STOCHASTIC MODELING OF STOCK PRICES ON THE GHANA STOCK EXCHANGE

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DECLARATION

Candidate's Declaration

This is to certify that, this thesis is the result of my own research work and that no part of it has been presented for another degree in this University or elsewhere.

SIGNATURE: …………………………. DATE: ………………………..….

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Supervisors' Declaration

We hereby certify that this thesis was prepared from the candidate's own work and supervised in accordance with guidelines on supervision of thesis laid down by the University of Ghana.

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(Co-Supervisor)

DEDICATION

This work is dedicated to the Almighty God for the divine wisdom and strength throughout this study, my parents Mr. David Amenong Anang and Miss Matilda Dede Larsey, siblings Joyce Leeghoi Quayesam and Yehowada Naadu Quayesam for their love, care and support.

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May the Almighty God richly bless you all

ABSTRACT

Financial models today rely on assumptions that make them insufficient in many real cases. The Geometric Brownian Motion model is assumed as a process for stock prices frequently. The study examined whether the behavior of weekly and monthly returns series of some selected equities listed on the Ghana Stock Exchange can be modeled with the Geometric Brownian Motion(GBM). The Augmented Dickey Fuller, Shapiro-Wilk, Ljung-box and some graphical methods are some statistical methods used to unveil the behavior of the returns. The study showed that only the monthly returns of Ghana Commercial Bank and Benso Oil Palm Plantation satisfied all the three assumptions of the Geometric Brownian Motion, however the Hurst exponent estimates showed that seven return series can be modeled by the Geometric Brownian Motion. To test the model, parameters were estimated for five equities and three subsequent months forecasts is made for which the accuracy of the estimation is measured by the error between the estimates and the actual price observed. The study showed that, the expected price of the equities modeled is close to the actual stock price realized on the Exchange even though some deviated slightly, however the entire actual prices observed lies within the estimated confidence interval. The study concluded that the monthly returns of the Ghana Stock Exchange is a better data set to be used for statistical inference as compared to the weekly returns and even though most of the returns exhibit long range dependency the Geometric Brownian Motion can be used for some equities on the Ghana Stock Exchange .

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LIST OF ABREVIATIONS

GBM ………………………………. Geometric Brownian Motion

BOPP ……………………………….. Benso Oil Palm Plantation

FML…………………………………. Fan Milk Limited

GCB…………………………………. Ghana Commercial Bank

GGBL………………………............ Guinness Ghana Breweries Ltd

GOIL…………………………………. Ghana Oil Company

HFC……………………………………HFC Bank

SCB…………………………………… Standard Chartered Bank

SIC…………………………………….. State Insurance Company

TLW…………………………………… Tallow Oil

TOTAL……………………………….. Total Petroleum Ghana

UNIL…………………………………….Unilever (Ghana) Ltd

UTB…………………………………….. Unique Trust Bank

EBG…………………………………….. Ecobank Ghana

EMF…………………………………….. Efficient Market Hypothesis

GSE…………………………………….. Ghana Stock Exchange

DEFINITIONS OF KEY WORDS

Stochastic process

A stochastic process is when there exist a variable whose value fluctuates in a random manner over time. In cases were the values change of the variable occur at only fixed time points is term as a discrete-time stochastic process while changes of values at any time is a continuous-time stochastic process

Markov property

Is a type of stochastic process in which the predictions of the future depends solely on the present information of the variable and is independent about the variables past history.

Wiener Process/ Brownian Motion

It is a stochastic process with Markov property which as a mean of variable changes as 0 and a rate of variance of 1 per time period. It was used to describe the movement of pollen surrounded by several shocks from other molecules.

Itô Process

It is a general Wiener process by which the f and g are functions of the underlying variable p and time t. Itô process algebraically is stated as

$$dp = f(p,t)dt+g(p,t)ds$$

Stock price

It is the price of a single share of a company that may be listed on an Exchange at a particular time. It is usually determine by the market expectation of the stock's performance.

Dividend

A bonus payment granted to a shareholder by the board of directors depending on the number of shares of a corporation an investor owns.

Rate of Return

The gains on an investment relative to the amount invested which is stated usually as a percentage.

Stationarity

The term is used to describe a time series whose mean and invariance over a period of time does not change or is constant.

White Noise

A time series which are independent and identically distributed and have uncertain variables with constant expectation and variance

Closing Price

It refers to the last transaction price of a stock in a trading day.

CHAPTER 1

1.0 INTRODUCTION

This chapter discusses the background information covering the Stock market, Geometric Brownian motion and defines the main concepts used in this study. It also presents the problem statement, the main assumptions underlying the Geometric Brownian Motion in the financial stock markets and states the aims, objectives and importance of the study.

1.1 BACKGROUND

The price of a financial asset evolves over time and wealth creation is the goal of every investor. One attractive area for investments is the stock market. A stock market can be described as a legal framework surrounding the trading of shares of many companies or organizations by which prices of publicly listed assets are determined by buy and sell instructions which arrive at random intervals and in random quantities. The Ghana Stock Exchange, being a developing market, is considered to be of interest to both native investors and foreigners to seize the chance to profit on the stock market. Nevertheless, this has not been the case for many potential investors with interest in the Ghanaian economy due to inability to make optimal investment decisions dependent on future expectations of the market performance and equity prices. Within the financial market, stock market performance and its operations have gained recognition as a significant viable investment field. When an equity market is on the ascendency, it is assumed that the economy is up and coming. It is often assumed as the primary measure of a country's economic advancement and strength. Rises in equity index and prices are usually linked with augmented business activities and investment; this affects the wealth of families and their consumption. The stock market's nature is mostly random, unpredictable and it does not take academic cleverness to realize that stock price movements have a random nature to them. We most likely find investors seeking to know the background and historical behavior of listed equities to assist investment decision and choice, based on the movements of the stock markets. Investors worldwide try to watch the movements of market indices and have always shown keen interest in trying to predict the share market trend (Otieno, 2015). Although stock trading is noted for its likelihood of yielding high returns, earnings of the stock brokers or market players in part depend on the degree of equity price fluctuations and other market interactions (Mettle et al, 2014). Stock market forecasts focus on developing a successful approach for predicting index values or stock prices. The ultimate purpose is to earn high return by means of well-defined transaction approaches which does not conflict with the laws of the market regulators. However, forecasting stock indices and stock prices is very difficult because of the market volatility that needs accurate forecast model. This makes earnings very volatile being associated with high risks and sometimes significant losses. Financial practitioners need a way to handle this uncertainty. A predictive model is unlikely to be accurate but an ability to capture the inherent volatility and essentially risk could salvage the situation.

In mathematics for finance, the subtleties of the price of assets are typically modeled by paths of some exogenously definite stochastic process demarcated on some underlying probability space. Stochastic models of stock prices allow one to speak of the probability that a stock price will be a certain value, breach a certain boundary or fluctuate within a specific band.

In 1827, the biologist Robert Brown discovered Brownian motion while observing pollen particles movement in water with a microscopic device, he noticed minute particles in the pollen grains executing a fidgety motion. Upon subsequent repetition of the experiment with particles of dust, he was able to infer that the motion was due to pollen being “animated” but the source of the motion remained inexplicable. In 1900, Louis Bachelier in his doctoral thesis "The theory of speculation" was the first one to provide a concept of Brownian motion which suggested a probabilistic explanation of the dynamics of asset price in the financial markets. He established the mathematics of the Brownian motion to model the time regulation of asset prices and also the probability of price changes by writing down the Chapman-Kolmogorov equation. The work of Bachelier was not acknowledged by the community of science prevailing at that time, perhaps because of its application to financial markets and also because it suffers from the impractical property that allows asset prices to be negative. However, Albert Einstein in 1905 sufficiently expounded the motion of pollen discovered by Brown using a probabilistic model. He perceived that if the locomotive energy of fluids was exact, the molecules of water moved haphazard. Thus, small particle would accept a random number of influences of random strength from random directions in any short time period. These haphazard collisions by the molecules of the fluid would cause a sufficiently small particle to travel exactly just as described by Brown.

The Brownian motion theory has since then found its applications in many fields including physics, astronomy, medicine (medical imaging), robotics, and others including our study area of interest (stock markets). Researchers of stock market until recently have been confronted with some difficulty that, while they can chart the pathway of the market on a day by day or minute by minute basis, it is very difficult for them to observe who specifically buys, who sells and how demand and supply influences the movement prices. There seem many fascinating theories existing about how different investor behavior makes the prices move but there is no experimental evidence to buttress the critical link between the investor decisions and the price dynamics.

Over a small period of time, asset prices are continually changing by random amounts due to the random influx of new information. The money market and the commodity market are attributed to follow Brownian motion with reason of random change in market trends (Karangwa, 2008). All financial asset and derivatives pricing models are similarly based on fundamental mathematical models on which the Brownian motion is constructed and described.

Geometric Brownian motion process is a continuous time stochastic process where the logarithms of the underlying variable mimic a Brownian motion. GBM is very popular in modeling stock prices and forecast future prices. The Geometric Brownian Motion was derived from watching light particles randomly crash into heavy particles. It was observed that the randomness did not change over time, which some mathematics resulted in a conclusion that the displacement of the particle was normally distributed over time, with mean and standard deviation depending solitary on the amount of time elapsed. Fitting a model in time series analysis can be very tough especially facing with random walk data. In financial modeling, the process of Geometric Brownian motion (GBM) or lognormal process can be a suitable model form for such time series. It is a variable changing process which imbibes the assumption that the logarithmic returns are independent and normally distributed. There are a great number of applications of GBM process in different areas such as forecasting prices of energy coal, river flow, strategic and planning decisions in a supply chain, electric power consumption and many others.

1.2 PROBLEM STATEMENT

Uncertainties are always found in economic relations and the Ghanaian financial market is not an exception. The equity market, Fixed Income market, the Ghana Alternate market and the general commodity market which are the main market mechanisms are all exposed to uncertainties, people therefore require ways that can help in making better informed decisions or choose the optimal business strategies among alternatives. Louis Bachelier (1900) being the first to propose the random walk movement model for varying market prices stated that prices behave just like the movement of pollens as describe by Brown which implies that the prior change in the variable (price) is unrelated to the forthcoming or former changes in the price path. Bachelier's himself accepted an important shortcoming of model which allows the price to become negative. Realistically, the owner of the share's liability of a share price of a corporation cannot be negative, because it is limited by the money invested or to what he pays to buy the equity. Presenting the Geometric Brownian Motion (GBM) which presumes that the logarithm of the share prices rather than the actual prices itself follows a Brownian motion thereby solving the negativity in the work of Bachelier. Samuelson (1965)

As an emerging stock market, people will only invest in equities if they are sure of their investment decision. The problem of which equities to invest in, with a good return on investment is a major challenge for all investors thus, we study the behavior of the major equities trading in the Ghanaian financial markets.

1.3 OBJECTIVES OF THE STUDY

1. This study will assess whether the Geometric Brownian motion may be used on the Ghanaian Stock markets as a model or not.
2. To model the stock price of some equities listed on the Stock Exchange if objective (1) above is applicable
3. To estimate parameters in the model based on existing data

4. To forecast prices in subsequent periods.

1.4 ASSUMPTIONS

(i) The stocks pay no dividends

(ii) No commissions are charged due to the sale or buy of stocks by stock brokers.

1.5 THE AIM OF THE STUDY

The study aims to test if the assumptions of the GBM as proposed by other researchers can be applied to some equities on the Ghana Stock Markets. Thereafter, the study aims at determining the efficiency of the Ghana financial markets and also to make forecast of stock prices in some future time. It also makes guidance to how investors can profit on the Exchange having a prior knowledge on the path of the stock for a particular equity.

1.6 SIGNIFICANCE OF STUDY

Although there are several studies in modeling stock prices and asset returns, there are few or no studies that have model stock prices on the Ghana Stock Exchange using the Geometric Brownian Motion. The results obtained from this study could help in understanding the behavior of the Ghanaian financial markets and improve market assessments as they vary over time. The findings could also be more useful to investors who make investment decisions based on risk analysis and optimized portfolios in a very short time. The findings of this research will inform potential investors to know where to invest their money since model and forecast will give idea on which stock will be doing well at particular future period.

1.7 GHANA STOCK MARKET

In February 1989, the concern of instituting a stock exchange in Ghana was tasked to a national committee set by the government at that time to foresee modalities in the direction of the establishment of an Exchange. The stock Exchange was founded in July, 1989 as a private entity limited by guarantee under the Companies code of 1963 and granted recognition in October, 1990. Transactions began on its floor in November 12 at the same time it was inaugurated. In April, 1994 the Exchange became a public company limited by guarantee. Some objectives of the Exchange are to make available the needed information to the public for the transaction of shares, bonds and other securities, to also regulate the quotations on the market in respect of securities of corporate bodies including government agencies, to control the dealings of members and their clients and to coordinate with stock exchange and other stock brokers in different countries.

Currently, the Ghana stock Exchange boasts of three markets which are the Fixed Income, Equity and the Ghana Alternative markets. Some of its major successes since its establishment include the facilitation of long term capital for expansion and growth, over GH 2.1 billion capital raised since 1990. It has also increased market capitalization from GH 3.05 million in 1990 to GH 62,183.49 million as at 31st October, 2015. It has further created wealth for investors and improved market efficiency through the introduction of electronic trading system (www. gse.com.gh).

1.8 ORGANISATION OF THE STUDY

This study is structured into five chapters as follows: The first chapter gives an overall introduction of the study; the second chapter discusses in brief on what has been published so far about this particular topic just to provide a conceptual framework for an enlightenment of the study. The third chapter expounds on the methods presented for the major concepts used in the study. Fourth chapters give an analysis of the data and interpretation. The fifth chapter provides summary, conclusion and recommendations on the findings and a section for appendix of results from the study.

CHAPTER 2

LITERATURE REVIEW

2.0 Introduction

This part of the study dwells on review of other studies that are related to our study of the geometric Brownian motion on financial market. Below are some review articles that are relevant to this study.

2.1 Evidence from the world

Data mining, regression analysis, moving averages, neural network, Autoregressive Integrated Moving Average (ARIMA) models to Markov Chain analysis is some of several different approaches that have been used to predict the trend of stock markets. Geometric Brownian motion model which this study aims at exploring is a prediction method based on probability forecasting approach and may be effectively used to analyze and predict the stock price in the stock market mechanisms.

Osborne (1959) introduced the Brownian motion to be used on common stock prices, it was revealed that worth of money of common stock can be viewed as an ensemble of decision in statistical equilibrium with properties that can be liken to the group of particles in statistical mechanics. The objective of his work was to indicate that the logarithm of common stock prices can be viewed as an ensemble of pronouncements in a statistical steady state and this ensemble of logarithms of prices varying each with time that has a close resemblance with the ensemble of a coordinates of large number of molecules.

The GBM uses are well known in the one of the most imperative concepts in contemporary financial theory; Black-Scholes Model. Black and Scholes (1973) mentioned in their option pricing formula for the first time in their paper, "The Pricing of Options and Corporate Liabilities". The paper states that in a market, if options are properly priced, constructing portfolios of short and long positions in options and the underlying stocks should not be possible to make sure profit. A theoretical valuation procedure for options using this principle is derived. One of many assumptions for their conceptual framework which is of interest for our research is that stock prices possess a geometric Brownian motion with drift and volatility constant.

A number of considerable studies have suggested the reason to generalize the GBM model by the possibly introducing jumps and allowing the path volatility to also follow a stochastic process. The first jump-diffusion model was introduced by Merton (1976). The work by Merton included an addition of a compound Poisson process as a model for the jumps to the Brownian motion part of the GBM model. In Merton's model the jumps occur randomly, with a certain average frequency at the time when the jump occurs in the logarithmic price.

Kanniainen (2008) argued that Geometric Brownian motion is customarily applied as a dynamic model of underlying project value in real option analysis, conceivably for reasons of tractability analysis. His paper considered how alterations between a state variable and cash flow are associated to project drift and volatility thus characterizing of future cash flow as a stochastic state variable. The model specified conditions necessary and sufficient for project volatility and drift to be varying with time, an important topic for the analysis of real option because project value and its variations can be estimated seldom from data only. The study further showed that project volatility could be caused to be mean- reverting by fixed cost and concluded that Geometric Brownian motion conditions can hardly be met; hence real option analysis should rather be centered on cash flow factor models instead of direct model of project value.

Hassett and Metcalf (1995) also argued that the processes of mean-reverting are suitable for most "real option" investment models and that the application of the GBM process is reasonable since mean-reversion possess two contrasting effects. Firstly, it conveys the investment trigger nearer and secondly it lessens the conditional volatility thereby lowers the probability of reaching that trigger. They concluded in their work that cumulative investment in general is unaffected by the use of a mean reversion process rather than Geometric Brownian motion.

Sarkar (2002) extended the model of Hassett and Metcalf (1995) by integrating a third factor of systematic risk on the mean-reversion process. The major finding is that, mean reversion process on investment does not general have a significant effect. Moreover, depending on different factors such as cost of living, interest rate and duration of project, there could either be positive or negative effect. It was found that the use of GBM process as an approximation to a mean-reverting process is generally not appropriate.

Lewis (2002) asserted that there are some problems with the use of the geometric Brownian motion. Proceeding with the positive side, the author mentioned that the GBM is consistent with securities having limited liabilities, has uncorrelated returns and is very tractable computationally. The challenge according to Lewis (2002) is that it relies on normal distribution, and that there are too many outliers for this assumption to hold. Furthermore, J.Zhu (2008) provided practical evidence of a company in some stock markets which issued a dual stock. In as much as the listed dual stocks have the same firm-specific risk, identical dividends and voting policies, they are however priced differently. The market price of risk and the expected between the two shares listed in the Shanghai and Shenzhen stock market were subjected to testing. The study adopted models with dynamic Geometric Brownian motion and also included multivariate GARCH models to consolidate the time-varying feature of volatility in stock return. The results suggested that the explanation to the difference in pricing of the dual stock is related to the difference in expected returns between the two shares.

Dmouj (2006) argued that, the Geometric Brownian motion appears to be a good method to model the prices of stock ideally in the future however in practice, the GBM model exhibits some inadequacies especially when it is applied over period of short time. The study concludes that over longer periods of time, the application of GBM in modeling stock price is more accurate.

In the work of Ladde et al. (2009), the authors employed a classical model to produce a modified stochastic linear model. The stock price data in this study was used to examine the precision of the prevailing Geometric Brownian motion model and subjected to standard statistical test. The experimental comparison between the GBM model and the constructed models was outlined through the Monte Carlo technique by the demonstration of the developed modified linear models with or without jumps under distinct data partitioning.

Bendkia & Giversen (2010) in their thesis work studied continuous time models within different stock market environments. The authors assumed that the modeling of continuous time processes may be altered whether an equity market is experiencing a crisis or a pre-crisis period. Among the continuous time processes family, the study covered the Geometric Brownian Motion (GBM) and the Constant Elasticity of Variance (CEV). After estimating and analyzing their respective parameters using the Maximum Likelihood Estimation method, this research test outcomes showed that there is no strong argument that could favor the addition of a discount factor i.e. CEV over the Black-Scholes based process, the GBM model.

Siaw Li & Djauhari (2013) in their work proposed that autocorrelated process in modern industry is ubiquitous and in practice, the elimination of the autocorrelation by the usage of appropriate models such as Box-Jenkins and other models as been the current norm to conduct process control operation with the focus on the residuals. In their work, the outcome showed that numerous time series are governed by the process of Geometric Brownian motion. The work stated therefore that they only need an appropriate transformation by using the properties of the GBM process and proceed to model the transformed data to generate the conditions needed in the traditional process control

Sagadavan & Djauhari (2013) presented that autocorrelation multivariate process is mostly encountered in real life where the current process is dependent on the prior process. This process type can be conducted on the residuals which transforms to univariate in nature and can be modeled by employing the traditional multivariate time series. With an appropriate transformation that will generate the conditions necessary for the traditional multivariate control, they concluded that the traditional multivariate process can be applied on the transformed time series data under the GBM process.

Gajda & Wyłomanska (2012) argued that one of the earliest systems that were used for asset prices description is a model proposed by Black- Scholes. It is conditioned on geometric Brownian motion properties and was used as a tool for pricing various financial instruments. However, when it comes to data description, geometric Brownian motion is not capable to capture many properties of present financial markets. Therefore they suggested another approach grounded on subordinated tempered stable geometric Brownian motion which is a combination of the popular geometric Brownian motion and inverse tempered stable subordinator. In this paper they introduce the process mentioned and present its main properties.

Fuentes, Gerig & Vicente (2009) proposed that numerous studies assume prices of stock to follow a random process referred as Geometric Brownian motion. Though arguably correct, the model however fails to elucidate the regular occurrence of extreme movement of prices. They presented a modification to the random model (GBM) based on evidence by adding a small but relevant changes to the standard deviation of the process. This in their view exactly expounds the probability of different price size changes together with the relative high frequency of extreme movements. They showed further that the fluctuations of the price can be described by a single curve and the process is the same for various stocks

Abidin & Jaffar (2012) reviewed that the Geometric Brownian motion and its application to predict share prices. Geometric Brownian motion formula is evaluated and examined to meet the fluctuation of share prices. Their paper elaborated on the randomness, drift and volatility involving the Geometric Brownian motion that can wisely help investment decision making by investors for short term investment.

Marathe & Ryan (2005) argued that the geometric Brownian motion process is regularly chosen as a model for such varied quantities in different fields of application such as natural resource prices, growth in demand for products or services and stock prices. They discoursed on procedures to ascertain whether a given time series charts the GBM process and analyzed approaches to remove seasonal variation from such a time series. They established with evidence that the GBM process for some of the data sets is likely to be appropriate based on the normality and independence criteria, nonetheless the assumption of GBM process for some set of data may not be appropriate. In their view, extreme care should be taken before assuming that the particular data set follows the GBM process.

Erlwein, Mitra & Roman (2011) presented that the Geometric Brownian motion is a customary modeling method for time series data in the financial settings. The assumption of constant parameters of the method is imperative in the criticism of the process owing to the fact that the volatility clustering or extreme behavior noted as some important features of financial time series cannot be captured. With these reasons outstanding, their study offered a methodology presided by a hidden Markov Chain where the GBM parameters are able to change between regimes. Consequently, a GBM in each state of the hidden Markov model was used for modeling the financial time series scenarios of portfolio optimization resulting in the CVAR of the portfolio being minimized by the suggested approach.

Mota (2012) argued that it is usually presumed that the price changes of the stocks are determined by Geometric Brownian motion with properties of the log return of the prices being normally distributed and independent of the previous return time series. In this paper, the study investigated the assumptions of normality and independence to verify the real world case of large number of company stock prices listed in the Nasdaq composite index. The log-returns of the prices stocks were subjected to tests to check the normality assumptions. The Shapiro-Wilks, Anderson-Darling and Kolmogorov-Smirnov test for normality were applied to several number of company stock returns. In view of this, the study advocates that a high proportion of company prices will fail the normality assumption from the Nasdaq composite index. The result was justified when different observations interval of both weekly and monthly prices were utilized. It was concluded that in testing the normality for the prices, the sample size of observation is more important than just considering the daily, weekly or even monthly prices of stock.

Abiden (2014) forecasted the future small sized corporations' closing price by employing the Geometric Brownian motion. The study suggested that the Geometric Brownian motion can be applied to estimate a maximum of two weeks daily closing price of stocks. The Mean Absolute Percentage Error (MAPE) was used to prove the accuracy of the method by the realization of low values. The study further concluded that in using the GBM, one week daily data is sufficient to be used to forecast the price of shares.

Sonono & Mashele (2015) presented that the movement in stock price is universally accepted to be difficult in their future prediction and still poses to be challenging whenever it is being attempted. Continuous time models were used to address the problem of the forecast of stock price fluctuation. Continuous time models such as Variance Gamma and the Geometric Brownian motion were in particular compared analytically in terms of the predicting direction and preciseness levels of stock price by employing Monte Carlo methods, Least Square Monte Carlo and Quasi Monte Carlo. The experimental tests proposed that either the Variance Gamma or GBM embedded in any Monte Carlo can be employed to estimate the movement of the stock price direction when evaluated with both the Mean-Absolute Percentage Error and hit ratio. In the jurisdiction of model performance, the findings suggested that the GBM model does better in the Quasi Monte Carlo while the Variance Gamma model does well in the Least Square Monte Carlo method.

2.2 Evidence from Africa

Otieno et al (2015) through a longitudinal case study design predicted the direction of the Safaricom shares listed on Nairobi Securities Exchange by applying a Markov Chain model. Grounded on initial state vector and probability transition matrix, a Markov Chain model was proposed which estimated that the share price of Safaricom would decrease, stabilize in value or increase with some respectively attached probabilities which are regardless of the present state of the share price in the long run.

Karangwa (2008) submitted a study that looked at the Geometric Brownian motion process with concerns to the South African financial market behavior. Experimental data of some major trading securities were selected from the market on daily, weekly and monthly basis of stock price time series. Using both graphical and statistical test methods, the assumptions underlying the GBM were tested to ascertain whether the GBM can be applied to the South African financial market. The study concludes that although some assumptions underlying the process of the GBM were violated, the GBM process cannot be rejected on the South African financial market and it can only be accepted in some instances if some parameters such as the Hurst exponent are added.

Fiarbrother (2012) argued that standard Geometric Brownian motion is the stock model underlying Black-Scholes famous option pricing formula. However numerous problems with certain features of this stock model did not follow some empirical stylized facts seen from the observation of actual assets prices. In particular, the constant parameter idea behind Geometric Brownian Motion is flawed thus the study proposed an alternate model which the parameters in the standard Geometric Brownian Motion change according to the Hidden Markov Process. The model is tested on South African data where the implied volatility surfaces produced appear to obey some of the empirical observation and theoretical ideas around the expected implied volatility surfaces.

2.3 Evidence from Ghana

Frimpong and Oteng-Abayie (2006) proposed models by exploring the GARCH(1,1), TGARCH(1,1), EGARCH(1,1) and the Random Walk to forecast conditional variance

(volatility) on the Ghana Stock Exchange, with the application of LL information , AICs and diagnostic checks for non-linearity, the models were judge based on their specification and performance in terms of forecast accuracy when compared. The Data Bank stock index was equally used for the various models. In relation to the characteristics of stock returns on developed stock markets around the globe, Data Bank stock index showed asymmetry effects, volatility clustering and leptokurtosis. In view of these, they further concluded the Random Walk is not appropriate for modeling the Data Bank stock index and based on the normality distribution assumption held for innovations, the GARCH(1,1) model performs better than the others in the study.

Dedu and Oduro (2013) examined the application of option pricing methodology to evaluate the corporate risk in a banking credit portfolio. The resulting probability of default of clients was stochastically modeled and the default probability of the corporate entities was evaluated using the Black-Scholes default probability which has an embedded assumption of the Geometric Brownian Motion.

Mettle et al. (2014) established a methodology for weekly transaction data of some equities selected from the Ghana Stock Exchange. The equity price fluctuations were specifically assumed as a stochastic process and possess Markov dependency with state transition probabilities matrices. The associated state space was represented as price appreciation, price stability and decrement in price which are respectively known from data. For optimal investment decision making on the GSE, they proposed a criteria grounded on realization of maximum transition probability, minimum mean return time and the maximum limiting distribution of price appreciation of the equities.

To the best of our knowledge, there is hardly any published work in Ghanaian and many Africa literatures that focus exclusively on stochastic modeling of stock prices using Geometric Brownian Motion on Stock Exchanges.

2.4 Review of theoretical Literature

2.4.1 Wiener Process

A variable S is formally said to follow a Wiener process if two properties required are met.

1. The differences in Δs during small interval of time Δt is $\Delta s = \varepsilon\sqrt{\Delta t}$ where ε possess an attribute of a standard normal distribution $N(0,1)$.
2. The figures of Δs for any two distinct short time (Δt) periods are independent.

It proceeds from the property (1) stated above that Δs is itself normally distributed with mean of $\Delta s = 0$ and standard deviation of $\Delta s = \sqrt{\Delta t}$.

The property (2) indicates that S satisfies a Markov process.

In a comparatively long interval of time T, the variation in values of S can be expressed as $S(T) - S(0)$. The aggregate change in S in K periods of small time measure Δt can be expressed $K = \frac{T}{\Delta t}$.

Subsequently $$S(T) - S(0) = \sum_{i=1}^{K} \varepsilon_i \sqrt{\Delta t}$$

as the $(i = 1, 2, \dots K)$ and ε_i are normally distributed with N $(0,1)$.

A variable p can be stated for a general Wiener process in terms of ds as

$$dp = f'dt + gds \tag{2.1}$$

denoting f', g notations as constants. The $f'dt$ term in equation (2.1) is the anticipated drift rate which is measured per unit time and the white noise or random nature that the path variable (p) trends are imbedded in the gds component. The product of the Wiener process and g is the extent of the noise or fluctuations.

The change (Δp) in the amount of p for short time period is $\Delta p = f'\Delta t + g\varepsilon\sqrt{\Delta t}$

subsequently, Δp is distributed normally with mean $\Delta p = f'\Delta t$ and standard deviation of $\Delta p = g\sqrt{\Delta t}$ since $\varepsilon \sim N(0,1)$.

2.4.2 Itô Process

This is a general Wiener process where f' and g parameters are functions of the causal variable of price (p) and time (t). Algebraically an Itô process can be represented as

$$dp = f'(p,t)dt + g(p,t)ds \tag{2.2}$$

In an Itô process, the anticipated drift rate and variability rate is likely to fluctuate over a short time period varying from t to $t + \Delta t$ and similarly for p to change to $p + \Delta p$ resulting in

$$\Delta p = f(t,p)\Delta t + g(t,p)\varepsilon\sqrt{\Delta t} \tag{2.3}$$

with time period between t and $t+\Delta t$, the assumption that the variability rate and drift rate of p stays constant is equivalent to $g(t,p)^2$ and $f(t,p)$ respectively.

2.4.3 Stock Price Process

To propose that the prices of a stock shadow a generalized Weiner process can be tempting. It will in effect indicate the possession of a constant anticipated drift rate and a fixed variance rate. A very important trait of prices of stock is that the anticipated rate of return investors will require from a stock is independent of the stock's price which might not be easily captured in the model.

If the expected rate of return for a rational investor requires 25% per annum on the price of the stock when price is GH₵ 1.00, then all other things being equal will similarly make the investor to expect the same 25% rate return when its price is GH₵ 3.50 or even GH₵ 5.00. Nevertheless, the constant expected drift rate assumption is not suitable and remarks that it should rather be substituted with a constant expected rate of return assumption by the investor.

If P denote the price value of a stock at time (t), then for some fixed parameter (u), it is assumed for the expected drift rate in P to be uP.

In a small period of time Δt , the anticipated change in price (P) is $uP\Delta t$ defining the u parameter to be the stock's rate of return.

In cases where the value of the stock price has zero volatility, then the stock model implies

$\Delta P = uP\Delta t$ as time change approaches $(\Delta t \to 0)$ or

$$dP = uPdt$$

$\frac{dP}{P} = udt$, integrating from 0 to T in terms of time results in

$P_t = P_0 \ell^{uT}$ with P_0 and P_T denoting stock prices at 0and T time respectively

When the variance rate is 0, it can be deduce from the above model that the price of the stock develops at a compounded continuous rate of u per unit time. However the price of the stock in reality shows volatility and there seems to be sufficient literature on volatility modeling.

A further assumption aside the expected rate of return is the constancy in the rate of variability in returns irrespective of the price of the stock within a small time interval.

Consequently, an investor who is probably not certain of the percentage rate of return when the stock price is GH₵ 5.00 will have the same uncertain attitude when the price is GH₵ 1.00. This advocates the standard deviation in price change to be directly proportional to the stock price within a small time period (Δt) leading the model to

$dP = uPdt + \sigma Pds$ similar to

$$\frac{dP}{P} = udt + \sigma ds \qquad (2.4)$$

The variable σ is price of the stock volatility while the variable u being the anticipated return rate of the stock. The model developed in relation to price behavior of stocks is known as the Geometric Brownian Motion.

2.4.4 Discrete Case of GBM

The model in its discrete time form is

$$\frac{\Delta P}{P} = \mu \Delta t + \sigma \varepsilon \sqrt{\Delta t}$$ similar to

$$\Delta P = \mu P \Delta t + \sigma P \varepsilon \sqrt{\Delta t} \qquad (2.5)$$

The variable ΔP is the difference in the price (P) of the stock in a short interval of time (Δt) and ε takes a standard normal distribution. The value of return expected is the $u\Delta t$ term and the stochastic component is the $\sigma\varepsilon\sqrt{\Delta t}$ term in the model.

In the practical world of stocks, the changes in prices are limited to discrete values where observation are only realized when transactions commerce on the Exchange when open for trading. For the purpose of modeling, it is however rational to consider a continuous variable - continuous time stochastic process of the stock price (Farid & Ashraf, 1995).

2.4.5 Efficient Market

The fluctuations of prices of stock in financial market settings are unveil by movement of uncertain values over a period of time. The motivation for the probable changes in stock behavior can be associated with the Efficient Market Hypothesis (Fama,1965) which essentially postulate that

1. The current price of a stock reflects wholly all the past history price.
2. Any new information on the stock causes the market to immediately react to the news.

This two statements infer that the stock price fluctuations follows a Markov process which can be explained that the future value of the stock rely solely on its current price thus it is alluded that GBM has a Markov property, see proof (Yang, 2015).

2.4.6 Asset Returns

The term refers to the gain or loss of a security in a particular period (www. Investopedia). In investment management or performance, there are two types of returns namely net return and total return. Net returns refer to the returns obtained from values accumulated and display only price gains, interest or income from dividends. The accumulated values can be transaction costs, management fees, taxes while total returns is the returns before the accumulated values such as all fees, expenses, taxes etc.

Let P_t denote the price of an asset at time (t). Assuming the asset pays no dividends for instant, then the following returns type are likely to occur

2.4.6.1 Simple Return for one period

An asset held for one period from $t-1$ to t would end in a simple gross return given as

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad (2.6)$$

2.4.6.2 Simple Return for multi-period

An asset held for more than one period, assuming j periods $t-j$ date and t will end up in a j period gross simple return. This is a geometric mean of the j one period gross return

$$R_t(j) = \left[\prod_{i=0}^{j-1}(1+R_{t-i})\right]^{1/j} - 1 \quad (2.7)$$

2.4.6.3 Continuously Compounded Return

The natural logarithm of the simple return also known as the log return of an asset is define as

$$R_t = \ln \frac{P_t}{P_{t-1}} \quad (2.8)$$

but with the inclusion of dividend payment by an asset periodically, the definitions must be modify to capture the dividend. Let D_t be the dividend paid between $t-1$ and t dates. Then the continuous return compounded at time t and the simple return respectively become

$$R_t = \ln(P_t + D_t) - \ln(P_{t-1}) \quad \text{and} \quad R_t = \frac{D_t + (P_t - P_{t-1})}{P_{t-1}}$$

2.4.7 Volatility

Volatility refers to measure of risk or uncertainty about the magnitude of fluctuations of an asset such as the price of a share. A relatively high volatility means that over a small time interval, the price of an asset or share can vary dramatically in the direction of either up or down while a relatively low volatility signify that the price of the asset over a short period of time will change steadily. Some explanations to different types of volatility are

2.4.7.1 Historical Volatility

It is a measure of previous or past behavior fluctuations in share price with respect to period of time. The reason for the determination of historical volatility is to have an over view of what the forthcoming volatility might be. There are several arguments over the best method for computing historic volatility however the standard deviation which is a statistical term that measures over a given time period the distribution of the samples is the most usual method .

$$Volatility = std\left(\ln \frac{R_t}{R_{t-1}}\right) \quad (2.9)$$

where $R_t, R_{t-1}, \ldots$ are stochastic processes which represent the price returns.

2.4.7.2 Implied Volatility

Implied Volatility also known as realized volatility is anticipated by the entire market and it is determined by the price the market is implying or suggesting by its activities. It is independent of the movement of the underlying asset and the implied volatility of the stock in the future will be centered on the market where the asset is being traded.

2.4.7.3 Forecast Volatility

Future volatility or forecast volatility is a measure of fluctuations that tries to forecast what the volatility will be in the subsequent periods (weeks, months) just to give a rough position of the underlying asset in the near future. Forecast volatility determination is a challenging task because it requires the combination of answering what influence does the past have on the future and figures in anticipated events ahead.

2.4.8 Decomposition of Returns

Prices that follow a Geometric Brownian Motion satisfy the independence condition and therefore their volatilities increase with the square root of time. The geometric returns can be decomposed into multiple periods. For instance, we consider two weeks or months period returns

$$R_{t,2} = In\left(\frac{P_t}{P_{t-2}}\right) = In\left(\frac{P_t}{P_{t-1}}\right) + In\left(\frac{P_{t-1}}{P_{t-2}}\right)$$

$$= R_t + R_{t-1}$$

The expectation and variance of the sum of two weeks or months returns can be decompose as

$$E(R_1+R_2)=E(R_1)+E(R_2)$$

$$Var(R_1+R_2)=Var(R_1)+Var(R_2)+2Cov(R_1,R_2)$$

If the returns follow a Geometric Brownian Motion then the returns are uncorrelated or independent as it is commonly believed, thus

$$Cov(R_1,R_2)=0$$

2.4.9 Lognormal Property

Assume that the value of f variable and g follow the Itô process.

$$dP=f(t,P)dt+g(t,P)ds$$

where f, g notations are functions of P and t with ds being a Wiener process. The P variable has a drift rate of f and a variance rate of g^2.

If function G of P follows the Itô's process then from Itô's lemma

$$dG=\left(\frac{\partial G}{\partial P}f+\frac{\partial G}{\partial t}+\frac{1}{2}\frac{\partial^2 G}{\partial P^2}g^2\right)dt+\frac{\partial G}{\partial P}g\,ds \qquad (2.10)$$

since G follows an Ito process, it also have drift rate and variance rate as

$\frac{\partial G}{\partial P}f+\frac{\partial G}{\partial t}+\frac{1}{2}\frac{\partial^2 G}{\partial P^2}g^2$ and a variance rate of $\left(\frac{\partial G}{\partial P}\right)^2 g^2$ see proof (Itô, 1951)

From equation (2.4), we proposed that

$$dP=uPdt+\sigma Pds$$

with u and σ assuming to be constant is a cogent model of stock price movement. From Itô's lemma, the process followed by the function G of P and t is

$$dG = \left(\frac{\partial G}{\partial P} uP + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial P^2} \sigma^2 P^2 \right) dt + \frac{\partial G}{\partial P} \sigma P dz$$

where P and G are exposed to the same uncertainty underlying the source.

If the natural logarithm of the variable is distributed normally then the variable is said to have a lognormal distribution.

denote $G = \ln P$ and taking derivatives with respect to P result in

$$\frac{\partial G}{\partial P} = \frac{1}{P}, \quad \frac{\partial^2 G}{\partial P^2} = \frac{-1}{P^2}, \quad \text{and} \quad \frac{\partial G}{\partial t} = 0$$

It follows that from Equation (2.10) upon substitution of derivatives, the process of G is

$$dG = \left(u - \frac{\sigma^2}{2} \right) dt + \sigma dz \qquad (2.11)$$

as u and σ are constants, the equation (2.11) indicates that $G = \ln P$ follows a general Wiener process with drift rate $\left(u - \frac{\sigma^2}{2} \right) T$ and constant variance rate σ^2.

The variations in $\ln P$ between 0 and some future time T is normally distributed with mean $\left(u - \frac{\sigma^2}{2} \right) T$ and variance $\sigma^2 T$. This means that

$$\ln P_T - \ln P_0 \sim N\left[\left(u - \frac{\sigma^2}{2}\right)T, \sigma^2 T\right]$$

where P_T is the stock price at future time T, P_0 is the stock price at time 0. The stock price model proposed that given its price today a stock price at time T is lognormally distributed.

CHAPTER 3

METHODOLOGY

3.0 Introduction

This study seeks to explore whether the Geometric Brownian Motion can be applied on the Ghana Stock exchange and if it can, the study will seek to model the stock prices of some companies listed on the Ghana Stock Exchange. Specifically, this chapter is concern with the description of the data set and the statistical guidelines that will be used in unveiling the price dynamism on the GSE.

3.1 Data Collection and Source

The data used for the study consist of time series of weekly and monthly closing price of some randomly selected equities on the GSE. The study chose the closing price on Fridays as the end of week price for the selected stocks and the closing price on the last working day of the month as the end of month price for the equities from the period of Feb 2014 to Feb 2016. However due to public holidays when the Exchange is not operational, the study chose the closing price of the recent working day before the holiday as the end of week or month price.

3.2 Description of data and method of analysis

In the calculation of weekly and monthly stock returns, the log returns are used for the reason that, unlike the simple discrete returns, log returns are time additive (Bendkia & Giversen, 2010) such that when the period j is needed, the sum of each of the consecutive weekly or monthly returns summed will equal the return of j period and also, statistical properties of the log returns are more tractable (Tsay, 2005).

$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$ where P_{t-1} is the stock price at time $t-1$ and P_t is the stock closing price at time t.

Each dataset was checked for unit root (that is if the variance and mean is not constant over time period).The study will first do a visual check by plotting the log returns and time to check for trend. To confirm whether the series yield a unit root or is stationary, a further check of stationarity is done by the study utilizing the Augmented Dickey Fuller (ADF) test. The study will employ both graphical and power test methods to check for the normality assumption in the returns data set. The study will also check if the returns series for a particular week or month is independent of the previous week or month's return. The procedure for testing for stationarity, normality, independence and the statistical models used are discussed below

3.3 Unit Root Test for non-stationarity

Several financial and economic time series data reveal behavior of non-stationarity or trending in the variance and mean. Prominent examples among such series are GDP, exchange rates and prices of assets. The determination of a suitable form of trend in a set of data is an essential task in economic analysis. Most time series have to be rendered stationary for accurate prediction reasons. One call of application for a unit root test is to decide if a non-stationary data should be differenced first to make it stationary. Another usage for the test is when it becomes necessary to determine whether a series is mean-reverting or not. A basic and preliminary test to investigate a series for unit root is a time series plot. The presence of variability in patterns such as seasonality, variance, mean and autocorrelation suggest a unit root in the data set. However an additional test is required to provide sufficient evidence to conclude on the visual test output.

Employing the Augmented Dickey-Fuller Test (ADF) proposed by Dickey and Fuller (1981) can also be used to find out if a univariate time series is stationary or not. The Kwiatkowski, Pillips, Schmidt and Shun (KPSS) can be used as an alternative test to the ADF. To unveil the behavior of returns in terms of the presence of a unit root, the Augmented Dickey-Fuller (ADF) unit root test was applied for this purpose.

3.4 Normality Test

Inference and interpretation from statistical analysis may not be consistent when the assumption of normal distribution is flawed for a given random data set. In several procedures in statistics, the significance of the distribution to be normal is irrefutable since it is essential in even future prediction and how naturally the data emanate. In determining whether a sample observed from a population with independent outcomes is normally distributed, the study relies on three familiar techniques in assessing a sample of time series to be emanating from a normal distribution. The graphical approach consists of Q-Q plot, histogram, box plot and others. Another approach is the numerical method which examines the kurtosis and skewness of the data set and a formal test approach comprising of Kolmogorov-Smirnov test, Lilliefors test, Shapiro-Wilk and Anderson-Darling test for normality.

3.4.1 Histogram

A constructive graphical representation of a normal curve placed over a histogram presents useful insight of the data. A bell-shaped “normal curve” indicates that the time series data point adhere to a normal distribution and any shape different from the perfect bell shape will make us doubt the assumption of normality. Since the histogram is not a very accurate way of detecting the normality of the data, the study further employs the Q-Q plot.

3.4.2 Q-Q Plot

Goodness of fit to a specified distribution can be assessed visually by the Q-Q plot which employs a graphical technique. The Q-Q plot performs two main tasks in the analysis of data. It is employed to ascertain if two distinct data sets possess the same distribution and also bearing the name normal probability plot it is used in checking for univariate normality which is of interest to the study.

The following procedures are the methodology behind the output when checking the normality in our return series.

Order the n original observations $R_1, R_2, R_3, \ldots R_n$ to obtain the order statistic

- compute n number of normal quantiles as $q_1, q_2, q_3, \ldots, q_n$ where q_i represent a standard normal quantile linked with the ith order statistic. The formula for q_i is

  $q_i = \phi^{-1}\left(\frac{i - \frac{3}{8}}{n + \frac{1}{4}}\right)$ where $\left(\frac{i - \frac{3}{8}}{n + \frac{1}{4}}\right)$ is " continuity correction" or "plotting position".

- Plot the points $(q_i, R_{i:n})$ and inspect linearity for plot. The plot will be a straight line if the data are univariate normal.

in checking the normality of observations of independent data, the graphical method is not adequate to grant decisive evidence that the normality assumption is sustain in certain cases due to personal judgments. To validate the graphical methods, the Shapiro-Wilk is use as a formal normality test before the final remarks with regards to the normality of the data is made.

3.4.3 Shapiro-Wilk Test

The Shapiro-Wilk is the ratio of the best estimator of the variance to the usual corrected sum of squares estimator of the variance (Shapiro and Wilk 1965).The core concept of the test method entails the information imbedded in a normal probability with information that is extracted from the estimator of the sample's standard deviation. The data set of the returns is independent and identically distributed (i.i.d) and might be normal having real unknown mean and standard deviation as a result of the test having a composite hypothesis.

The steps below are the procedures for computing the Shapiro-Wilk test statistic. Let $R_1, R_2, R_3, \ldots R_n$ be n number of sample returns of i.i.d random observations coming from a random ordered variable R with unknown mean $u \in \mathbb{R}$ and unknown variance $\sigma^2 > 0$.

Further, let

$S_n^2 = \sum_{i=1}^{n} \left(R_i - \bar{R} \right)^2$ represent unbiased estimator of $(n-1)\sigma^2$

the test statistic for the hypothesis of normality is

$$W = \frac{\left(a^1 R\right)^2}{S_n^2} = \frac{\left(\sum_{i=1}^{n} a_i R_i \right)^2}{\sum_{i=1}^{n} \left(R_i - \bar{R} \right)^2}$$ where

$a^1 = \left(a_1, a_2, \ldots, a_n\right) = \dfrac{m^1 v^{-1}}{\left(m^1 v^{-1} v^{-1} m\right)^{1/2}}$ is called W statistic.

$m' = (m_1, m_2, ..., m_n)$ denote the vector of anticipated values of the ordered random samples $R_1, R_2, R_3, ... R_n$ and $V = v_{ij}$ corresponding to $n \times n$ covariance matrix.

The test is based on the hypothesis that

H_o: The population has a normal distribution

H_a: The population does not have a normal distribution

The Test statistic satisfies $0 < W \leq 1$. Depending on n, the values of W that are near 1 will not reject the normality hypothesis but for lower W, the null hypothesis will be rejected depending on the p-value. The conventional level of significance for the study is α = 0.05. The study therefore fail to reject the null hypothesis if the p-value is greater than 0.05 or reject otherwise.

3.5 Independence

The Efficient Market Hypothesis contends that asset returns over time should be different from each other because of the random and independent arrival of new information to the market. This new information causes prices to adjust rapidly. Among other statistical test, the run test and autocorrelation test and can be employed to unveil independence in the return series. The study employ Ljung-Box test and the Hurst exponent as discuss below to verify the independence in price returns on selected equities listed on the Ghana Stock Exchange.

3.5.1 Ljung-Box Test for independence

There are quite several numbers of tests for independence or randomness. One popular test method for independence of a data set is the autocorrelation plots which the Ljung-Box itself is grounded on. Rather than testing independence at each different lag, it however test the entire randomness on number of lags specified. It is also refer to as "Portmanteau" test.

The Ljung-Box test hypothesis is stated as follows

H_o : The data are independently distributed

H_a : The data are not independently distributed (they exhibit serial correlation)

the test statistic is

$$Q = n(n+2)\sum_{k=1}^{h}\frac{\hat{p}_k^2}{n-k}$$

where n is the size of sample, $\hat{p}_k$ is the sample's autocorrelation at lag k and h is the number of lags being tested. For significance level α, the critical region of the hypothesis of independence is rejected if

$$Q = \chi^2_{1-\alpha,h}$$

where $\chi^2_{1-\alpha,h}$ is the α- quantile of the Chi-squared distribution with $h$degrees of freedom. The use of *p*-value is considered in the study for the independence of the stock returns.

3.6 Hurst Parameter

When dealing with financial time series, it is always crucial to check whether or not they are predictable before attempting to model and forecast their development. A measure of duration of long range dependence of the stochastic process in the study plays a pivotal role in the area of randomness. Estimation of the Hurst exponent for experimental data is used to check for the independence of time series and to inform on the presence of randomness or long memory (long range correlations) in the series. Initially proposed by Harold Ewin Hurst to establish a principle for regularities of the Nile water level, it currently has applications in medicine and interpreted as a measure for the trendiness in finance. There are several methods for estimating the Hurst exponent such as the variance-time analysis, Detrended Fluctuation Analysis (DFA), R/S-analysis and wavelet-based estimation. The study applied the Rescaled range statistic (R/S statistic) approach to estimate the Hurst exponent. The interval of the partial sums of deviations of the return series from its mean and rescaled by its standard deviation is the underlying idea behind the R/S method. A log-log plot of the number of accumulated series and the R/S statistic outputs a straight line with the gradient being the estimate for the Hurst exponent.

Consider market return series of $R_1, R_2, R_3, ..., R_n$ from the stock Exchange data. First, the scale data is normalized as follows

$$Z_r = (R_r - R_m),\ r = 1, 2, ... n$$ where R_m represent mean

A new series is calculated as follows

$$Y_r = (Z_1 + Z_r),\ r = 2, 3, ... n$$

Since Z (the deviations) has a mean of zero, the final value of Y, Y_n will be zero always. The modified scope will be equal

$$\Re_n = \max(Y_1, ..., Y_n) - \min(Y_1, ..., Y_n)$$

The modified scope $(\Re_n)$ will be always non-negative since Y will have maximum value always equal to zero or more than the minimum value which is equal to zero or less with mean 0. Using the principle of half rule in statistic, Hurst proposed

$\left(\frac{\Re}{S}\right)_n = an^H$ where S is the standard deviation of the return series, $\Re$ is revised range of the series, n is the observed sample number, a is a constant and H is the Hurst exponent to be estimated. Taking logarithm of the above relation results in

$$\log\left(\frac{\Re}{S}\right)_n = \log a + H\log(n)$$

By running a regression, the Hurst exponent (H) can be estimated. The parameter H (0<H<1) is a measure of duration of long-range dependence of the stochastic process. The study estimated the parameter using R package.

If the estimate of the Hurst exponent is equal or near 0.5, it implies that return series have an independent process. If value estimated is less than 0.5, the process is mean-reverting or temporal and subsequently, if the estimated value is greater than 0.5, it indicates the return series or process to have a long range memory.

3.7 Model Specification

The stock price is considered as a random variable since its value changes over time. The price of any stock in a future time is currently unknown at the moment. The GBM stock process developed from the theoretical literature review in equation (2.4) is

$$dP = uPdt + \sigma Pds \qquad 0 \le t \le T$$

where u represents expected rate of return and σ denotes volatility of the stock price. The function G of P and t is from Itô s lemma

$$dG = \left(\frac{dG}{dp} uP + \frac{dG}{dt} + \frac{1}{2} \frac{d^2G}{dp^2} \right) dt + \frac{dG}{dp} \sigma p ds \qquad (3.1)$$

let $G = \ln p$ and substituting the derivatives of $\frac{dG}{dp}, \frac{d^2G}{dp^2}$ and $\frac{dG}{dt}$ into the above equation

$$dG = \left(u - \frac{\sigma^2}{2} \right) dt + \sigma ds$$

but $G = \ln p$

$$d(\ln p) = \left(u - \frac{\sigma^2}{2} \right) dt + \sigma ds$$

integrating from t to $t+1$ for stochastic differential equation above such that $0 \le t \le t+1 \le T$

$$\ln \frac{P_{t+1}}{P_t} = \int_t^{t+1} \left(u - \frac{\sigma^2}{2} \right) dt + \int_t^{t+1} \sigma ds$$

$$\ln \frac{P_{t+1}}{P_t} = \left(u - \frac{\sigma^2}{2} \right) ((t+1) - t) + \sigma (S_{t+1} - S_t)$$

this gives recursive expression for P_{t+1} as

$$P_{t+1} = P_t \exp \left\{ \left(u - \frac{\sigma^2}{2} \right) ((t+1) - t) + \sigma (S_{t+1} - S_t) \right\}$$

assume equal time distance, let $\Delta t = (t+1) - 1$ and $\Delta s = S_{t+1} - S_t$.

also noticed that $\Delta s \sim N(0, \Delta t)$

substitute Δs by $\sqrt{\Delta t}.S_i$ where S_i are independent and follow standard normal random variables for $i = 1, 2, \ldots N-1$. The above model result in

$$P_{t+1} = P_t \exp\left\{\left(u - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}\right\} \qquad (3.2)$$

3.7.1 Expected Value of the Stock Price

It can be shown that taking the expectation of equation (13) results

$$E(p_{t+1}) = p_t \exp\left\{\left(u - \frac{\sigma^2}{2}\right)\Delta t\right\} \qquad (3.3)$$

3.7.2 Variance of the Stock Price

It can been shown also from the second moment and expectation of the stock price in equation (3.2) that the variance is

$$Var(P_{t+1}) = P_t^2 \exp\left(2u + \sigma^2 \Delta t\right).\left(\exp\left(\sigma^{2\Delta t}\right) - 1\right) \qquad (3.4)$$

3.7.3 Confidence Interval of the Stock Price

The (1-α)100% confidence interval is constructed using the expected value of the stock and the standard deviation of the stock price as

$$\left\{ E(P_{t+1}) - Z_{\alpha/2}.std(P_{t+1}),\ E(P_{t+1}) + Z_{\alpha/2}.std(P_{t+1}) \right\}$$

3.7.4 Parameters Estimation

The unknown parameters are estimated by the method of maximum likelihood. This estimation method assumes that the distribution of the observed process is known except for a finite number of parameters which are unknown. The parameters to be estimated are $\theta = (u, \sigma)$ in the process of the GBM. The unknown parameters (u, σ) are estimated by looking at the sample values and choosing our estimates of the unknown parameters for which the probability of getting the sample value is maximum.

After the independence and normality assumption have been satisfied, the log likelihood function is stated as

$$L(\theta) = \sum_{t=1}^{n} \ln\left(f_\theta(R_t)\right) \tag{3.5}$$

where the probability density function is

$$f_\theta(R_t) = \frac{1}{p_t \sigma \sqrt{2\pi(\Delta t)}} \exp\left(\frac{-\left[R_t - \left(u - \frac{1}{2}\sigma^2\right)\Delta t\right]^2}{2\sigma^2 \Delta t} \right) \tag{3.6}$$

The mean and the variance parameters are computed as

$$\hat{m} = \left(\hat{\mu} - \frac{1}{2}\hat{\sigma}^2\right)\Delta t \text{ and variance } \hat{s}^2 = \hat{\sigma}^2 \Delta t \tag{3.7}$$

The parameters of the GBM to be estimated are deduced from the estimates of m and s^2. To obtain an expression for the parameters m and s^2, the derivative of the above density function is taken with respect to these parameters and setting the resulting expressions obtained equal to zero. This results is stated as

$$\hat{m} = \sum_{k=1}^{n} \frac{R_t}{n} \quad \text{and} \quad \hat{s}^2 = \sum_{k=1}^{n} \left(\frac{R_t - \hat{m}}{n} \right)$$

for weekly returns $\Delta t = \frac{1}{52}$ and for the monthly returns $\Delta t = \frac{1}{12}$ and from Equation (3.7) , the estimates for $\hat{u}$ and $\hat{s}^2$ can be gotten.

3.8 Model Testing

The study will test the model by comparing the outcome of the forecasts to the actual prices of the equities observed on the GSE. The study will first estimate a six periods forecast of weekly or monthly price of the equities within our data. The study further used the actual stock price observed in the six period of our forecast as the current price of the stock and again forecast three subsequent periods where a comparison of the forecasted values (expected price) and the actual price of the stock is made. The study will also observe the fitness of actual price of the equities selected within our confidence interval.

3.8.1 Mean Square Error of Prediction

The study evaluates the performance of the model by computing the Mean Square Error (MSE) together with how best the actual stock price for the various periods fit in the estimated 95% confidence interval. The MSE is a measure of how neighboring a forecast moves away for the actual. The lesser the MSE value in terms of percentage usually below 10%, the closer the fit of the time series data to the model.

The formula below is used to compute the MSE as

$$MSE = \frac{\sum_{t=1}^{N} \left(P_t - \hat{P}_t \right)^2}{N}$$

where P_t denotes the price of the stock at time t and $\hat{P}_t$ is the estimated price at time.

CHAPTER 4

DATA ANALYSIS AND INTERPRETATION

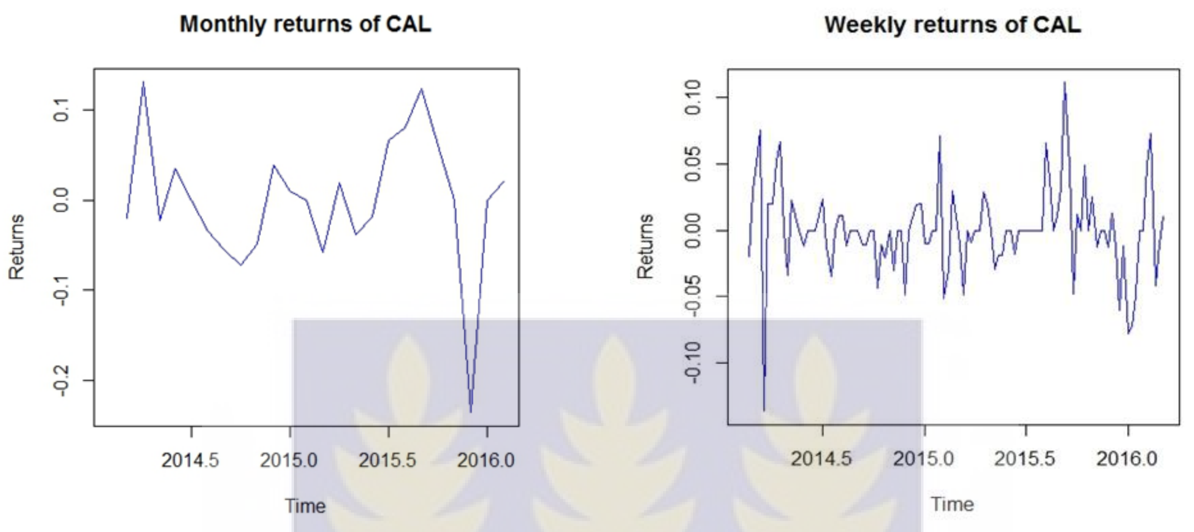
4.0 Introduction

To ascertain whether or not the Geometric Brownian motion may be used as a model for stock prices on the Ghana Stock Exchange, the assumptions underlying the process is studied. The study checks the data for stationarity, normality and the independence of each security selected.

The study first and foremost test for stationarity in the return series data using the sequence plots together with the Dickey-Fuller test. The study also examines the normality of the data set by employing the histogram, normal quantile-quantile plot (graphical method) and Shapiro-Wilk test method. The study further checks the independence behavior through the Ljung-Box statistic test of autocorrelations at a significance level of 5% for the first order autocorrelation. The Hurst exponent considered in the methodology is used in order to buttress our analysis when making the final decision. R console version Rx64 3.2.5 is used to obtain the various graphs, all statistical test and computations because of its interactive environment for data manipulation, statistical graphs and analysis that includes the creation of specific functions when needed.

4.1 Preliminary Analysis

The study choose the CAL equity and perform all the test methods on its weekly and monthly returns series as describe in the methodology and later extended the test methods to the other equities selected from the GSE. To investigate the presence of unit root or not, the weekly and monthly returns time series of the returns is plotted.



(a) (b)

Figure 4.1: Time series plot of weekly and monthly stock returns for CAL

The weekly time plot of the CAL equity shows in figure 4.1 (a) a kind of periodic time series with upward or downward fluctuations. The vertical changes of the series differ from one portion of the series to the other side indicating that the mean and variance are not constant but are changing with respect to time and therefore it is assumed to be not stationary.

Similarly, the monthly sequence plot for CAL equity appears in figure 4.1 (b) too have upward or downward trend which is also characterized by a non-constant mean and variance with respect to the time, thus an indication of a non-stationarity of the returns.

To confirm the presence of the unit root or non-stationarity of the returns, the Augmented Dickey-Fuller (ADF) test was performed.

Table 4.1: Augmented Dickey-Fuller test for Weekly and Monthly returns of CAL.

Test (Dickey-Fuller)	Value	*p*-value
Weekly	-3.3619	0.06455
Monthly	-3.5095	0.06257

The *p*-value for the Augmented Dickey-Fuller Test in both the weekly and monthly returns of the CAL equity is greater than the 0.05 level of significance hence the study fail to reject the null hypothesis of unit root (non-stationarity) at 5% level of significance and thus conclude that the both weekly and monthly returns of CAL are non-stationary over the period.

The study then tested the normality assumptions through the histogram. The graph of the histogram is displayed in the figure below.

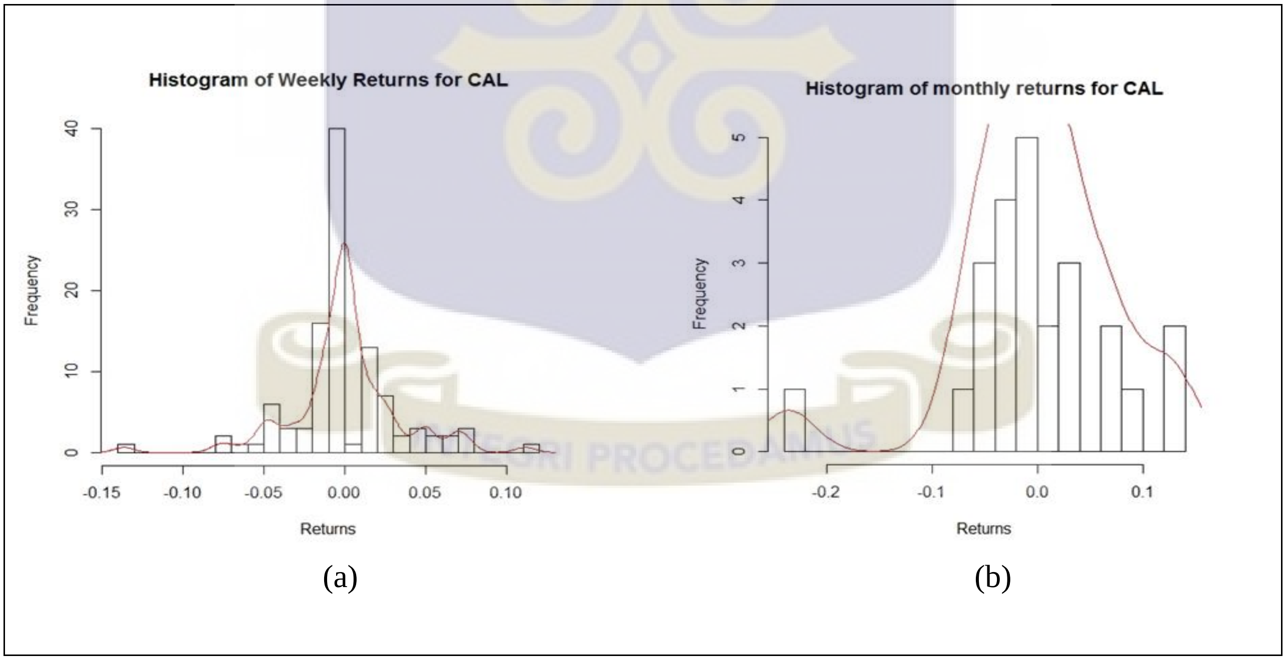


Figure 4.1.2: Histogram plot of weekly and monthly returns for CAL

The histogram for the weekly returns for CAL in Figure 4.2 (a) does not exhibit the bell shape, not symmetrical and data point are not equally distributed around the middle, thus the study assumes that weekly returns of CAL is not normally distributed. The monthly returns of CAL in Figure 4.2 (b) appear to have data point skewed to the right. Though a bell shape is formed on the extreme bars, it does not appear to be perfect. The study therefore has doubt of normality for the monthly returns of CAL. To confirm our assumption, the study employed the Q-Q plot as displayed in the figure below.

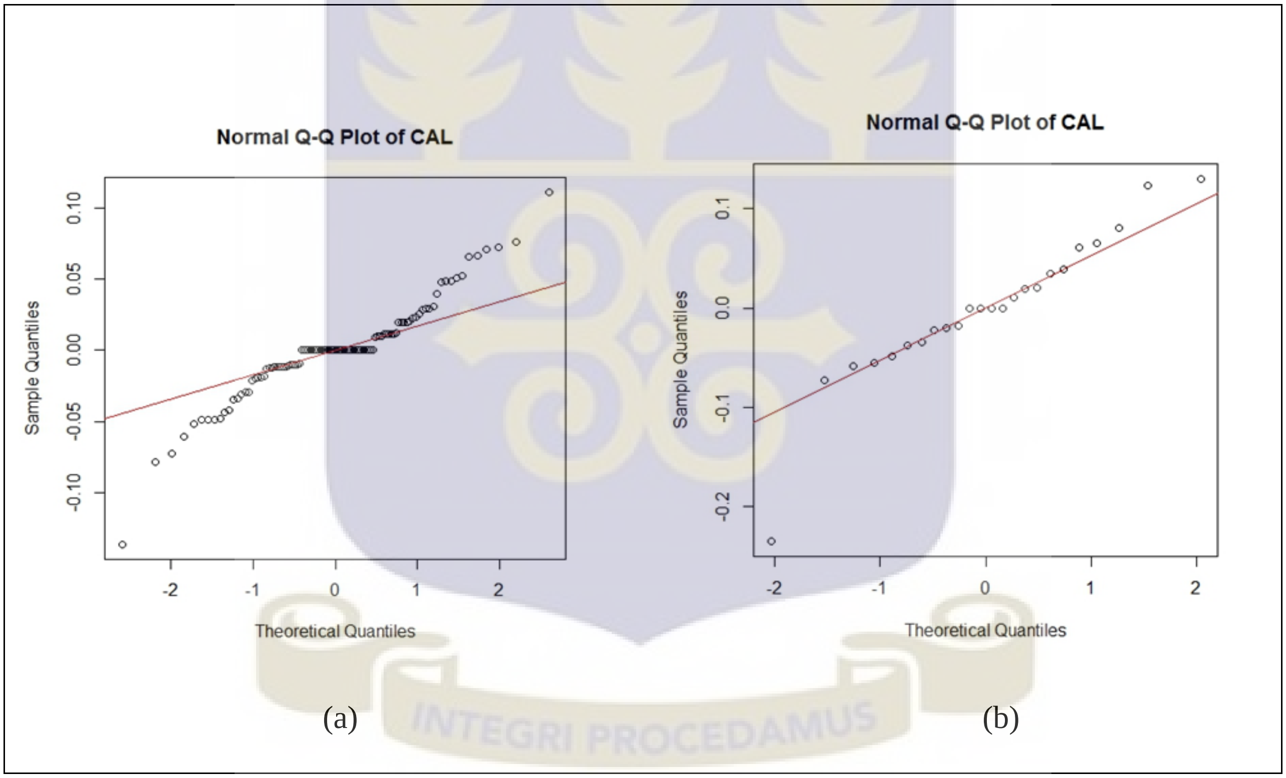


Figure 4.1.3: Q-Q plot for weekly and monthly returns for CAL

The weekly Q-Q plot for the returns of CAL displayed in figure 4.3 (a) shows data points not linear and moving away from the straight line. This is an indication that, the weekly returns does not follow the normal distribution. The study tends to agree with the result in figure 4.2(a) that the weekly returns of CAL are not normally distributed. The monthly Q-Q plot in figure 4.3 (b) shows most of its point close to the straight line and the middle of the plot looks like what would have been expected from a normal distribution data but the upper tail have some points move away from the line. Again our doubt of the normality of the monthly returns of CAL remains. To confirm that the returns follow the normal distribution, the study further performed the Shapiro-Wilk Test.

Table 4.1.2: Weekly and monthly results for Shapiro-Wilk Test for CAL

Test (Shapiro-Wilk)	**Value**	***P*-value**
Weekly	0.91776	5.644e-06
Monthly	0.90826	0.03233

As the Shapiro-Wilk test statistic for the weekly returns of CAL is 0.91776 and it's corresponding *p*-value is less than the 0.05 alpha level chosen, the study reject the null hypothesis and conclude that our weekly returns of CAL equity is not normally distributed. Similarly, having a Shapiro-Wilk test statistic for the monthly returns of CAL as 0.90826 and its corresponding *p*-value of 0.03233 which is also less than our 0.05 significance level, the study again reject the null hypothesis of normality and conclude that our monthly returns of CAL is also not normally distributed.

To check for independence in the return series, the study perform the Ljung-Box test with lag order of 1 to verify if the rate of returns on a week or month at time (t) to check if it correlates with the previous rate of return of the week or month at time ($t-1$). The results for the CAL equity is showed

Table 4.1.3: Weekly and monthly results of Ljung –Box Test for CAL

Test (Ljung-Box)	Value	*P*-value
Weekly	2.4655	0.1164
Monthly	0.8484	0.3570

As the computed *p*-value of 0.1164 in the weekly returns of CAL from Table 4.1.3 is above the 0.05 significance level chosen, the study therefore fail to reject the null hypothesis that the weekly returns is independent of the previous week returns and also the monthly returns for CAL has *p*-value of 0.357 which is more than 0.05 significance level, the study similarly fail to reject the null hypothesis that the monthly returns is independent of the previous month return.

To conclude on which equities follows a random process, the study estimate the Hurst exponent for both the weekly and monthly returns of CAL. The result is displayed below

Table 4.1.4: Hurst Exponent estimate for CAL

Security	Weekly	Monthly
CAL	0.4956917	0.5906153

A value of 0.4956917 from Table 4.1.4 is found to be the weekly estimate for the Hurst exponent. Since the value is close to 0.5, there is an assumption of independence from the Ljung-Box test, the study concludes that the weekly return of CAL follow a random walk. Consequently the Hurst exponent of the monthly returns of CAL has an estimate of 0.5906153 which is greater than the 0.5 significance of the random variation. The study therefore fails to accept that the monthly returns of CAL follow a random walk even though the Ljung-Box test in Table 4.3 showed independence.

4.2 Other Equities

The weekly and monthly logarithm returns of all the other selected equities that are listed on the GSE were all subjected to a similar test and analysis as CAL. The visual or sequence plots, histograms and Q-Q plot are displayed at the Appendix.

Table 4.2.1: Augmented Dickey Fuller test for weekly returns on other Equities

Equity	Value	*P*-value
BOPP	-3.4632	0.04875
EBG	-3.6649	0.03068
EGL	-5.3766	0.01000
FML	-3.3009	0.07471***
GCB	-3.9212	0.01566
GGBL	-3.5897	0.03742
GOIL	-5.0006	0.01000
HFC	-3.7900	0.02202
SCB	-3.2602	0.08149***
SIC	-5.0952	0.01000
TLW	-5.1855	0.01000
TOTAL	-3.9717	0.01321
UNIL	-4.0346	0.01016
UTB	-4.4100	0.01000

As computed in Table 4.2.1, the *p*-value of BOPP, EBG, EGL, GGBL, GOIL, HFC, SIC, TLW, UNIL, UTB are lower than the significance level (α=0.05) hence the study reject the null hypothesis that the series has a unit root in the weekly returns of these equities and therefore confirms stationarity of the weekly returns of the equities listed. However, the *p*-values of FML and SCB are higher than the significance level of 0.05 hence the study fail to reject the null hypothesis and conclude that FML and SCB weekly returns series has a unit root or is non-stationary.

Table 4.2.2: Augmented Dickey Fuller of monthly returns on other equities

Equity	Value	*P*-value
BOPP	-4.0424	0.02178
EBG	-2.8352	0.25420***
EGL	-1.6231	0.71600***
FML	-2.3903	0.42370***
GCB	-4.2176	0.01567
GGBL	-2.7409	0.29010***
GOIL	-3.2046	0.11350***
HFC	-2.4592	0.39750***
SCB	-2.6655	0.31890***
SIC	-1.5085	0.75960***
TLW	-3.9845	0.02380
TOTAL	-2.993	0.19410***
UNIL	-1.8382	0.63400***
UTB	-2.7873	0.27240***

The study fail to reject the assumption of the presence of a unit root in the monthly return series of EBG, EGL, FML, GGBL, GOIL, HFC, SCB, SIC, TOTAL, UNIL and UTB since their computed *p*-values shown in Table 4.2.2 is greater than the 5% significance level chosen by the study, nevertheless the *p*-values of the monthly returns of BOPP, GCB and TLW are less than 0.05 hence their returns for the month confirms stationarity.

Table 4.2.3: Shapiro-Wilk test for weekly returns of other equities

Equity	Value	*P*-value
BOPP	0.75346	4.088e-12
EBG	0.81410	2.701e-10
EGL	0.69867	1.641e-13
FML	0.53818	<2.2e-16
GCB	0.76555	8.868e-12
GGBL	0.58894	7.696e-16
GOIL	0.71122	3.297e-13
HFC	0.83833	1.863e-09
SCB	0.75406	4.245e-12
SIC	0.88374	1.223e-07
TLW	0.29662	<2.2e-16
TOTAL	0.46973	<2.2e-16
UNIL	0.55948	<2.2e-16
UTB	0.92594	1.608e-05

Since Table 4.2.3 shows that the computed *p*-values of all the selected equities are less than the 5% level of significance, the study rejects the null hypothesis of normality and conclude that the weekly returns of the equities are not normally distributed. There is not a perfect bell shaped in the weekly returns histograms and the data points emanating from the Q-Q plots are not linear as shown in the Appendix and therefore Shapiro-Wilk test further confirms our results.

Table 4.2.4: Shapiro-Wilk test for Monthly returns for other equities

Equity	Value	*P*-value
BOPP	0.95285	0.3120000***
EBG	0.97393	0.7635000***
EGL	0.90051	0.0220700
FML	0.71604	1.653e-05
GCB	0.92764	0.0863300***
GGBL	0.77297	0.0001099
GOIL	0.76642	8.737e-0.5
HFC	0.97824	0.8614000***
SCB	0.87726	0.0073240
SIC	0.94989	0.2695000***
TLW	0.64486	2.039e-06
TOTAL	0.64161	1.865e-06
UNIL	0.82887	0.0009083
UTB	0.89838	0.0198900

The monthly returns of some of the equities exhibited the characteristics of the normal distribution in the graphical procedures. The *p*-value of BOPP, EBG, GCB, HFC and SIC are greater than the 5% level of significance and therefore the study fail to reject the null hypothesis of normality which clarifies the doubts in the graphs. It can also be seen from the results on the Table 4.2.4 that the *p*-value of EGL, FML, GGBL, GOIL, SCB, TLW, TOTAL, UNIL and UTB are less than the significance level (α=0.05) hence the study rejects the null hypothesis of their monthly return series. There is also clear evidence from the graphical procedures shown in the Appendix that, there is not a perfect bell shaped formed in some of the monthly returns histograms and the shaped formed are also not symmetrical with data point equally distributed around the middle of the graph. Some return series also move away from the straight line in the monthly Q-Q plot shown in the Appendix which is not a characteristic of a normally distributed data points.

Table 4.2.5: Ljung-Box Test of independence of weekly returns for other equities

Equity	Value	*P*-value
BOPP	5.111400	0.023770***
EBG	2.078700	0.149400***
EGL	0.644620	0.422000***
FML	3.033400	0.081570***
GCB	1.997500	0.157600***
GGBL	10.974000	0.000924
GOIL	3.306000	0.069030***
HFC	1.322300	0.250200***
SCB	3.867800	0.049220
SIC	7.549200	0.006004
TLW	0.001202	0.972300***
TOTAL	0.546960	0.459600***
UNIL	0.352890	0.552500***
UTB	0.045452	0.831200***

From the result displayed in the Table 4.3.1, the computed *p*-value of the weekly returns of most of the selected equities are greater than the chosen significance level (α=0.05) and therefore the study fail to reject the null hypothesis of independence. The study concludes that the returns of these equities are independent on the previous week returns. Nevertheless, GGBL, SCB, SIC have *p*-value less than the 5% significance level, hence the study fail to reject the null hypothesis and conclude that there exist a serial correlation in the weekly returns of the three equities.

Table 4.2.6: Ljung-Box Test for independence of monthly data

Equity	Value	*P*-value
BOPP	2.4765	0.1156 ***
EBG	0.0223	0.8813 ***
EGL	1.9476	0.1629 ***
FML	1.3532	0.2447 ***
GCB	0.9554	0.3284 ***
GGBL	1.9379	0.1639 ***
GOIL	0.2738	0.6008 ***
HFC	2.0623	0.1510 ***
SCB	5.2706	0.0217
SIC	0.1341	0.7143 ***
TLW	0.1183	0.7309 ***
TOTAL	0.1224	0.7264 ***
UNIL	0.3529	0.5525 ***
UTB	0.0271	0.8692 ***

The computed *p*-value in Table 4.3.2 of almost all the monthly returns of the equities are greater than the chosen significance level (α=0.05) except SCB, the study therefore fail to reject the null hypothesis of independence for all the equities except of SCB which have a *p*-value less than α (5%) thus the study reject the null hypothesis of independence for SCB equity. It subsequently implies that there is correlation in the monthly returns of SCB.

Table 4.2.7: The Hurst exponent estimation

Equity	Weekly	Monthly
BOPP	0.6342395	0.5906153
EBG	0.6354775	0.6354775
EGL	0.5548624	0.5548624
FML	0.6315649	0.5530174
GCB	0.5538641	0.4813197 ***
GGBL	0.5975904	0.5621294
GOIL	0.5974339	0.4976206 ***
HFC	0.5736665	0.5873312
SCB	0.5957307	0.5778077
SIC	0.5974339	0.5663617
TLW	0.5056434 ***	0.4606062 ***
TOTAL	0.5052311 ***	0.4652753 ***
UNIL	0.6499680	0.6546573
UTB	0.5515071	0.4872178 ***

The estimates of the weekly returns of the Hurst exponent in Table 4.2.7 indicates that only two of the selected equities follow a random walk process which have their Hurst exponent equal or close to 0.5 (TLW and TOTAL) . The remaining weekly estimates for the Hurst exponent are greater than the H = 0.5 which indicates that those series exhibit long dependence.

The monthly returns of the equities in the same Table 4.2.7 shows five equities having their Hurst exponent equal or close to 0.5 upon approximation which signify the random walk process. Apart from GCB, GOIL, TLW, TOTAL and UTB , all the monthly returns of the other equities exhibit long range dependency since their Hurst parameter is greater than 0.5. The study summarizes findings about the assumptions underlying the Geometric Brownian motion process of the selected equities trading on the Ghana Stock Exchange.

Table 4.2.8.a: Summary results of selected Equities

Equity	Weekly	Monthly
BOPP	Stationary	Stationary
	Not Normally distributed	Normally distributed
	Independent	Independent
CAL	Non-stationary	Non-stationary
	Not Normally distributed	Not Normally distributed
	Independent	Independent
EBG	Stationary	Not-stationary
	Not Normally distributed	Normally distributed
	Independent	Independent
EGL	Stationary	Not-stationary
	Not Normal distributed	Not Normally distributed
	Independent	Independent
FML	Non-stationary	Not-stationary
	Not Normal distributed	Not Normal distributed
	Independent	Independent
GCB	Stationary	Stationary
	Not Normal distributed	Normal distributed
	Independent	Independent
GGBL	Stationary	Not-stationary
	Not Normal distributed	Not Normal distributed
	Independence	Independent
GOIL	Stationary	Not-stationary
	Not Normally distributed	Not Normally distributed
	Independent	Independent
HFC	Stationary	Not-stationary
	Not Normally distributed	Not Normally distributed
	Independent	Independent
SCB	Non-Stationary	Not-stationary
	Not Normally distributed	Not Normally distributed
	Not independent	Not Independent

Table 4.2.8.b: Summary results of selected Equities

Equity	Weekly	Monthly
SIC	Stationary	Not - stationary
	Not Normally distributed	Normally distributed
	Not independent	Independent
TLW	Stationary	Stationary
	Not Normally distributed	Not Normally distributed
	Independent	Independent
TOTAL	Stationary	Not - stationary
	Not Normally distributed	Not Normally distributed
	Independent	Independent
UNIL	Stationary	Not – stationary
	Not Normally distributed	Not Normally distributed
	Independent	Not independent
UTB	Stationary	Not stationary
	Not Normally distributed	Not Normally distributed
	Independent	Independent

From the summary in Table 4.2.8, the weekly returns of all the equities are not normally distributed which is contrary to the Geometric Brownian Motion assumption thereby modeling the weekly returns by the Geometric Brownian motion will not be appropriate. Among the selected equities from the exchange, the monthly returns of BOPP and GCB satisfy the Geometric Brownian Motion assumption. However, the returns of BOPP has Hurst exponent greater than 0.5. The value observed of the Hurst exponent estimate implies that the returns or prices of most of the equities can best be modeled by the fractal Geometric Brownian Motion since long memory is dominant.

4.3 Modeling of stock prices

The study proceeds to model the monthly prices of the Equities that satisfy the GBM conditions. Specifically, the study considers GCB, GOIL, TLW, TOTAL and UTB.

The parameter estimates of the stochastic model as proposed in the literature were estimated for each monthly return of the equity stated above.

Table 4.3.1: Parameter estimates

Equity	μ	σ
GCB	0.08499719	0.2646442
GOIL	-0.26525730	0.3090071
TLW	0.06631433	0.1683587
TOTAL	0.03200082	0.1770301
UTB	0.80071650	0.4410409

The estimates of the parameters in Table 4.3.1 show that GOIL has a negative value. Realization of a positive value of the rate of return (μ) signifies increase in profit and higher volatility (σ) implies that the share price fluctuates rapidly. UTB has the higher return and also a higher volatility despite a low share price compared to the other four.

Closing price at 31/08/2015 was used as the current price and the expected price forecast is made for the last trading day in Feb, 2016. The results is shown below

Table 4.3.2 : Summary of forecast for Equities

Equity	Expected Price	CI		Actual Price
		Lower	Upper	
GCB	4.20	2.65	5.76	3.71
GOIL	1.66	0.94	2.38	1.44
TLW	34.00	26.04	41.96	27.93
TOTAL	5.49	4.14	6.84	5.12
UTB	0.15	0.06	0.24	0.12

The expected price, confidence limits and the actual stock price displayed in the Table (4.3.2) shows that the prices forecasted are close to the actual prices realized in 6 months period. The actual prices also fall in the 95% confidence interval. UTB forecast price is the closest to the actual price compared.

The study further made forecast of three subsequent months and compare values of forecast with the actuals of all the five equities as shown below. Closing price at 29/02/2016 was used as the current price and the expected price forecasts is made for the last trading day in March, April and May, 2016

Table 4.4.1: Three months forecast of GCB

Month	Expected price	CI		Actual price
		Lower	Upper	
MARCH	3.71	3.16	4.27	3.65
APRIL	3.72	2.93	4.50	3.03
MAY	3.72	2.60	4.84	3.02

It can be seen from Table 4.4.1 that the monthly expected price of GCB forecasted did not change significantly. The forecast for March is close to the actual whilst that of April and May deviate slightly. However, the actual prices observed for the various months lies in our estimated confidence interval range.

Table 4.4.2: Three months forecast of GOIL

Month	Expected price	CI		Actual price
		Lower	Upper	
MARCH	1.41	1.16	1.72	1.45
APRIL	1.38	1.04	1.72	1.51
MAY	1.35	0.94	1.76	1.37

The expected price of GOIL in Table 4.4.2 indicates a steady decrease of the equity. Even though the forecast is close to the actual price, the GOIL equity experienced an increment in April and regress to 1.37 as compared to our predicted 1.35. Similarly the actual price observed for the various month fit in the confidence interval.

Table 4.4.3: Three months forecast of TLW

Month	Expected Price	CI		Actual Price
		Lower	Upper	
MARCH	28.08	25.41	30.76	27.92
APRIL	28.24	24.43	32.05	27.92
MAY	28.40	23.10	33.09	27.92

The forecasts displayed in Table 4.4.3 of TLW shows slight increments of the equity but our observed actual price for the various months did not change. Both the forecast and the actual price are relatively close with the actual price observed lying in our confidence range too.

Table 4.4.4: Three months forecast for TOTAL

MONTH	Expected Price	CI		Actual Price
		Lower	Upper	
MARCH	5.13	4.62	5.65	5.10
APRIL	5.15	4.42	5.88	4.90
MAY	5.16	4.26	6.06	4.08

The forecast prices in Table 4.4.5 shows that the closing price in March is close to the actual price whereas that of April and May deviate slightly from the actuals. Nonetheless our predicted confidence interval captured all the actual price of the various months.

Table 4.4.5: Three months forecast for UTB

MONTH	Expected Price	CI Lower	CI Upper	Actual Price
MARCH	0.13	0.10	0.16	0.11
APRIL	0.14	0.09	0.19	0.09
MAY	0.15	0.08	0.21	0.08

Similarly, the expected price displayed in the Table 4.4.6 as the March equity price close to the actual while that of April and May deviate slightly, again all the actual price for the forecasted months fall within the confidence interval.

4.4 Mean Square Error of prediction

The Mean Square Error of prediction showed that UTB as 2.6%, GOIL as 6.3%, TLW, GCB and TOTAL produced 11.9%, 32.3% and 41.0% respectively. The MSE less than 10% indicate the appropriateness of the model of those equities and the confidence interval fitness justify even those with higher MSE.

Table 4.5 Forecast for monthly returns of the Equities

Equity	Month	Expected Price	CI	
			Lower	Upper
	July	3.06	2.41	3.71
GCB	August	3.08	2.28	3.89
	September	3.11	2.17	4.04
	July	1.31	0.99	1.64
GOIL	August	1.28	0.89	1.67
	September	1.25	0.81	1.70
	July	28.23	24.42	32.04
TLW	August	28.39	23.69	33.10
	September	28.54	23.09	34.00
	July	4.10	3.52	4.68
TOTAL	August	4.11	3.40	4.83
	September	4.12	3.30	4.95
	July	0.09	0.06	0.12
UTB	August	0.10	0.05	0.14
	September	0.10	0.05	0.16

From Table 4.5, the study expects the price of GCB equity to appreciate in capital gains with the price not exceeding 4.04 in the next four months. It is also expected that the price of GOIL will drop slightly but the price wouldn't go down beyond 0.81 and even in case of any increment, the price will not exceed 1.70 from its current price in the next four months. The study expects the price of TLW to appreciate slightly with some probability to lie within 23.09 to 34.00 in the next four months. The study further envisages the price of TOTAL not to exceed 4.95 in the next four months even though it will appreciate by a very small margin from its current price. Finally, it is expected of UTB equity to also appreciate by 0.01 in the end of July and 0.02 in both August and September with the actual price lying within 0.05 to 0.16 in the next four months.

CHAPTER 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Introduction

The chapter presents a summary of the findings from the study, conclusions and recommendations as well as the areas for future research. Consequently, the main results and findings on the model used in the study are presented. To satisfy the objectives, the study employed some statistical test methods to check for a unit root as a characteristic of stock returns as indicated by some researchers and most importantly the critical assumptions of normality and independence of the process upon which the Geometric Brownian Motion model is built.

5.1 Summary

The presence of a variable mean and variance of the equity returns under the study was investigated using graphs and the ADF test. It was realized that a few weekly returns of three (3) equities out of the fifteen selected equities are non-stationary or has a unit root in the returns. However, the monthly returns of the same equities exhibited the presence of a unit root of twelve (12) out of fifteen (15) selected equities being non-stationary.

Aside the histogram and the Q-Q plots, the Shapiro-Wilk test for normality showed that none of the weekly returns of the selected equities follow a normal distribution. The graph of the histogram failed to produce the perfect bell-shaped "normal curve" and the data points of the entire weekly returns move away from the straight line in the Q-Q plots. It was realized that five (5) of the monthly returns of the selected equities are normally distributed having their *p*-value greater than the chosen α significance level associated with the Shapiro-Wilk test and their Q-Q plots in the appendix show their data points to closely lie on a straight line with just a few at the ends scattered.

To test for the independence in the weekly and monthly returns, the Ljung-Box test was used with a lag of 1. It showed that three (3) out of fifteen (15) weekly equities returns selected are not independent. Similarly the study realized that the monthly returns of the selected equities are all independent except one.

To confirm the use of the Geometric Brownian Motion to model the equities which satisfy all the GBM properties, the study used the Hurst exponent estimate. The estimates showed that two (2) of the selected weekly equities returns follow the Geometric Brownian process or random walk having their weekly returns of the Hurst exponent estimate close to 0.5 and five(5) of the monthly equities returns was detected to have their Hurst exponents close to 0.5 upon approximation, hence follow the Geometric Brownian motion process or random walk.

The study further focused on the five monthly returns of the equities that follow the GBM and estimated their parameters through the maximum likelihood estimation. The parameters estimates were used in a proposed model in the literature to forecast the expected value of the stock price in three reserve months. A comparison of our forecast prices and the actual observed prices showed a close relation of values though some deviate slightly from the actuals. However all the actual prices of the equities observed from the stock market fall within our 95 % confidence interval estimates.

The study tested the accuracy of the model by checking from the mean square error between the expected stock price value and the actual stock price value. Among the five equities estimated, GOIL and UTB have their MSE less than 10% whilst GCB,TLW and TOTAL have their MSE greater than 10%.

5.2 Conclusions

The objective of the study was to ascertain whether the Geometric Brownian motion can be used as a model on the GSE. The study analyzed the equities returns on the Ghanaian financial market for 15 listed companies on the Exchange and studied the underlying assumptions of the GBM process namely; stationarity, normality and independence.

The study showed that the monthly returns of the equities have of a unit root which is consistent with other researchers. Also none of the weekly returns of the equities was normally distributed unlike the monthly equities return which has a few being normally distributed. Finally, all the monthly equities returns are independent of the previous month return except one whereas the weekly equities return has three of the weekly returns dependent on the other weeks return. The study conclude that the monthly equities returns behavior is a better data set to draw inference and make valid conclusions from them than the weekly equities returns series of the GSE market and also confirm that the Ghana stock market is efficient due to the independence nature of both weekly and monthly returns from previous ones.

The study also showed that at least one assumption among the three assumptions of the GBM was violated except the monthly returns of BOPP and GCB. The study concludes that the behavior of the GSE market returns through the assumptions underlying the GBM process hold for some equities. In support of our conclusion, our findings with the use of the Hurst exponent estimates showed that seven (7) return series namely; the monthly return of GCB, GOIL and

UTB while both the weekly and monthly return of TLW and TOTAL can be modeled by the Geometric Brownian motion.

The study further showed that, the expected stock prices predicted from the model from the reserved data set are close when compared to the actual price values even though some deviate slightly but all the actual price observed on the market for the various months lie in the 95% confidence interval. The study conclude that the model performs well since the actual price observed lies in the 95% confidence interval and some of equities prices estimated have MSE is less 10%.

The study therefore concludes that investors can use the GBM process to help make decision and predict the stock prices on the Ghana Stock Exchange.

5.3 Recommendations

The study recommends that an attempt be made to extend or refine the model since the GBM might not be a proper method for some of the equities behavior listed on the GSE and therefore recommend that long memory assumption should be added to the assumptions underlying the price variation on the Ghana financial market.

The study also recommends the use of the monthly prices of returns as a data set for other researchers instead of the weekly returns since it will give a better statistical inference compared to the weekly returns for the reason of the normality assumption.

It is recommended to future researchers that since the parameters of the expected return and volatility are not constant, other stochastic models can be investigated to unveil their underlying behavior of returns on the GSE.

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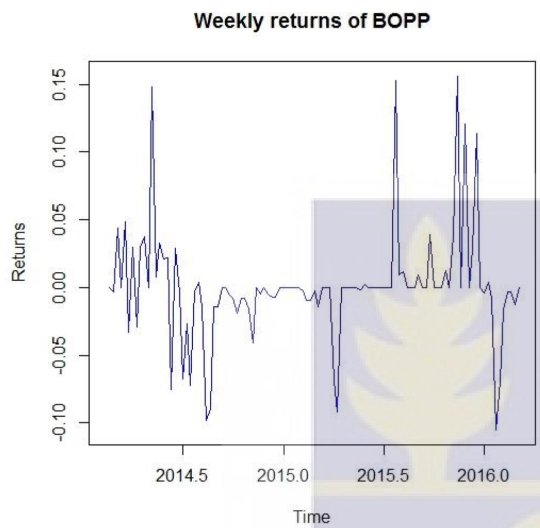
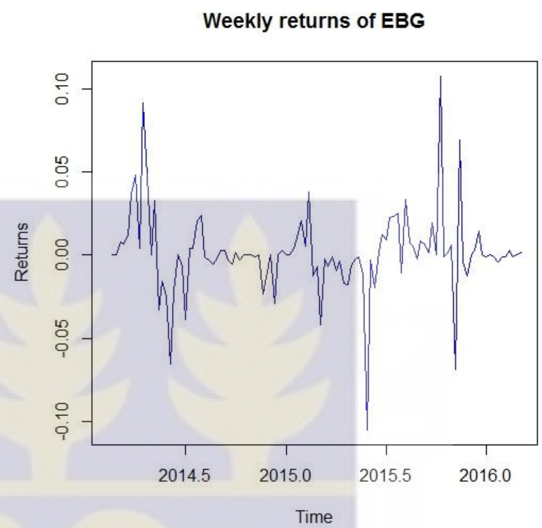
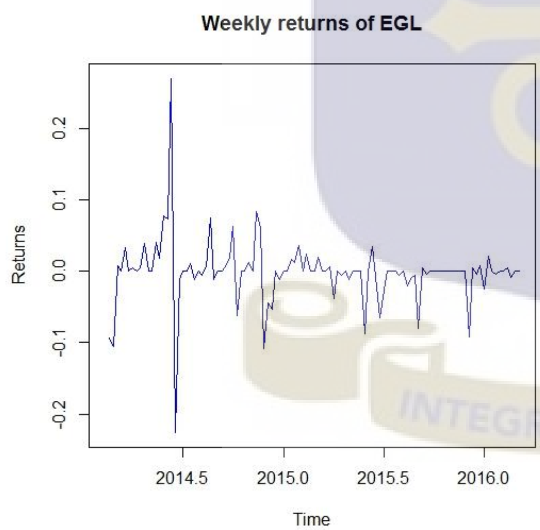
APPENDICES

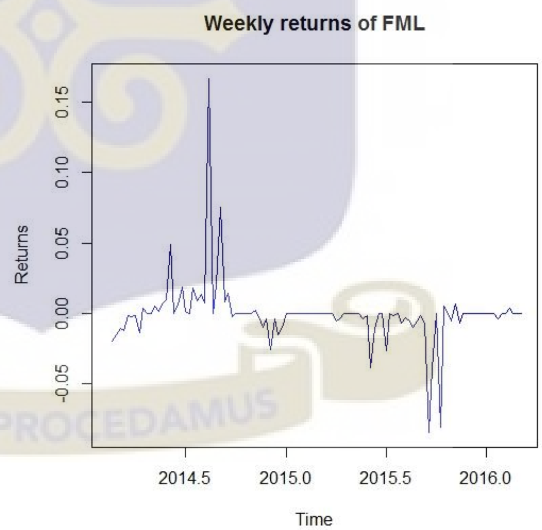
APPENDIX A

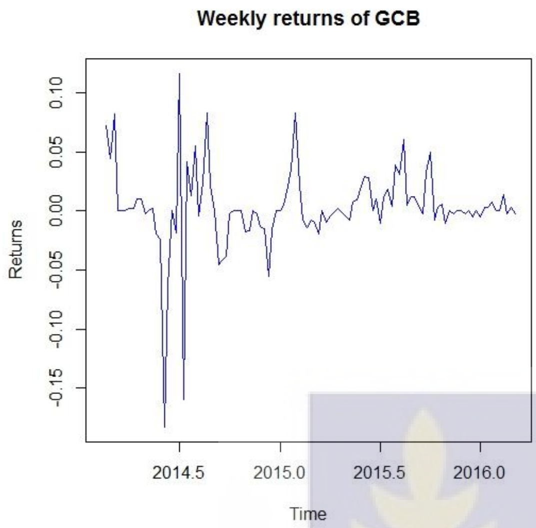
Visual plots of weekly returns of selected equities

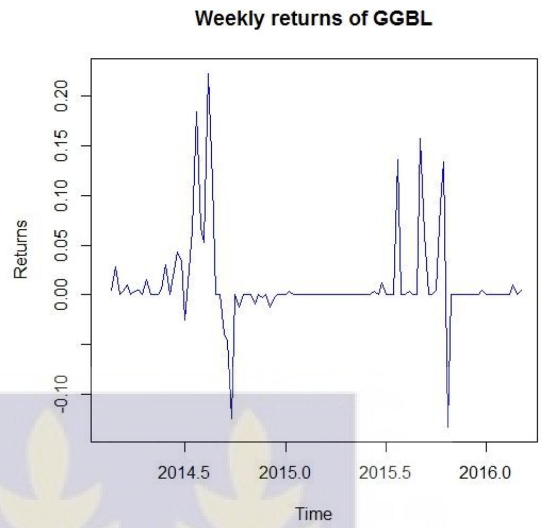


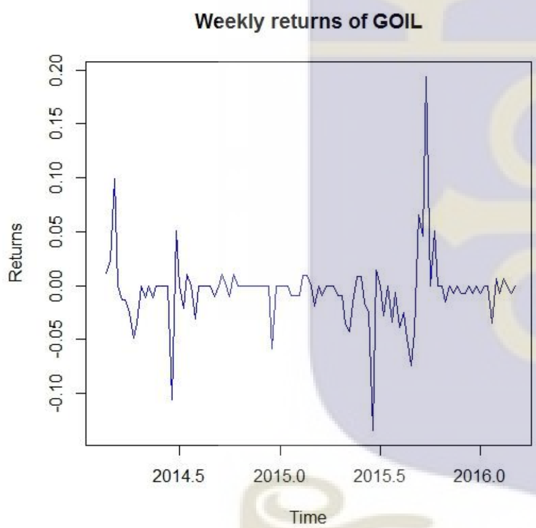


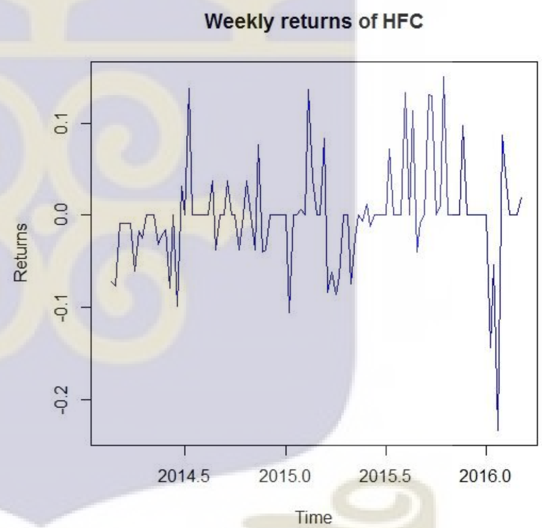


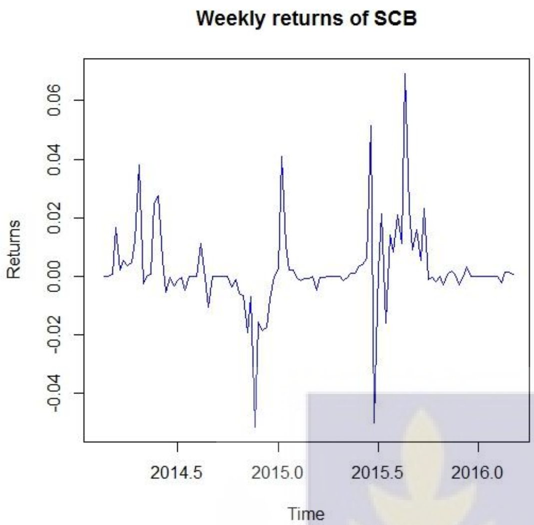


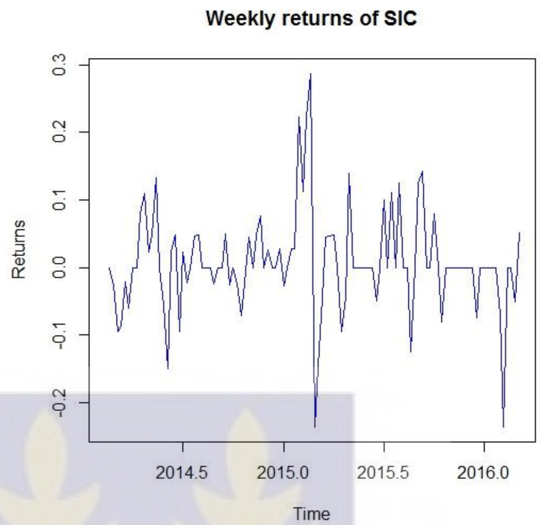

Weekly returns of GCB
Returns
Time
0.10
0.05
0.00
-0.05
-0.10
-0.15
2014.5
2015.0
2015.5
2016.0

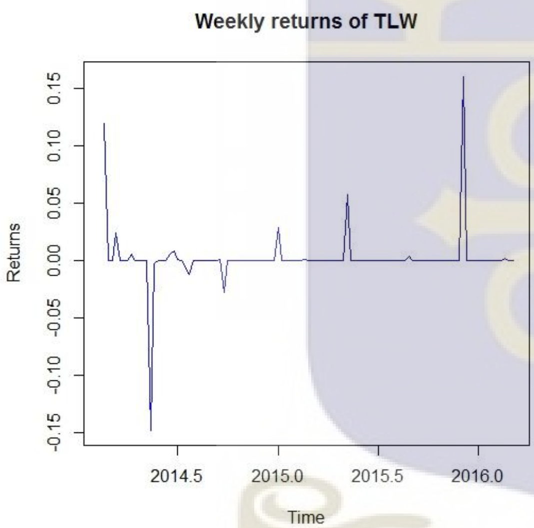

Weekly returns of GGBL
Returns
Time
0.20
0.15
0.10
0.05
0.00
-0.10
2014.5
2015.0
2015.5
2016.0

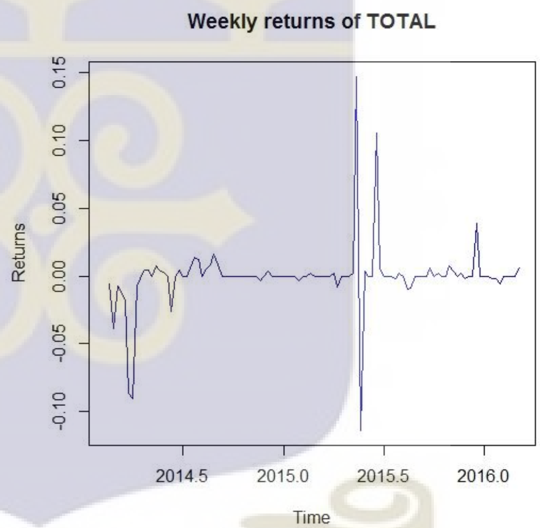

Weekly returns of GOIL
Returns
Time
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
2014.5
2015.0
2015.5
2016.0

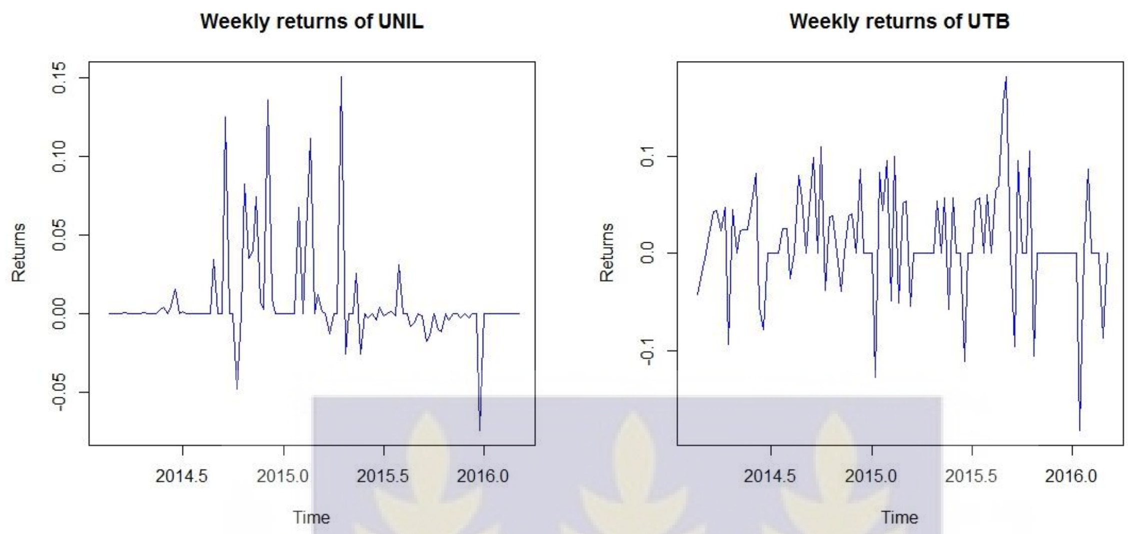

Weekly returns of HFC
Returns
Time
0.1
0.0
-0.1
-0.2
2014.5
2015.0
2015.5
2016.0


Weekly returns of SCB
Returns
Time
0.06
0.04
0.02
0.00
-0.02
-0.04
2014.5
2015.0
2015.5
2016.0


Weekly returns of SIC
Returns
Time
0.3
0.2
0.1
0.0
-0.1
-0.2
2014.5
2015.0
2015.5
2016.0

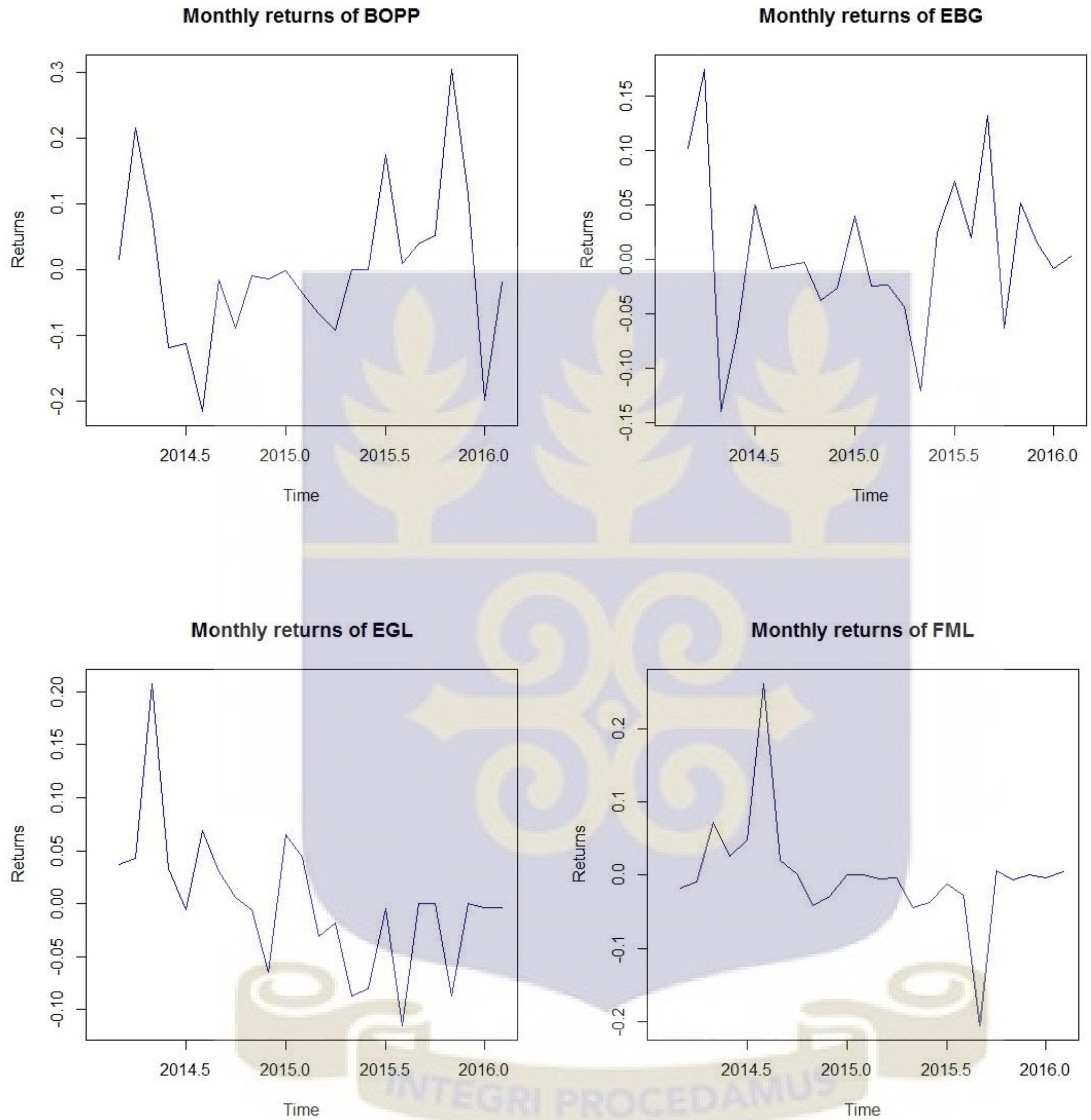

Weekly returns of TLW
Returns
Time
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
2014.5
2015.0
2015.5
2016.0

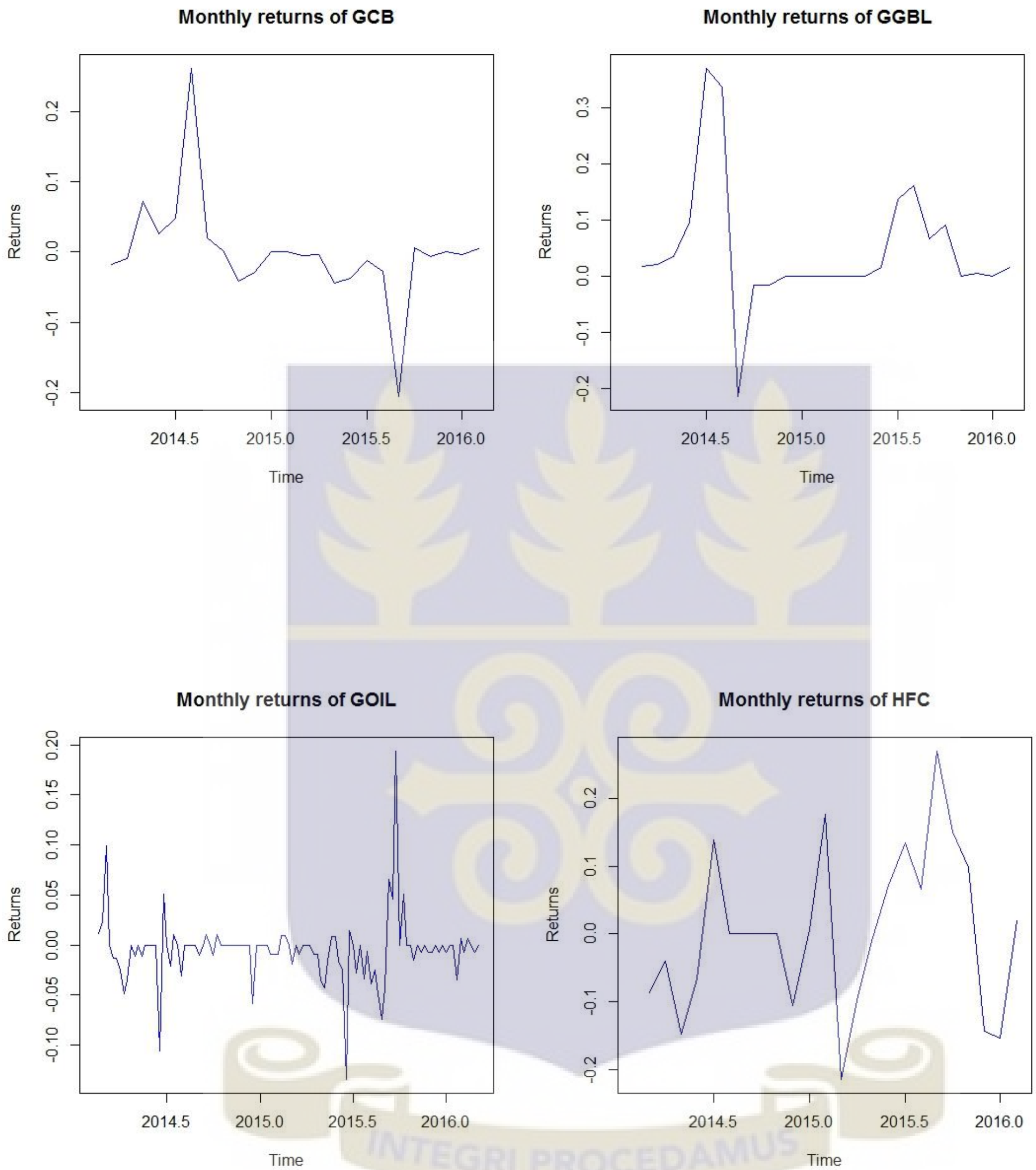

Weekly returns of TOTAL
Returns
Time
0.15
0.10
0.05
0.00
-0.05
-0.10
2014.5
2015.0
2015.5
2016.0

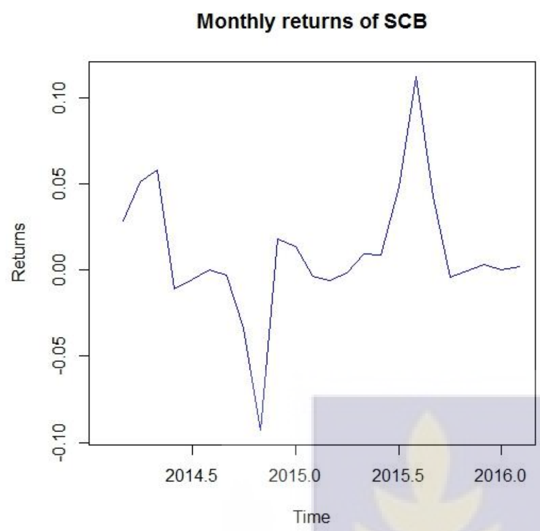

Weekly returns of UNIL
Returns
0.15
0.10
0.05
0.00
-0.05
2014.5
2015.0
2015.5
2016.0
Time
Weekly returns of UTB
Returns
0.1
0.0
-0.1
2014.5
2015.0
2015.5
2016.0
Time

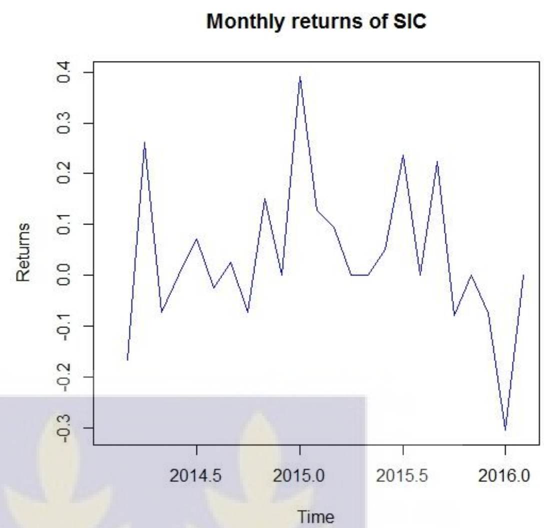
APPENDIX B

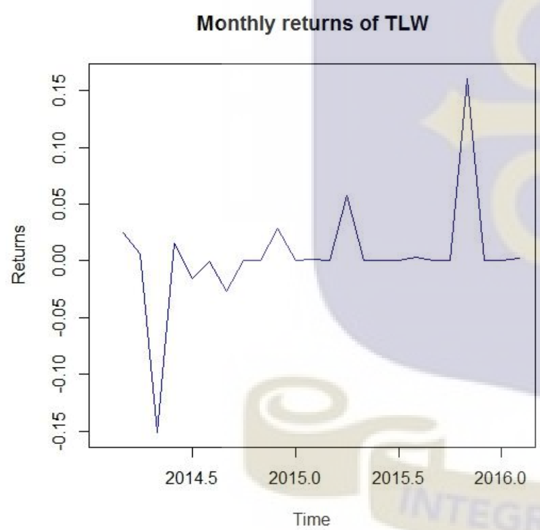
Visual plots of monthly returns of selected equities

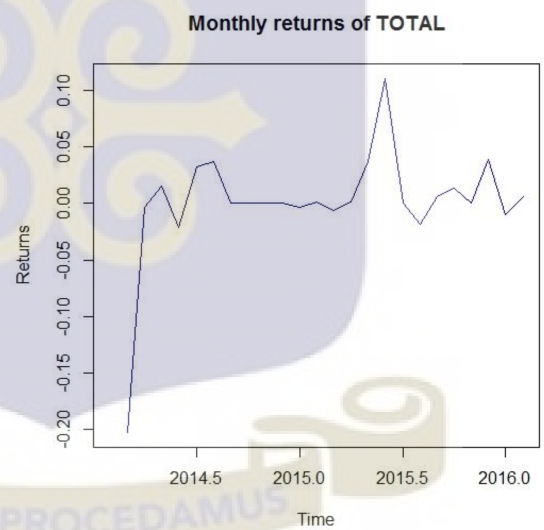


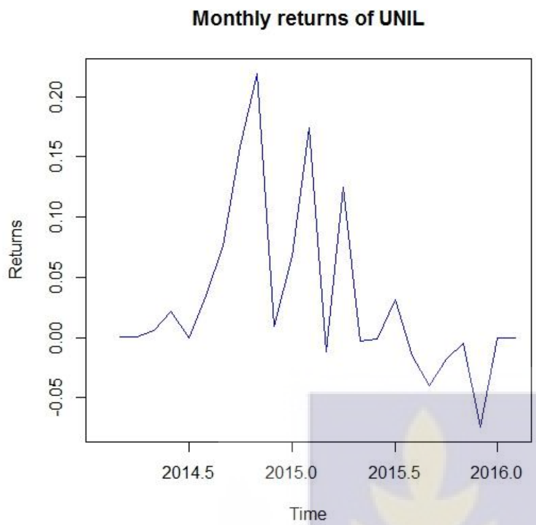

Monthly returns of GCB
Returns
Time
Monthly returns of GGBL
Returns
Time
Monthly returns of GOIL
Returns
Time
Monthly returns of HFC
Returns
Time

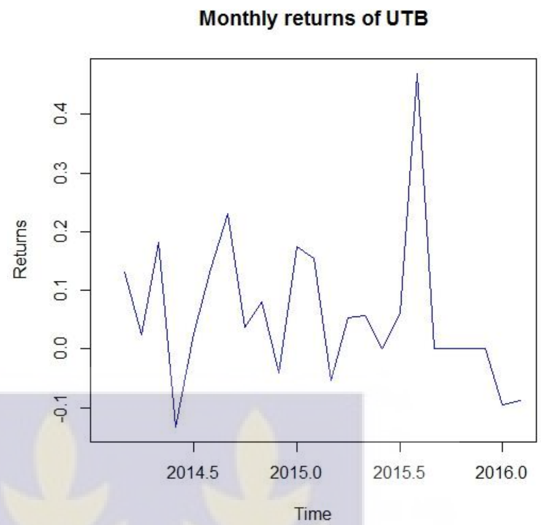

Monthly returns of SCB
Returns
0.10
0.05
0.00
-0.05
-0.10
2014.5
2015.0
2015.5
2016.0
Time


Monthly returns of SIC
Returns
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
2014.5
2015.0
2015.5
2016.0
Time


Monthly returns of TLW
Returns
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
2014.5
2015.0
2015.5
2016.0
Time

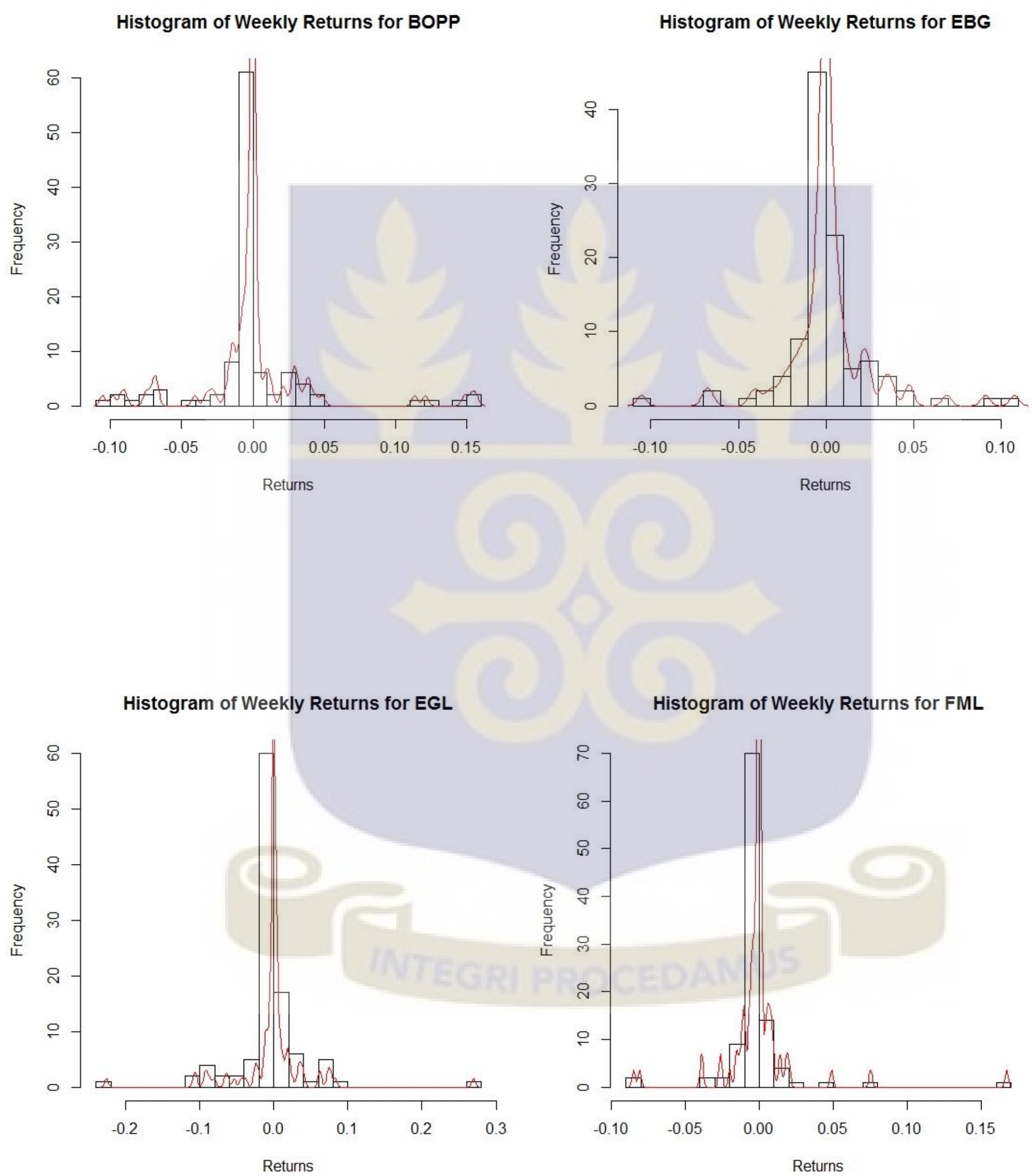

Monthly returns of TOTAL
Returns
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
2014.5
2015.0
2015.5
2016.0
Time

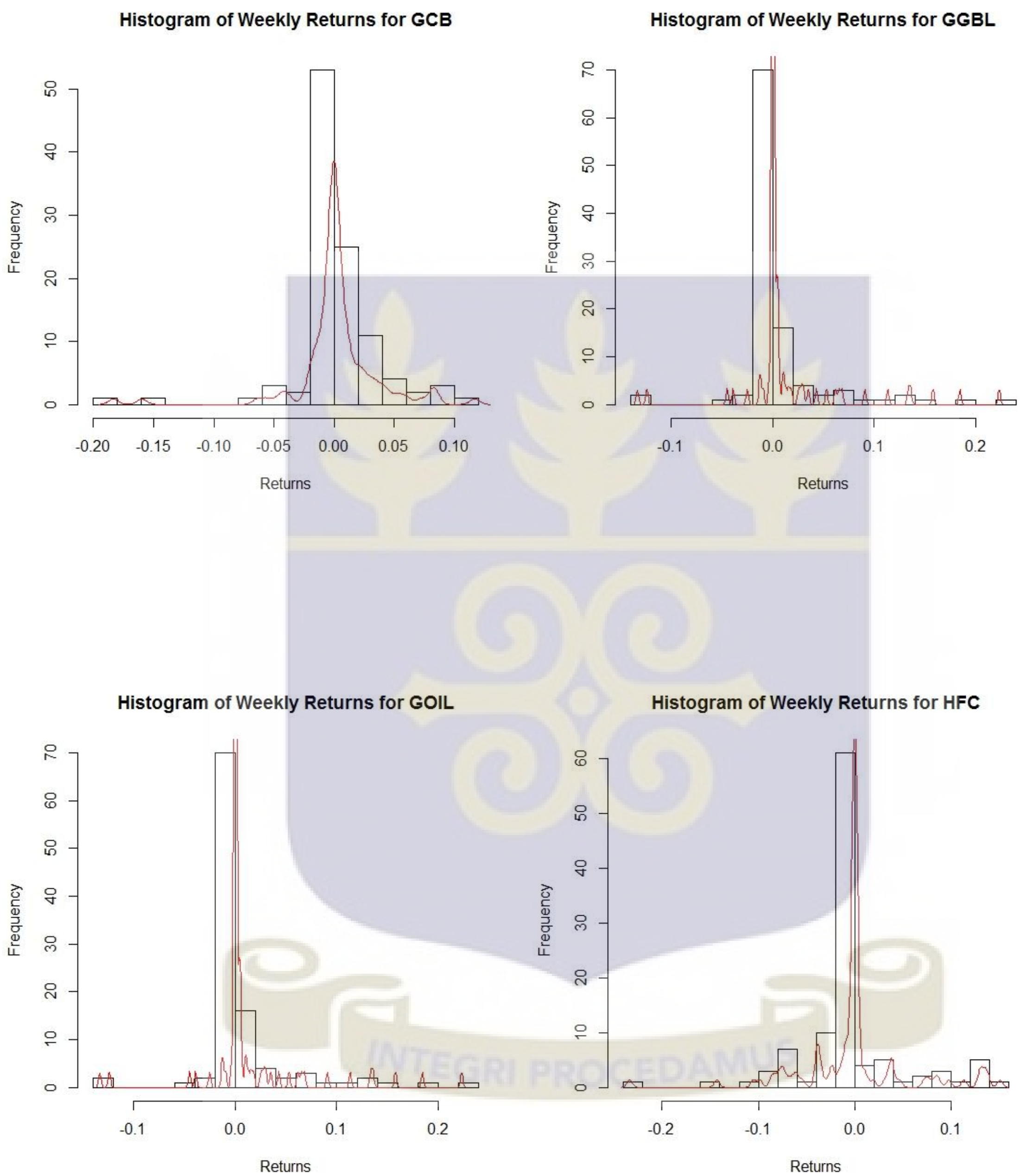

Monthly returns of UNIL
Returns
0.20
0.15
0.10
0.05
0.00
-0.05
2014.5
2015.0
2015.5
2016.0
Time

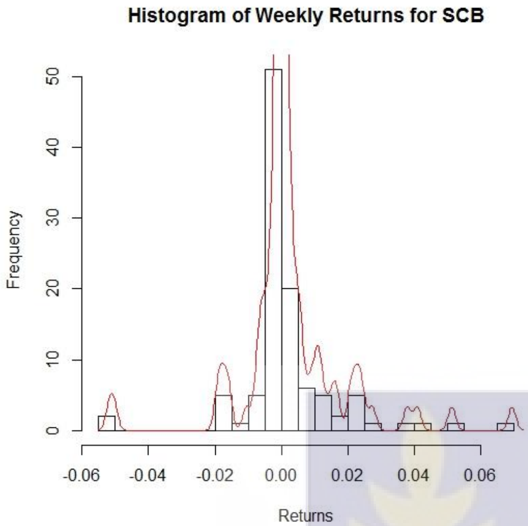

Monthly returns of UTB
Returns
0.4
0.3
0.2
0.1
0.0
-0.1
2014.5
2015.0
2015.5
2016.0
Time

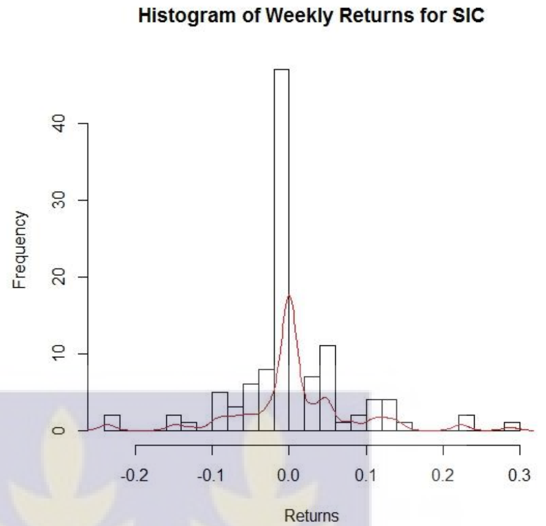
APPENDIX C

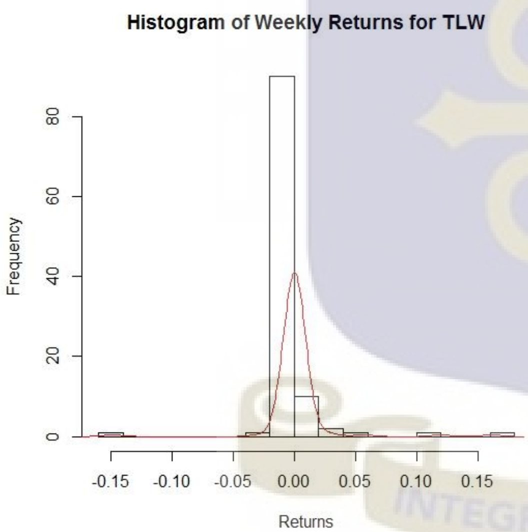
Histogram of weekly returns plots for selected equities

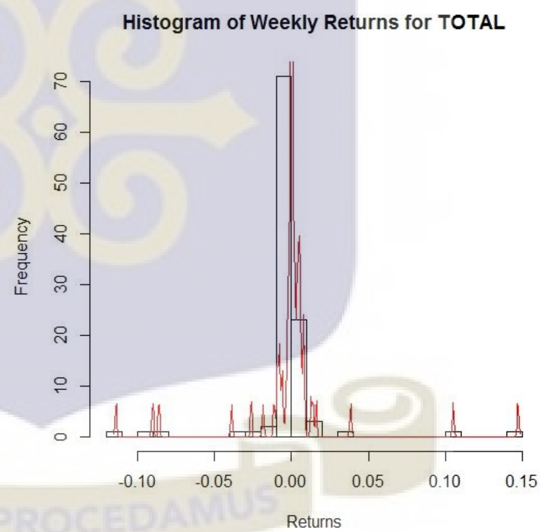


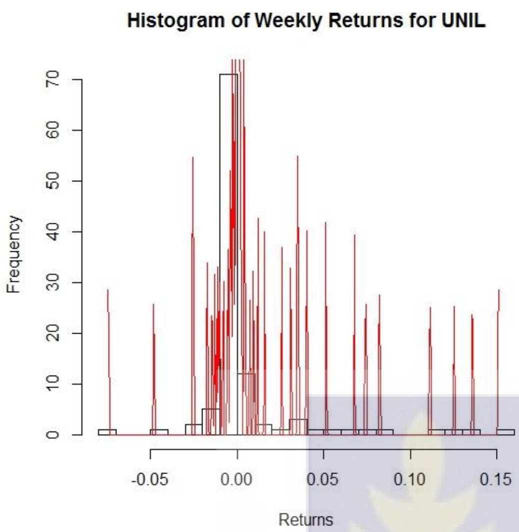

Histogram of Weekly Returns for GCB
Frequency
Returns
Histogram of Weekly Returns for GGBL
Frequency
Returns
Histogram of Weekly Returns for GOIL
Frequency
Returns
Histogram of Weekly Returns for HFC
Frequency
Returns

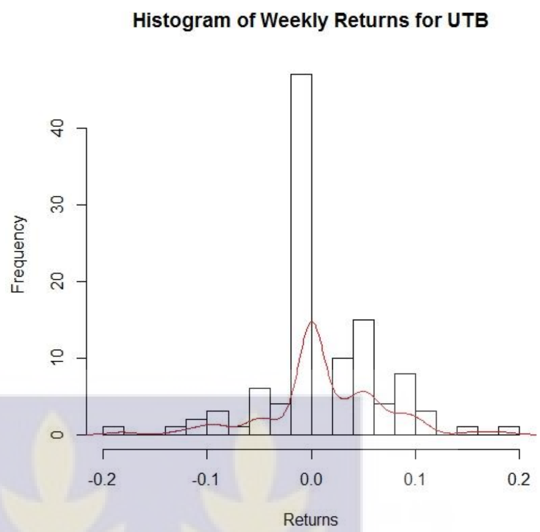

Histogram of Weekly Returns for SCB
Frequency
0
10
20
30
40
50
-0.06
-0.04
-0.02
0.00
0.02
0.04
0.06
Returns


Histogram of Weekly Returns for SIC
Frequency
0
10
20
30
40
-0.2
-0.1
0.0
0.1
0.2
0.3
Returns


Histogram of Weekly Returns for TLW
Frequency
0
20
40
60
80
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
Returns

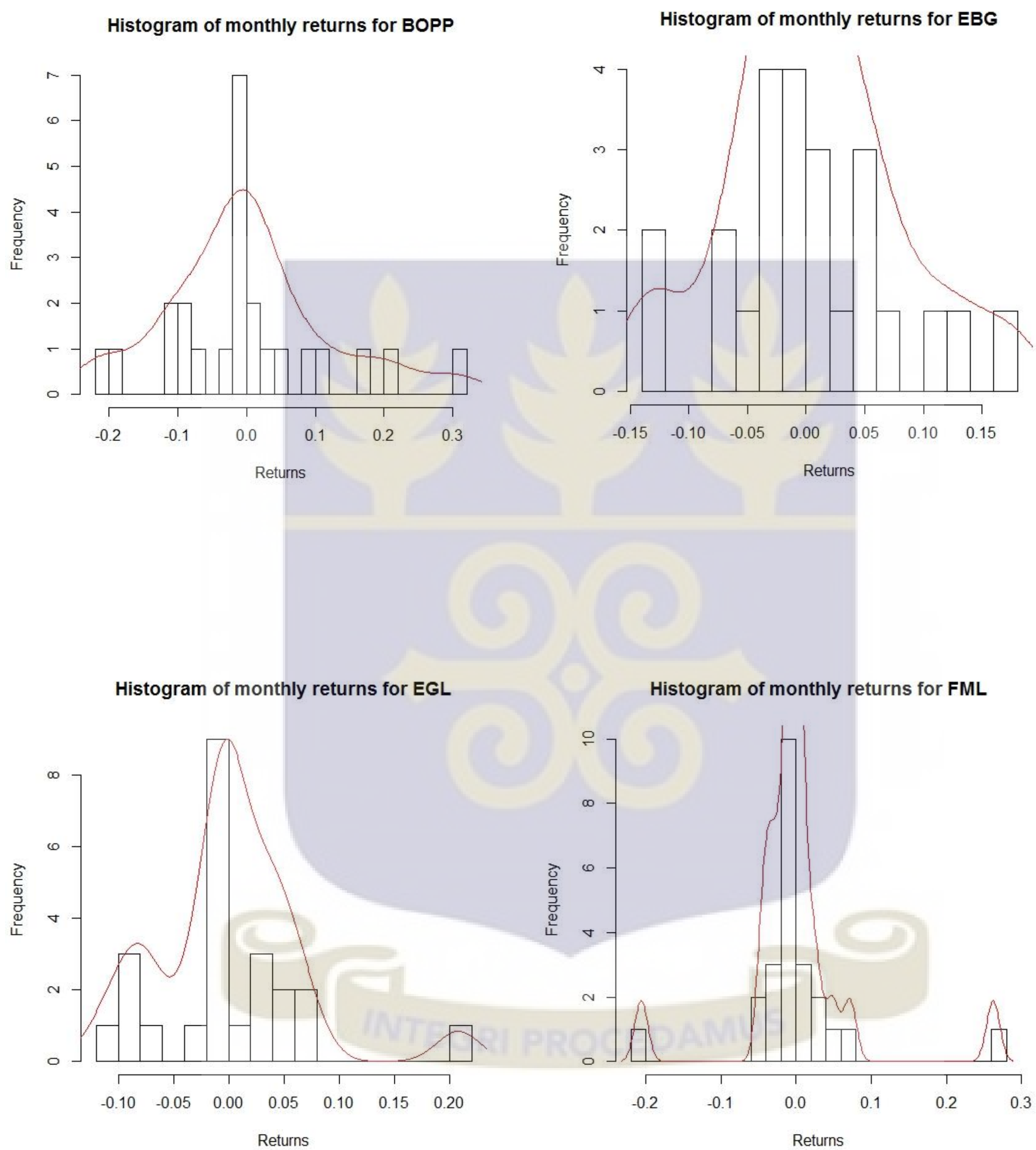

Histogram of Weekly Returns for TOTAL
Frequency
0
10
20
30
40
50
60
70
-0.10
-0.05
0.00
0.05
0.10
0.15
Returns

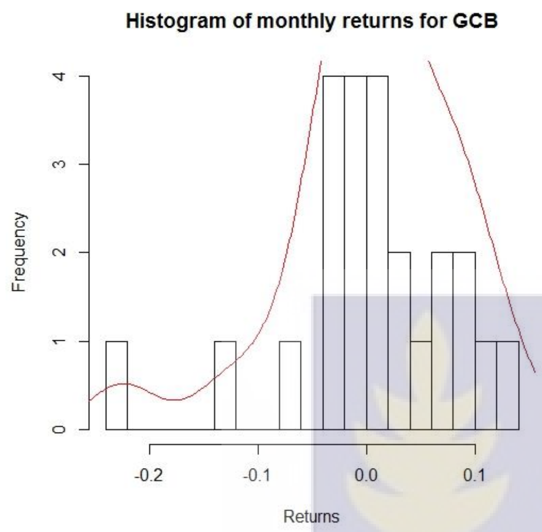

Histogram of Weekly Returns for UNIL
Frequency
0
10
20
30
40
50
60
70
-0.05
0.00
0.05
0.10
0.15
Returns

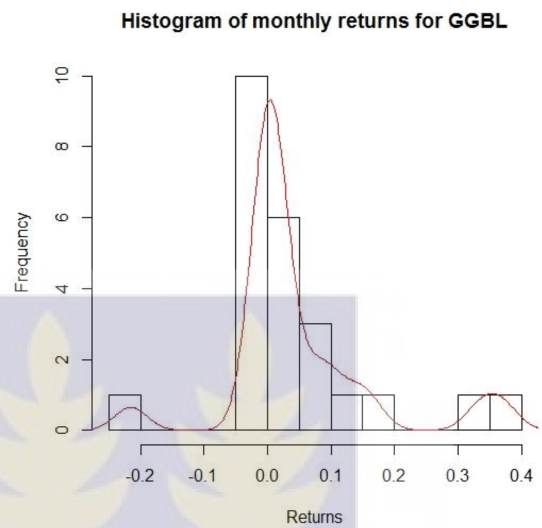

Histogram of Weekly Returns for UTB
Frequency
0
10
20
30
40
-0.2
-0.1
0.0
0.1
0.2
Returns

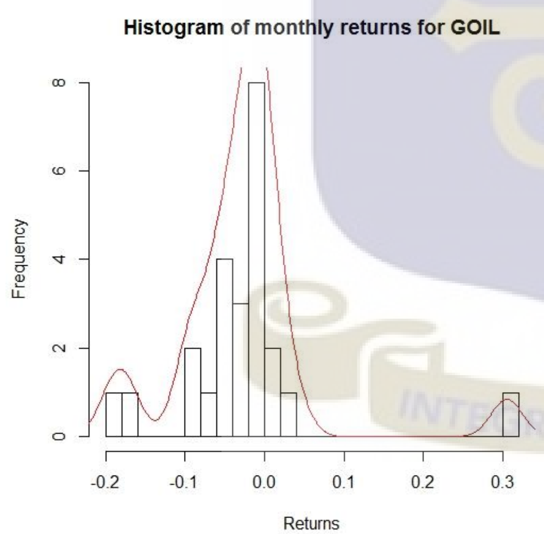
APPENDIX D

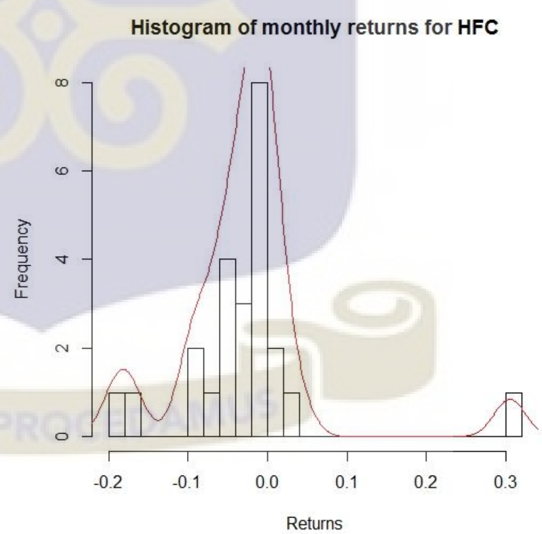
Histogram of monthly returns plots for selected equities

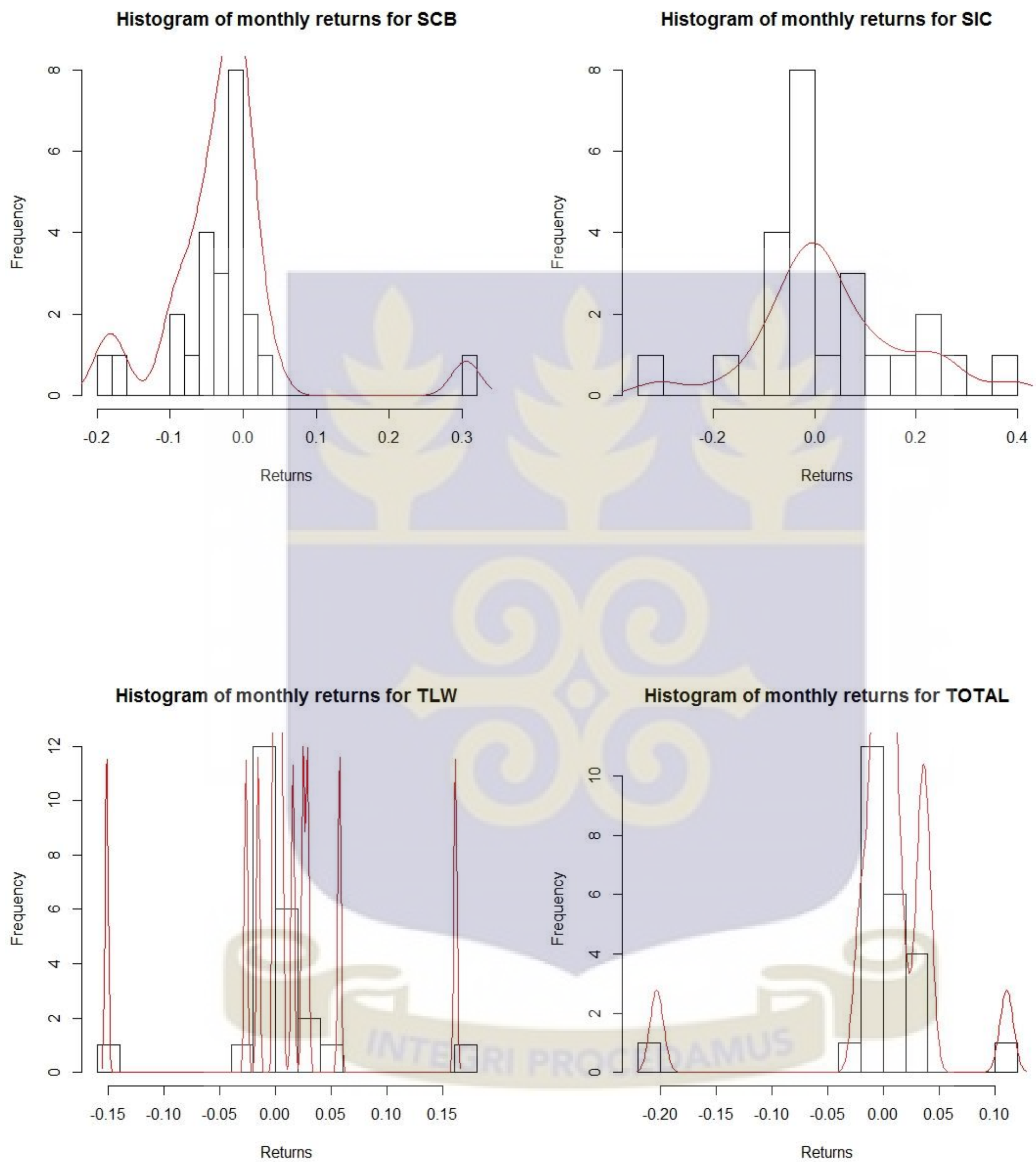


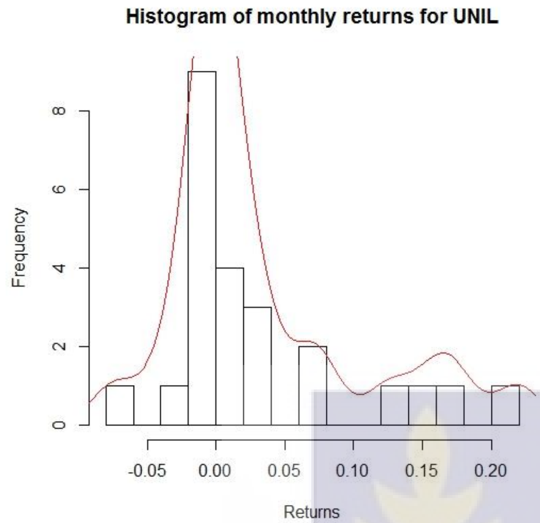

Histogram of monthly returns for GCB
Frequency
Returns
-0.2
-0.1
0.0
0.1

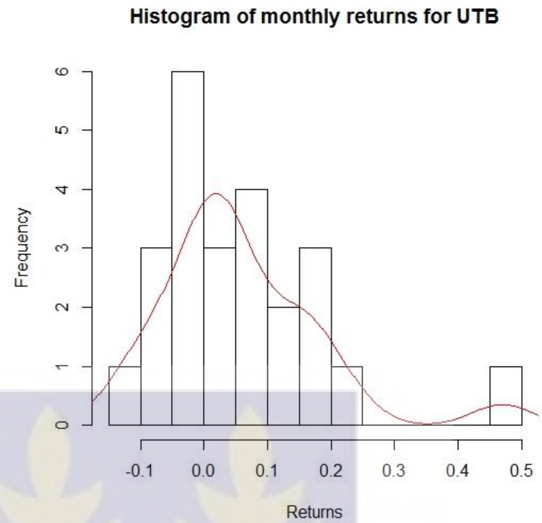

Histogram of monthly returns for GGBL
Frequency
Returns
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4


Histogram of monthly returns for GOIL
Frequency
Returns
-0.2
-0.1
0.0
0.1
0.2
0.3


Histogram of monthly returns for HFC
Frequency
Returns
-0.2
-0.1
0.0
0.1
0.2
0.3

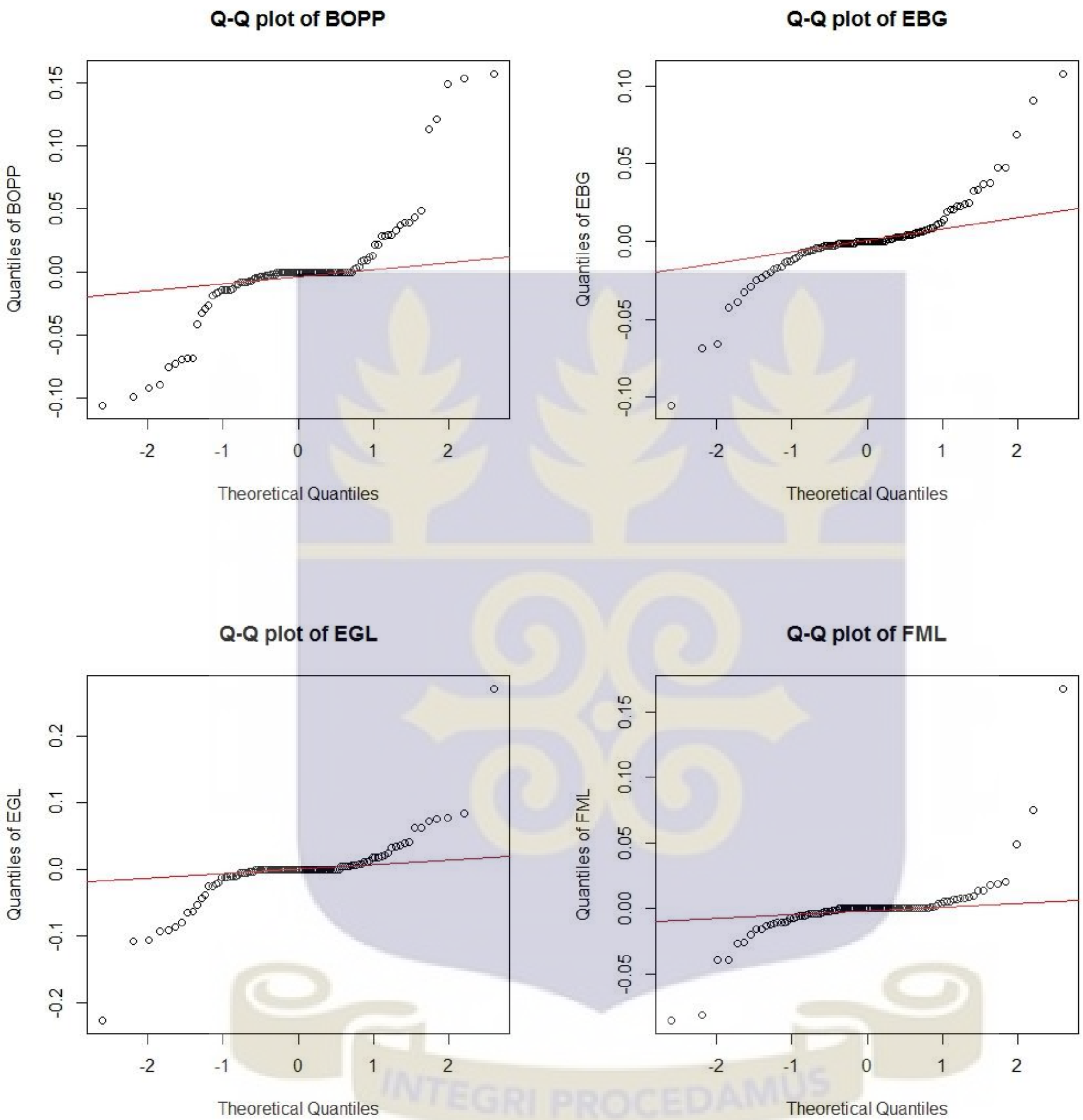

Histogram of monthly returns for SCB
Frequency
Returns
Histogram of monthly returns for SIC
Frequency
Returns
Histogram of monthly returns for TLW
Frequency
Returns
Histogram of monthly returns for TOTAL
Frequency
Returns

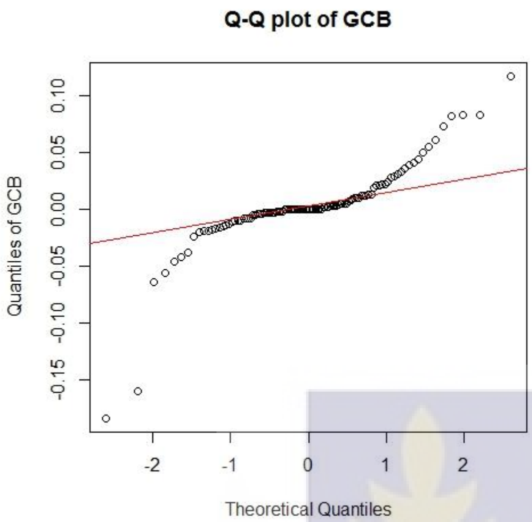

Histogram of monthly returns for UNIL
Frequency
0
2
4
6
8
-0.05
0.00
0.05
0.10
0.15
0.20
Returns

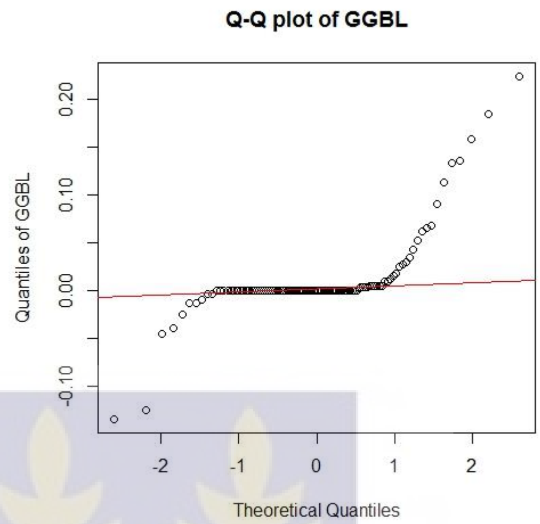

Histogram of monthly returns for UTB
Frequency
0
1
2
3
4
5
6
-0.1
0.0
0.1
0.2
0.3
0.4
0.5
Returns

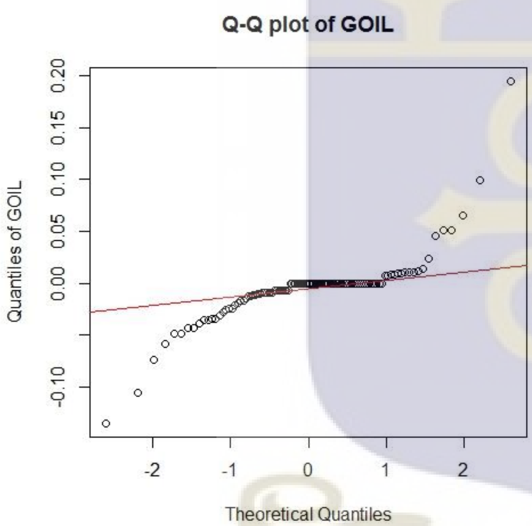
APPENDIX E

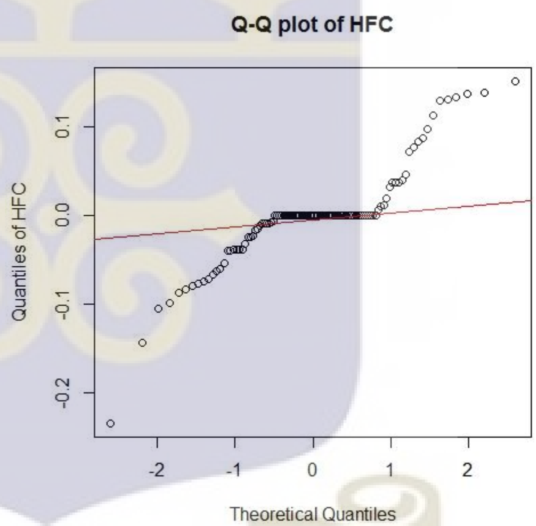
Weekly returns Q-Q plots for selected equities

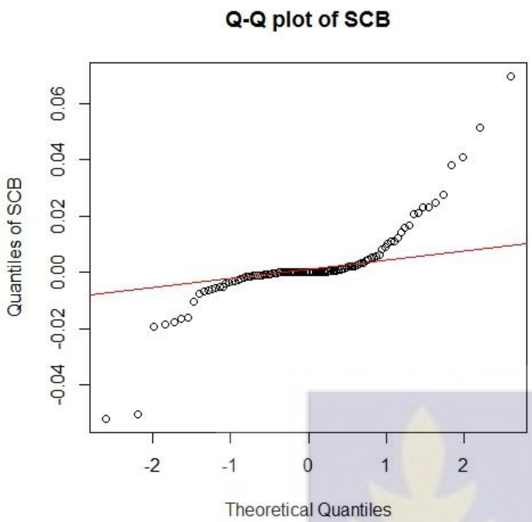


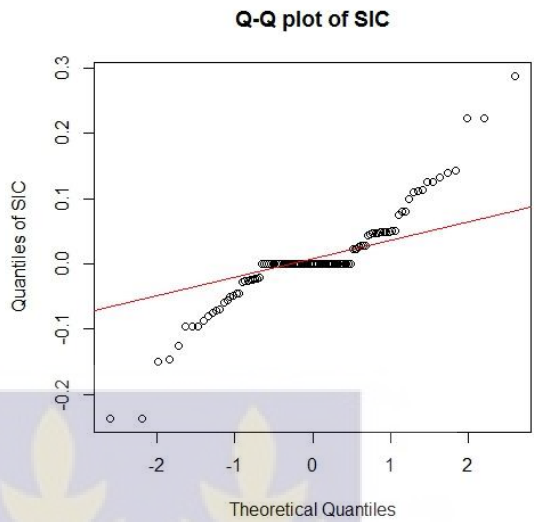

Q-Q plot of GCB
Quantiles of GCB
-0.15 -0.10 -0.05 0.00 0.05 0.10
-2 -1 0 1 2
Theoretical Quantiles

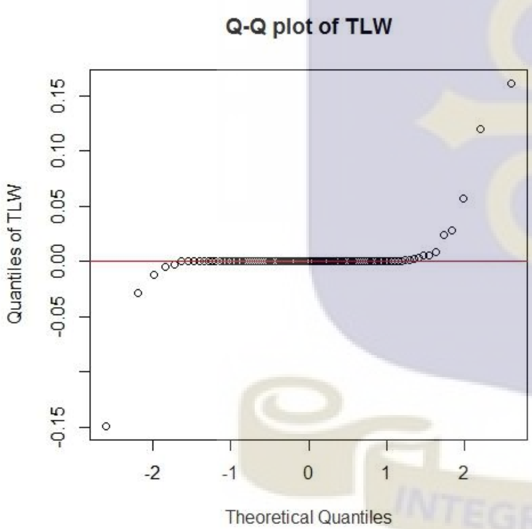

Q-Q plot of GGBL
Quantiles of GGBL
-0.10 0.00 0.10 0.20
-2 -1 0 1 2
Theoretical Quantiles

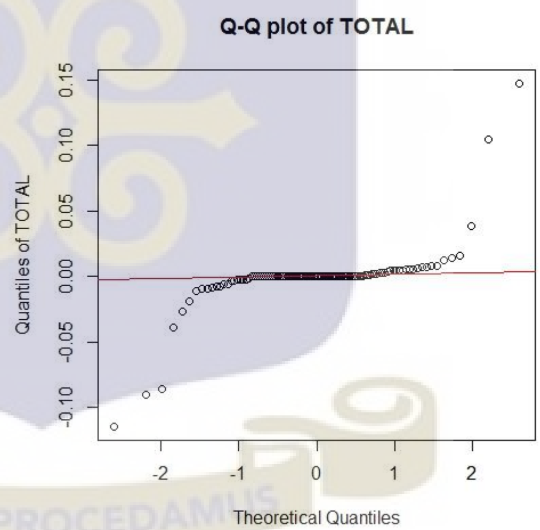

Q-Q plot of GOIL
Quantiles of GOIL
-0.10 0.00 0.05 0.10 0.15 0.20
-2 -1 0 1 2
Theoretical Quantiles

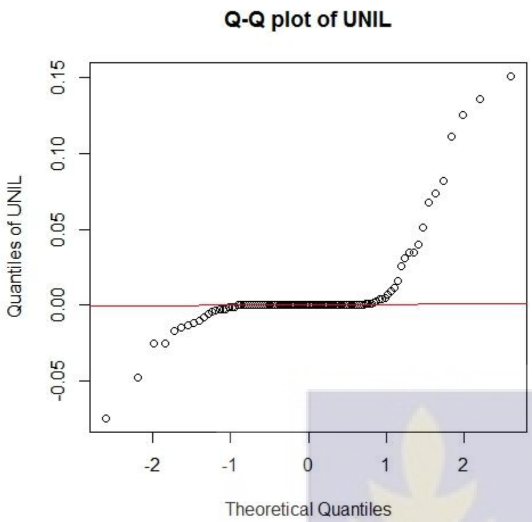

Q-Q plot of HFC
Quantiles of HFC
-0.2 -0.1 0.0 0.1
-2 -1 0 1 2
Theoretical Quantiles

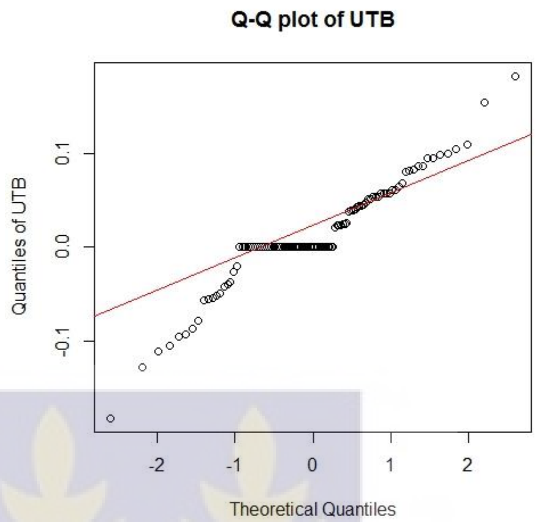

Q-Q plot of SCB
Quantiles of SCB
-0.04
-0.02
0.00
0.02
0.04
0.06
-2
-1
0
1
2
Theoretical Quantiles


Q-Q plot of SIC
Quantiles of SIC
-0.2
-0.1
0.0
0.1
0.2
0.3
-2
-1
0
1
2
Theoretical Quantiles


Q-Q plot of TLW
Quantiles of TLW
-0.15
-0.05
0.00
0.05
0.10
0.15
-2
-1
0
1
2
Theoretical Quantiles

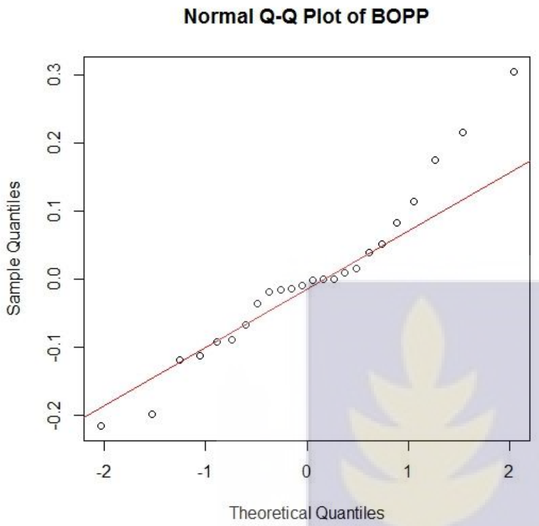

Q-Q plot of TOTAL
Quantiles of TOTAL
-0.10
-0.05
0.00
0.05
0.10
0.15
-2
-1
0
1
2
Theoretical Quantiles

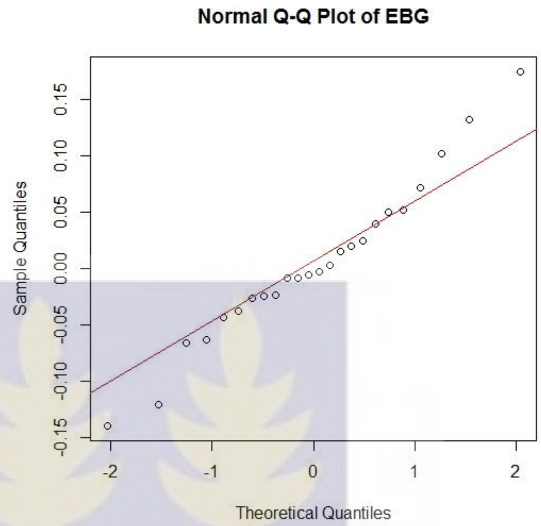

Q-Q plot of UNIL
Quantiles of UNIL
-0.05
0.00
0.05
0.10
0.15
-2
-1
0
1
2
Theoretical Quantiles

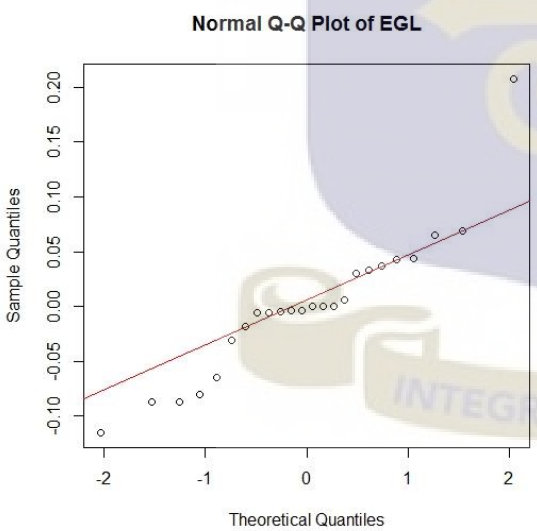

Q-Q plot of UTB
Quantiles of UTB
-0.1
0.0
0.1
-2
-1
0
1
2
Theoretical Quantiles

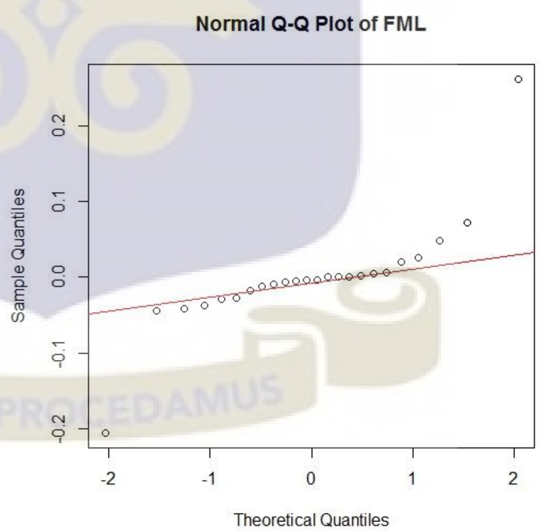
APPENDIX F

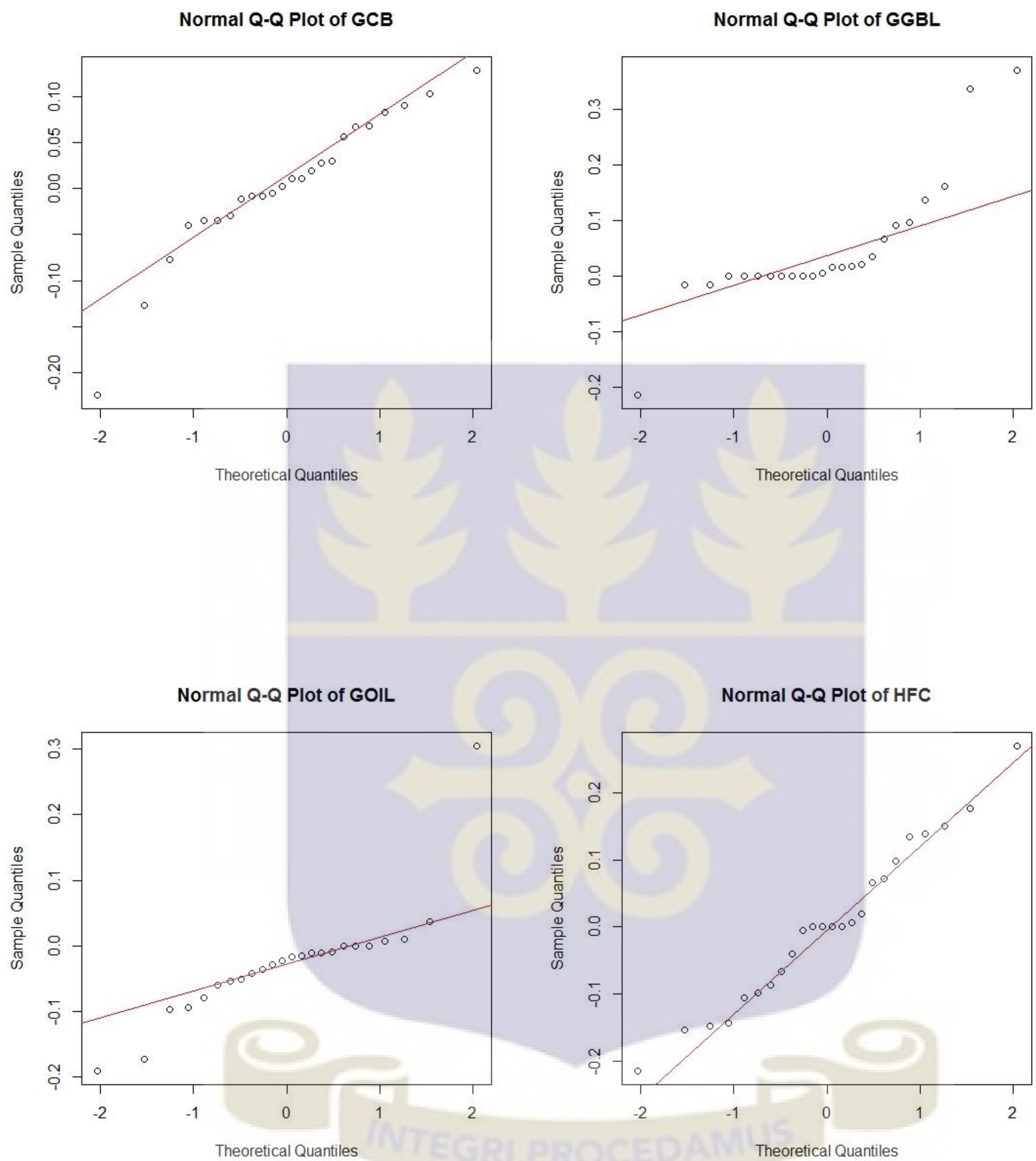
Monthly Q-Q plot of returns for selected equities

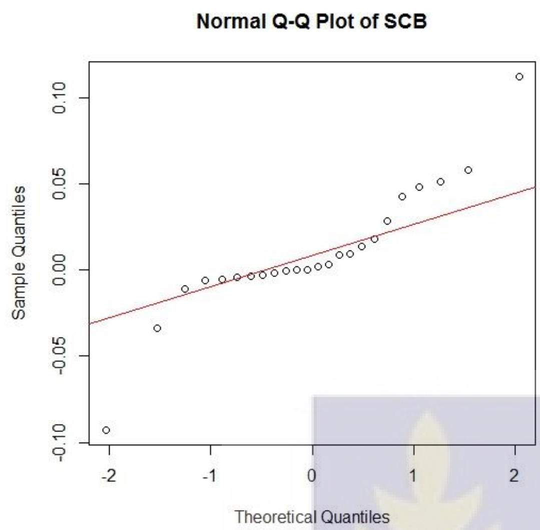


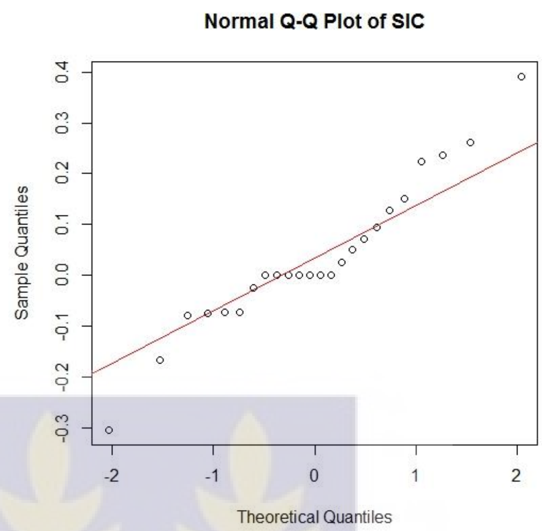


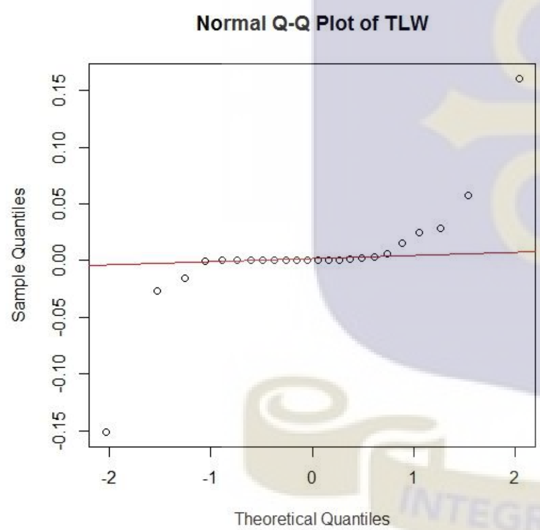


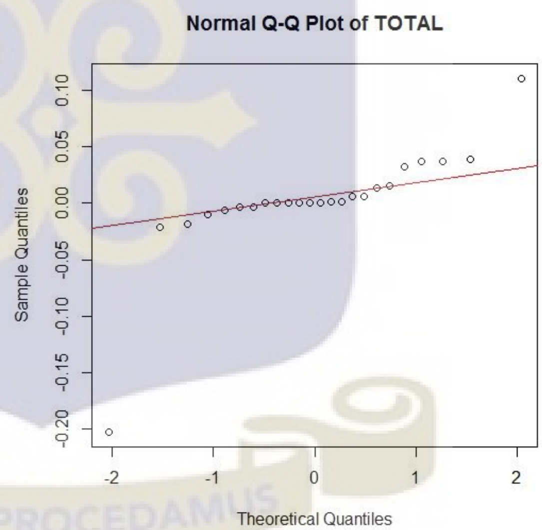


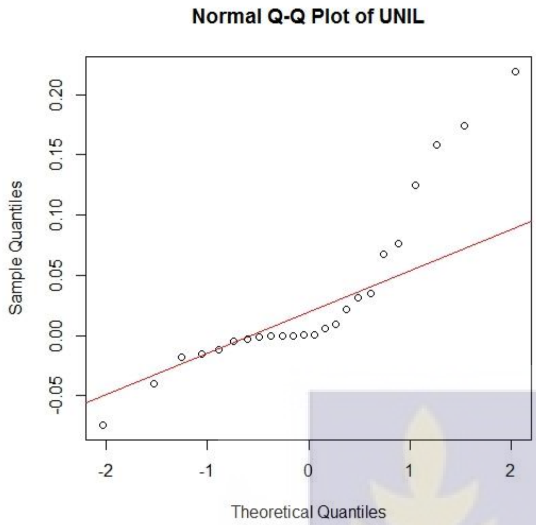

Normal Q-Q Plot of GCB
Normal Q-Q Plot of GGBL
Normal Q-Q Plot of GOIL
Normal Q-Q Plot of HFC
Sample Quantiles
Theoretical Quantiles

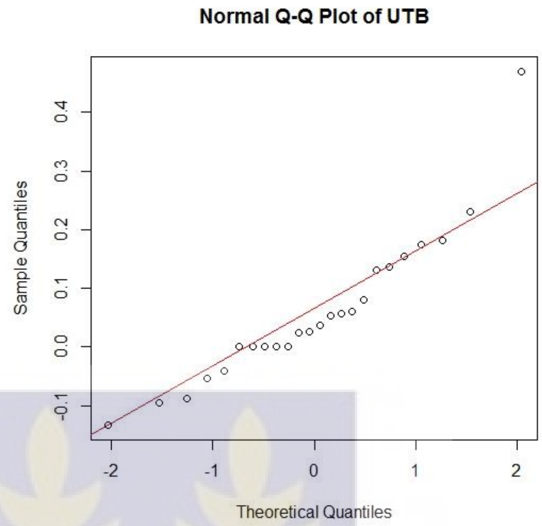

Normal Q-Q Plot of SCB
Sample Quantiles
Theoretical Quantiles
-0.10
-0.05
0.00
0.05
0.10
-2
-1
0
1
2


Normal Q-Q Plot of SIC
Sample Quantiles
Theoretical Quantiles
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
-2
-1
0
1
2


Normal Q-Q Plot of TLW
Sample Quantiles
Theoretical Quantiles
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
-2
-1
0
1
2


Normal Q-Q Plot of TOTAL
Sample Quantiles
Theoretical Quantiles
-0.20
-0.15
-0.10
-0.05
0.00
0.05
0.10
-2
-1
0
1
2


Normal Q-Q Plot of UNIL
Sample Quantiles
Theoretical Quantiles
-0.05
0.00
0.05
0.10
0.15
0.20
-2
-1
0
1
2


Normal Q-Q Plot of UTB
Sample Quantiles
Theoretical Quantiles
-0.1
0.0
0.1
0.2
0.3
0.4
-2
-1
0
1
2