

**EXTREME VALUE ANALYSIS OF TEMPERATURE AND RAINFALL:
CASE STUDY OF SOME SELECTED REGIONS IN GHANA**



**THIS THESIS IS SUBMITTED TO THE UNIVERSITY OF GHANA, LEGON IN
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STATISTICS**

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DECLARATION

This is the result of research work undertaken by Nkrumah Stephen under the supervision of Dr. Kwabena Doku-Amponsah and Dr. Richard Minkah. I declare that the entity of the work contained therein is my own work, that I am the authorship owner thereof (unless to the extent explicitly otherwise stated) and I have not previously in its entirety or in part submitted it for obtaining any qualification.

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ABSTRACT

Rainfall and temperature are important climatic factors for crop production and affects humans' basic needs such as health, shelter, food and water in Ghana. However extreme rainfall causes flooding which can lead to loss of lives, properties among others. Extreme temperature also causes drought, hot and cold spells among others which have adverse implications on human beings and agriculture. It is therefore important to know about occurrences of such extreme events and their chances of occurring. Extreme value theory gives us a statistical approach for the occurrences and the magnitude of these extreme cases or rare event that are beyond the scope of available data. In this thesis, we used "extreme value theory" to model the tail distributions of temperature and rainfall in Ghana. Daily data on rainfall and temperature was obtained and it spanned the period January 1960 to December 2012. "Extreme value theory" was used to estimate the extreme occurrence to be expected once every 5, 10, 20, 50 and 100 years respectively. It was observed that the extreme occurrences of temperature and rainfall can be modelled using Weibull and Frechet family of distributions in Ghana respectively. A maximum temperature of $34.7^{\circ}C$, $34.66^{\circ}C$, and $39.6^{\circ}C$ was predicted to occur in Accra, Ashanti and Northern regions respectively once every five years. Furthermore, it was estimated that an amount of 88.38mm, 85.61mm, and 81.98mm will be recorded in Accra, Ashanti, and Northern regions once every 5 years respectively. It was recommended that Government should employ all necessary legislative policies to help manage the issue of Global warming considering the substantial rise in temperature in the years ahead based on our forecasted predictions.

DEDICATION

This work is dedicated to the Most High God for His loving kindness and mercies towards my life.

Also to my parents, Mr. John Nkrumah and Mrs. Florence Nsiah for supporting my education. I am forever grateful for everything you have done for me. God richly bless you.



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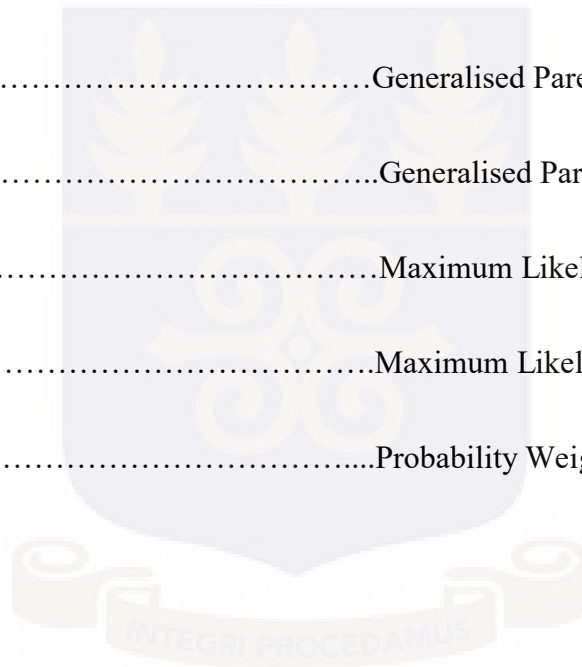
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LIST OF ABBREVIATIONS

D.F	Density Function
EVI	Extreme Value Index
EVT	Extreme Value Theory
GEV.....	Generalised Extreme Value
GEVD	Generalised Extreme Value Distribution
GP.....	Generalised Pareto
GPD.....	Generalised Pareto Distribution
ML.....	Maximum Likelihood
MLE.....	Maximum Likelihood Estimation
PWM.....	Probability Weighted Moment



CHAPTER ONE

INTRODUCTION

Extreme weather events such as heavy precipitation episodes affect our society through their adverse impact on agriculture and economy (Rosenzweig, 2001). For example the flood in August 2016 in Northern Region of Ghana which lasted ‘only’ for two days arguably killed several people, caused damages of roughly 3 billion Ghana cedis, and destroyed several crops and homes of Ghanaian citizens. Thus, knowing how often such extreme weather conditions occurs is of practical significance for planning and mitigation purposes. “Extreme Value Theory (EVT)” is extensively applied in hydrology and climatology. Results obtained from EVT in engineering practice play an essential role e.g. in flood management and water resources design (Katz & Philippe, 2002). One of several assumptions of EVT is the stationarity of time series. However, climate has a distinct seasonal cycle and as a result temperature and precipitation data with respect to the seasonal cycle cannot be considered to be stationary.

The seasonal evolution of extreme temperature and precipitation is in itself a subject of interest. Apart from its climatological significance, it can find practical use, for instance as input for flood models, amongst other parameters. The purpose of explicitly modeling the seasonal non-stationarity using daily temperature and precipitation data is to shed light on the seasonal behaviour of extreme temperature and extreme precipitation in Ghana. It is upon these reasons that this study would be focusing on applying the climate data in some selected regions in Ghana to EVT.

1.1 Background of the Study Area

The first weather observations were unquestionably made in the 1980's when the Aburi gardens was established though records from these times have not survived.

The Ghana meteorological department was established in the year 1937 but under the ministry of communications in the year 1957 the Ghana Meteorological Service was established.

The meteorological service was started with 14 stations, which had full range of meteorological instrumentation and equipment operated 24 hours a day measuring temperature, precipitation, wind and humidity among others with readings taken once or twice in a day.

There was an upgrade in the late 1990's to approximately 310 stations operating including 22 synoptic stations across the country.

Considering Ghana's distance from the equator, its climate is relatively mild but tropical. There are usually two rainy seasons across the country: From April to June and September through November. In the northern parts of Ghana however, March and April experience squalls with occasional showers until August and September. Thereafter, heavy precipitation sets in.

Available data from the meteorological service indicates that rainfall in Ghana varies from 83 to 220cm per year.

In addition, temperature readings are highest during March while lower temperature readings are experienced in August. The lowest temperature to ever be recorded in Ghana is 10°C (50°F) but the average temperature readings range from 21°C to 32°C ($70\text{-}90^{\circ}\text{F}$).

1.2 Research Problem and Justification

Concerns about extreme weather events have increased due to the emergence of global warming. Evidences of increasing “extreme weather events” for example severe cyclones, droughts, floods, heat waves and cold waves over the past few decades have been found by many researchers (Mayooran and Laheetharan, 2014).

The construction and design of certain projects, for example urban drainage systems, dams, water resources management and flood damage prevention require information on “spatial and temporal variability of extreme rainfall events” since they need a sufficient understanding of “extreme events of high return periods”.

The “return levels” of interest in most cases is usually not in the range of available data and hence “extreme value theory” is useful since it focuses on extreme data points. However, based on “statistical analysis of maximum precipitation or maximum temperature records where the parameters of a selected frequency distribution” can be estimated using available sample data, the estimation of extreme temperature or extreme rainfall is accomplished. Event magnitudes corresponding to “return periods” less than or greater than those of the recorded events are then estimated using the fitted distribution (Mayooran and Laheetharan., 2014).

Accurate estimation of low and high rainfall can help alleviate the damage caused by floods and droughts respectively in Ghana. In addition, accurate estimation of extreme (low or high) temperature and rainfall can help in various fields including health, agriculture among others.

The “distribution of annual extreme rainfalls at a single site” have a number of probability models developed to describe them (Tao, Nguyen & Bourque, 2002). In engineering practice however, choosing a suitable model still remain a major difficulty since finding which distribution should be used for the “frequency analysis of extreme rainfalls” has no general agreement. However, these distributions fit well where the bulk of the datasets are concentrated. Large bias can occur in the tails of the distribution where little or no observations are found. “Applications of extreme value theory” to temperature and rainfall data in some regions in Ghana was investigated to help identify an appropriate tail distributions for rainfall and temperature in Ghana.

1.3 Research Objectives

1. To find the appropriate distribution for the tails of the distributions of rainfall and temperature
2. To find out how extremes of rainfall relate to the extremes of temperature.
3. To find the exceedance probabilities for selected levels of rainfall and temperature
4. To obtain the return periods of extreme rainfall and temperature and their corresponding return levels for rainfall and temperature

1.4 Significance of the Study

Inter-Governmental Panel on Climate Change (IPCC) in 2010 reported that, intensity and frequency of natural disasters will rise due to climate change. “Extreme weather events” such

as floods, droughts, salinization/ contamination of agricultural water and lands supplies lead to variations in rainfall patterns and temperature with a likely decrease in agricultural productivity in already fragile areas, especially in sub-Saharan Africa and declining water availability and quality in semi-arid and arid regions.

In this thesis, we perform extreme value analysis of rainfall and temperature in Ghana to assess the return periods and exceedance probabilities of extreme weather conditions.

The study adds to the existing literature and also enriches the knowledge in this field of statistics on the application of EVT in climatology, specifically to rainfall and temperature.

1.5 Scope and Contributions of the Study

A very important issue whenever one is confronted with a data set is to determine which distribution the data are likely to have come from. In practice however, many methods are centered on the “empirical distribution function” which can lead to misleading conclusions particularly when dealing with tails of distribution functions, such as income distributions. The main objective of this thesis was to apply tools from “Extreme Value Theory” to model extreme occurrences of rainfall and temperature in some selected regions in Ghana. The main contributions of this thesis are as follows:

1. Data set for Greater Accra, Ashanti and Northern regions were assessed respectively to identify the extreme occurrence of temperature and rainfall readings.
2. Identifying the distributions to which the temperature and rainfall data belongs to was considered for the selected regions. This was done by applying the data to the “Generalised Extreme value” and the “Generalised Pareto distributions”.

3. Forecasting was however done in other to ascertain the various extreme occurrences to be expected in the next 5, 10, 20, 50 and 100 years respectively. The probabilities of exceedances for these extreme occurrences were also estimated.
4. The influence of the results in the field of Agriculture, Health and Environmental science (Global warming) and its impact on human life was extensively discussed.

1.6 Outline of the Study

In chapter 2, we provide an overview about the development of “extreme value theory” and we give an outline of the various applications of extreme value in climatology based on available literature. Chapter three is devoted to describing the Extreme Value methodology by considering the “Generalised Extreme Value” and the “Peak over Threshold” approach. In that chapter, we also introduce the various parameter estimation techniques and we review some methods of estimating the “Extreme Value Index” and choosing the threshold in the dataset above which threshold are fit to the “Generalised Pareto Distribution”.

In chapter four, we shall apply the methods presented in chapter 3 to real data (Temperature and Rainfall data) by fitting it to the “Generalised Extreme Value distribution” and the “Generalised Pareto distribution” in our bid to find the appropriate distribution to describe the dataset.

Chapter five presents the findings, conclusions and recommendations for interested bodies and future research.

CHAPTER TWO

LITERATURE REVIEW

This chapter reviews what other researchers have done on extreme rainfalls and extreme temperature by the scientific community that has served numerous reasons which include: estimation of life-threatening rainfalls for strategy resolutions; the evaluation of the rarity of observed rainfall and evaluating the approaches to evaluate strategy on rainfalls among others.

2.1 The Concept of Extreme Value Theory

“Extreme value theory entails the stochastic behaviour of the extreme values in a process”.

Fisher and Tippett (1928) indicated that, the maximal behaviour can be defined for a single process by the “three extreme value distributions; Gumbel, Weibull and Frechet”.

The “extreme value distributions” could be outlined back to the efforts by Bernoulli in 1709 as indicated by Kotz and Nadajarah (2000). Fuller in 1914 undoubtedly introduced the first “application of extreme value distributions”. Subsequently, several researchers particularly brought out useful “applications of extreme value distributions” to climate data from diverse regions in the world.

Nicolas Bernoulli deliberated on “the mean largest distance from the origin given n points lying randomly on a straight line of a fixed length t ” (Kotz and Nadajarah, 2000). A paper by Von Bortkiewicz (1922), which considered the “distribution of range in random samples from a normal distribution” started the systematic development of the general theory. The relevance of the paper by Bortkiewicz is characterised in the fact that “the concept of distribution of largest value” was for the first time introduced (Kotz and Nadajarah, 2000).

Fisher and Tippet in 1928 obtained a key result on the possible “limit laws of the sample maximum” which apparently created the impression that EVT was something very distinct and rather special from “classical central limit theory” (Beirlant, Goegebeur and Teugels, 2004).

A detailed basis for the Extreme Value Theory (EVT) was however presented in 1943 by Gnedenko and provided “sufficient and essential conditions for the weak convergence of the extreme order statistics” (Kotz and Nadajarah, 2000). Gnedenko (1943) incorporated and formalised the ideas into the basic assumption in EVT termed as the “Extreme Value Condition” (Kotz & Nadajarah, 2000). The Extreme Value Condition provides a “Semi-parametric model” solely for the tails of the distribution function.

Doctoral dissertation by Haan in 1970, on “Regular Variation and its Application to the Weak Convergence of Sample Extremes”, is an important contribution to the theoretical development of EVT (Beirlant, Goegebeur & Teugels, 2004).

Theoretical developments of the late 1930s and 1940s came with various papers which dealt with “practical applications of extreme value statistics in distributions of human lifetimes, flood analysis (Gumbel, 1941), material strength (Weibull, 1939) and Radioactive emission” as summarised by Kotz and Nadajarah (2000).

The EVT is a blend of various applications concerning regular varying functions, sophisticated “mathematical results on point processes and natural occurrences (such as corrosion, rainfall, droughts, floods and wind gusts among others)”. Therefore, the first to show interest in the development of EVT were hydrologists, engineers and theoretical probabilists. This theory has recently attracted mainstream statisticians.

Founders of statistical theory and probability; Laplace, Fermat and Pascal, among others were too engaged with the “general behaviour of statistical masses” to be interested in rare “extreme values”.

As early as 1709, Bernoulli worked on extreme value problems by discussing “the mean largest distance from the origin with a straight line of length t having n points on it at random” (Johnson, Kotz & Balakrishnan, 1995).

In 1922, exploration of EVT and systematic study started in Germany. Bortkiewicz (1922) presented a paper which dealt with the “distribution of the range of random samples from the Gaussian distribution”. Bortkiewicz’s contribution was the introduction of the “concept of distribution of largest values”. However, the concept of “expected value of the largest member of a sample of observations” was introduced by another German, Von Mises, from the Gaussian distribution in the year later (Mises, 1923). Basically, he studied samples from the Gaussian distribution with particular interest in the asymptotic distribution of extreme values.

A major first step in 1925 was initiated by (Tippet, 1925), when probabilities for various sample sizes and their corresponding tables of the largest values from a Gaussian distribution were presented along with the mean range of such samples. Frechet (1927) considered the “asymptotic distributions of largest values from a class of individual distributions” in the first paper he presented. A year later, the paper that is now regarded as the “basis of the asymptotic theory of extreme value distributions” was published by Fisher and Tippet (1928). They independently developed “Frechet’s asymptotic distribution and two others”. These three distributions were sufficient to define “extreme value distributions of all statistical distributions”. This result will be explored further in the subsequent chapter. Also, prior researcher’s encountered difficulties mainly because, they (Fisher and Tippet)

established that, “from Gaussian samples toward asymptote there is an extremely slow convergence of the distribution of the largest value”. Some useful and simple sufficient conditions for the weak convergence of the largest order statistic to each of “the three types of limit distributions” were specified by Von Mises (1936). A few years later, Gnedenko (1943) provided a rigorous foundation for the EVT and sufficient and necessary conditions for the “weak convergence of the extreme order statistics”. The theoretical developments before the mid-1940s were followed by a number of publications concerning “applications of extreme value statistics”. The first to study the “application of EVT” was Gumbel (1941) with his first application being the consideration of the largest duration of life. The author showed in the result that, EVT could be used to understand the statistical distribution of floods. Extensively, extreme value procedures have been applied to breaking strength of structural materials, other meteorological phenomena and to the outlying observations of statistical problem.

In 1958, an extensive bibliography of the developed literature was presented in Gumbel (1958). Several fields of application and theoretical developments have emerged since then and some of these recent developments are further discussed in the chapters to follow.

2.2 Extreme Temperature

A procedure for modeling of hot spells that is centered on “statistical theory of extreme values” has been suggested by Furrer, Kartz, Walter & Furrer (2010). The authors extended the “point process approach and geometric distribution to extreme value analysis” in order to model the duration, intensity and frequency of hot spells. They modeled the length of hot spells by a geometric distribution whereas annual frequency of hot spells was fitted with a Poisson distribution. The authors demonstrated the excesses over some high threshold by a

conditional Generalised Pareto distribution, and the outcome can be used to account for the temporal dependence of maximum daily temperatures within a hot spell (Furrer et al., 2010) .

In Coles (2001), statistical models for extreme values are presented. Exceedances over threshold can be modeled by Generalised Pareto (GP) distribution, and the declustering method” is used to model the dependence of exceedances. Block maxima, for which annual maxima or minima temperatures for an example, converges to Generalised Extreme Value (GEV) distribution. Return levels are obtained based on the estimation results of GP and GEV models. However, for non-stationary sequences, trends are analysed in the GP and GEV models using the generalised linear models (GLM) (Coles, 2001) .

Igor, Rychlik & Jesper (2006) also considered the “Peak over Threshold approach to estimate quantiles, “return periods and probabilities of extreme events”.

Bergstrom & Anders, (2002) reconstructed data in the period 1722-1998 for sea level air pressure and air temperature for Uppsala, Sweden based on the raw daily meteorological observations from, printed monthly bulletins, computer records and hand written registers.

In addition, trend in annual minima and maxima temperatures in Uppsala, has been investigated by Ryden (2010).

2.3 Extreme Rainfall

Nadarajah (2005) and Nadarajah and Withers (2001) presented an analysis of rainfall data in over fourteen locations spread through West Central Florida and sixteen locations across New Zealand respectively using extreme value theory. “Weibull distribution” was found to be the suitable distribution for describing annual maximal rainfall in Japan (Hirose, 1994). Additionally, the future behaviour of the extremes of rainfall was described and fitted to the

generalised extreme value distribution for five locations in South Korea from 1961 to 2001 (Nadajarah and Dongseok, 2007). Of the five locations considered, the most realistic model for four of them was the Gumbel distribution. Deka, Borah and Kakaty (2009) considered five extreme value distributions and derived the best distribution to describe the annual series of maximum rainfall data of nine distantly located stations in north east India for the period 1966 to 2007.

Varathan, Perera & Nalin (2010) found the “Gumbel distribution to be the best fitting model for describing the annual maximum rainfall in Colombo district”.

Rainfall data from 77 stations were analysed to determine the mean annual and inter- decadal rainfall variability and distribution in Ghana for the period 1981-2010 (Logah et al., 2013). They analysed the temporal variability of the decadal mean rainfall for the periods 1981-1990, 1991-2000 and 2001- 2010. The authors established that, there is variation on high and low rainfall distributions in the country. They also found that, the mean decadal rainfall amounts in Ghana is on a gradual increase from 1981 to 2010 (Logah et al., 2013).

Nkrumah, et al. (2014) used data from six meteorological stations selected from three rainfall distribution zones to explore the nature of variability and extent in the annual rainfall and pattern of the raining seasons in Ghana” from 1990 to 2008. They concluded that, there is consistent downward trend in the rainfall pattern at all the stations.

Several attempts have been made to study rainfall and temperature on the local level. However, none of the study reviewed used extreme value in estimating rainfall and temperature in Ghana. An attempt is made to study the tails of the “annual rainfall and temperature” data in some selected regions in Ghana.

CHAPTER THREE

EXREME VALUE THEORY

In order to draw information regarding particular measures in a “statistical distribution”, many statistical tools are available but this thesis looked to place emphasis on extreme values behaviour in the dataset under consideration. Assume the data are realisations of a sample $Y_1, Y_2, Y_3, \dots, Y_m$ of m “identical and independently distributed random variables” with the ordered data denoted by $Y_{1,m} \leq Y_{2,m} \leq \dots \leq Y_{m,m}$. The properties about the “quantile function; $Q(p) = \inf \{y : F(y) \geq p\}$ ” or a “distribution function; $F(y) = P(Y \leq y)$ ” are usually studied using the sampled data.

Based on the foundation of the “law of large numbers, the sample mean, $\bar{Y}$ is used as a consistent estimator of the expected value $E(Y)$ ”. Moreover, the “Central Limit Theorem (CLT) yields the asymptotic behaviour of the sample mean”. For sufficiently large sample sizes, this outcome can be used to find a “confidence interval” for $E(Y)$, a condition that is essential when applying the central limit theorem.

It can be just as essential in some cases to estimate tail probabilities. For example, supposing there can be hot spell if the temperature rises beyond $40^\circ C$, then it will be motivating to approximate the tail probability $p = (X > 40)$. The empirical distribution function defined by

$\hat{F}_m(y) = \frac{i}{m}$, if $y \in [y_{i,m}, y_{i+1,m})$ can however be used. Where $y_{i,m}$ represent the i th ordered sample value.

However, in situations where the mean $E(Y)$ or the second moment $E(Y^2)$ is infinite, both the classical theory and the central limit theorem are no longer applicable. In addition,

supposing we want to estimate $p = P(Y > y)$, where $y > y_{m,m}$ and $\hat{p}$ which is the estimate results to a value 0.

In what follows, we would introduce extreme value theory, which was the basis for our analysis of extreme values of temperature and rainfall in Ghana. Furthermore, the “Generalised Extreme Value theory” and the “Generalised Pareto distribution” would be introduced along with the parameter estimation procedures to be employed.

3.1 Extreme Value Theorem

Let $Y_1, Y_2, \dots, Y_n$ be an order of “independent random variables” with a common “distribution function” F . The focus of the model is on the statistical behaviour of

$$M_n = \max\{Y_1, Y_2, \dots, Y_n\}, \quad (3.1)$$

The Y_i generally denotes “values of a process measured on an even time scale”, say the temperature recorded hourly within a day and the M_n represents the daily maximum temperature.

In principle, we can obtain the rigorous distribution of M_n accurately for all values of n :

$$P\{M_n \leq z\} = P\{Y_1 \leq z, \dots, Y_n \leq z\} = P\{Y_1 \leq z\} \times \dots \times P\{Y_n \leq z\} = \{F(z)\}^n \quad (3.2)$$

However, the “distribution function” of F is usually unknown in practice and as a result very slight inconsistencies in the estimation of F can result in significant inconsistencies for F^n . Standard statistical techniques are mostly used to estimate F from the data observed. Therefore in EVT, F is anticipated to be unknown and on the foundation of extreme values,

the focus is on estimated “families of models for F^n when $n \rightarrow \infty$, which can then be estimated”.

Fisher and Tippett (1928) found the linear renormalisation of the variable M_n so that the function F^n does not degenerate to a point mass and this is stated in the theorem below.

The subsequent theorem gives a brief description of the “limit law for block maxima”, which is usually represented by M_n where n basically denote the block size.

“The complete range of likely limit distributions for M_n^* is given by extremal types theorem”.

Theorem 3.2.1 (Fisher and Tippett (1928), Gnedenko (1943)).

If there exist sequences of constants $\{b_n > 0\}$ and $\{a_n\}$ such that

$P\{(M_n - a_n)/b_n \leq z\} \xrightarrow{d} G(z)$ as $n \rightarrow \infty$, where G is a non-degenerate distribution function, then G belongs to one of the following families:

$$\begin{aligned}
 I : G(z) &= \exp \left\{ -\exp \left[-\left(\frac{z-a}{b} \right) \right] \right\}, & -\infty < z < \infty; \\
 II : G(z) &= \begin{cases} 0, & z \leq a, \\ \exp \left\{ -\left(\frac{z-a}{b} \right)^{-\alpha} \right\}, & z > a \end{cases} \\
 III : G(z) &= \begin{cases} \exp \left\{ -\left[-\left(\frac{z-a}{b} \right)^{\alpha} \right] \right\}, & z < a, \\ 1 & z \geq a, \end{cases}
 \end{aligned} \tag{3.3}$$

For parameters $b > 0$, a and $b > 0$ for families II and III.

These various distributions are jointly referred to as the “extreme value distribution”. “Gumbel, Frechet and Weibull families” represents type I, II and III respectively. The density function F however is contained in the “domain of attraction” G_γ thus $F \in D(G_\gamma)$.

Theorem 3.2.1 infers that, the corresponding “normalised variable M_n^* has a limiting distribution that must be one of the three types of extreme value distributions”, if M_n can be stabilised with appropriate sequences $\{b_n\}$ and $\{a_n\}$ (Coles, 2001).

3.2 Generalised Extreme Value Distribution

The various types of limit arising in the extremal type theorem have a behaviour that appears to be of distinct forms. The Frechet distribution density, G , decays polynomially whilst the Gumbel distribution, F , decays exponentially.

In applications, these various distributions give relatively different representations of “extreme value behaviour”. It used to be common in early applications to implement any of the “three families of extreme value theory” and approximate parameters of that distribution.

Nevertheless, a weakness of this method is that you require a technique to select which of the three families is best suitable. In addition, once a domain of attraction is selected, successive inferences assume this selection as accurate and do not permit for the errors involved in such a selection, although these errors may be significant (Coles 2001).

Jenkinson (1955) and Von mises (1930) proposed a three parameter model termed the Generalised Extreme Value model and the advantage of this is that, it is one family of model in which you are able to determine your domain of attraction based on the shape parameter.

The choice of the domain of attraction can be accurately made by allowing for the behaviour of the limit distribution G at $y_+ = \sup\{z : F(z) < 1\}$, its upper end point. The “Weibull distribution” has a finite upper end point whereas the “Frechet and Gumbel family of distributions” both have an infinite upper end point (Coles, 2001). The three models can be jointly presented in one family of models referred to as the “generalised extreme value family” with distribution functions of the form

$$G(z) = \exp \left\{ - \left[1 + \gamma \left(\frac{z - \mu}{\sigma} \right) \right]^{\frac{1}{\gamma}} \right\}, \quad (3.4)$$

Defined on $\{z : 1 + \gamma(z - \mu)/\sigma > 0\}$ and with parameters satisfying

$-\infty < \mu < \infty$, $-\infty < \gamma < \infty$ and $\sigma > 0$. The parameters μ , γ and σ are “location, shape and scale parameters” separately. The “three classes of extreme value distribution” are distinguished by the value of the shape parameter (γ). However, types II and III in Theorem 3.2.1 correspond to values of $\gamma > 0$ and $\gamma < 0$ respectively. In addition, the case of $\gamma = 0$ is interpreted as the limit as $\gamma \rightarrow 0$ of the “GEV distribution” and its “distribution function” is

$$G(z) = \exp \left[- \exp \left\{ - \left(\frac{z - \mu}{\sigma} \right) \right\} \right], \quad -\infty < z < \infty. \quad (3.5)$$

3.3 Asymptotic Models for Minima

When looking at problems such as temperature variations, it is mostly of interest to research on the tail behaviour of the minimum temperature recorded especially when the interest is to study the possibility of a cold weather and these practicalities involve models for particularly small, rather than very large observations.

Consider the minimum $\tilde{M}_n = \min\{Y_1, Y_2, \dots, Y_n\}$, where the Y_i denote a data point for the random variable Y . Supposing that Y_i is identical and independently distributed, similar arguments relate to $\tilde{M}_n$ as were applied to M_n , leading to a “limiting distribution” of an appropriately re-scaled variable. However, there is duality between minimum and maximum such that $\tilde{M}_n = \min\{Y_1, \dots, Y_n\}$ with $M_n = \max\{-Y_1, \dots, -Y_n\}$, that is to say $\tilde{M}_n = -M_n$ (Coles, 2001).

Therefore, for n large,

$$P\{\tilde{M}_n \leq z\} = P\{-M_n \leq z\} = P\{M_n \geq -z\} = 1 - P\{M_n \leq -z\} \quad (3.6)$$

$$P\{\tilde{M}_n \leq z\} \approx 1 - \exp\left\{-\left[1 + \gamma\left(\frac{-z - \mu}{\sigma}\right)\right]^{-1/\gamma}\right\} \quad (3.7)$$

$$P\{\tilde{M}_n \leq z\} \approx 1 - \exp\left\{-\left[1 - \gamma\left(\frac{-z - \tilde{\mu}}{\sigma}\right)\right]^{-1/\gamma}\right\}$$

on $\{z : 1 - \gamma(z - \tilde{\mu})/\sigma > 0\}$, where $\tilde{\mu} = -\mu$.

This is termed as the “Generalised Extreme Value distribution for the minima”.

Given that $y_1, y_2, \dots, y_m$ are observations from the “Generalised Extreme Value distribution for the minima, with parameters $(\tilde{\mu}, \sigma, \gamma)$, this indicates fitting the GEV distribution for maxima to the data $-y_1, -y_2, \dots, -y_m$ ”.

In some circumstances, “GEV distribution for minima” can be applied directly since modelling the “block minima” is suitable. Also, given data $y_1, y_2, \dots, y_m$ to model the

minima, the duality between the distributions for minima and maxima can be explored alternatively.

Apart from the sign correction $\hat{\mu} = -\hat{\mu}$, the “maximum likelihood estimate” of the parameters of this distribution corresponds exactly to that of the required “Generalized Extreme Value (GEV) distribution for minima”.

3.3.1 Parameter Estimation under the GEV

Under the Generalised extreme value distribution, the “sample of size n is divided into m blocks of size k , with $n = m \times k$ and k sufficiently large”. In the Annual Maxima method generally, a block represents a period of one year and the k being realisations per year. However, blocks can represent daily, weekly or monthly maxima. The largest observation in each block is carefully chosen to get a sample of m “independent sample maxima”. Assume Y to be the random variable denoting the maximum of a block with size k , then;

$$M_k = \max(Y_1, \dots, Y_k) \quad (3.8)$$

Bearing in mind m blocks, we find a collection of m sample maxima $(M_{1,k}, \dots, M_{m,k})$.

Subsequently, we can find estimates of the “Generalized Extreme Value (GEV)” for the Extreme Value Index (γ) , along with the location (μ) and the scale (σ) parameters. In literature, there are a number of estimation methods for the “Generalized Extreme Value Distribution (GEVD)” but we will consider the “Probability Weighted Moments (PWM)” and the “Maximum Likelihood (ML) method” in this subsection.

3.3.1.1 Maximum Likelihood Estimation

The parameters of the “Generalized Extreme Value (GEV) distribution” were estimated by “Maximum Likelihood method”. Adaptability of the Maximum Likelihood to changes in model structure as compared to other parameter estimation techniques makes it preferable. Although, when the model is altered, the estimating equations change, the core procedure is fundamentally unaffected. The Maximum Likelihood (ML) also has “an expedient set of off-the shelf large sample inference properties” (Coles, 2001).

Let $(Y_1, \dots, Y_m)$ be a random sample of the random variable Y taken from a “Generalized Extreme Value (GEV) distribution”.

The “log-likelihood function” for $\gamma \neq 0$ of an observed “random sample” $(y_1, \dots, y_m)$ is given by

$$l(\mu, \sigma, \gamma | z_1, \dots, z_m) = -m \log \sigma - \left(\frac{1}{\gamma} + 1 \right) \sum_{i=1}^m \log \left(1 + \gamma \frac{z_i - \mu}{\sigma} \right) - \sum_{i=1}^m \left(1 + \gamma \frac{z_i - \mu}{\sigma} \right)^{-\frac{1}{\gamma}}, \quad (3.9)$$

Provided that

$$1 + \gamma \frac{z_i - \mu}{\sigma} > 0, \quad i = 1, 2, \dots, m$$

However, when $\gamma = 0$ the “log-likelihood function” becomes

$$l(\mu, \sigma, \gamma | z_1, \dots, z_m) = -m \log \sigma - \sum_{i=1}^m \exp \left(-\frac{z_i - \mu}{\sigma} \right) - \sum_{i=1}^m \frac{z_i - \mu}{\sigma}, \quad (3.10)$$

The Maximum Likelihood estimators $(\hat{\mu}, \hat{\sigma}, \hat{\gamma})$ for the unknown parameters (μ, σ, γ) was found by maximizing (3.10).

We get the likelihood system of equations which has no explicit solution by differentiating expression (3.10). Consequently, this system of equations must be solved by numerical methods or iteratively.

The common regularity conditions fundamental to the “asymptotic properties of ML estimators” are not automatically applicable because the support of the GEV depends on unknown parameter values (Beirlant et al, 2004).

According to Smith (1985) as cited in Coles (2001), researched thoroughly into this problem and his discoveries are briefly discussed as follow;

- The “ML estimators” are not likely obtainable when $\gamma < -1$
- The “ML estimators” are regular, and therefore have a usual asymptotic properties when $\gamma > -0.5$.
- Also, the “ML estimators” do not have the common asymptotic properties, but is obtainable when $-1 < \gamma < 0.5$

3.3.1.2 Probability Weighted Moments (PWM)

“Probability-Weighted Moments (PWM) estimator” is also a very standard estimation method for the generalised extreme-value density function. PWM’s are generalisation of the usual moments of a “probability distribution”, which give increasing weight to the tail information. Parameters of several distributions such as; the Weibull, Gumbel and logistic density functions can be appropriately estimated using the “probability-weighted moments”. However, the “Gumbel distribution” is an exceptional case of the “generalised extreme-value distribution”, thus indicating that the PWM may be significant in the “generalised extreme-

value” d.f as well. Estimating parameters of the “generalised extreme value distribution” using the PWM is described in details by (Hosking, Wallis & Wood, 1985).

Definition: The “probability-weighted moments of a random variable Y with distribution function” F are the quantities:

$$M_{p,r,s} = E \left[Y^p \{F(y)\}^r \{1-F(y)\}^s \right], \quad p, r, s \in \mathfrak{R} \quad (3.11)$$

However in the context of “generalised extreme-value estimation”, we use “probability weighted moments” of the form

$$\beta_r = M_{1,r,0} = E \left[Y \{F(y)\}^r \right], \quad r = 0, 1, 2, 3, \dots \quad (3.12)$$

Given a “random sample” of size m from the distribution F , estimation of β_r is most conveniently based on the ordered sample $Y_{1:m} \geq Y_{2:m} \geq \dots \geq Y_{m:m}$ an “unbiased estimator” of β_r is given by the statistic

$$b_r = \frac{1}{m} \sum_{i=1}^m \left[\frac{(i-1)(i-2)(i-3)\dots(i-r)}{(m-1)(m-2)(m-3)\dots(m-r)} Y_{m-i+1:m} \right] \quad (3.13)$$

However, one can derive the PWMs for the generalised extreme value density function

$H_{\gamma,\mu,\sigma}$ (see Hosking et al., 1985 for proof)

$$\beta_r = \frac{1}{r+1} \left\{ \mu - \frac{\sigma}{\gamma} \left[1 - (r+1)^\gamma \Gamma(1-\gamma) \right] \right\} \quad (3.14)$$

for $\gamma < 1$ and $\gamma \neq 0$.

For $r = 0, 1, 2$ we have the following system of equations:

$$\beta_0 = \mu - \frac{\sigma}{\gamma} [1 - \Gamma(1 - \gamma)], \quad 2\beta_1 - \beta_0 = \frac{\sigma}{\gamma} \Gamma(1 - \gamma) (2^\gamma - \gamma) \quad \text{and} \quad \frac{3\beta_1 - \beta_0}{2\beta_1 - \beta_0} = \frac{1 - 3^\gamma}{1 - 2^\gamma} \quad (3.14a)$$

The PWM estimators $\hat{\theta} = (\hat{\gamma}, \hat{\mu}, \hat{\sigma})$ of the parameters are the solutions of the equation (3.14a) when the moments β_r are replaced by their empirical counterparts.

3.3.4 Estimation of other Parameters of Extreme Events

In this subsection, we introduce the inference for return levels, return periods and exceedance probability.

When considering “extreme values of a random variable”, say y_p , such that there is a probability, p , that y_p is exceeded in any given year, the “return level” of an extreme event is mostly of concern. Conversely, the “return level” related with the “return period” $1/p$ is the level that on the average is expected to be exceeded once every $1/p$ years. However, y_p is the “return level associated with the return period” $1/p$ in extreme value theory.

Assuming the “100-year return level” for temperature is found to be $50^\circ C$, then the probability of temperature surpassing $50^\circ C$ in any particular year is 0.01 (i.e. $1/100$).

The “rarity” of an “extreme event” is indicated by the quantity $1/p$, generally termed as the waiting time or return period for an event.

Let y_{1-p} be “the extreme quantile with order $(1-p)$ of the generalised extreme value distribution” fundamental to the random variable Y , well-defined in (3.1), with p

sufficiently small. Inverting the “generalised extreme value distribution” will lead to estimates of extreme quantiles. That is, the return level is obtained as;

$$y_{1-p} = G_{\gamma}^{-1}(1-p | \mu, \sigma) = \begin{cases} \mu + \frac{\sigma}{\gamma} \left\{ (-\log(1-p))^{-\gamma} - 1 \right\}, & \text{if } \gamma \neq 0 \\ \mu - \sigma \log(-\log(1-p)), & \text{if } \gamma = 0 \end{cases} \quad (3.15)$$

where (μ, σ, γ) is replaced by its equivalent maximum likelihood estimator.

Extreme quantiles can be expressed in terms of the “tail quantile function” with $t = \frac{1}{p}$:

$$U\left(\frac{1}{p}\right) = (1-p | \mu, \sigma) = y_{1-p} \quad (3.16)$$

Dealing with “Annual Maxima” in particular, the “T-period level, $U(T)$ ”, is given by;

$$T = \frac{1}{1 - G_{\gamma}(u | \mu, \sigma)} = \frac{1}{P(X > u)}, \quad (3.17)$$

where the denominator $P(X > u)$ is called exceedance probability. The mean value of a geometric random variable can also be used to interpret the return period.

Let N_u be the number of periods needed for the first time to exceed the level u . Therefore,

N_u is a “geometric random variable” having mean value $1/p_u$, with $p_u = P(N_u > u)$.

Therefore, we have $T = E(N_u)$.

If we are looking to obtain the quantiles of Y and make inference about the population principal to the m blocks of observations $Y_1, Y_2, \dots, Y_k$, with $Y \in F$, we then know the random variable Y defined in (3.8) is distributed as

$$F_Y \equiv F_{M_k} = F^k \square G_\gamma \quad (3.18)$$

Where k is length of the block

For the random variable Y , we denote by Y_{1-p} the quantile of order $(1-p)$ of the density function F underlying the random variable Y , i.e., a quantity such that $F(Y_{1-p}) = 1-p$.

Using now relation (3.17), we have, $F^k(Y_{1-p}) = G_\gamma(Y_{1-p} | \mu, \sigma)$ and then

$$Y_{1-p} \square G_\gamma^{-1}((1-p)^k | \mu, \sigma) = \begin{cases} \mu + \frac{\sigma}{\gamma} \left\{ \left(-\log(1-p)^k \right)^{-\gamma} - 1 \right\}, & \text{if } \gamma \neq 0 \\ \mu - \sigma \log \left(-\log(1-p)^k \right), & \text{if } \gamma = 0 \end{cases} \quad (3.19)$$

Again, estimates of (3.17) and (3.19) are obtained by replacing (μ, σ, γ) by its ML estimator.

The right endpoint of the generalised extreme value distribution is finite if $\gamma < 0$. Hence, we can approximate the finite right endpoint of the underlying density function F by

$$y^F = U(\infty) = Y_1 = \mu - \frac{\sigma}{\gamma} \quad (3.20)$$

3.4 Peaks-Over Threshold (POT) Method

The “Peaks Over Threshold (POT) method”, considers the distribution of exceedances over a certain threshold. The interest in the use of the “peaks over threshold” method is estimating the “distribution function” F_u of values of x above a certain threshold u .

The “distribution function” F_u is called the “conditional excess distribution function” and is defined as

$$F_u(x) = P(Y - u \leq x | Y > u) \quad 0 \leq x \leq y_F - u \quad (3.21)$$

where Y is a random variable, u is a threshold given, $x = y - u$ are the excesses and $y_F \leq \infty$ is the right endpoint of F . However, we can verify that F_u can be written in terms of F , that is

$$F_u(x) = \frac{F(u + x) - F(u)}{1 - F(u)} = \frac{F(y) - F(u)}{1 - F(u)} \quad (3.22)$$

Generally, the estimation of F in this interval generally poses no problem since the realizations of the random variable Y lie mainly between 0 and u . However, the estimation of the portion F_u might be difficult since we have very little observations in general within this area. EVT can prove very helpful at this point since it provides us with a dominant result about the conditional excess distribution function which is stated in theorem 3.4.1.

Determining a threshold is the first consideration for the POT approach and data beyond the threshold are fitted to the “Generalised Pareto Distribution (GPD)”, which is based on the excesses above a threshold and it has an asymptotic justification as the “Extreme Value Theory”. As the sample size increases and the threshold u gets large, the amounts by which

the observations exceed a threshold u (termed exceedances) should approximately follow the “Generalised Pareto Distribution”.

Let $Y_1, Y_2, \dots$ be a sequence of “identical and independently distributed random variables with a common distribution function F ” with u being a sufficiently high threshold value. We regard an extreme value of observations as one exceeding the threshold and the description of its asymptotic behaviour as a conditional probability. Thus if the underlying distribution F is known, then the “conditional distribution F_u ” can be obtained. However, as pointed out earlier, it is often the case that F is unknown and as a result, an asymptotic distribution for high threshold is needed. Balkema and de Hann (1974) and Pickands (1975) provides analogous results to the Fisher-Tippett theorem as follows;

Theorem 3.4.1 (Pickands (1975), Balkema and de Hann (1974))

For a large class of underlying “distribution function F ” the conditional excess “distribution function” $P\{M_n \leq z\}$, for n large, is well approximated by $P\{M_n \leq z\} \approx G(z)$, $n \rightarrow \infty$, where

$$G_{\gamma, \sigma}(z) = \begin{cases} 1 - \left(1 + \frac{\gamma}{\sigma} z\right)^{1/\gamma} & \text{if } \gamma \neq 0 \\ 1 - e^{-z/\sigma} & \text{if } \gamma = 0 \end{cases} \quad (3.23)$$

For $z \in [0, (x_F - u)]$ if $\gamma \geq 0$ and $z \in [0, -\sigma/\gamma]$ if $\gamma < 0$.

$G_{\gamma, \sigma}(z)$ is the “Generalised Pareto Distribution (GPD)”.

If x is defined as $x = u + z$, then the “GPD” can be expressed as

$$G_{\gamma, \sigma}(z) = 1 - (1 + \gamma(x - u)/\sigma)^{-1/\gamma} \quad (3.24)$$

Castillo and Hadi (1997) made the following remarks about the “Generalised Pareto Distribution”

- When $\gamma = 0$, the $GPD(\gamma, \sigma)$ reduces to the “exponential distribution” with mean $Exp(\sigma)$.
- When $\gamma = 1$, the $GPD(\gamma, \sigma)$ becomes “uniform”: $U[0, \sigma]$.
- When $\gamma < 0$, the $GPD(\gamma, \sigma)$ reduces to the “Pareto distribution” of the second kind.

The qualitative behaviour of the “Generalised Pareto distribution” is dominantly determined by the shape parameter γ due to the duality between the “GEV and generalised Pareto families just as it is for the GEV distribution”. If $\gamma < 0$ the distribution of excesses has an upper bound of $u - \tilde{\sigma}/\gamma$ (Beta type): If $\gamma > 0$, the distribution is “heavy tailed” and has no “upper limit” (Pareto type). The distribution is “light tailed” and unbounded if $\gamma = 0$ (Exponential type) which should again be interpreted by taking the limit $\gamma \rightarrow 0$.

From Theorem 3.4.1, if “block maxima” have an approximating distribution G within the “generalised Pareto family”, the threshold excesses have a corresponding approximate distribution. Furthermore, “parameters of the generalised Pareto distribution” of threshold excesses are determined distinctively by those of the associated “GEV distribution of block maxima”. In particular, the parameter γ in equation 3.6.1 is equal to that of the corresponding GEV distribution. Selecting a different, but still large, block size n would

affect the values of the GEV parameters, but not those of the corresponding “generalised Pareto distribution of threshold excesses”.

3.4.1 Parameter Estimation under the POT Method (Maximum Likelihood Estimation)

From the Balkema and de Hann theorem, the exceedances over a sufficiently large threshold, u , can be characterized by the GPD. In the literature, several methods are presented for estimating the parameters of the GPD. These include; ML, PWM, Method of moments, elementary percentile method, among others.

In this thesis, we make use of the ML and the PWM. In the case of ML, a likelihood function can be formed from the distribution (3.5) as

$$l(\gamma, \sigma_u | y_1, \dots, y_m) = \begin{cases} -m \log \sigma_u - \left(\frac{1}{\gamma} + 1 \right) \sum_{i=1}^m \log \left(1 + \frac{\gamma y_i}{\sigma_u} \right), & \gamma \neq 0 \\ -m \log \sigma_u - \frac{1}{\sigma_u} \sum_{i=1}^m y_i, & \gamma = 0 \end{cases} \quad (3.25)$$

where $1 + \frac{\gamma y_i}{\sigma_u} > 0$, $i = 1, 2, \dots, m$

Generally, we prefer a reparametrization of the “log-likelihood function” in (3.25) for computational purposes. The main objective of this reparametrization, is to get $\hat{\gamma}$ explicitly as a function of $\hat{\zeta}$ as introduced by Davison (1984) is obtained numerically through (3.10.9), but after replacing $\hat{\zeta}$ by $\frac{1}{m} \sum_{i=1}^m \log(1 + \hat{\zeta} y_i)$.

For $\gamma = 0$, $\hat{\sigma}_u = \bar{Y}$ which is the classical case of the exponential distribution.

Defining $\zeta = \frac{\gamma}{\sigma_u}$, the “log-likelihood function” may well be rewritten as

$$l(\gamma, \zeta | y_1, \dots, y_m) = -m \log \gamma + m \log \zeta - \left(\frac{1}{\gamma} + 1 \right) \sum_{i=1}^m \log(1 + \zeta y_i) \quad (3.26)$$

where $1 + \zeta y_i > 0$, $i = 1, 2, \dots, m$. Maximization of (3.26) with respect to γ and ζ yields the “ML estimator $(\hat{\gamma}, \hat{\zeta})$ ” of the parameters (γ, ζ) .

3.4.2 Estimation of other Parameters of Extreme Events

After obtaining estimates of the parameters γ and σ . Defining once again the “extreme quantile of order $(1-p)$ ” of the GPD underlying the excesses Y as Y_{1-p} , with p sufficiently small, estimates of “extreme quantiles” can be obtained by inverting the GPD

$$H_\gamma^{-1}(1-p | 0, \sigma_u) = \begin{cases} \frac{\sigma_u}{\gamma} (p^{-\gamma} - 1)^k, & \text{if } \gamma \neq 0 \\ -\sigma_u \log p, & \text{if } \gamma = 0 \end{cases} \quad (3.27)$$

where (γ, σ_u) are replaced by its ML estimator.

If $\gamma < 0$, the GPD has a finite endpoint and is given by

$$U_{H_\gamma}(\infty) = y_1 = \frac{\mu - \sigma_u}{\gamma} \quad (3.28)$$

Which can be estimated by replacing the (γ, σ_u) by its corresponding ML estimator.

However, with the “asymptotic normality of the ML estimators” as the basis, confidence intervals can be constructed for parameters of the GPD. The $100(1-\alpha)\%$ confidence interval for γ is

$$CI_{\gamma} = \left\{ \gamma : \log L_p(\gamma) \geq \log L_p(\hat{\gamma}) - \frac{\chi_{(1)}^2(1-\alpha)}{2} \right\} \quad (3.29)$$

3.4.3 Threshold Selection

It is still an unsolved problem as to the procedure for the choice of threshold u and it is not straightforward; indeed, a compromise has to be found between high values of u , where the bias of the estimators is smaller, and low values of u , where the variance is smaller.

The “mean excess function” is a tool generally used to aid the choice of u and also to determine the appropriateness of the “Generalised Pareto Distribution” model in practice. The mean excess function sometimes referred to as the “mean residual life function” of a random variable Y is defined as

$$M(u) = E[X - u | Y > u] \quad (3.30)$$

provided $E(Y) < \infty$.

Davison and Smith (1990) used the property that the linearity of the “mean excess function” illustrates the “Generalised Pareto Distribution” class to formulate a simple graphical check that data conform to a “Generalised Pareto Distribution model”.

If the mean excess plot is close to linear for high values of the threshold then there is no indication against use of a GPD model.

Let Y be a positive random variable with finite first moment and “distribution function F ”.

Then the “mean excess function” of Y for all $u > 0$ is

$$e(u) = E(Y - u | Y > u) = \frac{1}{\bar{F}(u)} \int_u^{y_F} \bar{F}(y) dy \quad (3.31)$$

where $y_F = \sup\{y : F(y) < 1\}$

The “mean excess plot” is the set of points $\{u, e(u)\}$ for all $u > 0$.

The empirical counterpart of the mean excess function based on the sample $(y_1, y_2, y_3, \dots, y_n)$

$$\text{is } \hat{e}(u) = \frac{\sum_{i=1}^n y_i 1_{(u, \infty)}(y_i)}{\sum_{i=1}^n 1_{(u, \infty)}(y_i)} - u \quad (3.32)$$

where $1_{(u, \infty)}(y) = 1$ for $y > u$, 0 otherwise

Generally, the expression (3.32) is obtained by replacing the theoretical average by its empirical counterpart that is to say by averaging the data that are larger than u and subtracting u (Beirlant et al, 2004).

The “mean excess plot” is estimated at the points $u = x_{m-p, m}$, $p = 1, 2, \dots, m-1$, the $(p+1)$

largest observation and the mean residual function then takes the form

$$\hat{e}_{p, m} = \hat{e}_m(x_{(m-p):m}) = \frac{1}{p} \sum_{j=1}^p x_{m-j+1, m} - x_{m-p, m}, p = 1, 2, \dots, m \quad (3.6.3.d)$$

If $F \in D(G)$ $\gamma > 0$ then it is easy to show that,

$$e_{\ln X}(\ln u) = E(\ln X - \ln u \mid X > u) \rightarrow \gamma \text{ as } u \rightarrow \infty$$

It can however be shown that the statistic $\hat{e}_{m, \ln X}(\ln x_{m-p, m})$ with $\hat{e}_{m, \ln X}$ denoting the empirical “mean excess function” of the log-transformed data, appears as the “Hill estimator” (Beirlant et al, 2004).

3.4.4 Statistical Choice of GPD Models

Exploratory data analysis is often used to test the “goodness-of-fit” of sample observations to specific target distributions. A few graphical tools have been extensively used to detect heavy-tailed behaviour or extremal behaviour in observed data

In view of the likelihood of modeling any combination of the “extreme value model parameters” (such as Temperature or Rainfall) as functions of time or other covariates, there is a wide range of models to choose from, and selecting the best fitting model becomes an essential issue.

We will employ the Quantile-Quantile plots, Probability-Probability plots, Mean Excess plots and Return Level plots to assess the quality of a fitted “Generalised Pareto model”.

Assuming a threshold u , an estimated model $\hat{H}$ and threshold excesses $y_{(1)} \leq \dots \leq y_{(k)}$, with $\hat{\gamma} \neq 0$, the Quantile plot which is a plot of the empirical quantiles from the data against the theoretical quantiles of $\hat{H}$ will consist of the pairs

$$\left\{ \left(H^{-1}(i/(k+1)), y_{(i)} \right), i = 1, 2, \dots, k \right\} \quad (3.33)$$

where $\hat{H}^{-1}(y) = u + \frac{\hat{\sigma}}{\hat{\gamma}} [y^{-\hat{\gamma}} - 1]$

Again the probability plot will consist the pairs $\left(i/(k+1), H(y_{(i)})\right); i=1, \dots, k$

where $\hat{H}(y) = 1 - \left(1 + \hat{\gamma} \frac{y}{\hat{\sigma}}\right)^{-1/\hat{\gamma}}$, provided $\hat{\gamma} \neq 0$. However, the plot is constructed by using

$1 - e^{-z/\sigma}$ in place of $1 - \left(1 + \frac{\gamma}{\sigma} z\right)^{1/\gamma}$.

For the GP model to be reasonable for modelling excesses of u , both the “quantile and probability plots” should entail points that are approximately linear (Coles, 2001). Also, a

“return level plot” which consist of the locus points $\{(m, \hat{y}_m)\}$ for large values of m , where

$\hat{y}_m$ is the estimated m -observation return level $\hat{y}_m = u + \frac{\hat{\sigma}}{\hat{\gamma}} \left[(m \hat{\zeta}_u)^{\hat{\gamma}} - 1 \right]$ which is modified if

$\hat{\gamma} = 0$.

3.5 Comparison of ML and PWM Estimation Methods for GEV Distribution

Hosking et al. (1985) performed a simulation study of the small-sample properties (in particular their study was concentrated on sample sizes $n=15, 25, 50, 100$) of the PWM estimators of the “generalised extreme-value distribution”, in comparison to ML and sextiles estimators. Sextiles estimators had been originally also used for the estimation of the parameters of “generalised extreme-value density functions”, still they have been proven to be inferior of the other methods, so they are not used anymore. As far as estimators of the parameter γ are concerned, all three estimation methods are equivalent for $n=100$, but for smaller sample sizes the “PWM estimator” has lower variance. Still, “PWM estimator” has in general larger bias than the other estimators, though the bias is small near the “critical value”

$\gamma = 0$. Similar results are obtained for the estimators of the scale and location parameters, though the differences in the variances are less pronounced.

Another quantity of interest in extreme-value analysis are (large) quantiles.

Estimators of quantiles can be simply obtained by substituting the estimators of the parameters θ to the quantile function of the generalised extreme-value distribution. So, when comparing (via simulation) the quantile estimators obtained by the three different estimation methods, Hosking et al. (1985) get the following findings. For $n=100$, all methods are comparable. For small samples the upper quantiles obtained by the PWM method are rather biased, but they are still preferable to the ML estimators, which have very large biases and variances. All the methods are very inaccurate when estimating extreme quantiles in small samples with $\gamma > 0$.

Another comparison of ML and PWM estimation methods was performed by Coles and Dixon (1999). They also compiled a simulation study to explore the small-sample properties of ML and PWM. Their results confirmed the results of Hosking et al. (1985), that is, for small sample sizes, the ML estimator is seen to be a poor competitor to the PWM estimator, in terms of both bias and mean square error. Still, while Hosking et al. (1985) were obviously in favor of the PWM estimators, Coles and Dixon (1999) share the view that as a general, all-round procedure for extreme value modelling, likelihood-based methods are preferable to any other. Still, despite the many advantages of maximum likelihood mentioned in the previous section, poor performance in small samples remains a serious criticism, since it is not uncommon in practice to need to make inferences about extremes with very few data – the rarity of extreme events means that even long observational periods may lead to very few data that can be incorporated into an extreme value model. So, Coles and Dixon (1999) tried to examine more closely the comparison between ML and PWM for estimating parameters of the generalised extreme-value density function with small datasets by exploiting the

simulated distributions of the parameter estimates. Understanding more clearly the failings of ML, Coles and Dixon (1999) explore the possibility of correcting for such deficiencies.



CHAPTER FOUR

RESULT AND DISCUSSION

Daily minimum and maximum temperature as well as the rainfall readings for all the ten (10) regions in Ghana were obtained from the Ghana Meteorological Agency in Accra. The daily temperature and rainfall readings were employed in the study and it spanned the period January 1960 to December 2012. The study focused on three key regions which takes care of the Southern, Central and Northern sectors of the country. The selection was purely subjective. These regions were Greater Accra (Southern), Ashanti region (Central) and Northern region (Northern). This chapter presents the analysis of the data and it begins with basic statistical analysis involving graphical tools and basic statistics. In addition, the extreme value analysis of both temperature and rainfall for these three major regions in Ghana are presented. The Generalized Extreme Value Model and the Threshold Exceedances model were applied to the data. The chapter further explored the “left and right tails of the Generalized Pareto Distribution” via the data just as in the case of the “GEV distribution”. The R software was employed in the data analysis.

4.1 Basic Statistics

This section of the study presents the summary statistics and assesses the stationarity of the data. Extreme observations were explored through scatter plots.

Table 4.1 presents the summary statistics of both the daily temperature and rainfall readings of the three selected regions.

Table 4.1: Summary statistics of Data

Variable		Region	Minimum	1 st Quartile	3 rd Quartile	Maximum	Median	Skewness
Temperature	Minimum	Accra	17.5	20.6	21.95	25.50	21.20	0.17
		Ashanti	17.00	19.17	20.40	22.30	20.00	-0.56
		Northern	17.00	19.60	21.00	24.40	20.00	-0.28
	Maximum	Accra	28.30	31.70	34.40	38.90	32.90	0.22
		Ashanti	28.30	31.70	34.40	38.90	32.90	0.22
		Northern	31.10	34.40	39.30	42.8	37.2	-0.09
Rainfall	Maximum	Accra	0.00	6.925	43.630	243.90	29.30	2.02
		Ashanti	0.00	6.00	21.30	167.90	9.10	2.36
		Northern	0.00	2.40	46.70	143.00	27.60	0.91

From Table 4.1, the minimum temperature in Greater Accra ranges from $17.5^{\circ}C$ to $25.5^{\circ}C$ with a median temperature reading of $21.2^{\circ}C$. The corresponding maximum temperature for the region ranged from $28.3^{\circ}C$ to $38.9^{\circ}C$ with a median temperature reading of $32.9^{\circ}C$. Northern region recorded high temperatures as compared to Greater Accra and Ashanti. Interestingly, the temperatures recorded in Greater Accra and Ashanti regions were approximately the same. The data was approximately normally distributed based on the observed skewness.

Also, the minimum rainfall readings observed in the three regions was 0.0mm. Greater Accra however recorded the highest rainfall of 243.9mm with Northern region recording the least

amount of rainfall. Again, the data on amount of rainfall was approximately normal. The scatter plots of the minimum and maximum daily temperature readings as well as that of the maximum rainfall observed in these regions are depicted in Appendix A.

Furthermore, the stationarity of the data was tested using the “augmented Dickey-Fuller (ADF)”. In this test the null hypothesis that data is nonstationary (has a unit root) is tested. Both the undifferenced and the first difference of the temperature and rainfall data were tested. Table 4.2 present the stationarity test on the undifferenced series for temperature and rainfall.

Table 4.2: ADF Stationarity Test on Temperature and Rainfall Data

Variable	Temperature				Rainfall	
	Minimum		Maximum		Maximum	
	t – stat	p – value	t – stat	p – value	t- stat	p-value
Accra	-6.97	0.000	-0.68	0.000	-8.77	0.000
Ashanti	-27.48	0.000	-0.68	0.000	-10.99	0.000
Northern	-0.56	0.000	-1.00	0.000	-6.97	0.000

From Table 4.2, the “null hypothesis of nonstationarity” was rejected for both the temperature and rainfall data for the selected regions (all p values are less than 5% level of significance).

4.2 Greater Accra

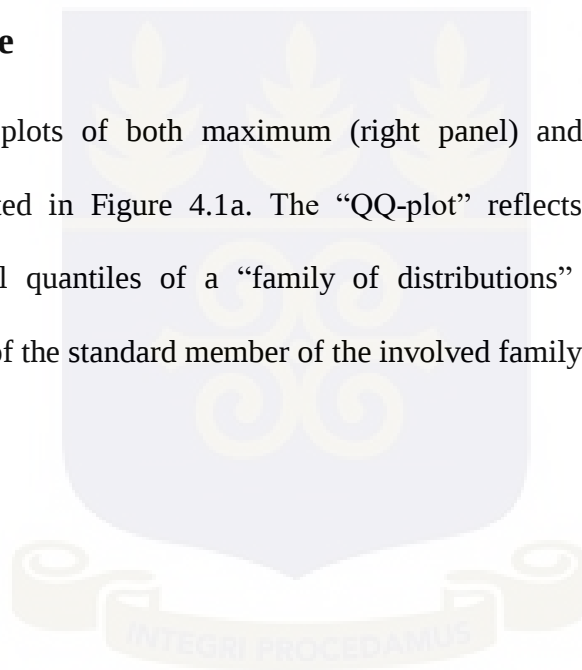
In this section of the study, the extreme value and the “Generalized Pareto models” were fitted to the temperature and rainfall data of Greater Accra.

4.2.1 Fitting the Extreme Value Model

This section addresses the “Generalized Extreme Value distribution (GEVd)” of the temperature and rainfall data in Greater Accra region. The Exponential QQ-plot was used to have a rough and quick confirmation of the plausible fit of the “GEVd” to the data.

4.2.1.1 Temperature

The exponential Q-Q plots of both maximum (right panel) and minimum (left panel) Temperature is presented in Figure 4.1a. The “QQ-plot” reflects the linear relationship between the theoretical quantiles of a “family of distributions” and the corresponding “theoretical quantiles” of the standard member of the involved family.



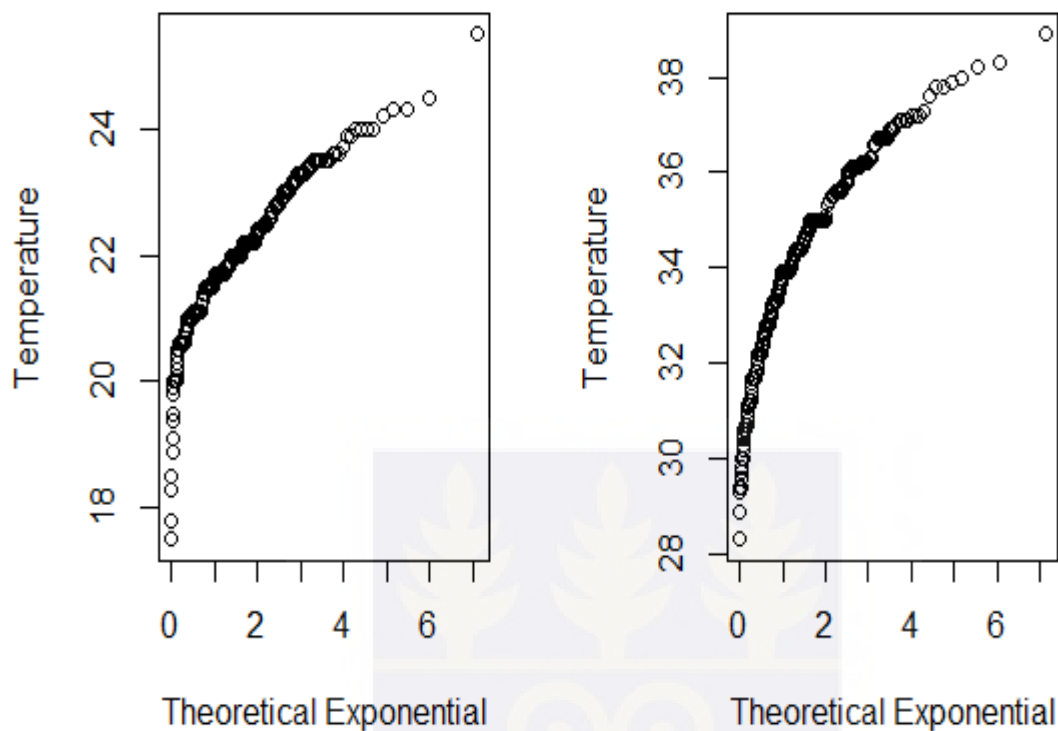


Figure 4.1a: Exponential Q-Q plot for minimum (Left panel) and maximum (right panel) Temperature

From Figure 4.1a, the “Q-Q plots” deviates from linearity at the tails of the distribution. For the upper tails, it exhibits a non-linear pattern, pointing to an underlying distribution function with a lighter right tail than the proposed “exponential distribution”. This indicates that the distributions belong to the “Weibull domain of attraction”.

Next we fit a GEV to the annual maximum and minimum temperature using the maximum likelihood. Tables 4.3a and 4.4a present the parameter estimates and their corresponding “standard errors” (in parentheses) for the minimum temperature and maximum temperature respectively.

Table 4.3a: GEV Parameter Estimate for Minimum Temperature

Parameter	Estimate	95% Confidence Interval
Location (μ)	-21.7443 (0.0464)	(-21.8352, -21.6535)
Scale (σ)	1.0673 (0.0301)	(1.0083, 1.1263)
Shape (γ)	-0.2315 (0.0123)	(-0.0555, -0.2075)



Table 4.4a: GEV Parameter estimate for Maximum Temperature

Parameter	Estimate	95% Confidence Interval
Location (μ)	32.279(0.0825)	(32.1172, 32.4407)
Scale (σ)	1.859 (0.059)	(1.7435, 1.9748)
Shape (γ)	-0.212 (0.0107)	(-0.2664, -0.1573)

The shape parameter (γ), that is the “Extreme Value Index (EVI)”, is negative in both cases and this suggests that the “underlying distribution” belongs to the “Weibull family of distributions”. In addition, the confidence interval exclude zero (0) and hence the distribution indeed belongs to the Weibull family of distributions

The “diagnostic plot”s for the GEV fit are presented in Figures 4.2a and 4.3a for minimum and maximum temperatures respectively.

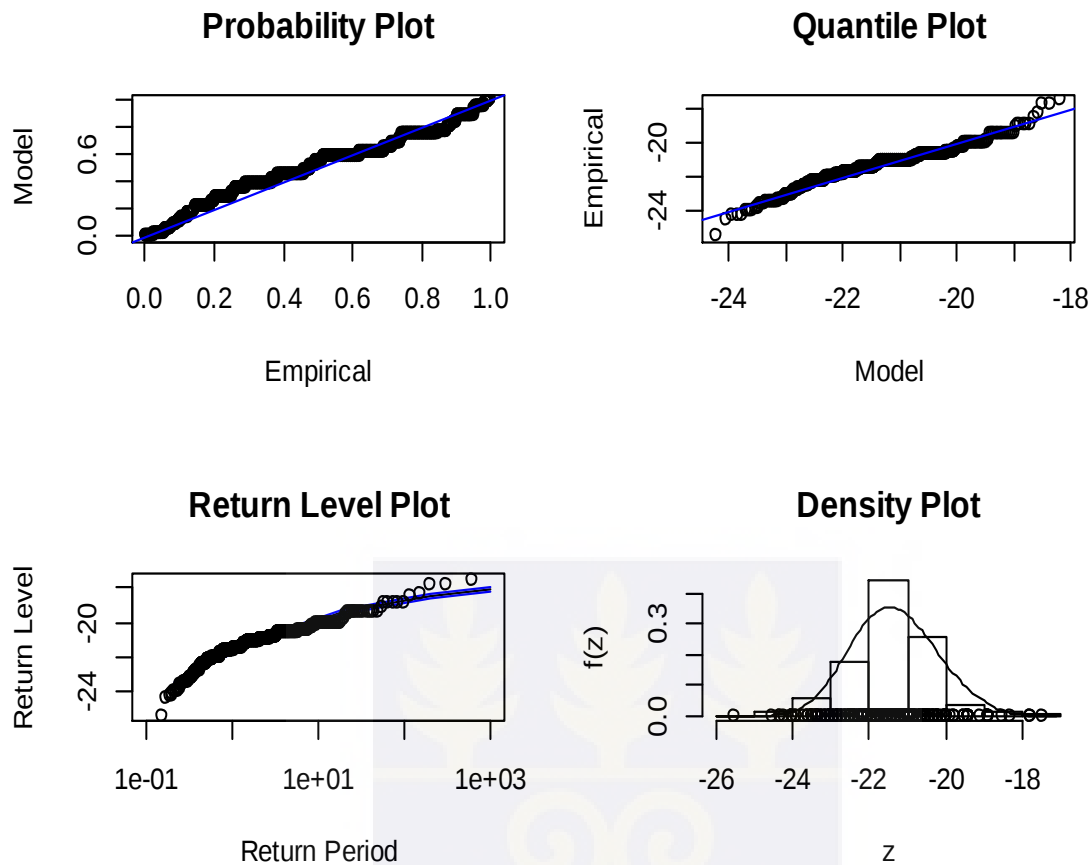


Figure 4.2a: Diagnostic plot for the fitted GEV model for minimum Temperature

From 4.2a, the linearity of the Q-Q and the PP plots indicate that, the model is valid. From the histogram, the density also appears to be consistent with the data points. We therefore conclude that the diagnostic plots indicate a good fit of the GEV model.

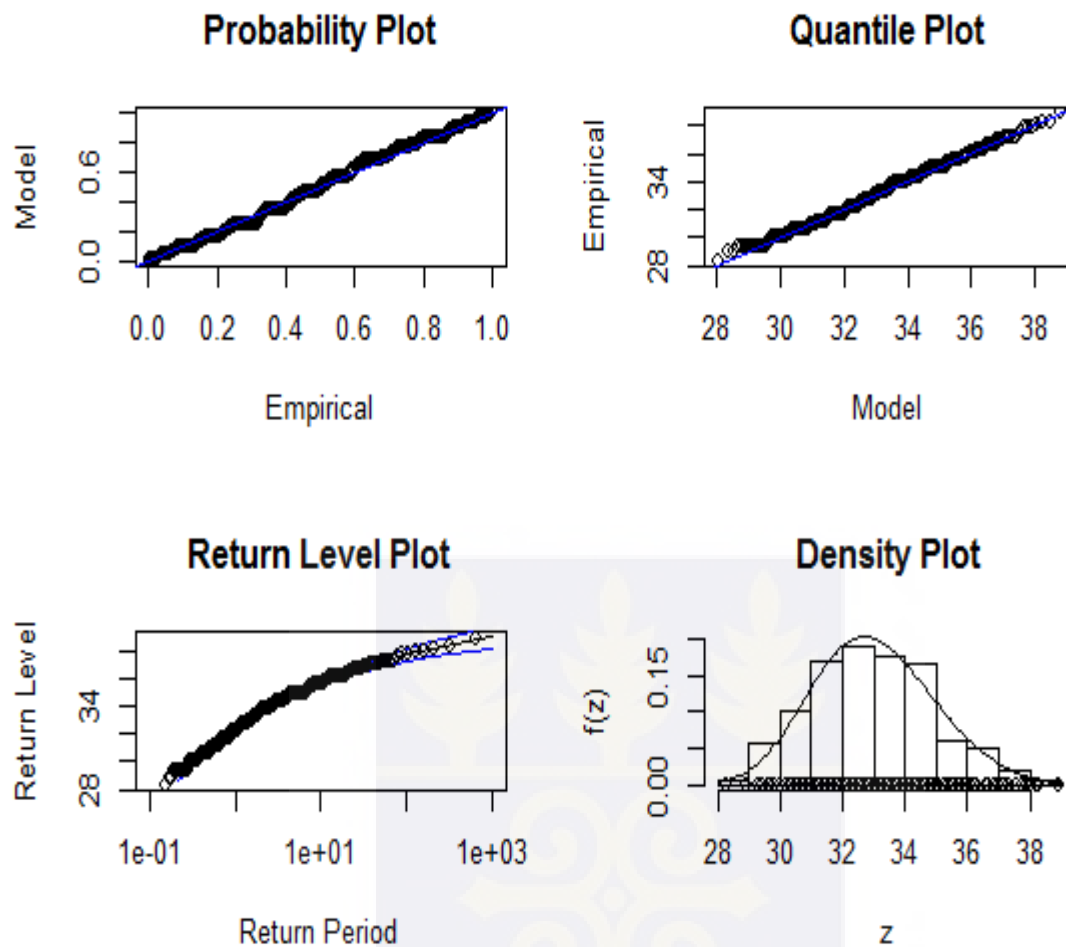


Figure 4.3a: Diagnostic plot for the fitted GEV model for maximum Temperature

From 4.3a, the Q-Q and PP plots appear to be linear indicating that, the model is valid. The histogram plots with density also appears to be approximately normal. Thus, the “GEV model” is appropriate for the data.

The return periods and exceedance probabilities for the tails of the distribution was further explored. Table 4.5a presents the probabilities of exceedances for some selected temperatures. The exceedance probabilities for the minimum temperature is interpreted as, probability that, the average temperature falls below the absolute value of the temperature.

Table 4.5a: Exceedance Probabilities estimates for Temperature

Maximum Temperature						
Temperature	34	35	36	37	38	39
Exceedance Probs	0.3002	0.1593	0.0713	0.0257	0.0068	0.0010
Min Temperature						
Temperature	-22	-21	-20	-19	-18	-17
Exceedance Probs	0.717	0.373	0.120	0.020	0.001	0.000

Negative values are interpreted in absolute value terms

From Table 4.5a, it was observed that, the probability of observing a temperature of 39°C was very small (0.0010). Thus, the maximum temperature within greater Accra region of Ghana is unlikely to exceed 39°C . However, the chances of observing a maximum temperature of 34°C or more daily is about 0.3002. Furthermore, there was a probability of 0.001 for the temperature to fall below 18°C daily. The return periods for daily minimum and maximum temperature was then estimated and presented in Table 4.6a.

The return periods measure how long, it is expected for a given temperature to be exceeded, once for a specified period (Salas and Fernandez, 1999). The return periods for the GEV model are in years. The periods considered in the study were 5, 10, 20, 50 and 100 years.

Table 4.6a: Return Periods estimates for Temperature

Maximum Temperature					
Temperature	34.7	35.6	36.4	37.2	37.7
Return Period (Years)	5	10	20	50	100
Minimum Temperature					
Temp	22.3	22.8	23.2	23.7	24.0
Return Period (Years)	5	10	20	50	100

Negative values are interpreted in absolute value terms

From Table 4.6a, a 5 year return period implies that, maximum temperature is expected to exceed 34.7°C once every 5 years. Also, Greater Accra is expected to experience maximum temperature of 35.6°C , 36.4°C , 37.2°C , and 37.7°C once every 10, 20, 50 and 100 years respectively. Furthermore, the 100 year return period for the minimum temperature means that, it is expected for the temperature to fall below 24°C on the average once every 100 years.

4.2.1.2 Rainfall

We fitted the extreme value model to the rainfall data in this section. The exponential Q-Q plot of maximum Rainfall for Greater Accra is presented in Figure 4.4a.

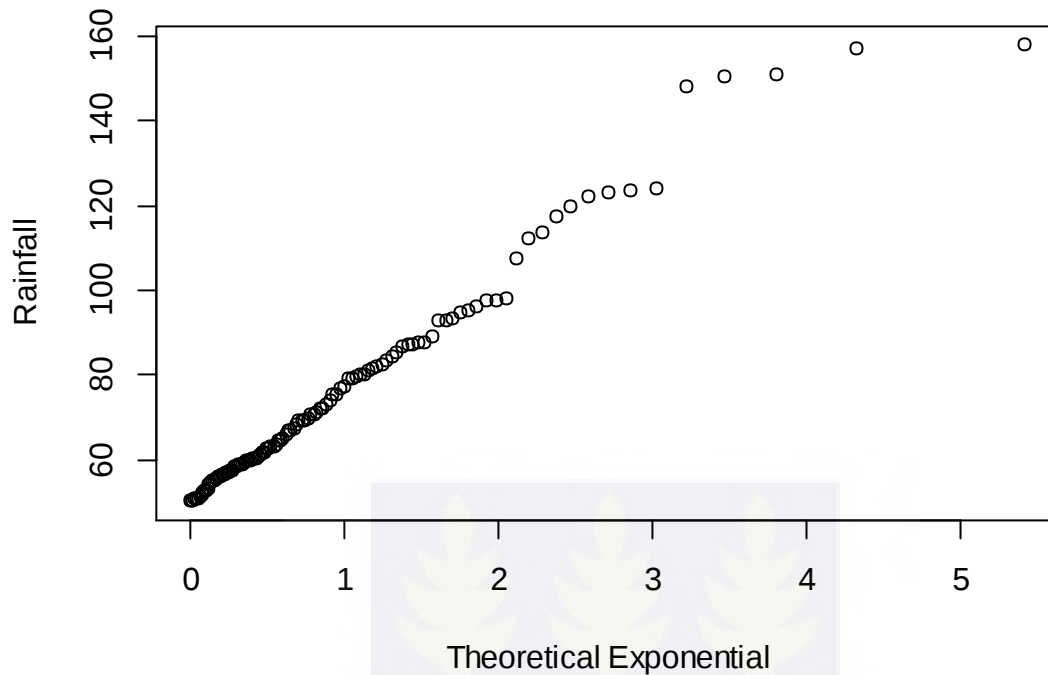


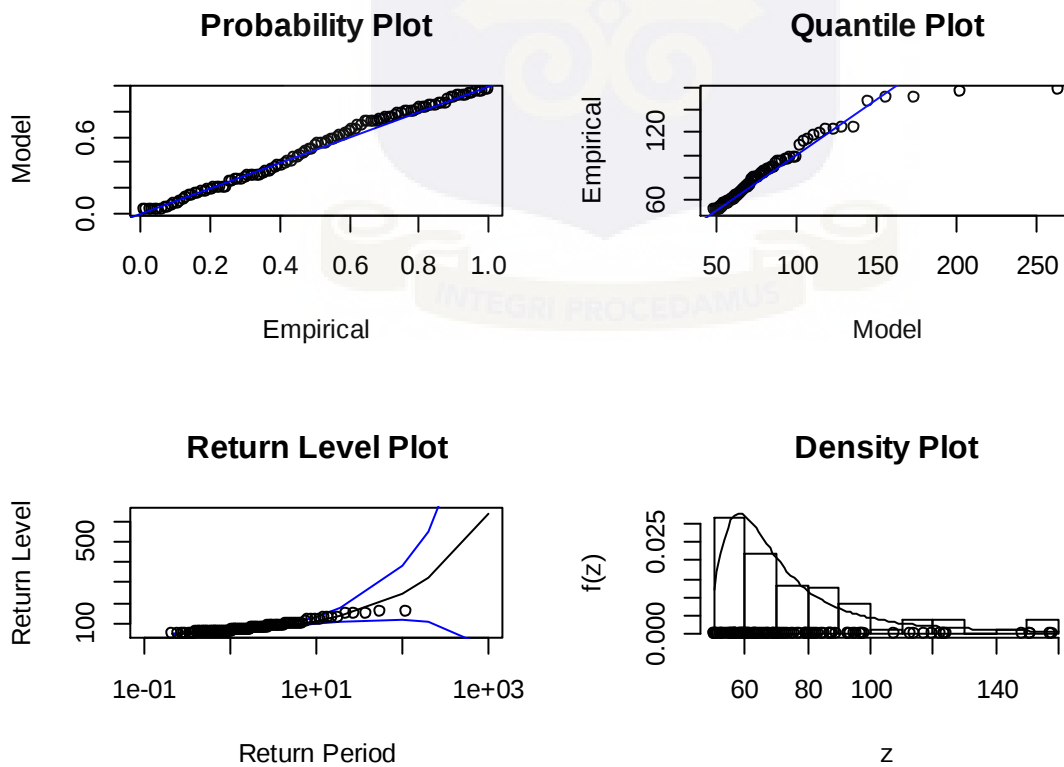
Figure 4.4a: Exponential Q-Q plot of maximum Rainfall

From Figure 4.4a, the Q-Q plot for maximum rainfall present more linear pattern than that for temperature. The exponential distribution is therefore not appropriate parametric family to be fitted to the rainfall data. We therefore fitted a GEV model to the data. The estimated GEV model for the maximum rainfall with the associated confidence intervals for the parameters is presented in Table 4.7a.

Table 4.7a: GEV Parameter estimate for Maximum Rainfall

Parameter	Estimate	95% Confidence Interval
Location (μ)	62.3323 (1.3935)	(59.6011, 65.0635)
Scale (σ)	12.1475 (1.2891)	(9.6209, 14.641)
Shape (γ)	0.4514 (0.1172)	(0.2216, 0.6811)

From Table 4.7a, the shape parameter is positive this suggests that the underlying distribution belongs to the “Frechet family of distributions”. The diagnostics plots for assessing the model are presented in Figure 4.5a.

**Figure 4.5a: Diagnostic plot for the fitted GEV model for maximum rainfall**

From Figure 4.5a, the histogram plot shows that the residuals are rightly skewed. However, the PP plot is approximately linear while from the Q-Q plot, there is a deviation for larger values. The return level plot confirms that the model departures are not significant indicating the appropriateness of the GEV fit. Estimates for probabilities of exceedances for some selected rainfall readings were computed. This is presented in Table 4.8a.

Table 4.8a: Exceedance Probabilities estimates for Rainfall

Rainfall (X) in mm						
X	50	100	150	200	250	300
$P(X > x)$	0.9795	0.1340	0.0396	0.0179	0.0100	0.0063

From Table 4.8a, the chances of experiencing a rainfall amount of 50 mm annually was 0.9795. Also, there was an observed chance of 0.134 of experiencing an amount of 100 mm rainfall reading annually. The return periods was also estimated and presented in Table 4.9a.

Table 4.9a: Return Periods estimates for Rainfall

Maximum Rainfall in mm					
Rainfall	88.38	109.74	138.27	192.05	250.08
Return Period (Years)	5	10	20	50	100

From Table 4.9a, the 100 year return period for the maximum rainfall in Accra means that, on average it is expected to record a rainfall of 250.08 mm once every 100 years.

4.2.2 Fitting the Generalised Pareto (GP) Model

In this section, the generalized Pareto model was fitted to both the temperature and rainfall data.

4.2.2.1 Temperature

The daily temperatures are considered as realization of a random variable having a “distribution function F ”. The “peak-over threshold method” considers an event as extreme (here daily temperatures) if it exceeds a high threshold μ . The excesses over the threshold μ are approximated with the “Generalized Pareto (GP) distribution”. The choice of μ is a very important issue for this method. It is a trade-off between a low μ with minimal variance and a high μ with a minimal bias. In this section of the study, a generalized Pareto model was fitted to the minimum and maximum temperature in Accra.

The Peak-Over Threshold Method; Threshold Selection

The mean residual life plot also known as “mean excess plot” and a threshold stability plot was used to aid in the selection of an appropriate threshold. For the “mean excess plot”, we select μ as the point to the right of which a linear pattern is observed. The second approach is to estimate the model at a range of thresholds and above a threshold μ_0 at which the asymptotic properties for the GP model is valid; the parameter estimates should be approximately stable, thus approximately constant, Coles (2001).

Figure 4.6a is the mean residual life plot for daily minimum (left panel) and maximum (right panel) temperature.

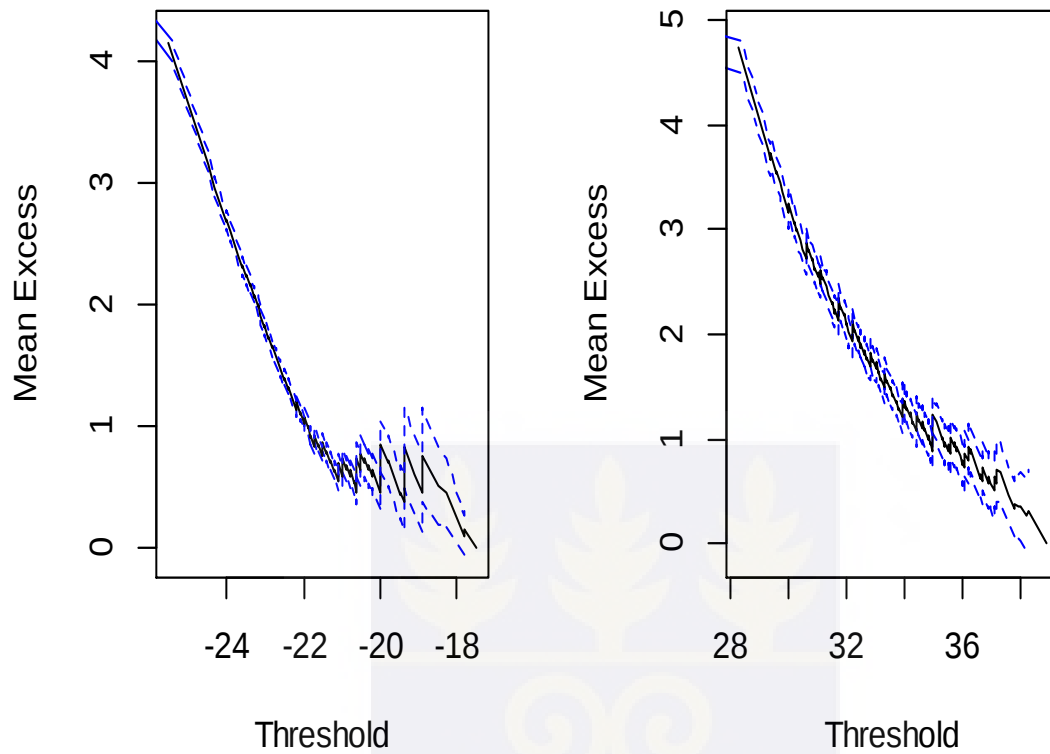


Figure 4.6a: Mean excess plot for minimum (left panel) and maximum (right panel) Temperature

From Figure 4.6a there is an observed linearity above $\mu = 28^{\circ}C$ for the maximum average monthly temperature. There was also evidence of linearity above $\mu = -26^{\circ}C$ for the minimum temperature. A threshold of $28^{\circ}C$ for the maximum temperature and $26^{\circ}C$ for the minimum temperature seems feasible. The decreasing nature in both cases indicates that, they both have a tail lighter than the exponential distribution. Thus, they belong to the GP distribution with Weibull domain of attraction. If the “mean excess plot” is close to linearity for high threshold values, then the data conform to a GP distribution (Ghosh and Reinck 2011).

Figures 4.7a and 4.8a present the threshold stability plot of the maximum and minimum daily temperatures respectively.

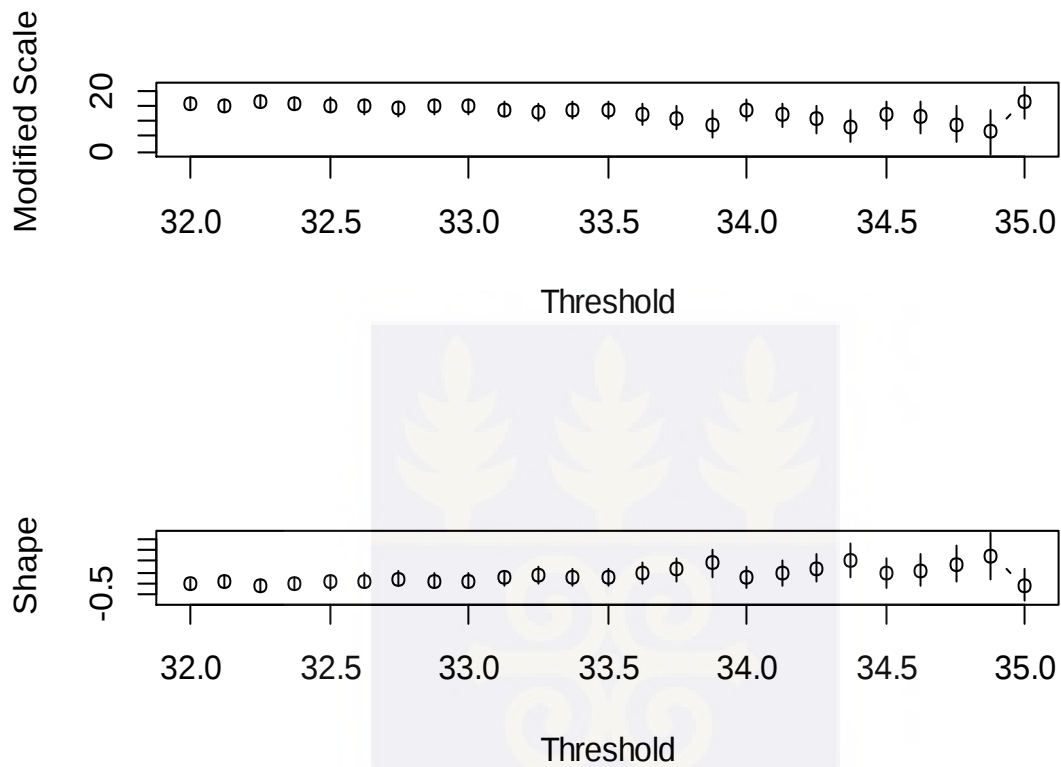


Figure 4.7a: Parameter estimates against threshold for maximum Temperature

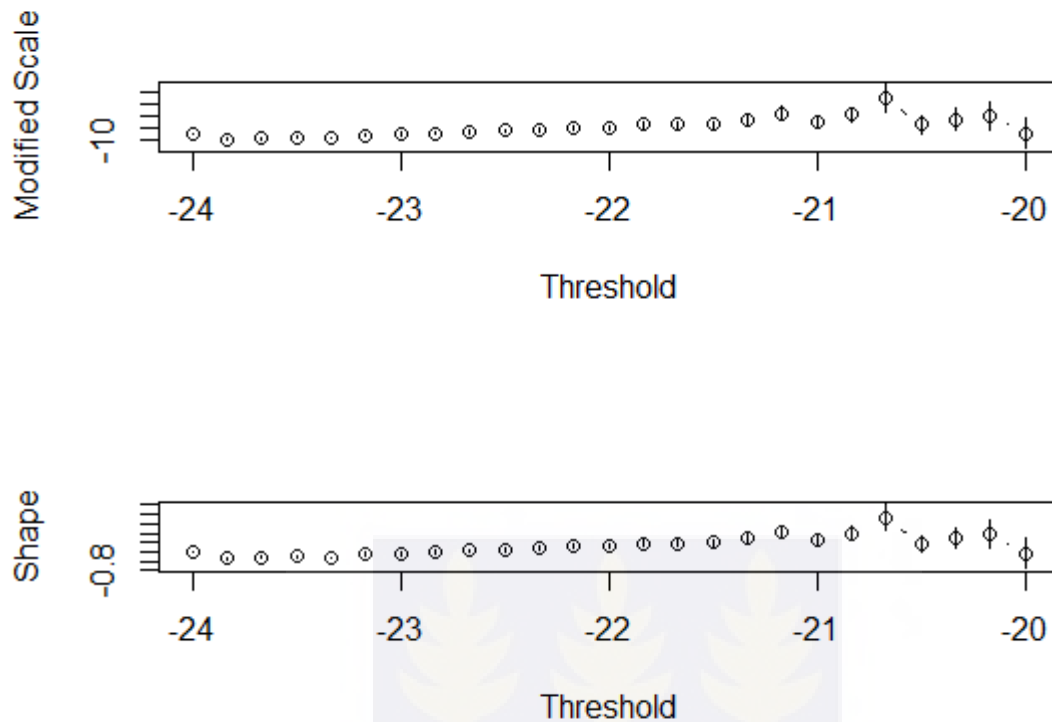


Figure 4.8a: Parameter estimates against threshold for minimum Temperature

From Figure 4.7a, the parameter estimates for the maximum temperature appear stable around $\mu = 34^{\circ}\text{C}$. Also, from Figure 4.8a, the threshold stability plot shows stability of the parameter estimates around $\mu = -21^{\circ}\text{C}$.

From the mean threshold plot and the plot of the threshold stability, a threshold of $\mu = 34.5^{\circ}\text{C}$ and $\mu = 20.6^{\circ}\text{C}$ were selected for both the maximum and minimum temperatures respectively.

Table 4.10a is the estimate of the GP for the maximum temperature for Accra. With the threshold of $\mu = 34.5^{\circ}\text{C}$ there were 145 exceedances which constitutes a proportion of 0.2294.

Table 4.10a: GP parameter estimate for maximum Temperature

Parameter	Estimate	95% Confidence Interval
Scale (σ)	1.5679 (0.1587)	(1.2569, 1.8789)
Shape (γ)	-0.3059 (0.0632)	(-0.4297, -0.1820)

From Table 4.10a, the shape parameter is negative which implies that, the usual asymptotic properties are not met, (Smith, 1985).

Table 4.11a is the estimate of the GP for the minimum temperature. With the threshold of $\mu = 20.6^\circ C$ there were 98 exceedances which constitutes a proportion of 0.1612.

Table 4.11a: GP Parameter Estimate for Minimum Temperature

Parameter	Estimate	95% Confidence Interval
Scale (σ)	0.8687 (0.1183)	(0.6368, 1.1006)
Shape (γ)	-0.1636 (0.0900)	(-0.3399, 0.0127)

From Table 4.11a, the shape parameter for the minimum temperature is also negative.

Figures 4.9a and 4.10a are the diagnostic plots of the fitted GP model for the maximum and minimum temperatures.

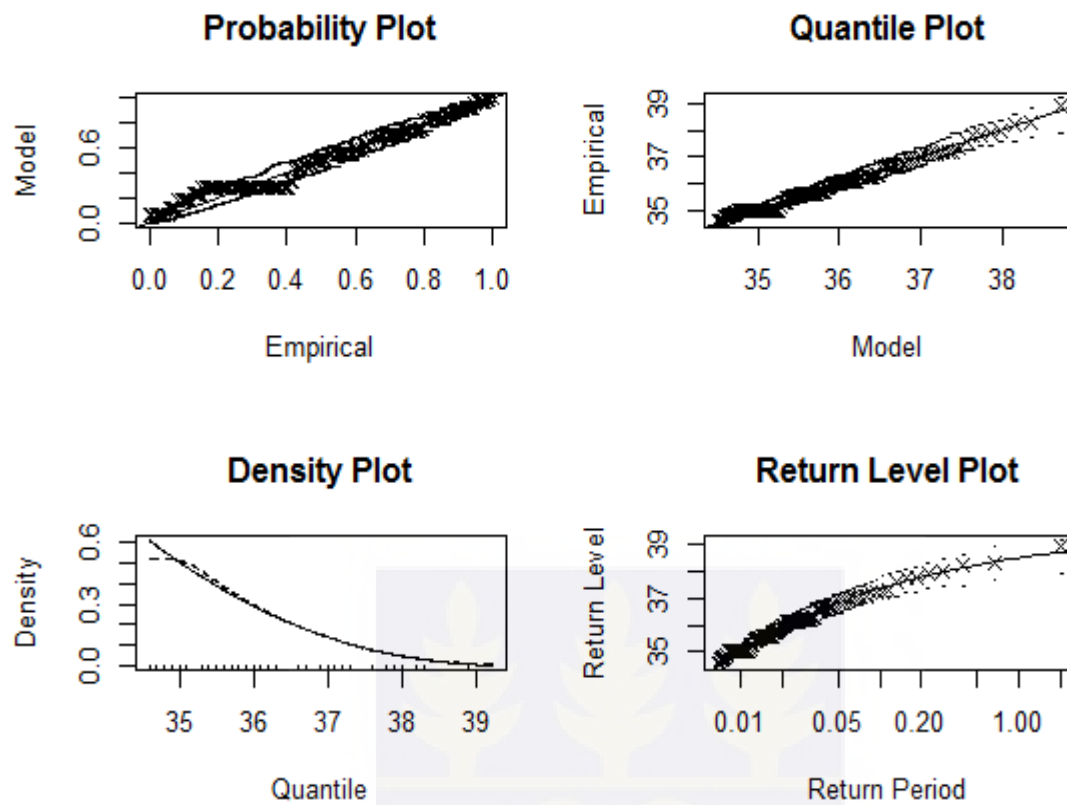


Figure 4.9a: Diagnostic plot for fitted GP for maximum Temperature

From Figure 4.9a, the goodness-of-fit in the Q-Q and PP plots seems convincing. The confidence intervals on the return level plot suggest that the model departures are not significant. Thus, the fitted generalized Pareto distribution is appropriate.

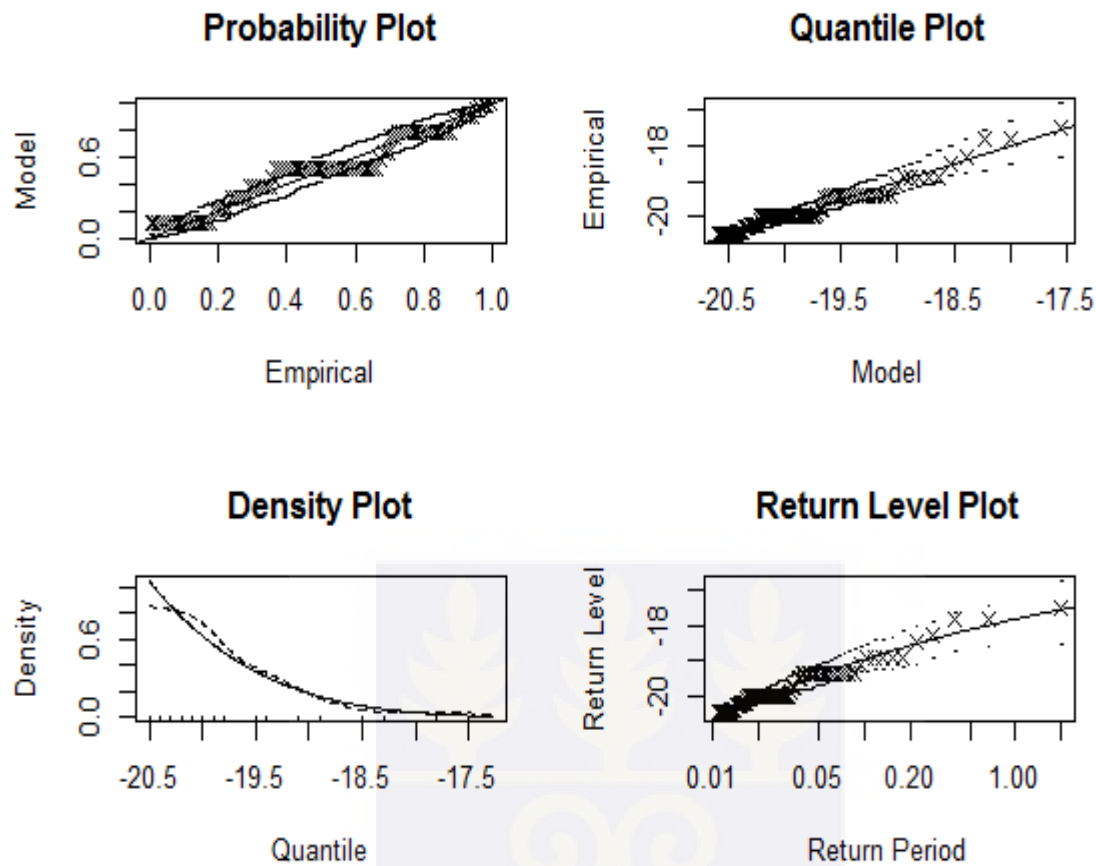


Figure 4.10a: Diagnostic plot for fitted GP for minimum Temperature

From Figure 4.10a, the Q-Q and PP plots are approximately linear as the deviations from the model are very small from the “confidence intervals” observed on the “return level plot”. Thus, the goodness-of-fit is satisfied and therefore the fitted generalised Pareto distribution is appropriate.

4.2.2.2 Rainfall

Figure 4.11a is the mean residual life plot for maximum rainfall readings observed.

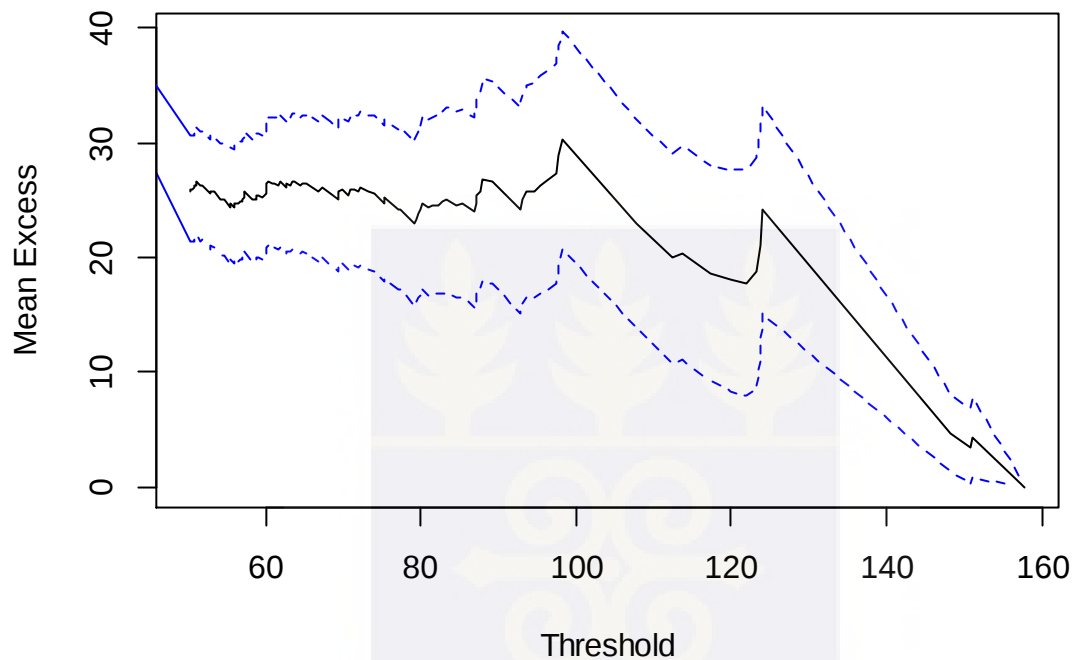


Figure 4.11a: Mean excess plot for maximum Rainfall

From Figure 4.11a there is an observed non-linearity above $\mu = 80\text{mm}$. A threshold of $\mu = 70$ seems feasible. To settle on an appropriate threshold, the threshold stability plot was also examined. This is presented in Figure 4.12a

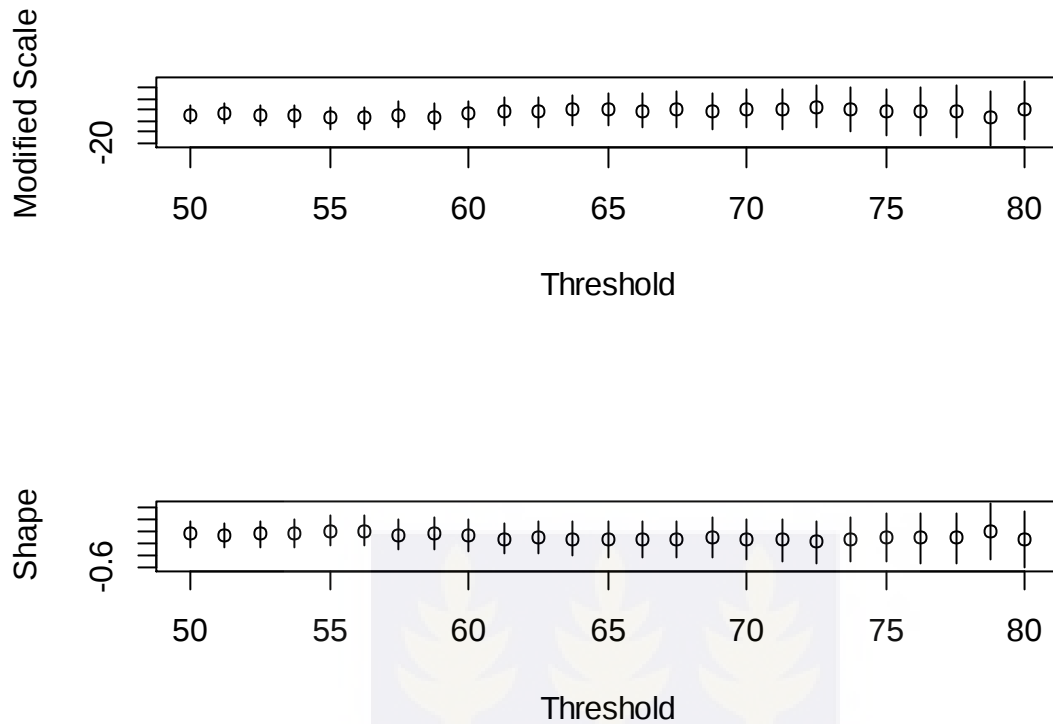


Figure 4.12a: Parameter estimates against threshold for maximum Rainfall

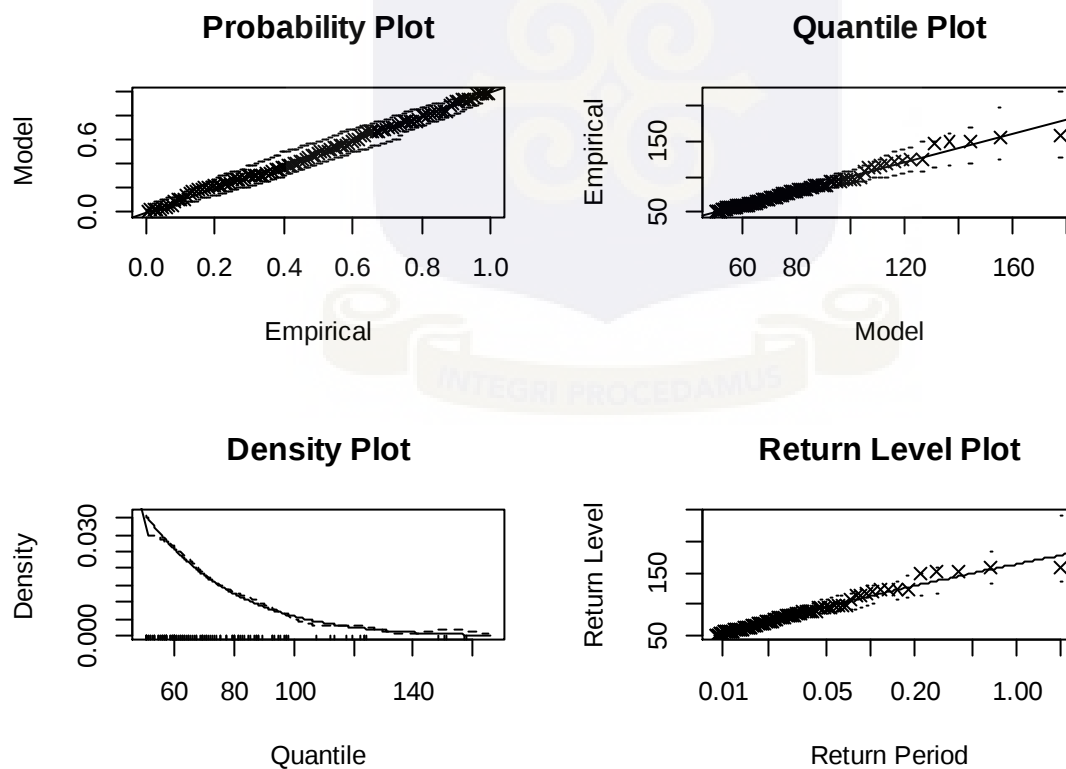
The threshold stability also suggested a threshold of $\mu = 70$ mm. Thus, from the mean threshold plot and the plot of the threshold stability, a threshold of $\mu = 60$ mm was selected. For this threshold, there were 77 exceedances which constitutes a proportion of 0.6814. Table 4.13a is the estimate of the GP for the maximum rainfall.

Table 4.12a: GP parameter estimate for Maximum Rainfall

Parameter	Estimate	95% Confidence Interval
Scale (σ)	27.8372 (4.9216)	(18.1911, 37.4833)
Shape (γ)	-0.0738 (0.1354)	(-0.3392, 0.1917)

From Table 4.12a, the shape parameter is negative and this is an indication that the normal heavy tailed Pareto distribution could have been considered.

Figure 4.13a is the diagnostic plot of the fitted GP model for the maximum rainfall.

**Figure 4.13a: Diagnostic plot for fitted GP for maximum Rainfall**

From Figure 4.13a, the Q-Q and PP plots is approximately linear as the deviations from the model is very small from the confidence intervals observed on the return level plot. Thus, the goodness-of-fit is satisfied and therefore the fitted generalized Pareto distribution is appropriate.

4.2.3 General Comments

Rainfall and temperature are important climatic factors for crop production in Ghana. Ghana, among other African countries' vulnerability is heightened by the fact that most of the economies in this region rely mainly on rain-fed agriculture and natural resources, which are very sensitive to climate change and variability. This section of the study explored the extreme distribution of the temperature and rainfall occurrences in Greater Accra region of Ghana. The return periods of extreme occurrences and probability of exceeding such occurrences were also estimated. It was observed that the extreme occurrences of temperature in the region can be modelled using the "Weibull family of distributions" while the "Frechet family of distributions" can be used to model the extreme rainfall.

A maximum temperature of 34.7°C is predicted to occur in the region once every five years. Furthermore, maximum temperatures of 35.6°C , 36.4°C , 37.2°C , and 37.7°C were predicted to be experienced one every 10, 20, 50, and 100 years respectively. Also, a minimum temperature of 22.3°C was estimated to occur once every five years while a minimum temperature of 24°C was predicted to occur once every 100 years in the region. The probability of exceeding a maximum temperature of 39°C was 0.001 which is very unlikely to be experienced in the region. However, there was a moderate likelihood of exceeding a maximum temperature of 34°C . Also, the likelihood of a minimum temperature falling below 18°C was very small.

The maximum average rainfall of Greater Accra recorded in the study period was 243.9 mm which was an extreme occurrence. An amount of 88.38 mm of rainfall reading was estimated for the region in the next five years and every five years after. Furthermore, a maximum amount of 250.1 mm of rainfall was also estimated for once every 100 years. In addition, the chances of observing a maximum amount of rainfall exceeding 100 mm was very small with a probability of 0.0.9795 of recording a rainfall of 50 mm.

Climate affects humans' basic needs, such as shelter, food, water and health. These needs are threatened by climate change with increased temperatures, changes in precipitation, and more frequent or intense extreme events. Among crop species, responses to temperatures differ throughout their life cycle and are primarily the phenological responses, that is, stages of plant development. For each species, a defined range of maximum and minimum temperatures form the boundaries of observable growth. Hatfield and Prueger (2011) established that increases of temperature may cause yield declines between 2.5% and 10% across a number of agronomic species throughout the 21st century. Interestingly, in the current study, average temperatures of Greater Accra is more likely to lie in the range 24°C and 30°C, which favors the agro industry. However, there was indications of observing temperatures beyond this range though very unlikely.

Furthermore, it has been demonstrated that weather is associated with changes in birth rates, and sperm counts, with outbreaks of bronchitis, pneumonia and influenza, and is related to other morbidity effects linked to high pollution levels and pollen concentrations. The high birth rate and breeding of mosquitos in Accra can be attributed to high temperatures recorded in this region. Although rainfall is critical in providing suitable habitats for mosquitoes to breed (Craig et al., 1999). Temperature is a key driver of several of the essential mosquito and parasite life history traits that combine to determine transmission intensity, including

mosquito development rate, biting rate, and development rate and survival of the parasite within the mosquito (Mordecai *et al.*, 2013).

Greater Accra recently has been experiencing flooding every year due to several factors for which the amount of rainfall is the key factor. The current study revealed that the daily amount of rainfall recorded in the region ranged from 0 inches to 243.9mm. Extreme occurrences beyond this range was recorded though. Thus measures needs to be put in place to handle such occurrences in the near future.

4.3 Ashanti Region

In this section of the study, we focused on the extreme occurrence of temperature and rainfall readings in the Ashanti region. The extreme value and the Generalized Pareto models were fitted to the temperature and rainfall data observed.

4.3.1 Fitting the Extreme Value Model

The extreme behaviour of temperature and rainfall was explored through the Generalized Extreme Value distribution (GEVd). The Exponential QQ-plot was used to have a rough and quick confirmation of the plausible fit of the GEVd to the data.

4.3.1.1 Temperature

Figure 4.1b is the exponential Q-Q plots of both minimum (left panel) and maximum (right panel) Temperature.

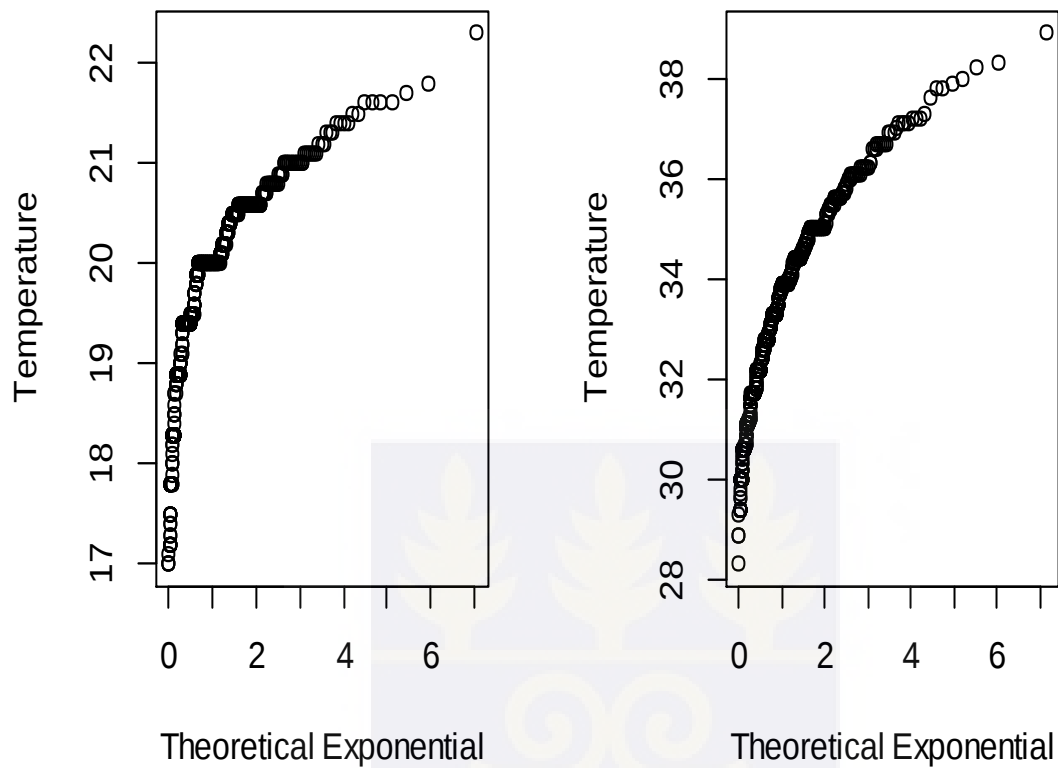


Figure 4.1b: Exponential Q-Q plot for minimum (left panel) and maximum (right panel) Temperature

From Figure 4.1b, the upper tail deviates from linearity pointing to an underlying distribution function with a “lighter right tail” than the “exponential distribution”. Thus the exponential family of distributions is not appropriate. In this vain, the “generalized extreme value distribution” was fitted to the data.

Tables 4.3b and 4.4b present the parameter estimates and their corresponding “standard errors” (in parentheses) for the minimum temperature and maximum temperature respectively.

Table 4.3b: GEV Parameter estimates for Minimum Temperature

Parameter	Estimate	95% Confidence Interval
Location (μ)	-20.0487 (0.0391)	(-20.1613, -20.0081)
Scale (σ)	0.8454 (0.0272)	(0.7921, 0.8987)
Shape (γ)	-0.1291 (0.0273)	(-0.1826, -0.0755)

Table 4.4b: GEV Parameter estimates for Maximum Temperature

Parameter	Estimate	95% Confidence Interval
Location (μ)	32.2790 (0.0825)	(32.1172, 32.4407)
Scale (σ)	1.8592 (0.0590)	(1.7435, 1.9748)
Shape (γ)	-0.2118 (0.0278)	(-0.2664, -0.1573)

The fitted generalised extreme distribution suggested that the extreme occurrences of both the average minimum and maximum temperatures observed in the region are in the Weibull family of distributions since $\gamma < 0$. Figures 4.2b present the diagnostic plot for the GEV fit for minimum temperature.

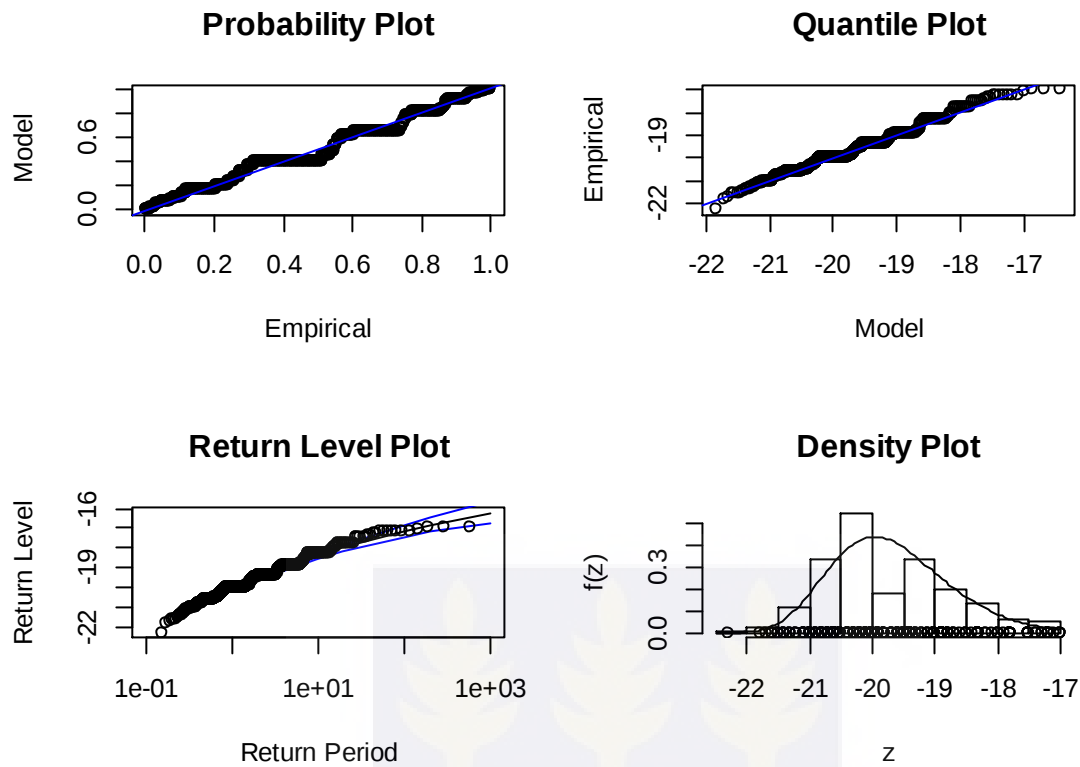


Figure 4.2b: Diagnostic plot for the fitted GEV model for minimum Temperature

From Figure 4.2b, the Q-Q and PP plots are approximately linear, an indication that the model is valid is. Thus the GEV model fit the data very well. The density plot shows that the normality of the residual is attained.

Figure 4.3b shows the diagnostic plots for the GEV fit for maximum temperature.

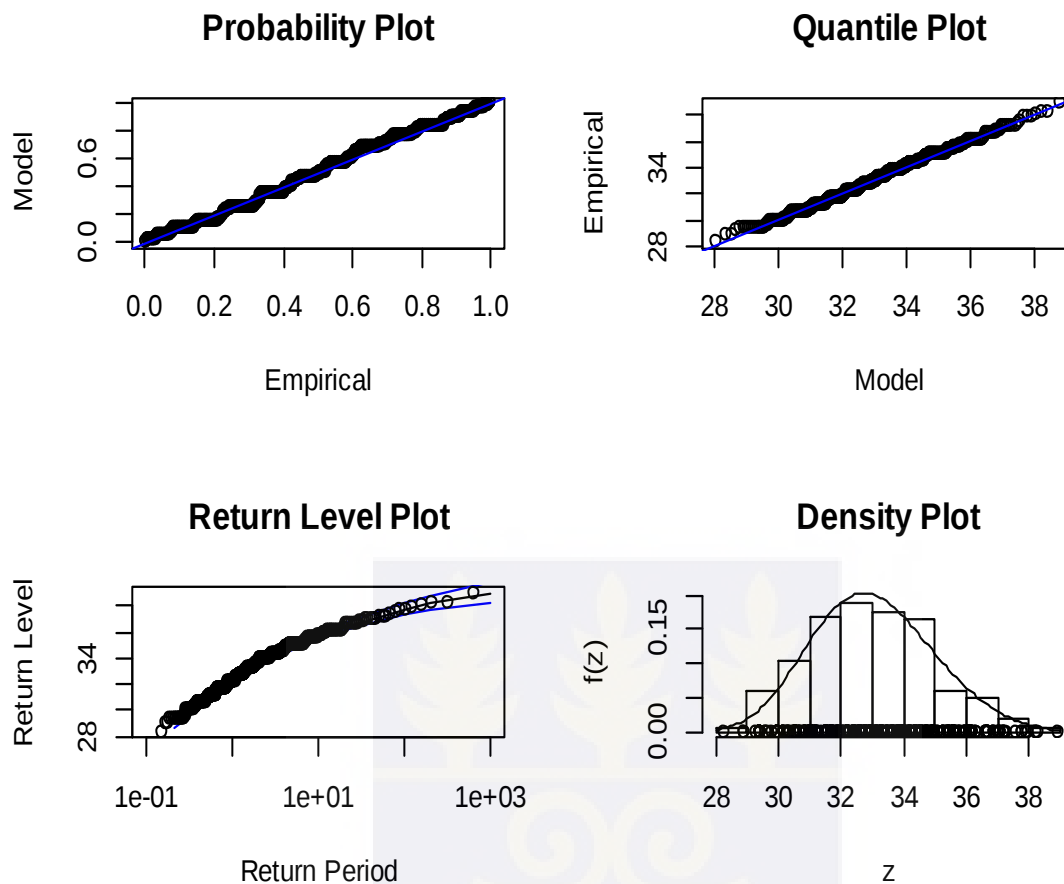


Figure 4.3b: Diagnostic plot for the fitted GEV model for maximum Temperature

From Figure 4.3b, it was concluded that the “GEV model” is appropriate for the data due to normality of the residuals depicted by the density plot. Also, the Q-Q and PP plots are linear despite the slight deviations.

After assessing and concluding on the appropriateness of the model, the “return periods” and “exceedance probabilities” for the tails of the distribution was estimated. The temperature experienced in the region ranged from 19.17°C to 38.9°C on a daily basis. The probabilities of exceedances for some selected temperatures is shown in Table 4.5b.

Table 4.5b: Exceedance Probabilities estimates for GEV fit Temperature

Maximum Temperature						
Temperature	34	35	36	37	38	39
Exceedance Probs	0.3001	0.1593	0.0713	0.0258	0.0068	0.0011
Minimum Temperature						
Temperature	-22	-21	-20	-19	-18	-17
Exceedance Probs	0.9993	0.9364	0.5951	0.2180	0.0501	0.0072

From Table 4.5b, it was observed that, temperatures of 36°C and above are very unlikely to be experienced in the region. Also, minimum temperatures of 22°C to 20°C are likely to be observed in the region. We further estimated the return periods for average annual minimum and maximum temperature. Table 4.6b presents the return periods for various average temperatures. The periods considered in the study were 5, 10, 20, 50 and 100 years.

Table 4.6b: Return Periods estimates of GEV fit for Temperature

Maximum Temperature					
Temperature	34.66	35.61	36.38	37.22	37.74
Return Period (Years)	5	10	20	50	100
Minimum Temperature					
Temperature	-18.93	-18.43	-18.00	-17.49	-17.15
Return Period (Years)	5	10	20	50	100

Negative values are interpreted in absolute value terms

From Table 4.6b, Ashanti region is expected to experience a maximum temperature of 34.66°C once every five years with a minimum temperature of 18.93°C . Also, a minimum temperature of 17.15°C will be experienced in the region once every 100 years.

4.3.1.2 Rainfall

Figure 4.4b is the exponential Q-Q plot for maximum Rainfall.

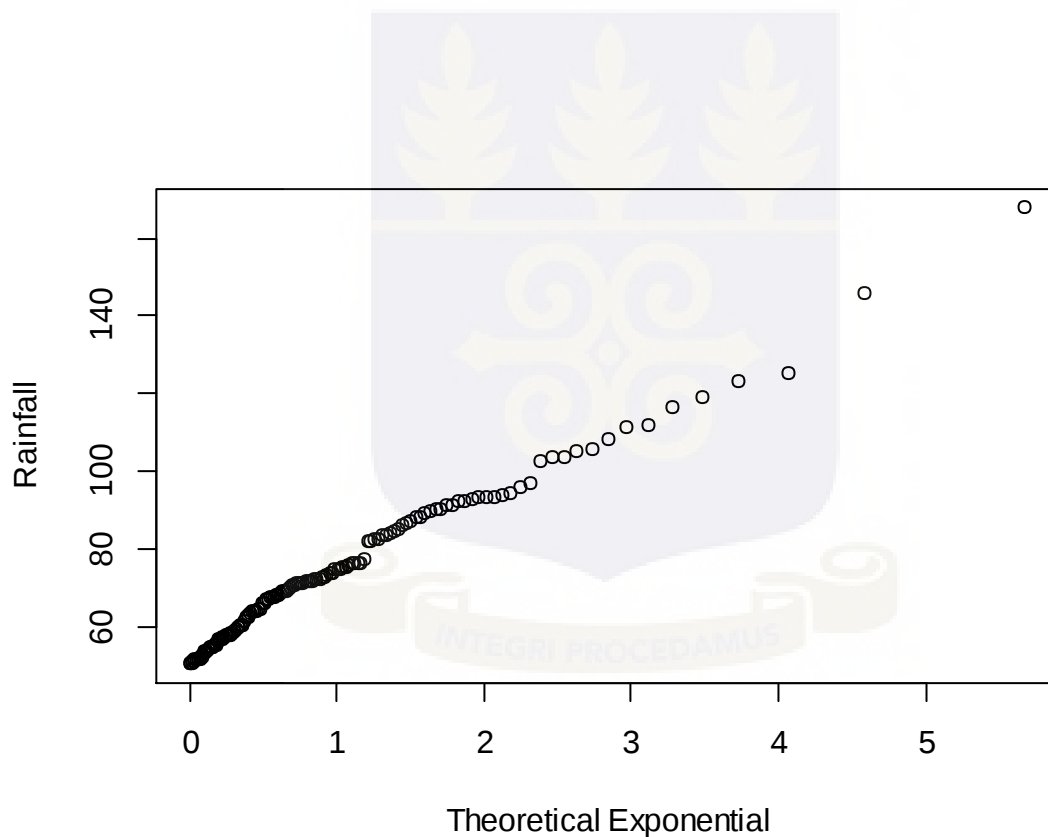


Figure 4.4b: Exponential Q-Q plot for maximum Rainfall

From Figure 4.4b, the plot of the maximum rainfall exhibits a roughly linear pattern for the plotted points when the “exponential model” was proposed as a “parametric model” to be fitted to the data. However, the lower tail of the plot deviates from linearity. A more lighter

tailed distribution will seem feasible as compared to the exponential distribution. A “GEV model” was fitted to the maximum rainfall data. The estimated GEV model with the associated confidence intervals for the parameters is presented in Table 4.7b.

Table 4.7b: GEV parameter estimate for Maximum Rainfall

Parameter	Estimate	95% Confidence Interval
Location (μ)	63.8614 (1.2261)	(59.6011, 65.0635)
Scale (σ)	12.2728 (1.0016)	(9.6209, 14.6741)
Shape (γ)	0.2163 (0.0920)	(0.2216, 0.6811)

From Table 4.7b, the shape parameter is positive this suggests that the underlying distribution belongs to the “Frechet family of distribution”. The diagnostic plots is presented in Figure 4.5b.



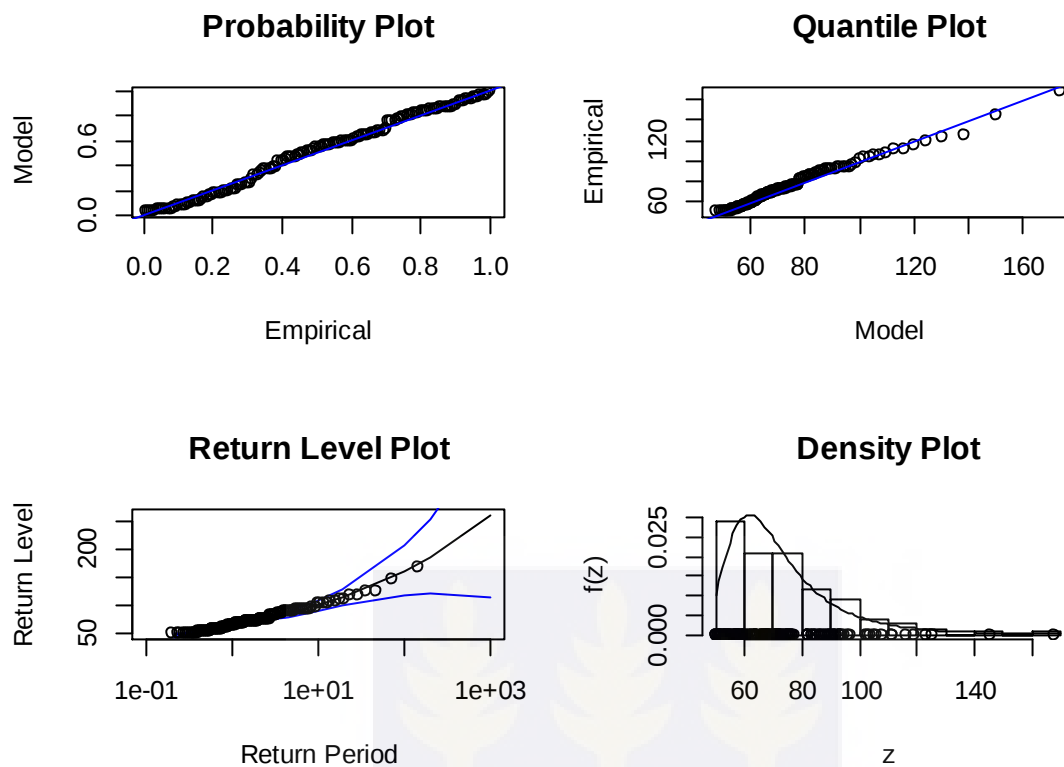


Figure 4.5b: Diagnostic plot for the fitted GEV model for maximum Rainfall

From Figure 4.5b, the PP plots is linear. However, the goodness-of-fit in the Q-Q plot seems unconvincing, but the confidence intervals on the return level plot suggest that the model departures are not large. Thus, the GEV model is valid. The exceedance probabilities exceedances for some selected rainfall readings was then estimated and presented in Table 4.8b.

Table 4.8b: Exceedance Probabilities estimates for GEV fit for Maximum Rainfall

Rainfall (X) in mm						
X	50	100	150	200	250	300
$P(X > x)$	0.9740	0.0974	0.0139	0.0035	0.0012	0.001

From Table 4.8b, there was 0.974 chance of experiencing a daily rainfall amount of 50 mm in the region. The maximum rainfall recorded by the region for over the period of the study was 167.9 mm. The study estimated 0.002 likelihood of observing a daily rainfall amount of 100 inches in the region. The return periods was also estimated.

Table 4.9b presents the return periods for various average rainfall in Accra.

Table 4.9b: Return Periods estimates of GEV fit for Maximum Rainfall

Maximum Rainfall					
Rain	85.61	99.44	114.99	139.08	160.59
Return Period (Years)	5	10	20	50	100

From Table 4.9b, Ashanti region is expected to record 85.61 mm amount of rainfall readings daily once every 5 years. A 10 year return period for the maximum rainfall in the region was 160.59 mm.

4.3.2 Fitting the Generalized Pareto (GP) Model

In this section, the generalized Pareto model was fitted to both the temperature and rainfall data.

4.3.2.1 Temperature

The generalized Pareto model was fitted to the average temperature. The mean residual life plot and a threshold stability plot was employed here as well in the selection of an appropriate threshold. Figure 4.6b is the mean residual life plot for monthly average minimum (left panel) and maximum (right panel) temperatures.

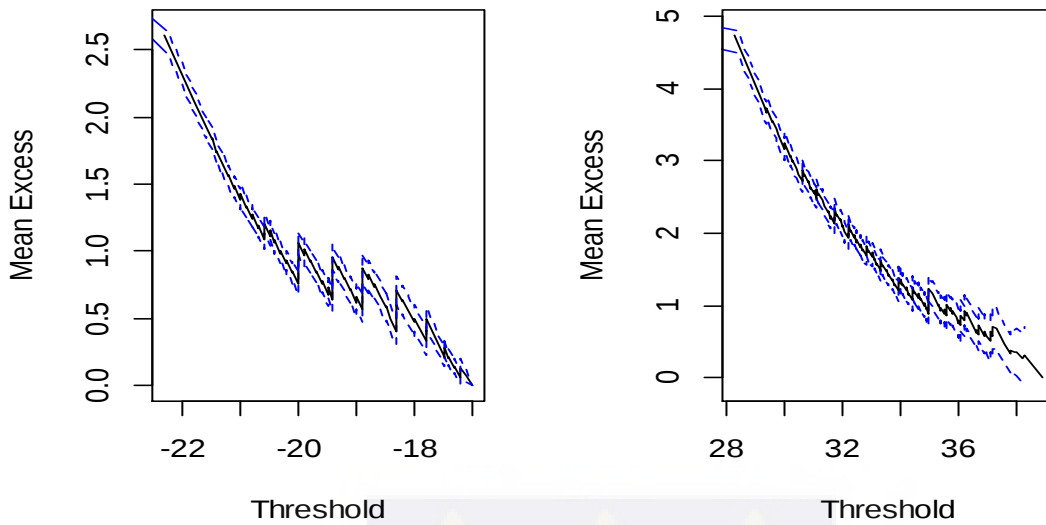


Figure 4.6b: Mean excess plot for minimum (left panel) and maximum (right panel) Temperature

From Figure 4.6b there is an observed linearity above $\mu = 34^{\circ}C$ for the maximum average monthly temperature. There was also evidence of linearity above $\mu = -21^{\circ}C$ for the minimum temperature. A threshold of $34^{\circ}C$ for the maximum temperature and $21^{\circ}C$ for the minimum temperature seems feasible. The decreasing nature in both cases indicates that, they both have a tail lighter than the exponential distribution. Thus, they both belong to the GP distribution with Weibull domain of attraction.

Figures 4.7b and 4.8b present the threshold stability plot of the maximum and minimum average temperatures respectively.

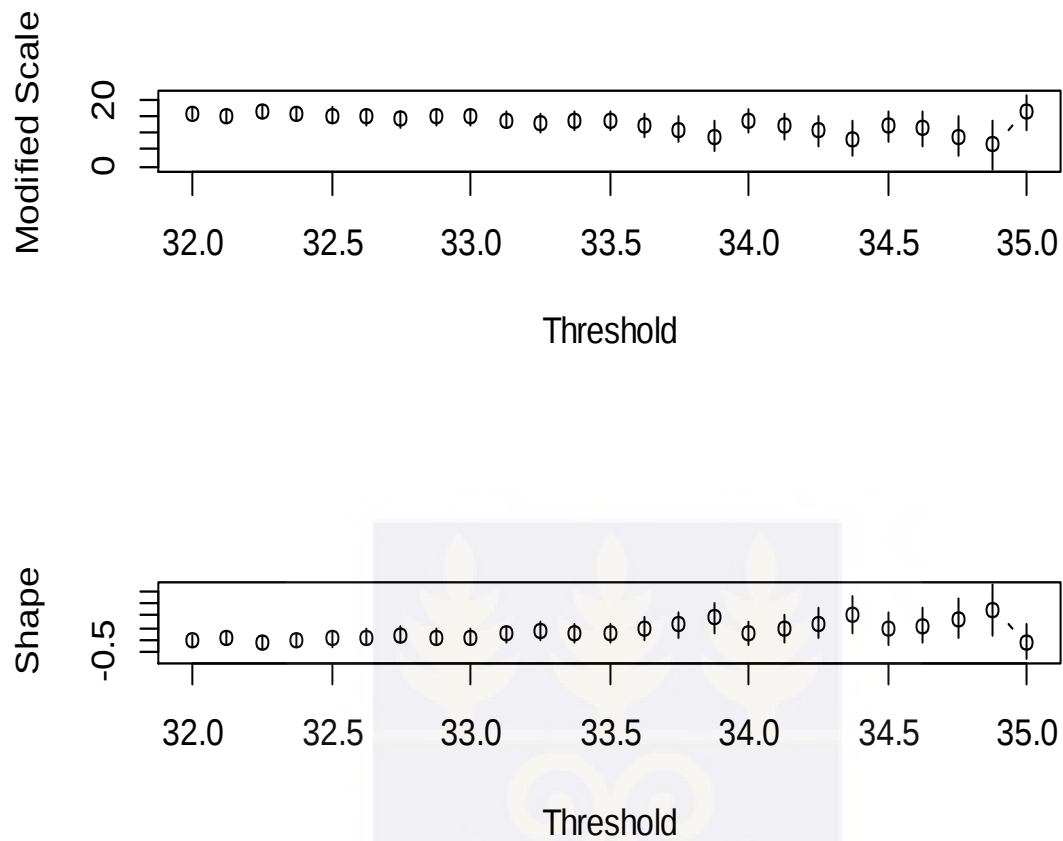


Figure 4.7b: Parameter estimates against threshold for maximum Temperature

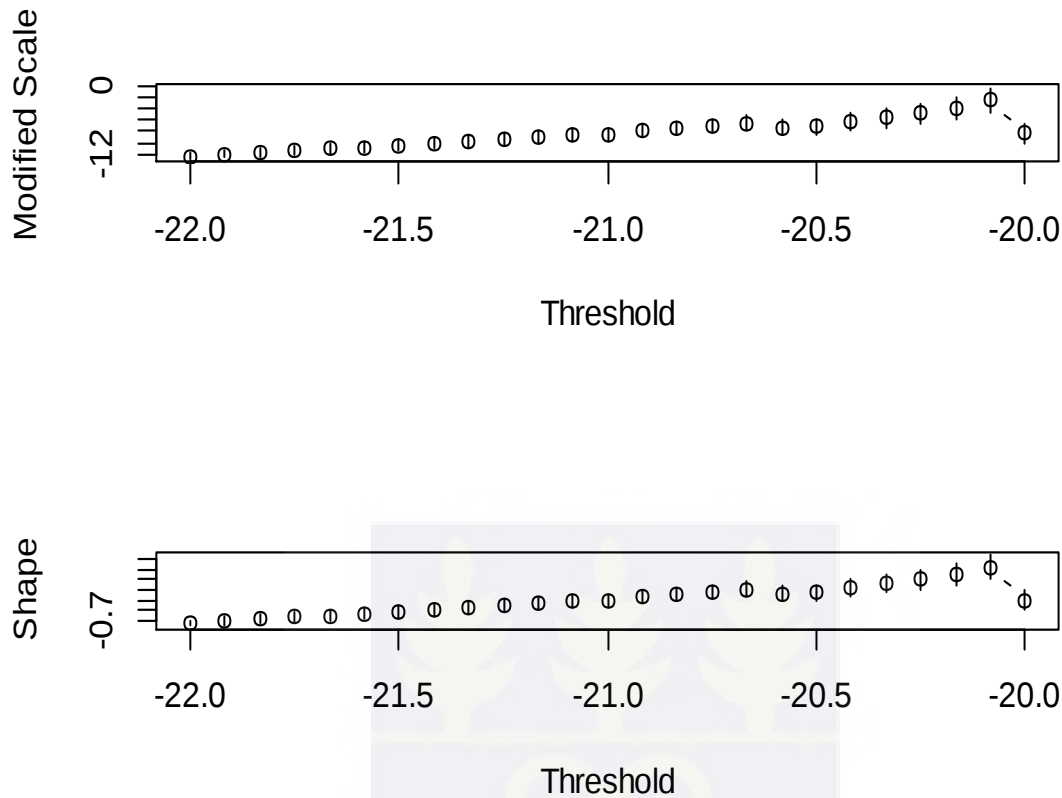


Figure 4.8b: Parameter estimates against threshold for minimum Temperature

From Figure 4.7b, the parameter estimates for the maximum temperature appear stable around $\mu = 33.5^{\circ}C$. Also, from Figure 4.8b, the threshold stability plot shows stability of the parameter estimates around $\mu = -21^{\circ}C$.

From the mean threshold plot and the plot of the threshold stability, a threshold of $\mu = 34.5^{\circ}C$ and $\mu = 19.6^{\circ}C$ were selected for both the maximum and minimum temperatures respectively.

Table 4.10b is the estimate of the GP for the maximum temperature. With the threshold of $\mu = 34.5^{\circ}C$ there were 145 exceedances which constitutes a proportion of 0.2294.

Table 4.10b: GP parameter estimate for maximum Temperature

Parameter	Estimate	95% Confidence Interval
Scale (σ)	1.5679 (0.1587)	(1.2569, 1.8789)
Shape (γ)	-0.3059 (0.0632)	(-0.4297, -0.1820)

From Table 4.10b, the shape parameter is negative. This is an indication that the GP distribution has a “tail lighter” than the exponential distribution, thus, the GP distribution belongs to the “Weibull domain of attraction”.

Table 4.11b is the estimate of the GP for the average minimum temperature. With the threshold of $\mu = 19.6^\circ C$ there were 248 exceedances which constitutes a proportion of 0.4306.

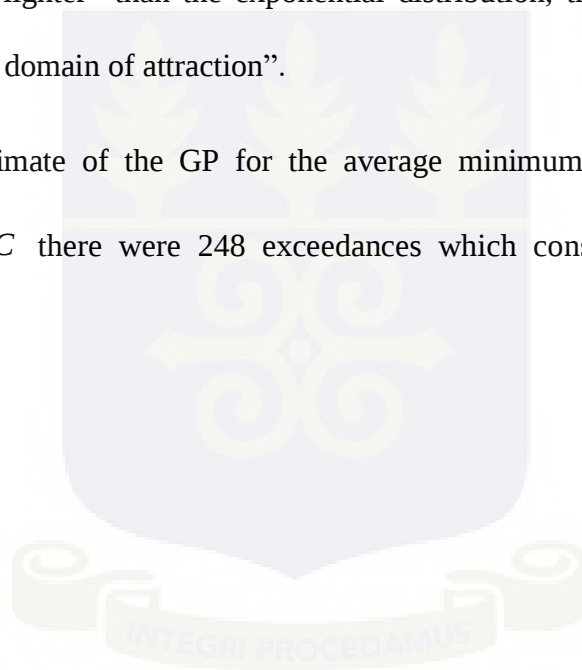
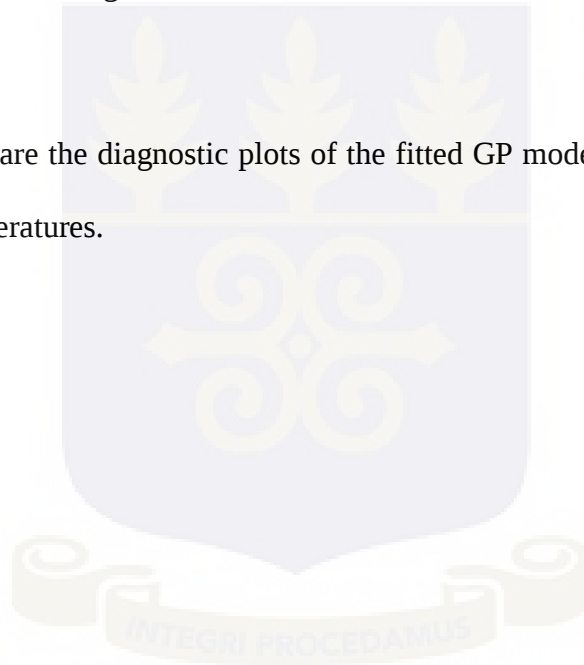


Table 4.11b: GP parameter estimate for Minimum Temperature

Parameter	Estimate	95% Confidence Interval
Scale (σ)	0.9964 (0.0966)	(0.8071, 1.1857)
Shape (γ)	-0.2624 (0.0751)	(-0.4096, -0.1152)

From Table 4.11b, it was observed that the appropriate distribution for the extreme minimum temperature occurrence is the light tailed Pareto distribution. This as a result of the negative shape parameter.

Figures 4.9b and 4.10b are the diagnostic plots of the fitted GP model for the maximum and minimum average temperatures.



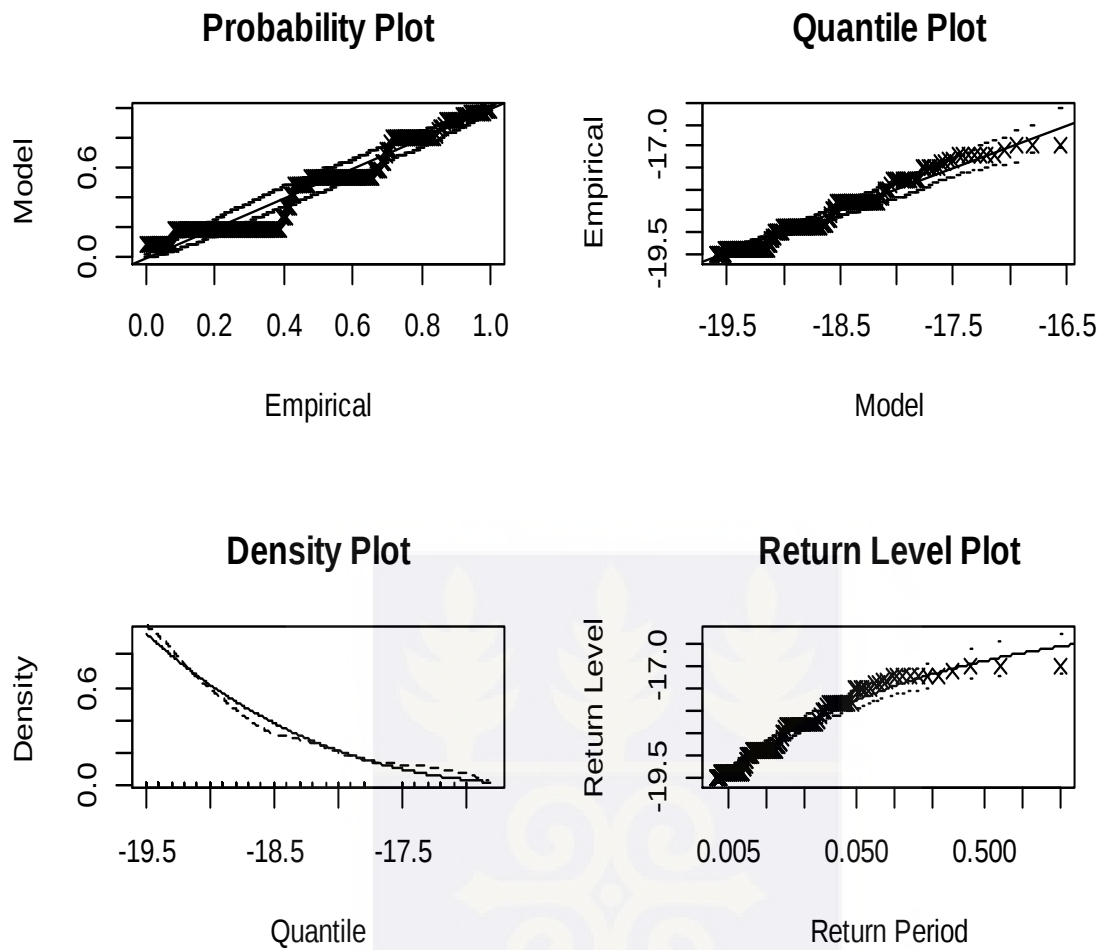


Figure 4.9b: Diagnostic plot for fitted GP for maximum Temperature

From Figure 4.9b, the linearity of the Q-Q and PP plots suggest valid goodness-of-fit. Furthermore, confidence intervals on the return level plot is an indication that the model departures are not large. Thus, the fitted generalized Pareto distribution is appropriate.

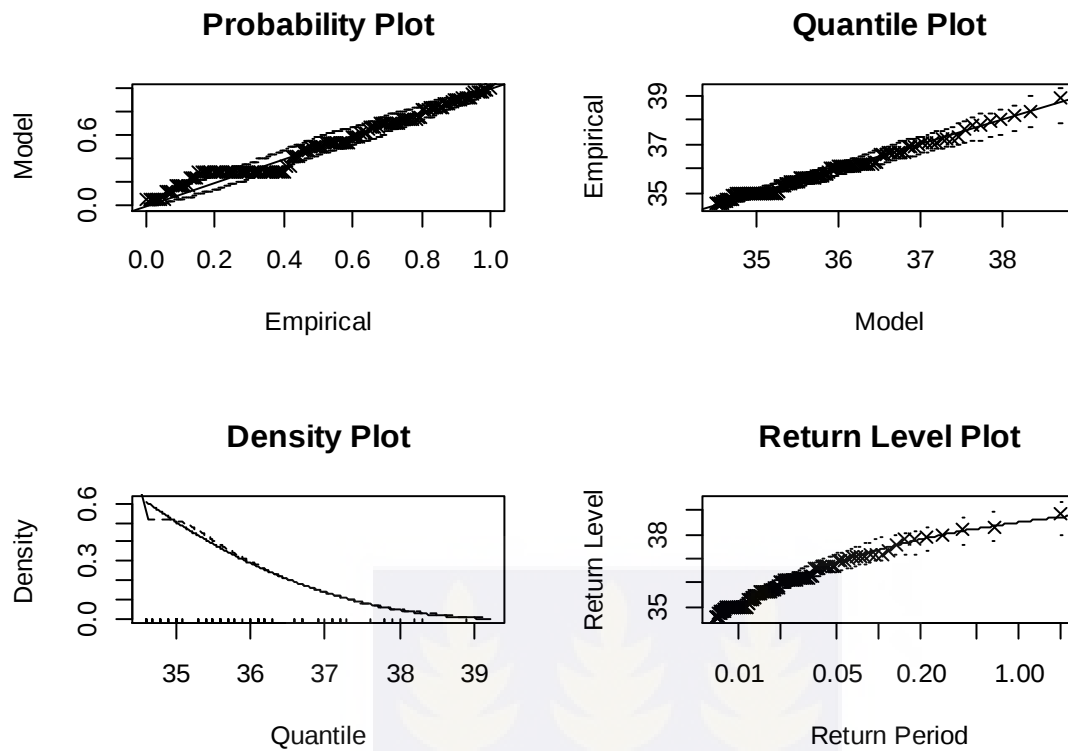


Figure 4.10b: Diagnostic plot for fitted GP for minimum Temperature

From Figure 4.10b, the Q-Q and PP plots are approximately linear with small deviations as depicted by the return level plot. The fitted generalized Pareto distribution is thus appropriate.

4.3.2.2 Rainfall

Figure 4.11b is the mean residual life plot for monthly average maximum rainfall readings.

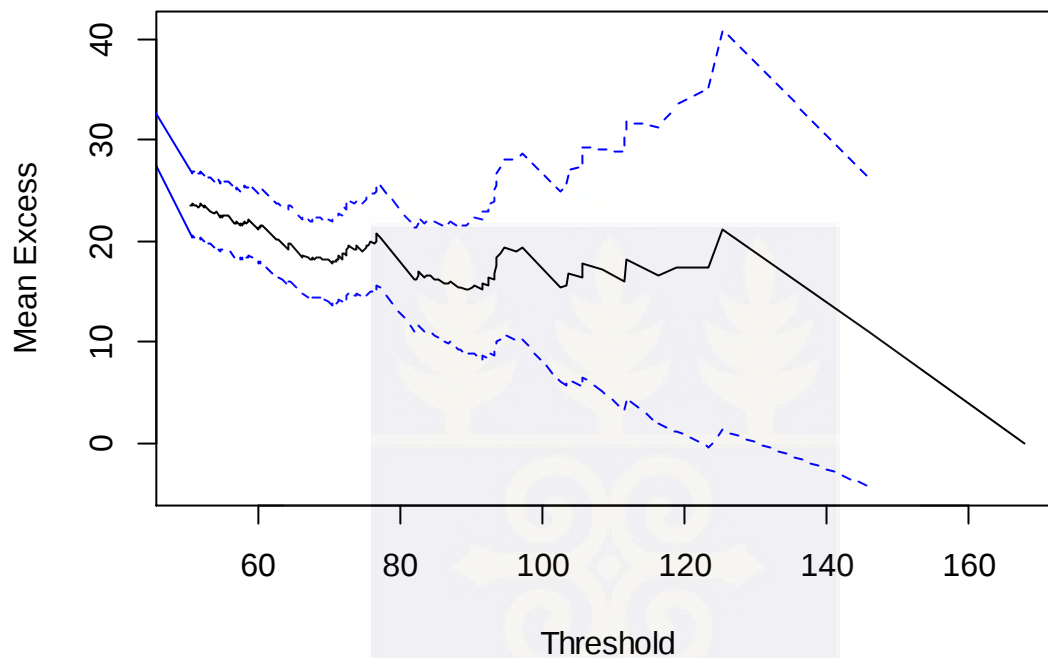


Figure 4.11b: Mean excess plot for maximum Rainfall

From Figure 4.11b the plot is decreasing with an observed non-linearity above $\mu = 60$ mm. A threshold of either $\mu = 70$ may be feasible. The threshold stability plot was also examined. This is presented in Figure 4.12b.

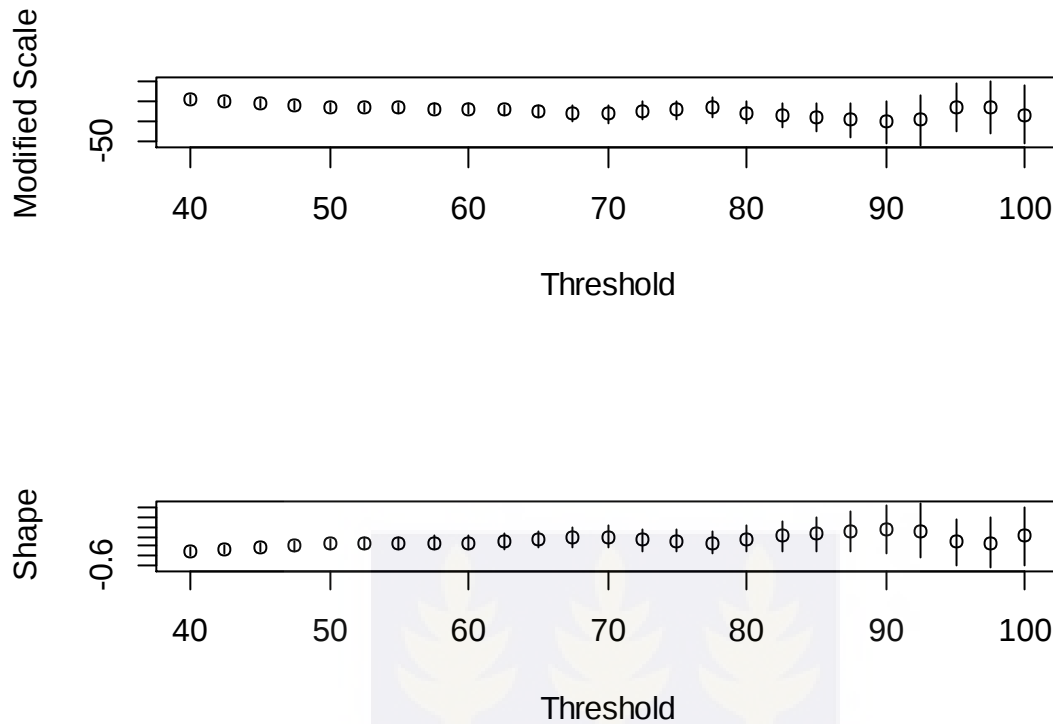


Figure 4.12b: Parameter estimates against threshold for maximum Rainfall

From Figure 4.12b, threshold of $\mu = 70$ inches was observed to be more feasible. Thus, from the mean threshold plot and the plot of the threshold stability, a threshold of $\mu = 70$ was selected. For this threshold, there were 74 exceedances which constitutes a proportion of 0.5068. Table 4.12b is the estimate of the GP for the average maximum rainfall.

Table 4.12b: GP parameter estimate for Maximum Rainfall

Parameter	Estimate	95% Confidence Interval
Scale (σ)	17.9901 (3.0549)	(12.0026, 23.9777)
Shape (γ)	0.0025 (0.1238)	(-0.2401, 0.2451)

From Table 4.12b, the shape parameter is positive, which is also an indication that, the GP distribution has lighter tail than the exponential distribution.

Figure 4.13b is the diagnostic plot of the fitted GP model for the maximum rainfall.

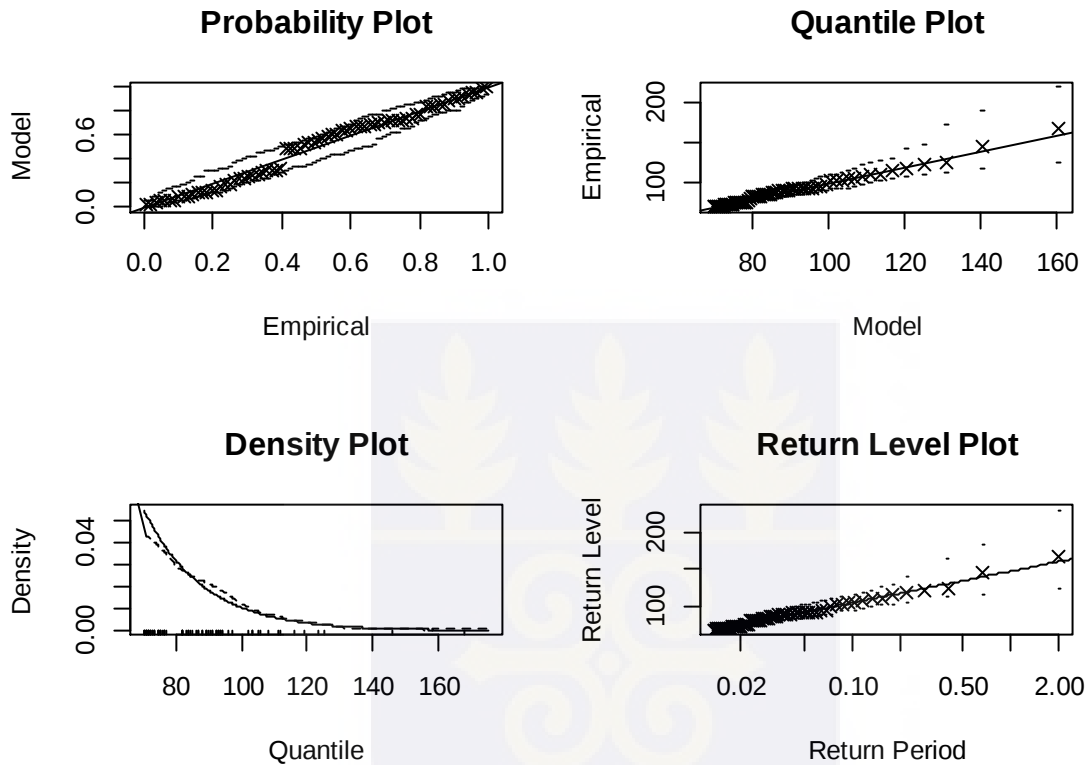


Figure 4.13b: Diagnostic plot for fitted GP for maximum Rainfall

From Figure 4.13b, the Q-Q and PP plots had slight deviations from linearity. This is confirmed by the “confidence intervals” observed on the “return level plot”. Thus, the goodness-of-fit is satisfied and therefore the fitted generalized Pareto distribution is appropriate.

4.3.3 General Comments

The extreme distribution of the temperature and rainfall occurrences in Ashanti region was explored in this section of the study. Interestingly, the extreme occurrences of temperature and rainfall also follows the Weibull and Frechet family of distributions respectively.

In this region, a maximum and minimum temperatures of 34.66°C and 19°C was estimated to occur once every five years. Also, estimated temperatures of 37.74°C and 17.15°C is predicted to occur in the region once every 100 years. Furthermore, the occurrence of maximum temperatures beyond 39°C every year was very unlikely. There was an estimated probabilities of 0.0011 and 0.072 of observing a maximum temperature beyond 39°C and below 17°C respectively.

The recorded maximum daily rainfall ever recorded over the study period was 169.9mm. It was estimated that a maximum daily rainfall amount of 85.61 mm will be recorded in the region every 5 years with an exceedance probability being small.

4.4 Northern Region

In this section of the study, the extreme value and the Generalized Pareto models were fitted to the extreme occurrence of temperature and rainfall readings in the Northern region.

4.4.1 Fitting the Extreme Value Model

The extreme occurrences of temperature and rainfall was explored through the Generalized Extreme Value distribution (GEVd). The Exponential QQ-plot was used to have a rough and quick confirmation of the plausible fit of the GEVd to the data.

4.4.1.1 Temperature

Figure 4.1c is the exponential Q-Q plots of both minimum (left panel) and maximum (right panel) Temperature.

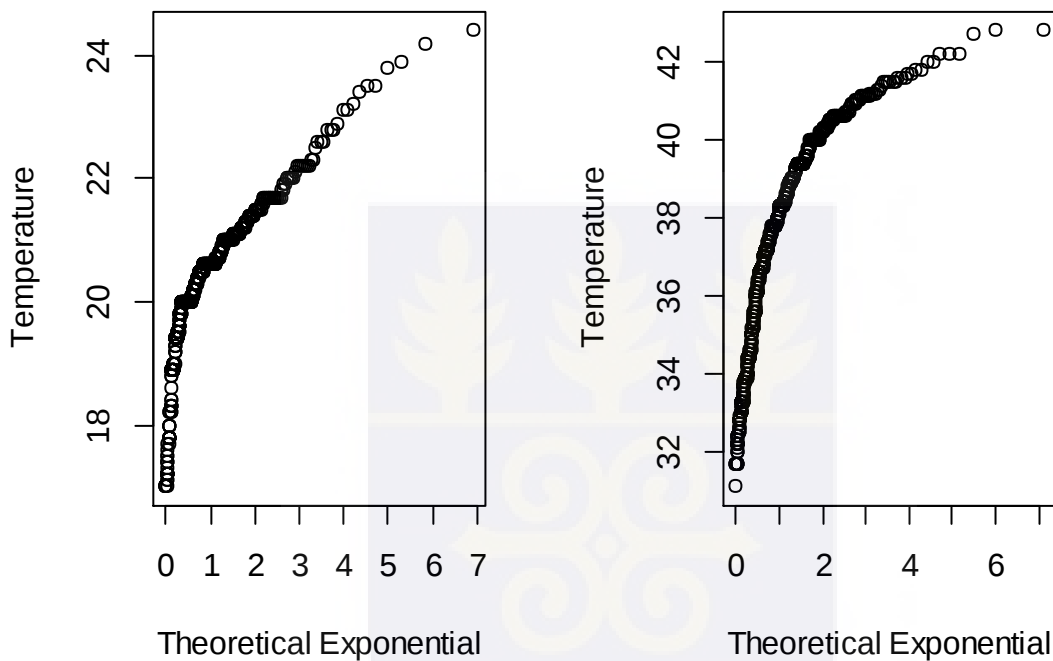


Figure 4.1c: Exponential Q-Q plot for minimum (left panel) and maximum (right panel) Temperature

From Figure 4.1c, the upper tails of the plot for both the minimum and maximum temperatures deviates from linearity pointing to an underlying distribution function with a “lighter right tail” than the “exponential distribution”. The generalized extreme value distribution was fitted to the data. The GEV models for both monthly maximum and minimum temperature were fitted using maximum likelihood. Tables 4.3c and 4.4c present the parameter estimates and their corresponding standard errors (in parentheses) for the minimum temperature and maximum temperature respectively.

Table 4.3c: GEV parameter estimate for Minimum Temperature

Parameter	Estimate	95% Interval	Confidence
Location (μ)	-20.6998 (0.0610)	(-20.8194, -20.5802)	
Scale (σ)	1.2614 (0.0408)	(1.1813, 1.3414)	
Shape (γ)	-0.2244 (0.0242)	(-0.2718, -0.1770)	

Table 4.4c: GEV parameter estimate for Maximum Temperature

Parameter	Estimate	95% Interval	Confidence
Location (μ)	36.088 (0.1393)	(35.8146, 36.3606)	
Scale (σ)	3.262 (0.09851)	(3.0686, 3.4547)	
Shape (γ)	-0.474 (0.0169)	(-0.5071, -0.4409)	

The shape parameter is negative for the distributions of both the minimum and maximum temperatures and this suggests that the underlying distribution belongs to the Weibull family of distributions.

Figures 4.2c and 4.3c are the diagnostic plots for the GEV fit for both minimum and maximum temperatures respectively.

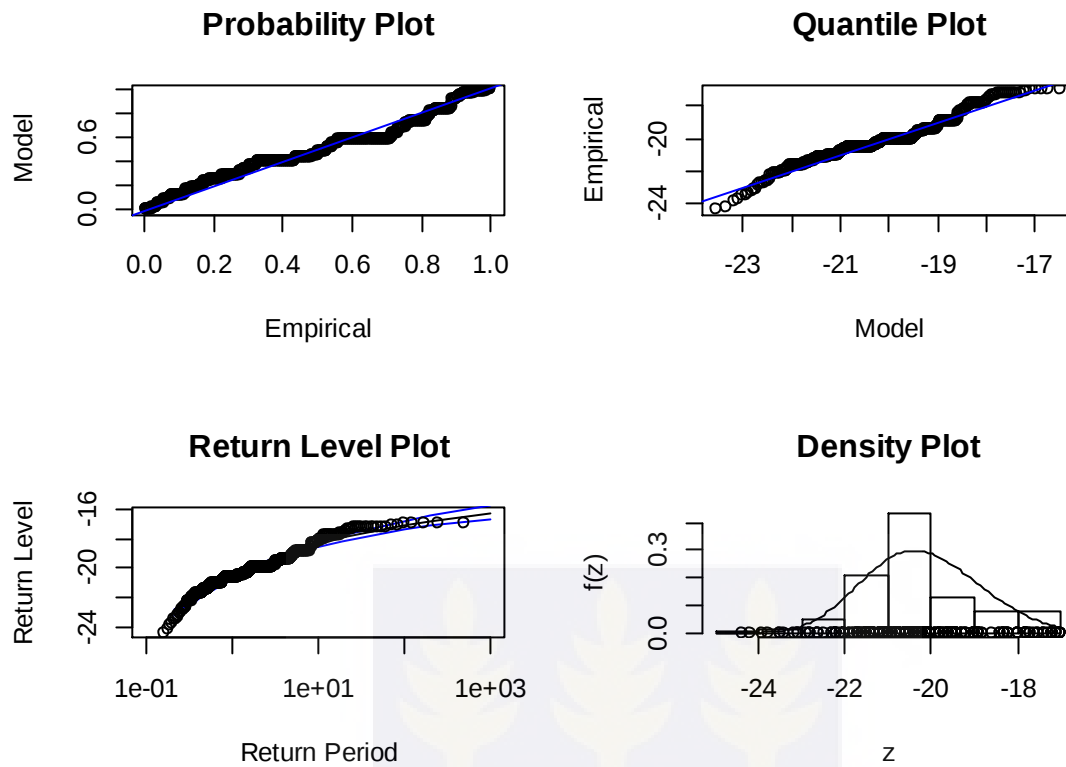


Figure 4.2c: Diagnostic plot for the fitted GEV model for minimum Temperature

From Figure 4.2c, deviations were observed from both Q-Q and PP plots. The deviations were significant though not that large as depicted by the return level plot. Also, the density plot shows that the residuals are approximately normal. Thus, the model is valid.

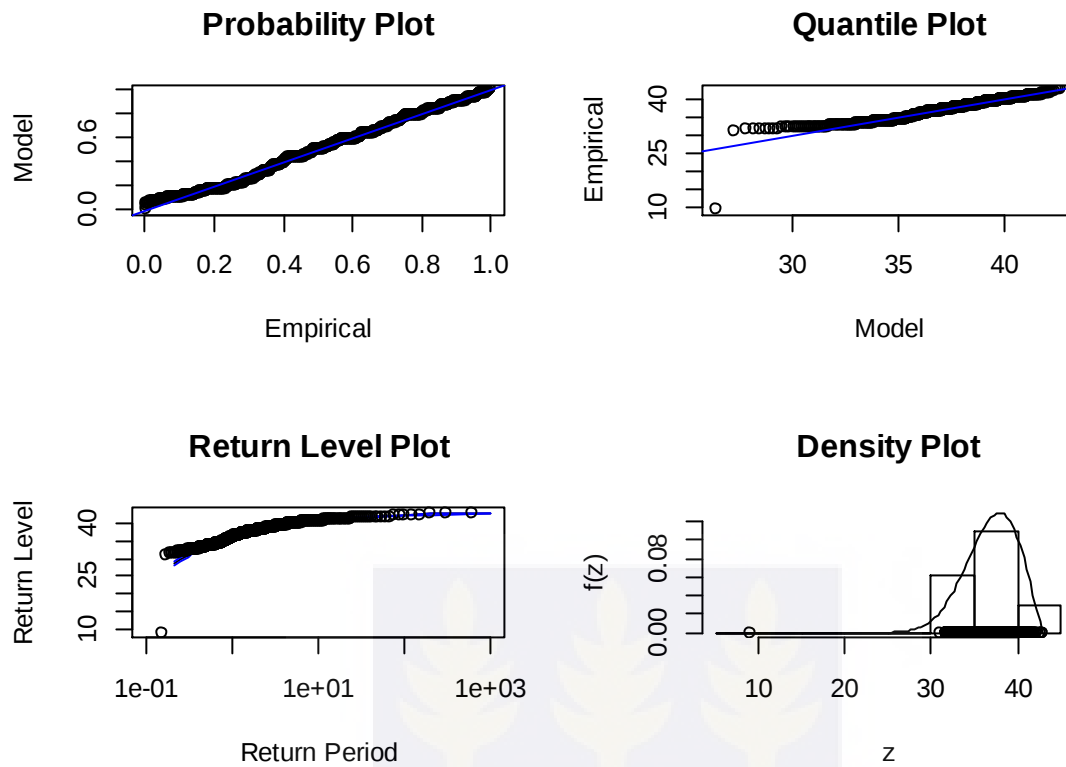


Figure 4.3c: Diagnostic plot for the fitted GEV model for maximum Temperature in Northern

From Figure 4.3c, the Q-Q plot shows some deviations at the lower tail but appears linear. The return level plot shows that these deviations are significant. However, on the average, the model is valid. The return periods and exceedance probabilities for the distributions was then estimated. Table 4.5c presents the probabilities of exceedances for some selected temperatures.

Table 4.5c: Exceedance Probabilities estimates for GEV fit for Temperature

Maximum Temperature						
Temperature	37	38	39	40	41	42
Exceedance Probs	0.5233	0.3954	0.2690	0.1562	0.0689	0.0159
Minimum Temperature						
Temperature	-22	-21	-20	-19	-18	-17
Exceedance Probs	0.9201	0.7166	0.4248	0.1820	0.0527	0.0083

From Table 4.5c, it was observed that, there was 0.5233 likelihood of observing a daily maximum temperature of 37°C in the northern region. The chances of observing a temperature above 42°C was very small. Also, there was 0.0083 chance of observing a minimum temperature of 17°C and chances of observing temperature below this was unlikely. However, a minimum temperature of 22°C was very likely. Table 4.6b presents the return periods for various average temperatures. The periods considered in the study were 5, 10, 20, 50 and 100 years.

Table 4.6c: Return Periods estimates of GEV fit for Temperature

Maximum Temperature					
Temperature	39.5897	40.6015	41.2861	41.8873	42.1923
Return Period (Years)	5	10	20	50	100
Minimum Temperature					
Temperature	-19.09	-18.47	-17.97	-17.42	-17.08
Return Period (Years)	5	10	20	50	100

Negative values are interpreted in absolute value terms

From Table 4.6c, the region is expected to experience a maximum temperature of 39.59°C once every five years with a minimum temperature of 19.1°C on the average. Again, the region will experience a maximum temperature of 42°C every 100 years.

4.4.1.2 Rainfall

The Exponential Q-Q plot of maximum Rainfall for is presented in Figure 4.4c.

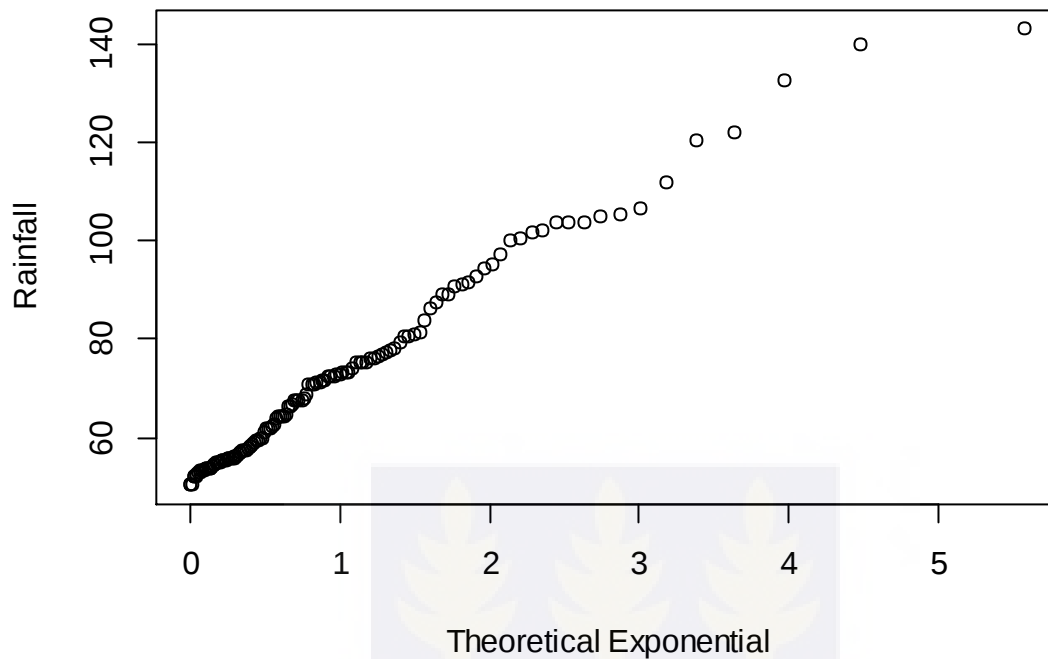


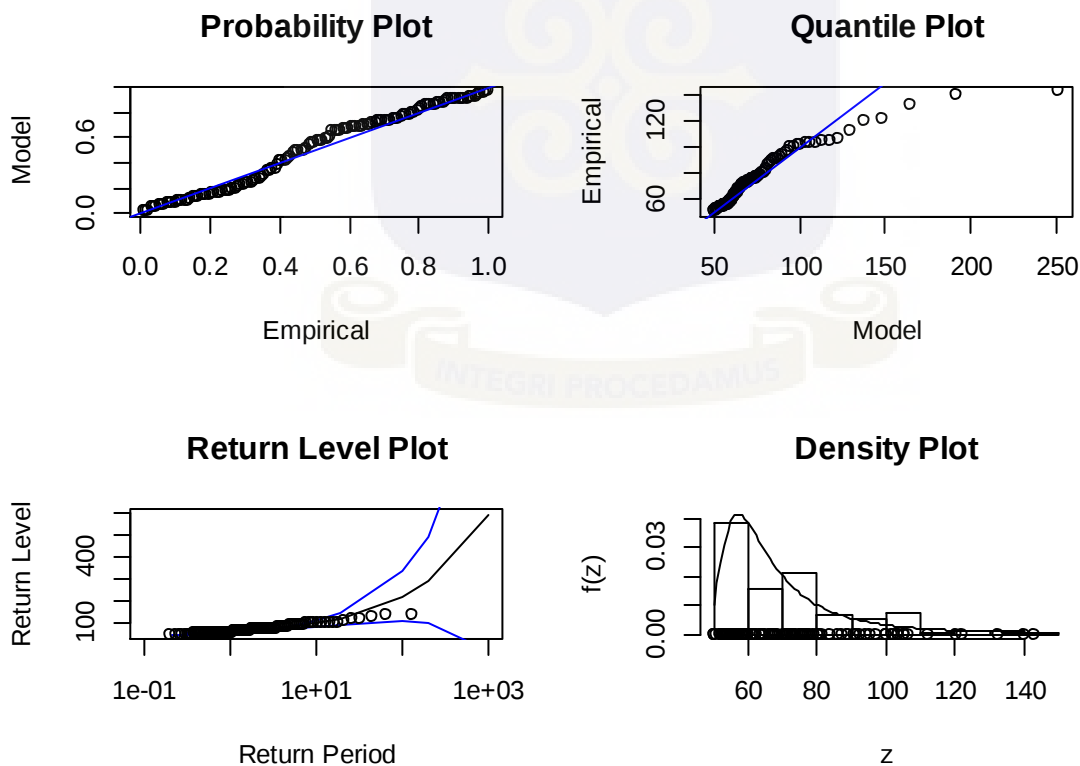
Figure 4.4c: Exponential Q-Q plot for minimum (left panel) and maximum (right panel) Rainfall

From Figure 4.4c, the plot of the maximum rainfall exhibits a roughly linear pattern for the plotted points, despite the slight deviation at the upper tail. The proposed exponential distribution thus appears to be inappropriate. The GEV model was rather fitted to the rainfall data. The estimated GEV model with the associated confidence intervals for the parameters is presented in Table 4.7c.

Table 4.7c: GEV parameter estimate for Maximum Rainfall

Parameter	Estimate	95% Interval	Confidence
Location (μ)	60.5174 (1.0690)	(58.4222, 62.6125)	
Scale (σ)	9.7901 (1.0033)	(7.8236, 11.7565)	
Shape (γ)	0.4774 (0.1191)	(0.2440, 0.7109)	

From Table 4.7c, the shape parameter is positive this suggests that the underlying distribution belongs to the Frechet family of distribution. The diagnostic plot is presented in Figure 4.5c.

**Figure 4.5c: Diagnostic plot for the fitted GEV model for maximum Temperature**

From Figure 4.5c, the return level plot indicate that the deviations depicted by both the Q-Q and PP plots is significant though not large. The goodness-of-fit therefore seems not convincing. However, we went ahead to estimate the probabilities of exceedances for some selected rainfall readings. This is presented in Table 4.8c

Table 4.8c: Exceedance Probabilities estimates for GEV fit Rainfall

Rainfall (X) in mm						
X	50	100	150	200	250	300
$P(X = x)$	0.9890	0.1002	0.0292	0.0134	0.0076	0.0049

From Table 4.8c, there was 0.989 chance of experiencing a rainfall amount of 50 mm on the average. The chances of recording an average amount of rainfall greater than 50 mm was very small. The return periods was also estimated. Table 4.9c presents the return periods for various average rainfall readings.

Table 4.9c: Return Periods estimates of GEV fit for Rainfall

Maximum Rainfall					
Rain	81.98	100.06	124.68	172.11	224.369
Return Period (Years)	5	10	20	50	100

From Table 4.9c, Northern region is expected to record 81.98 mm amount of rainfall readings once every 5 years. A 100 year return period for the maximum rainfall in the region is 224.37mm.

4.3.2 Fitting the Generalized Pareto (GP) Model

The generalized Pareto model was fitted to both the temperature and rainfall data in this section of the study

4.3.2.1 Temperature

The generalized Pareto model was fitted to the average temperature. The mean residual life plot and a threshold stability plot was employed also employed here as well in the selection of an appropriate threshold. Figure 4.6c is the mean residual life plot for monthly average minimum (left panel) and maximum (right panel) temperatures.



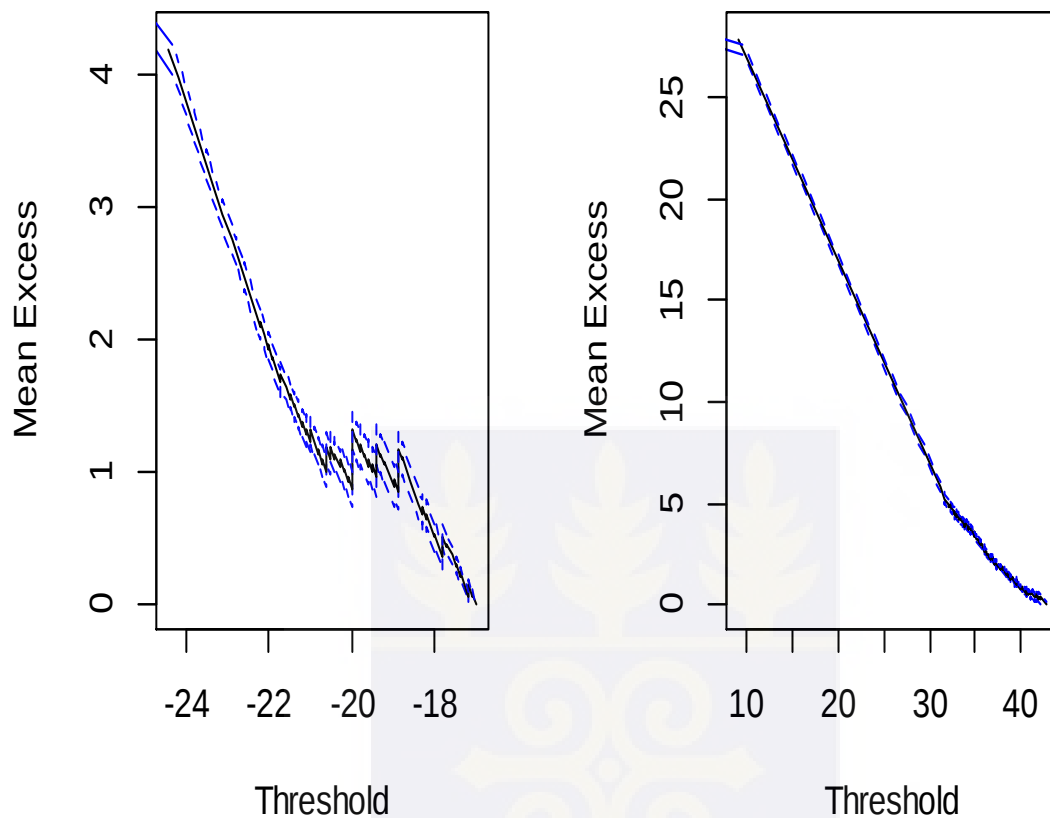


Figure 4.6c: Mean excess plot for minimum (left panel) and maximum (right panel) Temperature

From Figure 4.6c there is an observed linearity above $\mu = 30^{\circ}C$ for the maximum average monthly temperature. There was also evidence of linearity above $\mu = -21^{\circ}C$ for the minimum temperature. A threshold of $34^{\circ}C$ for the maximum temperature and $20^{\circ}C$ for the minimum temperature seems feasible.

Figures 4.7c and 4.8c present the threshold stability plot of the maximum and minimum average temperatures respectively.

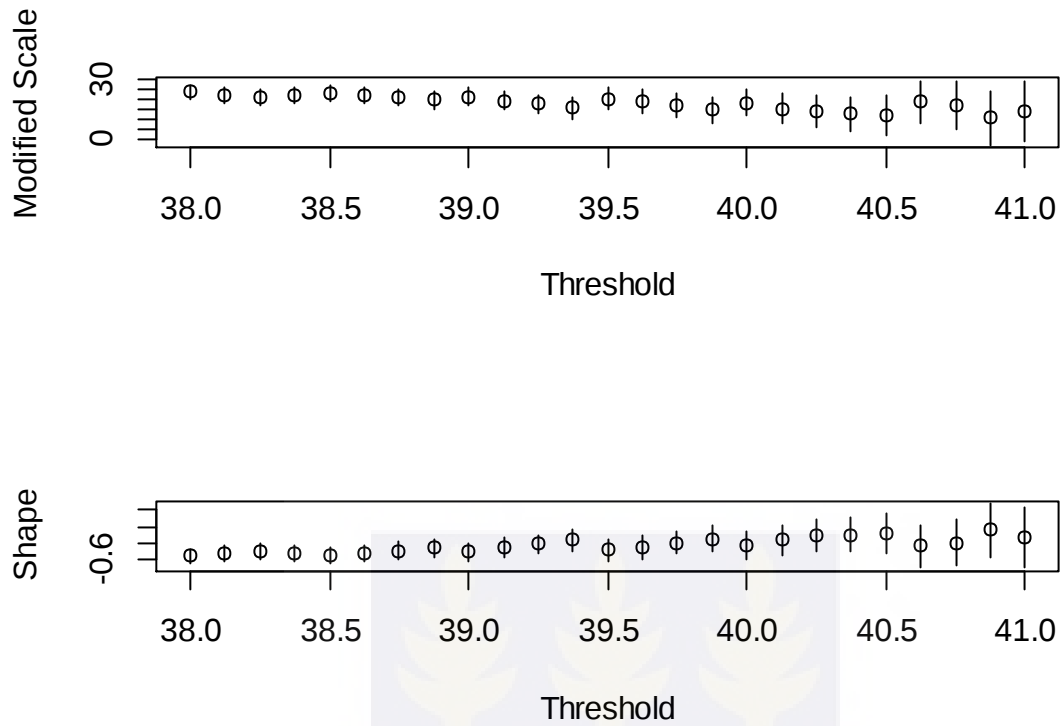


Figure 4.7c: Parameter estimates against threshold for maximum Temperature

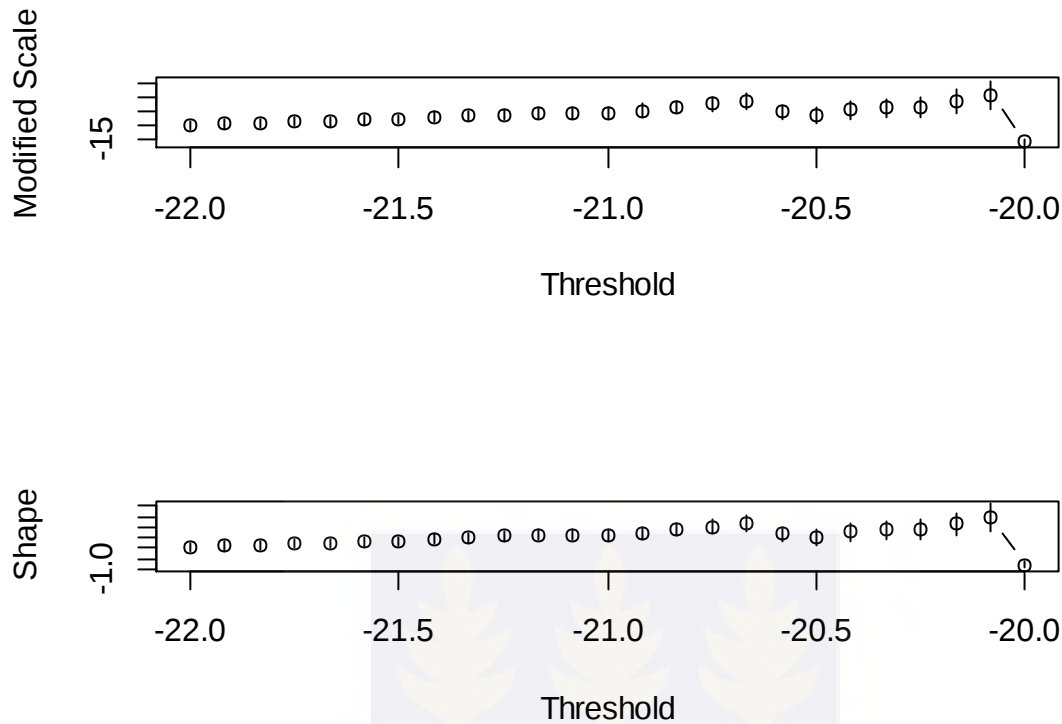


Figure 4.8c: Parameter estimates against threshold for minimum Temperature

From Figure 4.7c, the parameter estimates for the maximum temperature appear stable around $\mu = 39^{\circ}C$. Also, from Figure 4.8c, the threshold stability plot shows stability of the parameter estimates around $\mu = -21^{\circ}C$.

From the mean threshold plot and the plot of the threshold stability, a threshold of $\mu = 40.5^{\circ}C$ and $\mu = 20.7^{\circ}C$ were selected for both the maximum and minimum temperatures respectively.

Table 4.10c is the estimate of the GP for the average maximum temperature. With the threshold of $\mu = 40.5^{\circ}C$ there were 65 exceedances which constitutes a proportion of 0.1055.

Table 4.10c: GP parameter estimate for maximum Temperature

Parameter	Estimate	95% Confidence Interval
Scale (σ)	0.8986 (0.1535)	(0.5977, 1.1994)
Shape (γ)	-0.2793 (0.1225)	(-0.5193, -0.0393)

From Table 4.10c, the shape parameter is negative and this indicates that, the GP distribution has a tail lighter than the exponential distribution, thus, the GP distribution belongs to the Weibull domain of attraction.

Table 4.11c is the estimate of the GP for the average minimum temperature. With the threshold of $\mu = 20.7^\circ C$ there were 343 exceedances which constitutes a proportion of 0.6765.

Table 4.11c: GP parameter estimate for Minimum Temperature

Parameter	Estimate	95% Confidence Interval
Scale (σ)	1.3100 (0.1148)	(1.0849, 1.5350)
Shape (γ)	-0.1771 (0.0697)	(-0.3137, -0.0405)

From Table 4.11c, the shape parameter is negative. The negativity of the shape indicates that, the GP distribution has a tail lighter than the exponential distribution, thus, the GP distribution belongs to the Weibull domain of attraction.

Figures 4.9c and 4.10c are the diagnostic plots of the fitted GP model for the maximum and minimum average temperatures.

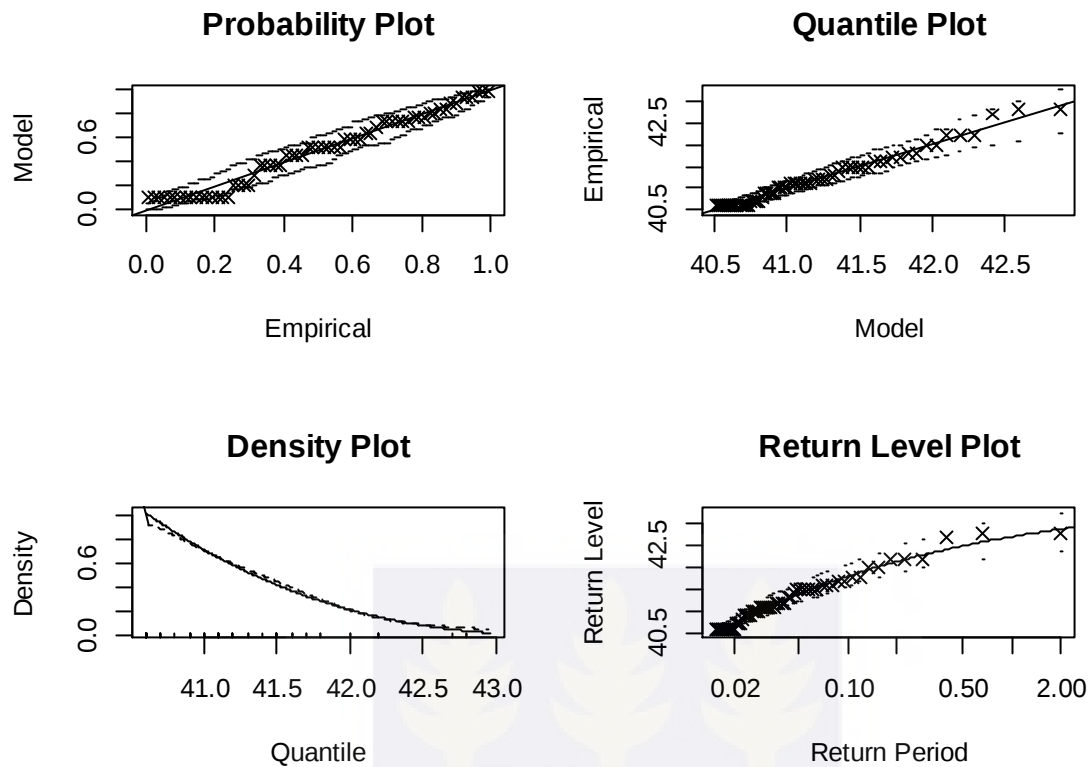


Figure 4.9c: Diagnostic plot for fitted GP for maximum Temperature

From Figure 4.9c, the linearity of the Q-Q and PP plots suggest valid goodness-of-fit. Furthermore, confidence intervals on the return level plot is an indication that the model departures are not large. Thus, the fitted generalized Pareto distribution is appropriate.

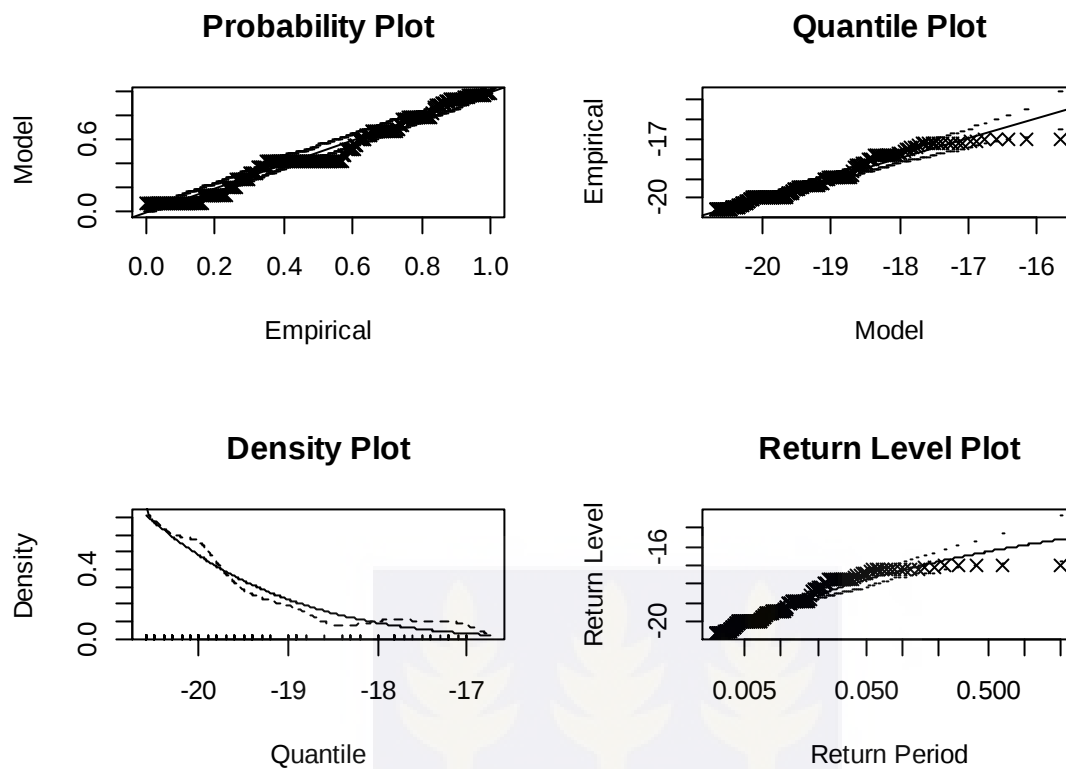


Figure 4.10c: Diagnostic plot for fitted GP for minimum Temperature

From Figure 4.10c, the linearity of the Q-Q and PP plots suggest valid goodness-of-fit. Furthermore, confidence intervals on the return level plot is an indication that the model departures are not large. Thus, the fitted generalized Pareto distribution is appropriate.

4.3.2.2 Rainfall

Figure 4.11b is the mean residual life plot for monthly average maximum rainfall readings.

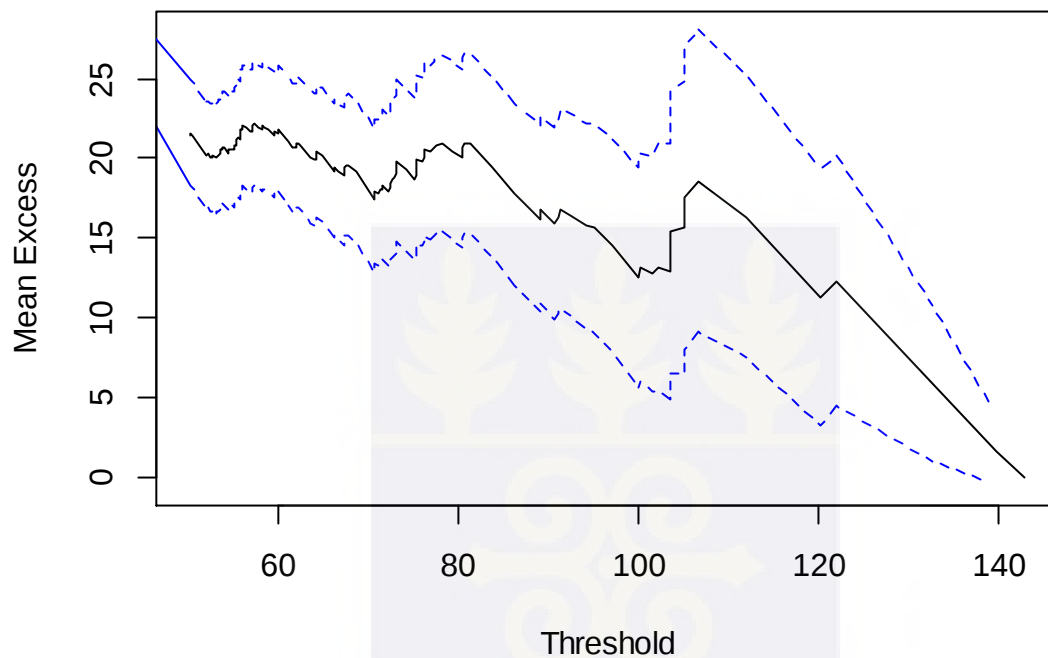


Figure 4.11c: Mean excess plot for maximum Rainfall

From Figure 4.11c the plot is decreasing with an observed non-linearity above $\mu = 70$ inches.

A threshold of either $\mu = 70$ may be feasible. The threshold stability plot was also examined.

This is presented in Figure 4.12c.

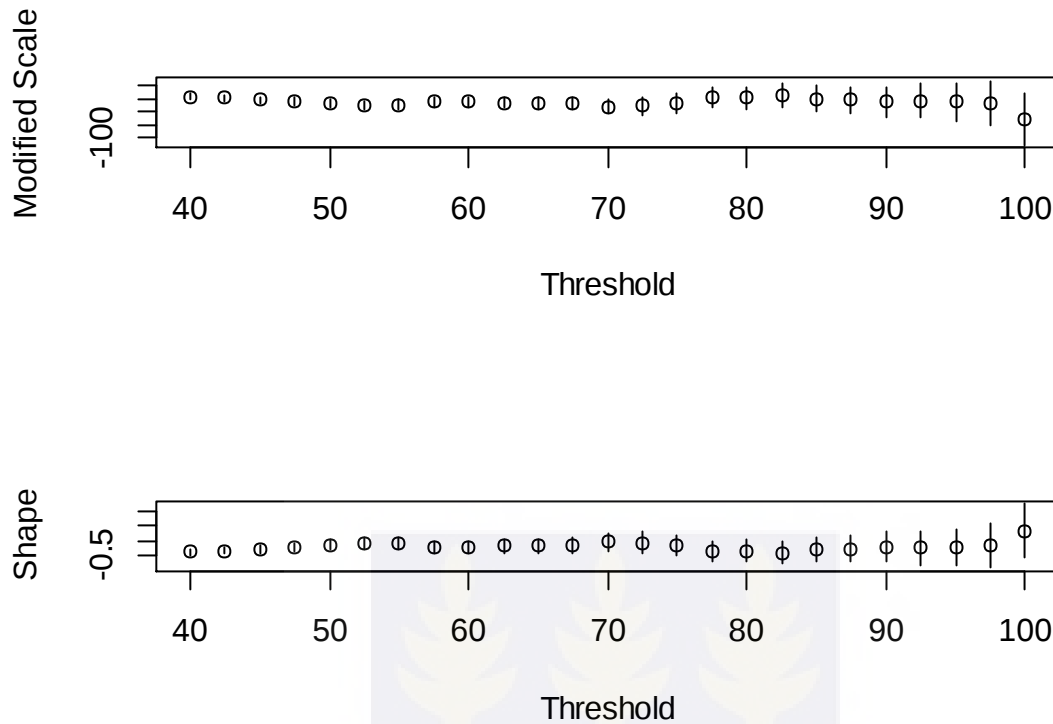


Figure 4.12c: Parameter estimates against threshold for maximum Rainfall

From Figure 4.12c, threshold of $\mu = 70$ inches was observed to be more feasible. Thus, from the mean threshold plot and the plot of the threshold stability, a threshold of $\mu = 60$ was selected. For this threshold, there were 81 exceedances which constitutes a proportion of 0.6136. Table 4.12c is the estimate of the GP for the average maximum rainfall.

Table 4.12c: GP parameter estimate for Maximum Rainfall

Parameter	Estimate	95% Confidence Interval
Scale (σ)	26.8341 (3.9725)	(19.0481, 34.6201)
Shape (γ)	-0.2129 (0.1006)	(-0.4101, -0.0157)

From Table 4.12c, it was observed that the appropriate distribution for the extreme minimum temperature occurrence is the to heavy tailed Pareto distribution. This as a result of the positive shape parameter. Figure 4.13c is the diagnostic plot of the fitted GP model for the maximum rainfall.

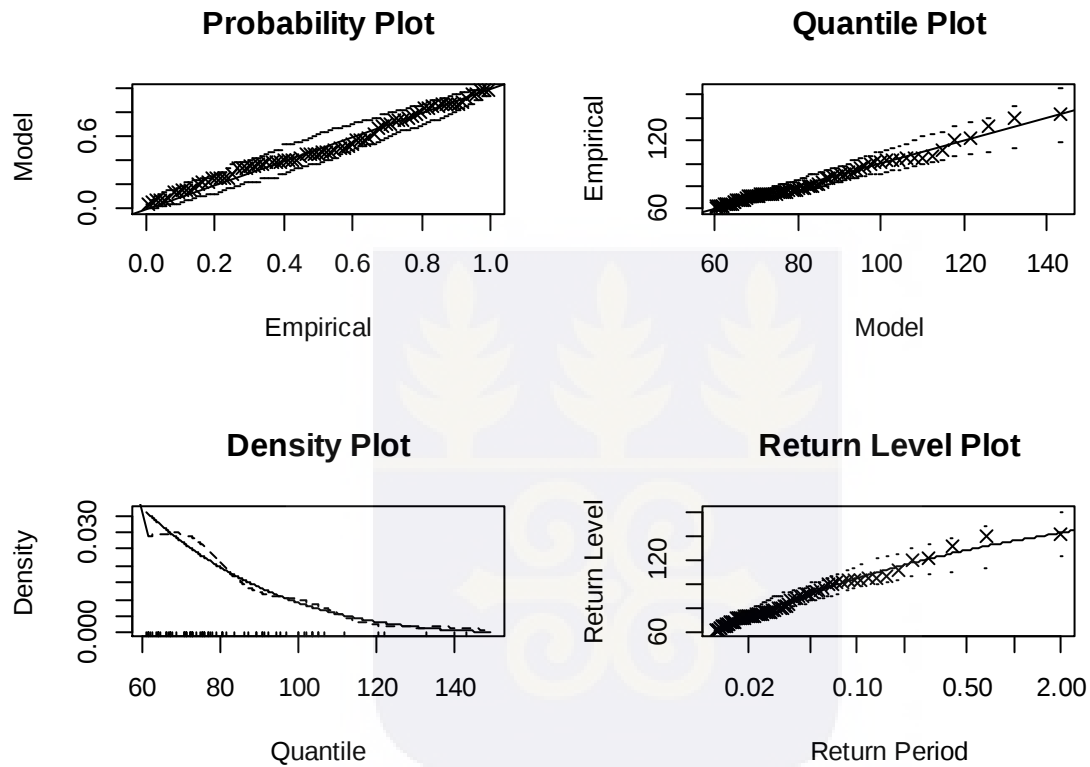


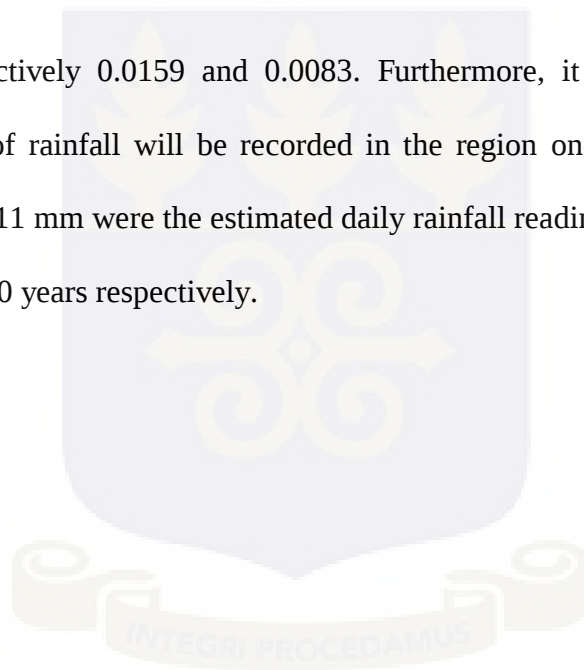
Figure 4.13c: Diagnostic plot for fitted GP for maximum Rainfall

From Figure 4.13c, the Q-Q and PP plots had slight deviations from linearity. This is confirmed by the confidence intervals observed on the return level plot. Thus, the fitted generalized Pareto distribution is appropriate.

4.4.3 General Comments

In this section, the extreme distribution of the temperature and rainfall occurrences in Northern region was explored. Again, the extreme occurrences of temperature and rainfall had the Weibull and Frechet family of distributions respectively.

The minimum and maximum temperatures of this region was 19.6°C and 42.8°C respectively. A maximum and minimum temperatures of 39.6°C and 19.1°C was estimated to occur once every five years with probability of exceeding such temperatures as 0.1562 and 0.1820 respectively. The exceedance for a maximum and minimum temperatures of 42°C and 17°C were respectively 0.0159 and 0.0083. Furthermore, it was estimated that an amount of 81.98 mm of rainfall will be recorded in the region once every 5 years. Also, 100.1, 124.68, and 172.11 mm were the estimated daily rainfall readings to be recorded in the once every 10, 20 and 50 years respectively.



CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATION

In this chapter, we draw conclusions from the extreme value analysis of climate data (i.e Temperature and Rainfall) from previous chapters. We end this chapter with some recommendations and raises a few issues that call for further investigation. The aim of the study was to;

1. find the appropriate distribution for the tails of the distributions of rainfall and temperature
2. find out how extremes of rainfall relates to the extremes of temperature.
3. find the exceedance probabilities for selected levels of rainfall and temperature
4. obtain the return periods of extreme rainfall and temperatures and their corresponding return levels for rainfall and temperature

The study was based on secondary data obtained from the Ghana meteorological Agency office in Accra. Daily temperature and rainfall readings for the period January 1960 to December 2012 was obtained. The study focused on three key regions which takes care of the Southern (Greater Accra), Central (Ashanti Region) and Northern (Northern region) sectors of the country.

Northern region recorded the highest temperature $42.8^{\circ}C$ as compared to $38.9^{\circ}C$ recorded by both Ashanti and Greater Accra Region.

It was observed that the extreme occurrences of temperature and rainfall can be modelled using Weibull and Frechet family of distributions in Ghana.

A maximum temperature of $34.7^{\circ}C$, $34.66^{\circ}C$, and $39.6^{\circ}C$ was predicted to occur in Accra, Ashanti and Northern regions respectively once every five years. The chances of exceeding a

temperature of 39°C in Accra was estimated as 0.001 with likelihood of exceeding a minimum temperature of 22°C being 0.014.

Also, the likelihood of exceeding a maximum and minimum temperatures of 39°C and 17°C for Ashanti are 0.0011 and 0.072 respectively. In addition, the exceedance probabilities for maximum (42°C) and minimum (17°C) temperatures for Northern region were 0.0159 and 0.0083 respectively.

Furthermore, it was estimated that an amount of 88.38mm, 85.61mm, and 81.98mm will be recorded in Accra, Ashanti, and Northern regions once every 5 years respectively.

RECOMMENDATIONS

As a result of the findings of this study, the following are recommended:

1. Combining time series with EVT for the case of applying EVT to climate data might be of interest.
2. It may be of interest to look at non-stationary extremes
3. Interested organizations can employ this technique as one of the means of forecasting extreme occurrences.
4. Multivariate Extreme Value Analysis, which include other factors such as relative humidity, wind speed should be employed to further investigate this problem.
5. Researchers are encouraged to look into other areas of EVT applications such as using Bayesian approach or extreme quantiles to further investigate this problem in Ghana.

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APPENDIX A

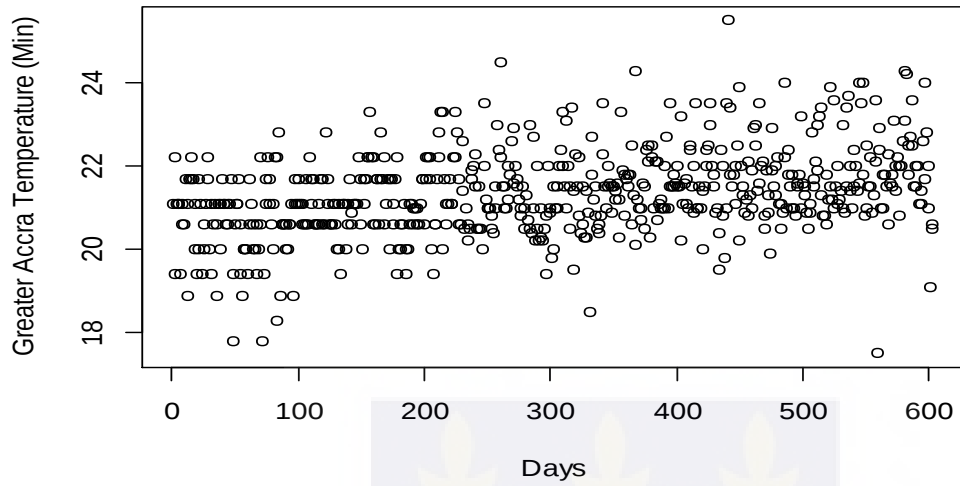


Figure 1A: Scatter plot for Minimum Temperature in the Greater Accra Region

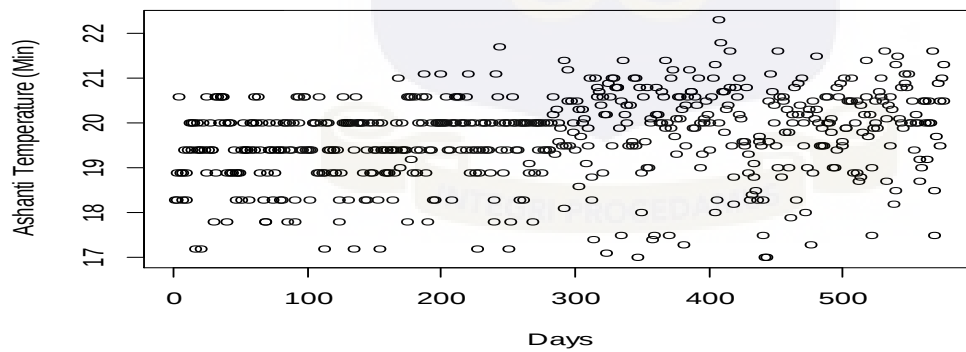


Figure 2A: Scatter plot for Minimum Temperature in the Ashanti Region

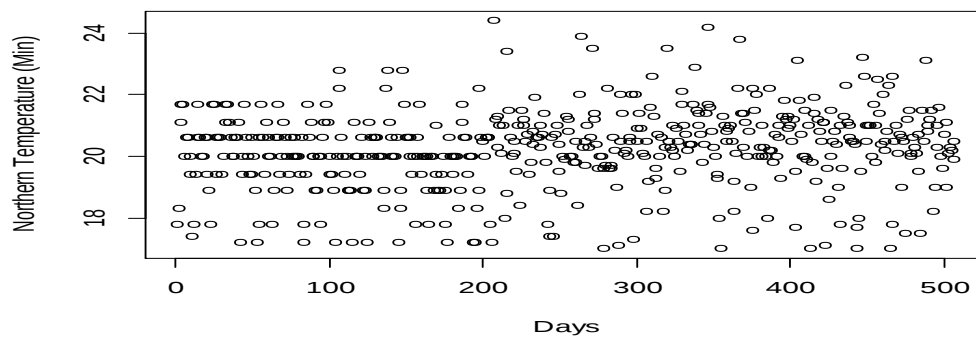


Figure 3A: Scatter plot for Minimum Temperature in the Northern Region

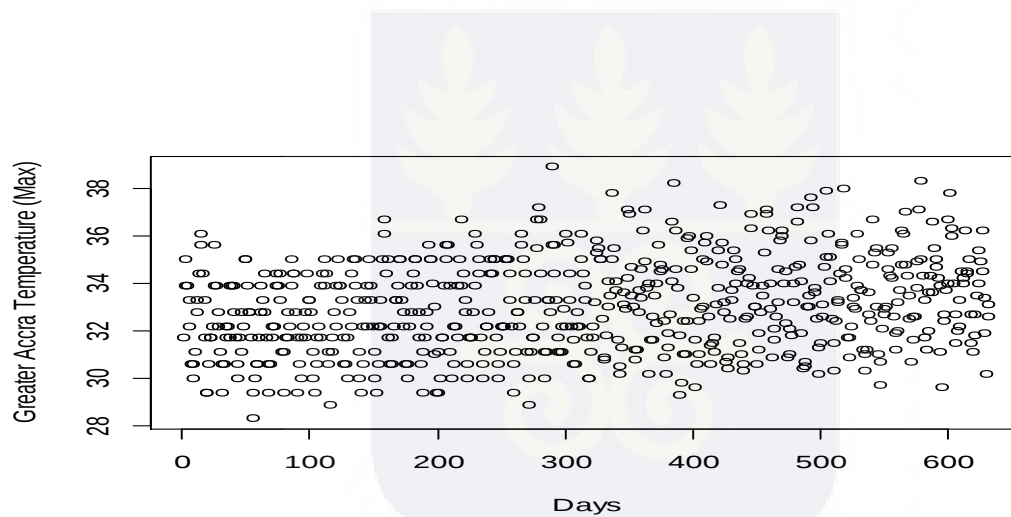


Figure 4A: Scatter plot for Maximum Temperature in the Greater Accra Region

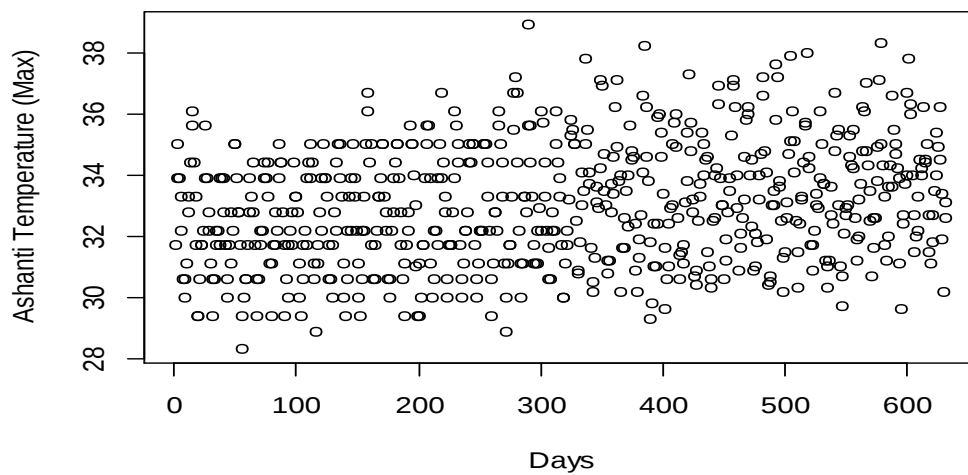


Figure 5A: Scatter plot for Maximum Temperature in the Ashanti Region

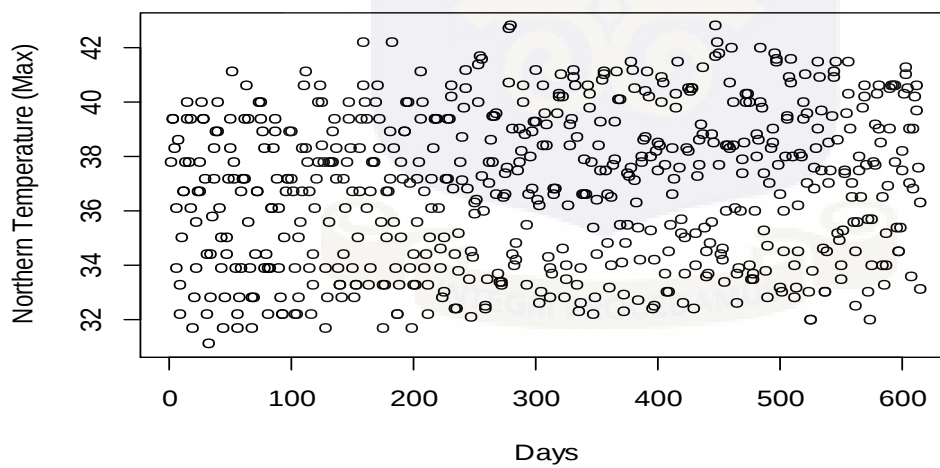


Figure 6A: Scatter plot for Maximum Temperature in the Northern Region

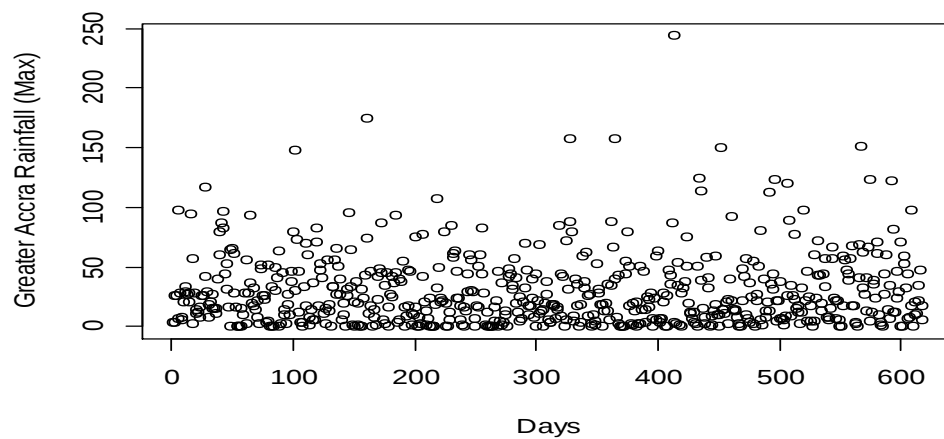


Figure 7A: Scatter plot for Maximum Rainfall in the Greater Accra Region

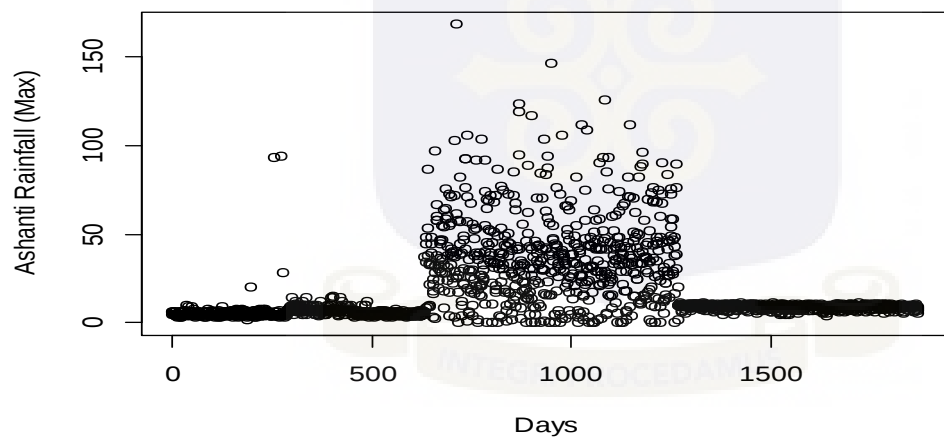


Figure 8A: Scatter plot for Maximum Rainfall in the Ashanti Region

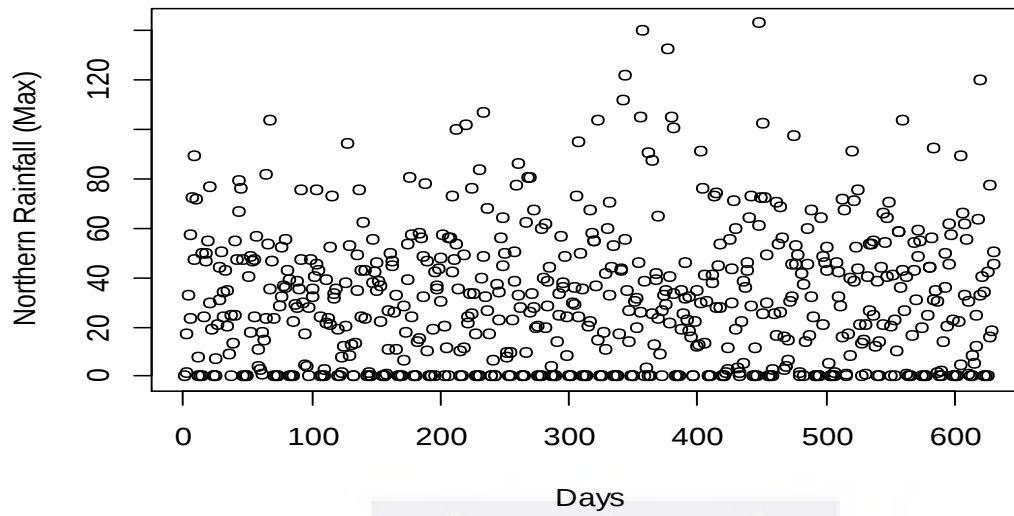


Figure 8A: Scatter plot for Maximum Rainfall in the Northern Region

