stable distributions and goodness of fit tests was also used to determine the best methods of estimating the four parameters of $\alpha$ -stable distribution. The analysis established that the return distribution of assets and exchange rates follow $\alpha$ -stable distribution and does not obey the Gaussian law. The next chapter presents summary, conclusion and recommendations of the research and also further studies areas.

# Chapter Five

# SUMMARY, CONCLUSION AND RECOMMENDATIONS

## 5.0 Introduction

This chapter presents summary of the findings from the study, and recommends rational measures for stakeholders, financial analysts and investors. It again recommends further study possibility areas for future researchers. The chapter provides the concluding statements of the research based on the findings.

## 5.1 Summary

The main purpose of this research was to investigate and model the return distributions of financial assets and exchange rates. The study considered four different financial data namely; Ghana stock exchange all-shares index from 2000 to 2010, US dollar to Ghana cedi exchange rate from 2001 to 2011, British pound sterling to Ghana cedi from 2001 to 2011 and Euro to Ghana cedi from 2002 to 2011, and the weekly indices and exchange rates were used for this study.

The study revealed that all normality tests rejected the returns distribution of the financial data being normally distributed and the Kolmogorov-Smirnov and Chi-square goodness of fit tests also rejected the data being Cauchy distributed. It was revealed that the returns of all the four financial data considered, come from a leptokurtic, heavy tails and asymmetry distribution, and the weekly logarithm returns of GSE All-Shares index, USD/GHC, GBP/GHC and EUR/GHC exchange rates are α-stable distributed with parameters $\alpha$, $\beta$, $\gamma$ and $\delta$. The Kolmogorov-Smirnov and Chi-square goodness of fit tests uniformly fail to reject the Maximum likelihood estimation method of estimating the four parameters of $\alpha-$stable distribution and produces the best fit to the financial data considered.

The financial data considered were modelled with $\alpha-$stable distribution with the following estimates; GSE All-Shares index $\alpha = 1.005 \pm 0.104$, $\beta = 0.31 \pm 0.135$, $\gamma = 0.002 \pm 0.0002$ and $\delta = 0.001 \pm 0.0002$, US dollar exchange rate $\alpha =$ $1.073 \pm 0.114$, $\beta = 0.394 \pm 0.147$, $\gamma = 6.6*10^{-4} \pm 7.62*10^{-5}$ and $\delta = 2.3*10^{-4} \pm 8.9*10^{-5}$, British pounds sterling exchange rate $\alpha = 1.289 \pm 0.129$, $\beta = 0.173 \pm 0.193$, $\gamma = 1.8*10^{-3} \pm 1.8*10^{-4}$ and $\delta = 6.1*10^{-4} \pm 2.5*10^{-4}$ and Euro exchange rate $\alpha =$ $1.236 \pm 0.126$, $\beta = 0.234 \pm 0.18$, $\gamma = 2.02*10^{-3} \pm 2.1*10^{-4}$ and $\delta = 6.87*10^{-4} \pm 2.8*10^{-4}$.

## 5.2 Conclusion

The findings from the study discovered that the daily returns distribution of Ghana Stock Exchange All-Shares index, US dollar, British pounds sterling and Euro exchange rates come from leptokurtic, heavy tailed and asymmetry distribution but cannot be modelled with $\alpha-$stable distribution, however the weekly logarithm return distribution of Ghana Stock Exchange All-Shares index, US dollar, British pounds sterling and Euro exchange rates follows $\alpha-$stable distribution. It was revealed that the weekly logarithm returns of the four financial data considered (GSE All-Shares, USD/GHC, GBP/GHC and EUR/GHC) do not obey the Gaussian law but $\alpha-$stable law. The study demonstrated that the Ghanaian financial data is not volatile as compared to the developed countries.

The study elucidated that among the three methods (MLE, Sample Quantile and Empirical Characteristic function) of estimating the four parameters of α-stable distribution, the

Maximum likelihood estimation method produces the best fit and more accurate estimates of the financial data considered. The returns of Ghana Stock Exchange All-Shares index, US dollar exchange rate, British pound sterling exchange rate and Euro exchange rate are successfully modelled with $\alpha-\text{stable}$ laws. These findings successfully help to achieve the set objectives for this study, and contradict findings of Gopikrishnan et al. (1999), Pagan, (1996), Coronel-Brizio & Hernandez-Montoya, (2005) who found out that the tail of some financial time series data have to be modelled with $\alpha > 2$.

The study shows that the Gaussian laws for modelling logarithm returns of Ghana financial data is wrong and that the study proposed stable laws for modelling logarithm returns of Ghana financial data. It can then be generally concluded that the weekly logarithm returns of financial data (GSE All-Shares index, currency exchange rates) of Ghana obey the laws of Stable distributions and not Gaussian laws, and therefore Stable laws should be used to manage risk and make outstanding and precise projections.

## 5.3 Recommendations

The following are recommendations to policy makers, stakeholders, investors and financial analysts;

1. It is recommended that weekly logarithm returns of Ghana financial data should be estimated with $\alpha-\text{stable}$ laws and avoids using Gaussian laws.
2. The bank of Ghana should encourage investment banks in currency swaps to enable greater volatility of the returns.
3. The bank of Ghana should put in place good economic and monetary policies that will make the financial market more efficient and not focus on policies that will make the market static.

4. The Ghana Stock Exchange should put in place policies that will encourage buying or selling shares or assets in Forward, Futures or Options than immediate buying or selling of an asset.
5. Economists and financial analysts should not assume the return distribution of assets or currency exchange rates but explore and discover their specific return distributions.

## 5.4 Further Studies

1. It is recommended that future studies should be conducted on the tail decay of the return distributions of Ghanaian financial data.

2. It is further recommended that a study should be carried out to assess the multivariate distribution of the returns of currency exchange rates using $\alpha-$stable distributions.

3. Further studies should be carried out with value at risk (VaR) models of financial data using stable laws.

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# APPENDIX A

| WEEKS | GSE Index 2000-2010 | USD/GHC 2001-2011 | GBP/GHC 2001-2011 | EUR/GHC 2002-2011 |
|---|---|---|---|---|
| 1 | 737.230 | 0.6800 | 0.9714 | 0.6498 |
| 2 | 739.017 | 0.6800 | 0.9714 | 0.6568 |
| 3 | 740.063 | 0.6860 | 0.9923 | 0.6589 |
| 4 | 740.677 | 0.6900 | 1.0060 | 0.6633 |
| 5 | 742.230 | 0.6900 | 1.0155 | 0.6660 |
| 6 | 743.140 | 0.6900 | 1.0155 | 0.6649 |
| 7 | 742.103 | 0.6900 | 1.0155 | 0.6687 |
| 8 | 741.170 | 0.6900 | 1.0155 | 0.6696 |
| 9 | 740.860 | 0.6995 | 1.0155 | 0.6696 |
| 10 | 748.587 | 0.7100 | 1.0162 | 0.6707 |
| 11 | 755.410 | 0.7100 | 1.0164 | 0.6713 |
| 12 | 759.947 | 0.7110 | 1.0165 | 0.6720 |
| 13 | 761.607 | 0.7141 | 1.0192 | 0.6726 |
| 14 | 763.707 | 0.7141 | 1.0195 | 0.6765 |
| 15 | 801.000 | 0.7141 | 1.0198 | 0.6810 |
| 16 | 864.380 | 0.7141 | 1.0198 | 0.6838 |
| 17 | 866.587 | 0.7141 | 1.0193 | 0.6936 |
| 18 | 854.520 | 0.7141 | 1.0188 | 0.7047 |
| 19 | 812.413 | 0.7141 | 1.0184 | 0.7148 |
| 20 | 809.480 | 0.7144 | 1.0185 | 0.7229 |
| 21 | 810.580 | 0.7145 | 1.0183 | 0.7276 |
| 22 | 812.633 | 0.7145 | 1.0180 | 0.7369 |
| 23 | 814.463 | 0.7145 | 1.0037 | 0.7518 |
| 24 | 816.130 | 0.7145 | 1.0023 | 0.7708 |
| 25 | 818.087 | 0.7145 | 1.0030 | 0.7823 |
| 26 | 817.070 | 0.7020 | 0.9925 | 0.7890 |
| 27 | 818.920 | 0.7019 | 0.9922 | 0.8040 |
| 28 | 821.330 | 0.7053 | 0.9925 | 0.8082 |
| 29 | 821.860 | 0.7057 | 0.9925 | 0.8034 |
| 30 | 821.873 | 0.7057 | 0.9930 | 0.7990 |
| 31 | 821.403 | 0.7057 | 0.9932 | 0.7916 |
| 32 | 820.410 | 0.7052 | 0.9952 | 0.7916 |
| 33 | 819.937 | 0.7092 | 1.0079 | 0.7945 |
| 34 | 822.403 | 0.7092 | 1.0092 | 0.7964 |
| 35 | 821.680 | 0.7107 | 1.0120 | 0.8070 |
| 36 | 835.133 | 0.7121 | 1.0142 | 0.8092 |
| 37 | 848.500 | 0.7122 | 1.0233 | 0.8130 |
| 38 | 866.633 | 0.7123 | 1.0245 | 0.8139 |
| 39 | 860.067 | 0.7123 | 1.0270 | 0.8142 |
| 40 | 855.983 | 0.7165 | 1.0348 | 0.8139 |
| 41 | 855.753 | 0.7198 | 1.0348 | 0.8139 |
| 42 | 849.563 | 0.7200 | 1.0288 | 0.8149 |
| 43 | 858.993 | 0.7200 | 1.0280 | 0.8261 |
| 44 | 858.363 | 0.7200 | 1.0297 | 0.8406 |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

| WEEKS | GSE Index 2000-2010 | USD/GHC 2001-2011 | GBP/GHC 2001-2011 | EUR/GHC 2002-2011 |
|---|---|---|---|---|
| 45 | 852.173 | 0.7225 | 1.0349 | 0.8575 |
| 46 | 851.663 | 0.7310 | 1.0388 | 0.8587 |
| 47 | 857.890 | 0.7318 | 1.0374 | 0.8608 |
| 48 | 863.730 | 0.7321 | 1.0351 | 0.8613 |
| 49 | 866.490 | 0.7332 | 1.0360 | 0.8599 |
| 50 | 865.667 | 0.7330 | 1.0357 | 0.8624 |
| 51 | 859.543 | 0.7327 | 1.0355 | 0.8675 |
| 52 | 857.980 | 0.7328 | 1.0364 | 0.8784 |
| 53 | 859.193 | 0.7330 | 1.0411 | 0.9010 |
| 54 | 861.553 | 0.7449 | 1.0538 | 0.9175 |
| 55 | 861.393 | 0.7461 | 1.0559 | 0.9272 |
| 56 | 858.167 | 0.7495 | 1.0561 | 0.9304 |
| 57 | 857.633 | 0.7583 | 1.0620 | 0.9297 |
| 58 | 858.217 | 0.7534 | 1.0648 | 0.9306 |
| 59 | 857.620 | 0.7664 | 1.0665 | 0.9315 |
| 60 | 863.653 | 0.7684 | 1.0697 | 0.9317 |
| 61 | 878.717 | 0.7699 | 1.0740 | 0.9306 |
| 62 | 892.220 | 0.7700 | 1.0821 | 0.9294 |
| 63 | 893.560 | 0.7701 | 1.0822 | 0.9289 |
| 64 | 897.890 | 0.7762 | 1.0895 | 0.9283 |
| 65 | 899.290 | 0.7793 | 1.0942 | 0.9292 |
| 66 | 896.443 | 0.7794 | 1.0965 | 0.9381 |
| 67 | 897.015 | 0.7795 | 1.1112 | 0.9691 |
| 68 | 897.650 | 0.7816 | 1.1236 | 0.9870 |
| 69 | 897.650 | 0.7898 | 1.1358 | 0.9982 |
| 70 | 897.997 | 0.7902 | 1.1390 | 1.0436 |
| 71 | 897.290 | 0.7902 | 1.1420 | 1.0104 |
| 72 | 897.740 | 0.7906 | 1.1457 | 1.0215 |
| 73 | 896.143 | 0.7906 | 1.1607 | 1.0274 |
| 74 | 894.730 | 0.8017 | 1.1816 | 1.0162 |
| 75 | 895.830 | 0.8048 | 1.2010 | 1.0051 |
| 76 | 896.897 | 0.8076 | 1.2123 | 0.9954 |
| 77 | 915.890 | 0.8087 | 1.2350 | 0.9894 |
| 78 | 932.020 | 0.8103 | 1.2405 | 0.9866 |
| 79 | 933.157 | 0.8085 | 1.2418 | 0.9815 |
| 80 | 934.350 | 0.8099 | 1.2418 | 0.9819 |
| 81 | 935.603 | 0.8104 | 1.2392 | 1.0200 |
| 82 | 935.563 | 0.8107 | 1.2408 | 0.9757 |
| 83 | 995.163 | 0.8149 | 1.2415 | 0.9715 |
| 84 | 939.250 | 0.8167 | 1.2480 | 0.9736 |
| 85 | 942.917 | 0.8209 | 1.2606 | 0.9763 |
| 86 | 946.680 | 0.8225 | 1.2609 | 0.9821 |
| 87 | 949.143 | 0.8270 | 1.2689 | 0.9903 |
| 88 | 953.097 | 0.8283 | 1.2740 | 1.0033 |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

| WEEKS | GSE Index 2000-2010 | USD/GHC 2001-2011 | GBP/GHC 2001-2011 | EUR/GHC 2002-2011 |
|---|---|---|---|---|
| 89 | 950.440 | 0.8301 | 1.2772 | 1.0170 |
| 90 | 951.353 | 0.8311 | 1.2785 | 1.0227 |
| 91 | 958.480 | 0.8311 | 1.2788 | 1.0255 |
| 92 | 958.207 | 0.8315 | 1.2791 | 1.0266 |
| 93 | 958.937 | 0.8372 | 1.2892 | 1.0463 |
| 94 | 961.113 | 0.8477 | 1.3071 | 1.0532 |
| 95 | 961.017 | 0.8605 | 1.3287 | 1.0745 |
| 96 | 961.010 | 0.8633 | 1.3206 | 1.0768 |
| 97 | 959.737 | 0.8659 | 1.3332 | 1.0798 |
| 98 | 957.330 | 0.8675 | 1.3336 | 1.1004 |
| 99 | 957.930 | 0.8680 | 1.3341 | 1.1482 |
| 100 | 958.267 | 0.8661 | 1.3360 | 1.1662 |
| 101 | 958.547 | 0.8674 | 1.3408 | 1.1680 |
| 102 | 958.250 | 0.8687 | 1.3493 | 1.1674 |
| 103 | 958.890 | 0.8696 | 1.3588 | 1.1675 |
| 104 | 956.893 | 0.8695 | 1.3694 | 1.1704 |
| 105 | 955.950 | 0.8700 | 1.3847 | 1.1685 |
| 106 | 956.440 | 0.8706 | 1.3865 | 1.1539 |
| 107 | 956.440 | 0.8705 | 1.3800 | 1.1283 |
| 108 | 956.530 | 0.8702 | 1.3637 | 1.1228 |
| 109 | 958.450 | 0.8695 | 1.3611 | 1.1244 |
| 110 | 961.440 | 0.8695 | 1.3615 | 1.1261 |
| 111 | 962.597 | 0.8699 | 1.2879 | 1.1160 |
| 112 | 965.345 | 0.8693 | 1.2812 | 1.1060 |
| 113 | 970.330 | 0.8698 | 1.3506 | 1.0962 |
| 114 | 983.020 | 0.8695 | 1.3488 | 1.0892 |
| 115 | 991.857 | 0.8688 | 1.3486 | 1.0895 |
| 116 | 1006.170 | 0.8676 | 1.3570 | 1.0878 |
| 117 | 1018.373 | 0.8700 | 1.3698 | 1.1035 |
| 118 | 1019.163 | 0.8723 | 1.3788 | 1.1170 |
| 119 | 1020.923 | 0.8735 | 1.3938 | 1.1202 |
| 120 | 1026.690 | 0.8730 | 1.3976 | 1.1223 |
| 121 | 1033.417 | 0.8739 | 1.4071 | 1.1237 |
| 122 | 1041.790 | 0.8769 | 1.4296 | 1.1271 |
| 123 | 1049.967 | 0.8823 | 1.4488 | 1.1276 |
| 124 | 1061.807 | 0.8829 | 1.4541 | 1.1319 |
| 125 | 1077.037 | 0.8856 | 1.4498 | 1.1253 |
| 126 | 1123.143 | 0.8859 | 1.4204 | 1.1067 |
| 127 | 1142.837 | 0.8839 | 1.3964 | 1.0955 |
| 128 | 1172.980 | 0.8782 | 1.3941 | 1.1105 |
| 129 | 1207.800 | 0.8756 | 1.3886 | 1.1145 |
| 130 | 1217.887 | 0.8773 | 1.3890 | 1.1101 |
| 131 | 1229.737 | 0.8786 | 1.3872 | 1.1048 |
| 132 | 1237.330 | 0.8920 | 1.3841 | 1.1101 |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

| WEEKS | GSE Index 2000-2010 | USD/GHC 2001-2011 | GBP/GHC 2001-2011 | EUR/GHC 2002-2011 |
|---|---|---|---|---|
| 133 | 1246.527 | 0.8906 | 1.3836 | 1.1140 |
| 134 | 1253.023 | 0.8835 | 1.3863 | 1.1174 |
| 135 | 1259.213 | 0.8831 | 1.3880 | 1.1176 |
| 136 | 1267.137 | 0.8833 | 1.3976 | 1.1181 |
| 137 | 1272.673 | 0.8828 | 1.4174 | 1.1226 |
| 138 | 1286.560 | 0.8847 | 1.4295 | 1.1387 |
| 139 | 1302.617 | 0.8898 | 1.4371 | 1.1580 |
| 140 | 1311.857 | 0.8915 | 1.4538 | 1.1732 |
| 141 | 1307.380 | 0.8926 | 1.4706 | 1.1678 |
| 142 | 1305.650 | 0.8997 | 1.4877 | 1.1935 |
| 143 | 1307.370 | 0.9196 | 1.5159 | 1.2070 |
| 144 | 1313.653 | 0.9091 | 1.5117 | 1.2146 |
| 145 | 1327.010 | 0.9118 | 1.5318 | 1.2134 |
| 146 | 1334.010 | 0.9102 | 1.5353 | 1.2118 |
| 147 | 1335.543 | 0.9098 | 1.5402 | 1.2136 |
| 148 | 1339.573 | 0.9148 | 1.5617 | 1.2167 |
| 149 | 1343.677 | 0.9294 | 1.6220 | 1.2154 |
| 150 | 1352.543 | 0.9398 | 1.6712 | 1.2109 |
| 151 | 1359.007 | 0.9410 | 1.6755 | 1.2083 |
| 152 | 1361.767 | 0.9391 | 1.6800 | 1.2051 |
| 153 | 1367.733 | 0.9393 | 1.6892 | 1.2001 |
| 154 | 1376.367 | 0.9383 | 1.7099 | 1.1992 |
| 155 | 1379.780 | 0.9326 | 1.7159 | 1.2058 |
| 156 | 1385.300 | 0.9258 | 1.6954 | 1.2051 |
| 157 | 1397.643 | 0.9223 | 1.6623 | 1.2072 |
| 158 | 1411.407 | 0.9236 | 1.6488 | 1.2125 |
| 159 | 1420.120 | 0.9232 | 1.6614 | 1.1733 |
| 160 | 1428.007 | 0.9236 | 1.6655 | 1.1919 |
| 161 | 1433.127 | 0.9204 | 1.6616 | 1.1910 |
| 162 | 1444.333 | 0.9201 | 1.6416 | 1.1895 |
| 163 | 1459.593 | 0.9201 | 1.6275 | 1.1887 |
| 164 | 1473.573 | 0.9200 | 1.6282 | 1.1845 |
| 165 | 1485.077 | 0.9196 | 1.6271 | 1.1756 |
| 166 | 1530.267 | 0.9236 | 1.6223 | 1.1685 |
| 167 | 1586.683 | 0.9276 | 1.6387 | 1.1564 |
| 168 | 1622.983 | 0.9288 | 1.6723 | 1.1274 |
| 169 | 1635.297 | 0.9305 | 1.6790 | 1.1249 |
| 170 | 1681.570 | 0.9318 | 1.6821 | 1.1220 |
| 171 | 1714.270 | 0.9330 | 1.6883 | 1.1231 |
| 172 | 1726.283 | 0.9326 | 1.6842 | 1.1132 |
| 173 | 1751.937 | 0.9318 | 1.6906 | 1.1126 |
| 174 | 1772.723 | 0.9310 | 1.6970 | 1.0998 |
| 175 | 1791.393 | 0.9269 | 1.6893 | 1.0950 |
| 176 | 1812.383 | 0.9205 | 1.6690 | 1.0917 |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

| WEEKS | GSE Index 2000-2010 | USD/GHC 2001-2011 | GBP/GHC 2001-2011 | EUR/GHC 2002-2011 |
|---|---|---|---|---|
| 177 | 1840.710 | 0.9220 | 1.6634 | 1.0940 |
| 178 | 1864.827 | 0.9236 | 1.6672 | 1.1054 |
| 179 | 1928.853 | 0.9234 | 1.6666 | 1.1036 |
| 180 | 1931.493 | 0.9213 | 1.6488 | 1.1030 |
| 181 | 1984.703 | 0.9208 | 1.6399 | 1.1115 |
| 182 | 2058.890 | 0.9225 | 1.6393 | 1.1174 |
| 183 | 2100.243 | 0.9176 | 1.6394 | 1.1175 |
| 184 | 2148.307 | 0.9207 | 1.6392 | 1.1126 |
| 185 | 2244.227 | 0.9191 | 1.6362 | 1.1035 |
| 186 | 2294.503 | 0.9191 | 1.6300 | 1.1042 |
| 187 | 2329.770 | 0.9221 | 1.6301 | 1.1055 |
| 188 | 2386.953 | 0.9258 | 1.6535 | 1.1045 |
| 189 | 2422.477 | 0.9295 | 1.6666 | 1.1067 |
| 190 | 2484.227 | 0.9304 | 1.6880 | 1.1037 |
| 191 | 2538.497 | 0.9280 | 1.6814 | 1.1029 |
| 192 | 2586.790 | 0.9314 | 1.6980 | 1.1033 |
| 193 | 2605.733 | 0.9295 | 1.7112 | 1.1041 |
| 194 | 2599.877 | 0.9361 | 1.7322 | 1.1035 |
| 195 | 2621.957 | 0.9294 | 1.7478 | 1.1029 |
| 196 | 2647.813 | 0.9259 | 1.7470 | 1.1002 |
| 197 | 2762.980 | 0.9223 | 1.7360 | 1.0988 |
| 198 | 2854.947 | 0.9243 | 1.7329 | 1.1000 |
| 199 | 2870.857 | 0.9265 | 1.7119 | 1.1012 |
| 200 | 2900.263 | 0.9273 | 1.7310 | 1.1046 |
| 201 | 2950.187 | 0.9265 | 1.7319 | 1.1053 |
| 202 | 2988.517 | 0.9291 | 1.7335 | 1.1070 |
| 203 | 3137.087 | 0.9266 | 1.7212 | 1.1048 |
| 204 | 3297.773 | 0.9270 | 1.7225 | 1.1051 |
| 205 | 3368.923 | 0.9269 | 1.7330 | 1.1051 |
| 206 | 3402.397 | 0.9245 | 1.7372 | 1.1172 |
| 207 | 3444.800 | 0.9226 | 1.7390 | 1.1031 |
| 208 | 3500.227 | 0.9212 | 1.7424 | 1.1041 |
| 209 | 3584.003 | 0.9180 | 1.7189 | 1.1051 |
| 210 | 3605.540 | 0.9184 | 1.7140 | 1.1052 |
| 211 | 3659.873 | 0.9213 | 1.7183 | 1.1070 |
| 212 | 3721.250 | 0.9222 | 1.7255 | 1.1069 |
| 213 | 3828.590 | 0.9205 | 1.7233 | 1.1892 |
| 214 | 3927.773 | 0.9170 | 1.7180 | 1.1295 |
| 215 | 4194.928 | 0.9184 | 1.7171 | 1.1503 |
| 216 | 4501.417 | 0.9209 | 1.6974 | 1.1510 |
| 217 | 4986.640 | 0.9212 | 1.6775 | 1.1582 |
| 218 | 5153.767 | 0.9204 | 1.6646 | 1.1649 |
| 219 | 5357.340 | 0.9200 | 1.6663 | 1.1635 |
| 220 | 5562.230 | 0.9203 | 1.6655 | 1.1643 |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

| WEEKS | GSE Index 2000-2010 | USD/GHC 2001-2011 | GBP/GHC 2001-2011 | EUR/GHC 2002-2011 |
|---|---|---|---|---|
| 221 | 5792.123 | 0.9209 | 1.6664 | 1.1675 |
| 222 | 6076.400 | 0.9174 | 1.5277 | 1.1701 |
| 223 | 6208.260 | 0.9183 | 1.5400 | 1.1800 |
| 224 | 6428.373 | 0.9176 | 1.5498 | 1.1805 |
| 225 | 6599.180 | 0.9173 | 1.5893 | 1.1820 |
| 226 | 6708.090 | 0.9157 | 1.5865 | 1.1862 |
| 227 | 6774.370 | 0.9135 | 1.5914 | 1.1864 |
| 228 | 6824.763 | 0.9141 | 1.6027 | 1.1862 |
| 229 | 6852.167 | 0.9138 | 1.6161 | 1.1836 |
| 230 | 6895.140 | 0.9135 | 1.6186 | 1.1851 |
| 231 | 6942.770 | 0.9128 | 1.6283 | 1.1837 |
| 232 | 7010.110 | 0.9122 | 1.6495 | 1.1860 |
| 233 | 7046.837 | 0.9162 | 1.6475 | 1.1936 |
| 234 | 7064.020 | 0.9182 | 1.6387 | 1.1955 |
| 235 | 7098.713 | 0.9192 | 1.6158 | 1.2097 |
| 236 | 7067.203 | 0.9204 | 1.6114 | 1.2265 |
| 237 | 7120.710 | 0.9254 | 1.6100 | 1.2348 |
| 238 | 7168.853 | 0.9284 | 1.6126 | 1.2171 |
| 239 | 7325.063 | 0.9290 | 1.6216 | 1.2149 |
| 240 | 7367.020 | 0.9295 | 1.6180 | 1.2159 |
| 241 | 7440.800 | 0.9338 | 1.6170 | 1.2150 |
| 242 | 7451.830 | 0.9394 | 1.6159 | 1.2173 |
| 243 | 7157.927 | 0.9411 | 1.6159 | 1.2172 |
| 244 | 7027.073 | 0.9398 | 1.6166 | 1.2200 |
| 245 | 7102.710 | 0.9393 | 1.6208 | 1.2395 |
| 246 | 7022.017 | 0.9320 | 1.6149 | 1.2300 |
| 247 | 6998.187 | 0.9323 | 1.6064 | 1.2285 |
| 248 | 7017.753 | 0.9323 | 1.6054 | 1.2301 |
| 249 | 6998.813 | 0.9311 | 1.6090 | 1.2380 |
| 250 | 6994.560 | 0.9313 | 1.6139 | 1.2390 |
| 251 | 6951.040 | 0.9288 | 1.6160 | 1.2312 |
| 252 | 6901.127 | 0.9263 | 1.6164 | 1.2562 |
| 253 | 6894.833 | 0.9280 | 1.6124 | 1.2723 |
| 254 | 6781.413 | 0.9316 | 1.6123 | 1.2729 |
| 255 | 6744.203 | 0.9319 | 1.6132 | 1.2731 |
| 256 | 6761.120 | 0.9286 | 1.6292 | 1.2710 |
| 257 | 6778.613 | 0.9250 | 1.6050 | 1.2680 |
| 258 | 6780.657 | 0.9277 | 1.6045 | 1.2657 |
| 259 | 6789.877 | 0.9282 | 1.6054 | 1.3002 |
| 260 | 6801.280 | 0.9276 | 1.6044 | 1.3456 |
| 261 | 6840.077 | 0.9269 | 1.6011 | 1.3776 |
| 262 | 6897.343 | 0.9265 | 1.6026 | 1.2770 |
| 263 | 6897.173 | 0.9265 | 1.6293 | 1.2761 |
| 264 | 6866.293 | 0.9280 | 1.6333 | 1.2728 |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

| WEEKS | GSE Index 2000-2010 | USD/GHC 2001-2011 | GBP/GHC 2001-2011 | EUR/GHC 2002-2011 |
|---|---|---|---|---|
| 265 | 6844.920 | 0.9278 | 1.6721 | 1.2744 |
| 266 | 6777.127 | 0.9288 | 1.6964 | 1.2818 |
| 267 | 6718.353 | 0.9291 | 1.6983 | 1.2894 |
| 268 | 6686.630 | 0.9302 | 1.6913 | 1.3002 |
| 269 | 6643.060 | 0.9252 | 1.6963 | 1.3193 |
| 270 | 6526.413 | 0.9257 | 1.7157 | 1.3364 |
| 271 | 6454.680 | 0.9266 | 1.7277 | 1.3459 |
| 272 | 6386.693 | 0.9288 | 1.7306 | 1.3566 |
| 273 | 6357.540 | 0.9301 | 1.7416 | 1.4025 |
| 274 | 6329.047 | 0.9320 | 1.7392 | 1.4131 |
| 275 | 6297.570 | 0.9353 | 1.7449 | 1.4188 |
| 276 | 6102.923 | 0.9400 | 1.7569 | 1.4353 |
| 277 | 6068.277 | 0.9399 | 1.7523 | 1.4447 |
| 278 | 6051.117 | 0.9403 | 1.7529 | 1.4465 |
| 279 | 6050.810 | 0.9435 | 1.7530 | 1.4336 |
| 280 | 6050.030 | 0.9390 | 1.7537 | 1.4150 |
| 281 | 6039.580 | 0.9383 | 1.7573 | 1.4252 |
| 282 | 6033.503 | 0.9401 | 1.7655 | 1.4361 |
| 283 | 6034.013 | 0.9432 | 1.7750 | 1.4401 |
| 284 | 5952.610 | 0.9439 | 1.7781 | 1.4348 |
| 285 | 5741.635 | 0.9445 | 1.7954 | 1.4376 |
| 286 | 5393.840 | 0.9438 | 1.8131 | 1.4496 |
| 287 | 5243.450 | 0.9452 | 1.8204 | 1.4458 |
| 288 | 5196.473 | 0.9389 | 1.8134 | 1.4472 |
| 289 | 5022.617 | 0.9309 | 1.8043 | 1.4781 |
| 290 | 4912.014 | 0.9415 | 1.8091 | 1.5050 |
| 291 | 4850.644 | 0.9439 | 1.8200 | 1.5144 |
| 292 | 4838.120 | 0.9462 | 1.8330 | 1.5190 |
| 293 | 4833.322 | 0.9473 | 1.8368 | 1.5423 |
| 294 | 4831.626 | 0.9483 | 1.8393 | 1.5566 |
| 295 | 4883.036 | 0.9521 | 1.8441 | 1.5671 |
| 296 | 4896.666 | 0.9494 | 1.8461 | 1.5721 |
| 297 | 4885.570 | 0.9422 | 1.8328 | 1.5696 |
| 298 | 4880.238 | 0.9451 | 1.8234 | 1.5732 |
| 299 | 4884.254 | 0.9466 | 1.8302 | 1.5810 |
| 300 | 4890.416 | 0.9484 | 1.9209 | 1.5823 |
| 301 | 4897.504 | 0.9487 | 1.8373 | 1.5945 |
| 302 | 4898.222 | 0.9487 | 1.8556 | 1.5621 |
| 303 | 4904.164 | 0.9487 | 1.8696 | 1.6291 |
| 304 | 4906.262 | 0.9485 | 1.8695 | 1.6335 |
| 305 | 4908.400 | 0.9485 | 1.8702 | 1.6417 |
| 306 | 4813.276 | 0.9484 | 1.8688 | 1.6716 |
| 307 | 4896.116 | 0.9466 | 1.8656 | 1.7006 |
| 308 | 4801.510 | 0.9470 | 1.8685 | 1.6952 |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

| WEEKS | GSE Index 2000-2010 | USD/GHC 2001-2011 | GBP/GHC 2001-2011 | EUR/GHC 2002-2011 |
|---|---|---|---|---|
| 309 | 4792.822 | 0.9460 | 1.8812 | 1.6316 |
| 310 | 4768.078 | 0.9424 | 1.8992 | 1.6485 |
| 311 | 4766.450 | 0.9412 | 1.8978 | 1.6720 |
| 312 | 4756.530 | 0.9412 | 1.8973 | 1.7300 |
| 313 | 4713.925 | 0.9417 | 1.8908 | 1.6652 |
| 314 | 4730.990 | 0.9425 | 1.8802 | 1.6703 |
| 315 | 4738.072 | 0.9455 | 1.8795 | 1.6569 |
| 316 | 4724.248 | 0.9497 | 1.8866 | 1.5999 |
| 317 | 4743.010 | 0.9532 | 1.9012 | 1.5652 |
| 318 | 4737.626 | 0.9551 | 1.9033 | 1.5547 |
| 319 | 4737.682 | 0.9555 | 1.9162 | 1.5444 |
| 320 | 4739.660 | 0.9579 | 1.9062 | 1.5421 |
| 321 | 4743.415 | 0.9623 | 1.9394 | 1.5377 |
| 322 | 4756.830 | 0.9681 | 1.9534 | 1.5365 |
| 323 | 4769.148 | 0.9820 | 2.0097 | 1.5868 |
| 324 | 4771.890 | 0.9814 | 2.0236 | 1.6862 |
| 325 | 4776.122 | 0.9814 | 2.0177 | 1.6819 |
| 326 | 4778.923 | 0.9849 | 2.0255 | 1.6975 |
| 327 | 4780.500 | 0.9873 | 2.0271 | 1.7157 |
| 328 | 4784.884 | 0.9898 | 2.0215 | 1.7188 |
| 329 | 4792.250 | 0.9904 | 1.9983 | 1.7263 |
| 330 | 4799.654 | 0.9893 | 1.9643 | 1.7113 |
| 331 | 4783.242 | 0.9891 | 1.9558 | 1.7226 |
| 332 | 4832.000 | 0.9842 | 1.9454 | 1.7126 |
| 333 | 4856.052 | 0.9931 | 1.9392 | 1.7446 |
| 334 | 4862.084 | 0.9893 | 1.9329 | 1.7602 |
| 335 | 4863.452 | 0.9888 | 1.9336 | 1.7601 |
| 336 | 4863.336 | 0.9875 | 1.9313 | 1.7832 |
| 337 | 4853.998 | 0.9874 | 1.9297 | 1.8422 |
| 338 | 4850.872 | 0.9876 | 1.9295 | 1.8603 |
| 339 | 4854.414 | 0.9881 | 1.9530 | 1.8554 |
| 340 | 4891.098 | 0.9908 | 1.9681 | 1.8505 |
| 341 | 4909.516 | 0.9901 | 1.9578 | 1.8561 |
| 342 | 4901.078 | 0.9900 | 1.9562 | 1.8664 |
| 343 | 4892.016 | 0.9925 | 1.9594 | 1.8921 |
| 344 | 4896.448 | 0.9988 | 1.9633 | 1.9191 |
| 345 | 4915.246 | 1.0041 | 1.9710 | 1.9459 |
| 346 | 4930.876 | 1.0051 | 1.9736 | 1.9747 |
| 347 | 4928.716 | 1.0073 | 1.9616 | 1.9994 |
| 348 | 4927.993 | 1.0106 | 1.9703 | 2.0235 |
| 349 | 4975.084 | 1.0140 | 1.9822 | 2.0379 |
| 350 | 4974.462 | 1.0157 | 1.9866 | 2.0360 |
| 351 | 4963.986 | 1.0247 | 2.0078 | 2.0202 |
| 352 | 4965.932 | 1.0356 | 2.0250 | 2.1446 |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

| WEEKS | GSE Index 2000-2010 | USD/GHC 2001-2011 | GBP/GHC 2001-2011 | EUR/GHC 2002-2011 |
|---|---|---|---|---|
| 353 | 4983.204 | 1.0507 | 2.0426 | 2.0574 |
| 354 | 4982.555 | 1.0528 | 2.0564 | 2.0706 |
| 355 | 4994.376 | 1.0535 | 2.0691 | 2.0841 |
| 356 | 5004.476 | 1.0637 | 2.0978 | 2.0993 |
| 357 | 5004.510 | 1.0755 | 2.0246 | 2.0779 |
| 358 | 4994.658 | 1.0850 | 2.1409 | 2.0800 |
| 359 | 5010.188 | 1.1036 | 2.1151 | 2.0985 |
| 360 | 5013.742 | 1.1191 | 2.1153 | 2.0876 |
| 361 | 5020.324 | 1.1259 | 2.1087 | 2.1042 |
| 362 | 5026.626 | 1.1341 | 2.1006 | 2.1187 |
| 363 | 5026.770 | 1.1432 | 2.0931 | 2.1241 |
| 364 | 5027.007 | 1.1504 | 2.0996 | 2.1241 |
| 365 | 5026.150 | 1.1560 | 2.0682 | 2.1271 |
| 366 | 5028.812 | 1.1756 | 2.0125 | 2.1407 |
| 367 | 5030.690 | 1.1893 | 1.9753 | 2.1402 |
| 368 | 5032.894 | 1.1955 | 1.9539 | 2.1348 |
| 369 | 5038.386 | 1.1946 | 1.9276 | 2.1423 |
| 370 | 5049.374 | 1.1943 | 1.8480 | 2.1387 |
| 371 | 5063.360 | 1.1962 | 1.8271 | 2.1466 |
| 372 | 5071.050 | 1.1981 | 1.8327 | 2.1516 |
| 373 | 5082.640 | 1.2245 | 1.8459 | 2.1830 |
| 374 | 5082.940 | 1.2449 | 1.8747 | 2.1620 |
| 375 | 5088.622 | 1.2548 | 1.8905 | 2.0896 |
| 376 | 5104.884 | 1.2524 | 1.8608 | 2.0826 |
| 377 | 5117.102 | 1.2766 | 1.9074 | 2.0619 |
| 378 | 5140.726 | 1.3041 | 1.9224 | 2.0828 |
| 379 | 5157.146 | 1.3298 | 1.8913 | 2.0716 |
| 380 | 5159.708 | 1.3355 | 1.8781 | 2.0147 |
| 381 | 5164.122 | 1.3370 | 1.9148 | 2.0180 |
| 382 | 5171.958 | 1.3498 | 1.9345 | 1.9856 |
| 383 | 5182.258 | 1.3741 | 1.9544 | 1.9772 |
| 384 | 5192.540 | 1.3942 | 1.9559 | 1.9740 |
| 385 | 5220.700 | 1.3986 | 1.9668 | 1.9717 |
| 386 | 5251.390 | 1.4088 | 1.9639 | 1.9802 |
| 387 | 5267.604 | 1.4106 | 1.9893 | 1.9815 |
| 388 | 5293.922 | 1.4107 | 2.0002 | 1.9659 |
| 389 | 5317.148 | 1.4112 | 2.0061 | 1.9502 |
| 390 | 5333.466 | 1.4123 | 2.0317 | 1.9570 |
| 391 | 5356.258 | 1.4263 | 2.0561 | 1.9430 |
| 392 | 5378.724 | 1.4370 | 1.9897 | 1.9256 |
| 393 | 5386.062 | 1.4419 | 2.1093 | 1.9114 |
| 394 | 5379.726 | 1.4456 | 2.1448 | 1.8436 |
| 395 | 5390.082 | 1.4491 | 2.1979 | 1.7873 |
| 396 | 5445.428 | 1.4515 | 2.2694 | 1.7687 |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

| WEEKS | GSE Index 2000-2010 | USD/GHC 2001-2011 | GBP/GHC 2001-2011 | EUR/GHC 2002-2011 |
|---|---|---|---|---|
| 397 | 5514.362 | 1.4558 | 2.2905 | 1.7534 |
| 398 | 5573.592 | 1.4670 | 2.3447 | 1.7556 |
| 399 | 5575.254 | 1.4733 | 2.3765 | 1.7555 |
| 400 | 5614.320 | 1.4769 | 2.3822 | 1.7620 |
| 401 | 5662.150 | 1.4711 | 2.3666 | 1.7745 |
| 402 | 5667.778 | 1.4746 | 2.3720 | 1.8026 |
| 403 | 5704.636 | 1.4768 | 2.3881 | 1.8713 |
| 404 | 5712.084 | 1.4801 | 2.4040 | 1.8471 |
| 405 | 5786.480 | 1.4843 | 2.4353 | 1.8603 |
| 406 | 5835.432 | 1.4852 | 2.4420 | 1.8642 |
| 407 | 5842.916 | 1.4819 | 2.4196 | 1.8525 |
| 408 | 5848.176 | 1.4788 | 2.4078 | 1.8309 |
| 409 | 5857.054 | 1.4762 | 2.3869 | 1.8389 |
| 410 | 5928.002 | 1.4752 | 2.3937 | 1.8355 |
| 411 | 6229.400 | 1.4757 | 2.4116 | 1.8383 |
| 412 | 6411.582 | 1.4762 | 2.4080 | 1.8573 |
| 413 | 6443.322 | 1.4756 | 2.3943 | 1.8793 |
| 414 | 6539.596 | 1.4741 | 2.3782 | 1.9099 |
| 415 | 6589.976 | 1.4712 | 2.3632 | 1.9265 |
| 416 | 6621.834 | 1.4635 | 2.3521 | 1.9394 |
| 417 | 6663.102 | 1.4619 | 2.3615 | 1.9535 |
| 418 | 6700.224 | 1.4624 | 2.3645 | 2.0371 |
| 419 | 6716.752 | 1.4621 | 2.3749 | 1.9827 |
| 420 | 6727.530 | 1.4639 | 2.3808 | 1.9840 |
| 421 | 6812.102 | 1.4641 | 2.3836 | 1.9722 |
| 422 | 6899.978 | 1.4622 | 2.3788 | 1.9579 |
| 423 | 6967.116 | 1.4645 | 2.3779 | 1.9592 |
| 424 | 7027.712 | 1.4660 | 2.3591 | 1.9605 |
| 425 | 7135.054 | 1.4664 | 2.3446 | 1.9558 |
| 426 | 7192.010 | 1.4658 | 2.3348 | 1.9600 |
| 427 | 7754.116 | 1.4697 | 2.3257 | 1.9558 |
| 428 | 8331.630 | 1.4610 | 2.3276 | 1.9595 |
| 429 | 8599.956 | 1.4587 | 2.3408 | 1.9920 |
| 430 | 8881.186 | 1.4524 | 2.3128 | 2.0365 |
| 431 | 9112.426 | 1.4812 | 2.3305 | 2.0389 |
| 432 | 9369.790 | 1.4519 | 2.2775 | 2.0185 |
| 433 | 9529.678 | 1.4478 | 2.2725 | 2.0204 |
| 434 | 9595.432 | 1.4527 | 2.2436 | 2.0286 |
| 435 | 9744.256 | 1.4533 | 2.2191 | 2.0449 |
| 436 | 9858.340 | 1.4604 | 2.2035 | 2.0565 |
| 437 | 10008.146 | 1.4521 | 2.2049 | 2.0907 |
| 438 | 10241.822 | 1.4476 | 2.1772 | 2.1141 |
| 439 | 10327.788 | 1.4441 | 2.1730 | 2.1179 |
| 440 | 10353.372 | 1.4434 | 2.1842 | 2.1276 |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

| WEEKS | GSE Index 2000-2010 | USD/GHC 2001-2011 | GBP/GHC 2001-2011 | EUR/GHC 2002-2011 |
|---|---|---|---|---|
| 441 | 10461.464 | 1.4421 | 2.1838 | 2.1313 |
| 442 | 10559.616 | 1.4427 | 2.1841 | 2.1357 |
| 443 | 10603.694 | 1.4423 | 2.1812 | 2.1412 |
| 444 | 10614.288 | 1.4396 | 2.1297 | 2.1331 |
| 445 | 10633.870 | 1.4362 | 2.0833 | 2.1303 |
| 446 | 10565.870 | 1.4346 | 2.0709 | 2.1314 |
| 447 | 10693.092 | 1.4369 | 2.0858 | 2.1484 |
| 448 | 10802.650 | 1.4376 | 2.0873 | 2.1447 |
| 449 | 10851.866 | 1.4386 | 2.0946 | 2.1413 |
| 450 | 10878.580 | 1.4398 | 2.1084 | 2.1402 |
| 451 | 10890.534 | 1.4401 | 2.1243 | 2.1373 |
| 452 | 10902.088 | 1.4412 | 2.1480 | 2.1342 |
| 453 | 10927.400 | 1.4481 | 2.2377 | 2.1326 |
| 454 | 10853.244 | 1.4503 | 2.2076 | 2.1274 |
| 455 | 10853.660 | 1.4496 | 2.2346 | 2.1240 |
| 456 | 10851.508 | 1.4563 | 2.2412 | 2.1271 |
| 457 | 10798.546 | 1.4481 | 2.2369 | 2.1377 |
| 458 | 10684.862 | 1.4497 | 2.2348 | 2.1392 |
| 459 | 10600.990 | 1.4500 | 2.2276 | 2.1372 |
| 460 | 10568.364 | 1.4500 | 2.2194 | 2.1315 |
| 461 | 10572.818 | 1.4499 | 2.2200 | 2.1699 |
| 462 | 10507.180 | 1.4498 | 2.2254 | 2.1215 |
| 463 | 10471.862 | 1.4493 | 2.2312 | 2.1451 |
| 464 | 10460.238 | 1.4480 | 2.3109 | 2.1557 |
| 465 | 10436.144 | 1.4469 | 2.2546 | 2.1869 |
| 466 | 10430.980 | 1.4463 | 2.2547 | 2.2010 |
| 467 | 10402.152 | 1.4459 | 2.2532 | 2.2087 |
| 468 | 10314.962 | 1.4775 | 2.3139 | 2.2084 |
| 469 | 10247.070 | 1.4524 | 2.2864 | 2.2224 |
| 470 | 10152.790 | 1.4651 | 2.2904 | 2.1896 |
| 471 | 10050.888 | 1.4531 | 2.2881 | 2.1977 |
| 472 | 9935.904 | 1.4540 | 2.2698 | 2.2125 |
| 473 | 9928.408 | 1.4549 | 2.2683 | 2.1885 |
| 474 | 9823.208 | 1.4624 | 2.2677 | 2.1657 |
| 475 | 9704.390 | 1.4687 | 2.2689 | |
| 476 | 9658.048 | 1.4815 | 2.2564 | |
| 477 | 9367.276 | 1.4868 | 2.2837 | |
| 478 | 9172.268 | 1.4883 | 2.3187 | |
| 479 | 8987.622 | 1.4996 | 2.3441 | |
| 480 | 8989.174 | 1.5166 | 2.3788 | |
| 481 | 8937.902 | 1.5228 | 2.3875 | |
| 482 | 8821.016 | 1.5150 | 2.3746 | |
| 483 | 8734.506 | 1.5140 | 2.3755 | |
| 484 | 8092.522 | 1.5138 | 2.3755 | |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

| WEEKS | GSE Index 2000-2010 | USD/GHC 2001-2011 | GBP/GHC 2001-2011 | EUR/GHC 2002-2011 |
|---|---|---|---|---|
| 485 | 7811.080 | 1.5175 | 2.3889 | |
| 486 | 7189.708 | 1.5215 | 2.3954 | |
| 487 | 6700.694 | 1.5263 | 2.4137 | |
| 488 | 6031.998 | 1.5289 | 2.4328 | |
| 489 | 5380.196 | 1.5183 | 2.4328 | |
| 490 | 5399.856 | 1.5308 | 2.4020 | |
| 491 | 5508.256 | 1.5296 | 2.4190 | |
| 492 | 5376.608 | 1.5281 | 2.4384 | |
| 493 | 5305.258 | 1.5268 | 2.4395 | |
| 494 | 5244.702 | 1.5215 | 2.4436 | |
| 495 | 5208.614 | 1.5199 | 2.4375 | |
| 496 | 5187.554 | 1.5176 | 2.4431 | |
| 497 | 5211.516 | 1.5165 | 2.4488 | |
| 498 | 5418.368 | 1.5159 | 2.4461 | |
| 499 | 6286.740 | 1.5168 | 2.4424 | |
| 500 | 6489.376 | 1.5174 | 2.4331 | |
| 501 | 6462.884 | 1.5175 | 2.4304 | |
| 502 | 6426.466 | 1.5177 | 2.4311 | |
| 503 | 6255.902 | 1.5173 | 2.4336 | |
| 504 | 6164.412 | 1.5152 | 2.4267 | |
| 505 | 6143.640 | 1.5155 | 2.4354 | |
| 506 | 6019.338 | 1.5182 | 2.3698 | |
| 507 | 5538.420 | 1.5215 | 2.3954 | |
| 508 | 5424.392 | 1.5233 | 2.4509 | |
| 509 | 5392.424 | 1.5221 | 2.4468 | |
| 510 | 5400.708 | 1.5258 | 2.4441 | |
| 511 | 5396.986 | 1.5364 | 2.4428 | |
| 512 | 5375.836 | 1.5558 | 2.4419 | |
| 513 | 5342.636 | 1.5826 | 2.4680 | |
| 514 | 5323.276 | 1.6052 | 2.4791 | |
| 515 | 5418.032 | 1.6146 | 2.4941 | |
| 516 | 5546.050 | 1.6107 | 2.4796 | |
| 517 | 5575.472 | 1.6011 | 2.4544 | |
| 518 | 5468.136 | 1.6042 | 2.4680 | |
| 519 | 5546.352 | 1.6160 | 2.4976 | |
| 520 | 5635.906 | 1.6210 | 2.5364 | |
| 521 | 5644.200 | 1.6394 | 2.5363 | |
| 522 | 5625.388 | 1.6535 | 2.5486 | |
| 523 | 5632.104 | 1.6537 | 2.5519 | |
| 524 | 5606.838 | 1.6559 | 2.5514 | |
| 525 | 5642.906 | | | |
| 526 | 5638.102 | | | |
| 527 | 5625.554 | | | |
| 528 | 5644.578 | | | |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

| WEEKS | GSE Index 2000-2010 | USD/GHC 2001-2011 | GBP/GHC 2001-2011 | EUR/GHC 2002-2011 |
|---|---|---|---|---|
| 529 | 5569.558 | | | |
| 530 | 5754.408 | | | |
| 531 | 5796.866 | | | |
| 532 | 5838.242 | | | |
| 533 | 5901.256 | | | |
| 534 | 6098.218 | | | |
| 535 | 6153.242 | | | |
| 536 | 6206.094 | | | |
| 537 | 6466.364 | | | |
| 538 | 6696.548 | | | |
| 539 | 7011.168 | | | |
| 540 | 7060.814 | | | |
| 541 | 7130.212 | | | |
| 542 | 7067.748 | | | |
| 543 | 6730.306 | | | |
| 544 | 6513.884 | | | |
| 545 | 6698.816 | | | |
| 546 | 6575.998 | | | |
| 547 | 6490.928 | | | |
| 548 | 6240.194 | | | |
| 549 | 6261.264 | | | |
| 550 | 6365.214 | | | |
| 551 | 6266.136 | | | |
| 552 | 6384.854 | | | |
| 553 | 6537.062 | | | |
| 554 | 6649.158 | | | |
| 555 | 6757.654 | | | |
| 556 | 6765.902 | | | |
| 557 | 6769.602 | | | |
| 558 | 6725.136 | | | |
| 559 | 6835.102 | | | |
| 560 | 6781.376 | | | |
| 561 | 6789.772 | | | |
| 562 | 6799.806 | | | |
| 563 | 6819.670 | | | |
| 564 | 6843.320 | | | |
| 565 | 6822.322 | | | |
| 566 | 6915.920 | | | |
| 567 | 7012.594 | | | |
| 568 | 7111.224 | | | |
| 569 | 7191.962 | | | |
| 570 | 7182.450 | | | |
| 571 | 7264.597 | | | |

Source: Bank of Ghana (2001-2011) and GSE (2000-2010)

# APPENDIX B

```
## Install stable 5.3 package and nortest package
library(stable)
library(stats)
library(MASS)
library(nortest)

## Load Data into R
usd <- read.csv("usdweek.csv") ## example: reading USD/GHC exchange rate

## Ploting times series
X <- ts(data, frequency = n, start=c(year,1))
plot.ts(X, main="title of the graph")

## Plotting Histogram and normal density curve
x <- usd$logreturn
mn <- mean(x)
stdev <- sd(x)
 hist(x, seq(-0.05, 0.05, 0.002), prob=TRUE, col ="pink", freq =FALSE, main=" title")
 lines(density(x, bw=0.001),col="green", lwd=3,)
curve(dnorm(x, mean = mn, sd= stdev), add=TRUE, col="red",lwd=2, xaxt="n")
  rug(x)     # show the actual data points
mtext(paste("mean ", round(mean(x),1), "; sd ", round(sd(x),1), "; N ", length(x),sep=""),
side=1, cex=.75)
```

```
## Density plots
x<- usd$logreturn
theta <- stable.fit(x,1,0) # maximum likelihood estimates for stable distribution
mn <- mean(x)
stdev <- sd(x)
stable.density.plot(x, theta, xrange = range(x), main=" title of the plot")
curve(dnorm(x, mean = mn, sd= stdev), add=TRUE, col="black",lwd=3, xaxt="n")
curve(dcauchy(x, location = mn, scale = stdev), add=TRUE, col="blue", lwd=3)
grid()

## Diagnostic tests
stable.diag(x, xrange = range(x), continuous = TRUE, main=" title" )
ppstable(x, theta, var.stabilized = FALSE, param = 0)
qqstable(x, theta, param = 0, ptwise.ci = TRUE, col="blue")
zzstable(x, theta, param = 0)
stable.density.plot(x, theta, param=0, xrange =
range(x),width=2*diff(quantile(usd,c(.25,.75)))*length(usd)^(-1/3),show.legend =
TRUE, continuous = TRUE, main="stable density plot" ))

qqnorm(x,col="green") # normal qq-plot
qqline(x,lwd=2)
grid()

## Estimating the parameters of stable distribution
stable.fit(x,1,0)  # MLE
```

```
stable.fit(x,2,0)  # Sample quantile

stable.fit(x,3,0)  # ECF


## Test for Normality

shapiro.test(x)

ks.test(x, "pnorm", mean = mean(x), sd = sqrt(var(x)))

ad.test(x)

chisq.test(x,dnorm(x,mean=mean(x), sd=sd(x)))


## Goodness of fit test

theta <- stable.fit(x,1,0)

stable.ks.gof(x,theta,0,0) # K-S goodness of fit test

stable.chisq( x, theta, 0, n.classes=5) # Chi-square goodnes of fit test


chisq.test(x, dcauchy(x, location = mean(x), scale=sd(x))) # Goodness of fit to Cauchy
distribution


ks.test(x, "pcauchy") # Goodness of fit to Cauchy distribution
```