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Group lending with covariate risk

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ABSTRACT

Group-based lending with joint liability has been a major tool microfinance institutions (“MFIs”) have employed to improve lending feasibility. The related theoretical literature typically assumes borrowers face independent risk. This paper examines how covariate risk affects the usefulness of joint liability lending, in the hidden-information setting of Stiglitz and Weiss (1981) and Ghatak (2000). In a benchmark setting where all agents face the same degree of covariate risk, greater correlation renders group lending less effective; this is because the effective rate of joint liability is reduced when borrowers are more likely to fail together. We focus on a setting where the extensive and intensive margins are distinguished: some agents face independent risk while others face correlated risk. We find that an intermediate prevalence of correlated risk can lead to lower outreach than both a low and a high prevalence. Thus, reaching a market with mixed covariate risk profiles, e.g. farmers and micro-entrepreneurs, can be harder than reaching markets with a single profile of either kind. Assuming limited ability of lenders to use information on borrower correlatedness, we find that higher outreach is often achievable by separately servicing correlated and non-correlated borrowers. This can help explain the existence of specialized institutions such as agricultural banks versus standard microenterprise-focused MFIs.

1. Introduction

The spread of microfinance over the past five decades is one of the more remarkable recent developments in the developing world. As the 2006 Nobel Peace Prize press release puts it, “loans to poor people without any financial security had appeared to be an impossible idea”. From near non-existence fifty years ago, microfinance grew to reach an estimated 211 million clients by the end of 2013 (Reed, 2015). Further, much of this financial inclusion is increasingly accomplished without subsidies (Cull et al., 2009), suggesting that gains from trade are being realized.

The puzzle of the takeoff of microcredit has drawn the attention of economists. Much attention has naturally focused on innovations within the movement that could help explain the enhanced ability to lend in environments where reliance on collateral is ineffective. Among these, group-based joint liability lending (“group lending”) has been at the forefront. Economists have shown theoretically how requiring borrowers to co-sign each others’ loans within a formal group can harness local information and enforcement capabilities to repair credit markets without use of collateral.¹ And while group lending is not used universally by MFIs,² its wide use persists.

One gap in this theoretical literature is that it typically ignores covariate risk, assuming for convenience that borrowers’ project returns are uncorrelated. Yet covariate risk may clearly play an important role when borrowers are asked to bear liability for each other’s loans. As Ghatak (2000) has conjectured, covariate risk may partially or completely undo the benefits of group lending. The goal of this paper is to analyze more formally the role of covariate risk in the effectiveness of group lending.

This topic is pertinent because covariate risk is likely a reality for many microcredit borrowers. At the very least, borrowers in the same group typically face many of the same aggregate economic conditions — national, regional, and local. Covariate risk is especially endemic in agriculture, which employs a large fraction of the world’s poorest, given common dependence on commodity prices and weather and pest conditions (Besley, 1994; Mahajan and Ramana, 2004; Zeller, 2006; IFC, 2012, 2014). And while most microcredit is extended for non-agricultural enterprise, group-based microcredit is used for agricultural loans as well.

We analyze this issue in a hidden information context based on Stiglitz and Weiss (1981) and Ghatak (2000). In this setting, agents require capital to fund risky projects that differ in privately observed

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E-mail addresses: ahlin@msu.edu (C. Ahlin), gdebrah@ug.edu.gh (G. Debrah).¹ See Stiglitz (1990), Besley and Coate (1995), Ghatak (1999, 2000), and Rai and Sjöström (2004), among others.² De Quidt et al. (2018) and Ahlin and Suandi (2019) document a recent decline in group lending among MFIs and offer several explanations.

second-order risk. Collateral is not available and liability is limited to project returns. Offering everyone individual loans at the same interest rate would lead safe borrowers to repay more in expectation due to their lower likelihood of default; the resulting safe-to-risky cross-subsidy could overwhelm the gains from borrowing for safe agents, causing them to opt out of the market.

Group lending can restore the market in this context if there is good local information, i.e. if agents know each others' risk (Ghatak, 1999, 2000). Key to this result is that agents respond to joint liability by forming groups with self-similar agents. This implies that safe borrowers are liable for safe partners, and risky for risky; but bearing liability for safer partners gives safe borrowers an implicit discount in their repayment burden. This discount can be enough to draw them into the market. Thus, group lending achieves full outreach (and full efficiency) over a larger parameter space than individual lending.

Our first step is to introduce covariate risk into this setting. We do so in such a way that all homogeneous pairs of agents have the same correlation coefficient $\rho \in [0, 1]$ for project returns. This leads to a clear result formalizing Ghatak's conjecture (2000): correlation transforms the contractual rate of joint liability, c , into an "effective" degree of joint liability, $c(1 - \rho)$. Clearly, greater correlation mutes any joint liability stipulation, and perfect correlation effectively eliminates joint liability altogether. This result provides a possible explanation for the stylized fact that agriculture has largely been missed by the microfinance movement (Mahajan and Ramana, 2004; Zeller, 2006; Morvant-Roux, 2011; IFC, 2012, 2014): highly correlated risk makes one key tool of a microlender ineffective.³

Our main focus is on a setting where covariate risk applies to some potential borrowers more than others. For example, many rural settings contain a mix of farmers and microentrepreneurs. We introduce a second dimension of privately observed heterogeneity independent of risk-type, related to correlatedness. A fraction $k \in (0, 1)$ are "correlated" at rate ρ with each other; a fraction $1 - k$ are "uncorrelated" with everyone. We thus allow covariate risk to vary on the extensive margin (k) and intensive margin (ρ). In the matching equilibrium, again agents form groups with self-similar agents, both in amount of risk and in risk correlatedness.⁴

Unsurprisingly, full outreach is harder to attain the greater is covariate risk on either margin. When expected covariate risk ($k\rho$) gets high enough, joint liability is useless in achieving universal outreach. There is one exception, a discontinuity at $k = 1$: full outreach becomes easier to achieve when everyone faces correlated risk than when a large fraction do. That is, it is easier to reach fully a population all of whom are correlated than a population in which a sufficiently high fraction are. This result is due to a novel cross-subsidy that arises in this setting. Not only do safe cross-subsidize risky, but uncorrelated borrowers cross-subsidize correlated borrowers. Uncorrelated borrowers face effective liability rate c , while correlated borrowers face a discounted effective rate, $c(1 - \rho)$. When $k = 1$ there is only the standard safe-to-risky cross-subsidy; when k drops just below 1, both cross-subsidies are in operation, making the marginal (safe-uncorrelated) agents discontinuously harder to reach.

There is a related non-monotonicity in maximal outreach (as opposed to whether maximum outreach is attainable). For certain parameter configurations, outreach is higher when all are correlated or when few are correlated than when a significant fraction are. This is driven by the fact that as covariate risk becomes more prevalent ($k \nearrow$), the uncorrelated-to-correlated cross-subsidy increases (from the standpoint

³ Other rationales given for the difficulty of agricultural microlending include low profitability, low density and thus high transactions costs, and longer gestation periods making typical microfinance products – short-term contracts with weekly installments – unattractive (Mahajan and Ramana, 2004; Morvant-Roux, 2011).

⁴ For evidence consistent with this behavior, see Ahlin (2020).

of the uncorrelated), but fewer borrowers are paying it ($1 - k \searrow$) and thus at risk of dropping out. At some k , this cross-subsidy can get high enough to drive away the safe-uncorrelated; but as k continues to increase toward 1, the excluded safe-uncorrelated group drops in size toward 0. Computation suggests that this non-monotonicity is a fairly robust feature of the model.

Thus, outreach and efficiency can be lowest when lending to a population in which the prevalence of correlated risk is intermediate. This suggests that outreach could be improved if the lender could separate the two populations, correlated and uncorrelated, and offer exclusive contracts to each. Doing so would require observing correlation-type; we next assume this is possible but restrict usage of this knowledge in several ways. We find that separately servicing the correlated and uncorrelated populations is better under some parameters, while pooling them is better under others. Separately servicing them eliminates the uncorrelated-to-correlated cross-subsidy, helping attract the safe-uncorrelated; but the loss of this cross-subsidy hurts the safe-correlated, who in addition are helped less by joint liability because they are in a universally correlated pool. While either separate servicing or pooling can dominate under specific parameters, when we look at expected outreach over a uniform distribution on key parameters, we find computationally that separating borrowers dominates.

On a positive level, this result offers an explanation for why very few MFIs seek to reach both farmers and microentrepreneurs; instead traditional MFIs tend to eschew farmers, while a number of agricultural banks service farmers (Mahajan and Ramana, 2004; Morvant-Roux, 2011). Segmenting markets in this way may raise total outreach, and in particular, among the uncorrelated (i.e. microentrepreneurs). On a normative level, it suggests that the goal of some banks of having a mixed agriculture and non-agriculture portfolio (ibid.) may need to be approached with caution. The cross-subsidies across borrowers with very different covariate risk profiles may limit outreach. If banks do pursue this approach, our results suggest the preferability of servicing these two types of clients separately, that is, distinguishing between the two in contract rates rather than offering the same terms to all.⁵ See Section 2 for more discussion.

The rest of the paper is organized as follows. Section 2 discusses related literature. Section 3 presents the basic model and known results on individual lending and group lending with uncorrelated risk. In Section 4 we introduce covariate risk and analyze the case in which all face the same correlation. Section 5 separates correlated risk into extensive and intensive margins and shows how outreach can be non-monotonic along the extensive margin. Section 6 follows up by analyzing the question of whether and when outreach can be improved by servicing correlated and uncorrelated borrowers separately. Section 7 concludes. Proofs are contained in Online Appendix B.

2. Literature review

To our knowledge, most models of group lending with joint liability either assume risk is independent across borrowers or allow for correlated risk but do not analyze its implications.⁶ There are some notable exceptions. In discussing extensions to his work, Ghatak (2000) conjectures that positive correlation between borrowers will make joint liability less effective. Our work formally confirms his conjecture when correlated risk is universal, and goes beyond it by characterizing a setting in which correlated risk is heterogeneous in the population.

Ahlin and Townsend (2007a,b) analyze how correlated risk affects both selection into contracts and repayment behavior of groups, and check for predicted correlations in Thai data. This paper follows them

⁵ A related point on the importance of risk pricing is made by IFC (2014).

⁶ For example, see (Stiglitz, 1990; Besley and Coate, 1995; Ghatak, 1999, 2000; Laffont, 2003; Rai and Sjöström, 2004; Baland et al., 2013; Ahlin, 2015; De Quidt et al., 2016; Maitra et al., 2017).

in its modeling of correlated risk. The main difference is that we allow contract terms to adjust to the presence and nature of correlated risk, while those papers take a more cross-sectional approach by analyzing the effects of correlated risk holding fixed contract parameters.⁷

Ahlin and Townsend (2007a,b) also do not analyze heterogeneity in correlated risk. Ahlin (2020) does, but develops implications for matching patterns only, not for outreach and efficiency.

Katzur and Lensink (2012) show that covariate risk can raise outreach in the same setting as ours. Critical for their result is the assumption that safe borrowers are substantially more highly correlated than risky borrowers. This turns the uncorrelated-to-correlated cross-subsidy into something close to a risky-to-safe cross-subsidy, which can counteract the safe-to-risky cross-subsidy inherent in this environment. We take a different tack, assuming for the sake of a benchmark that the presence of covariate risk is the same across safe and risky borrowers, rather than predominantly a safe-borrower phenomenon.

Thus, to our knowledge this paper breaks ground in examining how neutral covariate risk affects outreach in equilibrium with lenders accounting for it, and especially along extensive and intensive margins.

One aspect of covariate risk in the lending context is that the lender needs to pursue sufficient geographic diversification or insurance in order to survive systemic shocks (Besley, 1994; Mahajan and Ramana, 2004; Zeller, 2006; CGAP/IFAD, 2006). Our paper abstracts from this issue by assuming a risk-neutral lender, which presumes the lender has solved this diversification/insurance problem.

The findings also intersect research on lending for agriculture in developing countries. Several authors encourage MFIs to attempt lending to a diversity of borrowers rather than excluding farmers, as is often done (Mahajan and Ramana, 2004; CGAP/IFAD, 2006; Morvant-Roux, 2011); rationales include farmers' need for credit, and that lenders can benefit from a more diverse portfolio. Others recommend segmenting the agricultural market, not only between agricultural and non-agricultural borrowers but also within agriculture (IFC, 2012); the rationale here is the need to tailor contracts to specifics of the crop and business environment. Our results offer a different, information-based rationale for segmenting the agricultural market from the rest. However, this does not necessarily contradict the lending diversification strategy; the important feature of segmentation in our model is that different contracts tailored to risk characteristics are offered to the two types, which can be accomplished by a single MFI serving two different markets (IFC, 2014). In practice, however, given that MFIs seem to value unified product offerings, separate MFIs or perhaps MFI affiliates may be the most natural approach.

3. Baseline model and results

3.1. Economic environment

The model is based on a canonical lending model under hidden information (Stiglitz and Weiss, 1981; Ghatak, 2000). The lender is uninformed about borrower risk; its resulting inability to price for risk can lead to a truncated and adversely selected market.⁸

⁷ This explains key differences in results. For example, Ahlin and Townsend (2007a) find that higher correlated risk can be associated with higher repayment rates, all else equal, in the setting of this paper. This follows because higher correlated risk raises borrower payoffs and thus draws in marginal types, who are relatively safe. This effect can be found in our paper, under next-best outreach in the heterogeneous-correlation case (see Section 5.2): only safe-uncorrelated borrowers drop out, so uncorrelated borrowers are all risky while correlated borrowers are both risky and safe. But the analysis here focuses on effects when contract rates adjust to the degree of correlation.

⁸ Adverse selection is a slight misnomer in this context since borrowers' projects all have the same expected returns. Any inefficiency comes from market truncation, i.e. exclusion of borrowers — a so-called “lemons” problem.

There is a measure-one continuum of risk-neutral agents. All agents are endowed with 1 unit of labor and a project that requires 1 unit of labor and 1 one unit of capital. Since agents have no capital, they need external financing to undertake the project. They can earn a payoff $u \geq 0$ using only their labor.

Agents' projects are risky, either succeeding or failing. The probability of success differs across agents: safe agents succeed with probability p_s , risky agents with probability p_r , $0 < p_r < p_s < 1$. Project riskiness is the agent's private information. It is common knowledge that a fraction $\theta \in (0, 1)$ of agents is risky and $1 - \theta$ is safe.

The project of a safe (risky) agent pays $R_s > 0$ ($R_r > 0$) when successful, and 0 otherwise. As in Stiglitz and Weiss (1981), all projects are assumed to have the same expected return: $\bar{R} \equiv p_s R_s = p_r R_r$. This is a key assumption: while some projects succeed more often ($p_s > p_r$), they pay less when successful ($R_s < R_r$), and thus yield the same expected return (\bar{R}). Thus heterogeneity is in second-order risk.

There is a single risk-neutral lender that maximizes borrower surplus subject to earning at least an expected gross rate of return $v > 0$. We call this the zero-profit or break-even constraint. Any financing contract must satisfy a limited liability constraint ensuring that agents pay nothing more than project proceeds: R_s or R_r when successful, 0 when unsuccessful. Further, the lender can verify failure or success but not exact output. That is, the lender can distinguish between $\{0\}$ and $\{R_s, R_r\}$, but not within $\{R_s, R_r\}$. Together these assumptions make debt contracts the only feasible contracts. We restrict attention to deterministic contracts, and assume that borrowers repay whenever able; there are no enforcement problems. Thus the model is tailored to focus on debt financing in the presence of limited liability and hidden information about risk, but not hidden action or enforcement limitations.

Parameters are assumed to satisfy

$$\bar{R} > u + v .$$

This implies that agents' projects have a higher expected return to the unit of capital, net of the cost of labor ($\bar{R} - u$) than the rate of return required by the lender (v). Every project carried out thus contains gains from trade, and a fully efficient market would fund all projects.⁹ Related, borrower surplus is strictly increasing in the number of projects funded if the lenders' rate of return equals v , as it will in equilibrium. Thus, throughout the paper we focus on comparisons of outreach, i.e. total projects funded, but the conclusions are identical for borrower surplus and total surplus.

Key notation includes two summary statistics of the rate of return parameters:

$$\mathcal{N} \equiv \frac{\bar{R} - u}{v} \quad \text{and} \quad \mathcal{G} \equiv \frac{\bar{R}}{v} .$$

\mathcal{N} and \mathcal{G} give the return to capital in borrower projects, net and gross of the opportunity cost of labor, respectively, relative to the lender's required rate of return. Thus they can be interpreted as net and gross excess rates of return of the agents' projects. Previous assumptions imply that $\mathcal{G} \geq \mathcal{N} > 1$.

The population average of any function $g(p)$ is denoted by $\overline{g(p)}$, with $\overline{g(p)} \equiv \theta g(p_r) + (1 - \theta)g(p_s)$.

For example, \bar{p} denotes the average risk-type, $\theta p_r + (1 - \theta)p_s$, and \bar{p}^2 the mean squared-type, $\theta p_r^2 + (1 - \theta)p_s^2$.

⁹ In the absence of asymmetric information, the lender would charge gross interest rates $r = v/p_s$ to safe agents and $r' = v/p_r$ to risky, and all agents would borrow and earn $\bar{R} - v$.

3.2. Individual lending

In this and the following subsection, we review several results from the literature that are foundational to our own. Consider first individual loan contracts. Given gross interest rate r , a safe agent that borrows to undertake the project will earn $\bar{R} - p_s r$ in expectation, and a risky agent $\bar{R} - p_r r$. (Limited liability implies nothing is due after failure.) Note that the safe borrower earns less than the risky, due to a higher probability of repayment. If the lender funds all agents, it must charge an interest rate that breaks even given average success probabilities, i.e. satisfying $\bar{p}r \geq v$. Combining the minimum such interest rate, $r = v/\bar{p}$, with the safe agent's borrowing payoff gives the condition for safe agents to prefer borrowing as

$$\bar{R} - p_s \frac{v}{\bar{p}} \geq u \iff \mathcal{N} \geq \frac{p_s}{\bar{p}} \equiv B_1.$$

This interest rate must also be affordable by any successful borrower, i.e. $v/\bar{p} \leq R_s, R_r$, equivalently $\mathcal{G} \geq p_s/\bar{p} \equiv C_1$, which is guaranteed by $\mathcal{N} \geq B_1$.

Thus, for full inclusion and maximal borrower surplus, the net excess return to capital in borrower projects must not only exceed 1, the relevant cutoff under full information, it must exceed p_s/\bar{p} . Safe borrowers cross-subsidize risky, since they repay the same interest rate with higher probability; if this cross-subsidy exceeds the surplus from borrowing, safe agents opt out. In this case, the lender lends to risky agents, charging $r = v/p_r$ to break even and leave risky borrowers all surplus from their projects; but maximal outreach is not attained, and surplus is sacrificed as safe agents' projects are left unfunded.

3.3. Group lending with uncorrelated risk

Ghatak (1999, 2000) showed that group lending with joint liability may increase outreach and borrower surplus in this setting (see also (Van Tassel, 1999)). A critical assumption is strong local information: an agent's project risk is assumed to be known by all other agents, though not the lender. Remarkably, joint liability lending enables the lender to harness this local information for potentially increased outreach.

Here we follow Ghatak (1999, 2000) and the refinements in Gangopadhyay et al. (2005) and Ahlin and Waters (2016). A joint liability contract for a borrowing group of two¹⁰ stipulates that in addition to owing gross interest rate r for the borrower's own loan, she owes an additional joint liability rate $c \geq 0$ on her partner's loan if her partner fails. Project outcomes are uncorrelated across borrowers. The expected payoff under this contract for a borrower of type $\tau \in \{r, s\}$ matched with a borrower of type $\tau' \in \{r, s\}$ is

$$\bar{R} - p_\tau r - p_{\tau'}(1 - p_{\tau'})c = \bar{R} - p_\tau \tilde{r}(r, c, \tau'), \text{ where } \tilde{r}(r, c, \tau') \equiv r + (1 - p_{\tau'})c.$$

This first expression reflects the fact that a borrower repays r after her own success, and pays an additional liability payment c when she succeeds and her partner fails. The second expression writes the payoff in terms of an "implicit interest rate" \tilde{r} capturing the total expected payment of a successful borrower: r for her own loan, and c for her partner's loan with probability $(1 - p_{\tau'})$.

The implicit interest rate \tilde{r} depends on contract parameters r and c , but also on partner type τ' . Partner type is assumed to be determined in a frictionless transferable-utility matching game (necessary for which is strong local information). As Ghatak (2000) has shown, the unique equilibrium involves homogeneous groups: safe matched with safe, risky with risky. As a result, the implicit interest rate for a borrower of type τ becomes $\tilde{r}(r, c, \tau' = \tau) = r + (1 - p_\tau)c$. That is, a borrower's implicit interest rate is a function of her own type. Further, safe borrowers receive a discount in their implicit interest rate compared to risky

¹⁰ Groups of two are standard in the literature; Baland et al. (2013) and Ahlin (2015) study larger groups.

borrowers, equal to $\tilde{r}(r, c, \tau' = r) - \tilde{r}(r, c, \tau' = s) = (p_s - p_r)c$. This discount comes through lower expected bailout payments after success, due to having safer partners; thus it is proportional to the difference in success probabilities $(p_s - p_r)$ and the magnitude of liability, c .

Thus, even though the uninformed lender cannot meaningfully vary r and c across borrowers, by leaving group formation to informed agents it allows partner type to vary across borrowers in such a way as to provide safe-borrower discounts. These discounts come through joint liability plus decentralized, homogeneous matching, which results in safer borrowers bearing liability for safer partners.

Incorporating the matching equilibrium, the optimal contract and conditions for maximal outreach can be derived. Attention is restricted to a single offered contract rather than a menu; this is common practice for microlenders, and Ahlin (2015) and Ahlin and Waters (2016) show in similar contexts that it is without loss of generality. Following Gangopadhyay et al. (2005), we also impose the constraint that contracts can require no more than full liability, that is, $c \leq r$. This can be interpreted as a monotonicity constraint as in Innes (1990).

Maximal outreach can be found by checking whether the contract that maximizes the safe-borrower payoff is able to attract safe agents. The key idea is that joint liability falls relatively more heavily on risky borrowers, since they have risky partners; related, the safe-borrower discount in implicit interest rate is proportional to c . Thus, to attract safe agents, the lender seeks to earn its return using c as heavily as possible: liability is full ($c = r$) when this is affordable, and otherwise partial ($c < r$) but set to the maximum affordable level. This leads to the following result (Proposition 3, (Ahlin and Waters, 2016)).

Proposition 0. *Let*

$$B_{2,0} \equiv \frac{p_s(2 - p_s)}{p(2 - p)} \quad \text{and} \quad C_{2,0} \equiv \frac{2p_s}{p(2 - p)}.$$

The group contract that maximizes borrower surplus subject to monotonicity, borrower limited liability, and lender breaking even achieves full outreach and maximal borrower surplus iff

$$\mathcal{N} \geq \begin{cases} B_1 - \frac{B_1 - B_{2,0}}{C_{2,0} - C_1} (\mathcal{G} - C_1) & \text{for } \mathcal{G} \in [C_1, C_{2,0}] \\ B_{2,0} & \text{for } \mathcal{G} \geq C_{2,0} \end{cases}.$$

Otherwise, only risky agents borrow.

The result is pictured in Fig. 1 of the next Section. The subscript "2,0" here refers to groups of size 2 and a fraction 0 of the population facing correlated risk. It is straightforward to verify that $B_{2,0} < B_1 = C_1 < C_{2,0}$. Thus group lending makes inclusion of all borrowers feasible over a larger parameter space than individual lending, i.e. for lower net excess returns \mathcal{N} . But $B_{2,0} > 1$, so there remain cases where full outreach is not realized even though it would be under full information.

4. Group lending with universal correlated risk

We next extend the model to study how well group lending works in the presence of covariate risk. In this section, risk correlation is universal; in the next section it affects some borrowers more than others.

To understand the effect of covariate risk ceteris paribus, its inclusion in the model must preserve individuals' probabilities of success. Fix two borrowers (i, j) with probabilities of success (p_i, p_j) . Any joint distribution of the two borrowers' outcomes can be written in the following one-parameter form.

	Event Probabilities	
	j Succeeds	j Fails
i Succeeds	$p_i p_j + \epsilon$	$p_i(1 - p_j) - \epsilon$
i Fails	$(1 - p_i)p_j - \epsilon$	$(1 - p_i)(1 - p_j) + \epsilon$

The no-correlation case corresponds to $\epsilon = 0$, while a positive ϵ gives positive correlation.

Thus, for every pair of agents (i, j) there is an ϵ that determines the degree of correlation. Of course, ϵ could differ across pairs of agents in complex ways. A natural simplifying assumption is that ϵ depends only on the success probabilities (p_i, p_j) of the given agents (i, j) , and further, that all pairs of agents face a similar degree of correlated risk. We pursue these simplifications in two ways.

In the “constant-mass” case, all pairs of borrowers have the same probability mass added to symmetric-outcome events and subtracted from asymmetric-outcome events. That is, every pair’s joint outcome distribution involves the same ϵ . In the “constant-correlation” case, we assume the same correlation coefficient across all (homogeneous) borrowing pairs. This requires ϵ to depend on p_i and p_j . One can think of the constant-mass case as analogous to a uniform absolute increase in comovement, and the constant-correlation case to a uniform relative increase. The two cases are similar, so we analyze the constant-correlation case throughout the body of the paper and the constant-mass case in Online Appendix A.

We maintain the assumption that borrower types are unknown to the lender. The nature of correlated risk and all population risk parameters are common knowledge.

In the constant-correlation case, the ϵ varies with success probabilities as follows:

$$\epsilon(p_i, p_j) = \check{\rho} \cdot \min\{p_i(1 - p_j), p_j(1 - p_i)\} .$$

We assume that $\check{\rho} \in [0, 1]$, which guarantees that all event probabilities lie between zero and one.¹¹ Thus the joint distribution for all-safe groups is perturbed by $\check{\rho} \cdot p_s(1 - p_s)$ and for all-risky groups by $\check{\rho} \cdot p_r(1 - p_r)$. The result in both cases is a correlation coefficient of project output equal to $\check{\rho}$.¹²

The expected payoff under a joint liability contract for a borrower of type $\tau \in \{r, s\}$ matched with a borrower of type $\tau' \in \{r, s\}$ is

$$\bar{R} - p_\tau r - [p_\tau(1 - p_{\tau'}) - \check{\rho} \min\{p_\tau(1 - p_{\tau'}), p_{\tau'}(1 - p_\tau)\}] c . \tag{1}$$

The key difference from the independent-risk case is that a borrower has a lower chance of facing liability for her partner, since when she succeeds her partner is more likely to have succeeded too. The payoff simplifies significantly given that homogeneous matching obtains here as well.

Lemma 1. *In the constant-correlation case of correlated risk, given contract offer (r, c) satisfying monotonicity, affordability, and $c > 0$, a matching equilibrium exists, and in any equilibrium almost every group is homogeneous in risk-type.*

As in the no-correlation case, safe borrowers have safer partners and so receive a discounted implicit interest rate. Payoffs and implicit interest rate \tilde{r} of a borrower of type τ can now be written as

$$\bar{R} - p_\tau r - p_\tau(1 - p_\tau)(1 - \check{\rho})c = \bar{R} - p_\tau \tilde{r}(r, c, \tau) ,$$

$$\text{where } \tilde{r}(r, c, \tau) \equiv r + (1 - p_\tau)(1 - \check{\rho})c .$$

The safe-borrower discount in implicit interest rate (i.e. $\tilde{r}(c, r, \tau = r) - \tilde{r}(c, r, \tau = s)$) is now

$$(p_s - p_r)c(1 - \check{\rho}) .$$

These expressions differ from those of the independent-risk case (Section 3.3) only in that here an “effective” liability rate $c' \equiv c(1 - \check{\rho})$ takes the place of the contractual joint liability rate c . It is transparent that

correlated risk acts to dampen the effective degree of joint liability; by making it less likely that one borrower is in a position to bail out the other, correlation weakens any contractual joint liability stipulation. And since it is the contractual joint liability rate c that is constrained by monotonicity and affordability, the lender cannot compensate for higher correlated risk by raising the degree of joint liability. Correlation thus reduces the lender’s ability to improve risk-pricing using joint liability.

To show this, we derive conditions for full outreach to be attainable. Since safe borrowers earn less than risky under any monotonic contract, the best chance for including all borrowers is to tailor contract parameters toward safe borrowers. The solution to the following program is the maximal safe-borrower payoff subject to monotonicity,¹³ affordability,¹⁴ and the lender zero-profit constraint assuming all borrow:

$$\begin{aligned} \max_{r,c} \quad & \bar{R} - p_s r - p_s(1 - p_s)(1 - \check{\rho})c \\ \text{s.t.} \quad & 0 \leq c \leq r \\ & r + c \leq R_s \\ & \bar{p}r + \overline{p(1-p)}(1 - \check{\rho})c \geq v \end{aligned} \tag{2}$$

For the reasons discussed in Section 3.3, safe borrowers do best when joint liability is used as heavily as possible. Hence, when affordability is not a binding constraint, i.e. when G is high enough, monotonicity binds and liability is full: $c = r = \frac{v}{p(2-p) - \check{\rho}p(1-p)}$. When full liability is not affordable, liability is partial but set to the maximum affordable:

$$(r, c) = \left(v \frac{p_s - (1 - \check{\rho})\overline{p(1-p)}G}{p_s [p^2 + \check{\rho}p(1-p)]}, v \frac{\overline{p}G - p_s}{p_s [p^2 + \check{\rho}p(1-p)]} \right) .$$

Comparing the resulting safe borrowing payoffs with the outside option u gives the following result.

Proposition 1. *Let*

$$B_{2,1} \equiv \frac{p_s(2 - p_s) - \check{\rho}p_s(1 - p_s)}{p(2 - p) - \check{\rho}p(1 - p)} \quad \text{and} \quad C_{2,1} \equiv \frac{2p_s}{p(2 - p) - \check{\rho}p(1 - p)} .$$

Consider the constant-correlation case, in which $\epsilon(p_i, p_j) = \check{\rho} \cdot \min\{p_i(1 - p_j), p_j(1 - p_i)\}$, with $\check{\rho} \in [0, 1]$. The group contract that maximizes borrower surplus subject to monotonicity, borrower limited liability, and lender breaking even achieves full outreach and maximal borrower surplus iff

$$\mathcal{N} \geq \begin{cases} B_1 - \frac{B_1 - B_{2,1}}{C_{2,1} - C_1}(G - C_1) & \text{for } G \in [C_1, C_{2,1}] \\ B_{2,1} & \text{for } G \geq C_{2,1} \end{cases} .$$

Otherwise, only risky agents borrow.

The subscript “2,1” refers to groups of size 2 and a fraction 1 of the population facing correlated risk.

Fig. 1 illustrates the results of Propositions 0 and 1 using the same risk parameters (θ, p_s, p_r) as in Ahlin and Waters (2016). Since $G \geq \mathcal{N}$, the relevant parameter space lies east of the (dash-dotted) 45-degree line. Group lending achieves full outreach in the entire region above the solid boundaries for correlation coefficients $\check{\rho} = 0.5$, $\check{\rho}' = 0.2$, and $\check{\rho}'' = 0.8$, respectively (Proposition 1). For comparison, above the dashed line is the region where group lending achieves full outreach under independent risk (Proposition 0); and above the dotted line is where individual lending achieves full outreach (for any covariate risk structure). In each case, below the boundary all risky and no safe agents borrow. One can see that, as in the baseline case, group lending expands the parameter space over which full outreach is feasible compared

¹¹ A breve is used to clearly differentiate correlation $\check{\rho}$ from probability of success p .

¹² See Ahlin and Townsend (2007b). As they show, for mixed groups (safe-risky) the correlation coefficient under this specification is $\check{\rho} \cdot \bar{\rho}$, where $\bar{\rho} = \sqrt{p_r(1 - p_s) / [p_s(1 - p_r)]}$, the maximum correlation possible for such a group.

¹³ For brevity, we lump the non-negativity of c into what we call “monotonicity” throughout the paper: $0 \leq c \leq r$.

¹⁴ By “affordability” we mean limited liability after success. Limited liability after failure, which guarantees that nothing is owed, is embedded in the payoffs as written.

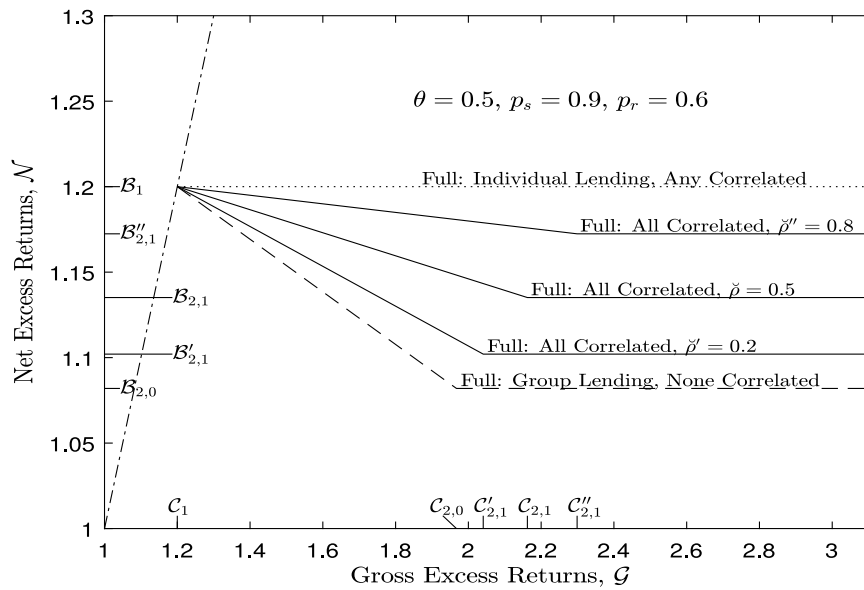


Fig. 1. Full-outreach space, universal correlation.

to individual lending ($B_{2,1}, B'_{2,1}, B''_{2,1} < B_1$). However, correlated risk reduces its positive impact ($B_{2,0} < B'_{2,1} < B_{2,1} < B''_{2,1}$).

More generally, the higher the correlation, the smaller the parameter space over which full outreach is attainable; this follows since $B_{2,1}$ and $C_{2,1}$ are both increasing in $\tilde{\rho}$. As $\tilde{\rho} \rightarrow 1$, the parameter space becomes identical to the one for individual lending. Thus, correlated risk makes group lending less effective, and perfect correlation makes group lending perfectly ineffective.

In sum, covariate risk blunts a key tool of the lender for improving risk-pricing. It thus makes full outreach and maximal borrower surplus harder to attain.

5. Group lending with heterogeneous correlated risk

Some environments are characterized by significant heterogeneity in the amount of correlated risk faced by different borrowers. For example, consider a lender serving borrowers from a pool of farmers and a pool of micro-entrepreneurs. The farmers would typically face a greater degree of correlated risk – from common dependence on weather, pests and diseases, and market prices – than the small business owners.

In this section, we augment the constant-correlation model of Section 4 with an additional dimension of heterogeneity related to correlated risk, denoted σ . There are two types of agents along this dimension. Agents of type $\sigma = A$ face risk that is independent from the risk of all other agents; they are “uncorrelated”. Agents of type $\sigma = B$ face correlated risk with all other type-B agents; they are the “correlated” agents.¹⁵ Thus, the joint probability distribution perturbation ϵ (see beginning of Section 4) varies with agent types as follows:

$$\epsilon(p_i, \sigma_i, p_j, \sigma_j) = \tilde{\rho} \cdot \min\{p_i(1 - p_j), p_j(1 - p_i)\} \cdot \mathbf{1}\{\sigma_i = \sigma_j = B\},$$

where $\mathbf{1}$ is the indicator function and $\tilde{\rho} \in [0, 1]$.

We assume that an agent’s risk-type $\tau \in \{r, s\}$ is independent of her correlation-type $\sigma \in \{A, B\}$. That is, a fraction $k \in (0, 1)$ of both safe and risky agents are correlated, and $(1 - k)$ are uncorrelated. This is common knowledge. As with risk-type, an agent’s correlation-type σ is assumed to be observed by all agents but unobserved by the lender.

The expected payoff under joint liability contract (r, c) for a borrower of type $(\tau, \sigma) \in \{r, s\} \times \{A, B\}$ matched with a borrower of type (τ', σ') is

$$\bar{R} - p_\tau r - [p_\tau(1 - p_{\tau'}) - \tilde{\rho} \min\{p_\tau(1 - p_{\tau'}), p_{\tau'}(1 - p_\tau)\} \mathbf{1}\{\sigma = \sigma' = B\}] c. \tag{3}$$

Only pairs of type-B borrowers face correlated risk.

Homogeneous matching occurs along both dimensions of individual heterogeneity:

Lemma 2. *In the heterogeneous, constant-correlation case of correlated risk with $\tilde{\rho} > 0$, given contract offer (r, c) satisfying monotonicity, affordability, and $c > 0$, a matching equilibrium exists, and in any equilibrium almost every group is homogeneous in both risk-type τ and correlation-type σ .*

Thus, there can be at most four types of groups: safe-correlated (both borrowers of type $\{s, B\}$), safe-uncorrelated (both $\{s, A\}$), risky-correlated (both $\{r, B\}$), and risky-uncorrelated (both $\{r, A\}$). Mixed groups do not occur in equilibrium. The intuition is that correlated risk lowers the effective degree of joint liability, which raises borrower payoffs. Borrowers thus prefer correlated risk within groups, and type-B borrowers are able to create it by matching together, leaving type-A borrowers to match together.¹⁶ Within correlated-risk type, standard forces lead to safe–safe and risky–risky matches.

Given homogeneous matching, the payoff for an uncorrelated (type-A) borrower of type $\tau \in \{r, s\}$ is

$$\bar{R} - p_\tau r - p_\tau(1 - p_\tau)c$$

and for a correlated (type-B) borrower of type $\tau \in \{r, s\}$ is

$$\bar{R} - p_\tau r - p_\tau(1 - p_\tau)(1 - \tilde{\rho})c.$$

The challenges to lending in this setting are multiple. There continues to be the cross-subsidy from safe to risky borrowers endemic to this environment, which group lending is able to reduce to the extent that risk is uncorrelated. But there is also a second kind of cross-subsidy under group lending, from uncorrelated to correlated groups. It arises

¹⁵ The model could easily be extended to allow both types of agents to face correlated risk, one less than the other, with similar results.

¹⁶ Ahlin (2020) finds higher correlated risk within Thai microcredit groups than random matching would predict, even controlling for occupation.

because correlated groups avoid bailing each other out more often than uncorrelated groups; they face a lower effective rate of liability, $c(1 - \check{\rho})$ instead of c . This is the key novelty: with heterogeneity in correlated risk, joint liability introduces a new and possibly poorly targeted cross-subsidy that can work against lending outreach.

5.1. Achieving full outreach

In this setting, the type of group that earns the least from borrowing is the safe-uncorrelated. They pay extra both for being safe and for being uncorrelated. Thus, the goal of full outreach is best attained by tailoring contracts to favor safe-uncorrelated borrowers, which is accomplished via the solution to the following:

$$\begin{aligned} \max_{r,c} \quad & \bar{R} - p_s r - p_s(1 - p_s)c \\ \text{s.t.} \quad & 0 \leq c \leq r \\ & r + c \leq R_s \\ & \bar{p}r + \overline{p(1-p)}(1 - k\check{\rho})c \geq v \end{aligned} \tag{4}$$

The zero-profit constraint assumes all agents borrow and features something like an expected correlation: the fraction of correlated borrowers k multiplied by the degree of correlation $\check{\rho}$.

In this context, joint liability is a two-edged sword: it lowers the safe-to-risky cross-subsidy, but it introduces a new uncorrelated-to-correlated cross-subsidy, both in proportion to its magnitude, c . When correlation is sufficiently high on intensive and extensive margins, i.e. for $k\check{\rho}$ high enough, joint liability works against the goal of full outreach: the uncorrelated-to-correlated cross-subsidy that it creates outweighs its reduction in the safe-to-risky cross-subsidy for the marginal safe-uncorrelated group. In this case, joint liability is abandoned, and group lending offers no advantage over individual lending in achieving full outreach.¹⁷

On the other hand, when correlated risk is not too widespread and significant, i.e. for $k\check{\rho}$ low enough, joint liability does more good than harm and should be used as heavily as possible. In this case, when affordability is not a binding constraint, monotonicity binds and liability is full: $c = r = \frac{v}{p(2-p) - k\check{\rho}p(1-p)}$. When full liability is not affordable, liability is partial but set to the maximum affordable: $(r, c) = \left(v \frac{p_s - (1-k\check{\rho})p(1-p)G}{p_s[p^2 + k\check{\rho}p(1-p)]}, v \frac{\bar{p}G - p_s}{p_s[p^2 + k\check{\rho}p(1-p)]} \right)$. This leads to the following result.

Proposition 2. *Let*

$$\begin{aligned} B_{2,k} &\equiv \frac{p_s(2 - p_s)}{p(2 - p) - k\check{\rho}p(1 - p)}, & C_{2,k} &\equiv \frac{2p_s}{p(2 - p) - k\check{\rho}p(1 - p)}, \\ \text{and} \quad \kappa_0 &\equiv \frac{p(p_s - p)}{p(1 - p)}. \end{aligned}$$

Consider the heterogeneous, constant-correlation case, in which

$$e(p_i, \sigma_i, p_j, \sigma_j) = \check{\rho} \cdot \min\{p_i(1 - p_j), p_j(1 - p_i)\} \cdot \mathbf{1}\{\sigma_i = \sigma_j = B\},$$

with $\check{\rho} \in [0, 1]$. The group contract that maximizes borrower surplus subject to monotonicity, borrower limited liability, and lender breaking even achieves full outreach and maximal borrower surplus iff

$$k\check{\rho} \leq \kappa_0 \quad \text{and} \quad \mathcal{N} \geq \begin{cases} B_1 - \frac{B_1 - B_{2,k}}{C_{2,k} - C_1}(G - C_1) & \text{for } G \in [C_1, C_{2,k}] \\ B_{2,k} & \text{for } G \geq C_{2,k} \end{cases}$$

or

$$k\check{\rho} \geq \kappa_0 \quad \text{and} \quad \mathcal{N} \geq B_1.$$

¹⁷ It may still dominate individual lending in outreach by enabling a lesser outreach goal; see next section.

Otherwise, full outreach cannot be achieved and safe-uncorrelated agents do not borrow.

In the latter case ($k\check{\rho} \geq \kappa_0$), joint liability does not help in achieving full outreach.

The subscript “2,k” refers to groups of size 2 and a fraction k of the population facing correlated risk.

Fig. 2 illustrates the results of the Proposition when $\check{\rho} = 0.5$, for $k = 0.25$ and $k' = 0.75$; the parameter spaces for which full outreach is achieved for these values of k lie above the respective solid boundaries. The parameter region boundaries for full outreach using group lending in the polar cases of no correlation (Proposition 0) and universal correlation (Proposition 1) are graphed using dashed lines, and the parameter region for individual lending using a dotted line.

One might expect the conditions for achieving full outreach to span, as k varies, the range set by the polar cases of no correlated risk and universal correlated risk.¹⁸ This is true in the neighborhood of zero prevalence of correlated risk: $\lim_{k \rightarrow 0} B_{2,k} = B_{2,0}$ and $\lim_{k \rightarrow 0} C_{2,k} = C_{2,0}$. But it is not in the neighborhood of universal prevalence: $\lim_{k \rightarrow 1} C_{2,k} = C_{2,1}$, but $\lim_{k \rightarrow 1} B_{2,k} > B_{2,1}$. That is, it is easier to achieve full outreach if everyone faces correlated risk than if not quite everyone does. This is apparent in Fig. 2: full outreach is more difficult to achieve when 75% of agents are correlated than when all are.

This counterintuitive result occurs because only in a mixed population is there the additional cross-subsidy from uncorrelated to correlated borrowers. The marginal agents for full outreach in the mixed case are the safe-uncorrelated, who are paying two cross-subsidies, safe-to-risky and uncorrelated-to-correlated. By contrast, the marginal agents in a universally correlated population are the safe, who pay only one cross-subsidy, safe-to-risky. In sum, it is easier to fully reach a universally correlated population than a mixed population with k high enough (holding fixed $\check{\rho}$).

A more intuitive implication is that full outreach is attainable over a smaller parameter space the greater is correlated risk extensively (k) or intensively ($\check{\rho}$); this follows since $B_{2,k}$ and $C_{2,k}$ both increase in k and in $\check{\rho}$. The effect is capped when $k\check{\rho}$ is high enough (note that $\kappa_0 \in (0, 1)$), in which case joint liability is abandoned if the goal is full outreach, replaced by individual liability lending, which is unaffected by correlated risk.

Putting these two implications together, full outreach gets harder to attain as k increases from 0, but becomes discontinuously easier at the upper limit where all are correlated.

5.2. Achieving maximal outreach

The previous section provides the conditions for full outreach to be possible. This Section shows how much outreach is possible when full outreach is not. Together, the two Sections pin down maximal outreach over the entire parameter space.

In the no-correlation and universal-correlation cases, since there are only two types of agents ($\{r, s\}$), there are only two possible outcomes: the lender attracting all borrowers, or only risky. The situation is more complicated in the mixed-correlation case, since there are four types of agents ($\{r, s\} \times \{A, B\}$). Falling short of full outreach means losing one type of agent, but not necessarily reaching only the risky.

Given that the safe-uncorrelated are the hardest to attract, they cannot be reached if full outreach is not possible. One can show that the safe-correlated are the next-hardest to attract with any monotonic contract iff¹⁹

$$\check{\rho} \leq \frac{(p_s - p_r)(2 - p_s - p_r)}{p_s(1 - p_s)}. \tag{A1}$$

¹⁸ Note that this Section assumes $k \in (0, 1)$, so the limits of none correlated and all correlated are not covered in Proposition 2.

¹⁹ Given $\check{\rho} \leq 1$, this assumption is non-binding if $p_s \geq p_r(2 - p_r)$, as in the cases graphed in the Figures.

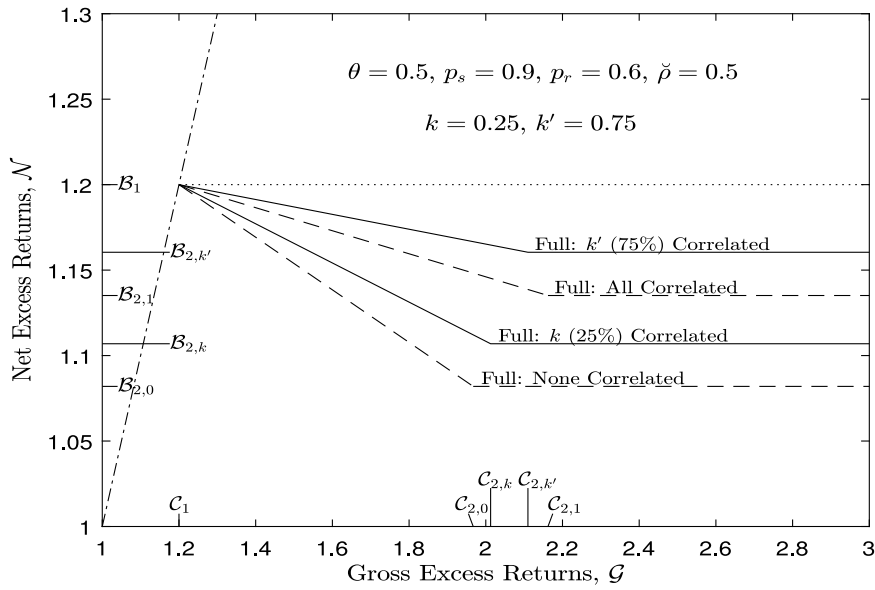


Fig. 2. Full-outreach space, heterogeneous correlatedness.

The goal of “next-best” outreach, by which we mean lending to all but safe-uncorrelated agents, is thus best attained under Assumption (A1) by tailoring contracts to favor safe-correlated borrowers, assuming that all but safe-uncorrelated agents borrow. This corresponds to a mass of borrowers equal to $n_p \equiv \theta + (1 - \theta)k$. Let $\theta' \equiv \theta/n_p$, the fraction of risky borrowers in this truncated population, and $k' \equiv k/n_p$, the fraction of correlated borrowers. We also denote the average of $g(p)$ in this truncated population with $\overline{g(p)'}:$

$$\overline{g(p)'} \equiv \theta' g(p_r) + (1 - \theta') g(p_s).$$

For example, $\overline{p'}$ denotes the average risk-type among all but the safe-uncorrelated, $\theta' p_r + (1 - \theta') p_s$.

Given the truncated set of borrowers serviced by the lender, the zero-profit constraint becomes

$$\theta p_r r + (1 - \theta) k p_s r + [\theta k p_r (1 - p_r) (1 - \check{p}) + \theta (1 - k) p_r (1 - p_r) + (1 - \theta) k p_s (1 - p_s) (1 - \check{p})] c \geq n_p v.$$

The best contract for safe-correlated agents is the solution to the following, which incorporates this zero-profit constraint after dividing both sides by n_p and simplifying:

$$\begin{aligned} \max_{r,c} \quad & \bar{R} - p_s r - p_s (1 - p_s) (1 - \check{p}) c \\ \text{s.t.} \quad & 0 \leq c \leq r \\ & r + c \leq R_s \\ & \overline{p'} r + \left[\overline{p(1-p)'} - k' \check{p} \overline{p(1-p)} \right] c \geq v \end{aligned} \tag{5}$$

In this context, joint liability is an unambiguously useful tool that is used as heavily as possible: it lowers the safe-to-risky cross-subsidy and creates an uncorrelated-to-correlated cross-subsidy, both of which help safe-correlated agents. When affordability is not a binding constraint, liability is full: $c = r = \frac{v}{\overline{p(2-p)'} - k' \check{p} \overline{p(1-p)}}$. When full liability is not affordable, liability is partial but set to the maximum affordable: $(r, c) = \left(v \frac{p_s - G[\overline{p(1-p)'} - k' \check{p} \overline{p(1-p)}]}{p_s [p_s^2 + k' \check{p} \overline{p(1-p)}]}, v \frac{\overline{p'} G - p_s}{p_s [p_s^2 + k' \check{p} \overline{p(1-p)}]} \right)$. This leads to the following result.

Proposition 3. Let

$$\begin{aligned} B_{1,k}^{nb} \equiv C_{1,k}^{nb} & \equiv \frac{p_s}{p'}, & B_{2,k}^{nb} & \equiv \frac{p_s(2-p_s) - \check{p} p_s(1-p_s)}{p(2-p)' - k' \check{p} \overline{p(1-p)}}, \\ \text{and} & & C_{2,k}^{nb} & \equiv \frac{2p_s}{p(2-p)' - k' \check{p} \overline{p(1-p)}}. \end{aligned}$$

Consider the heterogeneous, constant-correlation case described in Proposition 2. Assume that \check{p} satisfies Assumption (A1) and that \mathcal{N} does not satisfy the conditions of Proposition 2 for full outreach. The group contract that maximizes borrower surplus subject to monotonicity, borrower limited liability, and lender breaking even achieves next-best outreach, i.e. financial inclusion of all but safe-uncorrelated agents, iff

$$\mathcal{N} \geq \begin{cases} B_{1,k}^{nb} - \frac{B_{1,k}^{nb} - B_{2,k}^{nb}}{C_{2,k}^{nb} - C_{1,k}^{nb}} (G - C_{1,k}^{nb}) & \text{for } G \in [C_{1,k}^{nb}, C_{2,k}^{nb}] \\ B_{2,k}^{nb} & \text{for } G \geq C_{2,k}^{nb} \end{cases}$$

Otherwise, only risky agents borrow.

Propositions 2 and 3 together map out the full range of outcomes for outreach in this case of mixed-correlation, under the mild Assumption (A1). To sum up, if \mathcal{N} and G are high enough (see Proposition 2 for conditions), full outreach is attained. If not, it may still be possible to attract all but the safe-uncorrelated agents (see Proposition 3 for conditions).²⁰ Otherwise, only risky borrow.²¹

Fig. 3 augments Fig. 2, graphing the results of Propositions 2 and 3 together for $k = 0.25$ and $k' = 0.75$. The conditions for next-best outreach when $k = 0.25$ are met when parameters fall above the piecewise linear dotted boundary that kinks at $(C_{2,k}^{nb}, B_{2,k}^{nb})$, but

²⁰ The conditions on \mathcal{N} and G in Proposition 3 are not necessarily weaker than those in Proposition 2; that is, next-best outreach is not always easier to attain than full outreach. One force for next-best outreach being more attainable than full outreach is that it targets the safe-correlated rather than the safe-uncorrelated, which all else equal is easier due to the uncorrelated-to-correlated cross-subsidy. But a force making next-best outreach less attainable is that it involves proportionately fewer safe agents borrowing ($\theta' > \theta$), so each safe agent shoulders a bigger safe-to-risky cross-subsidy. The latter force can outweigh the former — as it does when \check{p} is small enough.

²¹ Propositions 2 and 3 together enable us to check whether group lending (i.e. $c > 0$) offers any improvement in outreach, at any set of parameters. We do not have a full characterization of this parameter space, but some features are clear. By Proposition 2, when $k\check{p}$ is not too high, group lending expands the (G, \mathcal{N}) parameter space over which full outreach is achievable. Proposition 3 can be used to show that when this condition on $k\check{p}$ does not hold, group lending can often attract at least some safe borrowers (the safe-correlated) where individual lending would reach only safe; it certainly does for k and G high enough. But there are cases where group lending does not increase maximal outreach anywhere in (G, \mathcal{N}) space, i.e. when θ is low and k is intermediate.

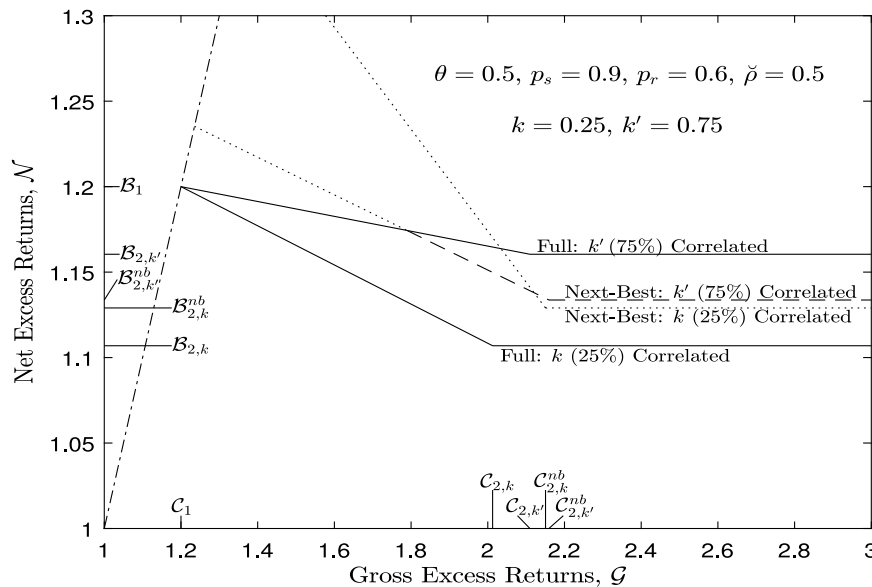


Fig. 3. Outreach space, heterogeneous correlatedness.

below the solid $k = 0.25$ boundary. Clearly no parameters satisfy this condition, since the solid boundary is below this dotted boundary. That is, next-best outreach is never relevant when $k = 0.25$, since full outreach is easier to attain; thus, either full outreach is attained or only risky agents borrow. Consider next $k' = 0.75$. Necessary for next-best outreach is that parameters fall above the piecewise linear boundary that is alternately dotted and dashed; the dashed line is used when this boundary lies below the (solid) boundary corresponding to full outreach in this case. Thus when $k' = 0.75$, full outreach is achieved if parameters lie above the solid $k' = 0.75$ boundary; next-best outreach is achieved, i.e. all but safe-uncorrelated borrow, if parameters lie below this solid boundary but above the dashed boundary; and elsewhere only risky borrow.

5.3. Pitfalls of moderately prevalent correlated risk

Here we show that outreach can be lowest when correlated risk is neither ubiquitous nor absent. That is, outreach can be non-monotonic with respect to the fraction of the population that is correlated, k .²²

Corollary 1. Consider the heterogeneous, constant-correlation case described in Proposition 2. Assume that $\check{\rho}$ satisfies Assumption (A1), that $G \geq \max\{C_{2,1}, \lim_{k \rightarrow 0} C_{2,k}^{nb}\}$, and that $N \in (B_{2,1}, \min\{B_1, \lim_{k \rightarrow 1} B_{2,k}\})$. Borrower outreach is non-monotonic in k : approaching full as k approaches 0 or 1, but bounded away from full for k in some interval (a, b) with $0 < a < b < 1$.

Corollary 1 shows that under reasonable conditions,²³ the fewest borrowers can be reached when the fraction of borrowers facing correlated risk is intermediate. In these cases, outreach would be higher if correlated risk were more widespread — despite the fact that correlated risk in general works against lending outreach.

The basic idea is that full outreach is possible for k low enough but becomes impossible for k high enough, at which point next-best outreach is attained.²⁴ Thus, outreach declines from full to next-best

when k passes a threshold. But outreach rises as k increases beyond this threshold, since the excluded group under next-best outreach – the safe-uncorrelated – is shrinking with k . In fact, full outreach is approached as k approaches 1, since the safe-uncorrelated are vanishing. Thus, while it is discontinuously hard to achieve full outreach as k drops below 1, outreach itself is continuous there.

Fig. 4 graphs outreach to safe agents against k , in the Left Panel when $N = 1.14$ and G is sufficiently big. Outreach is full until k moves just past 0.5, then drops as the safe-uncorrelated are lost and linearly increases toward full as k approaches 1 and the safe-uncorrelated decline in number. This N -value is chosen to fit the Proposition's assumptions. Fig. 4, Right Panel, displays expected outreach to safe agents under a wider range of N -values, i.e. N chosen at random from the uniform distribution on $[1.05, 1.2]$. Clearly, non-monotonicity with respect to k survives this generalization, and appears to be a relatively general property of the extensiveness of correlated risk.

In sum, it can be when correlated risk is moderately prevalent that fewest borrowers are reached.

6. Separating borrowers by correlatedness

The results of the previous section carry some interesting implications. Consider a lender facing a mixed population, k correlated and $(1-k)$ uncorrelated. Would the lender achieve better results if it serviced these populations separately? In cases where outreach is minimized when k is intermediate, achievable outreach would seem to be higher if the two populations could be separated into populations with $k = 0$ and $k = 1$, respectively.

Given our assumptions that both dimensions of borrower types are unobserved, the only possibility for separating the borrowers is through borrower self-selection across a menu of contract offers. However, as mentioned earlier, this will not help in our setting due to the zero-sum nature of payments from borrowers to lender, which imply that separating does no better than pooling.²⁵

²² A non-monotonicity result can also be shown on the intensive margin, but it is more fragile.

²³ The assumed interval for N is non-empty as long as $\check{\rho} \in (0, 1)$.

²⁴ This is true for the range of N focused on in the Proposition. For N above the assumed range, full outreach is achievable for any k ; for $N \leq B_{2,0}$, well below the assumed range, only risky agents borrow regardless of k . For

$N \in (B_{2,0}, B_{2,1})$, i.e. below the assumed range, there may be a local non-monotonicity, but only risky agents can be attracted for k high enough, so the general pattern is one of decline in outreach with the extent of correlated risk.

²⁵ To sketch the reasoning, let M be the contract selected by the hardest-to-reach set of agents, i.e. the agents with the lowest payoffs under any feasible

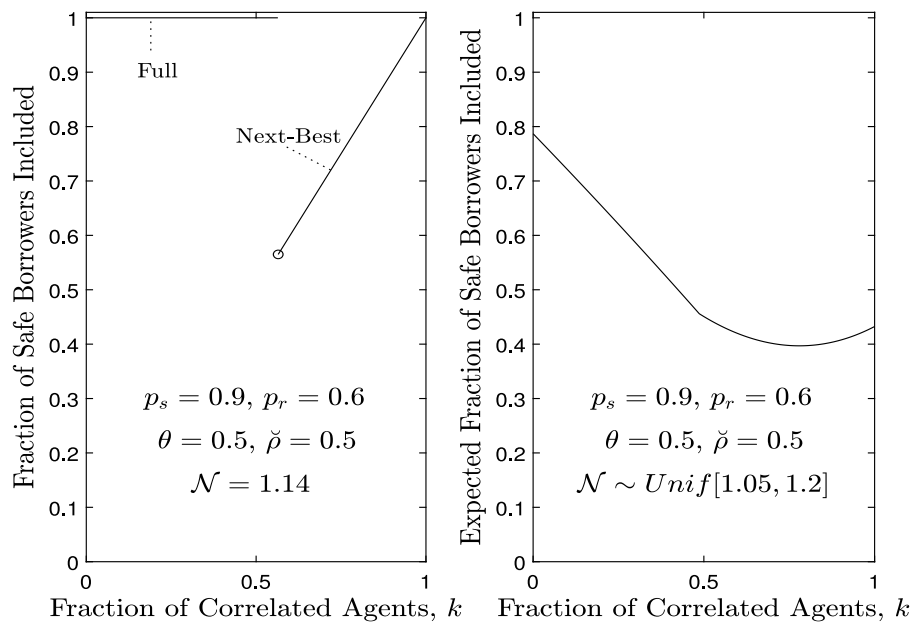


Fig. 4. Outreach or expected outreach as k varies.

Separating could only help, then, if it meant the ability to offer different, exclusive contracts to the correlated and uncorrelated, respectively. For this, borrower correlation-type must be observable. Full observability would not only enable the lender to condition contract terms on borrower correlation-type, but also to make stipulations about the matching process, for example, requiring correlated agents to match with uncorrelated in order to lower or eliminate within-group correlation.

In this section, we proceed by assuming that the lender can make limited use of correlation-type observability. Specifically, it can make separate contract offers to correlated and uncorrelated agents, but must offer the same contract to all borrowers of the same observable type (thus offering at most two contracts); must break even with each contract offered, i.e. on each subpopulation separately; and cannot interfere with matching. These limitations may be rationalized in richer settings. Offering observationally similar borrowers different contracts may raise fairness concerns; cross-subsidization across observable populations may not survive competition (McIntosh and Wydick, 2005); and interfering with matching may have negative consequences in matching markets with unobservables or markets with correlation between type dimensions. Further, given institutional constraints on microfinance practice that appear to push toward simplicity, it is interesting to see how far the lender can get with a straightforward and limited use of information. We therefore assume these limitations, but discuss later how conclusions would change if the lender could use information more freely.

Thus, we examine the desirability of separately serving borrowers by correlatedness; risk-type remains unobserved. The lender may offer a different contract to each type A and B , each of which must break even. One way to interpret this setup is as separate banks serving two

contract. By incentive compatibility, other agents must choose contracts that earn them at least as much as M would, and (by the zero-sum feature) that lead to no greater expected repayment than under M . But then the lender can achieve the same outreach and break even by offering only M , i.e. by pooling. Thus, self-selected separation of the populations will not help. See Ahlin (2015) and Ahlin and Waters (2016) for formal proofs that pooling is sufficient in similar settings. (If borrowers were risk-averse while the lender was risk-neutral, as in standard analyzes of pooling and screening, this zero-sum feature would not hold.)

different markets, e.g. an agricultural bank and a standard MFI focusing mainly on the self-employed. One can also think of it as the same lender operating in two different types of markets but unable to cross-subsidize across them, perhaps due to competition. Pooling refers to a single, break-even contract offered to the entire population, as analyzed in the previous section.

The conditions under which full outreach is easier to achieve when lending separately to universally-correlated borrowers and uncorrelated borrowers than when lending to the mixed pool are outlined next.

Corollary 2. Let $\kappa_a \equiv \frac{\kappa_0}{1+(1-p_s)(1-\beta)}$ and $\kappa_b = \frac{\kappa_0}{1-(1-p_s)(1-\beta)}$, where κ_0 is defined in Proposition 2; these parameters satisfy $0 < \kappa_a \leq \kappa_b < 1$. Consider the heterogeneous, constant-correlation case described in Proposition 2. Let \mathcal{P} and \mathcal{S} be the $(\mathcal{G}, \mathcal{N})$ parameter spaces over which full outreach is attainable under pooling and separating, respectively. Then $\mathcal{S} \subset \mathcal{P}$ if $k < \kappa_a$ and $\mathcal{P} \subset \mathcal{S}$ if $k > \kappa_b$.

Thus if there are few correlated (k low), pooling all borrowers achieves full outreach over a larger parameter space. When correlated risk is more pervasive (k high), achieving full outreach is more widely feasible when populations A and B are serviced separately. Neither pooling nor servicing separately always dominates. Intuitively, separating borrowers avoids the uncorrelated-to-correlated cross-subsidy, which is increasingly onerous for safe-uncorrelated borrowers as k rises. However, separating maximizes the safe-to-risky cross-subsidy that safe-correlated borrowers must pay, by putting them in a universally correlated borrowing pool where joint liability is less effective; and its elimination of the uncorrelated-to-correlated cross-subsidy also hurts the safe-correlated.

The solid lines in Figs. 5 and 6 illustrate the result. Under the assumed parameters, $\kappa_a = 0.52$ and $\kappa_b = 0.57$. In Fig. 5, $k = 0.25$, so pooling achieves full outreach over a larger parameter space by Corollary 2. One can see this by noting that Propositions 0 and 1 give the conditions for attracting all borrowers while breaking even in uncorrelated and correlated pools, respectively; these conditions are reflected in the “All Correlated” and “None Correlated” solid boundaries. For full outreach under separating, what matters is the upper of these two boundaries. The conditions for attracting all borrowers in a mixed pool, from Proposition 2, are reflected in the “ k Correlated” solid boundary. Between this boundary and the “All Correlated” boundary,

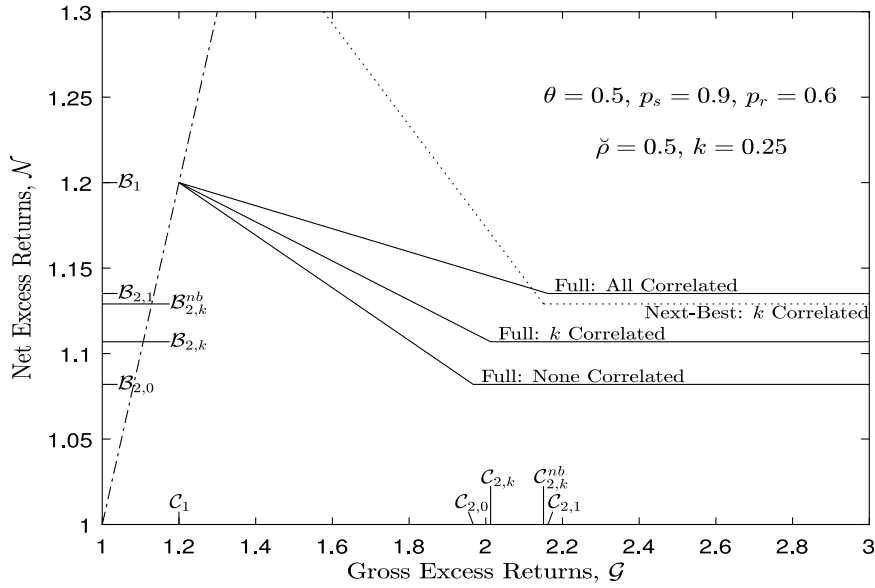


Fig. 5. Outreach space for k low.

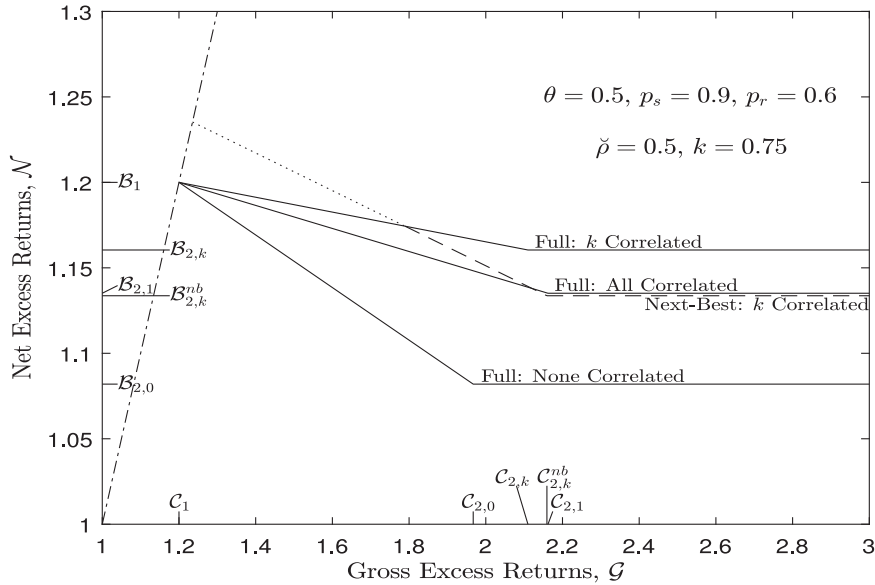


Fig. 6. Outreach space for k high.

a pooling approach leads to full outreach, while a separating approach loses the safe borrowers from the correlated pool. (Above the “All Correlated” boundary both approaches give full outreach; below the “ k Correlated” boundary, both fall short.) But when $k = 0.75$, as in Fig. 6, separating is fully effective over the larger parameter space by Corollary 2. This can be seen since the “ k Correlated” boundary for full outreach is higher than the other solid boundaries; thus conditions for full outreach are more stringent under pooling than for both the universally correlated and uncorrelated pools.

We next turn to examining comparisons of maximal outreach, which incorporates optimal outcomes when full outreach is not achievable. The comparison between separating and pooling becomes a bit more complicated, but the conclusion that either can dominate is not changed. What are the tradeoffs involved? Again, separately servicing correlated and uncorrelated borrowers is beneficial for the safe-uncorrelated, since it eliminates the uncorrelated-to-correlated cross-subsidy and sharpens group lending’s ability to lower the safe-to-risky cross-subsidy they pay (in an uncorrelated pool). But it hurts

the safe-correlated, by eliminating the uncorrelated-to-correlated cross-subsidy they receive and by further blunting group lending’s ability to lower the safe-to-risky cross-subsidy they pay (in a pool of universal correlation).

We first illustrate the issues graphically before discussing some computational results. Assumption (A1) is maintained throughout. In Fig. 5, where $k = 0.25$, next-best outreach is irrelevant, never achievable when full outreach is not ($B_{2,k} < B_{2,k}^{nb}$). Consider how outreach changes as \mathcal{N} increases, for \mathcal{G} high enough ($\mathcal{G} \geq C_{2,1}$). If $\mathcal{N} \in (1, B_{2,0})$, both pooling and separating attract only risky. If $\mathcal{N} \in [B_{2,0}, B_{2,k})$, separating attracts all the uncorrelated and the risky-correlated, excluding only the safe-correlated, while pooling attracts only risky; thus separating dominates. If $\mathcal{N} \in [B_{2,k}, B_{2,1})$, pooling achieves full outreach, while separating excludes the safe-correlated; thus pooling dominates. Finally, if $\mathcal{N} \geq B_{2,1}$, both approaches attain full outreach. See Fig. 7, Left Panel, which graphs maximal outreach against \mathcal{N} .

Next, consider outreach as \mathcal{N} varies in Fig. 6, where $k = 0.75$, assuming \mathcal{G} is high enough. In this case, next-best outreach is attainable

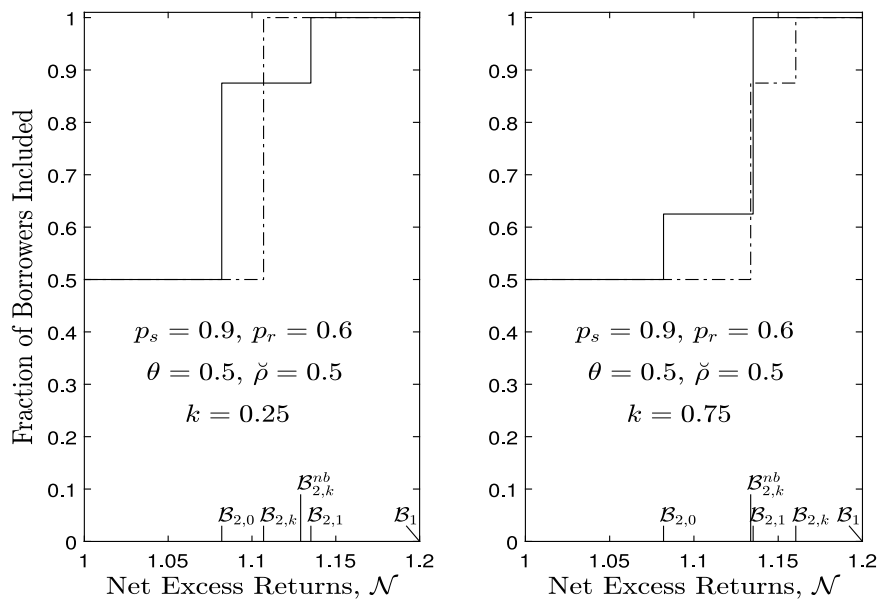


Fig. 7. Outreach: separating (solid) vs. pooling (dashed).

in the pooling case for some parameters where full outreach is not. If $\mathcal{N} \in (1, B_{2,0})$, both pooling and separating attract only risky. If $\mathcal{N} \in [B_{2,0}, B_{2,k}^{nb})$, separating excludes only the safe-correlated, while pooling attracts only risky; thus separating dominates. If $\mathcal{N} \in [B_{2,k}^{nb}, B_{2,1})$, pooling achieves next-best outreach, excluding only the safe-uncorrelated, while separating excludes the safe-correlated; pooling dominates, since the majority of safe are correlated when $k = 0.75$. If $\mathcal{N} \in [B_{2,1}, B_{2,k})$, pooling achieves next-best outreach while separating achieves full outreach; thus separating dominates again. Finally, if $\mathcal{N} \geq B_{2,k}$, both approaches attain full outreach. See Fig. 7, Right Panel.

From the Left Panel of Fig. 7, with $k = 0.25$, we see that separating and pooling dominate over similarly-sized ranges of \mathcal{N} , $[B_{2,0}, B_{2,k})$ and $[B_{2,k}, B_{2,1})$, respectively. However, the additional outreach available under separating, when it dominates, is significantly greater than the additional outreach available under pooling, when it dominates. This is because separating does better at reaching the 75% of safe agents who are uncorrelated, while pooling does relatively better at reaching the remaining 25%, the safe-correlated. In the Right Panel with $k = 0.75$, we see that separating dominates over a much more significant range of \mathcal{N} . So, while pooling has a significant advantage when it does dominate – effective as it is at reaching the 75% of safe agents who are correlated – it dominates over only a small range.

While either approach may dominate given specific parameters, separating appears to do better more often. To compare more formally, we calculate expected outreach under the two regimes, separating and pooling, assuming that any \mathcal{N} between 1.05 and $B_1 (= 1.2)$ is equally likely. As Fig. 8 shows, separating always dominates pooling. Intuitively, as described above, when k is low pooling dominates over a larger range of \mathcal{N} but does not dramatically improve outreach, and when k is high pooling dominates over a smaller range of \mathcal{N} .

In sum, even a very restricted ability to separately serve borrowers with substantially different covariate risk profiles appears likely to raise outreach and borrower surplus significantly.

How would this comparison change if the lender were less restricted in making use of information on borrower correlation-type? If the lender could offer different contracts to similar borrowers or cross-subsidize between populations, separating would do even better. When $\mathcal{N} \in (B_{2,0}, B_{2,1})$, the uncorrelated population are all inframarginal borrowers under the break-even contract offered to all uncorrelated agents. Thus, they can be charged higher rates in order to cross-subsidize correlated borrowers, who may borrow in a separate pool or partially “contaminate” the uncorrelated pool as a subset are offered

the uncorrelated contract. This will accentuate the outreach advantages of separating relative to the more constrained case analyzed above. A similar point can be made about intervening in matching. Since types A and B are uncorrelated with each other, the lender can reduce the prevalence of covariate risk within groups by requiring cross-type matching. If $k \leq 1/2$, within-group correlation could be entirely eliminated as all type- B agents match with type- A partners; all groups would be uncorrelated, and there would be no need to separate. If $k > 1/2$, however, there would be at least $2k - 1$ groups with correlated risk, as not all type- B agents could find type- A partners. The lender could essentially choose any fraction of correlated groups within $[2k - 1, k]$. But for any fraction of correlated groups on $(0, 1)$, separating achieves higher expected outreach in computations above (See Fig. 8). Thus, separating seems likely to retain its advantages even in this case of matching intervention, at least when correlated risk is sufficiently widespread ($k > 1/2$).

In general, then, when information allows it, lending to the populations separately appears to allow greater outreach. The benefits of separation of the pool of borrowers into those with correlated risk and those with independent risk may help explain and justify the existence of agricultural development banks separate from MFIs serving self-employed borrowers. Besley (1994), IFC (2012), and Mahajan and Ramana (2004) all acknowledge the fact that covariate risk makes agricultural financing unattractive to MFIs. Mahajan and Ramana (2004) support this stylized fact by citing the Grameen Bank and BRI of Indonesia as typical of MFIs in focusing exclusively on rural areas but not on agricultural lending, as against the Bank for Agriculture and Agricultural Cooperatives (BAAC) of Thailand, which focuses exclusively on lending to agricultural workers.

7. Conclusion

We derive and characterize the optimal lending contracts for group-based joint liability lending with covariate risk. When correlated risk is universal, the parameter space for efficient lending and full outreach shrinks as the amount of correlation increases, and approaches that of individual lending as correlation approaches perfect. Correlated risk reduces the effective rate of joint liability, lowering the lender’s ability to improve risk-pricing and borrower outreach via group lending.

Extending the model to analyze heterogeneous correlatedness of borrowers highlights a novel cross-subsidy, uncorrelated-to-correlated, that can work against outreach. Outreach can be non-monotonic in the

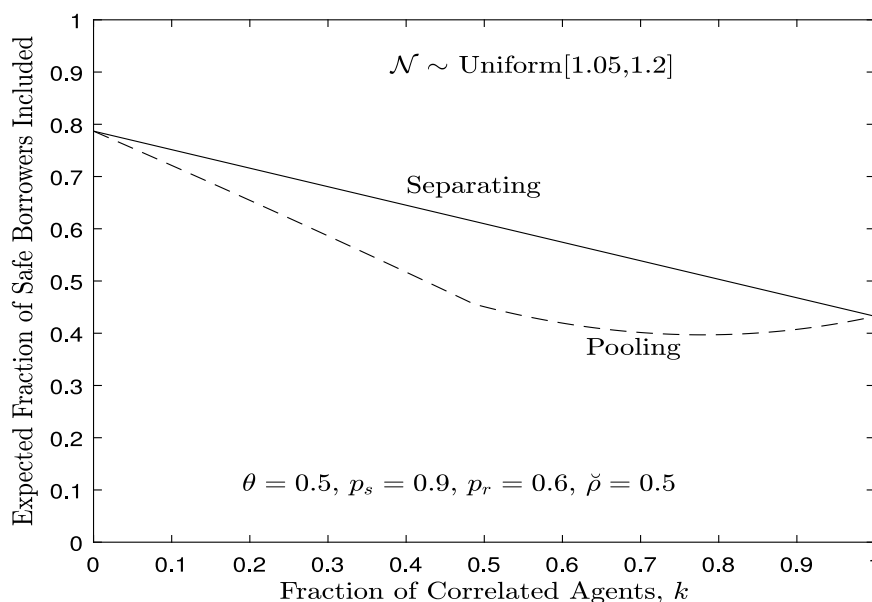


Fig. 8. Expected outreach: separating vs. pooling.

prevalence of correlated risk. That is, outreach can be higher when most or few borrowers are correlated than when a moderate number are. This non-monotonicity arises because the uncorrelated-to-correlated cross-subsidy that marginal agents must shoulder increases with the prevalence of correlated risk, but the number of marginal agents at risk of being lost due to this cross-subsidy declines.

The results imply that it is often best to separate borrowers by correlatedness when possible, offering separate contracts to different pools, even when these separated pools of borrowers cannot be mixed or used to cross-subsidize each other. This offers one explanation for the general failure of MFIs to reach farmers and the existence of dedicated agricultural banks, and suggests that this segmentation can often be best for lending outreach and efficiency.

Data availability

No data was used for the research described in the article.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.jdeveco.2022.102855>.

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