



UNIVERSITY OF GHANA

**FUNDING AND ASSET ALLOCATION POLICY OF A PARTIALLY FUNDED
DEFINED BENEFIT PUBLIC PENSION PLAN:**

A CASE OF SSNIT.

FRANCIS KUDITCHER

(10443012)

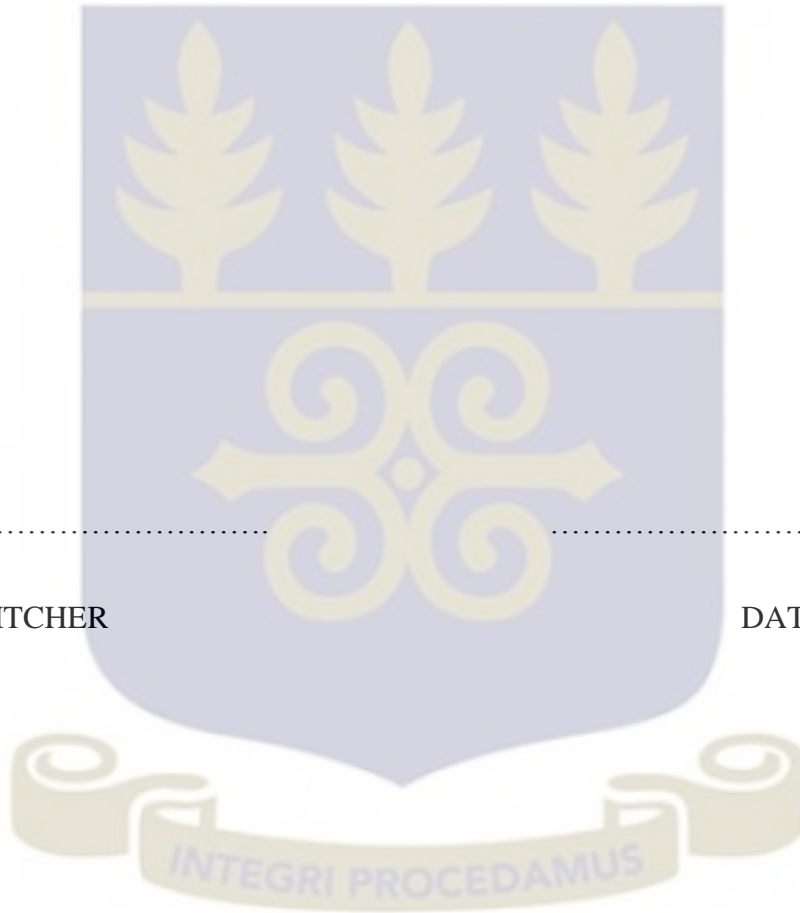
**THIS THESIS IS SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES,
UNIVERSITY OF GHANA IN PARTIAL FULFILMENT OF THE
REQUIREMENT FOR THE AWARD OF THE MASTER OF PHILOSOPHY
DEGREE IN RISK MANAGEMENT AND INSURANCE**

JUNE, 2015

DECLARATION

I do hereby declare that this work is the result of my own research and has not been presented by anyone for any academic award in this or any other University. All references used in the work have been duly acknowledged.

I bear sole responsibility for any shortcomings of this work.



FRANCIS KUDITCHER

DATE

(10443012)

CERTIFICATION

I hereby certify that this dissertation is supervised in accordance with the procedures laid down by the University.

.....
DR. ERIC DEI OFOSU-HENE

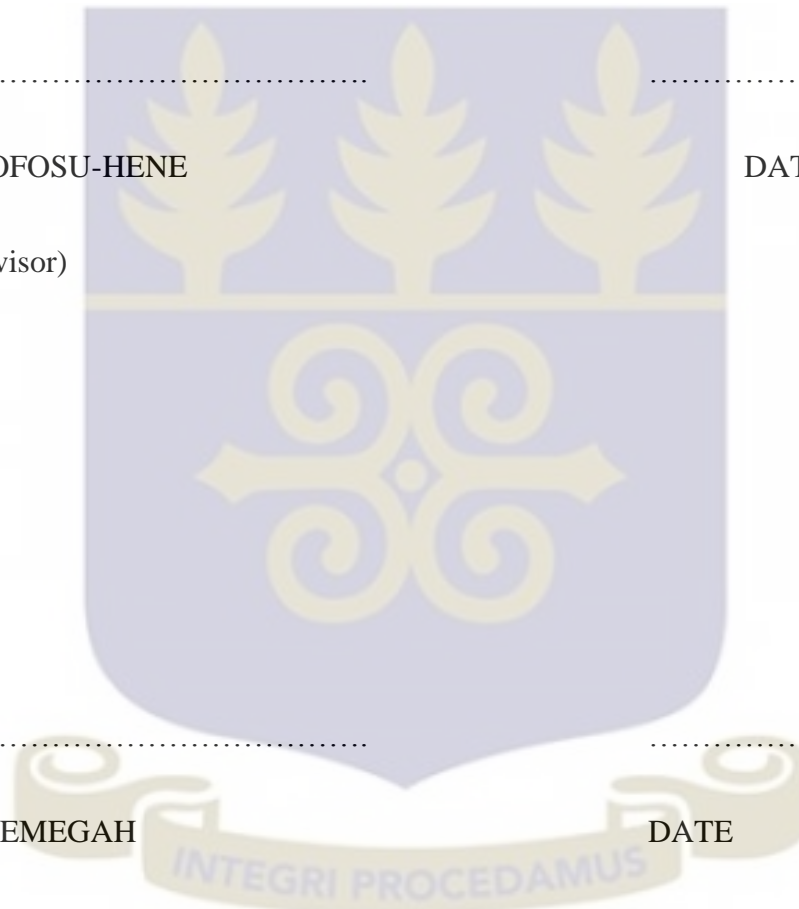
(Principal Supervisor)

DATE

.....
DR. ALBERT GEMEGAH

(Co-Supervisor)

DATE



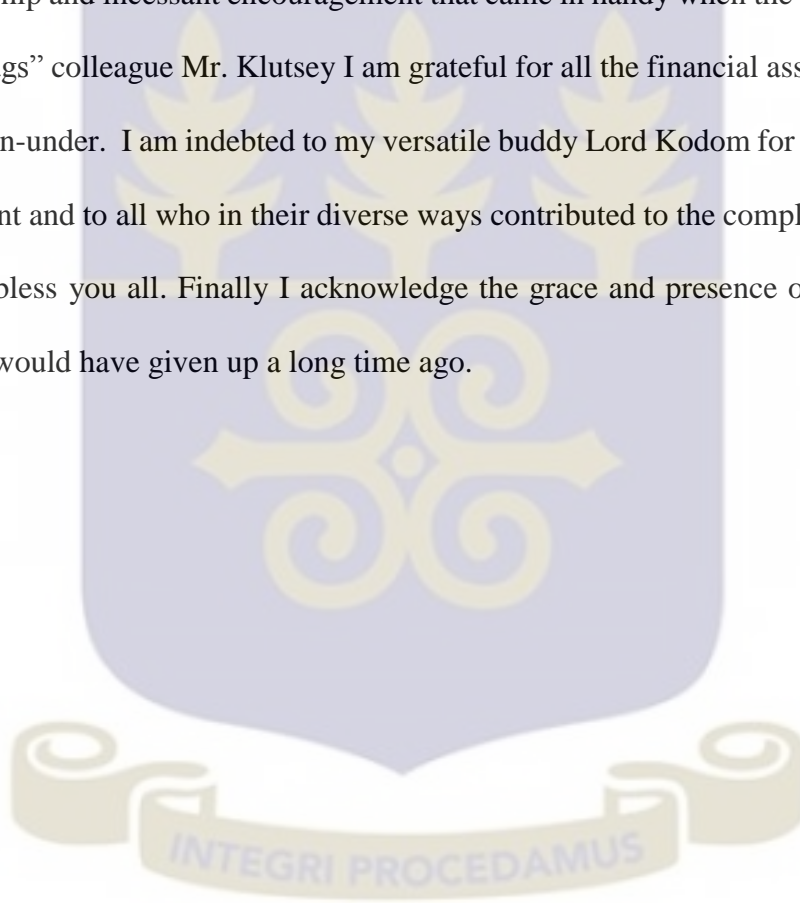
DEDICATION

This work is dedicated to my father Nene Akpotosu III and to my son Isaiah.



ACKNOWLEDGEMENT

I acknowledge the immeasurable assistance from Dr. Eric Dei Ofosu-Hene, Dr. Albert Gemegah and Dr. Charles Andoh, whose constant encouragement and support helped me complete this work. I acknowledge the patience, love and care from my partner Marjorie and all my siblings who had to bear with me during execution of this work. To my friend Ms. Imran, I am very thankful for your companionship and incessant encouragement that came in handy when the stress built up and to my “money bags” colleague Mr. Klutsey I am grateful for all the financial assistance I received when I went down-under. I am indebted to my versatile buddy Lord Kodom for data, Caleb Boadi for encouragement and to all who in their diverse ways contributed to the completion of this work. I pray that God bless you all. Finally I acknowledge the grace and presence of God in my life, without which I would have given up a long time ago.



ABSTRACT

Employing the VAR (1) model to predict future asset returns and generating multiple scenarios of these returns using the Monte Carlo simulation technique with 10,000 iterations we analyze the normal contribution rate and associated total pension costs for different strategic asset allocations of the SSNIT scheme and in the process we constrain the shortfall risk by integrating a conditional value at risk limit.

Using MatLab software we iterate to identify the optimal normal contribution rate that will fund the scheme at a minimum volatility of annual contribution rates coupled with a strategic asset allocation policy after we had identified the deterministic contribution rate that can fund the plan.

Our results indicate that the SSNIT scheme can operate at a lower normal contribution rate than the current normal contribution rate but given the rate of growth of salaries a latent plan failure going forward was identified.

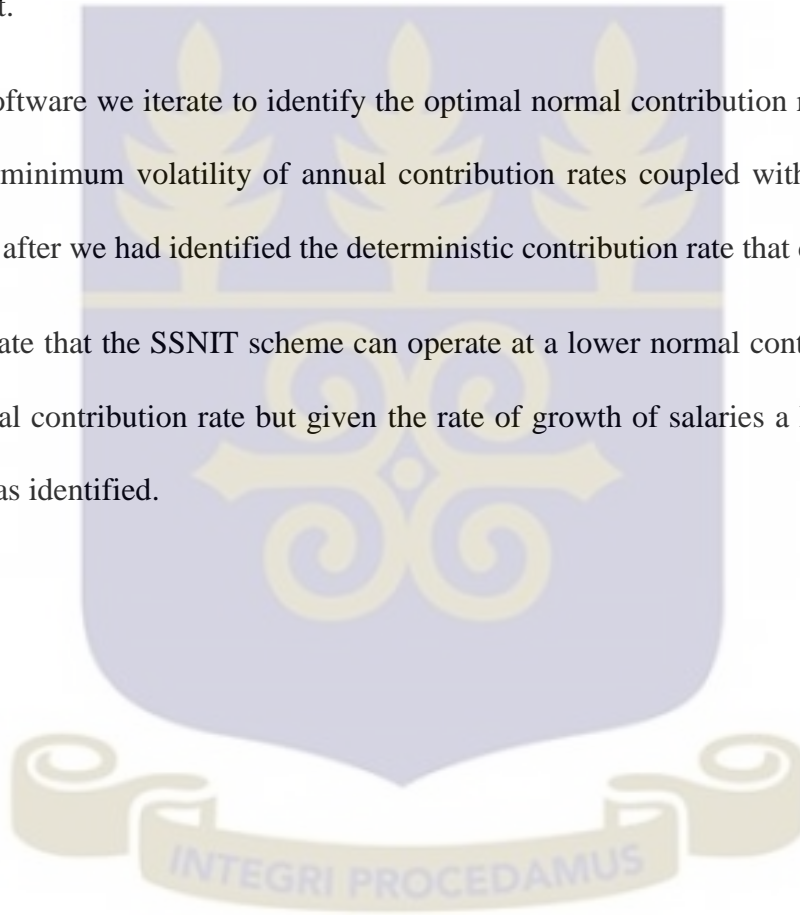


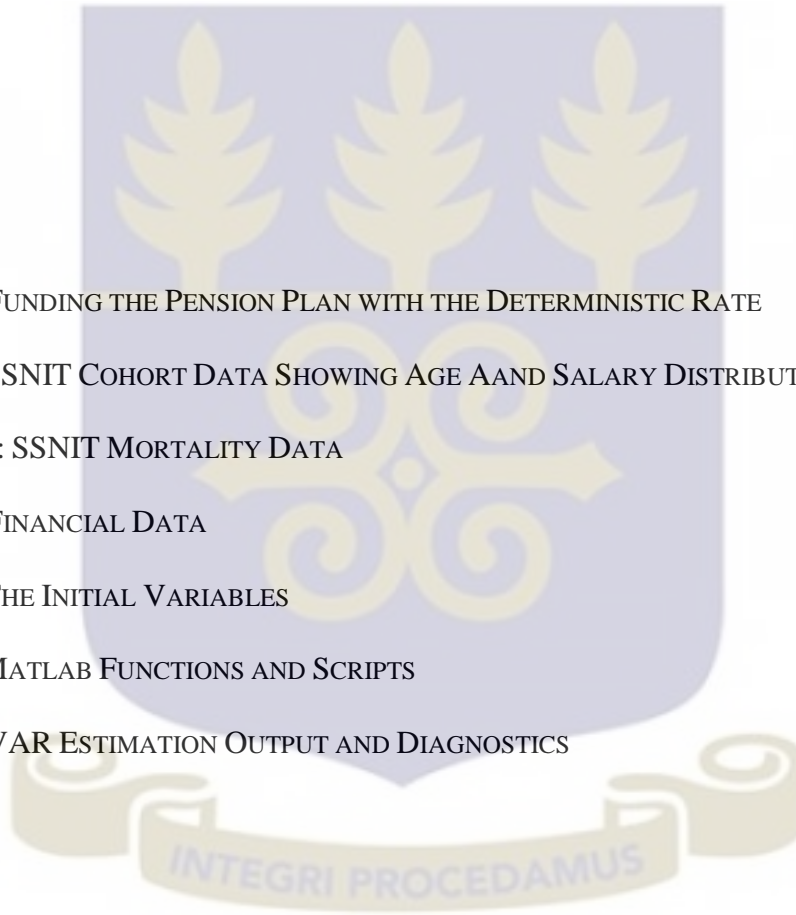
TABLE OF CONTENTS

DECLARATION	I
CERTIFICATION	II
DEDICATION	III
ACKNOWLEDGEMENT	IV
ABSTRACT	V
TABLE OF CONTENTS	VI
LIST OF FIGURES	X
LIST OF TABLES	XI
LIST OF ABBREVIATIONS	XII
CHAPTER ONE: INTRODUCTION	1
1.1 INTRODUCTION	1
1.2 BACKGROUND TO THE STUDY	1
1.3 STATEMENT OF THE PROBLEM	4
1.4 PURPOSE OF THE RESEARCH	5
1.5 RESEARCH QUESTIONS	6
1.6 OBJECTIVES OF THE STUDY	6
1.7 SIGNIFICANCE OF THE RESEARCH	6
1.8 SCOPE AND LIMITATIONS OF THE RESEARCH	7
1.9 OUTLINE OF THE THESIS	7

CHAPTER TWO: LITERATURE REVIEW	8
2.1 INTRODUCTION	8
2.2 PENSIONS	8
2.2.1 Purpose of Pensions	8
2.2.2 Types of Pension Schemes	10
2.2.3 Funding Methods of Pension Schemes	13
2.3 THE PENSIONS SYSTEM IN GHANA	14
2.3.1 The First of the Three Pillars of Ghana’s Pension Scheme	14
2.3.2 The Second of the Three Pillars of Ghana’s Pension Scheme	15
2.3.3 The Third of the Three Pillars of Ghana’s Pension Scheme	16
2.4 CONTROL OF RISK IN A DEFINED BENEFIT SCHEME	17
2.4.1 Risk Control by Total Pension Cost and Pension Underfunding	18
2.5 ASSET LIABILITY MODELLING	21
2.6 CHAPTER SUMMARY	22
CHAPTER THREE: RESEARCH METHODOLOGY	23
3.1 INTRODUCTION	23
3.2 AREA OF STUDY	24
3.3 DATA FOR THE STUDY	24
3.4 COHORT DYNAMICS AND ASSUMPTIONS	25
3.5 GENERAL METHODOLOGY OR OUR APPROACH	26
3.5.1 Projecting Total Salary Payments in Each Year	26
3.5.2 Projecting Total Pension Payments for each year	26

3.5.3 Steps to Achieving Objectives	28
3.6 DERIVING THE DETERMINISTIC CONTRIBUTION RATE	30
3.7 THE PENSION FUND PROBLEM	30
3.8 THE OPTIMAL FUNDING AND ASSET ALLOCATION POLICY	33
CHAPTER FOUR: DATA COLLECTION AND ANALYSIS	35
4.1 INTRODUCTION	35
4.1.1 SSNIT Cohort Data	35
4.1.2 Salary Growth Rate	37
4.2 DETERMINISTIC CONTRIBUTION RATE TO FULLY FUND THE PLAN	38
4.3 PREDICTING FUTURE RETURNS	40
4.4 WORST CASE PENSION COSTS	42
4.5 CONTRIBUTION RATE VOLATILITY	47
4.6 THE OPTIMAL RATE OF CONTRIBUTION AND INVESTMENT POLICY	52
4.7 FURTHER RESULTS	53
4.7.1 Sensitivity of Results to Spread Period	54
4.7.2 Sensitivity of Results to Discount Rate	54
4.7.3 Sensitivity of Results to Salary Growth Rate	55
4.8 CHAPTER SUMMARY	56

CHAPTER FIVE: SUMMARY, CONCLUSION AND RECOMMENDATIONS	58
5.1 INTRODUCTION	58
5.2 DISCUSSION OF RESULTS	58
5.3 CONCLUSION	59
5.4 RECOMMENDATIONS	60
REFERENCES	63
APPENDICES	73
APPENDIX A: FUNDING THE PENSION PLAN WITH THE DETERMINISTIC RATE	73
APPENDIX B: SSNIT COHORT DATA SHOWING AGE AAND SALARY DISTRIBUTION	74
APPENDIX C: SSNIT MORTALITY DATA	75
APPENDIX D: FINANCIAL DATA	76
APPENDIX E: THE INITIAL VARIABLES	78
APPENDIX F: MATLAB FUNCTIONS AND SCRIPTS	79
APPENDIX G: VAR ESTIMATION OUTPUT AND DIAGNOSTICS	88



LIST OF FIGURES

Figure 4. 1: Curtate Age Distribution of Plan Cohort at Base Year 2014	36
Figure 4. 2: Average Annual Salary Distribution of Plan Cohort at Base Year 2014	36
Figure 4. 3: Cohort Life Expectancies	37
Figure 4. 4: Rates Comparison - Wage Growth with GDP Growth & Inflation (1990-2014)	38
Figure 4. 5: Funding the Plan at the Deterministic Rate	39
Figure 4. 6: Asset Returns Behavior over the Historic and Projection Periods	42
Figure 4. 7: Behavior of 1%-CVaR of Total Pension Cost with Increasing Contribution Rates	43
Figure 4. 8: Worst Case Pension Cost Outcomes – The Overall Picture	45
Figure 4. 9: Annual Contribution Rate Volatility Behavior	47
Figure 4. 10: Volatility Profile for 0 - 10% Contribution Rate and up to 50% Equity Weight	49
Figure 4. 11: Volatility Profile for 10 - 40% Contribution Rate and up to 50% Equity Weight	50
Figure 4. 12: Segment of Volatility Profile for Equity Weight beyond 50%	50
Figure 4. 13: Volatility Trading Returns	53



LIST OF TABLES

Table 1.1: The Evolution of Pension Schemes in Ghana	2
Table 4. 1: Summary of Historic and Projected Asset Return Characteristics	41
Table 4. 2: Spread Period Sensitivity of Optimal Results	54
Table 4. 3: Discount Rate Sensitivity of the Optimal Results	55
Table 4. 4: Influence of Salary Growth on Optimal Contribution and Investment Strategy	56



LIST OF ABBREVIATIONS

ALM	Asset Liability Modelling
CR	Normal Contribution Rate
CVaR	Conditional Value at Risk
DB	Defined Benefit
DC	Defined Contribution
GOG	Government of Ghana
ILO	International Labor Organization
LCH	Life Cycle Hypothesis
NPRA	National Pensions Regulatory Authority
OECD	Organization for Economic Cooperation and Development
PAYG	Pay As You Go
PNDC	Peoples' National Defense Council
SSNIT	Social Security and National Insurance Trust
T-Bills	GOG 91 Day Treasury Bills
TPC	Total Pension Cost
VaR	Value at Risk
VAR	Vector Auto Regression

CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

The introductory chapter of this work explores the history of the SSNIT pension scheme from its inception to its current state and what this work intends to do.

1.2 BACKGROUND TO THE STUDY

Old age and disability impinge on the economic security of human beings probably due to the loss of earning power at an advanced age. According to Valdes-Prieto (1997), all countries of the world are concerned with the issue of old age economic security and the probability of default of pension promises that may throw older people into poverty. Thus there is a growing interest in pension reforms to ensure sustainability of schemes.

The Social Security and National Insurance Trust (SSNIT) is the public pension provider in Ghana and it provides a defined benefit (DB) pension scheme which is partially funded by a contribution rate of 11% of employee basic salary during the years of employment of a pensioner. This is the first tier pension provision that the SSNIT is currently mandated to manage by the National Pensions Act 766 of 2008.

Hitherto, there has been several pension reforms in the country since 1946 when formal pensions system were introduced into the country with the main cause for the reforms being high administrative cost, low competency, low coverage and inadequate investment returns (Stewart & Yermo, 2009). According to information on the National Pensions Regulatory Authority (NPRA) website, during the period before independence the colonial government introduced a pension scheme to take care of

the retirement benefits of employees that have served under the colonial administration. After independence the CAP 30 pension scheme of 1960 was introduced to cater for university and certified teachers and all government workers. The CAP 30 was a noncontributory scheme that was mainly financed by tax revenues generated by the government. The CAP 30 had problems because a large number of workers were excluded from it and thus in 1965, in order to widen the coverage of pension provision, the Social Security Act (Act 279) was passed to cater for those organizations who had a minimum of 5 workers and who were not covered under the CAP 30. This Act 279 brought into being a social security scheme which was a provident fund that catered for old age, invalidity and survivor's pension by providing a lump sum benefit to them. In 1972 Act 279 was amended by NCRD 127 thus reforming the social security scheme in place to the Social Security and National Insurance Trust (SSNIT). In 1991, by PNDC Law 247, the scheme was yet again transformed from being a provident fund to being a pension scheme. The current transformation of the pension scheme to the three tier pension scheme was by the National Pensions Act 766 of 2008. The current 3-Tier pension scheme is expected to be an improvement on all previous pension schemes. The table below shows the evolution of pension schemes in Ghana.

Table 1.1: The Evolution of Pension Schemes in Ghana

Pension Scheme	Year	Act or Law behind formation
CAP 30	1950-2004	Pension Ordinance No. 42
SSNIT Provident Fund	1965	Parliamentary Act 279
SSNIT Provident Fund Scheme	1970-1991	NRCD 127
SSNIT Pension Scheme	1991- 2008	PNDC Law 247
Three-Tier Pension scheme	2008-present	National Pensions Act 766

Source: Adapted from Kumado & Gockel (2003)

The first tier of the new scheme is the basic national social security scheme, which incorporates an improved system of SSNIT benefits and is compulsory for all employees in the public and private sectors, it pays only monthly pensions and related benefits such as survivors' benefit, while the second tier is the occupational pension scheme, which is also compulsory for all employees but it is privately managed, and designed to give contributors higher lump-sum benefits than currently available under the SSNIT or Cap 30 pension schemes; the third tier on the other hand is a voluntary provident fund and personal pension scheme, aided by tax benefit incentives for employees in the informal sector. It is also to provide additional funds for employees in the formal sector who want to make voluntary contributions to enhance their pension benefits.

The current Pensions Law (Act 766) which provided for the establishment of the contributory three-tier pension scheme also caters for a Pension Regulatory Authority which will give approval, regulation and monitoring of Trustees, Pension Fund Managers, Custodians and other institutions relating to pension matters. It will also give government advice on overall policy concerning pension issues in Ghana.

According to the World Bank Report (2010) Ghana's population statistics have shown a continuous rise in the old-age dependency ratio with the 2010 population census indicating an old age dependency ratio of 6.62 which is a rise from the 5.18 of 1967. The steady rise of this ratio is as a result of the continuous growth in the population of old-aged dependents due to declining fertility rates over the years with people giving birth to fewer children as compared to the baby-boom age of the 1960s. This baby-boom cohorts are now in their old age with a smaller cohort of the working class to cater for them. With this trend, there are concerns that most of the population will enter into retirement with inadequate resources (Gustman & Steinmeier, 1998). Hence the continual reforms of the pension scheme to ensure that old-age benefit promises can be effectively discharged when they become due.

1.3 STATEMENT OF THE PROBLEM

According to Dam (2014) there is a growing attention to pensions in Africa due to economic developments on the continent and that this growing interest is supported by international organizations like the International Labor Organization (ILO), the World Bank as well as other bilateral development partners. This has resulted in reforms in pension systems in African countries, including Ghana, and has engendered an increase in publications describing, comparing and evaluating these reforms.

A well-functioning pension system can be argued to be the one that will be able to meet liabilities as they become due thus delivering on all promised benefit obligations. One reason that this is needed, according to Dam (2014), is that it prevents old-age poverty and reduces the obligation of children having to support their parents, thereby allowing younger generations more time to invest in their education; the success of a well-functioning pension system also impinges on the success of a well-functioning financial market. (Barrientos, 2013).

Ghana has a high dependency ratio like many other African countries, with a total labor force of 12 million out of the total population of 25 million according to 2012 figures from the World Bank. Only one fifth of this labor force is working in the formal sector and it is this portion of the labor force that is covered by the first pillar of the three tier pension scheme which is managed by SSNIT.

Prior to the 2010 Pensions Reforms in Ghana, SSNIT was being funded by a higher contribution rate from employee basic salaries but after the reforms which brought into play the three tier pension scheme, SSNIT has had to do with a reduced contribution rate of 11% accruing from the first pillar. On 4th January 2014, it was carried in the Ghanaian media that SSNIT has increased pension payments by 100% and on 26th September, 2014, it was also reported in the media that SSNIT funds are drying up and may thus require higher contribution rates in order to be able to deliver on promised benefit

payments in future (source: The Business and Financial Times Online). This may be due to the two main weaknesses identified by OECD (2009) and Kpessa (2011) as;

1. Very low coverage as it is only the formal sector that is covered by the first and second pillars;
2. Inadequate investment returns due to the absence of corporate bond markets and the fragility of the private sector and national financial markets.

This is a worrisome issue to current and future plan beneficiaries and it thus calls for analysis to ascertain the optimal funding and asset allocation policy that while extracting optimal contribution rates from current and future employees, will be investing in strategies that yield such that promised benefit payments will be delivered when they become due.

The total contributions to the scheme in a given period expressed as a ratio of the total payroll size and the total return on investment for the period defines the contribution rate applicable to that period and it varies due to the experience of the investments on the market therefore the problem is to minimize this contribution rate volatility in the face of market risk.

Also to the best of our knowledge, none of the previous works of this nature has been conducted and published in the Ghanaian context and it is thus desirable to contextualize this analysis in Ghana using the SSNIT as a case and thereby add to the increasing publications on this subject.

1.4 PURPOSE OF THE RESEARCH

The purpose of this research is to extend previous knowledge and publication in the analysis of contribution rate risk in a defined benefit (DB) scheme that is coupled with an asset-liability management setting with different stochastic assets in the Ghanaian context using the SSNIT as a case study.

1.5 RESEARCH QUESTIONS

1. What is the deterministic rate of contributions required to fully fund future pension liabilities over a forty-five year time span?
2. What is the optimal combination of contribution rate and investment strategy that will minimize contribution rate volatility given a worst case plan cost?

1.6 OBJECTIVES OF THE STUDY

1. To estimate the rate of contributions required to fully fund future pension liabilities over a forty-five year time span for current and future employees in a deterministic setting;
2. To identify the minimum-volatility optimal contribution and investment strategy while restricting worst case plan costs.

1.7 SIGNIFICANCE OF THE RESEARCH

In the light of extant research in other contexts, which is skewed towards contribution rate and longevity risks associated with pensions, this research will draw significance from the context in which it is based since no such research is available from Ghana to the best of our knowledge and also it will contribute to the growing body of knowledge in pensions risk management.

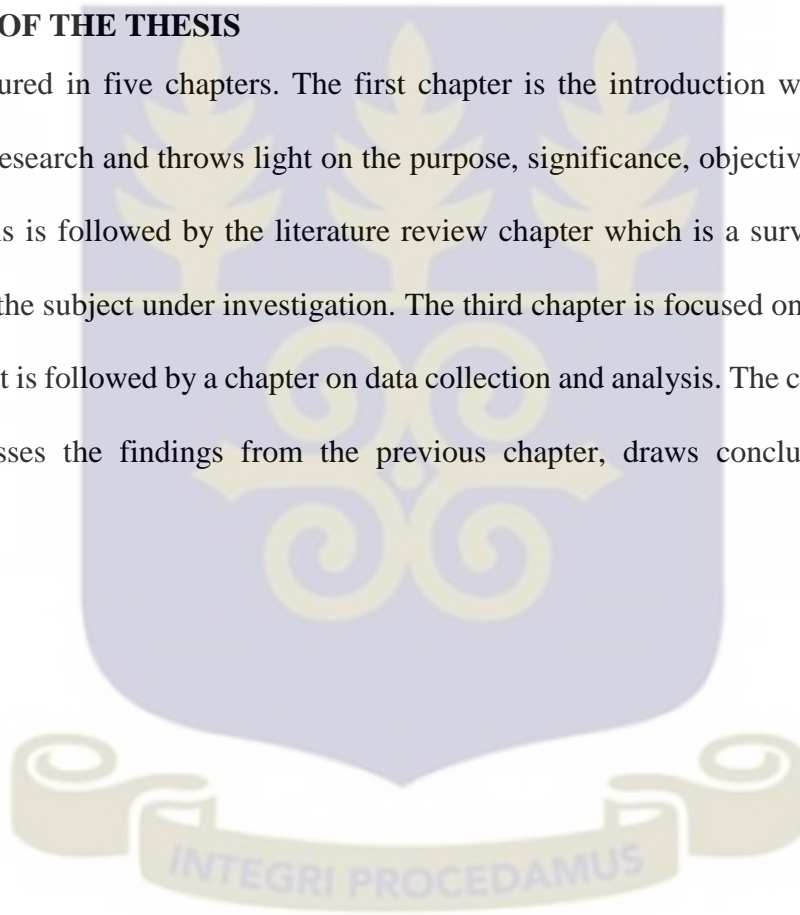
This work will give SSNIT an alternative view as to how their contribution rate and investment strategies could be and it will serve as a guide for their asset allocation strategy. This work will also form the basis of further research for the National Pensions' Regulatory Authority as they fashion out policies for the efficient management of pensions in Ghana. We expect this work to be of interest to public policy makers as it throws light on the possibility of the SSNIT scheme going bankrupt. The intentional pervasive use of MatLab coding should also interest researchers and graduate students as it can engender interest in multidimensional analysis.

1.8 SCOPE AND LIMITATIONS OF THE RESEARCH

The scope of this work is limited to SSNIT and the first tier pension scheme, it does not assess the solvency of SSNIT against the new solvency regime enshrined in the Pensions Reform Act. Unavailability of enough data, researcher's budget constraint and lack of a powerful computer that can iterate the MatLab algorithms limited this work.

1.9 OUTLINE OF THE THESIS

This work is structured in five chapters. The first chapter is the introduction which explores the background of the research and throws light on the purpose, significance, objectives and limitations of the research. This is followed by the literature review chapter which is a survey and review of extant literature on the subject under investigation. The third chapter is focused on the methodology of the research and it is followed by a chapter on data collection and analysis. The concluding chapter of the work discusses the findings from the previous chapter, draws conclusions and makes recommendations.



CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter of the study deals with the conceptual and empirical literature relevant to the research and it reviews works done in similar studies.

2.2 PENSIONS

A pension is a long-term investment that aims to build a fund for an employee's retirement in a tax efficient way. This can then be used to provide an income when the pay packets stop by reason of mandatory age retirement, ill-health or any disability that ensures the stoppage of income earning ability. It is an arrangement to prevent old age poverty and to ensure that in order to retire in good health there are funds available to support activities of the years when income earning ability stops.

Pension promises involve explicitly and relatively well-documented set of statements regarding future contingent payment and employment. The pension promises are inherently long-term compensation arrangements in which a worker earns ("accrues") a right to an eventual pension benefit, payable after the employee attains certain eligibility criteria, with benefits frequently commencing at retirement and continuing until death.

2.2.1 Purpose of Pensions

The beginning of social security legislation is related to the social desire to avoid poverty in the aged population and to insure income when one is too old to work; both reflect the charitable instincts and the social trend of the elderly living independently of their families, who have often scattered to other regions and no longer directly support them.

According to Samuelson (1975), Aaron & Reischauer (1998), the principal reason for the state to provide pensions is the belief that many citizens do not accumulate enough funds for retirement and as such there is the need to help households achieve life resources allocation by smoothing consumption over life as propounded in the life cycle hypothesis (LCH) by transferring resources from working life to post-retirement, when income earning stops. This is because many segments of society are viewed to be unsophisticated enough to be able to set up appropriate post-retirement arrangements. Thus pension systems are established to prevent the state from having to support large segments of retirees (Blahous, 2000). Thus at a macro level, a pension plan permits individuals to adopt a life-cycle model of consumption and hence protect myopic and unsophisticated individuals (Modigliani & Ando, 1957). This generates savings during working life to provide for postretirement.

Pensions have also been used as a tool for attracting and retaining workers with certain qualifications, for eliciting higher work effort for achieving desired goals and also for prompting age retirement (Hurd, 1990). Employers hold pensions as deferred wages that are payable only if an employee exhibits desirable traits such as honesty and hence companies are able to elicit the desired characteristics by offering matching contributions to ensure that their employees go on retirement without anxiety. Company pensions encompass three aspects: insurance (for old age income earning disability), compensation (as a reward for a faithful career service), and severance (payment to allow termination). Corporate pensions and health insurance growth arose as a *bribe* to forestall labor agitation without huge increases in wages. They helped to defer income and reduce the effect of high marginal tax rates for salaried employees (Bertocchi, Schwartz & Ziemba, 2010)

Lastly, from economic theory, countries require savings for capital formation, and individuals require savings to support themselves in the phase of their lives when they do not earn income. Using a variety of inducements (such as tax deferrals and tax credits) and compulsory contribution rules,

governments encourage citizens to increase their rate of savings. The greater the need for such savings, the higher the contribution rate required and potentially the higher the benefits. Thus pensions are also a vehicle for capital formation (Choi, Laibson, Madrian, & Metrick, 2002).

2.2.2 Types of Pension Schemes

Pension schemes can be differentiated by their benefit obligations and the way they fund the discharge such obligations. Although employer pension schemes vary in design, they can be broadly categorized into two types based on the promised benefits as; defined contribution (DC) and defined benefit (DB) schemes, with financing methods described as; unfunded (“pay-as-you-go”) and funded or contributory.

Defined Contribution and Defined Benefit plans differ significantly with respect to the risks faced by employees and employers, the flexibility of funding, the sensitivity of benefits to inflation, and the importance of governmental supervision.

2.2.2.1 The Defined Contribution Scheme

Under a defined contribution (DC) plan each employee has an account into which the employer and in some cases the employee make regular contributions. Benefit levels depend on the total contributions and investment earnings of the accumulation in the account. Often the employee has some choice regarding the type of assets in which the accumulation is invested and can easily find out what its value is at any time. Defined contribution plans are, in effect, tax-deferred savings accounts in trust for the employees, and they are by definition fully funded. The DC arrangement is the conceptually simpler retirement plan when compared with the defined benefit arrangement. In

DC pension plans, participants, sponsors, or both make pre-specified contributions either in absolute currency or as a fraction of salary. These contributions may also be partially or totally voluntary. The contributions are invested in assets. The ultimate pension is uncertain (before to retirement) because it depends entirely on investment performance of the accumulated contributions on the market. Accordingly, two plan participants with identical contributions but dissimilar investment portfolios can receive widely different pensions. Also, two plan participants with identical contribution profiles can receive different pensions over different time periods.

Contributions are tax-deductible, and investment income accrues tax-free. Usually the employee is given a choice as to how his account is to be invested. In principle, contributions may be invested in any security, although in practice most plans limit investment options to various stock, bond, and money-market funds. At retirement, the employee either receives a lump sum or an annuity, the magnitude of which is dependent upon the accumulated value of the funds in the retirement account. Thus all of the investment risk is borne by the employee; the retirement account is by definition fully funded, and the sponsor has no obligation beyond making periodic contributions. The DC plan valuation is achieved by simply measuring the market value of the assets held in the retirement account. The actual size of the plan at retirement or the retirement annuity will depend upon the realized investment performance of the retirement fund, the interest rate at retirement, and the ultimate wage path of the employee.

2.2.2.2 The Defined Benefit Scheme

Benefit entitlement in a defined benefit plan is determined by a formula which takes into account years of service and salary of the participant. DB plan formulas also take into account the Social Security benefits to which a plan participant is entitled. In the traditional DB plans, the plan sponsor

(employer) promises benefits and funds the plan during the working cycle of the participant (employee) and thus the sponsor bears most of the risk.

The main purpose of the defined benefit pension plan is the provision of regular payments in either absolute currency or as a fraction of a salary measure, for example, a defined proportion of the final salary or an average of some past years of salary. The pension benefit payout could be in either nominal or real terms. In DB pension schemes, employees, sponsors, or both make contributions to the plan that can change over time. The employee may be unaware of the formula linking contributions to benefits, however there is a unique “budget constraint” that connects a given replacement rate to contributions, returns, and benefits (Asad-Syed, Muralidhar, & Van der Wouden, 1998). According to Bateman, Kingston, & Piggott (2001) DB plans rely on a demographic sensitive pooling of investment and liability risk, which they referred to as the social allocation of risk. Corporate DB pension formulas traditionally define annuities based on the product of the number of years of service and some accrual factor. The inclusion of years of service in the formula creates nonlinear growth in pensions as one ages according to Godoy & Valdes-Prieto (1997), thereby leading individuals to stay with the pension-providing institution because the cost of leaving increases over time. They argue that this creates a perverse incentive to stay and they have provided a general condemnation of all DB plans by suggesting that “the defined benefit approach exposes workers to the risk of a low final wage on which benefits are based and must pay an implicit fee to the agents that provide the guarantee on investment returns.” This condemnation is unwarranted according to Bertocchi, Schwartz & Ziemba (2004) because a DB plan is only a promise to provide a guaranteed replacement rate and thus the formula for achieving that can be varied. They argued that if the rate of return is assured, the replacement rate can be guaranteed, given salary growth. They also contend that previous pension reform research has not recognized that good pension design can create a close link between DB and DC plans.

2.2.3 Funding Methods of Pension Schemes

There are several methods to fund DB or DC plans. Presently, social security systems are typically PAYG DB schemes in which current participants are required to make contributions used to pay the benefits of retirees. In a pure PAYG plan, there is no accumulation of funds because all contributions are disbursed to service pensions. However, corporate or occupational DB or DC schemes tend to be funded (partially and fully).

Funding requires an accumulation of funds before retirement to service future liabilities. Funds are invested in either marketable or nonmarketable assets. As a result, some combination of contributions and asset returns is used to service pensions. Some researchers (Logue & Rader, 1997) have suggested that DC plans are always fully funded. This comment is true in the sense that the pension is entirely paid out of whatever capital has been accumulated, but it can be misinterpreted to suggest that the sponsor is indifferent to the size of the pension. Many researchers incorrectly assume that both sponsors and retirees are indifferent to the annuity paid out in DC plans but it is of essence in a DC plan for participants to have a target replacement rate (though it is not guaranteed) and to select their contribution and investment policy to achieve that target. As a result, other researchers (Muralidhar & Van der Wouden, 1998) have suggested that DC plans are underfunded if the assets in the plan are insufficient to deliver a target replacement rate.

DB plans spread investment risk across a large number of individuals of different ages and over different time horizons. These plans pool risk within a cohort and across cohorts. The plan sponsor, who generally bears the investment risk of the plan, has a much longer time frame and a much higher risk-bearing capacity than individuals in the plan. In other words, whereas in a DC plan the time horizon is the life of one individual, in DB plans the time horizon is much longer (if not infinite). For these reasons, DB plans on average can take on more risk and generate higher returns, and their asset

allocation policy tends to take longer to change than can individual plans (Orszag & Stiglitz, 2001). Also by pooling assets, DB plans incur lower costs for managing assets. On the other hand, DC plans enable individuals to fashion their portfolio to the risk they wish to bear and allow for a better matching to their preferences. Individuals participate in all the gains and losses of their plan but incur higher costs in managing their assets.

2.3 THE PENSIONS SYSTEM IN GHANA

Ghana's current pension system is based on the three tier concept that is known in many OECD countries. This system came into force in 2008 after being initiated in 2004 with the creation of a Presidential Commission on Pensions to look into the reformation of the pension system. The promulgation of the new Pensions Law, the National Pensions Act, 2008 (Act 766) on 12th December, 2008 brought the current system in being.

This new Pensions Law also made provision for the establishment of a Pensions Regulatory Authority, currently known as the National Pensions Regulatory Authority (NPRA). The Authority is to approve, regulate and monitor Pension Fund Managers, Trustees, Custodians and other institutions relating to pension matters. It is to also advise government on the overall policy on pension matters in the country.

2.3.1 The First of the Three Pillars of Ghana's Pension Scheme

The first pillar is a basic national social security and mandatory scheme. It is partially funded by all employees in the formal labour market by levying a percentage of monthly salary and it provides retirees, survivors and the disabled in case of a minimum level of contribution years with monthly

annuities. There is a fund into which levies are paid and benefits withdrawn. This scheme is managed by SSNIT with the NPRA as regulatory body.

The total contribution to the first tier is 13.5% of employee monthly salary and it is made up of a 1% levy on employee salary matched by a 12.5% contribution by the employer. This is remitted to SSNIT but 2.5% of this contribution is a levy for the National Health Insurance Scheme, thus SSNIT retains 11% for the 1st tier pension fund.

Benefits under this scheme is a superannuation pension, which is a regular monthly income payable for the rest of the pensioner's life. This is triggered by age retirement at age 60 or voluntary retirement at age 55 with condition that a member has contributed to the scheme for not less than 15 years in aggregate. There is guaranteed payments till age 75 of the pensioner beyond which pension payments stop should death occur to the pensioner. If a member dies before retirement or during retirement but before attainment of age 75 then a lump sum benefit is payable to the deceased's nominated beneficiary.

2.3.2 The Second of the Three Pillars of Ghana's Pension Scheme

The second pillar is the occupational pension's scheme, and it is also salary related and mandatory for all employees in the formal labour market. It is a defined contribution scheme that pays lump-sum only. It is fully funded by contributions of employer and employees. With the NPRA as the supervisor and regulator, the second-tier pension scheme is managed by Trustees, Pensions Fund Managers and Pension Fund Custodians.

The total contribution rate to the second mandatory pillar is 5%, made up of a 4.5% levy on employee monthly salary and matched by a 0.5% contribution by the employer. This is remitted to the private

fund managers, custodians or trustees nominated by the employer and employees for the mandatory second tier lump sum benefits.

The lump sum benefit is paid on attainment of the compulsory age retirement at 60 or on attainment of age 50 without employment or on retirement due to ill health. On death of a member to the scheme, the nominated beneficiary receives the benefit.

2.3.3 The Third of the Three Pillars of Ghana's Pension Scheme

The third pillar is the private pensions' scheme in the form of a Personal Pension Scheme and Provident Fund. It is based on voluntary contributions that is tax exempt up to 16.5%. Benefits are based on defined contribution and are fully funded. The main objective of this pillar is to foster savings among informal sector workers and to offer supplementary benefits to participants of the first two tiers. The administration of this pillar is also carried out by pensions fund managers, trustees and custodians and also similar to the other pillars, the third pillar is regulated and supervised by the NPRA. The third tier is voluntary hence contribution rates differ and in the instance of formal employment, it is often the case that a contribution rate of 10% of employee monthly salary is matched equally by a contribution from the employer and remitted to the private fund managers.

In all cases, contributions up to 35% are tax-free. The first and second tier compulsory schemes have a full tax exemption, that is contribution of 18.5% and thus the third tier voluntary scheme has a maximum tax exemption of 16.5%. However, contributors on the third tier who do not contribute to the mandatory scheme are allowed the full 35% tax exemption. Pension benefits and investment income are tax exempt and withdrawals by beneficiaries on the death of a member is tax exempt.

2.4 CONTROL OF RISK IN A DEFINED BENEFIT SCHEME

The operation of DB pension plans involve risk and whatever happens in the financial markets, these plans are responsible for provision of guaranteed retirement income, thus the inherent volatility of the financial markets make sponsors of DB schemes to manage pension risk (Cox, Lin, Tian, & Yu, 2013). Extreme negative market events are a major factor that triggers pension risk, a recent example being the late 2000s recession that reduced the average funding ratio of U.S. defined benefit pension plans to 75% at the end of 2008 and though this ratio went up to 82% by the end of 2009, it was still so low as to make the plans require additional contributions (Plantanakis & Sutcliffe, 2015).

Volatility of the funding level and the contribution rate of a DB scheme are influenced by a variety of factors including the amortization strategy and amortization period, the delay period and the frequency of valuation (Huang & Cairns, 2006). According to Lee (1986), Haberman, Butt & Megaloudi (2000), pension plan managers are usually concerned with the volatility of contribution rates (contribution rate risk) and volatility of funding levels (solvency risk). Contribution rate risk refers to the extent of deviation of contributions from the normal plan cost and is related to stability of the plan whereas solvency risk refers to the safety of the plan and is a measure of deviations of the fund from the actuarial liability (Josa-Fombellida & Rincon-Zapatero, 2004).

Maurer, Mitchell, and Rogalla (2009) added that when managing pension plans, it is the wish of sponsors and beneficiaries that the assets of the plan will be adequate to match promised benefit payments while preferably ensuring smooth rates of contribution with time. Thus by pursuing a stable contributions strategy, the DB pension scheme will be able to minimize risk (Lee, 1986).

Huang & Cairns (2006) investigated the control of defined-benefit pension plans and found that conventionally, the rates of contribution for defined-benefit plans are set without reference to the funding levels and without making allowance for current interest rates and therefore they considered

how to use market information to reduce volatility in contribution rates. They proposed a model where interest rates were stochastic and rates of return were random by extending previous works in two ways; first by introducing a model for short-term interest rates which could be used to help control volatility in contribution rates and secondly by modelling three assets (equity, bonds and cash) instead of the usual single asset modelling to allow for comparison of different asset strategies. They developed a formulae for unconditional means and variances and discussed how variability can be controlled by setting rates of contribution with reference to current interest rates and funding levels.

2.4.1 Risk Control by Total Pension Cost and Pension Underfunding

Extant literature suggests that one way of managing pension risk is to control total pension cost, which is all costs and penalties associated with normal contributions, withdrawals and supplementary contributions (Maurer et al. 2009) and another way is to manage pension underfunding. Pension underfunding is defined by Haberman, Butt and Megaloudi (2000) as shortfall of pension assets to cover pension liabilities.

In the vein of control of total pension costs, Delong, Gerard, & Haberman (2008), dealt with the contribution and asset allocation strategies in the accumulation phase of a defined benefit plan by considering a single cohort of workers and investigating a retirement plan in which an accumulated fund is converted into a life annuity. They postulated that the price of an annuity in the future, and as a result, the liability of the fund, is uncertain but the fund manager has control over contribution rate and investment strategies and is interested in covering the random claim. They thus considered two mean-variance optimization problems with an additional constraint on the expected value of the terminal surplus of the fund. The financial market used in their work was restricted to a risk-free asset with a constant force of interest and a risky asset whose price was driven by a Lévy process whiles

the evolution of mortality intensity was described by a stochastic differential equation driven by a Brownian motion. They then applied techniques from stochastic control theory in order to find optimal strategies.

By considering a continuous time dynamic pension funding model in which plan benefit liabilities are random and given by a geometric Brownian process, Josa-Fombellida & Rincon-Zapatero (2004) studied three different situations regarding the investment decisions taken by the sponsoring employer in a defined benefit pension plan: in the first, the fund invests in a constant, risk-free instrument; in the second, the promoter invests in a portfolio with a risk-free security and a number of risky assets and; finally, the rate of return was supposed to be stochastic. They studied cases where the optimal behavior led to a spread method of funding by modelling the preferences of the plan manager such that the main objective is to minimize both the solvency risk and the contribution rate risk by minimizing a convex combination of the risks and the squared-difference between pension funds and liabilities.

Maurer et al., (2009) went a step further by imposing a Conditional Value at Risk (CVaR) constraint to control excessively high total pension cost when they minimized the variance of plan contributions. By means of a Monte Carlo framework, they analysed the rewards and risks of switching from an unfunded DB pension scheme to a funded one for German civil servants making room for different strategic investment patterns and contribution rates. They integrated a CVaR restriction on overall plan costs into their objective of controlling the volatility in annual contributions. After evaluating the rate of contribution that would fully fund projected pension liabilities for present and future members of the scheme, they identified the investment strategy and optimal rate of contribution that minimizes the volatility in annual contribution rates while controlling worst case plan costs. They also analysed the time-path of worst case contribution rates and expected plan cost to investigate the

possibility of reduced contribution rates. Their results showed that transitioning toward a funded defined benefit pension scheme can be beneficial to German civil servants and taxpayers.

Papers that focus on the control of pension underfunding include Haberman, Butt, & Megaloudi (2000), which investigated the stability of contributions and the security of a DB pension fund by assessing two types of risk; that is contribution rate risk and solvency risk in the defined benefit pension plan by presenting a stochastic investment model for the DB plan in the presence of independent and identically distributed real rates of return. Haberman et al. (2000) used the spread method under which the unfunded liability is spread into the future to adjust surpluses and deficiencies. By introducing a performance criterion to deal with the simultaneous minimisation of these two risks using the spread period and the fraction of the unfunded liability paid off, they were able to provide numerical optimal values for the unfunded liability paid off and the spread period. Their results led to conclusions about the optimal contribution rate subject to the constraints needed for the convergence of the performance criterion.

Another paper that looked at the control of pension risk via the control of pension underfunding is by Josa-Fombellida & Rincon-Zapatero (2006). In this paper the optimal control of a dynamic pension fund is studied. The pension plan sponsor continuously manages time-varying funds to cover the pension liabilities to employees at the age of retirement. By allowing the manager to choose the contribution rate to the fund, the proportion of investment in any given number of risky assets and a fixed income security, the problem they formulated was that of maximizing the probability that the fund assets achieve some prescribed goal before some undesirable lower value, or ruin point is first reached. They investigated the problem of minimizing the expected discounted cost of reaching a ruin point and finally analysed the maximization of utility function when the fund can suddenly terminate. The main finding of their paper was that optimal investment policies were of constant

proportionality type.

Cox, Lin, Tian & Yu (2013), on the other hand controlled both total pension cost and pension underfunding when they proposed a model for a DB scheme to minimize total funding volatility while controlling expected total pension cost and funding downside risk throughout the life of a pension cohort by imposing a constraint to specify expected total pension cost and a CVaR constraint to control downside risk from pension underfunding. They investigated the scheme's optimal rate of contributions and asset allocation strategies given random mortality evolutions and projected stochastic asset returns and then proposed that to manage longevity risk the scheme can use either the excess-risk hedging strategy or the ground up hedging strategy. Their numerical examples demonstrated that the scheme transfers more unexpected longevity risk with the excess-risk strategy due to its lower total hedge cost and more attractive structure.

2.5 ASSET LIABILITY MODELLING

This study is conducted primarily by matching pension fund experience at a decision moment with the liability at that moment hence literature on asset-liability modelling is of relevance.

Asset-Liability Management is defined by the Society of Actuaries (1998) as follows “Asset Liability Management (ALM) is the practice of managing a business so that decisions on assets and liabilities are coordinated. It can be defined as the ongoing process of formulating, implementing, monitoring and revising strategies related to assets and liabilities in an attempt to achieve financial objectives for a given set of tolerances and constraints.... ALM is relevant to, and critical for, the sound management of the finances of any institution that invest to meet liabilities” (Society of Actuaries, 1998). Thus ALM involves the coordinated management of assets and liabilities, optimizing the asset mix that effectively offsets each liability decision moment with the optimal asset combinations.

According to Plantanakis & Sutcliffe (2015) there are various of methods for deriving the optimal ALM strategies for pension funds, and they fall into four main categories; (1) stochastic programming as used by Kusy & Ziemba (1986), Kouwenberg (2001), Kouwenberg & Zenios (2006) and Geyer & Ziemba (2008); (2) dynamic programming as in Rudolf & Ziemba (2003) and Gao (2008); (3) portfolio theory, as in Sharpe & Tint (1990); and (4) stochastic simulation, as in Boender (1997).

The extant ALM perspective involves on the one hand a model that generates scenarios, and on the other hand a mathematical programming model that uses the information from the scenarios generated to find the optimal portfolio of assets that satisfy some given set of constraints. The behavior of the assets and liabilities are generated from a great number of scenarios conditioned on the environment in which the asset and liability decision moments are situated (Platanakis & Sutcliffe, 2014).

The methods used to obtain the scenario generation model may be based on cascade structures following the work of Mulvey & Thorlacius (1998), and of Wilkie (1986, 1995); time series with an Autoregressive Vector Error Correction Model following the line set out by Boender et al. (1995, 1998), Dert (1995, 1998), Kim, Malz & Mina (1999) and Platanakis & Sutcliffe (2014); and also time series with Vector Auto Regressive models as in Kouwenberg & Zenios (2001) and Mitchell, Maurer & Rogalla (2009).

2.6 CHAPTER SUMMARY

In this chapter, we have looked at extant literature pertinent to the research we are conducting. We have reviewed literature on the pension system in Ghana with a focus on its evolution and metamorphosis to the current three-tier structure which mimics the global trend in pensions' administration. Risk control of defined benefit plans in ALM settings were also reviewed.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 INTRODUCTION

Traditional studies have usually used a second order polynomial cost function as a plan manager's objective function in minimizing risk, typically measuring squared deviations of present rates of contribution and levels of funding levels from their mean values as a measure for contribution rate and solvency risk respectively (Haberman & Sung, 1994, 2005; Haberman 1997a, b; Owardally & Haberman, 2004; Ngwira & Gerard, 2007).

Our criterion for a risk measure is the Conditional Value at Risk (CVaR) criterion. According to Artzner, Delbaen, Eber & Heath (1997, 1999), Rockafellar & Uryasev (2002), the CVaR of a parameter at the α percent confidence level is defined as the parameter's expected value conditioned that its realized value surpasses the Value at Risk for that level.

Bogentoft, Romeijn & Uryasev (2001), included CVaR constraints on funding levels for a pension plan manager's objective of seeking to minimize cost. Haberman et al (2003a, b) investigated the trade-off between CVaR on plan solvency (mean shortfall risk) and excess contribution rate risk. Maurer, Mitchell, & Rogalla (2008a, b, 2009) also employed the CVaR criterion by directly minimizing the CVaR of total pension costs in their investigation of funding, asset allocation and risk budgeting.

We identify rate of contribution and asset allocation strategy that minimize contribution rate risk, as in Haberman, Butt, & Megaloudi (2000) and Mitchell, Maurer & Rogalla (2009), by concentrating on the CVaR of the stochastic present value of plan costs in order to compare different intertemporal

contribution and investment paths on a common basis. We can then define the maximum admissible CVaR of plan cost.

Maurer, Mitchell, & Rogalla (2009), showed that focusing on cost control only require a high regular contribution rate while low regular contribution rates and low risk asset allocations results from focusing only on contribution rate volatility, thus combining the two approaches can produce low contribution rates accompanied by minimum volatility equity portfolios that keep total pension costs under control. We will therefore follow a similar method.

3.2 AREA OF STUDY

The study area for the research is Ghana and specifically the SSNIT pension's scheme which caters for the 1st Tier public pension provision.

3.3 DATA FOR THE STUDY

The SSNIT pension's office provided us with anonymized data of active members on the scheme, showing the age distribution, number of members, their average contributions in months out of a year and their average annual salary. Our analysis is based on the 2014 cohort as the basis population on which we conduct our assessment since it is the most recent available data. This data represents the entire formal sector employees in the country that are on the SSNIT's 1st Tier Pension Scheme.

The total membership of the cohort is 1,099,091 with an average age of 37.8years earning an average annual salary of GHS 10,267.30. The data was not gender-segregated.

Data was also collected on Treasury bill rates (bonds) from the Bank of Ghana and the Ghana Stock Exchange Composite Index (previously All Share Index) returns data from the Ghana Stock

Exchange for the period 1990 to 2014. We also collected GDP growth rate, inflation and daily minimum wage data for the same period as the SSNIT data.

In this study all reference to our bond assets refers to the GOG 91-day Treasury Bills issued by the Bank of Ghana and all reference to our equity assets refers to the Ghana Stock Exchange Composite Index (formerly called the GSE All Share Index).

3.4 COHORT DYNAMICS AND ASSUMPTIONS

This analysis is conducted on a closed cohort of scheme members made up of current employees. Unfunded past service liabilities are incorporated into the current valuations by applying the average number of service years to the entire cohort. Projecting the cohort population entails simulating the progression of salary and age for the existing members, as well as making assumptions about new entrants. According to Mitchell, Maurer & Rogalla (2009) employee turnover is insignificant for reasons other than age retirement, therefore we made the assumption that all workers will remain active until age 60 and retire with certainty. Any vacated position in the cohort is assumed to be filled by a new recruit with the same entry age and employment dynamics as the person who vacated the position. Thus each new entrant's starting age is set to the empirical mean age of new entrants, accounting for service credited as prior creditable years or average time spent in position-related education. New hires receive the age-related remuneration for their positions.

To value the employee pension liability profiles, we use scheme specific mortality tables of the SSNIT scheme. Salary growth, investment returns and inflation have impacts significantly on pension plan valuation and therefore must be taken into account when predicting future benefit accruals (Hustead & Mitchell, 2001). Pension liabilities are sensitive to the chosen discount rate used in their valuation due to the long durations they usually have. We adopt the economic approach in our

analysis of pension valuation so we discount liabilities at the rate on the 7-year quasi risk-free government bonds reflecting the government's financing costs. We set the discount rate at 23% in our base case as it is the rate on the 7-year government bond.

3.5 GENERAL METHODOLOGY OR OUR APPROACH

The pension plan of the cohort runs for 45 years and starting from the year 2014 we simulate cohort behavior up to the year 2059 by which time the last members of the cohort will proceed on pension. The primary software we used was MatLab 2013a, supported by MS Excel and the econometric software E-Views was used to cross validate the VAR model generated in MatLab.

3.5.1 Projecting Total Salary Payments in Each Year

To achieve our objectives we first of all generate the projected total salary payments in each of the 45 years of the cohort. To do this, from the base year, we multiply the average annual salary of each age in the cohort by the corresponding number of members at that age to get the total salary payments for that age at the base year. We do this for all ages in the cohort and then sum the total salary payments for the ages to arrive at the total salary payments for the year. We then apply the salary growth rate to each total salary payments of each age remaining in the cohort after each period (a year) and sum the results again to achieve the total salary payments for the next year. We repeat this procedure for the 45 years to generate a vector of total salary payments for each year.

3.5.2 Projecting Total Pension Payments for each year

From the projected salaries for each year, we take the average of the best three years' salaries of the group going into pension to calculate their pension benefits. By our approach, this best three years'

salary coincides with the last three salary payments prior to retirement. The total pension benefit for group going into pension each year is then calculated by multiplying the average of the best-three-years' salary by their pension right by the annuity factor at age 60 and by the total number going into pension. This also generates a vector of projected total pension payments for each of the 45 years.

The pension payment to the i^{th} individual on the SSNIT scheme is given by

$$Pension_{i,t} = 0.60181 \cdot S_{60,i} \cdot \bar{a}_{60} \quad \text{--- (1)}$$

Where 0.60181 is the average pension right accrued by members on the SSNIT scheme (from SSNIT data), $S_{60,i}$ is the average of the best-three-years-salaries and \bar{a}_{60} is the scheme specific annuity factor at age 60, which we approximated by;

$$\bar{a}_{60} = \sum_{t=0}^{14} (1-r)^t + (1-r)^{15} \cdot {}_{15}P_{60} \cdot \sum_{t=0}^{24} (1-r)^t \cdot {}_tP_{75} \quad \text{--- (2)}$$

Where r is the level discount rate chosen to reflect the Government's cost of borrowing, ${}_{15}P_{60}$ is the probability that a person aged 60 will survive the next 15 years and ${}_tP_{75}$ is the probability that a person aged 75 years will survive the next t years. These survival probabilities are scheme specific and was taken from the SSNIT mortality tables (see Appendix C). For the SSNIT scheme, the first 15 years payment of pension benefits is guaranteed and that is captured by the first summation in equation (2) and thereafter payment of pension benefits is contingent on the pensioner being alive after age 75 years. It is also assumed that by age 100 years all pensioners will not be alive to collect benefits, and this dynamic is captured by the second summation in equation (3) (see Promislow, 2011). We generated the two vectors from above in Microsoft Excel and called them initial variables (refer to Appendix E) in our MatLab codes (refer to Appendix F MatLab codes line 2). We load the

liabilities with 1.1% of the total salary payments as plan expenses; this loading is informed by data in the SSNIT Annual Reports from the same period as our data.

3.5.3 Steps to Achieving Objectives

For the first objective we discounted the two vectors in the initial variable matrix to the base year (2014) with our chosen discount rate and by taking the ratio of the present value of the total pension payments to the present value of the total salary payments we achieve our first objective evaluated as “det_con_rate” in our MatLab codes (refer to Appendix F line 11). Then we run the MatLab function for the VAR modelling and forecasting of asset returns which we modified from the routines created by Cesa-Bianchi (2015) and perform lag selection and stability diagnostics on the model to verify if our assumption of an ordinary least squares VAR(1) fits the historical data (refer to Appendix G).

For the second objective we conduct a 3-dimensional analysis in the x - y - z coordinate plane by running loops in our MatLab routines that results in the generation of volatilities (z -coordinate) specific to a normal contribution rate (y -coordinate) and equity weight (x -coordinate) combination, these routines also evaluate the associated worst case plan cost as another z -value in the same plane.

The first 10 lines of code (refer Appendix F) initially empties the workspace of all variables and then loads the initial variables from MS Excel separating them into their individual vectors, the required discount rate for all iterations is then set along with the penalty factor and spread period for the penalization and amortization of any over- or underfunding. The annuity factor corresponding to the chosen spread period is the evaluated. The total pension liability for each year is the evaluated as the discounted value of the future stream of pension payments up to that year (refer to code lines 12 to 15 of Appendix F).

We then move on to the asset case, where we call the forecasted asset data from the VAR function (code line 16) and generate an 11 by 2 matrix of asset weight combinations. We then use a Monte Carlo framework with 10,000 simulations to generate correlated asset paths that maintains the attributes of the historical asset data for the 45 years of our projection period (refer to code line 18 in Appendix F). From the steps mentioned so far, we have generated the global parameters that will be employed in our iterations to find the optimal contribution rate and asset allocation strategy.

Code lines 19 to 66 (refer to Appendix F) in our MatLab codes executes the iterations in search of the optimal values. We start our iterations by choosing a normal contribution rate of 0% and an asset allocation strategy of 0% in equity. We run this combination across all the 10,000 correlated asset paths generated by the Monte Carlo simulation and in so doing we generate a distribution of total pension costs from which we evaluate the 1%-CVaR of total pension costs relative to the chosen normal contribution rate and asset allocation and we also generate the associated average annual volatility by the difference between the average annual contribution for each year and the chosen normal contribution rate, squaring these differences and summing them to generate the volatility.

Now keeping the same normal contribution rate, we loop this routine across all possible equity weights in our asset allocation strategy, that is, from 0% equity weight we move to 1%, then 2% and so on till we get to 100% equity weight. Then we move on to the next normal contribution rate of 1% and rerun all the iterations. We continue in steps of 1% normal contribution rate till we get to 40% normal contribution rate as was done in Maurer et al. (2009). At the end of the iterations, we have a 41 by 101 matrix of 1%-CVaR values and a similar matrix of volatilities. We then pick the minimum 1%-CVaR and minimum volatility using code line 67 and 68 (see Appendix F) and then go into our generated matrices to locate the coordinates of minimum in the matrix as the normal contribution rate and asset allocation, we then go into the 1%-CVaR matrix to check if the CVaR associated with the minimum volatility is less than the maximum permissible CVaR constraint in our objective function.

3.6 DERIVING THE DETERMINISTIC CONTRIBUTION RATE

To derive the deterministic contribution rate that will fully fund promised benefit obligations, we analyzed pension liability patterns by expressing projected total pension costs as a fraction (percentage) of active participants' payroll by the method known as the level percentage payroll method (see McGill, Brown, Haley, & Schieber, 2005). Starting with the current population of active contributors to the SSNIT scheme and projecting it over the next forty-five years, the present values of salary payments in the future was calculated as well as future pension payments. For the forty-fifth year, we conduct a discontinuance valuation and compute the outstanding projected pension liabilities. The ratio of the present value of total pension liabilities to the present value of total salary payments is the deterministic annual contribution rate required to fully fund pension liabilities expressed as a percentage of active employees' payroll (see Maurer, Mitchell, & Rogalla, 2009).

$$Det_Con_Rate = \frac{\textit{Present Value of Future Pension Payments}}{\textit{Present Value of Future Total Salary Payments}} \text{ --- (3)}$$

Where *Det_Con_Rate* is the deterministic contribution rate required to fund the plan.

3.7 THE PENSION FUND PROBLEM

Along the lines of Bogentoft, Romeijn, & Uryasev (2001), we consider the pension fund as conducting three activities as follows; (i) collection of premiums from active plan participants (employees); (ii) investment of the funds; and (iii) payment of pensions to retired employees. We set our objective that, at each time unit, the total value of all assets should be greater than the present value of the fund's future stream of liabilities and our problem will then be the setting at each decision

moment of an appropriate contribution rate and an appropriate investment strategy for funds available to the pension fund.

With stochastic investment returns, we need to minimize the volatility of annual contributions to the plan and at the same time ensure control of plan costs. To this end, a normal contribution rate CR is chosen (as a fixed fraction of salaries) as well as a constant investment strategy by allocation of a fixed percentage of plan assets to equities and the rest to bond. We will assume that at the start of each decision period, t , the fund is endowed with the regular contributions RC_t based on the normal contribution rate and salary payments in t . These funds will be used to pay for pensions due at time t , and any remaining amount will be invested according to the asset allocation plan.

At the end of every period (decision moment) the plan's situation is assessed by comparing the current plan liabilities measured by PBO_{t+1} with the current plan assets A_{t+1} . If A_{t+1} is greater than PBO_{t+1} then lower contribution rates to the plan can be permitted but if A_{t+1} is less than PBO_{t+1} then extra contributions will be necessary to re-establish the equilibrium between assets and liabilities. The shortfall is amortized by employing the spread method described by Haberman (1997a) to determine how much additional costs must be added to cover the underfunding over a specified time period.

The fund's liability at the end of period t (at the beginning of period $t+1$), PBO_{t+1} is calculated as follows;

$$PBO_{t+1} = \sum_i \frac{0.60181 \cdot S_{60,i} \cdot \bar{a}_{60}}{(1+r)^{60-Age_{i,t+1}}} \quad (4)$$

Where for each employee i aged $Age_{i,t+1}$, $S_{60,i}$ is the projected average of three-best-years' salary at retirement at age 60, \bar{a}_{60} is the immediate pension annuity factor at age 60 specific to the plan and r is the discount rate.

We will define $UL_{t+1} = PBO_{t+1} - A_{t+1}$ as the unfunded liability at the end of period t which is to be recovered over a pre-specified time horizon m , the spread period (see Haberman, 1997a) and say that the total contribution to the plan in time $t+1$, C_{t+1} , will be the sum of the regular contributions at $t+1$, RC_{t+1} , and an adjustment payment ADJ_{t+1} , that is,

$$C_{t+1} = RC_{t+1} + ADJ_{t+1} \text{ --- (5)}$$

Where

$$ADJ_{t+1} = ANF_{m,r} \cdot UL_{t+1} \text{ --- (6)}$$

and

$$ANF_{m,r} = \left(\sum_{i=0}^{m-1} \frac{1}{(1+r)^i} \right)^{-1} \text{ --- (7)}$$

A_{t+1} is the stochastic present value of investments in equities and bond at period $t+1$ that is obtained by projecting future asset returns on investments in equities and bonds. In order to forecast A_{t+1} , we will employ the first order vector autoregressive (VAR) model which is a model that is widely used in academic literature to model stochastic asset returns (Hoevenaars, Molenaar, & Steenkamp, 2003; Campbell, Chan, & Viceira, 2003; Campbell and Viceira, 2005).

We let A_t be a vector that contains the returns of equities and bond. We will assume that the dynamics of A_t are governed by an unrestricted VAR (1) according to

$$A_{t+1} = a + bA_t + \varepsilon_{t+1} \text{ --- (8)}$$

with $\varepsilon_{t+1} \sim N(0, \delta_{\varepsilon\varepsilon}^2)$ in consonance with Maurer et al. (2009).

3.8 THE OPTIMAL FUNDING AND ASSET ALLOCATION POLICY

To derive the optimal contribution and investment policy for the scheme, a Monte Carlo simulation with 10,000 iterations is run Maurer et al., (2009). The optimal policy parameters – that is the normal contribution rate CR and equity weight X – is computed with the objective to minimize the variance of contributions around the normal contribution rate over time under the constraint that the CVaR at the one-percent level of the stochastic present value of total pension costs (TPC) does not exceed a pre-specified level, c, which is chosen as the worst total pension cost from the deterministic case.

Thus the objective function is;

$$\min_{CR, x} E \left[\sum_{t=1}^{T-1} \left(\frac{C_t}{Salary\ Payments_t} - CR \right)^2 \right] \text{----- (9)}$$

subject to: CVaR_{1%}(TPC) ≤ c

Where the stochastic present value of total pension costs TPC is calculated as was done in Maurer, Mitchell, and Rogalla (2008a, b) as;

$$TPC = \sum_{t=1}^T \frac{RC_t + (1 + penaltyfactor) ADJ_t}{(1 + r)^t} \text{---- (10a) } \quad \text{if } ADJ_t > 0$$

or

$$TPC = \sum_{t=1}^T \frac{RC_t + (1 - penaltyfactor) ADJ_t}{(1 + r)^t} \text{---- (10b) } \quad \text{if } ADJ_t < 0$$

The discount rate r in all the equations above is fixed to reflect the government’s financing cost and RC_t is the regular contributions in time t based on the fixed normal contribution rate CR and the

payroll size in time t and ADJ_t is the adjustment payment in time t as calculated from (6) above. The $\alpha\%$ -CVaR of TPC is defined as;

$$CVaR_{\alpha\%}(TPC) = E[TPC | TPC > VaR_{\alpha\%}(TPC)] \text{ --- (11)}$$

From equation (7) the c in the objective function constraint is interpreted as the worst case plan cost and can be estimated deterministically by analyzing pension liability patterns and employing the aggregate level percentage payroll method as was done by McGill et al. (2005).



CHAPTER FOUR

DATA COLLECTION AND ANALYSIS

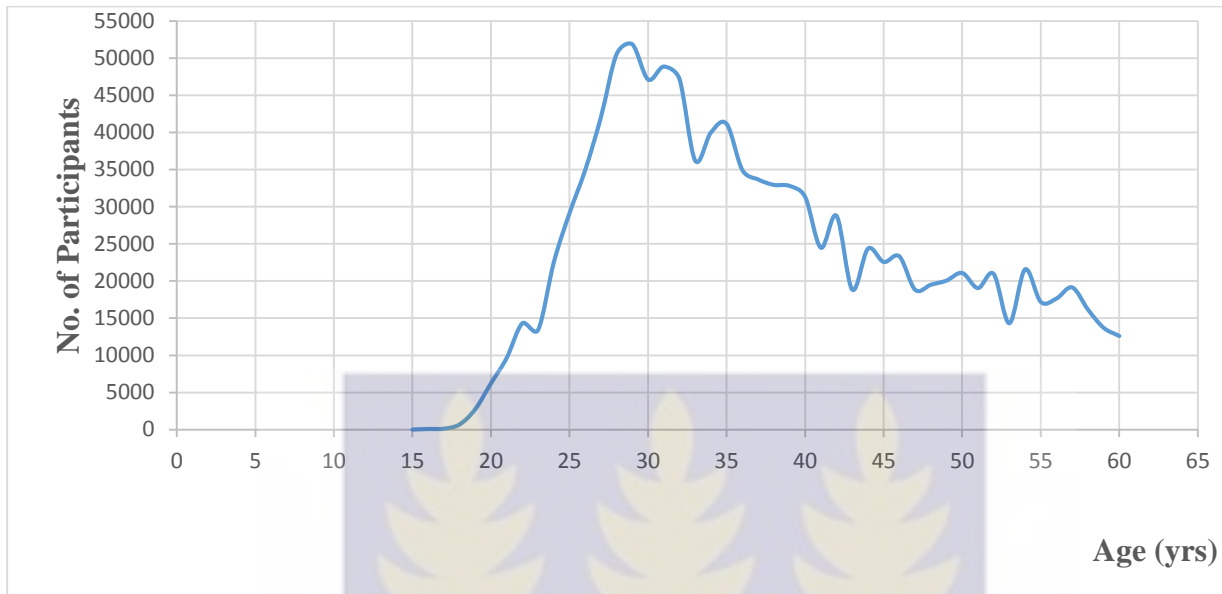
4.1 INTRODUCTION

The pension plan manager is free to choose the spread period and the normal contribution rate applicable to the plan at the outset of the plan. For each contribution rate and spread period the plan manager chooses, he is faced with a myriad of total pension costs and annual contribution rate volatilities given the investment strategies he will adopt in the course of the plan's existence. In this chapter we are going to analyze the data collected after we have put it through the rigors of the methodologies described in the previous chapter.

4.1.1 SSNIT Cohort Data

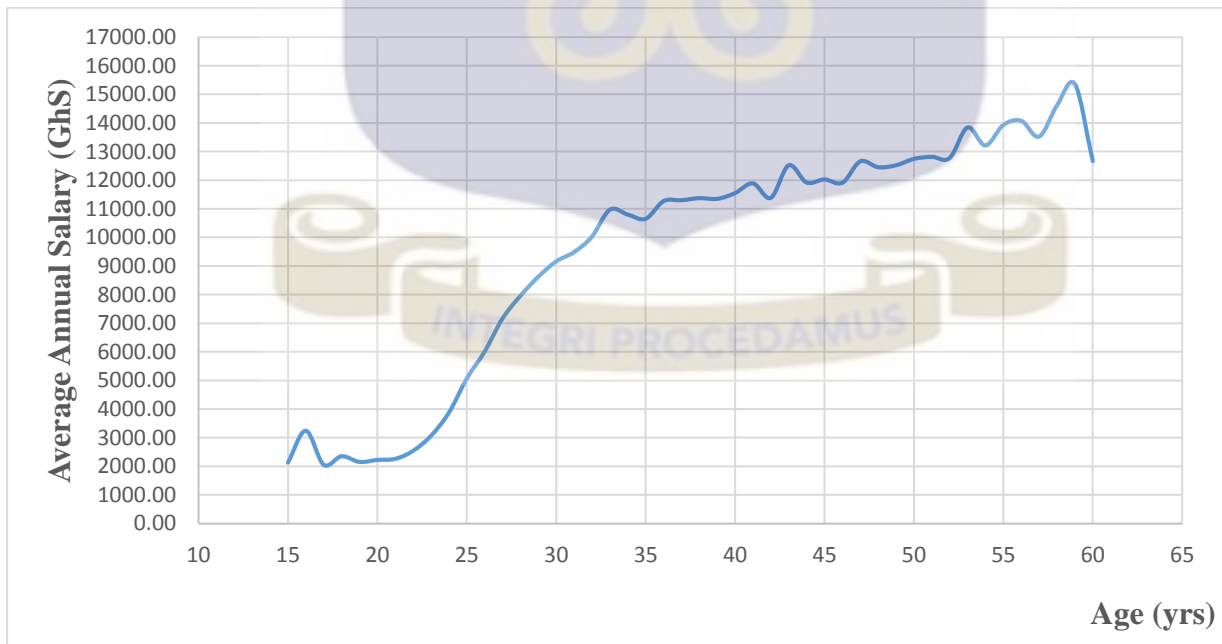
The cohort data from SSNIT (see Appendix B) has a mean age of 37.8 years and an overall average annual salary of GhS10267.30. The distribution of the age and average annual salaries for the base year of 2014 is as shown below in Figure 4.1 and Figure 4.2 respectively. The age of entry into the scheme is 15years and age-retirement is at 60years. For the purpose of this study we adopt the assumption of Maurer et al. (2009) that all withdrawals from the scheme apart from age retirement are not significant and that all plan participants will retire with certainty. This assumption is tenable because the mortality distribution from SSNIT shows that all the ages in the chosen cohort are expected to live beyond age 60 years. This is evidenced from the life expectancy at all the ages in the cohort that from 15yrs to 60 years. Figure 4.3 below shows the age distribution of the cohort with their life expectancies superimposed to show their expected terminal ages.

Figure 4. 1: Curtate Age Distribution of Plan Cohort at Base Year 2014



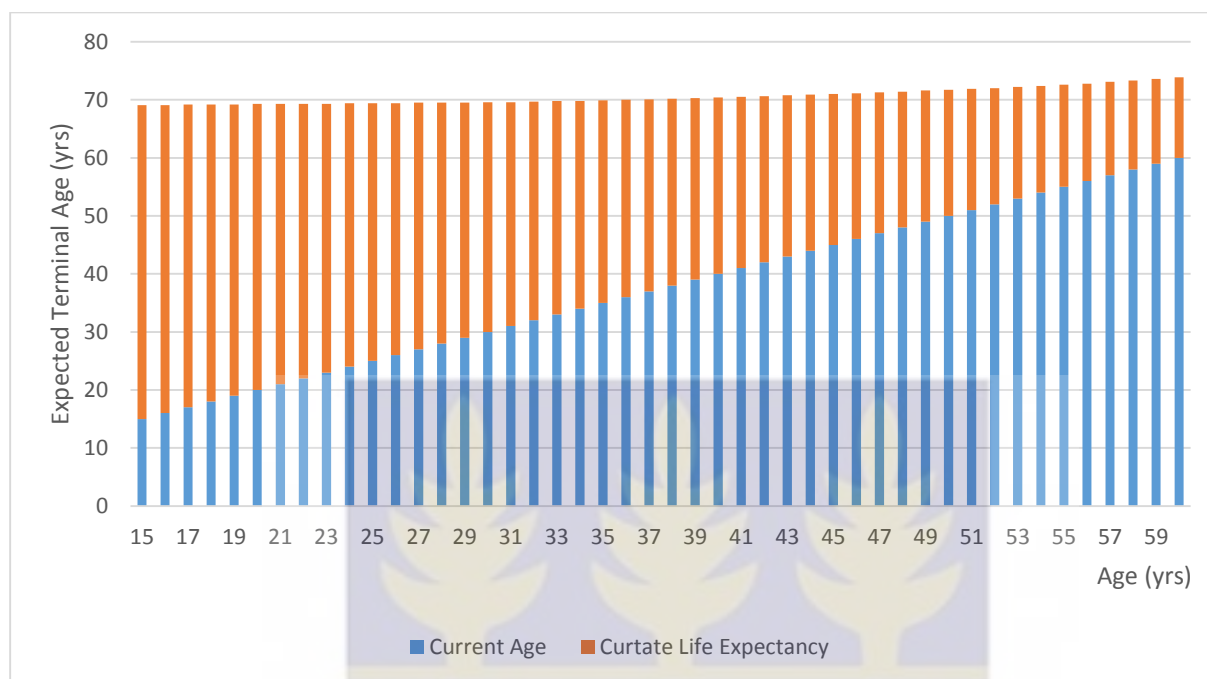
Source: Author's Computations.

Figure 4. 2: Average Annual Salary Distribution of Plan Cohort at Base Year 2014



Source: Author's Computations.

Figure 4. 3: Cohort Life Expectancies



Source: Author’s Computations.

4.1.2 Salary Growth Rate

The daily minimum wage growth rate was found to be unusually higher than the GDP growth rate and this is intuitively not sustainable because the wage growth rate feeds directly into the GDP growth rate hence we surmise that there are other exogenous forces causing this phenomena. We found that the wage growth was also highly volatile (refer to Figure 4.4) recording a volatility of 23.07% as against GDP’s 2.45% and inflation’s volatility of 13.61%. We therefore assumed that any prediction of wage growth rate (given the data length that we have) will be spurious hence we took the mean real growth rate for the historical period, evaluated as the average of the difference between the wage growth rate and inflation. The value obtained was 7.09% which we rounded off upwards to 8% for our computations (refer to Appendix D for the full wage-GDP-inflation distribution).

Figure 4. 4: Rates Comparison - Wage Growth with GDP Growth & Inflation (1990-2014)

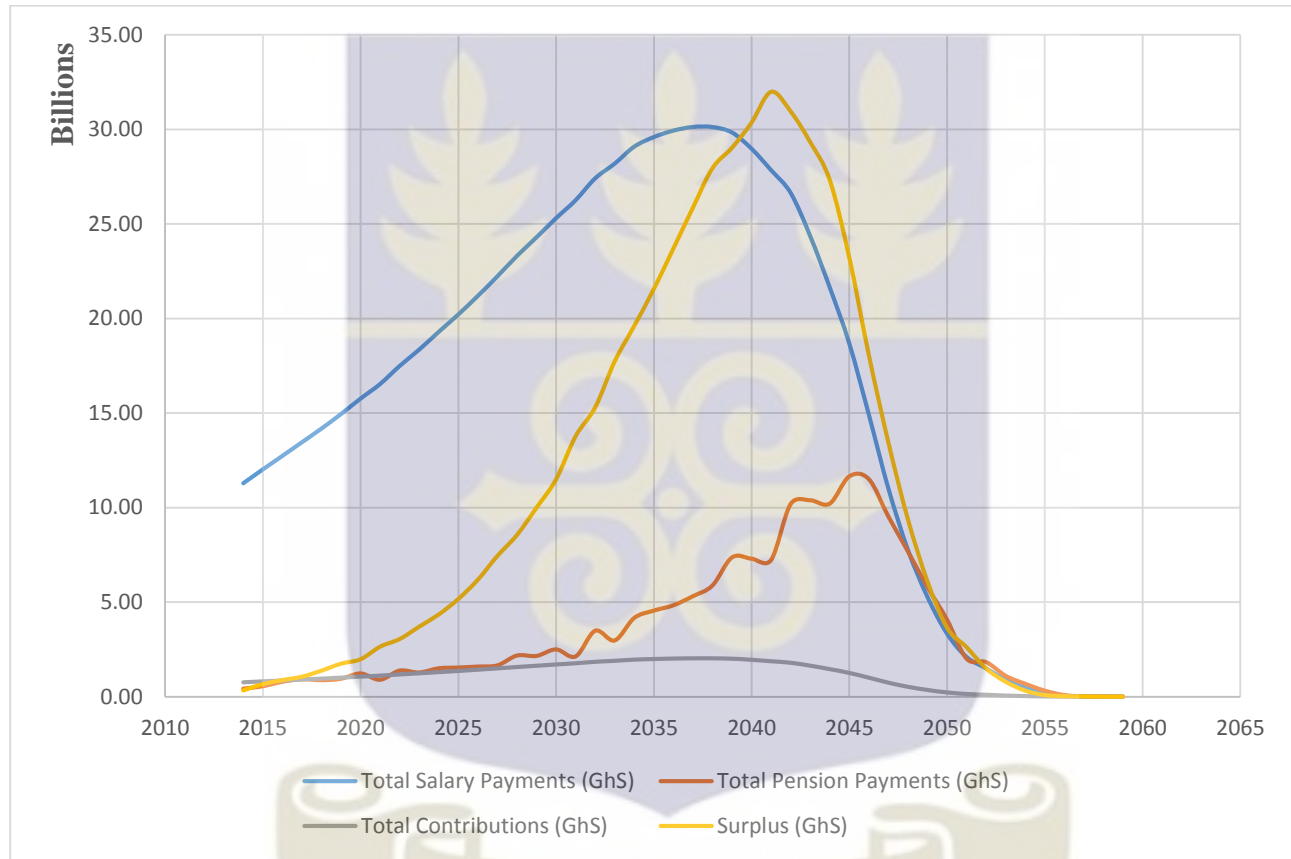
Source: Author's Computations.

4.2 DETERMINISTIC CONTRIBUTION RATE TO FULLY FUND THE PLAN

A plan manager seeking to fund their pension scheme must analyze pension liability patterns so as to evaluate the rate of contribution adequate to fund plan liabilities. To derive the plan's contribution rate, we used the aggregate level percentage of payroll method as employed by McGill et al (2005) and Maurer et al (2009) which expresses projected plan costs as a percentage of plan participants' payroll and thus by applying equation (3) from above using the discount rate of 23%, we found that a 6.73% level annual contribution rate will fully fund the pension plan in the deterministic setting of a constant discount rate of 23%. Figure 4.3 below shows the behavior of the plan through time with this deterministic rate (refer to Appendix A for more details). The surplus column shows the amount leftover when contributions are collected and pensions are paid out for the year. This surplus is then

accumulated by the rate used to discount the present values. We see that there is no terminal surplus or deficit showing that the plan has been fully funded by the chosen contribution rate without any surplus value remaining at the deterministic discount rate.

Figure 4. 5: Funding the Plan at the Deterministic Rate



Source: Author's Computations.

For each year the total liabilities are taken out from the sum of the contributions for that year and the accumulated surplus from the previous year and we notice from the graph that at the plan terminus the surplus goes to zero.

4.3 PREDICTING FUTURE RETURNS

To show the calibration of economic factor values and asset return system of equations, Zenios (2003) and Boender (1997) modelled the inherent risk factors by scenarios rather than probability distributions. Zenios (2003) considered a small set of broad asset classes: deposits, bonds, real estate and stocks and following Boender (1997) calibrated the VAR model with wage growth rate. Zenios (2003) cautioned that the specification of the VAR model should be chosen with care contending that although some inter temporal relationships between asset returns may be weakly significant based on past data, these relationships may not be useful in generating scenarios for financial optimizations with a long time horizon hence to avoid spurious and unstable predictability of returns, he did not use lagged variables of the assets to explain the assets' returns but retained memory for the time series of wage growth rate and return on deposits only and modelled the whole system by a first order VAR model. Maurer et al. on the other hand retained memory for all the asset classes they used for their VAR (1) model. We can infer from Zenios approach that when we make the assumption that the assets will retain their structure going into the future, then we can simply take the mean and standard deviation of the historical asset returns and simulate many future scenarios (say 10,000) by the Monte Carlo method for analysis.

What we did with the data we had, was to fit a VAR (1) model in MatLab (refer to Appendix F) and using the Cholesky decomposition of the variance-covariance matrix from the model we generated correlated normally distributed random errors which was then added to the model to forecast the future returns of the assets. Table 4.1 below summarizes the attributes of the historic returns data and the forecasted returns data for comparison. We can infer from the table that the general structure of the returns from the historic data is maintained in the forecasted returns.

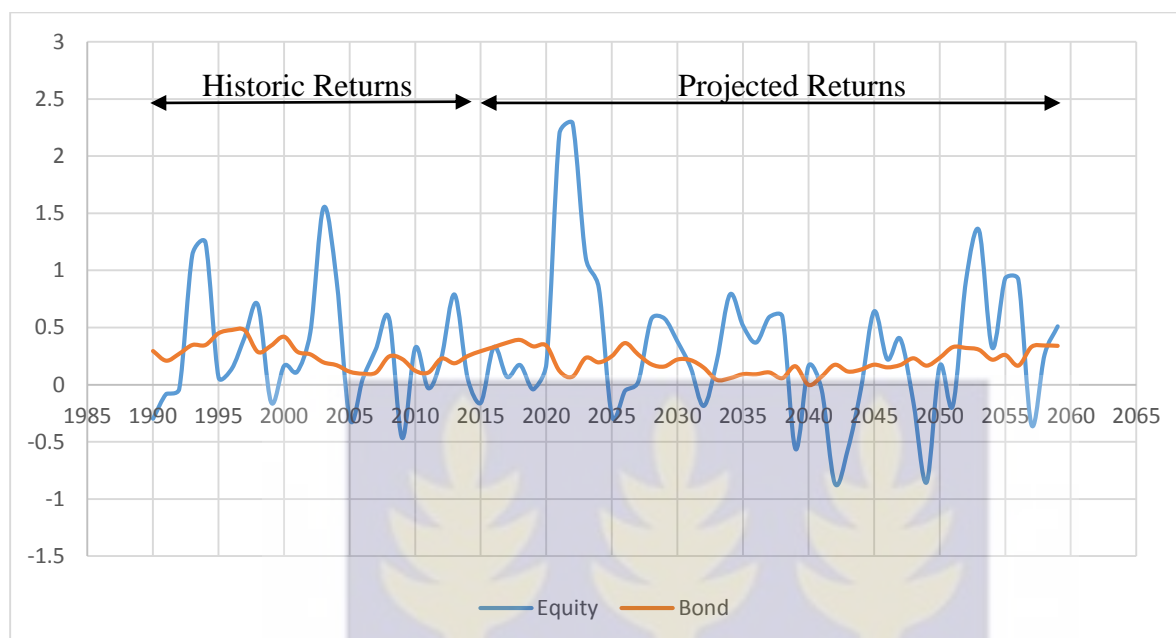
Table 4. 1: Summary of Historic and Projected Asset Return Characteristics

	<u>Expected Returns</u>	<u>Standard Deviations</u>	<u>Correlations</u>	
<i>Historical</i>				
Equity	31.53%	49.66%	Equity	Bond
Bond	26.13%	11.27%	1	0.053
<i>Forecasted</i>				
Equity	31.49%	62.76%	1	
Bond	20.87%	10.12%	-0.17	1

Source: Author's Computations.

We see that the correlation between the equity and bond is minimal (less than 20%) in both periods and in the forecasted period the correlation is negative, which is good for investment from a risk adjustment point of view. Also the expected return and volatility of the equity is higher than the expected return and volatility of the bond in both the historic and forecast periods. These observations are in concurrence with literature which posits that equity is more volatile than bonds and that equity is expected to do better than bonds in the long run (see Jorion & Goetzmann (1999), Keim & Ziemba (2000), Dimson, March & Staunton (2002), and Ziemba (2003). This relationship is seen in Figure 4.6 below, thus we posit that our asset returns forecast is good for analysis (refer to Appendix G for complete VAR Output).

The VAR (1) model was generated again with the econometric software EViews and the coefficients from both the EViews model and MatLab model were an exact match so we tested for the lag selection criteria and performed stability diagnostics in EViews. The lag selection criteria confirmed that a VAR (1) fits our data and the stability test also confirmed that an unrestricted VAR (1) model fits the data. The forecast and historical returns showed continuity in behavior and we attribute this to the Cholesky factors introduced in the forecasting given that we had a good VAR (1) model to begin with (see Appendix D for individual nature of historical and forecasted asset returns).

Figure 4. 6: Asset Returns Behavior over the Historic and Projection Periods

Source: Author's Computations.

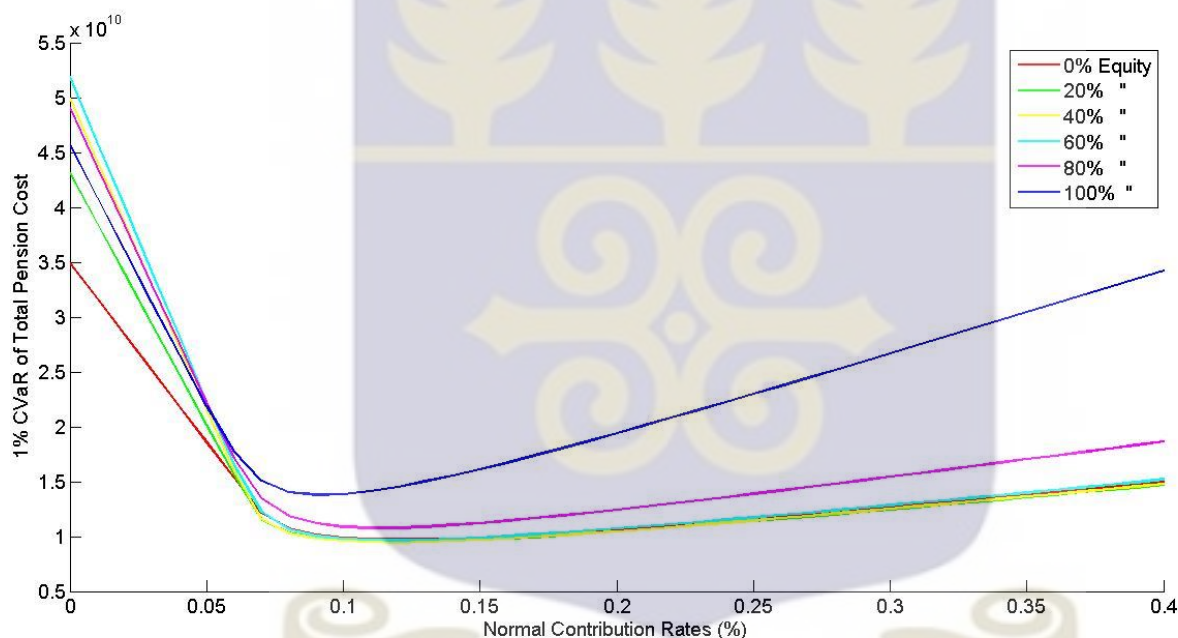
4.4 WORST CASE PENSION COSTS

To obtain optimal plan parameters, Monte Carlo simulation with 10,000 iterations was generated and the 1%-CVaR of total pension cost was constrained not to exceed Gh\$45.14 billion; this amount was chosen along the lines of Maurer et al. (2009) from the highest pension liability (worst case cost) in the deterministic case.

For the base case of the worst case pension plan costs, we assumed a spread of 20 years with 20% penalty factors for any adjustment payments in line with the works of Haberman (1997a) and Maurer et al. (2009), a discount rate of 23% to reflect the government's cost of borrowing on the longest term maturity bonds in the market to discount future pension payments, a salary growth rate of 8% annually. The growth in salaries in our data averages 24% but intuitively this growth is unrealistic in the face of the fact that the GDP growth rate is less than 10% on average hence we subtracted the current level of inflation from this growth rate to arrive at the 8%.

Consistent with the findings of Maurer et al (2009), we also found that the 1% conditional value at risk of total pension cost initially decreases to a point and then it rises steadily as the normal contribution rate is increased from 0% to 40% for all asset combinations. Figure 4.5 below shows this phenomenon; it is a plot of the 1%-CVaR of total pension cost against normal contribution rate for selected asset allocations beginning with 0% equity weight and continuing in steps of 10% equity weight additions.

Figure 4. 7: Behavior of 1%-CVaR of Total Pension Cost with Increasing Contribution Rates

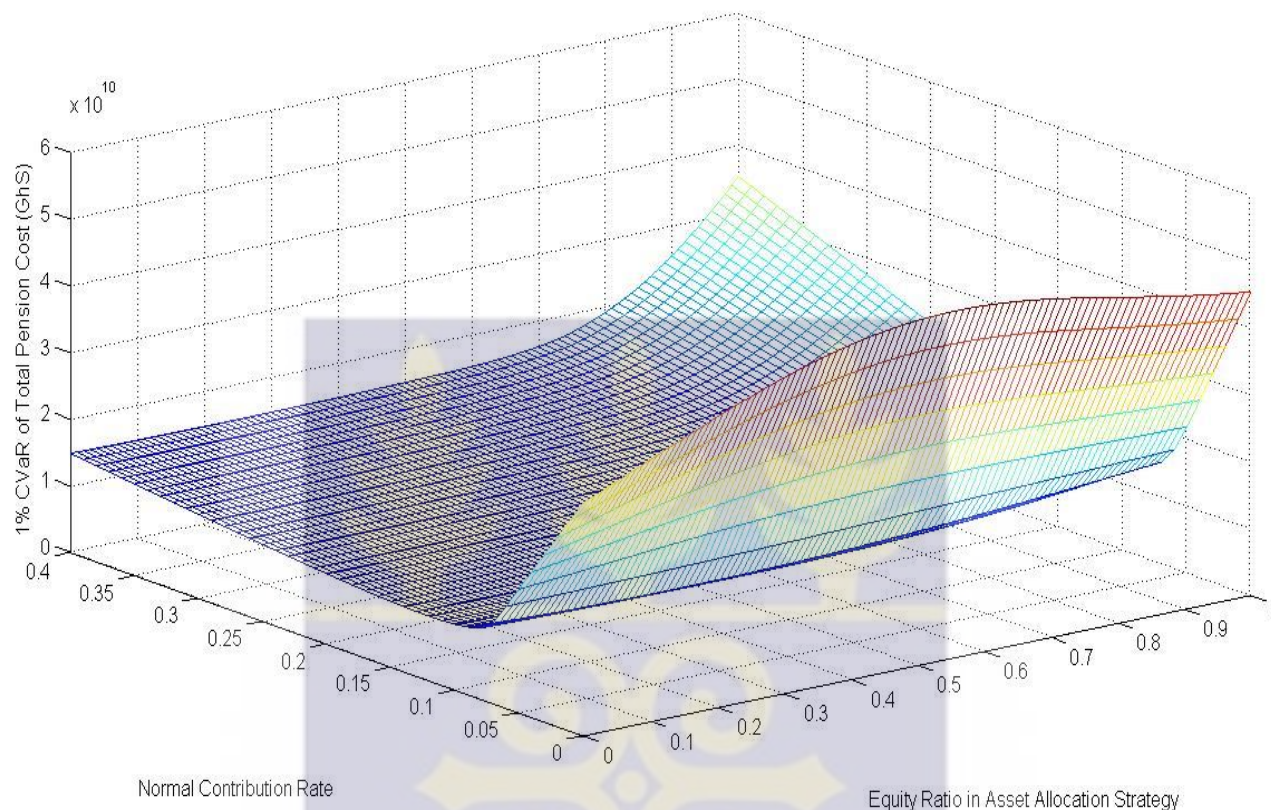


Notes: Assumption of 8% salary growth rate, 20% penalty factor and 20 years spread period.
 Source: Author’s Computations.

We notice from Figure 4.5 above that the minimum worst case plan costs occurs between 5% and 15% normal contribution rates for each asset allocation strategy and in fact the overall minimum worst case plan cost occurs at 12% normal contribution rate. The total worst case pension cost is GhS34.89billion if the normal contribution rate is set at 0% and all plan funds are invested in bonds. Increasing the normal contribution rate but keeping the same asset allocation strategy has the effect

of decreasing the total pension costs up to a normal contribution rate of 12% and thereafter the cost increases. This phenomenon was explained by Maurer et al (2009) as the effect of the penalties levied on supplementary payments. They argued that with normal contribution rate set at 0%, all pension payments and the amortized adjustment payments must be financed by the penalized supplementary contributions, thus higher normal contributions that impose no additional penalty reduces the amount of the more expensive supplementary payments and hence reduces the worst case pension costs. Furthermore, when the normal contributions are set higher than a certain threshold the plan exhibits structural overfunding, thus requiring negative supplementary payments, meaning reducing future contributions and even withdrawing funds if possible; this has the effect of increasing the worst case plan costs because the supplementary payments are levied. In our case keeping all investments in bonds, the overall plan cost falls to GhS9.73billion at a contribution rate of 12% and finally for a high contribution rate of 40% with all plan assets still invested in bonds, the overall worst case plan cost rise to GhS14.91billion.

We also found out that up to a normal contribution rate of 5%, adding more equities to the investment mix causes the overall pension cost to rise and then fall as the equity weight crosses over the 50% mark, whereas beyond a normal contribution rate of 6% the plan cost initially reduces up to a point as more equity weight is added to the asset mix. Recall that the deterministic contribution rate that can fund the plan was found to be 6.73%, thus for low contribution rates of 0% to 5%, the plan is chronically underfunded and has to rely on the expensive supplementary payments hence the rise in the pension cost but this mitigated by the expected high returns of the equity investments, thus by intuition as the equity weight increases, the expected high returns of equity causes reductions in the need for the supplementary payments until it gets to the point where the equity input has so much reduced the need for supplementary inputs that it causes the total pension cost to fall.

Figure 4. 8: Worst Case Pension Cost Outcomes – The Overall Picture

Notes: Assumption of 8% salary growth rate, 20% penalty factor and 20 years spread period.
Source: Author's Computations.

What we are showing here is that the supplementary payments may be the culprits in the rise of the pension cost when normal contributions are between 0% and 5% and we see that as the normal contribution increases between 0% and 5%, the gradients of the rise and fall of the pension cost reduces and this can be explained by the assistance that the increasing equity weight gets from the increasing normal contribution rate in curbing the need for supplementary payments. Now beyond the 5% normal contribution rate, we see that as we increase equity weight from 0%, the worst case pension cost falls up to a point and then picks and rises steadily. Maurer et al. (2009) explained this as the effect of diversification that as equity is added to a pure bond portfolio, (recall our predicted assets are negatively correlated albeit to a low degree), the capital market risk of the pure bond

investment is reduced as the portfolio becomes more diversified. Now, this is the risk that will kick-in the supplementary payments therefore if it is reduced it follows that less supplementary transactions will be executed thus causing the worst case pension cost to reduce. As we continue to add equity, the well diversified portfolio threshold will be breached, and we will be replacing the bond capital market risk with the more volatile equity thus reenergizing the need (size and likelihood) for supplementary payments and hence the rise in worst case pension cost after its fall when we began to add equity.

For a low normal contribution rate of 0%, minimum worst case pension cost of GhS34.89billion is achieved with all plan assets invested in bonds as noted above whereas pension cost is GhS45.53billion if all plan assets are invested in equities at the same normal contribution rate of 0%.

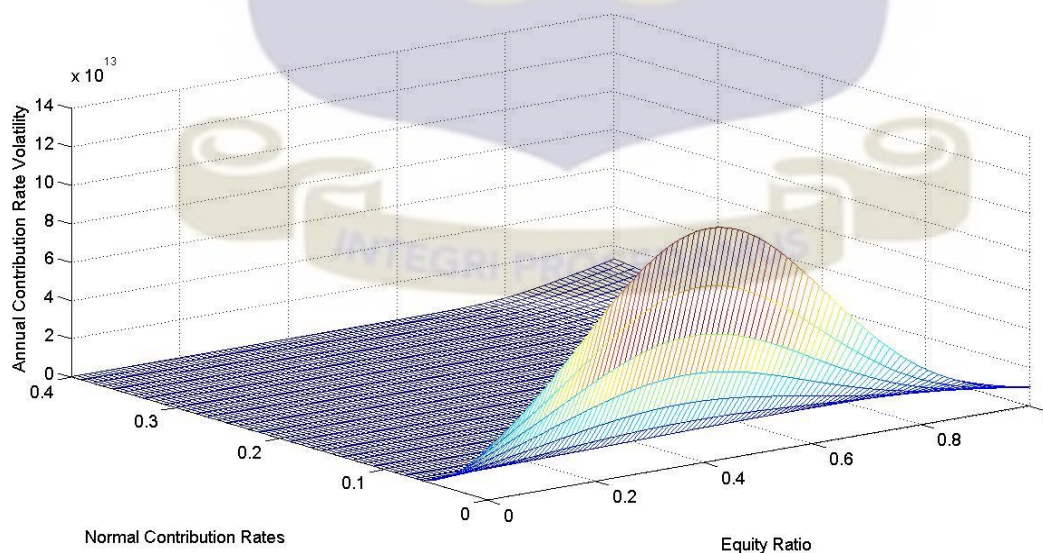
For a high normal contribution rate of 40%, minimum worst case plan cost of GhS14.70billion is achieved with a 23% equity weight allocation in assets whereas worst case plan cost is GhS35.34billion if all plan assets were invested in bond. The overall minimum worst case total pension cost is GhS9.47billion and it is achieved by selecting a normal contribution rate of 12% and investing 33% of plan assets in equities.

We summarize the foregoing results for the base case and conclude that the plan manager who wants to minimize overall worst case pension cost will select a normal contribution rate of 12% and invest 33% of plan assets in equities given the assumptions that salaries will grow at a constant rate of 8% and the discount rate is chosen at 23% using 20 years to spread the amortization of any over- or underfunding situation that occurs in the plan.

4.5 CONTRIBUTION RATE VOLATILITY

The contribution rate volatility is the deviation of the annual contribution rates (defined as the percentage of salary payments equaling total pension costs per year) from the normal contribution rate set for the plan. A zero volatility annual contribution rate implies that the total pension cost for the year is exactly the same as expected, and this happens in the deterministic case earlier evaluated. Different normal contribution rates paired with different asset allocations generates different contribution rate volatilities. Figure 4.7 below depicts the behavior of the average annual contribution rate volatilities for the plan. The first thing we notice from the figure is the rise and fall of volatility as equity allocation increases from 0% towards 100% for normal contribution rates between 0% and 5%. We intuitively expect the volatility of the supplementary payments to be higher than the volatility of the assets because the volatility of supplementary payments will be influenced by its size, frequency and type (whether a positive payment or a negative payment).

Figure 4. 9: Annual Contribution Rate Volatility Behavior



Notes: Assumption of 8% salary growth rate, 20% penalty factor and 20 years spread period.
 Source: Author's Computations.

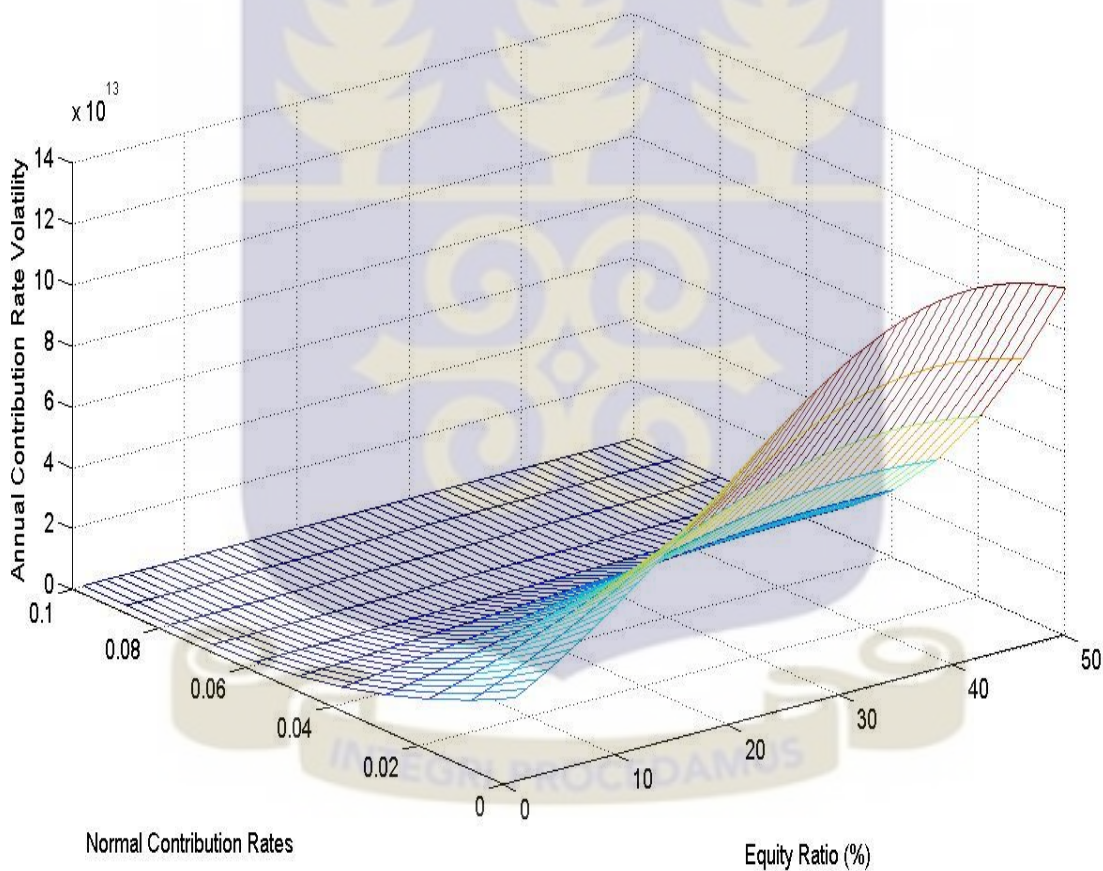
When we consider a low normal contribution rate of 0%, it implies that the plan is being funded primarily from supplementary payments and as we add equity to the investment mix at low levels (say below 50% equity from our figure) the volatility of the equity goes to boost the volatility in the system since supplementary payments are still dominant but beyond a certain level of equity participation (about 50% from our figure) the high expected returns of equity reduces the need for the supplementary payments leading to the replacement of the higher volatilities of the supplementary transactions with the comparatively lower volatilities of the equities thus causing the risen volatility of the annual contribution rates to decline. The equity volatility dominance in this case is also hampered by its domination of the bond volatilities, because as the investment portfolio becomes well diversified we expect the overall portfolio volatility to decline thus giving the volatility of the supplementary payments a field-day hence the rise before the fall in the annual contribution rate volatilities. If this analysis is true then we expect that with increasing normal contribution rates that seems to improve the funding situation of the plan, the peak volatilities of the annual contribution rates will decline and see this happening in figure 4.7 above where the peaks decay altogether at a normal contribution rate of 6%.

We also noticed from the data that for the same equity allocation across all possible allocations, the volatility of annual contribution rate drops as normal contributions are increased from 0% to 8% and this drop persists up to 13% normal contribution rate for some high equity allocations, thereafter the volatility rises in all asset allocation strategies. Figures 4.8, 4.9 and 4.10 are segments of figure 4.7 elucidating this phenomenon.

According to Maurer et al (2009), this phenomenon is intuitive. They argued that the zero volatility minimum optimal contribution rate in the absence of capital market risks is the deterministic rate (6.73% in our case) hence the volatility of the normal contribution rate is expected to converge to this

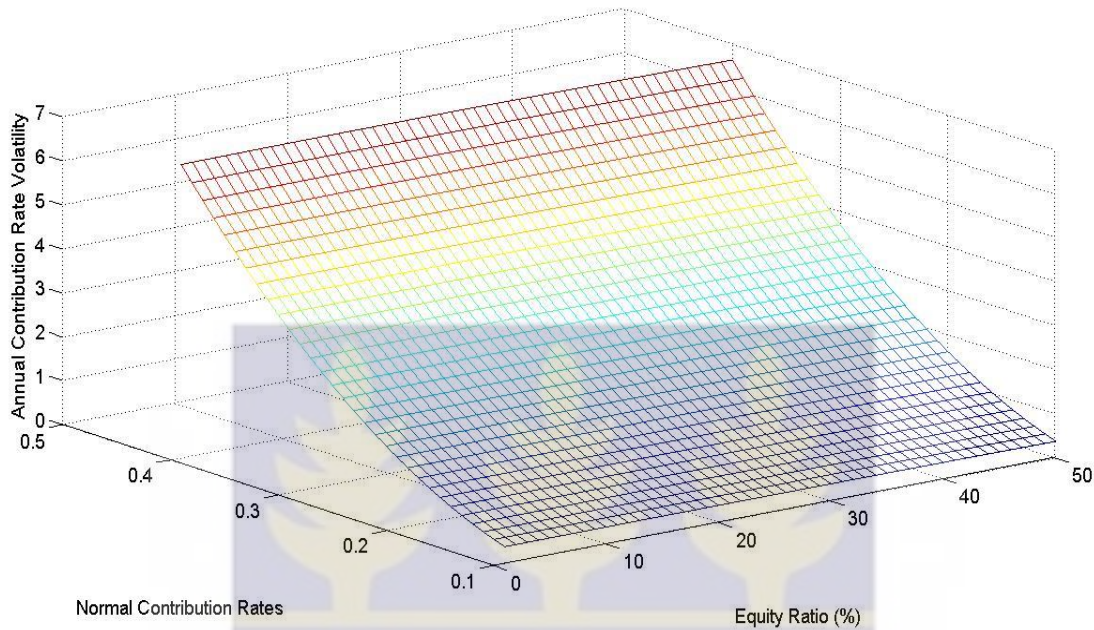
minimum as the normal contribution rate is increased from 0% towards 6.73% but partaking in risky investment strategies changes this minimum volatility normal contribution rate from the deterministic rate at the price of some positive contribution rate volatility related to investment risks. This explains the rapid fall in volatilities up to 8% normal contribution rates in our study (refer Figure 4.8 below).

Figure 4. 10: Volatility Profile for 0 - 10% Contribution Rate and up to 50% Equity Weight



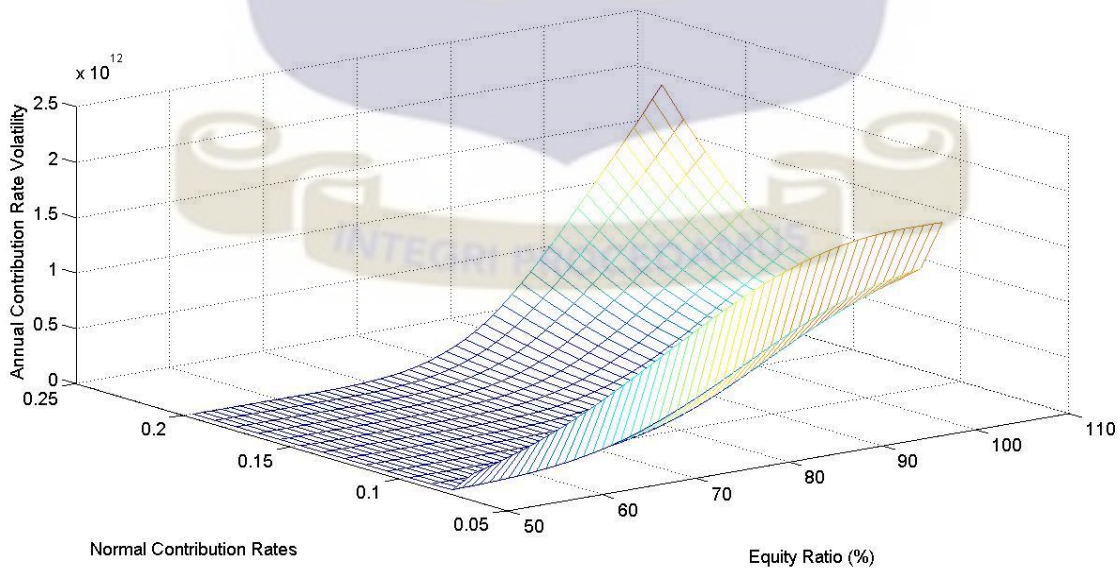
Notes: Assumption of 8% salary growth rate, 20% penalty factor and 20 years spread period.
Source: Author's Computations.

Figure 4. 11: Volatility Profile for 10 - 40% Contribution Rate and up to 50% Equity Weight



Notes: Assumption of 8% salary growth rate, 20% penalty factor and 20 years spread period.
Source: Author's Computations.

Figure 4. 12: Segment of Volatility Profile for Equity Weight beyond 50%



Notes: Assumption of 8% salary growth rate, 20% penalty factor and 20 years spread period.
Source: Author's Computations.

Maurer et al (2009) further argued that choosing too high a normal contribution rate leads to continuous reductions in actual contributions due to the amortization of asset surpluses, which in turn increases contribution rate volatility. Hence the rise in the volatilities when normal contribution rates are beyond 10% in our case (refer Figure 4.9 and 4.10 above).

Analyzing the results further we noted that the effect of adding equity to the investment portfolio depends on the funding scenario of the plan just as Maurer et al. (2009) did but in our case the effect on the overall scheme is “funnel-like” (refer to Figures 4.8, 4.9 and 4.10 taken together). The effect of increasing the equity weight when the plan is susceptible to chronic underfunding was discussed at the beginning of this section, now when the plan is adequately funded or moderately overfunded, adding equity at the same normal contribution rate causes the volatility to fall initially and then rise again. This happens because when the plan is adequately funded, there is minimal need for supplementary payments and the addition of equity causes portfolio diversification which in turn causes the capital market risk to decline up to a point and also as noted by Maurer et al. (2009), the higher expected equity returns improves the funding situation of the plan thereby further reducing the need for supplementary payments but if too much equity is added, the investment portfolio loses the advantage of portfolio diversification it opens up the plan to the shocks faced by equity on the capital market. This increases the need for supplementary transactions which, in addition to the high volatilities introduced by the high equity fraction, altogether causes the volatility of the annual contribution rate to rise, but this increase in volatility is more gradual for normal contribution rates between 8% and 11% than for other normal contribution rates. This pattern is also intuitive, in that, for low contribution rates that results in the plan being chronically underfunded and also for high contribution rates that result in the plan being chronically overfunded, supplementary payments and

withdrawals are triggered more frequently thus generating more volatility in addition to the increasing volatility introduced by the increasing equity investments. On the other hand, if the plan is sufficiently funded, as put forward by Maurer et al (2009), there is no need for earning excess investment returns, supplementary payments and withdrawals are less frequent and whatever volatility introduced by the increasing addition of equities is reduced by the amortization of over- and underfunding as the spreading of inflows can substantially reduce the impact of capital market volatility.

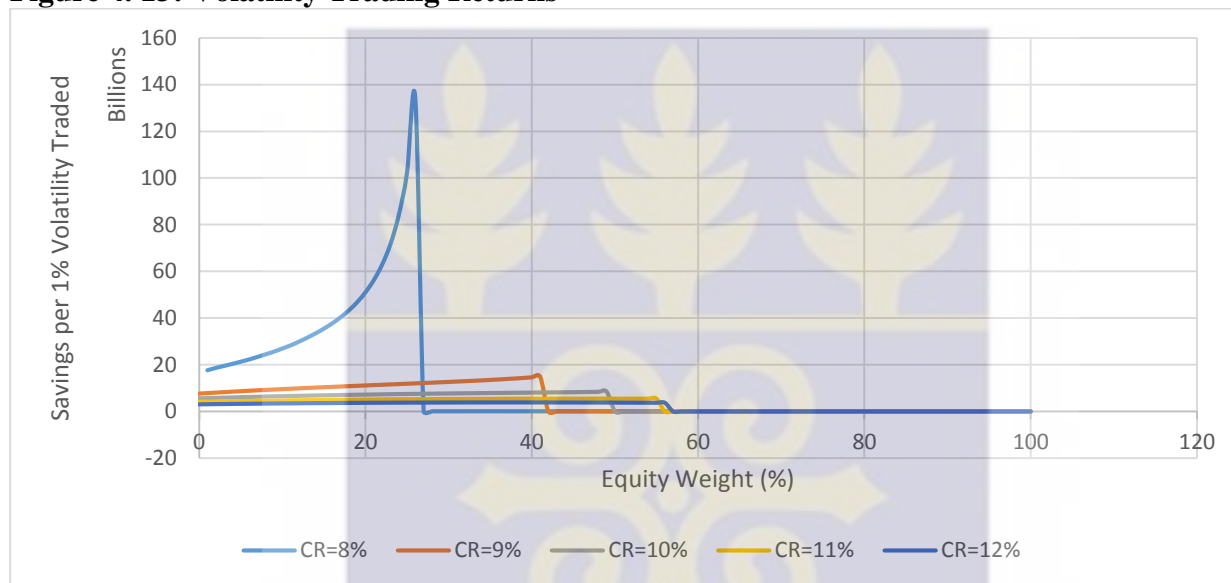
In summary, for our base case assumptions, the lowest volatility of the annual contributions is 22.75% and it occurs at a normal contribution rate of 8% with all of plan assets invested in bonds.

4.6 THE OPTIMAL RATE OF CONTRIBUTION AND INVESTMENT POLICY

The discussion so far has established that the pension plan manager who wants to minimize volatility of annual contributions around an optimal normal contribution rate will choose a normal contribution rate of 8% and invest all plan assets in bonds with a worst case plan cost of GhS10.71billion and a volatility of 22.75% whereas a choice of normal contribution rate of 12% with 33% of plan assets invested in equity has a lower worst case plan cost of GhS9.47billion albeit with a higher volatility of 55.10%. The plan manager's dilemma is to choose a contribution rate that in as much as it offers a low volatility, will also cash in on the potential savings available in trading some volatility for savings. The plan manager's optimal decision in this case is to choose the normal contribution rate and asset allocation strategy that maximizes savings per volatility increased. Here the plan manager's initial optimal point is the minimum volatility point and so he will consider differences between the worst case plan cost of subsequent strategies as "savings" and the difference between the associated volatilities as "trade". Then the final optimal choice will be the strategy in the optimal set that gives the highest "savings" to "trade" value expressed as the ratio of "savings" to "trade". If the plan

manager decides to choose the 12%, low worst case plan cost strategy by investing 33% in equity, the savings per 1% volatility traded is GhS3.85billion. The maximum savings per volatility traded occurs at a normal contribution rate of 8% and 26% plan assets invested in equities with a savings per 1% of volatility traded being GhS 133 billion (refer Figure 4.11) below.

Figure 4. 13: Volatility Trading Returns



Source: Author’s Computations.

4.7 FURTHER RESULTS

We now turn our attention to the investigation of the effect of different spread periods, different discount rates and different salary growth rates on the optimal contribution and investment strategy. The spread was chosen in periods of 1, 10, 30, 40 years and also included infinity. The results are shown in Table 4.2 below and the interest rate sensitivity of our results is shown in Table 4.3, finally in choosing salary growth rates of 0%, 16% and 24% we show the effect of salary growth rate on the optimal solution, and the effects of different salary growth rates on the optimal solution are summarized in Table 4.4 below.

4.7.1 Sensitivity of Results to Spread Period

From Table 4.2 below, it is seen that varying the spread period beyond 10 years has no appreciable effect on the optimal contribution rate and investment strategy. It is only when a 1 year spread period is used that a high worst case pension cost is realized; we also see a trend of monotonic decrease in the optimal volatilities. According to Maurer et al. (2009), this is due to the reduction in the impact of short term capital market fluctuations when the funding deviations of the plan are spread over a longer period.

Table 4. 2: Spread Period Sensitivity of Optimal Results

	Spread period m (in years)					
	1	10	20*	30	40	∞
Normal contributions (%)	11	8	8	8	8	8
Equity weight (%)	55	26	26	26	26	26
Bond Weight (%)	45	74	74	74	74	74
Contribution Rate Volatility p.a. (%)	101.31	24.35	23.05	22.91	22.90	22.89
1% - CVaR pension costs (in GhS bn)	22.84	10.87	10.30	10.24	10.23	10.23

Notes: Optimal contribution rate and investment strategy for various spread periods. 8% annual salary growth rate, discount rate of 23% and 20% penalty factors on funding deviations. Source: Author's Computations.

4.7.2 Sensitivity of Results to Discount Rate

We next select different discount rates to see the effect on the optimal solution. When the discount rate is changed it affects all the present values in our computations including the annuity factor at age 60 thus its relative impact on the pension liabilities is expected to be more than the relative impact on the salary payments. In Table 4.3 below, we chose a low discount rate of 8% following the work of Novy-Marx & Rauh (2008) as this rate is also close to the current real discount rate on the 7-year-bond being used to proxy the Government's cost of borrowing. We also chose a high discount rate of

30% to see what happens if the Government's cost of borrowing goes that high. In all what we are seeking is to find the effect around our results of a higher and a lower discount rate.

Table 4. 3: Discount Rate Sensitivity of the Optimal Results

	Interest Rate (%)		
	8	23*	30
Normal contributions (%)	20	8	8
Equity weight (%)	0	26	43
Bond Weight (%)	100	74	57
Contribution Rate Volatility p.a. (%)	313.37	23.05	22.93
1% - CVaR pension costs (in GhS bn)	88.21	10.30	5.74
Max. permissible 1%-CVaR (in GhS bn)	155.93	45.14	30.87

Notes: Optimal contribution rate and investment strategy for different discount rates. Spread period of 20 years, discount rate of 23% and 8% annual salary growth rate. Source: Author's Computations.

We infer from Table 4.3 that discount rate shows an inverse relationship with volatility and this is intuitive because the size and frequency of payments influences the volatility in the plan and lower interest rates generate higher present values hence the lower interest rates will tend to magnify the inherent volatilities and vice versa that's why we see the volatility in the 30% discount rate scenario lower than the 8% discount rate scenario even though we observe a higher equity participation at the higher discount rates.

4.7.3 Sensitivity of Results to Salary Growth Rate

To see the effect of salary growth rate on our optimal value, we selected salary growth rates of 0%, 16%, 24% and 32%. The choice of these rates is informed by the historic nominal salary growth rate distribution from the data, where the 0% is the minimum growth rate while 28% is the average growth rate in the salaries, thus we are wrapping a 4% band around the mean salary growth rate to see the

effect. From Table 4.4 below, we see a row of maximum permissible 1%-CVaR, this corresponds to the worst case plan costs in the deterministic case that is used as the constraint on worst case plan costs. This value is dependent on the liability profile of the plan which in turn is dependent on the salary experience and the discount rate used in the valuation hence we report this value as it changes. We notice, as we had intuitively expected that the growth rate of salaries highly impacts the optimal contribution rate and asset allocation strategy of the plan. It significantly affects all the parameters of the optimal strategy. Higher growth rates at the same plan parameters magnifies the volatility of annual contribution rate and shifts all other values higher.

Table 4. 4: Influence of Salary Growth on Optimal Contribution and Investment Strategy

	Salary growth rate per annum (%)			
	0	8*	16	24
Normal contributions (%)	8	8	10	17
Equity weight (%)	46	26	24	36
Bond Weight (%)	54	74	76	64
Contribution Rate Volatility p.a. (%)	22.61	23.05	41.63	173.20
1% - CVaR pension costs (in GhS bn)	6.16	10.30	22.56	70.92
Max. permissible 1%-CVaR (in GhS bn)	5.61	45.14	355	2677

Notes: Optimal contribution rate and investment strategy for various salary growth rates. Spread period of 20 years, discount rate of 23% and 20% penalty factors on funding deviations. Source: Author's Computations.

4.8 CHAPTER SUMMARY

In this chapter, we have analyzed the results from the application of the methodologies of the previous chapter on our dataset and have found that the optimal contribution and asset allocation strategy of the scheme is 8% normal contribution rate and a static equity allocation of 26% in investments. Prior to that, we found that 6.73% level annual contribution rate is the required rate to fund the scheme in a deterministic setting. Further to the above results, we also found that the chosen spread period, if greater than 10 years, does not affect the optimal contribution and asset allocation strategy much but

there is significant volatility inflation with changing discount rates with high volatility inflation occurring with lower discount rates and vice versa. The projected salary growth rate was found to totally affect the optimal contribution and asset allocation strategy shifting both equity weight in investments and optimal contribution rates higher with higher growth rates. The worst case total pension costs too increased with the increasing salary growth rate.



CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

In this final chapter, we present our conclusions and advance recommendations for optimal funding and asset allocation policy of SSNIT and we also identify gaps to guide future research in this area of study.

5.2 DISCUSSION OF RESULTS

We pointed out from the previous chapter that the plan manager is free to choose the spread and the normal contribution rate at the outset of the scheme and must subsequently adopt an investment strategy to execute the plan. Our results showed an optimal normal contribution rate of 8% per annum accompanied by a static 26% equity weight in plan investments throughout the cohort's preretirement period.

Consistent with the works of Maurer et al. (2009) and Haberman et al. (2000), we have seen that the spread period used to amortize any over- and underfunding, if chosen above 10 years, did not have much effect on the optimal normal contribution rate and asset allocation strategy, this is because in spread period jumps of 10years, the optimal normal contribution rate remained at 8% and the equity weights also did not change. Our results showed a high sensitivity to salary growth rate indicating that at the outset of the plan, the projected salary growth rate of the scheme members must be gotten right. The plan parameters showed high sensitivity to discount rates.

5.3 CONCLUSION

We have shown in this study that using long range strategic contribution and investment patterns, the SSNIT scheme can operate with a low contribution rate of 8% with minimal contribution rate risk while being constrained by a worst case plan cost. Our results is representative of the dynamics inherent in the SSNIT scheme since the chosen cohort will be part of the entire scheme for the next forty five years but in as much as the chosen cohort is expected to represent the general dynamics of the SSNIT fund, we caution the generalization of these results to the SSNIT scheme across board. Rather, different closed cohorts picked over different time periods be studied and then some form of averaging applied to describe the entire plan. This is a gap for future research. Another gap for research is to open the cohort to accept new hires in the evaluation of the plan parameters. From the sensitivity analysis conducted we see that the scheme is highly susceptible to interest rate risk hence strategic long term planning has to have a wide band of tolerance limits to accommodate the scheme in order to absorb the shocks from the highly volatile capital market the scheme is exposed to.

From our data the average historical (1990 to 2014) salary growth rate is about 28% and from our model this requires a normal contribution rate of over 17% and yet SSNIT is running on 11%. This points to a latent danger of plan failure due to chronic underfunding going forward but our model also shows that at high discount rates the normal contribution rate required to fund the defined benefit plan reduces as observed by Mohan & Zhang (2014), hence we surmise that SSNIT has been safe so far due to the high discount rates in the economy that is balancing out the effect of the high salary growth rate but given that the salary growth rate is leading both the inflation and GDP growth rates (refer to Figure 4.4) this danger is still highly latent. A scenario where improvements in the economy causes the discount rates to go down while keeping the salary growth rate trend as it is, will spell accelerated failure for the SSNIT scheme.

5.4 RECOMMENDATIONS

We recommend that SSNIT take a second look at the current normal contribution rate of 11% because from our data as stated above, the average nominal salary growth rate over the period of 1990 to 2014 is about 28% and this growth rate requires a high contribution rate of more than 17% to sustain the plan even though we contend that such a high salary growth rate is not sustainable given the growth rate of our GDP but it is a caution that must be taken into account.

Given that the scheme can operate with a reduced contribution rate of 8%, we recommend that at the current normal contribution rate, the scheme should improve the benefit promises under the scheme.

Our results is very significant for policy and we recommend that policymakers, the Government of Ghana, the NPRA and SSNIT adopt it and make funds available for further research in this direction because it has unearthed a latent possibility of plan failure if wage growth rate continues the way it has averaged over the past 25 years hence policy makers are to take note and fashion out policies to arrest the highly volatile wage growth rate given the rate of GDP growth we are experiencing.

The Government and policy makers are also enjoined not to tackle only one side of the dual high discount rate and salary growth rate phenomenon of the economy, but rather the two must be tackled hand in hand, because any economic policy that aims at reducing the general discount rates in the economy that is not concurrently supported by a reduction in the rate of growth of salaries will accelerate the failure of the SSNIT scheme and given that the scheme is a social intervention, its failure will put a lot of pressure on the Government and people of this country.

Pension fund assets are long term strategic investment instruments hence it bodes well for them to be shocked by long term macroeconomic factors, we therefore recommend to the Government of Ghana to create and sustain an enabling environment for long term strategic asset trade to thrive since currently the only long term instrument for our 45 year valuation was the 7-year GOG bonds and the

shocks from such a shorter term instrument on a longer term investment is more frequent than if a longer term instrument was available. This is needed to improve the stability of the long term investment and reduce its risk exposure. This will greatly improve the forecasts and planning of the scheme.

We also recommend that SSNIT should make its data readily available for more academic research to be conducted on the scheme since access to the SSNIT data was a major limitation of this work. Our recommendation here includes making their data available on the internet for easy access since they are not in competition with any entity in the operation of the first tier scheme.

To the University of Ghana we recommend that the training of graduate students in the use of 3-dimensional analytic software packages like MatLab and R be increased to enhance capacity in multi-dimensional analysis. And also high powered computer workstations be made available to researchers to aid them in such analysis because we were limited by computer power in this research that is why we did not seek our optimal values in fractions of the unit percentage as this requires high computer resources to run the iterations, for example to report contribution rates and associated asset allocation policies to the first decimal of the percentage will require $401 \times 1001 \times 10000 \times 46$ (184 billion) iterations and this figure jumps to $4001 \times 10001 \times 10000 \times 46$ (18 trillion) iterations if you intend to refine the results to the second decimal place, hence our recommendation for high powered computer systems

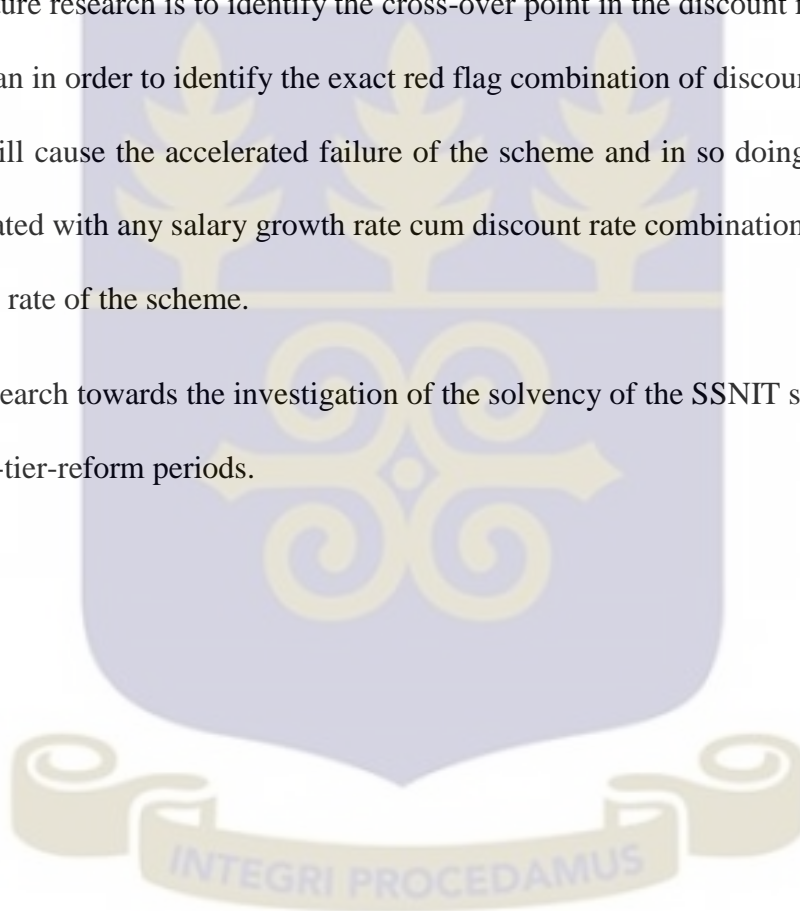
To the Government and the pensions' regulator once again, we recommend a holistic re-assessment of the entire first tier pension plan parameters to ensure that plan participants are not being short-changed for their contributions in the light of our results indicating that at the current promised benefit obligations, lower contributions could be taken even though we suspect there are pensions in payment that requires the sustenance of the scheme as it is.

Finally we direct future research to look into alternate time paths and dynamic asset allocation strategies for an optimal volatility-minimizing normal contribution rate for the scheme.

We also direct that this research should be improved by searching for the optimal values in-between percentage points; that is to say finding the trailing decimals that follow our optimal values, this will require improvement of our codes which we readily make available in the Appendices.

Another area for future research is to identify the cross-over point in the discount rate-salary growth rate effect on the plan in order to identify the exact red flag combination of discount rates and salary growth rates that will cause the accelerated failure of the scheme and in so doing identify the ruin probabilities associated with any salary growth rate cum discount rate combination given the current normal contribution rate of the scheme.

We direct future research towards the investigation of the solvency of the SSNIT scheme comparing the prior and post 3-tier-reform periods.



REFERENCES

- Aaron, H. J., & Reischauer, R. D. (1998). *Countdown to Reform*. New York: Century Foundation Press.
- Artzner, P., Delbaen, F., Eber, J., & Heath, D. (1997). Thinking Coherently. *Risk* 10 (11), 68 – 72.
- Artzner, P., Delbaen, F., Eber, J., & Heath, D. (1999). Coherent Measures of Risk. *Mathematical Finance* 9 (3), 203 – 228.
- Asad-Syed, Kemal, Arun Muralidhar, and Ronald J. P. van der Wouden. 1998. Determination of Replacement Rates for Savings Schemes. *Investment Management Department, Model Development Paper 1*. The World Bank.
- Barrientos, A., Møller, V., Saboia, J., Lloyd-Sherlock, P., & Mase, J. (2013). ‘Growing’ Social Protection in Developing Countries: Lessons from Brazil and South Africa. *Development Southern Africa*, 30(1), 54-68.
- Bateman, H., Kingston, G., & Piggott, J. (2001). *Forced Saving: Mandating Private Retirement Incomes*. Cambridge University Press.
- Bertocchi, M., Schwartz, S. L., & Ziemba, W. T. (2010). *Optimizing the Aging, Retirement, and Pensions Dilemma* (Vol. 477). John Wiley & Sons.

Blahous, C. P. (2000). *Reforming Social Security for Ourselves and Our Posterity*. Greenwood Publishing Group.

Boender, C.G.E. (1997). A Hybrid Simulation-Optimization Scenario Model for Asset-Liability Management. *European Journal of Operations Research*, 99, 126-135.

Bogentoft, E., Romeijn, H.E., & Uryasev, S. (2001). Asset/Liability Management for Pension Funds Using CVaR Constraints. *Journal of Risk Finance* 2 (3), 57-71.

Campbell, J.Y., Chan, Y.L., & Viceira, L.M. (2003). A Multivariate Model for Strategic Asset Allocation. *Journal of Financial Economics* 67, 41-80.

Campbell, J.Y., & Viceira, L.M. (2005). The Term Structure of the Risk-Return-Trade-Off. *Financial Analysts Journal* 61, 34-44.

Cesa-Bianchi, A. (2015, May 28). *Ambrogio Cesa-Bianchi*. Retrieved from A Toolbox for VAR Analysis: <https://sites.google.com/site/ambropo/MatlabCodes>

Choi, J. J., Laibson, D., Madrian, B. C., & Metrick, A. (2002). Defined Contribution Pensions: Plan Rules, Participant Choices, and the Path of Least Resistance. In *Tax Policy and the Economy, Volume 16* (pp. 67-114). MIT Press.

Cox, S.H., Lin, Y., Tian, R., & Yu, J. (2013). Managing Capital Market and Longevity Risks in a Defined Benefit Pension Plan. *Journal of Risk and Insurance* 80, 585-619.

Dam, J. V. (2014, November 18). *Towards a Three-Pillar Pensions System in Ghana* . Retrieved from The Actuary: <http://www.theactuary.com/features/2014/04/towards-a-three-pillar-pensions-system-in-ghana/>

Delong, L., Gerrard, R., & Haberman, S. (2008). Mean-Variance Optimization Problems for an Accumulation Phase in a Defined Benefit Plan. *Insurance: Mathematics and Economics*, 42: 107-118.

Dert, C. L. (1995). *Asset Liability Management for Pension Funds: A Multistage Chance Constrained Programming Approach*. (Doctoral dissertation, Erasmus School of Economics (ESE)).

Dert, C. L. (1998). A Dynamic Model for Asset Liability Management for Defined Benefit Pension Funds. *Worldwide Asset and Liability Modeling*, 10, 1-1.

Dimson, E., Marsh, P., & Staunton, M. (2009). *Triumph of the Optimists: 101 years of Global Investment returns*. Princeton University Press.

Gao, J. (2008). Stochastic Optimal Control of DC Pension Funds. *Insurance: Mathematics and Economics*, 42, 1159-1164.

Geyer, A., & Ziemba, W.T. (2008). The Innovest Austrian Pension Fund Financial Planning Model InnoALM. *Operations Research*, 56, 797–810.

Godoy, O., & Valdes-Prieto, S. (1997). Democracy and Pensions in Chile: Experience with Two Systems. *The Economics of Pensions: Principles, Policies and International Experience*. Cambridge, 58-91.

Haberman, S. (1997a). Stochastic Investment Returns and Contribution Rate Risk in a Defined Benefit Pension Scheme. *Insurance: Mathematics and Economics* 19, 127 – 139.

Haberman, S. (1997b). Risk in a Defined Benefit Pension Scheme. *Singapore International Insurance and Actuarial Journal* 1, 93–103.

Haberman, S., Butt, Z., & Megaloudi, C. (2000). Contribution and Solvency Risk in a Defined Benefit Pension Scheme. *Insurance: Mathematics and Economics* 27, 237-259.

Haberman, S., Day, C., Fogarty, D., Khorasane, M.Z., McWhirter, M., Nash, N., Ngwira, B., Wright, I.D., & Yakoubov, Y. (2003a). 'A Stochastic Approach to Risk Management and Decision Making in Defined Benefit Pension Schemes. *British Actuarial Journal* 9 (3), 493-618.

Haberman, S., Khorasane, M.Z., Ngwira, B., & Wright, I.D. (2003b). Risk Measurement and Management of Defined Benefit Pension Schemes: A Stochastic Approach. *IMA. Journal of Management Mathematics* 14, 111-128.

Haberman S., & Sung, J.H. (1994). Dynamic Approaches to Pension Funding. *Insurance: Mathematics and Economics* 15, 151-162.

Haberman S., & Sung, J.H. (2005). Optimal Pension Funding Dynamics over Infinite Control Horizon when Stochastic Rates of Return are Stationary. *Insurance: Mathematics and Economics* 36, 103-116.

Keim, D. B., & Ziemba, W. T. (Eds.). (2000). *Security Market Imperfections in Worldwide Equity Markets*. Cambridge, UK: Cambridge University Press.

Hoevenaars, R.P., Molenaar, R.D., & Steenkamp, T.B. (2003). Simulation for the Long Run. In: Scherer B. (Ed.). *Asset Liability Management Tools*. London: Risk Books, 177-210.

Huang, C., & Cairns, J.G. (2006). On Control of Defined-Benefit Pension Plans. *Insurance: Mathematics and Economics* 38, 113-131.

Hurd, M. D. (1990). The Joint Retirement Decision of Husbands and Wives. In *Issues in the Economics of Aging* (pp. 231-258). University of Chicago Press.

Jorion, P., & Goetzmann, W. N. (1999). Global Stock Markets in the Twentieth Century. *Journal of Finance*, 953-980.

Josa-Fombellida, R., & Rincón-Zapatero, J. P. (2004). Optimal Risk Management in Defined Benefit Stochastic Pension Funds. *Insurance: Mathematics and Economics*, 34: 489-503.

Josa-Fombellida, R., & Rincón-Zapatero, J.P. (2006). Optimal Investment Decisions with a Liability: The Case of Defined Benefit Pension Plans. *Insurance: Mathematics and Economics*, 39, 81-98.

Kim, J., Malz, A. M., & Mina, J. (1999). *Longrun: Technical Document*. New York: RiskMetrics Group.

Kouwenberg, R. (2001). Scenario Generation and Stochastic Programming Models for Asset Liability Management. *European Journal of Operational Research*. 34: 279-292.

Kouwenberg, R., & Zenios, S. (2006). Stochastic Programming Models for Asset Liability Management, *Handbook of Asset and Liability Management*, 1, 253–303.

Kpessa, M. W. (2010). The Politics of Retirement Income Security Policy in Ghana: Historical Trajectories and Transformative Capabilities. *African Journal of Political Science and International Relations*, 92-102.

Kumado, K., & Gockel, A. (2003). A Study on Social security in Ghana. Accra: Trades Union Congress.

Kusy, M.L., & Ziemba, W.T. (1986). A Bank Asset and Liability Management Model, *Operations Research*, 34, 356-376.

Lee, E.M. (1986). *An Introduction to Pension Schemes*. Institute and Faculty of Actuaries, London, UK.

Logue, D.E., & Rader, J.S. (1997). *Managing Pension Plans: A Comprehensive Guide to Improving Plan Performance* (Financial Management Association Survey and Synthesis Series), Cambridge: Harvard Business School Press.

Maurer, R., Mitchell, O.S., & Rogalla, R., (2008a). The Victory of Hope over Angst? Funding, Asset Allocation, and Risk Taking in German Public Sector Pension Reform. In: Broeders, D., Eijffinger, S., Houben, A. (Eds.). *Frontiers in Pension Finance*. Cheltenham: Edward Elgar, 51-79.

Maurer, R., Mitchell, O.S., & Rogalla, R., (2008b). Reforming German Civil Servant Pensions: Funding Policy, Investment Strategy, and Intertemporal Risk Budgeting. PRC Working Paper, 2008-09.

Maurer, R., Mitchell, O. S., & Rogalla, R. (2009). Managing Contribution and Capital Market Risk in a Funded Public Defined Benefit Plan: Impact of CVaR Cost Constraints. *Insurance: Mathematics and Economics*, 45: 25-34.

McGill, D.M., Brown, K.N., Haley, J.J., and Schieber, S.J. (2005). *Fundamentals of Private Pensions*. 9th Ed. Oxford: Oxford University Press.

Modigliani, F. & Ando, A. (1957). The Life Cycle Hypothesis of Saving: Aggregate Implications and Tests. *American Economic Review*, 53, 55-84.

Mohan, N., & Zhang, T. (2014). An Analysis of Risk-Taking Behavior for Public Defined Benefit Pension Plans. *Journal of Banking and Finance*, 40, 403-419.

Mulvey, J. M., & Thorlacius, A. E. (1998). The Towers Perrin Global Capital Market Scenario Generation System. *Worldwide Asset and Liability Modeling*, 10, 286.

New Pension Scheme Drying SSNIT Coffers. (2014, September 27). Retrieved from Business and Financial Times Website: <http://thebftonline.com/content/new-pension-scheme-drying-ssnit-coffers>.

Ngwira, B., & Gerrard, R. (2007). Stochastic Pension Fund Control in the Presence of Poisson Jumps. *Insurance: Mathematics and Economics* 40, 283-292.

Novy-Marx, R., & Rauh, J. D. (2008). The Intergenerational Transfer of Public Pension Promises (Research paper, No. 08-14). *Chicago, IL, School of Business*.

OECD (2013). *Pension Markets in Focus*. OECD.

Orszag, P., & Stiglitz, J. (2001). Rethinking Pension Reform: 10 Myths About Social Security Systems. In R. Holzmann, J. Stiglitz, L. Fox, E. James, & P. Orszag, *New Ideas About Old*

Age Security: Toward Sustainable Pension Systems in 21st Century (pp. 17-62). Washington, DC: World Bank.

Owadally, M.I., & Haberman, S. (2004). Efficient Amortization of Actuarial Gains/Losses and Optimal Funding in Pension Plans. *North American Actuarial Journal* 8(1), 21 – 36.

Plantanakis, E., & Sutcliffe, C. (2015). Asset Liability Modelling and Pension Schemes: The Application of Robust Optimization to USS. *European Journal of Finance*, Forthcoming. Available at SSRN: <http://ssrn.com/abstract=2422303>

Promislow, D. S. (2011). *Fundamentals of Actuarial Mathematics* (2nd ed.). Toronto: John Wiley and Sons Ltd.

Rockafellar R.T., & Uryasev, S. (2002). Conditional Value-at-Risk for General Loss Distributions. *Journal of Banking and Finance* 26, 1443-1471.

Rudolf, M., & Ziemba, W.T. (2004). Intertemporal Surplus Management. *Journal of Economic Dynamics and Control*, 28, 975-990

Samuelson, P. A. (1975). Optimum Social Security in a Life-Cycle Growth Model. *International Economic Review*, 539-544.

Rockafellar, R. T., & Uryasev, S. (2000). Optimization of Conditional Value-at-Risk. *Journal of risk*, 2, 21-42.

Sharpe, W.F., & Tint, L. (1990). Liabilities – A New Approach. *Journal of Portfolio Management*, 16, 5-10.

Stewart, F., & Yermo, J. (2009). Pension Fund Governance. *OECD Journal: Financial Market Trends*, 2008(2), 1-42.

Wilkie, A.D. (1986). A Stochastic Investment Model for Actuarial Use. *Trans. Faculty Actuaries* 39, 341–373.

Wilkie, A. D. (1995). More on a Stochastic Asset Model for Actuarial Use. *British Actuarial Journal*, 1(05), 777-964.

Valdés-Prieto, S. (1997). Financing a Reform toward Funding. In S. Valdés-Prieto, ed., *The Economics of Pensions: Principles, Policies, and International Experience*. Cambridge: Cambridge University Press.

Ziemba, W. T. (2003). *The Stochastic Programming Approach to Asset, Liability, and Wealth Management*. Research Foundation of AIMR, Scorpion Publications.

APPENDICES

APPENDIX A: FUNDING THE PENSION PLAN WITH THE DETERMINISTIC RATE

Year	Simulated Total Cohort Salary (GhS)	Projected Total Pension Liability (GhS)	Annual Contributions @ 6.73% (GhS)	Surplus after Paying Pension Liability
2014	11284691551.66	427300179.34	759356132.45	332055953.10
2015	12014939635.20	556987288.49	808495123.79	659936657.61
2016	12730325850.61	791953749.09	856634047.86	876402387.63
2017	13450887796.62	928623025.75	905121251.07	1054473162.11
2018	14174631980.85	875065535.89	953822589.71	1375759043.21
2019	14943538560.91	952105272.57	1005562942.92	1745641293.50
2020	15758816509.76	1229653340.32	1060423663.51	1977909114.19
2021	16530879526.67	900410071.30	1112376415.95	2644794555.11
2022	17486192117.12	1382126658.04	1176660182.20	3047630826.95
2023	18349690990.99	1279743829.87	1234765728.31	3703607815.58
2024	19290223391.81	1504487657.35	1298054923.50	4349004879.32
2025	20206703325.66	1542981843.21	1359725608.51	5166019766.86
2026	21190418377.53	1595154068.88	1425920599.64	6184970843.99
2027	22225002605.27	1657219514.70	1495538619.26	7445833242.67
2028	23302657803.26	2176970730.20	1568054919.73	8549459078.02
2029	24285449884.89	2155937870.30	1634187803.44	9994084599.10
2030	25298076518.66	2501498643.10	1702328278.59	11493553692.39
2031	26247500817.16	2129529919.54	1766215816.86	13773756938.96
2032	27404198198.40	3489371721.18	1844050930.54	15296400244.29
2033	28183113353.37	2974768686.31	1896464768.96	17736268383.12
2034	29080461002.70	4165075416.09	1956848026.88	19607382722.03
2035	29585065640.93	4550699897.45	1990803286.07	21557184136.72
2036	29927110652.46	4831594379.20	2013819774.90	23697561883.86
2037	30120652133.53	5312860626.91	2026843339.60	25861983829.84
2038	30116770513.28	5878675583.22	2026582142.19	27958146669.68
2039	29828210024.76	7363392702.02	2007164670.70	29032292372.38
2040	28971370586.61	7296716755.79	1949507243.47	30362510105.70
2041	27837713720.87	7227677840.90	1873222544.94	31991432134.05
2042	26636594615.50	10184832141.12	1792398257.07	30957027640.83
2043	24356592353.43	10384207858.10	1638975038.39	29331911178.52
2044	21638622546.77	10204281726.15	1456080625.10	27330049648.53
2045	18675448150.33	11643019495.45	1256686194.23	23229627766.47
2046	14917089279.97	11501238861.50	1003783149.16	18074986440.42
2047	11008280208.26	9524100893.27	740756186.87	13448888615.32
2048	7760035965.65	7684017036.50	522179172.69	9380295133.03
2049	5272919228.43	5794827424.88	354819051.43	6097754640.18
2050	3335915556.37	4074499230.66	224476488.65	3650215465.41
2051	2096912523.68	2012398684.86	141102960.30	2618469297.90
2052	1496432981.09	1830116315.50	100696200.32	1491297121.23
2053	883005687.48	1103137830.04	59418175.56	790575804.64
2054	479514670.60	684173811.24	32266934.73	320501363.20
2055	192268469.67	321093981.46	12937902.72	86060597.99
2056	57050578.49	85331940.50	3838980.13	24361575.15
2057	16449197.71	19442688.14	1106879.98	11628929.28
2058	9574001.87	12198102.29	644242.42	2749723.14
2059	1020556.89	3450833.56	68674.11	0.01

Notes: Exact deterministic rate is 0.067290819. Source: Author's Calculations.

APPENDIX B: SSNIT COHORT DATA SHOWING AGE AAND SALARY DISTRIBUTION

Year	Age	No. of People	Avg. ann. Salary	Total Salary
2014	15	15	2131.46	31971.8835
2014	16	90	3243.96	291955.95
2014	17	135	2052.89	277139.651
2014	18	701	2354.33	1650382.6
2014	19	2760	2153.36	5943269.46
2014	20	6239	2224.36	13877788.3
2014	21	9649	2261.86	21824714.2
2014	22	14327	2543.94	36446984
2014	23	13439	3069.18	41246774.5
2014	24	22528	3876.04	87319318.7
2014	25	29144	5068.67	147721182
2014	26	34964	6011.98	210202722
2014	27	42072	7168.60	301597251
2014	28	50554	7961.89	402505225
2014	29	51866	8628.09	447504267
2014	30	47140	9163.08	431947643
2014	31	48879	9487.59	463744044
2014	32	47196	10030.81	473414156
2014	33	36221	10970.65	397368022
2014	34	39997	10802.45	432065593
2014	35	41165	10651.58	438472332
2014	36	34958	11269.01	393942017
2014	37	33681	11300.52	380612679
2014	38	32949	11375.17	374800476
2014	39	32813	11350.24	372435425
2014	40	31331	11551.42	361917446
2014	41	24491	11890.35	291206586
2014	42	28767	11384.80	327506628
2014	43	18846	12523.11	236010531
2014	44	24359	11920.97	290382860
2014	45	22570	12030.10	271519244
2014	46	23324	11913.07	277860515
2014	47	18834	12660.09	238440192
2014	48	19504	12454.85	242919355
2014	49	20067	12523.15	251302011
2014	50	21088	12746.45	268797159
2014	51	19060	12817.84	244308107
2014	52	20965	12775.18	267831544
2014	53	14327	13845.46	198363920
2014	54	21575	13215.21	285118091
2014	55	17197	13932.30	239593729
2014	56	17649	14077.65	248456410
2014	57	19148	13524.69	258970745
2014	58	16179	14614.88	236454144
2014	59	13720	15360.17	210741560
2014	60	12608	12670.32	159747445

Source: SSNIT Pension Scheme

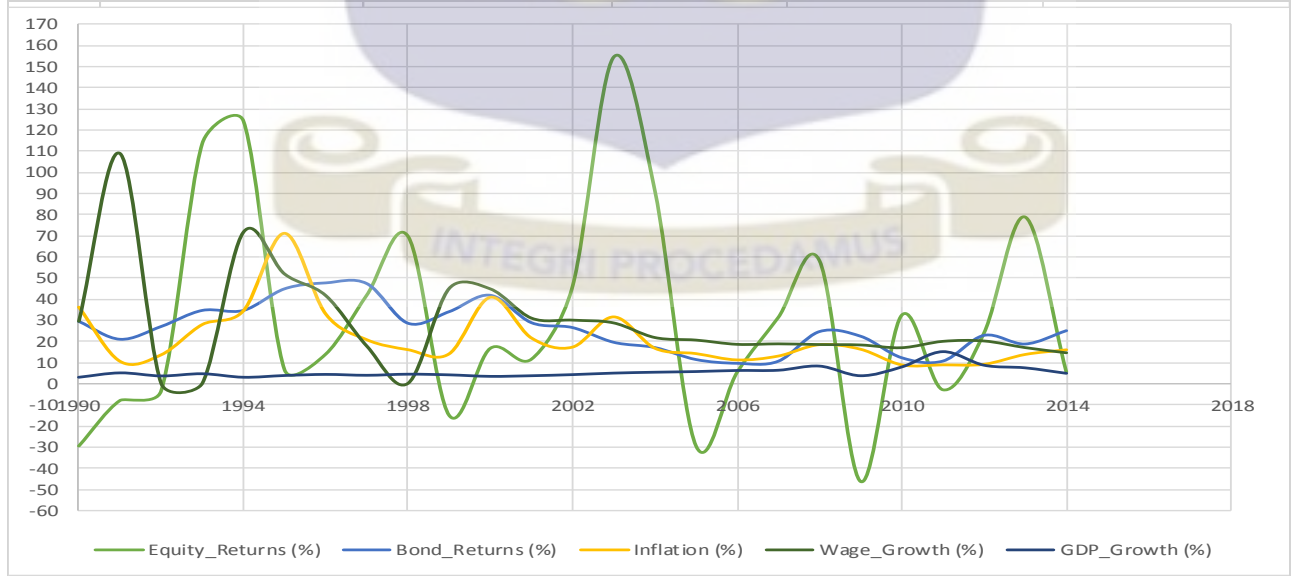
APPENDIX C: SSNIT MORTALITY DATA

Age	Unisex Graduated Mortality				Curtate ex	Complete ex	Age	Unisex Graduated Mortality				Curtate ex	Complete ex
	qx	px	tpx	Avg tpx				qx	px	tpx	Avg tpx		
			1	1									
15	0.00045	0.99955	0.99955	0.99978	54.1	54.6	58	0.01825	0.98175	0.82741	0.8351	15.3	15.8
16	0.00049	0.99951	0.99906	0.99931	53.1	53.6	59	0.02081	0.97919	0.81019	0.8188	14.6	15.1
17	0.00053	0.99947	0.99853	0.9988	52.2	52.7	60	0.02368	0.97632	0.79101	0.8006	13.9	14.4
18	0.00057	0.99943	0.99797	0.99825	51.2	51.7	61	0.02684	0.97316	0.76978	0.7804	13.3	13.8
19	0.00061	0.99939	0.99736	0.99766	50.2	50.7	62	0.03031	0.96969	0.74644	0.75811	12.6	13.1
20	0.00065	0.99935	0.99671	0.99703	49.3	49.8	63	0.03408	0.96592	0.721	0.73372	12	12.5
21	0.00065	0.99935	0.99606	0.99639	48.3	48.8	64	0.03815	0.96185	0.6935	0.70725	11.4	11.9
22	0.00065	0.99935	0.99541	0.99574	47.3	47.8	65	0.04252	0.95748	0.66401	0.67875	10.9	11.4
23	0.00065	0.99935	0.99476	0.99509	46.3	46.8	66	0.04718	0.95282	0.63268	0.64835	10.4	10.9
24	0.00066	0.99934	0.99411	0.99443	45.4	45.9	67	0.05213	0.94787	0.5997	0.61619	9.9	10.4
25	0.00066	0.99934	0.99345	0.99378	44.4	44.9	68	0.05738	0.94262	0.56529	0.5825	9.4	9.9
26	0.00067	0.99933	0.99279	0.99312	43.4	43.9	69	0.06292	0.93708	0.52972	0.54751	9	9.5
27	0.00082	0.99918	0.99198	0.99238	42.5	43	70	0.06687	0.93313	0.4943	0.51201	8.6	9.1
28	0.00097	0.99903	0.99101	0.99149	41.5	42	71	0.0718	0.9282	0.45881	0.47655	8.2	8.7
29	0.00114	0.99886	0.98988	0.99045	40.5	41	72	0.07658	0.92342	0.42367	0.44124	7.9	8.4
30	0.00133	0.99867	0.98857	0.98922	39.6	40.1	73	0.08112	0.91888	0.3893	0.40649	7.5	8
31	0.00152	0.99848	0.98706	0.98782	38.6	39.1	74	0.08545	0.91455	0.35604	0.37267	7.2	7.7
32	0.00173	0.99827	0.98535	0.98621	37.7	38.2	75	0.0895	0.9105	0.32417	0.34011	6.9	7.4
33	0.00195	0.99805	0.98343	0.98439	36.8	37.3	76	0.09398	0.90602	0.29371	0.30894	6.5	7
34	0.00218	0.99782	0.98128	0.98236	35.8	36.3	77	0.09901	0.90099	0.26463	0.27917	6.2	6.7
35	0.00242	0.99758	0.97891	0.98009	34.9	35.4	78	0.10508	0.89492	0.23682	0.25072	5.9	6.4
36	0.00267	0.99733	0.97629	0.9776	34	34.5	79	0.11217	0.88783	0.21026	0.22354	5.6	6.1
37	0.00292	0.99708	0.97344	0.97486	33.1	33.6	80	0.12014	0.87986	0.185	0.19763	5.3	5.8
38	0.00318	0.99682	0.97034	0.97189	32.2	32.7	81	0.12872	0.87128	0.16118	0.17309	5	5.5
39	0.00345	0.99655	0.96699	0.96867	31.3	31.8	82	0.13698	0.86302	0.1391	0.15014	4.8	5.2
40	0.00371	0.99629	0.9634	0.9652	30.4	30.9	83	0.1442	0.8558	0.11905	0.12907	4.5	5
41	0.00398	0.99602	0.95956	0.96148	29.5	30	84	0.15032	0.84968	0.10115	0.1101	4.3	4.8
42	0.00426	0.99574	0.95548	0.95752	28.6	29.1	85	0.15958	0.84042	0.08501	0.09308	4	4.5
43	0.00453	0.99547	0.95115	0.95331	27.8	28.3	86	0.16905	0.83095	0.07064	0.07782	3.8	4.3
44	0.00482	0.99518	0.94656	0.94885	26.9	27.4	87	0.1797	0.8203	0.05794	0.06429	3.6	4
45	0.00511	0.99489	0.94172	0.94414	26	26.5	88	0.19192	0.80808	0.04682	0.05238	3.3	3.8
46	0.00542	0.99458	0.93662	0.93917	25.1	25.6	89	0.20536	0.79464	0.03721	0.04202	3.1	3.6
47	0.00577	0.99423	0.93121	0.93391	24.3	24.8	90	0.21931	0.78069	0.02905	0.03313	2.9	3.4
48	0.00616	0.99384	0.92548	0.92835	23.4	23.9	91	0.23165	0.76835	0.02232	0.02568	2.7	3.2
49	0.00662	0.99338	0.91936	0.92242	22.6	23.1	92	0.24387	0.75613	0.01688	0.0196	2.6	3
50	0.00717	0.99283	0.91277	0.91606	21.7	22.2	93	0.25574	0.74426	0.01256	0.01472	2.4	2.9
51	0.00783	0.99217	0.90562	0.90919	20.9	21.4	94	0.26719	0.73281	0.0092	0.01088	2.2	2.7
52	0.00865	0.99135	0.89778	0.9017	20	20.5	95	0.2781	0.7219	0.00664	0.00792	2	2.5
53	0.00965	0.99035	0.88912	0.89345	19.2	19.7	96	0.28461	0.71539	0.00475	0.0057	1.8	2.2
54	0.01086	0.98914	0.87946	0.88429	18.4	18.9	97	0.29114	0.70886	0.00337	0.00406	1.6	1.9
55	0.0123	0.9877	0.86864	0.87405	17.6	18.1	98	0.29768	0.70232	0.00237	0.00287	1.2	1.4
56	0.014	0.986	0.85648	0.86256	16.8	17.3	99	0.30422	0.69578	0.00165	0.00201	0.7	0.8
57	0.01598	0.98402	0.84279	0.84964	16.1	16.6	100	1					

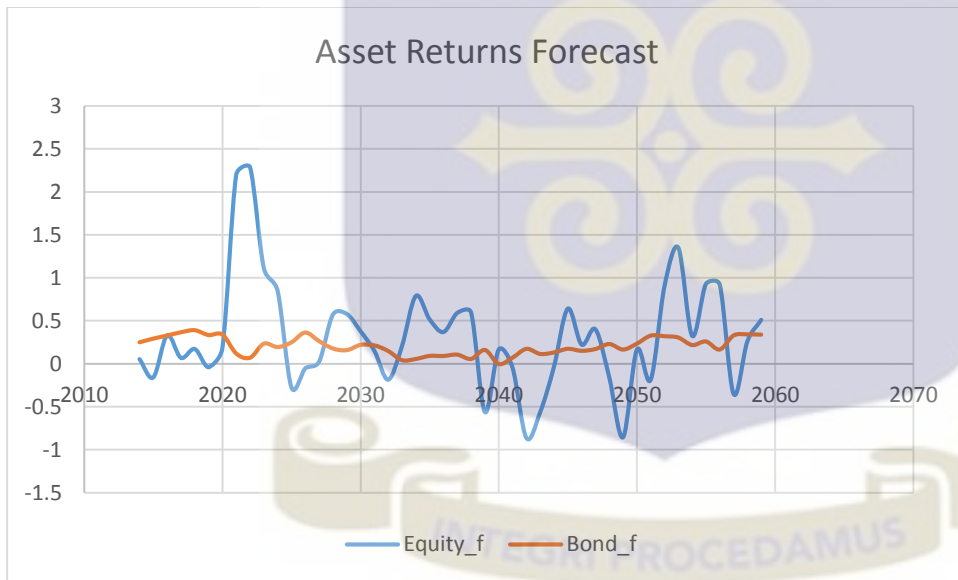
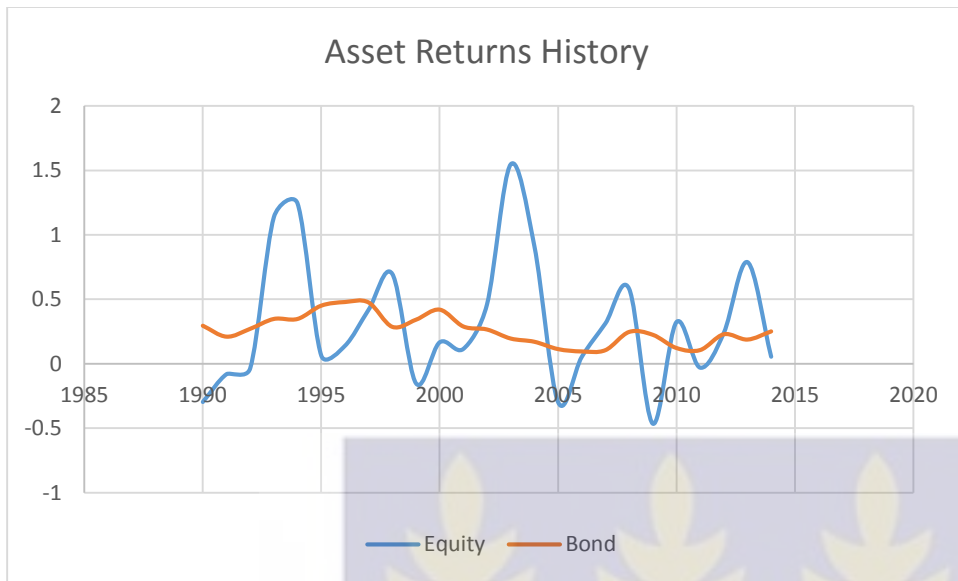
Source: SSNIT Pension Scheme

APPENDIX D: FINANCIAL DATA

Year	Equity_Returns (%)	Bond_Returns (%)	Inflation (%)	Wage_Growth (%)	GDP_Growth (%)
1990	-29.75	29.53	35.90	29.41	3.30
1991	-8.18	21.05	10.26	109.09	5.30
1992	-3.62	27.13	13.33	0.00	3.90
1993	113.73	34.78	27.66	0.00	4.90
1994	124.35	34.78	34.18	71.74	3.30
1995	6.33	45.06	70.82	51.90	4.10
1996	13.82	47.88	32.66	41.67	4.60
1997	41.85	47.53	20.46	17.65	4.20
1998	69.69	28.67	15.75	0.00	4.70
1999	-15.22	34.18	13.80	45.00	4.40
2000	16.55	41.99	40.54	44.83	3.70
2001	11.42	28.94	21.29	30.95	4.00
2002	45.96	26.60	16.99	30.00	4.50
2003	154.67	19.60	31.27	28.67	5.20
2004	91.33	17.10	16.44	21.74	5.60
2005	-29.72	11.40	13.91	20.54	5.90
2006	5.21	9.60	10.92	18.52	6.40
2007	31.21	10.60	12.75	18.75	6.50
2008	58.16	24.70	18.13	18.42	8.40
2009	-46.58	22.50	15.97	18.22	4.00
2010	32.25	12.25	8.58	16.92	8.00
2011	-3.10	10.67	8.58	19.94	15.00
2012	23.81	22.90	8.84	20.11	8.80
2013	78.81	18.80	13.50	16.96	7.60
2014	5.40	25.12	15.70	14.50	5.10
mu	31.53	26.13	21.13	28.22	5.66
sigma	49.66	11.27	13.61	23.07	2.45
c.v	1.57	0.43	0.64	0.82	0.43
mse	9.93	2.25	2.72	4.61	0.49
max	154.67	47.88	70.82	109.09	15.00
min	-46.58	9.60	8.58	0.00	3.30



Source: Author's Compilation



APPENDIX E: THE INITIAL VARIABLES

Total Salary	TPL_per_year
11,284,691,551.66	427,300,179.34
12,014,939,635.20	556,987,288.49
12,730,325,850.61	791,953,749.09
13,450,887,796.62	928,623,025.75
14,174,631,980.85	875,065,535.89
14,943,538,560.91	952,105,272.57
15,758,816,509.76	1,229,653,340.32
16,530,879,526.67	900,410,071.30
17,486,192,117.12	1,382,126,658.04
18,349,690,990.99	1,279,743,829.87
19,290,223,391.81	1,504,487,657.35
20,206,703,325.66	1,542,981,843.21
21,190,418,377.53	1,595,154,068.88
22,225,002,605.27	1,657,219,514.70
23,302,657,803.26	2,176,970,730.20
24,285,449,884.89	2,155,937,870.30
25,298,076,518.66	2,501,498,643.10
26,247,500,817.16	2,129,529,919.54
27,404,198,198.40	3,489,371,721.18
28,183,113,353.37	2,974,768,686.31
29,080,461,002.70	4,165,075,416.09
29,585,065,640.93	4,550,699,897.45
29,927,110,652.46	4,831,594,379.20
30,120,652,133.53	5,312,860,626.91
30,116,770,513.28	5,878,675,583.22
29,828,210,024.76	7,363,392,702.02
28,971,370,586.61	7,296,716,755.79
27,837,713,720.87	7,227,677,840.90
26,636,594,615.50	10,184,832,141.12
24,356,592,353.43	10,384,207,858.10
21,638,622,546.77	10,204,281,726.15
18,675,448,150.33	11,643,019,495.45
14,917,089,279.97	11,501,238,861.50
11,008,280,208.26	9,524,100,893.27
7,760,035,965.65	7,684,017,036.50
5,272,919,228.43	5,794,827,424.88
3,335,915,556.37	4,074,499,230.66
2,096,912,523.68	2,012,398,684.86
1,496,432,981.09	1,830,116,315.50
883,005,687.48	1,103,137,830.04
479,514,670.60	684,173,811.24
192,268,469.67	321,093,981.46
57,050,578.49	85,331,940.50
16,449,197.71	19,442,688.14
9,574,001.87	12,198,102.29
1,020,556.89	3,450,833.56

APPENDIX F: MATLAB FUNCTIONS AND SCRIPTS

% This VAR function was adapted and modified from the MatLab Codes freely supplied by Ambrogio Cesa-Bianchi on the internet @ <https://sites.google.com/site/ambropo/MatlabCodes>. It has to be run with the whole Cesa-Bianchi Toolbox in the MatLab path.

```
function [VAR, VARopt] = VARmodel(ENDO,nlag,const,EXOG,nlag_ex)
% =====
% Perform vector autogressive (VAR) estimation with OLS and Forecast
% =====
% [VAR, VARopt] = VARmodel(ENDO,nlag,const,)
% -----
% INPUT
% - Asset: an (nobs x nvar) matrix of y-vectors
% - nlag: lag length
% -----
% OPTIONAL INPUT
% - const: 0 no constant; 1 constant; 2 constant and trend; 3 constant,
%   trend, and trend^2 [dflt = 0]
% - EXOG: optional matrix of variables (nobs x nvar_ex)
% - nlag_ex: number of lags for exogenous variables [dflt = 0]
% -----
% OUTPUT
% - VAR: structure including VAR estimation results
% - VARopt: structure including VAR options (see VARoption)
% =====
% Ambrogio Cesa Bianchi, March 2015
% ambrogio.cesabianchi@gmail.com
% Note: this code is a modified version of of the vare.m function of James
% P. LeSage
% The Forecasting portion below is our modification

% Representation -->  $Y = Y(-1)*F' + u$ 

% Note: compared to Eviews, there is a difference in the estimation of the
% constant when lag is > 2. This is because Eviews initialize the trend
% with the number of lags (i.e., when lag=2, the trend is [2 3 ...T]),
% while VARmakexy.m initialize the trend always with 1.

% I thank Jan Capek for spotting and addressing a compatibility issue with
% Matlab R2014a

%% Check inputs
%=====
ENDO=xlsread('Assets','AssetMaster','B2:C26'); %Our input
nlag=1; %Our input
const=1; %Our input
```

```

[nobs, nvar] = size(ENDO);

% Create VARopt and update it
VARopt = VARoption;
VAR.ENDO = ENDO;
VAR.nlag = nlag;

% Check if there are constant, trend, both, or none
if ~exist('const','var')
    const = 1;
end
VAR.const = const;

% Check if there are exogenous variables
if exist('EXOG','var')
    [nobs2, nvar_ex] = size(EXOG);
    % Check that ENDO and EXOG are conformable
    if (nobs2 ~= nobs)
        error('var: nobs in EXOG-matrix not the same as y-matrix');
    end
    clear nobs2
    % Check if there is lag order of EXOG, otherwise set it to 0
    if ~exist('nlag_ex','var')
        nlag_ex = 0;
    end
    VAR.EXOG = EXOG;
else
    nvar_ex = 0;
    nlag_ex = 0;
    VAR.EXOG = [];
end

%% Save some parameters and create data matrices
%=====
nobs2 = nobs - max(nlag,nlag_ex);
VAR.nobs = nobs2;
VAR.nvar = nvar;
VAR.nvar_ex = nvar_ex;
VAR.nlag = nlag;
VAR.nlag_ex = nlag_ex;
ncoeff = nvar*nlag;
VAR.ncoeff = ncoeff;
ncoeff_ex = nvar_ex*(nlag_ex+1);
ntotcoeff = ncoeff + ncoeff_ex + const;
VAR.ntotcoeff = ntotcoeff;
VAR.const = const;

```

```
% Create independent vector and lagged dependent matrix
[Y, X] = VARmakexy(ENDO,nlag,const);
```

```
% Create (lagged) exogenous matrix
```

```
if nvar_ex>0
    X_EX = VARmakelags(EXOG,nlag_ex);
    if nlag == nlag_ex
        X = [X X_EX];
    elseif nlag > nlag_ex
        diff = nlag - nlag_ex;
        X_EX = X_EX(diff+1:end,:);
        X = [X X_EX];
    elseif nlag < nlag_ex
        diff = nlag_ex - nlag;
        Y = Y(diff+1:end,:);
        X = [X(diff+1:end,:) X_EX];
    end
end
```

```
%% OLS estimation equation by equation
```

```
%=====
```

```
for j=1:nvar;
    Yvec = Y(:,j);
    OLSout = OLSmodel(Yvec,X,0);
    aux = ['eq' num2str(j)];
    eval( ['VAR.' aux '.beta = OLSout.beta;'] ); % bhats
    eval( ['VAR.' aux '.tstat = OLSout.tstat;'] ); % t-stats
    % compute t-probs
    tstat = zeros(ncoeff,1);
    tstat = OLSout.tstat;
    tout = tdis_prb(tstat,nobse-ncoeff);
    eval( ['VAR.' aux '.tprob = tout;'] ); % t-probs
    eval( ['VAR.' aux '.resid = OLSout.resid;'] );% resid
    eval( ['VAR.' aux '.yhat = OLSout.yhat;'] ); % yhats
    eval( ['VAR.' aux '.y = Yvec;'] ); % actual y
    eval( ['VAR.' aux '.rsqr = OLSout.rsqr;'] ); % r-squared
    eval( ['VAR.' aux '.rbar = OLSout.rbar;'] ); % r-adjusted
    eval( ['VAR.' aux '.sige = OLSout.sige;'] ); % standard error
end
```

```

%% Compute the matrix of coefficients & VCV
%=====
Ft = (X'*X)\(X'*Y);
VAR.Ft = Ft;
SIGMA = (1/(nobse-ntotcoeff))*(Y-X*Ft)*(Y-X*Ft); % adjusted for # of estimated coeff per
equation
VAR.sigma = SIGMA;
VAR.residuals = Y - X*Ft;
VAR.X = X;
VAR.Y = Y;
if nvar_ex > 0
    VAR.X_EX = X_EX;
end

%% Companion matrix of Ft' and max eigenvalue
%=====
F = Ft';
Fcomp = [F(:,1+const:nvar*nlag+const); eye(nvar*(nlag-1)) zeros(nvar*(nlag-1),nvar)];
VAR.Fcomp = Fcomp;
VAR.maxEig = max(abs(eig(Fcomp)));

%% Initialize other results
%=====
VAR.invA = []; % inverse of teh A matrix (need identification: see VARir/VARfevd)
VAR.S = []; % Orthonormal matrix (need identification: see SR)

%Here is where our modifications start to integrate forecasting using the lower Cholesky
%decomposition of the generated residual covariance matrix

%% Generate lower Cholesky factor
%=====
Cholesky = chol(SIGMA,'lower');
VAR.Cholesky=Cholesky;

%% Generate correlated random errors
%=====
Randerrors=randn(45,2);
VAR.Randerrors=Randerrors;
Cranderrors=Cholesky*Randerrors';
Cranderrors=Cranderrors';
VAR.Cranderrors=Cranderrors;

%% Forecast asset with model
%=====

```

```

for i=1:45
    Equity_f(1)=ENDO(25,1);
    Bond_f(1)=ENDO(25,2);
    Equity_f(i+1)=Ft(1,1)+Ft(2,1)*Equity_f(i)+Ft(3,1)*TBondf(i)+Cranderrors(i,1);
    Bond_f(i+1)=Ft(1,2)+Ft(2,2)*Equity_f(i)+Ft(3,2)* Bond_f(i)+Cranderrors(i,2);

end

Equity=Equity_f;
Bond=Bond_f;
VAR.Equity=Equity;
VAR.Bond=Bond;
VAR.Forecasted_Assets=[Equity',Bond'];
end

```

Optimization Routines

```
%clean out the workspace variables
```

```
1. clear
```

```
%load initial variables which is a matrix of the projected total salaries and associated
%projected total pension liabilities evaluated in excel
```

```
2. Initial_Variables=xlsread('RawData','Initial_Variables','A2:B47');
```

```
3. total_salary=Initial_Variables(:,1);
```

```
4. tpl=Initial_Variables(:,2);
```

```
%set discount rate, penalty factor and spread period in years and evaluate spread annuity factor
```

```
5. drate=0.23;
```

```
6. penalty=0.20;
```

```
7. spreadperiod=20;
```

```
8. spread=ones(1,spreadperiod);
```

```
9. anf=pvvar(spread,drate);
```

```
10. anf=anf^-1;
```

```
%the deterministic contribution rate evaluated as a ratio of the present value of total pension
%liabilities to present value of total salary payments
```

```
11. det_con_rate=pvvar(tpl,drate)/pvvar(total_salary,drate);
```

```
%calculate discounted values of promised benefit obligations at time (t) from the yearly pension
%liabilities
```

```
12. pbo=zeros(1,46);
```

```
13. for i=1:46
```

```
14. pbo(i)=pvvar(tpl(i:46),drate);
```

```
15. end;
```

%load projected asset returns generated with E-Views data and generate associated weights in steps
%of unit percentages

16. Assets=VAR.Forecasted_Assets;

17. W=[(0:0.01:1);1-(0:0.01:1)];

%generate a monte carlo simulation with 10000 correlated asset paths

18. myseries=portsim(mean(Assets),cov(Assets),46,1,10000);

%taking one contribution rate at a time starting from 0% and one asset strategy starting at 0% equity
%weight, we run the iterations 10,000 times to generate the distribution of total pension costs for that
%combination and take the 1%-CVaR and associated volatility. We repeat with the same contribution
%rate but different equity weights till we exhaust the possible equity weight combinations and then
%proceed to the next contribution rate

%crbase is the normal contribution rate set between 0% and 40% in steps of 1% and w refers to the
%equity weight set from 0% to 100% in steps of 1% j is the 10,000 46x2 matrixes of the possible
%outcomes of the asset returns from the monte carlo simulation the stack order is the j-loop runs
%before the w-loop then the crbase loop rport generates the yearly portfolio returns of each monte
%carlo scenario

19. crbase=0:0.01:0.4;

20. for c=1:size(crbase,2)

21. for w=1:101

22. for j=1:10000;

23. rport=W(:,w)*myseries(:,j)';

%calculate rct, regular contributions in each year and
%invest, the amount remaining in each year for investment

24. rct=zeros(1,46);

25. invest=zeros(1,46);

26. for i=1:46

27. rct(i)=crbase(c)*total_salary(i);

28. invest(i)=rct(i)-tpl(i);

29. end;

%fund experience is the fund size after investment for each year

30. fund=zeros(1,46);

31. for i=2:46

32. fund(i)=(1+rport(i))*(fund(i-1)+invest(i-1));

33. end;

%calculate the unfunded liability and adjustments

34. ult=zeros(1,46);

35. for i=1:46

36. ult(i)=pbo(i)-fund(i);

37. end;

38. adjt=zeros(1,46);

39. adjt1=zeros(1,46);

```

40. for i=1:46
    adjt(i)=anf*ult(i);
41. if adjt(i)<0
    adjt1(i)=(1-penalty)*adjt(i);
    else
    adjt1(i)=(1+penalty)*adjt(i);
42. end;
43. end;

```

%evaluate the total pension costs for each year and divide by salary payments for that year to
 %obtain volatile contribution rate for the year and set up penalized adjustment payments

```

44. for i=1:46
45. if adjt(i)>=-rct(i);
    ct(i)=rct(i)+adjt(i);
46. else
    ct(i)=0;
47. end;
48. if adjt1(i)>=-rct(i);
    ct1(i)=rct(i)+adjt1(i);
49. else
    ct1(i)=0;
50. end;
51. conrate(:,i)=ct(i)/total_salary(i);
52. end;
53. conrate2(:,j)=conrate;
    for i=1:46
    agconrate(i,:)=mean(conrate2(i,:));
    end;
    for i=1:46
    dev(i)=agconrate(i)-crbase(c);
    devsq(i)=dev(i)^2;
    end;
54. volat=sum(devsq);
55. tpc(j)=pvvar(ct1,drate);
56. end;
57. volat2(w)=volat;
58. average_conrate(:,w)=agconrate;
59. VaR_tpc(w)=quantile(tpc,0.01);
60. CVaR_tpc(w)=mean(tpc(tpc >= VaR_tpc(w)));
61. Exp_tpc(w)=mean(tpc);
62. end;
63. volatility(c,:)=volat2;

```

%accumulate computations into a matrix representing the results from
 %the different normal contribution rates for a single weight combination

```

64. CVaR(c,:)=CVaR_tpc;
65. Exp_Pen_Cost(c,:)=Exp_tpc;

```

```
66. end;
```

```
%search for the minimum volatility and minimum CVaR values
```

```
67. Minimum_Volatility_Value=min(min(volatility));
```

```
68. Minimum_CVaR_Value=min(min(CVaR));
```

Scripts for Graphs

```
%script for Behavior of 1%-CVaR of Total Pension Cost with Increasing Contribution Rates
```

```
%graph
```

```
for b=1:10:101
```

```
hold on
```

```
plot(crbase,CVaR(:,b))
```

```
title('Behavior of 1%-CVaR with Increasing Normal Contribution Rates','FontSize',14)
```

```
xlabel('Normal Contribution Rates (%)','FontSize',14)
```

```
ylabel('1% CVaR of Total Pension Cost','FontSize',14)
```

```
legend('Equity Weights','0%','10%','20%','30%',...,'80%','90%','100%','FontSize',14)
```

```
set(gca,'FontSize',14)
```

```
hold off
```

```
end
```

```
%script for Worst Case Pension Cost Outcomes – The Overall Picture graph
```

```
mesh(W(1,:),crbase,CVaR)
```

```
xlabel('Equity Ratio (%)','FontSize',14)
```

```
ylabel('Normal Contribution Rates','FontSize',14)
```

```
zlabel('1% CVaR of Total Pension Cost','FontSize',14)
```

```
set(gca,'FontSize',14)
```

```
%script for Annual Contribution Rate Volatility Behavior graph
```

```
mesh(W(1,:),crbase,volatility)
```

```
xlabel('Equity Ratio (%)','FontSize',14)
```

```
ylabel('Normal Contribution Rates','FontSize',14)
```

```
zlabel('Annual Contribution Rate Volatility','FontSize',14)
```

```
set(gca,'FontSize',14)
```

%script for *Volatility Profile for 0 - 10% Contribution Rate and up to 50% Equity Weight* graph

```
risk2=volatility(1:11,:);
cr2=crbase(1:11);
w2=1:1:50;
risk3=risk2(:,1:50);
mesh(w2,cr2,risk3)
```

```
xlabel('Equity Ratio (%)','FontSize',14)
ylabel('Normal Contribution Rates','FontSize',14)
zlabel('Annual Contribution Rate Volatility','FontSize',14)
set(gca,'FontSize',14)
```

%script for *Volatility Profile for 10 - 40% Contribution Rate and up to 50% Equity Weight* graph

```
risk2=volatility(11:40,:);
cr2=crbase(11:40);
w2=1:1:50;
risk3=risk2(:,1:50);
mesh(w2,cr2,risk3)
```

```
xlabel('Equity Ratio (%)','FontSize',14)
ylabel('Normal Contribution Rates','FontSize',14)
zlabel('Annual Contribution Rate Volatility','FontSize',14)
set(gca,'FontSize',14)
```

%script for *Segment of Volatility Profile for Equity Weight beyond 50%* graph

```
risk2=volatility(7:21,:);
cr2=crbase(7:21);
w2=51:1:101;
risk3=risk2(:,51:101);
mesh(w2,cr2,risk3)
```

```
xlabel('Equity Ratio (%)','FontSize',14)
ylabel('Normal Contribution Rates','FontSize',14)
zlabel('Annual Contribution Rate Volatility','FontSize',14)
set(gca,'FontSize',14)
```

APPENDIX G: VAR ESTIMATION OUTPUT AND DIAGNOSTICS

%VAR function call command in MatLab using the modified Cesa-Bianchi (2015) Toolboxes

```
>> VAR=VARmodel_with_Forecast;
>> VARopt=VARoption;
>> Summary=VARprint(VAR,VARopt);
```

%VAR coefficients of the model, covariance matrix and its Cholesky factors

Reduced form VAR estimation:

	Equity	Bond
c	0.1495	0.0568
Equity(-1)	0.1470	0.0155
Bond(-1)	0.5478	0.7566

Variance-Covariance Matrix:

0.2636	-0.0028
-0.0028	0.0063

Lower Cholesky Decomposition Matrix:

0.5134	0.0000
-0.0054	0.0790

Diagnostics in EViews Output

Roots of Characteristic Polynomial

Endogenous variables: EQUITY BOND

Exogenous variables: C

Lag specification: 1 1

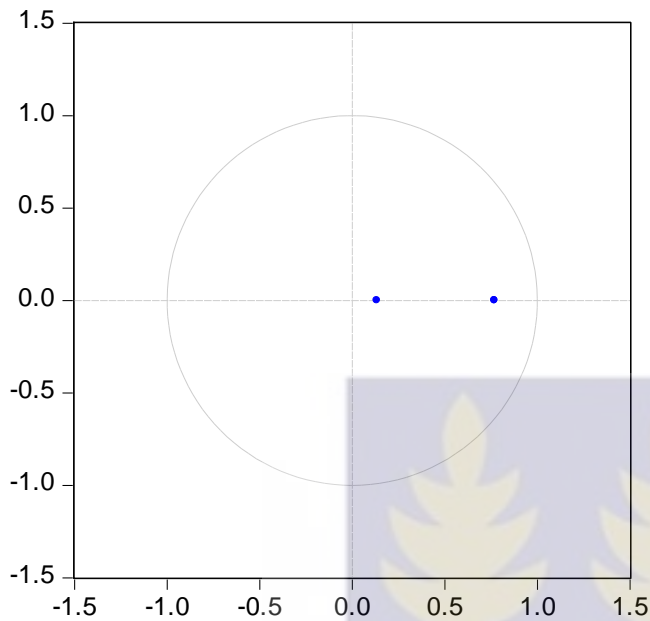
Date: 07/10/15 Time: 17:28

Root	Modulus
0.770229	0.770229
0.133301	0.133301

No root lies outside the unit circle.

VAR satisfies the stability condition.

Inverse Roots of AR Characteristic Polynomial



VAR Lag Order Selection Criteria

Endogenous variables: EQUITY BOND

Exogenous variables: C

Date: 07/10/15 Time: 17:33

Sample: 1990 2014

Included observations: 23

Lag	LogL	LR	FPE	AIC	SC	HQ
0	0.423056	NA	0.003932	0.137126	0.235864	0.161958
1	11.32943	18.96760*	0.002163*	-0.463429*	-0.167213*	-0.388931*
2	13.57062	3.507956	0.002549	-0.310489	0.183204	-0.186326

* indicates lag order selected by the criterion

LR: sequential modified LR test statistic (each test at 5% level)

FPE: Final prediction error

AIC: Akaike information criterion

SC: Schwarz information criterion

HQ: Hannan-Quinn information criterion