UNIVERSITY OF GHANA

APPLICATION OF BÜHLMANNS-STRAUB CREDIBILITY THEORY TO CLAIM HISTORIES OF NON-LIFE MARINE INSURERS IN GHANA

BY

PAUL KWAME ADJORLOLO

(10599328)

THIS THESIS IS SUBMITTED TO THE DEPARTMENT OF STATISTICS AND ACTUARIAL SCIENCE, UNIVERSITY OF GHANA, LEGON IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF MASTER OF PHILOSOPHY IN ACTUARIAL SCIENCE

JULY, 2019

DECLARATION

Candidate's Declaration

I, Paul Kwame Adjorlolo, hereby declare that this thesis is my own work, design and execution, submitted for the award of a Master's degree in Philosophy and that, to the best of my knowledge it contains no material that has been submitted to this institution or any other university except where duly acknowledged in the text.

Signature ………………………………… Date: ……………………

Paul Kwame Adjorlolo

(**10599328**)

Supervisors' Declaration

We hereby certify that this thesis was prepared from the candidate's own effort and supervised in accordance with the University of Ghana's guidelines on supervision of theses.

Signature………………………………… Date………………………

Dr. Anani Lotsi

(Supervisor)

Signature ………………………………… Date………………………

Dr. Felix O. Mettle

(Co-Supervisor)

ABSTRACT

The study sought to demonstrate how credibility claim costs without the consideration of claim frequency and claim severities underlined by different risk profiles underestimate claim costs or premiums charged policyholders by non-life insurance companies. We used secondary data of non-life marine insurers in Ghana, claim histories that range from the period of 2013 to 2018. The claim histories included claim sizes, claim counts and policy counts. Bühlmans-Straub Credibility theory model was used in estimating credibility weights, credibility claim costs, credibility claim frequencies and credibility claim severities and subsequently find the credibility frequency-severity claim cost as the product of credibility claim frequency and severity for the individual and respective risk classes. We compared the estimates of the claim costs or premiums and have observed that the credibility claim costs underestimates claim costs or the average claim costs compared to the credibility frequency-severity claim costs for most of the risk classes. This is an indication of how a lack of consideration for variability or unstableness of claim frequencies and severities with different risk profiles undermine claim costs estimated through credibility rate makings in insurance. The study recommends that credibility ratemaking by insurance companies based on inadequate claim history and or with enough class risk variation should include credibility risk frequency and severity for the determination of credibility risk premiums.

DEDICATION

To all women who care for children in any state or form.

ACKNOWLEDGMENTS

My sincere gratitude goes to my supervisors; Dr. Anani Lotsi and Dr. Felix O. Mettle for their patience and guidance and immeasurable input into this work. My sincere gratitude also goes to Dr. Gabriel Kallah-Dagadu for his advice and encouragement. My genuine appreciation also extends to all the lecturers at the Department of Statistics and Actuarial Science. I'm most grateful to my parents (Mr. Edward Newlove Tsatsu and Mrs. Christiana Agbenyegah) for their counseling and prayer, siblings for their support especially Dr. Samuel Adjorlolo and the wife, Mrs. Rita Adjorlolo and my classmate and friends for their supports especially Mr. Freeman Kofi Ahorkonu. Am also grateful to all members of the Sogakofe Church of Christ for the love they showed me in hard times whilst pursuing academic success.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

MI:	Marine Insurance
IACG:	International Awareness of Coordinating Group
NIC:	National Insurance Commission
GIA:	Ghana Insurers Association
IUMI:	International Union of Marine Insurers
GIBA:	Ghana Insurance Brokers Association
FPSO:	Floating Production Storage and Offloading
IIM:	The Insurance Institute of Michigan
EPV:	Expected Process Variances
VHM:	Variance of the Hypothetical Means

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

Modern insurance is described as a two-party contract normally between an insurance company and an insured who an insurance company accepts to pay a settled amount when a loss occurred in exchange for premiums (Mishra, 2004). Under the maritime law (Act 1906), marine insurance is described as; "A contract whereby the insurer undertakes to indemnify the insured, in the manner and to the extent thereby agreed, against marine losses, that is to say, the losses incident to marine adventure". Insurance reduces the risk of total loss to policyholders' as at when there is an insured hazard. Consequently, insurance companies function as managers of premiums gathered from the citizens who purchased insurance policy and also operate as allocators after offering them insurance, i.e paying for losses to those who encounter accidental misfortunes from the premiums collected from insuring citizens.

In view of the growing hazards on our seas, the need for specialized sea-related insurance has become more critical. The catastrophic effects of happenings at sea may be similar or more heartrending than those in the air. The need for insurance emerged from the desire of business persons to take calculated risks. Hitherto, such risks were encountered when sailing ships were sent off on hazardous sea voyages to trade with other countries far from their sources of destination without any form of financial compensation. This gave birth to marine insurance as the mother of all insurances. Marine Insurance (MI) intended to cover loss or damage to ships, cargo, terminals and any other type of ocean transport for which property is transferred, obtained or kept between the point of origin and the final destination also protects importers and exporters against loss, theft or damage to goods or property conveyed by sea including other waterways and rivers. Ship-owners are also protected against loss or damage to hull, machinery of the vessel and legal liability of the ship-owners, while on the high seas (Zogbenu, 2018).

Africa's marine insurance penetration nonetheless stays disappointingly low. Whilst worldwide changes have crept up by way of 2% as indicated with the aid of the global International Union of Marine Insurers' (IUMI's); Africa's numbers are still at a baffling 2.4%. Local players must take advantage of the lucky breaks which accompany the increasing worldwide exchange. Africa must also desire to position resources into growing the capacity in marine protection. There are not many markets like South Africa and Mauritius at the mainland with enough chronicles of marine products and local talents (IUMI, 2018). Ghana, in recent years has made striking effort to improving the marine insurance industry; the International Awareness of Coordinating Group (IACG) which comprises representatives of the National Insurance Commission (NIC), the Ghana Insurers Association (GIA), the Ghana Insurance Brokers Association (GIBA), held a workshop on marine insurance market. Among other relevant stakeholders who participated in an all-important workshop were freight forwarders, cargo haulers, trans-ocean importers and exporters, trade regulators and policymakers, among others, to deliberate on the way forward for marine insurance.

The workshop which was held on June 2018 seeks to provide a single platform on which concerns and perceived misgivings from all stakeholders would be addressed; right from 'the anchor to the masthead' until premiums are duly collected and claims paid when they fall due'**.** This would to a very large extent streamline and strengthen the marine insurance sector in Ghana. The procurement of specialized vessels like the Floating Production Storage and Offloading (FPSO) Kwame Nkrumah together with other cargo vessels, there is no way one could gloss over the need for various types of marine insurance policies to manage risks. And in Ghana, Marine Laws are in place to essentially protect local importers who will be disadvantaged in the event of offshore claims - especially in the event of litigation.

1.2 An Overview of Marine Insurance

For people to have a right knowledge of the outlook on maritime protection, it becomes important to hint the events of company transactions and some of the international locations of the sector. Seas have been the fundamental transport systems of alternate used within Man's early records evidently observed throughout in sacred and secular history. A couple of nations have claimed the honour of getting advanced the association of compensation as insurance or risk management. Be that as it is able to, the fine evidence proves that the Jews during the season when they were expelled from France in the remaining pieces of work of the 12th century presented this type of insurance plan for the protection of their belongings among their deportation from France. Preferred average and Bottomry Bonds have been typically the two oblique varieties of cutting-edge coverage practiced and had been utilized early inside the records of mass-marketplace (Winter, 1919).

A lawful widespread of maritime laws by which all parties on an ocean journey have an extraordinary proportion of any losses incurred from a deliberate forfeiture of part of shipments or the ship or part of those to save lives is known as the law of the general average. While the general average draws its beginnings in the ancient oceanic law, some aspects of the admiralty regulation form part of most countries' laws (Johnson, 2013). Bottomry is an out-dated bond agreement described as a loan in which a vessel is mortgaged in one of these manners that if the vessel had been lost, money lend by a lender is lost too, but if the ship arrived securely at the port of vacation spot, the borrower pay back the mortgage loan to the lender and with an interest agreed upon in advance (Winter, 1919). Besides the loans, a ship can be promised as a Respondentia Bond and cargo hypothecated and the documents carrying out the terms of the contract were known as Bottomry Bonds.

1.2.1 Modern Marine Insurance

In fact, a Bottomry bond was an inverse type of the current marine insurance scheme; contemporary time marine insurers receive premium against loss of the ship and shipments, plus interest and other costs, which amount the underwriter agreed to pay the insured as compensation in the case of a ship or cargo being lost as a consequence of the risks of insurance or marine adventure. In addition, under current commercial use, credits used on ships are produced through the implementation of hypothecated bonds on ships , Which, on the other hand, is covered by an insurance policy that must be paid to the lender (Winter, 1919). Since its advent in the 13th century, modern marine insurance has experienced tremendous growth and has had legal corroboration throughout the globe. For instance, the Hanseatic backing across countries including Northern Europe's emerging merchants had an insurance centre in Bruges, known as the first insurance chamber. The town of Barcelona also laid down the first statute recorded for insuring vessels in 1432. The Lombards started marine insurance by progressing amounts on Bottomry loans. The creation of Lloyd's of London in the 16th century became the insurers, traders, ship owners ' first meeting place and others interested in insuring their cargoes and boats including those that are ready to trade in the marine enterprises and at the time, Lloyds of London became the world's biggest insurance industry, particularly marine protection (Kyriaki, 2007).

1.3 The Ghanaian Insurance Industry

In Ghana, the National Insurance Commission (NIC) is the only establishment commanded to control and supervise insurance against future losses in the nation according to the insurance laws (Act 724, 2006). Ghana's insurance companies are now divided into three categories: general and composite and life insurance. Loss adjusters, insurance brokers, actuarial firms and dealers are also in Ghana. Globally, insurance provided the risk transfer mechanism in the early stages of the insurance industry, but now the industry is helping to channel resources appropriately to promote business operations in the economy (Curtis & Eric, 2013).

one element of the non-life insurance sector in Ghana that's also beginning to grab attention is the maritime insurance industry due to the recent enforcement of section 37 of the Insurance Act, 2006 (Act 724) ; this act compelled local dealers to ensure their products locally for the industry to take advantage of premium income gains (NIC, 2015). The implementation of this amendment has gone all the way to positively influence the country’s economic business industries**.**

1.4 Statement of the Problem

The non-life insurance companies like any other type of insurance business do not only act as risk managers but also serve as intermediaries that mobilize funds for banks which are also lend to business firms who also invest in the economy. The economy of Ghana like any other country will be affected if insurance companies do not charge risks the right premium in a competitive insurance business because insurance companies would not have enough funds to pay for losses or enough to invest in the economy(Mazviona, Dube, & Sakahuhwa, 2017). This can see many insurance companies out of business and the collapse of the insurance business can be disastrous to any economy, particularly in developing countries. As a consequence of these problems faced by insurance companies especially when there is little claim history available it is necessary to determine the best procedures of estimating fair premiums to be charged policyholders by these insurance companies.

The researcher’s interest in this study is to use Bühlmans-Straub credibility theory as an empirical approach to estimating and comparing credibility frequency-severity premiums or claim cost to credibility premiums (claim cost) for policyholders of some selected leading maritime insurance companies in Ghana taking into consideration the heterogeneous and limited past claim histories.

1.5 Objectives of the Study

The primary target of this research or study is to find the credibility frequency-severity premiums or claim costs for policyholders for each of the non-life marine insurance companies (risk classes) using claim frequencies and severities experienced by applying Bühlmans-Straub Credibility Theory model and also compare the result with credibility claim cost or premiums using annual claim costs estimated based on the same method and condition.

1.5.1 The Specific Objectives

The specific objective of the study is to;

1. estimate the average claim cost of the individual non-life marine insurance companies
2. estimate the factor of credibility for each non-life insurance companies
3. estimate credibility premiums based on claim costs and credibility frequency-severity premium for policyholders of the risk classes or individual non-life marine insurance companies for the period 2018 using claim histories between the period of 2013 to 2018.
4. Compare the two credibility estimates; the credibility frequency-severity risk claim costs and the credibility risk claim cost.

1.6 Research Questions

Three questions seem crucial in finding the credibility claim costs. Thus,

1. How much variation is there in the claim costs or frequencies or severities of the companies?
2. How homogeneous are the risk classes in terms of claim frequency and claim severity?
3. How significant are the credibility factors of the Bühlmans-Straub model in doing experience rating?

1.7 Justification

This research will open the debate on the need to integrate the frequency and severity of claims in a suitable and fair determination of Ghanaian non-life insurance policies ' credibility premiums. The Study can also serve as a point of reference for decision-making in evaluating and defining equilibrium between premiums granted based on experience rating and collective risk rating organizations. The results of this research will be helpful to players and stakeholders in the industry of maritime business, in particular the National Insurance Corporation (NIC) in its quest to determine premiums among competitive non-life insurance firms taking into consideration the intrinsic random risk files of each non-life marine insurance company that influence the size of claim costs, frequency and severity of these claims. The implementation of Bühlmanns-Straub's credibility methodology to selling maritime insurance products based on the frequency and severity of claims that includes individual risk profiles making the marine insurance company more competitive, leading to general industrial growth and also helping to reduce the insolvency of maritime industrial businesses.

1.8 Scope and Limitations

This study aimed at determining the best balance between experience ratings and class ratings incorporating the frequency and severity of claim histories. The data considered for this research is claim amounts, claim counts and policy counts for policyholders of maritime insurers reported by the marine insurance companies to the NIC from 2013 to 2018. Bühlmans-Straub credibility theory model was used to estimate the credibility claim cost using only annual claim cost with their respective policy counts and credibility frequency-severity claim counts or premiums using claim frequencies and severities with their respective policy counts and claim counts respectively. From these two estimates the best balance was observed. One of the limitations of this study is that the claim histories used for the analysis did not cover all non-life marine insurance companies in Ghana but limited to only 15 non-life marine insurance companies. The study did not factor inflation and other macroeconomic factors that might have been having an effect on risk rating into the analysis.

1.9 Organization of the Thesis

The study is structured into five major sections. The first section gives background of the research and brief overview of marine insurance. The second chapter focuses on literature and model assessment of other associated papers. The methodology and fundamental hypotheses are discussed in the third chapter. Analysis and presentation of results including interpretation of results are completed in the fourth chapter and discussion of research findings, conclusions and recommendations in chapter five.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter includes the review of insurance claim costs, underwriters' profitability for assuming risk, class and experience rating of insurance as well as credibility theory. The section also includes an appropriate empirical evaluation of the theory of credibility and its creation, determination of the factors of credibility and application of the credibility theory to the pricing of insurance products and the assessment of credibility premiums. The evaluation of the credibility views expressed by other authors, such as the credibility model of Bühlmann, the merits, and demerits or constraints of these models, is considered in this chapter with the aim of adequately using this understanding to determine claim costs of maritime insurance in Ghana.

2.2 Insurance Claim Costs

To be allowed to enter into an agreement with an insurer, an insured must pay a premium. The price of claim or premium may vary among individuals or insureds depending on how dangerous they are or how dangerous the social group to which they belong is perceived. Information is essential for insurance organizations to be able to determine the appropriate premium for individuals or groups of individuals to insure. Underwriters pooled the premiums from policyholders and use this pool of money received by the insurers is used to pay loss cases as they occur (Thiery & Schoubroeck, 2006).

“An insurance claim is a formal request to an insurance company for coverage or compensation for a covered loss or policy event. The insurance company validates the claim and, once approved, issues payment to the insured or an approved interested party on behalf of the insured ” (Kagen, 2018). In principle for an insurance provider to fulfil and cover all outstanding claims before the end of the accounting period, it is necessary for the insurer to render arrangements for claims from monies earned within such an accounting period. Shapland (2007) defined outstanding claim provision as “a sum conveyed in the liability area of a hazard or risk bearing element's balance sheet as a result of claims incurred preceding a given bookkeeping date, where liability, as utilized in the definition, is the actual sum of losses insureds made on an insurance company and that will, at last, be paid by a risk bearing company before the end of a given bookkeeping date”.

Insurance is the transfer of potential losses from policyholders to insurance companies or insurers for a predetermined claim price, which in effect offers the economic insurance coverage service in the event of a loss under policy cover. The desire of a reasonable premium is of paramount relevance for an insurer regarding the retaining of present clients, gaining new enterprise and also to have competitive advantage when the policy targets a big population and also extends over a significant region of property (Emanuelsson, 2011). Claim payment could be very crucial to the execution of any insurance coverage, which is why the NIC requires insurance companies to have an extremely good claim provision on their accounting books. Regulators are particularly concerned about the economic situation of insurers, given that the security of insureds and the security of investors is of paramount importance to regulators and stakeholders by ensuring that the security net supplier gives an accurate picture of their solvency situation (Asare & Opokutakyi, 2017).

According to Asare and Opokutakyi (2017) in determining the solvency factors of non-life insurance firms in Ghana, the absence of appropriate provision for enough claim reserves is significantly the primary mechanistic explanation of the bankrupt insurance industry, exposing investors and various associates to enormous losses and being unable to fulfil their share of insured contracts. Genuinely, insolvent insurance firms are not allowed to continue offering protection because they do not have the financial support to maintain their legally binding obligations to their clients (Cheung, 1997). Notwithstanding the relevance of this concern, little is learned about all the concepts of the topic in the non-life insurance market in Ghana, particularly in the maritime insurance sector.

Andoh and Aboagye (2014) also researched the factors that influence property insurance, market-based claims provision errors. The research used the information on published and paid-out claims for the 2000-2010 period. The authors discovered that the claim reserves or provision errors of the underwriters are random cross-sectional over companies, meaning that insurers behave separately and are not affected by trends of sector and competition. Insurance firms bankruptcy could be avoided if the pricing or assessment of claim costs for pricing insurance products for subsequent instant periods dependent on past knowledge of personal hazards for insurers taking considering the risks’ features of each business and the relationship between individual risk premiums and the collective claim cost (Asare et al., 2017).

Many researchers used distinct techniques for exactly estimating insurance firms' prices required to charge policyholders in order to fully cover claim reserves when risks occur. The actuarial accuracy measures are one of these distinct techniques and are normally employed depending on the data available on claims and based on what assumptions are allowable. Zhang (2004) suggested a Bayesian non-linear hierarchical model that tends to tackle some of the major problems faced by insurance companies in anticipating the outstanding claim sizes for which they will finally be required to redeem. This approach requires taking into account previous understanding and expert and technical opinion to be included in the assessment by judgmentally selected priori distributions. Viaene et al., (2007) similarly, in their investigation, it was found out that the assessment of the likelihood of claim is critical since a loss protection company can use these assessments to allocate claim reserve boundaries and discounts based on the threat features of an individual client or to develop strategies to detect false claims. Consequently, Meulbroek (2000) argued that loss protection Companies must treat corporate governance as development of associated variables and opportunities. Boland (2007) in his research, he said, in order to cope with allegations incidental events emerging in advance underwriters must utilize prognostic techniques for managing the level of loss, indicating that back-up plans are required to identify techniques for anticipating claims and appropriately charge a premium for covering claims created by those risks. The recovery schemes need to search for suitable solutions to capturing the risk features of policyholders influencing claim frequencies and severities and, consequently, distinguishing persons with poor risk, with a greater affinity to periodic claims on insurance companies.

As stated by Weisberg (1983), insurance organizations are endeavouring to assess sensitive expenses of insurance arrangements depending on the announced losses for specific types of hazards. The estimates must be based on previous claims histories; claim severity and the frequency of losses so as to determine the trends that occur in order to be able to take appropriate measures to mitigate losses and the consequences associated with the losses.

2.3 Profitability of Underwriters for Assuming Risk

Fixed premium firms are provided by insurers as a commercial insurance service with the main objective of profiting. The fixed premium is a predetermined insurance premium on the basis of agreements between underwriters and policyholders. In these agreements, an insurer continues the hazard section via charging a fixed price to the insured relying on the regular assessment of losses. The hazard is either accepted or reinsured or backed up or securitized or mixed of the two against the regular losses of these operations. Additionally, the peril risk claim cost is needed to pay policyholders, shareholders and investors. Because the insurance firms are required to make a profit, in theory of economics the charges to the policyholders would be larger than that of mutual insurance premiums that are cover only the full cost of losses (Ravichandran, 2001).

Drewry (1998), Haiss (2006) and Marrewijk (2002) all, in their study have made significant attempt to enumerate some problems and the causes undermining the performance and the profitability of the marine insurance industry. King (2008) suggested that the marine insurance industries must charge competitive and reasonable premiums as a solution to the problems of marine insurance. However, the suggestion by King (2008) did not really provide enough information on how policy or underwriters must go about the rate making in order to achieve competitive and reasonable premiums especially in situations where there is only a little information on risks to be considered or rated. Petersson (2010) stated that, because of the uneven nature of the hazards concerned, it is most hard to decide on maritime insurance claim costs. Hsrrington and Niehaus (2003) also mentioned that for insurance companies to price practical rates due to competition, they should rate premiums that may contend with the expected cost of claims and administrative prices. The authors also pointed out that if Policyholders ' risk is homogeneous, the company can prospectively do equal treatment. Potential equal treatment with a variable degree of risk for insured persons, however, may generate adverse selection for insurance companies. In this manner, less dangerous policyholders instead of paying same premium as higher-risk policyholders they moved to a more cautious treatment scheduled organisation. It is obvious that insurance firms cannot just enough rely on experience and expertise or by mere direct calculation of pure premiums as reasonable premiums for the competitive market without proper statistical analysis of risks facing individual exposures of the insurance policy. The study of risks leading to premium determination must include the study of claim severities as well as claim frequencies which all affect the financial position of insurance firms. Lawrence et al., (2017) emphasized in summary that bankruptcy of insurers could be avoided if suitable statistical and mathematical techniques are used to estimate premiums priced by insurers for selling insurance products in the future based on individual or class risks' claim information and other comparable risks deemed relevant.

2.4 Class and Experience Rating of Insurance

Risks with almost uniform characteristics or heterogeneous but with restricted reliable claim information, have been difficult to rate. Therefore, an additional and related claim data from similar risks that are not necessarily the same may be useful in assessing those risks whose estimated premiums may lie between the average claim cost and collective claim cost. This is very necessary for estimating fair premiums especially for contracts with limited historical information and cannot be depended upon for estimating premium for the next immediate year. Bhulman and Gisler (2005) demonstrated with diagrammatic illustrations of the difficulties to rely on limited available data and the problem of allocating collective or grand risk premium to less risky policyholders of non-life insurance contracts instead of doing experience rating or determining premiums by attaching some level of credibility to policyholders of a certain class of risks.

The authors assumed a claim information including gross premiums on a portfolio comprising of nine health contracts (appropriate and applicable to marine insurance contracts) over a period of 5 years. Thus, contract 1, 4 and 7 were assumed to have had a premium volume of 2000 per year, and contract 2, 5 and 8 had a premium volume of 20000 per year whilst the contracts 3, 6, and 9 had a premium volume of 200000 per year all in CHF(Swiss franc). Their loss ratios were calculated as the ratio of the aggregate claim to the gross premium of each contract over the five years expressed as a percentage.

Figure 2.1 demonstrates the average loss ratio observed over the five years. From the Figure, the average loss ratio for contracts is 50 per cent for 1 to 3, 75 per cent average loss portions for contract 4 to 6 and 100% average loss proportion for contracts 7 to 9.

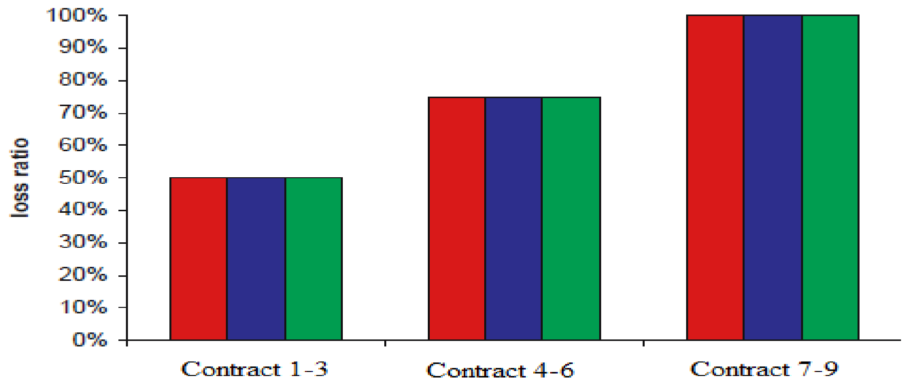

Figure 2.1 Average loss ratio of a portfolio of nine contracts

Source: Bühlmann and Gisler (2005).

The 75 per cent average loss proportion is the make back premium for covering claims whereas 25 per cent of the average loss ratio of insurance costs is needed for administrative costs as well as for various charges. Estimates of new claim costs for each policyholder had to be determined for the insurance policy for the next immediate period. In studying these agreements on the basis of their individual loss ratios over the last five years, two deductions can be made.

a) The variations observed in the loss ratios among the policyholders or individual contracts are completely as a result of the randomness of the claims and not due to the inherent different risk profiles of each these of contracts. This deduction makes no contract better or risky than the other. Hence, the best estimated loss ratio that can be made for the next immediate period for a given contract, is the 75 per cent average loss proportion. Therefore, a change to the current premium level of individual contracts is unnecessary.

b) The variations in the measured ratios of losses are not due to the random nature of claims but are systematic. The differences are due to individual risk profiles fluctuating over the agreements. This deduction makes some contracts better than others. Hence premiums for the first three contracts ought to be reduced by one-third of the 75 per cent average loss proportion, while the last three ought to be increased by one-third of the 75 per cent average loss proportion. The authors also considered the ratios of annual losses over the five-year period independently for the small, medium and large contracts for an increasing examination.

Figure 2.2 shows the annual loss ratios of agreements 1, 4 and 7, each of which has a premium amount of 2 000 Swiss francs per year.

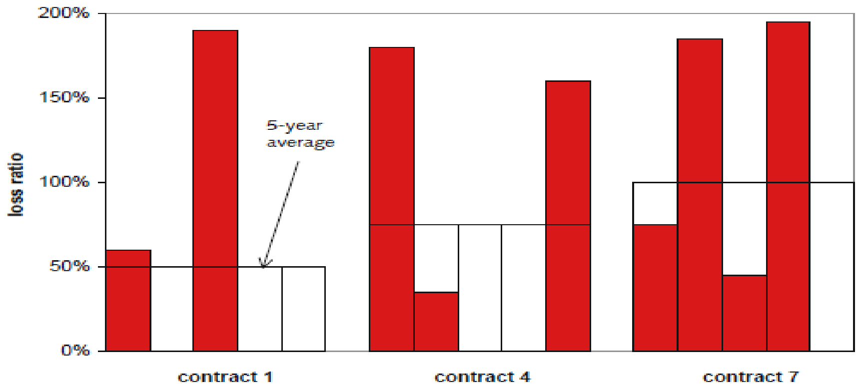

Figure 2.2 Yearly loss ratios of the small contracts

Source: Bühlmann and Gisler (2005).

From Figure 2.2, the authors identified a high variability in the annual average loss proportions and wonder whether the differences observed in the contracts of the 5-year loss ratios, irregular fluctuations should be attributed as they were, this truth decreases trust in the average reliability as a long-term average.

Figure 2.2 shows the annual loss ratios of contracts 2, 5 and 8, each of which has a premium amount of 2 000 Swiss francs per year.

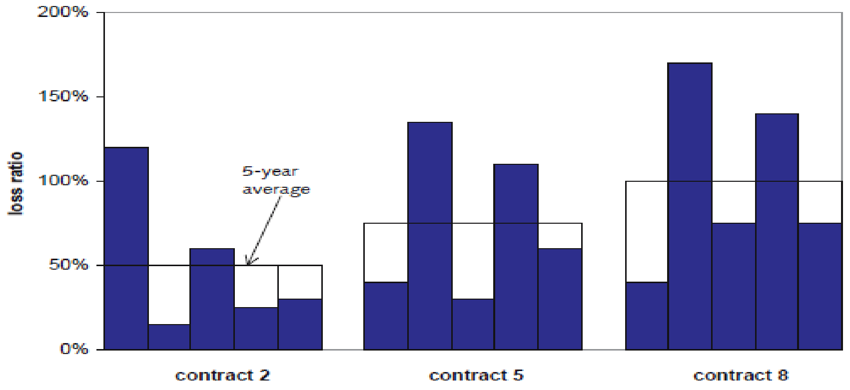

Figure 2.3 Yearly loss ratios of the medium-sized contracts

Source: Bühlmann and Gisler (2005).

Figure 2.3 also shows a similar picture as figure 2.2. The annual fluctuation is still significant, but it is difficult to accept the causes of the differences between the proportions of the contract to be by random nature of claims.

Figure 2.4 shows the yearly or annual loss ratios for the biggest contracts 3, 6 and 9, all having 200000 Swiss francs as their annual premium volume.

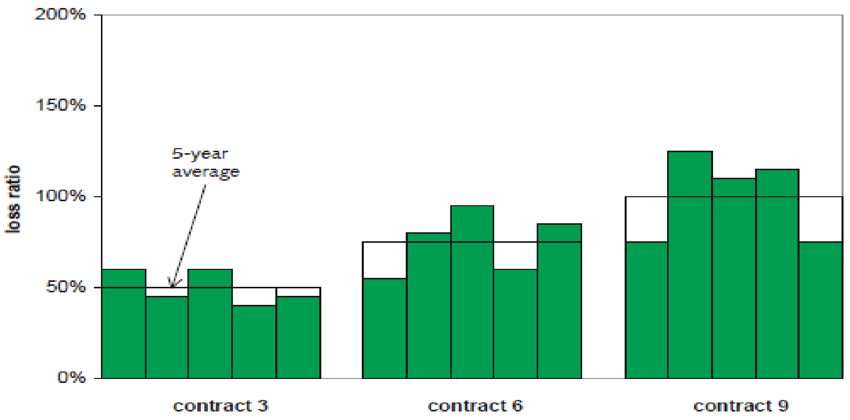

Figure 2.4 Yearly loss ratios of the large contracts

Source: Bühlmann and Gisler (2005).

From Figure 2.4, it seems certain that the risk profiles of the three contracts show real and clear contrasts. Taking into account contracts 1, 2 and 3 shown in Figure 2.5, the average loss ratio for each contract over the course of 5 years is 50 per cent.

Figure 2.5 demonstrates the annual loss proportions of the contracts 1, 2 and 3.

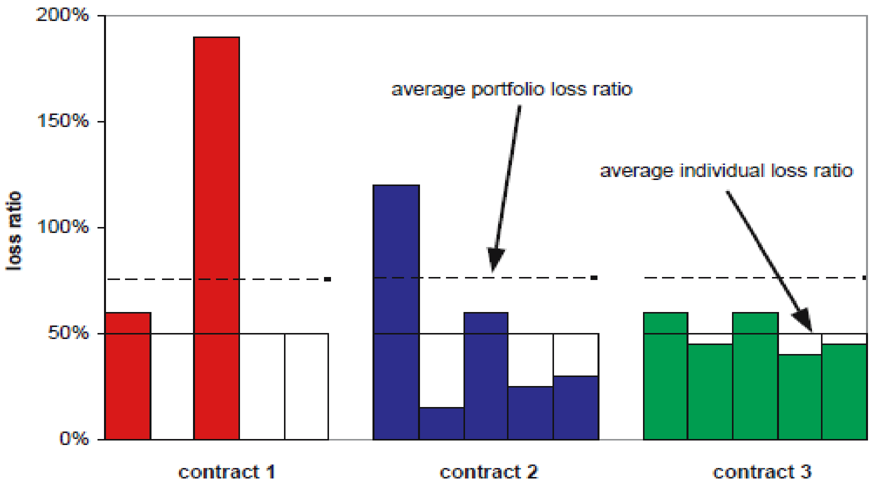

Figure 2.5 Yearly loss ratios of contracts 1-3

Source: Bühlmann and Gisler (2005).

Notwithstanding, consideration of the accessible data without further critical examination, the best prediction for each of the three agreements should not be the same. Thus, should a forecast of an average loss proportion of 50 per cent for the individual contracts or 75 per cent over the whole portfolio be utilized as an estimate? For contract 1, there is little certainty that its own measured average loss ratio is accurate as an estimate owing to the elevated variability in its annual loss ratios; we would be inclined to use the observed average loss ratio over a total of nine contracts ; 75 per cent as the estimate. For contract 3, the information from its claim history proves to be reliable and one would feel favourably arranged to use as an estimate a projected value in the neighbourhood of its 50 per cent average loss ratio. In addition, our prediction for contract 2 would most probably be between agreements 1 and 3.

Rate makings that are not based on statistical or mathematical reasoning may inherently involve a great deal of bias in the operation that may cast doubt on the insurers' forecasted and charged premiums. This is because such procedures do not adequately incorporate or account for variation or the different risk levels that are associated with each contract neither do the procedures determine the reliability of the data used for forecasting premiums. The solution to the issue is to determine and use suitable statistical and mathematical techniques to capture and describe the contract variation as well as the reliability of previous claim documents used in forecast premiums. The credibility theory, based on the literature of other authors, offers a sound mathematical and statistical foundation for finding a solution to the practical challenge of estimating reasonable insurance premiums and establishes guidelines for finding a solution to the arguments described in figure 2.1, 2.2, 2.3, 2.4 and 2.5.The methodology employed allowed the researcher to estimate and compare the claim severity and frequency including premiums of policyholders of some selected leading marine insurance companies in Ghana.

2.5 Credibility Theory

Credibility model is a set of processes for estimating claim costs for claims of contracts in advance before they occurred. The process establishes claim cost risks utilizing two fixings: information from individual risks and that of the collateral or collective information, for instance, information from distinct sources considered relevant (Waters, 1994)**.** Klugman and Panjer (1949) also explained the concept of credibility as a collection of quantitative tools enabling an insurance firm to assess the risk or group of hazards (modify future premiums based on prior experience). The authors added that if a policyholder's experience is reliably higher than assumed in the fundamental manual rate, the policyholder may request a rate decrease. The authors explained that the insured may claim that the grand rate reflects the anticipated experience of the entire rating class and that the risks are definitely homogeneous. However, no rating technique is flawless or perfect, and risk levels are somewhat heterogeneous after all the endorsement criteria are shown. Thus, few policyholders could be considered as better hazards and preferred in the basic manual or collective rate over those projected rate. A comparable rationale suggests that a rate increase should be linked to a bad risk, but in this situation the policyholder will certainly not ask for a rate increase. Due to value thoughts and the economics of the situation, an expansion may be crucial in a period of time.

2.5.1 Application of Credibility theory

The theory of credibility was originally developed in the mid-twentieth century by actuaries from North America for quite a while. Mowbray (1914) puts it into a practical used for computing premium and is presently known as the American theory of credibility. Whitney (1918) and different authors scrutinized a great deal the concept of the theory. The author suggested that insurance losses are of an erratic or random nature and subsequently the supposition of a "fixed effect" model was not valid. Moreover, the model further is met with the question of partial legitimacy as it was hard to decide on the assessment of weights for claim histories and after the World War II revolution; "Whitney's random impact model" came into being.

Bühlmann (1967) suggested credibility claim costs with a empirical structure that formalized the credibility theory assumption and norms and also proposed approaches to record Bühlmann's credibility variables with equal exposure units in the credibility display. Bühlmann and Straub (1970) established the formula of credibility estimates with no underlying distribution with the ultimate objective that no previous distribution as an assumption of claim history and further clarified in his job the few aspects of using the equation of credibility estimates. Given the result that Bühlmann (1967) produced, there have been wide-ranging efforts by scientists over the past few decades to expand the model to increasingly expanding instances and gradually handle complex circumstances. Gerber and Jones (1975) for example, were dedicated to models of credibility with time dependence. Hachemeister (1975)**,** Jewell (1975) used the approach and showed that exponential family distributions are the best straight choice for Bayesian credibility estimates obtained using the mean square error estimates. In addition, the real leap forward noted a large portion of the research moving to Jewell (1975) development of Bayesian estimation techniques. The generalized linear model approach was subsequently used by Nelder and Verall (1997) to create credibility features incorporating the model of random effects. In actuarial science, this has provided a broad range of apps, including claim reservation and risk rating.

2.5.2 The Credibility Factor

The factor of credibility is a percentage of how much reliance the risk firm is set up to bring on the individual's own data. It indicates how valid an item or estimate may be or may be suitable. Whitney (1918) gave a weighted average of risk class claims and collective risk class claims for a future estimated claim known as a credibility claim cost. The load associated with the risk groups under account was tended to be the credibility. Dorweiler (1934) and Bailey (1945) had an objective of achieving credibility for experience rating using risk-related weights, but Bühlmann (1967 suggested methods for processing Bühlmann's credibility variables with equivalent exposure units and equal time frames in the credibility model. Bühlmann and Straub (1970) improved the method of Bühlmann by allowing an unbalanced unit presentation. The authors also displayed their model for evaluating credibility by considering and comparing weights for each risk depending on claim frequencies or the number of exposures for each of the time periods. Jewell (1973) tended to increase the credibility of a multivariate version by minimizing the mean square error (MSE). Hachemeister (1975) presented Bühlmann and Straub's credibility for regression extension where he tended to a credibility matrix. For experimental design models, Bühlmann and Jewell (1987) showed different levels or hierarchical credibility. Couret & Venter (2008) introduced multi-dimensional credibility for an anticipated danger as one factor is accepted as a linear mixture of more than one variable over a few time frames in perspective of the covariance conditions between variables, the Expected Process Variances (EPVs) and the Variance of Hypothetical Means (VHMs) for exposure units.

An actuary is fascinated by a revised premium for the next instant time frame. All things considered, for the next time frame, the actuary must locate the premium for each risk using past claim histories. Credibility finds an arched combination with a weighted average between the class risk premium and the collective risk premium (Finan, 2016). The question here is how much weight of credibility should be given to the risk premiums of the individual class and the collective risk premium? This weight or factor that normally referred to as credibility factor lies in the ranges of zero to one. Furthermore, in credibility theory, zero credibility implies that the data provided is too little to be used for the rating of experience. The projected credibility premium is known as the credibility premium. The vital aspect of estimates of credibility is that it has a linear function of class mean and collateral or collective risk. The update of premium is often taken into consideration as more risk information are subsequently obtained.

2.5.3 Bühlmann and Straub's Credibility Theory

The Bayesian concepts and processes were introduced into actuarial practiced in the early 1960s when Bühlmann Straub (1967, 1969) laid the basis for the empirical Bayes approach to legitimacy, which is still widely used in the insurance business. Within the actuarial field, Bayesian method is used in distinct areas. The minimum value of the quadratic loss function is the credibility premiums such as the Bühlmann or Bühlmann-Straub credibility premiums and the others. Bühlmann (1967) displayed the strategy to credibility as a direct ability to assess and predict the anticipated claims for future periods, using previous information of claim histories for class risk or collective class of risks.

The Bühlmann model is simplest of the credibility models since it viably necessitates that the past model is the simplest of models of credibility since past claims of a policyholder require independent and identically distributed parts compared to each prior year. Vital useful trouble with this supposition is that it does not take into account varieties of exposure or size (Klugman, Panjer & Willmot, 2004). The accompanying inquiries by these authors further uncover the down to earth trouble of the Bühlmann model; for instance, What if the policyholder's first year claims experienced represented only a part of a year owing to an uncommon policyholder's anniversary? What if a shift in benefits happened partly through a policy year? For group insurance, what if the group size has altered over time? The Bühlmann and Straub model enables the inclusion of weights or volumes in the variance expressions which is a solution to the constraints of the Bühlmann model. This is a first generalization of the contract-related independent and identically distributed hypothesis, extending the field of application.

In this study, the researchers' interest is to apply Buhlmann-Straub model to determine and predict credibility premium of each marine insurance company in Ghana for the next immediate year $(n+1)$ based on claim frequencies and severities given past records (1, 2, 3,…….., n) by combining the weighted average of the mean of the class risks or individual premiums and the manual or grand mean premium considering the risk profile of these companies. This will help insurance companies and other players including investors have a broader view of risks in terms of frequency or severity. This will also offer players and stakeholders the opportunity of comparing the cost of insurance among marine insurance companies in Ghana.

In summary the Review torched on these broad areas of the study such as; Insurance Claims and Premiums which is very key to risk bearers in determining the prospective premium for insureds. Another area considered was Underwriters' Profitability for Assuming Risk of insureds which is one of the major reasons for insurance business which is always considered for setting rates. The review also considered rates for homogeneous and heterogeneous risks; the Class and Experience Ratings of Insurance including Credibility Theory and its applications. Finally the researcher considered Bühlmann and Straub Credibility model for estimating credibility frequency-severity premium which considered the variation or unstableness of claim frequencies and severities in estimating premium for risks.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

The chapter explains the methodology employed in achieving the objective of the study. In this chapter the type and sources of data as well as the size of data used in the evaluation were explained. The Bühlmann and Straub's Credibility theory model employed in the analysis and estimation of credibility premiums as well as the assumptions, theorems and lemma underlying the model were also discussed. R-software was employed in analysing the claim data. The data and codes written by the researcher can be found at the appendix.

3.2 Type and Sources of data

The study depends considerably on secondary data acquired from the NIC, the body in Ghana mandated by way of law to make sure high quality administration, supervision, rules and the management of insurance commercial enterprises. The information obtained from the NIC was claimed histories of policyholders disclosed through the various non-life marine insurance companies operating in the country. Generally, secondary data most often benefit from cost savings and overcomes some problems in achieving a wider population connected with collecting the most significant information. The advantage of using secondary data is less cost of obtaining the data and less consuming time for accessing the data. However, the use of secondary data exposes this research to the risk of replicating any defective data reported due to human or system errors. The resources and time available for this research mean cannot be eliminated completely. The researcher however, finds comfort in using the data from the National Insurance Commission due to the reputation of the commission.

3.3 Target Population

The marine insurance claim histories produced by policyholders of non-life marine insurance companies in Ghana is the target population considered for this study. The claim histories include the claim amounts or sizes, the exposure units or policy counts (the number of individual or group of policyholders for policy years) and finally claim count by the exposure units in the year reported to the National Insurance Commission by the non-life insurance companies in Ghana.

3.4 The Sample data

Purposive and convenient sampling methods of non-life insurance companies were considered. There are 26 non-life insurance institutions licensed by the National Insurance Commission as at 2018, and out of this number 15 met the prescribed standards of working and reporting annual claims continuously for a minimum of six years to the National Insurance Commission. The Years of operation in the non-life marine insurance business in Ghana played a principal role in determining which insurance organizations to be covered in the sample for the research analysis. The proposed model employed for this research requires understanding about the variability existing in the claim quantities; within and across the individual insurance companies. The claim sizes of the marine insurance organizations in relation to the total non-life portfolio or policy count, the number of claims made by policyholders were additionally considered. The researcher considered reported marine insurance claim histories to the National Insurance Commission from the period 2013 to 2018. In other to ensure that variability within the reported claim amount was largely due to the associated risks of the marine insurance portfolio, the researcher employed a technique to reduce the exogenous effects on the claim data reported by the insurance company by dividing the claim sizes or counts by their respective risk volumes. All the 15 sampled non-life marine insurance companies reported claim experiences consecutively for six accounting years to the NIC.

3.5 Bühlmann and Straub's Credibility for Multiple Exposures

The researcher estimated the basic average or risk premium per policy count or exposure units for policyholders of each marine insurance company. The researcher also used Bühlmann and Straub's Credibility model in estimating credibility premium for policyholders using their annual claim costs as well as credibility premium based on credibility claim severities and credibility claim frequencies by assuming that risk profiles or parameters of claim frequencies follow a poison distribution.

The Bühlmann and Straub's Credibility model generally estimates premium for class risks of companies for the next immediate period ($n+1$) given observed aggregate claims and their respective exposure units for the periods $j=1,\ 2,\ 3,\ldots\ldots.,n$ for each risk class or company. A risk's exposure to loss may vary from period to period and the number of years of observations may also differ over various risks. For these situations, let S_{ij} be the aggregate claim amount in the year j for a risk class or company i such that for each of the observed aggregate claims of the company i at period j, there is a corresponding weight W_{ij} representing exposures or exposure units in the year j for risk i and N_{ij} representing a number of claims in a year j for risk or company i. The Bühlmann and Straub's Credibility theory also known as Empirical Bayesian Credibility Theory (II) makes use of a risk parameter Θ which is a random variable and does not observe any specific statistical distribution. In this estimation process, the risk classes stand for the individual companies ($i = 1, 2, 3, \ldots., I$) which experiences or pay claims over n-years, ($j = 1, 2, 3, \ldots.., n$).

Given the fixed I risk classes (companies), we use the notation Y_{ij} for claim cost and X_{ij} for claim severities and F_{ij} for claim frequencies for the class risk (insurance company) i in the year j, whilst W_{ij} is the associated weight for Y_{ij} and F_{ij} but N_{ij} is the associated weights for X_{ij}. Our interest is to estimate and compare the credibility risk premiums for policyholders of each insurance company whose claims' history and exposures units and claim counts are under study. Thus, the researcher seeks to factor severity and frequencies of claims made by policyholders on each of the insurance companies, which are the two major underlying risks that undermine the fair determination of premium for policyholders by insurance companies.

In the calculation of premiums, the expected values of claim frequencies and severities are estimated or determined and modeled separately. Conditional independence is an assumption which is frequently appropriate in many situations in insurance practice. We can now estimate credibility frequency' $E[\mathrm{F}_{n+1}/\Theta_i]$ and credibility severity; $E[\mathrm{X}_{n+1}/\Theta_i]$ separately and then multiply the two estimators to get the frequency-severity credibility estimate; $P_{n+1}(\Theta_i)$.

3.6 Credibility Estimate of Claim Costs

The assumptions we do make for Bühlmann and Straub's Credibility model are most conveniently expressed in a manner which makes the model less restrictive than was the case for Bühlmann Credibility model itself and these assumptions are made not about the claims' variables themselves, but about the variables representing aggregate gross claim sizes per unit weight, Y_{ij}. The Y_{ij} can also be described as claim cost for company i at year j scaled to account for the average claim size for policyholders of company i made in year j.

Thus,

$$Y_{ij} = \frac{S_{ij}}{W_{ij}} \qquad 3.1$$

$$i = 1, 2, 3, \ldots\ldots\ldots\ldots, I, \; j = 1, 2, 3, \ldots\ldots, n$$

Where

$Y_{ij} =$ Aggregate claim per policy count of company i in the year j

$S_{ij} =$ Aggregate claim for company i in the year j

$W_{ij} =$ Policy count for company i in the year j

3.6.1 Assumptions of Model under Claim Cost

(1) Given the risk profile Θ_i associated with company i, the Y_{ij}, $j = 1, 2, 3, \ldots.$, n are independent but not identically distributed either conditionally and unconditionally

(2) $E[\mathrm{Y}_{ij} \mid \Theta_i]$ does not depend on *j*

(3) $W_{ij} Var[\mathrm{Y}_{ij} \mid \Theta_i]$ does not depend on *j*.

(4) For different class risks or companies $i \neq d$ the pairs of claim costs ($\Theta_{i,} Y_{il}$) and ($\Theta_d, \mathrm{Y}_{dk}$) are independent

(5) The risk parameters Θ_i ($i = 1, 2, 3, \ldots\ldots.., I$) are independently and identically distributed.

Under the assumptions 1, 2, and 3 for company i we defined,

$$m(\Theta_i) = \mathrm{E}[\mathrm{Y}_i / \Theta_i] = \mathbf{E}[\mathrm{Y}_{ij} \mid \Theta_i] \tag{3.2}$$

$$s^2(\Theta_i) = W_{ij} \mathbf{Var}[\mathrm{Y}_{ij} \mid \Theta_i] \tag{3.3}$$

The points, (2) and (3) under assumption **(3.6.1)** for Bühlmann and Straub showed that the claim sizes per unit of exposure from year to year for a particular company have constant mean but non constant variance.

Point (5) implies that $E[\mathrm{m}(\Theta_i)] = \mathrm{E}[\mathrm{m}(\Theta)]$ and $\mathrm{var}[\mathrm{m}(\Theta_i)] = \mathrm{var}[\mathrm{m}(\Theta)]$ or $E[\mathrm{s}^2(\Theta_i)] = \mathrm{E}[\mathrm{s}^2(\Theta)]$ because none of the structure parameters, $E[\mathrm{m}(\Theta_i)]$, $\mathrm{var}[\mathrm{m}(\Theta_i)]$, and $E[\mathrm{s}^2(\Theta_i)]$ depends on i. Thus, none of them is risk or company specific.

Thus, for each risk class or company, the aggregate claims per policy count in any given year have mean $m(\Theta_i)$ and variance $s^2(\Theta_i)$ where, Θ_i is the risk parameter or profile for a risk class or an insurance company. To find the credibility estimator, $m_{n+1}(\Theta_i)$ which is a linear function of the observations $Y_{i1}, Y_{i2}, \ldots, Y_{in}$ with minimum mean square error from the pure premium; there is the need to choose $a_{i0}, a_{i1}, \ldots, a_{in}$ that will optimize the estimator of $m_{n+1}(\Theta_i)$ given by

$$m_{n+1}(\Theta_i) = \alpha_{i0} + \alpha_{i1}Y_{i1} + \alpha_{i2}Y_{i2} + \cdots + \alpha_{in}Y_{in} = \alpha_{i0} + \sum_{j}^{n} \alpha_{ij}Y_{ij} \qquad 3.4$$

Lemma 3.1 With the set-up and notation of Bühlmann and Straub's credibility model given the company or risk i and time periods $j = 1, 2, \ldots\ldots, n$ we have

(i) $E[Y_{ij}] = E[\bar{Y}_i / \Theta_i] = E[m(\Theta)]$

(ii) $E[Y_{ij}\, m(\Theta_i)] = E[m^2(\Theta_i)]$

(iii) $E[Y_{ij}\, Y_{ik}] = E[m^2(\Theta)]$ *For* $j \neq k$;

(iv) $E[Y_{ij}^2] = \frac{1}{W_{ij}} E[s^2(\Theta)] + E[m^2(\Theta)].$

In the case of Bühlmann Straub's model, the constants $\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in}$ are not equal because Y_{ij} $(j = 1, 2, \ldots\ldots, n)$ are not identically distributed.

Theorem 3.1 Let $Y_{i1}, Y_{i2}, \ldots, Y_{in}$ be a sequence of random variables, each of whose distribution depends on a risk parameter Θ_i, and which given Θ_i are independent with

$$W_{ij}\, Var[Y_{ij} \mid \Theta_i] = s^2(\Theta_i) \qquad 3.5$$

Then the mean square error of the estimator, $\alpha_{i0} + \sum_{j=1}^{n} \alpha_{ij}Y_{ij}$ from $m(\Theta_i)$ given by

$$E\left[\left(m(\Theta_i)-\mathrm{m}_{n+1}(\Theta_i)\right)^2\right]=E\left[\left(m(\Theta_i)-\alpha_{i0}-\sum_{j=1}^{n}\alpha_{ij}Y_{ij}\right)^2\right] \quad 3.6$$

and is minimized and given by

$$m_{n+1}(\Theta_i)=Z_i\bar{Y}_i+(1-Z_i)E[m(\Theta)] \quad 3.7$$

where

$\bar{Y}_i =$ Average claim or hypothetical mean for company i

$Z_i =$ Credibility factor for claim cost histories of company i

$E[m(\Theta)]$ =Collective risk mean for claim costs

To obtain the values of α_{i0} and α_{ij}, ($j=1$, 2, ….., n) that minimize the estimate in equation (3.4), there is the need to take partial derivative of equation (3. 6) and equating them to zero and solving for α_{i0} and α_{ij} simultaneously by applying **lemma (3.1)** and **theorem (3.1)** to obtain equation (3.7).

The estimate $m_{n+1}(\Theta_i)$ in equation (3.3) is the credibility premium or claim cost for the company; i for the next immediate period $n+1$. We now consider how to estimate the three structural parameters $\mathrm{E}[\mathrm{m}(\Theta)]$, $\mathrm{E}[\mathrm{s}^2(\Theta)]$ and $\mathrm{var}[\mathrm{m}(\Theta)]$ using data from a collective of I (fixed) comparable risks or companies.

3.6.2 Estimation of the Structural Parameters

The formula for inhomogeneous credibility estimator involves the three structural parameters, $\mathrm{E}[\mathrm{m}(\Theta)]$, $\mathrm{var}[\mathrm{m}(\Theta)]$ and $\mathrm{E}[\mathrm{s}^2(\Theta)]$. These three parameters are also unknown and must be estimated from the data of the collective risks. Various articles are to be found in the actuarial literature about the estimation of such parameters example (Frees & Wang, 2005) and (Gisler, 2005). However, Bühlmann's approach of estimating parameters is convenient and widely used by researchers in estimating parameters since it provides an estimate that has small expected square deviations from its parameters. The Bühlmann's credibility estimators are based on the variance measures named Expected Process Variance; $\mathrm{E}[s^2(\Theta)]$ and Variance of the Hypothetical Means; $\mathrm{var}[m(\Theta)]$. These two variance measures are squared functions. According to literature and course work done on "Actuarial Applications" by Bühlmann and Gisler (2005), the Variance of the Hypothesized Means must not be negative but in case it happened, the credibility factor is assumed to be zero for that risk classes.

NOTE: the researcher adopted the statistical convention of using a clear point ($\bullet$) in place of a subscript to indicate summation over that subscript.

The credibility factors Z_i for the experience of each company can be estimated as,

$$\bar{Y}_i = \frac{\sum_{n=1}^{n} W_{ij} Y_{ij}}{W_{i\bullet}} \tag{3.8}$$

$$Z_i = \frac{W_{i\bullet}}{W_{i\bullet} + \frac{E[s^2(\Theta)]}{\mathrm{var}[m(\Theta)]}} \tag{3.9}$$

$$Z_i = [0,1]$$

$$\bar{Y} = E[m(\Theta)] = \frac{\sum_{i=1}^{I} W_{i\bullet}\bar{Y}_i}{W_{\bullet\bullet}} \quad 3.10$$

where

$$W_{i\bullet} = \sum_{j=1}^{n} W_{ij}$$

$$W_{\bullet\bullet} = \sum_{i=1}^{I}\sum_{j=1}^{n} W_{ij}$$

The parameters are therefore estimated as

$$E[\mathrm{s}^2(\Theta)] = \frac{1}{I(n-1)}\sum_{i}^{I}\sum_{j}^{n} W_{ij}(\mathrm{Y}_{ij} - \bar{Y}_i)^2 \quad 3.11$$

$$\mathrm{var}[m(\Theta)] = \frac{1}{W^*}\{\frac{1}{In-1}\sum_{i=1}^{I}\sum_{j=1}^{n}\mathrm{W}_{ij}(\mathrm{Y}_{ij} - \bar{Y})^2 - \frac{1}{I}\sum_{i=1}^{I}\frac{1}{n-1}\sum_{j=1}^{n}\mathrm{W}_{ij}(\mathrm{Y}_{ij} - \bar{Y}_i)^2\} \quad 3.12$$

where

$$W^* = \frac{1}{In-1}\sum_{i=1}^{I} W_{i\bullet}(1 - \frac{W_{i\bullet}}{W_{\bullet\bullet}})$$

3.7 Credibility Estimate of Claim Severities

The assumptions made under claim costs for Bühlmann and Straub's Credibility model remained the same under claim severities. X_{ij} represents the aggregate claim experience per claim count for company i over n years. The X_{ij}, also known as claim severity for company i at time j and is scaled to account for the average claim size by policyholders of company i who made claim in year j.

Thus,

$$X_{ij} = \frac{S_{ij}}{N_{ij}}, \qquad 3.13$$

$i = 1, 2, 3, \ldots\ldots\ldots, I, \; j = 1, 2, 3, \ldots\ldots, n$

Where

$X_{ij} =$ Annual claim severity for company i in year j

$S_{ij} =$ Aggregate claim size for company i in year j

$N_{ij} =$ Claim count for company i as a risk class in year j

3.7.1 Assumptions of Model under Claim Severities

(1) Given the risk profile Θ_i associated with company i, the X_{ij}, $j = 1, 2,$ $3, \ldots$, n are independent but not identically distributed either conditionally and unconditionally

(2) $E[X_{ij} \mid \Theta_i]$ does not depend on j

(3) $N_{ij} Var[X_{ij} \mid \Theta_i]$ does not depend on j.

(4) For different risks or companies $i \neq d$ the pairs of claim severities ($\Theta_{i,} X_{il}$) and (Θ_d, X_{dk}) are independent

(5) The risk parameters Θ_i ($j = 1, 2, 3, \ldots\ldots, I$) are independently and identically distributed.

Under the assumptions 1, 2, and 3 for company i we defined,

$$\mu(\Theta_i) = \mathrm{E}[\mathrm{X}_i / \Theta_i] = \mathbf{E}[X_{ij} \mid \Theta_i] \qquad 3.14$$

$$s^2(\Theta_i) = N_{ij} \mathbf{Var}[X_{ij} \mid \Theta_i] \qquad 3.15$$

Point (2) and (3) under Assumption **(3.7.1)** showed under this model that the claims per claim count from year to year for a risk or particular company have constant mean but non constant variance. Point (5) of assumption **(3.7.1)** implies that $E[(\mu\Theta_i)] = \mathrm{E}[\mu(\Theta)]$ and $\mathrm{var}[\mu(\Theta_i)] = \mathrm{var}[\mu(\Theta)]$ or $E[\mathrm{s}^2(\Theta_i)] = \mathrm{E}[\mathrm{s}^2(\Theta)]$ because none of the structure parameters, $E[\mu(\Theta)]$, $\mathrm{var}[\mu(\Theta)]$, and $E[\mathrm{s}^2(\Theta)]$ depends on i. Thus, none of them is risk or company specific. Thus, for each risk class or company, the aggregate claims per claim counts in any given year have mean $\mu(\Theta_i)$ and variance $s^2(\Theta_i)$ where, Θ_i is the risk parameter or profile for a risk class or non-life marine insurance company.

To find an estimator, $\mu_{n+1}(\Theta_i)$ which is the linear function of the observations $X_{i1}, X_{i2}, \ldots, X_{in}$ with minimum mean square error from the average claim severity; $\mu(\Theta_i)$ there is the need to choose $a_{i0}, a_{i1}, \ldots, a_{in}$ that will optimise the estimator of $\mu_{n+1}(\Theta_i)$ given by;

$$\mu_{n+1}(\Theta_i) = a_{i0} + a_{i1}X_{i1} + a_{i2}X_{i2} + \cdots + a_{in}X_{in} = a_{i0} + \sum_{j}^{n} a_{ij}X_{ij} \qquad 3.16$$

Applying Lemma 3.1, Bühlmann and Straub's credibility model given the company or risk i and time periods, $j = 1, 2, \ldots\ldots, n$ we have

(i) $E[X_{ij}] = E[\bar{X}_i / \Theta_i] = E[\mu(\Theta)]$

(ii) $E[X_{ij}\mu(\Theta_i)] = E[\mu^2(\Theta_i)]$

(iii) $E[X_{ij}X_{ik}] = E[\mu^2(\Theta)]$ *For* $j \neq k$;

(iv) $E[X_{ij}^2] = \frac{1}{N_{ij}} E[s^2(\Theta)] + E[\mu^2(\Theta)].$

Under Bühlmann-Straub's model, the constants $\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in}$ are not equal because X_{ij} ($j=1, 2, \ldots\ldots, n$) are not identically distributed.

Applying **Theorem 3.1** let $X_1, X_2, \ldots, X_n$ be a sequence of random variables, each of whose distribution depends on a parameter Θ_i, and which, given Θ_i, are independent, with

$$\mathrm{N}_{ij} Var[X_{ij} \mid \Theta] = s^2(\Theta_i) \qquad 3.17$$

Then the mean square error of the estimator, $\alpha_0 + \sum_{j=1}^{n} \alpha_{ij} X_{ij}$ from $\mu(\Theta_i)$ for which

$$E\left[\left(\mu(\Theta_i) - \mu_{n+1}(\Theta_i)\right)^2\right] = E\left[\left(\mu(\Theta_i) - \alpha_{i0} - \sum_{j=1}^{n} \alpha_{ij} X_{ij}\right)^2\right] \qquad 3.18$$

is minimized and given by

$$\mu_{n+1}(\Theta_i) = Z_i \bar{X}_i + (1 - Z_i) E[\mu(\Theta)] \qquad 3.19$$

Where

$\bar{X}_i =$ The average claim per claim count for company i

$Z_i =$ Credibility factor of claim history for company i

$E[\mu(\Theta)] =$ Collective risk mean for claim costs

To obtain the values of α_{i0} and α_{ij}, ($j=1, 2, \ldots..,$ n) that will minimize the estimate in equation (3.16) and (3.18) there is the need to take partial derivative of equation (3.18) and equating them to zero and solving for α_{i0}

and α_{ij} simultaneously by applying lemma (3.1) and theorem (3.1) to obtain equation (3.19).

The estimate $\mu_{n+1}(\Theta_i)$ in equation (3.11) is the credibility claim severity for company i for the next immediate period $n+1$

Now, the estimation of the three structural parameters $\mathrm{E}[\mu(\Theta)]$, $\mathrm{E}[s^2(\Theta)]$ and $\mathrm{var}[\mu(\Theta)]$ using data from a collective of I (fixed) comparable risks or companies.

3.7.2 Estimation of the Structural Parameters

The formulation for the heterogeneous credibility estimator entails the three structural parameters, $\mathrm{E}[\mu(\Theta)]$, $\mathrm{var}[\mu(\Theta)]$ and $\mathrm{E}[s^2(\Theta)]$. These three parameters are also unknown and must be estimated from the data of the collective. Bühlmann Staub's approach of estimating parameters is convenient and widely used by researchers in estimating parameters since it provides an estimate that has small expected square deviations from their parameters.

The Bühlmann's credibility estimators are based on the variance measures named Expected Process Variance; $\mathrm{E}[s^2(\Theta)]$ for the claim severities and Variance of the Hypothetical Means $\mathrm{var}[\mu(\Theta)]$ for the claim severities. These two variance measures are squared functions and must not be negative else the credibility factor is assumed to be zero for the risk classes.

To be able to determine credibility factors Z_i for the claim histories of each company.

$$\bar{X}_i = \frac{\sum_{j=1}^{n} N_{ij} X_{ij}}{N_{i\bullet}} \quad 3.20$$

The factor

$$Z_i = \frac{N_{i\bullet}}{N_{i\bullet} + \frac{E[s^2(\Theta)]}{\text{var}[\text{m}(\Theta)]}} \quad 3.21$$

$$Z_i = [0,1]$$

$$E[\mu(\Theta)] = \frac{\sum_{i=1}^{I} N_{i\bullet} \bar{X}_i}{N_{\bullet\bullet}} \quad 3.22$$

we estimate the parameters as;

$$E[\text{s}^2(\Theta)] = \frac{1}{I(n-1)} \sum_{i}^{I} \sum_{j}^{n} \text{N}_{ij} (\text{X}_{ij} - \bar{X}_i)^2 \quad 3.23$$

$$\text{var}[\mu(\Theta)] = \frac{1}{N^*} \{ \frac{1}{In-1} \sum_{i=1}^{I} \sum_{j=1}^{n} \text{N}_{ij} (\text{X}_{ij} - \bar{\text{x}})^2 - \frac{1}{I} \sum_{i=1}^{I} \frac{1}{n-1} \sum_{j=1}^{n} \text{N}_{ij} (\text{X}_{ij} - \bar{\text{X}}_i)^2 \} \quad 3.24$$

Where $N^* = \frac{1}{In-1} \sum_{i=1}^{I} N_{i\bullet} (1 - \frac{N_{i\bullet}}{N_{\bullet\bullet}})$.

3.8 Credibility Estimate of Claim Frequencies

Claim frequencies play a useful role in calculating a multivariate tariff or listing charges for policyholders of unique businesses in relation to Bonus—Malus structures. A common hypothesis is that the declare numbers can be represented as random variables with a conditional Poisson distribution (Buhlmann and Gisler, 2005).

Consider a portfolio of risk groups and relevant observations, as shown below:

$$F_{ij} = \frac{N_{ij}}{W_{ij}} \quad 3.25$$

N_{ij} = Annual claims frequency for company i in year j

N_{ij} = Number of claims of risk i in year j

W_{ij} = Associated weight

3.8.1 Model Assumptions under Claim Frequency

Similar to the Bühlmann-Straub model we have;

The risk i is characterized by its individual risk profile θ_i, which is itself the realization of a random variable Θ_i, and we assume that:

(i) Given Θ_i the N_{ij} (j = 1, 2,…….,n) are independent and Poisson distributed with Poisson parameter,

$$\lambda_{ij}(\Theta_i) = W_{ij}\Theta_i\lambda \quad 3.26$$

(ii) The pairs (Θ_1, N_1), (Θ_2, N_2) . . . are independent, and Θ_1, Θ_2 ….., are independent and identicall y distributed with E [Θ_i] = 1.

Θ reflects the frequency structure between the risks, whereas λ is the overall frequency level for all class risks or companies.

Given the model assumptions (3.8.1) we have;

$$E[\mathrm{F}_{ij} / \Theta_i] = \Theta_i\lambda \quad 3.27$$

$$w_{ij}\ \mathrm{var}[\mathrm{F}_{ij} / \Theta_i] = \Theta_i\lambda \quad 3.28$$

The conditions of the Bühlmann-Straub model (Model Assumptions 3.6.1) are therefore satisfied for the claim frequencies with,

$$\Theta_i \lambda = \lambda(\Theta_i) \qquad 3.29$$

$$s^2(\Theta_i) = \Theta_i \lambda = \lambda(\Theta_i) \qquad 3.30$$

Just as it was the case under claim costs or severities for each risk class or company, the aggregate claim Number or count per exposure in any given year have mean $\lambda(\Theta_i)$ and variance $\lambda(\Theta_i)$ where Θ_i is the risk parameter or profile for a risk class or an insurance company i.

To find an estimator, $\lambda_{n+1}(\Theta_i)$ which is the linear function of the observations $F_{i\mathbf{1}}, F_{i\mathbf{2}}, \ldots, F_{in}$ with minimum mean square error from the $\lambda(\Theta_i)$; there is the need to choose $\alpha_{i\mathbf{0}}, \alpha_{i\mathbf{1}}, \ldots, \alpha_{in}$ that will optimize the estimator; $\lambda_{n+1}(\Theta_i)$ given by

$$\lambda_{n+1}(\Theta_i) = \alpha_{i\mathbf{0}} + \alpha_{i1}F_{i\mathbf{1}} + \alpha_{i2}F_{i\mathbf{2}} + \cdots + \alpha_{in}F_{in} = \alpha_{i0} + \sum_j^n \alpha_{ij}F_{ij} \qquad 3.31$$

Applying **Lemma 3.1** in conjunction with the assumptions under claim frequency, With the set-up and notation of Bühlmann -Straub's Credibility model with regard to company i and time periods $j = 1, 2, \ldots\ldots$, n we have

(i) $E[F_{ij}] = E[F_i / \Theta_i] = E[\lambda(\Theta)] = \lambda$

(ii) $E[F_{ij}\, \lambda(\Theta_i)] = E[\lambda^2(\Theta)]$

(iii) $E[F_{ij}\, F_{ik}] = E[\lambda^2(\Theta)]$ *For* $j \neq k$;

(iv) $E[F_{ij}^2] = \dfrac{1}{W_{ij}} E[s^2(\Theta)] + E[\lambda^2(\Theta)] = \dfrac{1}{W_{ij}} \lambda + E[\lambda^2(\Theta)]$

Under Bühlmann-Straub's model, the constants $\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in}$ are not equal because F_{ij} ($j = 1, 2, \ldots\ldots$, n) are not conditionally or unconditionally identically distributed.

Applying **Theorem 3.1** ,Let $F_1, \mathrm{F}_2, \ldots, \mathrm{F}_n$ be a sequence of random variables, each of whose distribution depends on a parameter Θ_i , and which, given Θ_i ,are independent, with

$$\mathrm{W}_{ij}\, Var[\mathrm{F}_{ij} \mid \Theta] = \lambda(\Theta_i),\ \mathrm{j} = 1, \ldots, n \qquad 3.32$$

Then the mean square error of the estimator $\lambda_{n+1}(\Theta_i) = \alpha_0 + \sum_{j=1}^{n} \alpha_{ij} F_{ij}$ of $\lambda(\Theta_i)$ given by

$$E\left[\left(\lambda(\Theta_i) - \lambda_{n+1}(\Theta_i)\right)^2\right] = E\left[\left(\lambda(\Theta_i) - \alpha_{i0} - \sum_{j=1}^{n} \alpha_{ij} F_{ij}\right)^2\right] \qquad 3.33$$

is minimized and given by

$$\lambda_{n+1}(\Theta_i) = Z_i \bar{F}_i + (1 - Z_i)\lambda \qquad 3.34$$

where

$\bar{F}_i =$ The average claim frequency for company i

$Z_i =$ Credibility factor of claim frequencies for company i

$\lambda =$ The collective mean for claim frequencies

$$\bar{F}_i = \lambda(\Theta_i) = \frac{\sum_{j=1}^{n} W_{ij} F_{ij}}{W_{i\bullet}} \qquad 3.35$$

$$\lambda = \frac{\sum_{i=1}^{I} wi_{\bullet} \bar{F}_i}{w_{\bullet\bullet}} \qquad 3.36$$

$$Z_i = \frac{w_{i\bullet}}{w_{i\bullet} + k} \qquad 3.37$$

$$Z_i = [0,1]$$

where

$$k = \frac{E[\Theta\lambda]}{\mathrm{var}[\Theta\lambda]} = \frac{\lambda}{\lambda^2\, \mathrm{var}[\Theta]} = \frac{1}{\lambda\, \mathrm{var}[\Theta]}$$

To obtain the values of α_{i0} and α_{ij}, ($j=1,\ 2,\ \ldots..,\ n$) that minimized the estimate in equation (3.31), there is the need to take partial derivative of equation (3.33) with respect to α_{i0} and α_{ij} equating the partial derivative to zero and solving for α_{i0} and α_{ij} simultaneously by applying lemma (3.1) and theorem (3.1) to obtain equation (3.34). The estimate $\lambda_{n+1}(\Theta_i)$ in equation (3.34) is the credibility claim frequency for company i for the next immediate period $n+1$

Now considering how to estimate the three structural parameters λ and $\text{var}[\lambda(\Theta_i)]$ using data from a collective of I (fixed) comparable risks or companies.

3.8.2 Estimation of the Structural Parameter

The formula for inhomogeneous credibility estimator under poison model also involves the three structural parameters;

Properties

(1) $\text{E}[\mu(\Theta)] = E[\lambda(\Theta)] = \lambda$

(2) $E[\text{s}^2(\Theta)] = E[\lambda(\Theta)] = \lambda$

(3) $\text{var}[\mu(\Theta_i)] = \text{var}[\lambda(\Theta_i)] = \lambda^2 \text{ var}[\Theta_i]$

These three parameters under the properties above are also unknown and must be estimated from the data of the class risks using the Bühlmann’s parameter estimation approach.

$$\text{var}[\lambda(\Theta)] = \lambda^2 \text{ var}[\Theta] = \lambda^2 (\frac{1}{I}\sum_{i=1}^{I}(\bar{\text{F}}_i - \bar{\text{F}})^2) \qquad 3.38$$

where $\bar{F} = \frac{1}{I}\sum_{i=1}^{I}\bar{F}_i$

3.9 Credibility Frequency-Severity Estimates

Premium estimation in terms of claim frequency and severity help factor the risk components that directly or indirectly affect the investment position of insurance companies, claim reserves and also help reduce adverse selection in insurance. According to Gisler (2005) factorization or product of claim frequency and claim severity is allowable. Based on this principle the credibility frequency-severity premium or claim count is estimated.

Thus,

$$P_{n+1}(\Theta_i) = \lambda_{n+1}(\Theta_i).\mu_{n+1}(\Theta_i) \qquad 3.39$$

where

$P_{n+1}(\Theta_i) =$ the credibility frequency-severity estimate for company i for the next period $n+1$ obtained as the product of the credibility frequency and severity all for the period $n+1$.

$\lambda_{n+1}(\Theta_i) =$ Credibility frequency for company i in period $n+1$.

$\mu_{n+1}(\Theta_i) =$ Credibility severity of company i in the period $n+1$.

CHAPTER FOUR

DATA ANALYSIS

4.1 Introduction

This chapter discusses and explains the descriptive analysis of claim sizes, policy counts and claim counts including estimation of structural parameters under claim frequencies and claim severities and subsequently estimate credibility weights and premiums (claim or loss cost) including credibility frequencies and severities and by extension credibility frequency-severity premiums for policyholders of each non-life marine insurance company. The researcher under this chapter presents bar charts of estimated premiums showing the extent of variation in the pure premiums of individual class risks and also sort to compare the three different estimated premiums(the pure premiums, credibility risk premiums and credibility frequency-severity premiums) and thereby projecting the importance of estimating premiums based on claim frequencies and severities since this incorporated inherent risks both in terms of the claim frequencies and severities by policyholders who experience these risks and to whom obligation have to be met by insurers.

4.2 The Sample Period

The greater facts we have got as a result of an increase in the sample period for the relevant risks, the greater emphasis is placed on the precise hazard. In this study, the risk studied is the claim sizes, policy counts and claim counts made by marine insurance policyholders on marine insurers in Ghana. The number of relevant risk classes used in this analysis is 15 non- life marine insurance businesses that had reported claim history to the National insurance commission in Ghana. Ghana's economy recorded notably low and strong inflationary figures, interbank interest rate and exchange rates for the period 2013 to 2018 hence the outcome of this research will not be much affected by these external economic factors.

4.3 Descriptive Analysis of the Claim History

The sample information collected from the National Insurance Commission for the period 2013 to 2018 include claim sizes, claim counts and policy counts for policyholders of 15 marine insurance enterprises in Ghana. The descriptive assessment of sample information, claim size, policy count and claim count are provided in Tables 4.1, 4.2 and 4.3 respectively.

Table 4.1 displayed the mean, maximum and minimum observation, the standard deviation of claim counts data of each insurance company. Skewness and kurtosis, which are indications of symmetry and peakness respectively of the distribution of the reported claim size data for the companies, are also displayed.

Table 4.1: Descriptive Statistics of Claim Sizes for the Non-Life Insurance Companies

Company	Mean	Max.	Min.	Std.	sk'nes	Kurt.
Activa Int. Insurance	402517	582619	121728	163371	-0.555	-1.232
Allianz Insurance	349172	1829787	24704	725733	1.358	-0.089
Donewell	68563.5	139653	1158	49424.6	0.040	-1.640
Enterprise Insurance	279881	374831	212357	61554.6	0.267	-1.661
Ghana Union Assurr.	302029	393340	206921	90239.3	-0.020	-2.249
Glico General Ins.	188107	717007	25818	263361	1.264	-0.240
Hollard Insurance	220037	595226	30414	207852	0.757	-1.042
NSIA Ghana Ins.	58656.7	133006	2002	55805.5	0.299	-1.964
Phoenix Insurance	22135.7	52212	8693	19070	0.614	-1.719
Provident Ins.	19900	75561	3000	27671.6	1.272	-0.224
Quality Insurance	39416.3	65972	6597	27390	-0.052	-2.204
RegencyNem Ins.	558878	921565	24221	411294	-0.496	-1.952
SIC Insurance	1815258	4437325	311473	1536010	0.578	-1.354
Star Assurance	244300	427874	2023	171708	-0.123	-1.885
Unique Insurance	38604.3	89320	9131	36539.6	0.463	-1.939

From Table 4.1, SIC Insurance Company Limited shows the highest average of GHC1, 815, 257.7 whilst Provident Insurance Company Limited had GHC19, 900 as the lowest average claim size. The maximum claim size of GHC4, 437, 325 is recorded by SIC Insurance Company Limited. The risk classes with a positive coefficient of skewness show that the distributions of their observations are skewed to the right whilst those with negative skewness have most of their observations skewed to the left. However, the negative kurtosis depicted a platykurtic distribution for the claim sizes history of the risk classes or non-life marine insurance companies.

Table 4.2 displayed the summary statistics for the policyholders of each Non-life marine insurance company. It presents the mean, the maximum and minimum observations, the standard deviation, including skewness and kurtosis that depict the symmetry and peakness of the policy count data respectively for each risk class or non-life marine insurance company.

Table 4.2: Descriptive Statistics of The policy Count for the Non-Life Insurance Companies

Company	Mean	Max.	Min.	Std.	sk'nes	Kurt.
Activa Int. Insurance	108.333	158	72	30.982	0.371	-1.522
Allianz Insurance	63.5	154	30	45.654	1.19	-0.357
Donewell	25.833	48	11	13.906	0.378	-1.598
Enterprise Insurance	1647.67	2702	1163	561.28	0.898	-0.748
Ghana Union Assurr.	166.333	225	54	63.108	-0.691	-1.105
Glico General Ins.	135.833	163	93	34.354	-0.399	-1.999
Hollard Insurance	126.333	207	43	84.455	-0.003	-2.298
NSIA Ghana Ins.	28.667	54	13	14.828	0.532	-1.28
Phoenix Insurance	106.833	202	54	59.647	0.597	-1.665
Provident Insurance	39.333	82	24	21.538	1.182	-0.357
Quality Insurance	23.167	47	3	15.613	0.204	-1.587
RegencyNem Ins.	54.667	145	11	51.644	0.667	-1.255
SIC Insurance	1297.5	1896	790	458.35	0.087	-2.075
Star Assurance	177.333	222	85	54.021	-0.695	-1.394
Unique Insurance	79.833	114	24	31.928	-0.638	-1.176

Table 4.2 showed that, among the sampled Insurance companies, Star Assurance Company Limited shows the highest average of 177.333 policy count whilst Quality Insurance Company Limited has recorded 23.167 as the lowest average policy count. The maximum policy count, 2702 is recorded by Enterprise Insurance Company Limited whilst Quality Insurance Company Limited recorded 3 as the lowest minimum value among all the risk classes or non-life marine insurance companies. Risk classes with a positive coefficient

of skewness suggest that their observations are skewed to the right whilst those with negative skewness have their observations skewed to the left. The negative kurtosis for all risk classes depicted a platykurtic distribution for the policy counts for all the marine insurance companies.

Table 4.3 displayed the descriptive statistics; the mean, maximum and minimum observations, the standard deviation, for claim counts data for insureds of each of the marine insurance companies. The symmetry and peakness of the distributions of the claim count given by the values of skewness and kurtosis respectively are also displayed in Table 4.3.

Table 4.3: Descriptive Statistics of Claim Counts for the Non-Life Insurance Companies

Company	Mean	Max.	Min.	Std.	sk'nes	Kurt.
Activa Int. Insurance	9	25	1	8.809	0.79	-1.002
Allianz Insurance	1.333	2	1	0.516	0.538	-1.958
Donewell	1.5	2	1	0.548	0	-2.306
Enterprise Insurance	138	154	121	13.416	-0.097	-1.997
Ghana Union Assurance	8.167	11	1	3.764	-0.989	-0.693
Hollard Insurance	5.5	8	4	1.643	0.451	-1.757
NSIA Ghana Insurance	2.667	4	2	0.817	0.476	-1.583
Phoenix Insurance	2.333	5	1	1.751	0.51	-1.799
Provident Insurance	1.667	3	1	0.817	0.476	-1.583
Quality Insurance	2.167	4	1	1.169	0.371	-1.618
RegencyNem Ins. Ghana	8	18	1	7.043	0.192	-1.932
SIC Insurance	28.333	117	3	44.889	1.189	-0.413
Star Assurance	3.167	4	1	1.329	-0.67	-1.621
Unique Insurance	1.833	4	1	1.169	0.881	-0.904

From Table 4.3, Enterprise Insurance Company Limited recorded the highest average of 138 claim count whilst Allianz Insurance Company Limited recorded 1.333 as the lowest average claim count. The maximum claim count, 154 claim count is recorded by Enterprise Insurance Company Limited. The risk classes with a negative coefficient of skewness show that their observations are skewed to the left whilst those risk classes with a positive coefficient of skewness are positively skewed. The negative kurtosis of the claim counts of the risk classes depicted a platykurtic distribution for the claim counts of the risk classes or marine insurance companies.

4.4 Buhlmann-Straub Credibility Parameter Estimates

The Buhlmann and Straub credibility model made no assumption about any type of statistical distribution for the risk parameters (Θ_i) for claim cost and claim severity as well as claim frequency. It however uses the linear combination of the experience rating and class rating shown by equation (3.7), (3.19) and (3.34) with credibility factors; Z_i for the claim cost, Severity and Frequency gave by equation (3.9), (3.21) and (3.37) respectively.

4.4.1 Variability within Class Risks

Two elements appear crucial in finding proper balance between collective class rating and individual class ratings: How homogeneous are the class risks? If all of the risks in a category are equal and have identical expected price for losses then the class rating is preferred. Thus, using class rate known as collective or collateral risk means recorded under this research for claim costs as $E[m(\Theta)]$ and claim severities as $E[\mu(\Theta)]$ and for claim frequencies as λ. On the other side, if there is significant variation within the anticipated results of risks within the class, then the individual experience rating has to receive distinctly more weight. Each risk in the class has its own individual risk mean called its hypothetical means recorded for claim costs as $m(\Theta_i)$ claim severities as $\mu(\Theta_i)$ and claim frequencies as $\lambda(\Theta_i)$ respectively. The Variance of the Hypothetical Means (VHM), recorded for claim costs as $\text{var}[m(\Theta)]$, claim severities as $\mathrm{var}[\mu(\Theta)]$ and claim frequencies as $\mathrm{var}[\lambda(\Theta_i)]$ respectively across risk classes in the class are statistical measures for the homogeneity or heterogeneity, within the class.

A small variance of the Hypothetical means (VHM) indicates more class homogeneity and consequently, more weight for the class rate. A large VHM indicates more class heterogeneity and consequently, less weight going to the class rate.

Secondly, how much variation (Process Variance) represented as $s^2(\Theta_i)$ is there in an individual risk loss experienced? If there is a large amount of variation expected in the actual losses experienced by individual class risks, then the actual losses observed as Y_{ij}, X_{ij} or F_{ij} are far from their expected values; $m(\Theta_i)$, $\mu(\Theta_i)$ and $\lambda(\Theta_i)$ respectively and not very useful for individual experience rating. Therefore, less weight should be allocated to the individual experience rating in this situation. That is, the process variance (PV), which is the variance of a risk's random experience of its expected value, is a measure of variability in the losses experienced by individual risk. The Expected Value of the Process Variance (EPV) which is the average value of the process variance over the entire class of risks denoted under this study as $E[s^2(\Theta)]$. The smaller the process variances of the risks of a class ($s^2(\Theta_i)$) the smaller the Expected value of the Process Variance; $E[\mathrm{s}^2(\Theta)]$ and the higher the credibility factor or weight of the individual risk experience or risk means compared to the collective risk mean of the class ratings.

The result of experience rating implies a100% credibility to the means of individual risk classes or companies. This could be the case if the expected process variance within all risk is very small and the variance of the hypothetical means between the risk classes or the companies is large. The opposite, which is the class rating, implies 100% credibility to the collective risk means or 0% credibility to the individual class risk means.

Figure 4.1 displayed the annual risk means or annual claim costs the period (2013-2018) for the individual risks or companies including the collective risk mean.

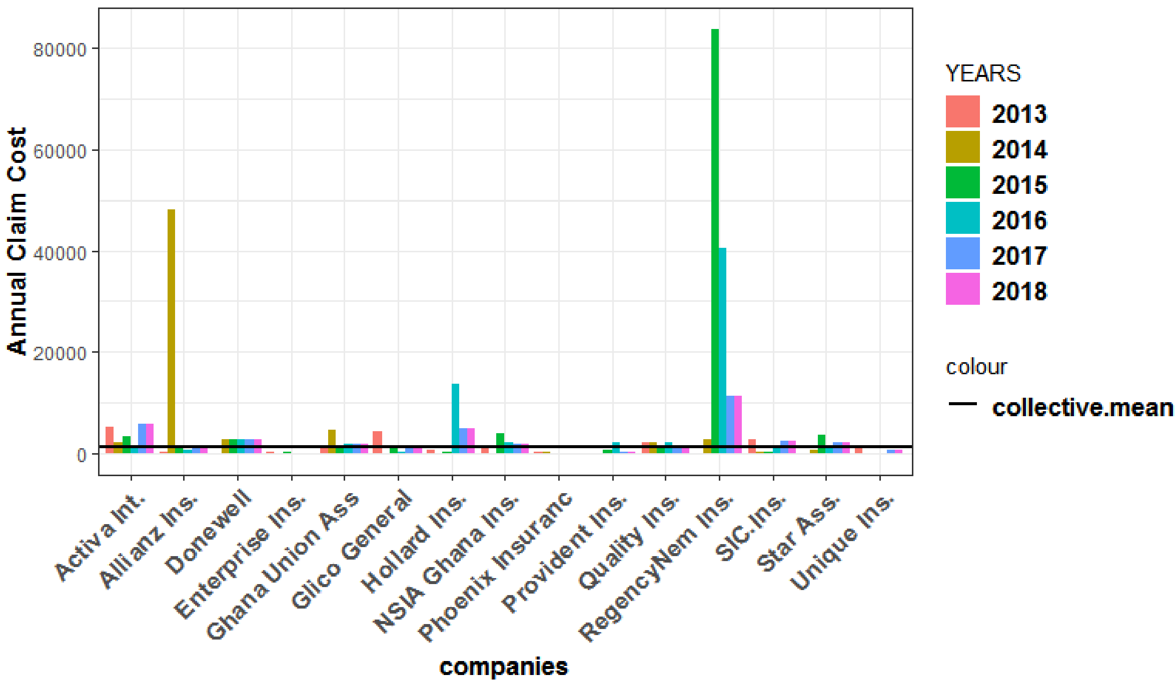

Figure 4.1: Bar Chart of Annual Claim cost for Marine Insurance Companies.

Figure 4.1 shows the variation that existed in the annual claim cost for policyholders of risk classes from the collective risk mean. RegencyNem Insurance Company Limited, Hollard Insurance Company Limited, Allianz

Insurance Company Limited and Activa Insurance Company Limited have shown that there exists a wide variation in their annual claim costs. This variation is also observed in class risks below the collective risk means. The insurance companies cannot set the same premium nor allow policyholders to pay the same premium since others may have a high tendency of making claim.

Table 4.4 displayed total policy count, the weight of average claim cost or experience rating and the credibility premium for the year, 2019 for policyholders of the non-life marine insurance companies. Whilst the estimated structure parameters of the claim cost are presented in Table 4.5.

Table 4.4: Policy Count and Credibility Factors and Estimates for Claim Cost for the Marine Insurance Companies

Marine Insurance company	$W_{i\bullet}$	$m(\Theta_i)$	Z_i	$m_{n+1}(\Theta_i)$
Activa Int. Insurance	650	3715.54	0.169	1567.248
Allianz Insurance	381	5498.77	0.107	1595.708
Donewell	155	2654.07	0.046	1199.711
Enterprise Insurance	9886	169.865	0.756	403.6203
Ghana Union Assurance	998	1815.8	0.239	1292.788
Glico General Insurance	815	1384.84	0.204	1181.078
Hollard Insurance	758	1741.72	0.192	1246.722
NSIA Ghana Insurance	172	2046.16	0.051	1175.936
Phoenix Insurance	641	207.198	0.168	974.5657
Provident Insurance	236	505.932	0.069	1085.988
Quality Insurance	139	1701.42	0.042	1152.887
RegencyNem Insurance	328	10223.4	0.093	1977.841
SIC Insurance	7785	1399.04	0.710	1320.609
Star Assurance	1064	1377.63	0.250	1191.212
Unique Insurance	479	483.562	0.131	1044.605

Table 4.5: Estimated Structure Parameters for Annual Claim Cost

$E[\mathrm{m}(\Theta)]$	$E[\mathrm{s}^2(\Theta)]$	$\mathrm{var}[\mathrm{m}(\Theta)]$
1 128.96	2 586 182 605.24	811 737.34

In Table 4.4, the total policy counts ($W_{i\bullet}$), credibility weights (Z_i), average claim cost; $\mathrm{m}(\Theta_i)$ and the Credibility claim costs or premiums; $m_{n+1}(\Theta_i)$ for each class risks or company are displayed.

The credibility weights for the claim histories have shown that most of the class risks have less than 50% credibility given more weight to the collective class risk mean. Only enterprise insurance company limited and SIC insurance company limited have weights of 75.6% and 71.0% respectively above 50% attach to their claim cost history.

The estimated unbiased collective class mean or risk premium; $E[\mathrm{m}(\Theta)]$ the expected process variance; $E[s^2(\Theta)]$ and the variance of the hypothetical means; $\mathrm{var}[\mathrm{m}(\Theta)]$ given as GHC1, 128.96, GHC2 586, 182, 605.24 and GHC811, 737.34 respectively, given by the equations (3.6), (3.7) and (3.18) respectively and are displayed in Table 4.5. The risks with less credibility have their average claim costs or pure premium $m(\Theta_i)$ decreased or increased closer towards the value of the collective risk mean or premium $E[\mathrm{m}(\Theta)]$ as a result linear combination between $m(\Theta_i)$ and $E[\mathrm{m}(\Theta)]$ as balanced premiums recorded as credibility premiums; $m_{n+1}(\Theta_i)$ recorded in Table 4.4.

Figure 4.2 displayed average claim cost and the collective risk mean over 6 years period. It also compares credibility premiums of risks to their respective pure premiums and collective risk mean premium for the year, 2019.

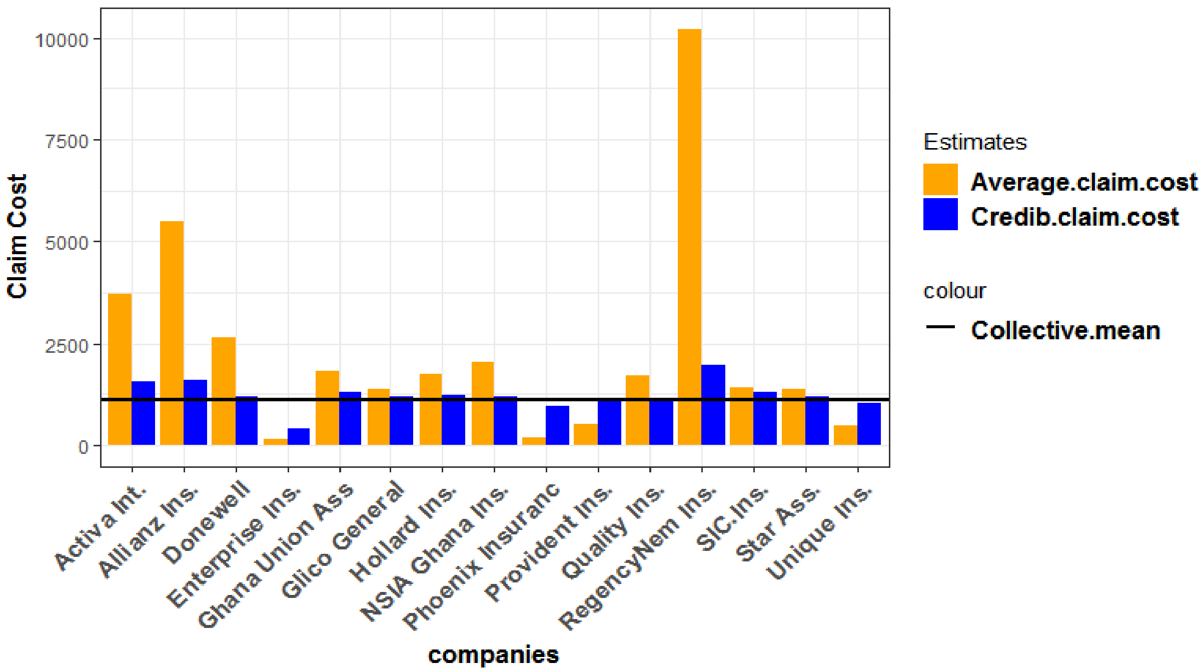

Figure 4.2: Bar plot of Average Claim costs and Credibility Estimates of risks.

As shown by Figure 4.2**,** over the six (6) years period, the experience rating for most of the risks or insurance companies except Enterprise Insurance Company Limited, phoenix Insurance Company Limited, provident Insurance Company Limited and unique Insurance Company Limited tend to have risk means higher than the collective risk mean. The risk mean of RegencyNem Insurance Company Limited compared to other risks or insurance companies has shown the highest deviation from the collective risk mean. The differences in the risk means cannot be said to be completely as a result of the random nature of claims but also due to the systematic nature of the claims. Thus, the differences are also as result of fluctuating individual risk profiles making some risks better risks than others and have their risk means even above the collective risk mean.

It can also be observed that apart from Enterprise Insurance Company Limited and SIC Insurance Company Limited that have their credibility claim cost or premiums closed to their experience rates (average claim costs), the rest of the class risks or marine insurance companies have their credibility premiums closed to the collective risk mean or premium far from their experience ratings and this is clearly observed in RegencyNem Insurance Company Limited.

4.5 Premium under Claim Frequency and Claim Severity

According to the Insurance Institute of Michigan (2019), insurance agencies basically do three things with the charges accrued. First, they pool the cash to pay claims. Second, insurance agencies pay for fees involved in selling and presenting insurance products. Third, insurance businesses make investments cash and earnings from investments assist maintain the value of insurance to policyholders. Underwriting operations encompass claims and related charges, expenses involved in sales and management, dividends to policyholders, state taxes and licensing prices, however exclude funding profits. Insurance organizations normally do now not make a benefit from their underwriting operations. The proceeds from invested funds in capital and surplus bills, in conjunction with money set apart for reserves, permit insurance organizations to keep insurance operations when underwriting losses exceed the amount amassed from premiums.

The insurance rate setting is a complicated technique. Insurance can’t be priced like most products, because the money people pay is intended to help cowl the expenses of unexpected events, such as marine incidents or auto injuries or fires. Due to the fact charges are paid earlier, the fee needs to be determined before the real costs are recognized. Whilst there are numerous factors considered in price making, rates essentially are depending on two trends: the frequency of claims and the severity of claims. A high claim frequency means large claim reserves denying insurance companies of investment opportunity thereby denying them of profit. This directly or indirectly contributes to the investment risks of insurance companies. However, according to Kagen (2018), there are instances where claim severity or average severity increases and largely affect losses for insurers while claim frequency is relatively stable over the life of a policy.

The claim severity is used by an insurance company to determine the premium that it must charge in order to break even with regard to policyholders who have experienced losses. The insurer then adds a percentage or loading to this premium to take into account any profit that it would like to make. The rise in frequency and severity of losses that aggravate the impact on volume and sizes of claims of the insurance companies and must subsequently be translated into the determination of premiums. The product of claim frequency and severity represents pure premium, thus the amount of money that the insurer will need to pay as an estimated loss over the life of the policy.

Claim frequencies and severities are affected by risk profiles of each class risk which vary across risks or insurance companies coupled with limited claim history. Hence, the researcher employs BuhlmannS-Straub’s Credibility Theory in estimating credibility average claim frequency and severity for the next immediate years, 2019. The product of these two estimates produces the premium known in this study as the credibility frequency-severity risk premium. Table 4.6: displayed claim count, average severity, weights and credibility severity for the year, 2019 for the risks or non-life marine insurance companies whilst Table 4.8 displayed the estimates of the structure parameters under claim severities.

Table 4.6: Claim Count and Credibility factors and estimates of claim Severities for the Marine Insurance Companies

Company	$N_{i\bullet}$	$\mu(\Theta_i)$	Z_i	$\mu_{n+1}(\Theta_i)$
Activa International Insurance	54	44724.15	0.5304	41356.225
Allianz Insurance	8	261878.9	0.14334	56502.788
Donewell	9	45709	0.15842	27488.721
Enterprise Insurance	828	2028.121	0.94541	7092.323
Ghana Union Assurance	49	36983.08	0.50614	34886.66
Glico General Insurance	20	56432.05	0.29494	33551.454
Hollard Insurance	33	40006.67	0.40836	33489.003
NSIA Ghana Insurance	16	21996.25	0.25074	25660.332
Phoenix Insurance	14	9486.714	0.2265	19824.896
Provident Insurance	10	11940	0.17298	21833.553
Quality Insurance	13	18192.15	0.21378	24233.977
RegencyNem Insurance	48	69859.77	0.50099	54249.55
SIC Insurance	170	64067.92	0.78049	56170.591
Star Assurance	19	77147.37	0.28439	37856.48
Unique Insurance	11	21056.91	0.18704	22396.001

Table 4.7: Parameter estimates for the Claim Severities

Parameters	$E[\mu(\Theta)]$	$E[s^2(\Theta)]$	$\text{var}[\mu(\Theta)]$
Estimates	21232.51	45296617020	947416672

In Table 4.7, the sum of claim counts per risk ($N_{i\bullet}$), credibility weights (Z_i), the average claim severity; $\mu(\Theta_i)$ and the Buhlmann-Straub Credibility claim severities; $\mu_{n+1}(\Theta_i)$ are displayed. Whilst the estimated unbiased overall or collective class mean for claim severity ($E[\mu(\Theta)]$), the expected process variance ($E[s^2(\Theta)]$) and the variance of the hypothetical means ($\text{var}[\mu(\Theta)]$) given as 21232.51, 45296617020 and 947416672 respectively, given by the equations (3.22), (3.23) and (3.24) respectively and are displayed in table 4.8. Companies with credibility weights closer to one (100% credibility) have their estimated balanced credibility severities ($\mu_{n+1}(\Theta_i)$) closer to their average severities ($\mu(\Theta_i)$) and vice versa.

Table 4.8: displays claim count, average claim frequency, weights and estimated credibility claim frequencies for the year, 2019 for the risks or non-life marine insurance companies whilst Table 4.9 displays the estimates of the structure parameters under claim frequency.

Table 4.8: Credibility Claim Frequencies Estimates for the Marine Insurance Companies

Company	$W_{i\bullet}$	$\lambda(\Theta_i)$	Z_i	$\lambda_{n+1}(\Theta_i)$
Activa Int. Insurance	650	0.0831	0.0439	0.0545
Allianz Insurance	381	0.0210	0.0262	0.0523
Donewell	155	0.0581	0.0108	0.0532
Enterprise Insurance	9886	0.0838	0.4110	0.0657
Ghana Union Assurance	998	0.0491	0.0658	0.0529
Glico General Insurance	815	0.0245	0.0544	0.0516
Hollard Insurance	758	0.0435	0.0508	0.0527
NSIA Ghana Insurance	172	0.0930	0.0120	0.0536
Phoenix Insurance	641	0.0218	0.0433	0.0518
Provident Insurance	236	0.0424	0.0164	0.0530
Quality Insurance	139	0.0935	0.0097	0.0536
RegencyNem Insurance	328	0.1463	0.0226	0.0553
SIC Insurance	7785	0.0218	0.3546	0.0421
Star Assurance	1064	0.0179	0.0699	0.0507
Unique Insurance	479	0.0230	0.0327	0.0522

In Table 4.9, the sum of policy counts per risk ($W_{i\bullet}$), credibility weights (Z_i), the average claim frequency ($\lambda(\Theta)$) and the Buhlmann-Straub Credibility claim frequency; $\lambda_{n+1}(\Theta)$ are displayed. The expected process variance $E[\lambda(\Theta)]$ and the variance of the hypothetical means of claim frequencies; $\text{var}[\lambda(\Theta)]$ are given as 0.05317 and 0.00000375 respectively, given by the equations (3.36) and (3.38) respectively, displayed in Table 4.9. The estimated unbiased collective class mean or frequency (λ) and the expected process variance $E[\lambda(\Theta)]$ are equal because the claim counts of the class risks follow a poison distribution.

Table 4.9: Parameter estimates for the Claim Frequency

Parameters	$E[\lambda(\Theta)]$	$\mathrm{var}[\lambda(\Theta)]$
Estimates	0.05317	0.00000375

Table 4.10 displays the credibility frequency-severity claim cost or premium. It displays the product of credibility claim frequency and that of credibility claim severity displayed in Table 4.6 and 4.7 respectively. The product of the two estimates gives the credibility frequency-severity claim premium.

Table 4.10: Credibility frequency-severity risk premium

Company	$P_{n+1}(\Theta_i)$
Activa Int. Insurance	2253.2092
Allianz Insurance	2956.706
Donewell	1463.0605
Enterprise Insurance	466.2564
Ghana Union Assurance	1845.6106
Glico General Insurance	1731.7122
Hollard Insurance	1764.2586
NSIA Ghana Insurance	1376.6535
Phoenix Insurance	1027.2253
Provident Insurance	1157.0504
Quality Insurance	1298.048
RegencyNem Insurance	2998.8773
SIC Insurance	2362.4816
Star Assurance	1919.4829
Unique Insurance	1168.6948

Figure 4.3 shows a bar chart plot of the average claim cost, credibility claim cost and the credibility frequency-severity claim cost as well as the collective claim cost.

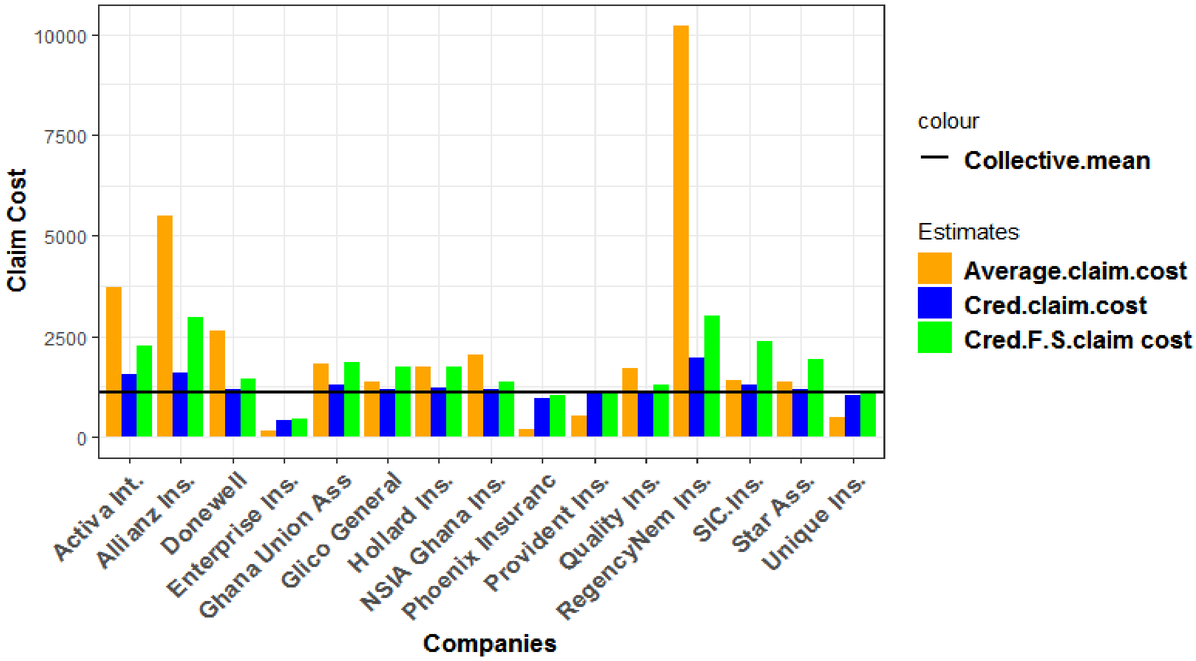

Figure 4.3: Bar Chart plot of Average Claim Cost and Credibility Estimates.

Figure 4.3 compares four different premiums estimated under this research work; The pure risk premiums or average claim cost ($m(\Theta_i)$) and the credibility claim cost for individual class risks, collective risk means ($E[\mathrm{m}(\Theta)]$) for all class histories displayed in table 4.4 and 4.5 respectively and the Credibility frequency-severity risk premium or claim cost ($P_{n+1}(\Theta_i)$) for the period; 2019 displayed in Table 4.10. The credibility frequency-severity claim cost is expected to be approximately the same as the credibility claim cost given that the claim frequencies and severities are stable over accounting periods for each class risk.

From Figure 4.3, comparing the two credibility estimates, the credibility frequency-severity claim costs for the policyholders be considered a better estimate as a better balance between the average claim cost and the collective risk mean than the credibility claim cost. Thus, it accounted for more variations in the risks or losses than it was in the case of the credibility claim cost.

CHAPTER FIVE

DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

This chapter discusses the major findings, conclusions made and recommendations drawn from the study about the Buhlmann-Straub Credibility claim costs or premiums for policyholders of a fifteen sampled non-life marine insurance companies in Ghana.

5.2 Discussion

Descriptive analysis revealed that, SIC Insurance Company Limited recorded the highest average of GHC1, 815, 257.7 whilst Provident Insurance Company Limited had GHC19, 900 as the lowest average claim size. A maximum claim size of GHC4, 437, 325 was recorded by SIC Insurance Company Limited. For policy counts, Star Assurance Company Limited recorded the highest average of 177.333 policy count whilst Quality Insurance Company Limited has recorded 23.167 as the lowest average policy count. A maximum policy count of 2702 was recorded by Enterprise Insurance Company Limited**.** But in terms of claim counts, Enterprise Insurance Company Limited recorded the highest average of 138 claim count whilst Allianz Insurance Company Limited recorded 1.333 as the lowest average claim count. The maximum claim count (154) was recorded by Enterprise Insurance Company Limited. From table 4.1, 4.2 and 4.3**,** it is observed that each of the non-life marine insurance companies recorded a claim size greater than at least one Ghana cedi and registered at least one policy and recorded at least one claim count over the six years (2013-2018) period.

The Buhlmann and Straub Credibility Theory model was used in estimating claim cost or premium to charge policyholders by the risk classes for the year, 2019 without considering the frequency and severity of the anticipated claims or losses. The average of the hypothetical means or the expected average claim costs ($E[\mathrm{m}(\Theta)]$) over the risk classes or non-life marine insurance companies is recorded for the policyholders as $E[\mathrm{m}(\Theta)]$ = GHS1, 128.96 shown in Table 4.5. The expected hypothetical mean (collective risk mean) can be used as a premium to be charged policyholders if the annul claim costs recorded by the risk classes are random and there are no significant differences in the risks profiles or parameters; Θ_i of the risk classes. This can only be the case if the variance of the hypothetical means is very small whilst the expected variance of the individual risk classes is large and is coupled with the fact that there is no enough claim history for risk classes or companies (Bühlmann, H., 2005). Inadequate claim history with a small variance of the hypothetical means and expected process variance reduces the credibility of the claim histories to zero and cannot be suitable for experience rating.

In Table 4.4 apart from enterprise insurance company limited and SIC insurance company limited that have weights,75.6% and 71.0% respectively, the rest of the risk classes have very small credibility factor or weights and deserve to charge their clients premiums closer to the collective risk mean or do the class rating. However, since the variance of the hypothetical means is not zero, a balance between the individual class rating and the collective class rating using the Buhlmann-Straub Credibility Theory became necessary and useful in estimating credibility pure risk premiums or claim costs; $m_{n+1}(\Theta_i)$ for the year 2019 recorded in Table 4.4.

Figure 4.2 also compares the experience ratings ($m(\Theta_i)$), collective class rating; $E[m(\Theta)]$ and the Buhlmann -Straub Credibility risk premiums or claim cost; $m_{n+1}(\Theta_i)$ which is a linear combination of the weights of the experience ratings and the class rating. The credibility factors and the estimated credibility risk severities; $\mu_{n+1}(\Theta_i)$ are also recorded in Table 4.6 whilst Table 4.7 records the estimated unbiased overall or collective class mean for the claim severities ($E[\mu(\Theta)]$), the expected process variance; $E[s^2(\Theta)]$ and the variance of the hypothetical means; $\mathrm{var}[\mu(\Theta)]$ given as 21232.51, 45296617020 and 947416672 respectively.

In Table 4.8, credibility weights (Z_i), the average claim frequency ($\lambda(\Theta_i)$) and the Buhlmann Straub Credibility claim frequency; $\lambda_{n+1}(\Theta_i)$ are displayed. The expected process variance $E[\lambda(\Theta)]$ and the variance of the hypothetical means for the claim frequencies; $\mathrm{var}[\lambda(\Theta)]$ are given as 0.054855 and 0.001327 respectively recorded in Table 4.9.

The collective risk frequency (λ) and the expected process variance $E[\lambda(\Theta)]$ are equal because the risk profiles; Θ_i underpinning the claim frequencies for class risks have claim counts following poison distribution with $E[\Theta_i]=1$.

Credibility claim severities and credibility claim frequency are determined so as to estimate the severity and frequency of losses for the next immediate period ($n+1$) or year; 2019 for each class of risks based on their risk profiles and subsequently takes the product of the two credibility estimates to obtain credibility frequency-severity claim cost or premium; $P_{n+1}(\Theta_i)$ for the year; 2019 recorded in Table 4.10. From Figure 4.3 of this study, a comparison between the average claim cost (experience ratings); $m(\Theta_i)$ and collective class rating; $E[m(\Theta)]$ shows that the average claim cost for most risks are greater than the collective risk mean. In a competitive insurance market, premiums through experience ratings in this case will leave most of the insurers to face anti-selection whilst premium based on the class rating (collective risk mean) will also leave most insurers to face huge losses because the irregular claim costs across risk classes depicted by figure 3.1 and 3.2 showed that most of the underwriters are facing bad risks and therefore cannot charge or be allowed to charge equal premium.

Fellingham, Tolley & Herzog (2005) Compares Credibility Estimates of Health Insurance Claims Costs for an insurance company located in the Midwest using mixed and hierarchical Bayesian credibility models and affirmed that all the estimates underestimated the claim cost. However, the comparison between the two linear combinations of weights of the experience ratings and the class ratings (the credibility claim cost and the credibility frequency-severity claim cost) shown by figure 4.3 shows that the credibility frequency-severity premiums; $P_{n+1}(\Theta_i)$ accounted for more of the variations accompanying the losses than credibility claim cost that failed to consider the unstableness of the claim severity and frequencies. This shows that the minimum premium determination using inadequate claim history among individual class risks must incorporate the frequency and severity of losses so as to be able to adequately cover losses transferred by policyholders to insurers. This vindicates the assertions by Kagen (2018), Zogbenu (2018) and IIM (2019) concerning how variability or unstableness of claim frequencies and severities influenced claims or losses faced by insurance companies. It is therefore, the researcher's contention that claim frequencies and severities be considered in determining credibility estimates as minimum rates for pricing insurance.

5.3 Conclusion

In accordance with the objectives of this study, the researcher estimated the average claim cost (risk premiums), the variability in the annual claim costs, annual claim severities and annual claim frequencies reported in table 4.5, 4.7 and 4.9 respectively based on which the credibility factors for the claim histories of each insurance company or class risks were estimated including the credibility premium or claim cost, credibility severity and frequency using the Bühlmanns-Straub Credibility Theory model reported in table 4.4, 4.6 and 4.8 respectively and subsequently estimated the credibility frequency-severity premium or claim cost.

Comparing the credibility frequency-severity claim cost to the credibility claim cost using a cluster of bar charts as shown by Figure 4.3, as balances between the average claim cost and the collective risk means, the researcher asserts that the credibility frequency-severity claim cost accounted for more variability in the claim histories than the credibility claim cost hence the frequency-severity claim cost can be considered a more better minimum estimate or premium to be charged policyholders compared to the credibility claim cost for the year, 2019.

5.4 Recommendations

I. Insurers interested in doing experience rating but having inadequate claim history can adopt Bühlmans-Straub Credibility Theory modeling to finding a balance between their average claim cost or premium and the collective risk premium over an entire or sizable number of classes trading in similar risks.

II. In order to fairly cover for losses and adequately determine claim reserves as well as funds for investments, insurers should also apply Bühlmans-Straub Credibility Theory models in estimating average claim frequencies and claim severities since variation in claim frequencies and claim severities are not normally stable during the policy year(s).

III. In order for underwriters to overcome liquidity and solvency challenges whilst overcoming anti-selections, the researcher strongly recommends that insurers price insurance policies using credibility frequency-severity risk claim cost (premiums) based on claim histories recorded in their books as well as similar or related claim histories from other insurers.

IV. There is also the need for regulators (NIC) to reconsider certain policies concerning the pricing of marine insurance policies including claim reserves such as the fixed premium and reserves imposed on insurers and rather introduce a regular update of premium and claim reserve based on the dynamics of claim histories recorded by the insurers.

REFERENCES

Asare, B. L., Evans, T., & Opokutakyi, R. (2017). Analysis of Factors Affecting Outstanding Claim Provision of Non-Life Insurance Firms in Ghana Analysis of Factors Affecting Outstanding Claim Provision of Non-Life Insurance Firms in Ghana, (August). https://doi.org/10.9790/5933-0804032634

Assurance, S. (2016). Annual report & financial statements 2016.

Boland, J. (2007). *Statistical and Probabilistic Methods in Actuarial Science*. London: Chapman&Hall/CRC.

Bu¨hlmann, H. & Jewell, W. (1987). Hierarchical credibility revis- ited. Mitteilungen der Vereinigung Schweizerischer Versicherungsmath- ematiker, 35–54.

Bühlmann, Straub E., G., & Schadensätze, F. (1970). Credibility for loss ratios. *Communications of the Association Swiss Actuaries*, 111–133.

Bühlmann. (1967). Experience rating and credibility. *ASTIN Bull*, 4, 119.

Bühlmann & Gisler. (2005). Course in Credibility Theory and its Applications. *Springer Berlin Heidelberg New York*.

Cheung, D., & Cheung, D. (1997). Estimating IBNR Reserves with Robust Statistics by.

Couret, J., & Venter, G. (2008). Using Multi-Dimensional Credibility to Estimate Class Frequency Vectors in Workers Compensation. *ASTIN Bulletin*, *38*(01), 73–85. https://doi.org/10.2143/AST.38.1.2030403

Curtis, E. (2013). Determinants of Profitability of Insurance Firms in Ghana. *International Journal of Business and Social Research (IJBSR) No. 3, 3*.

Drewry. (1998). *Marine Insurance Issues, Practices and cost*. london.

Emanuelsson, P. (2011). Construction of rating territories for water-damage claims. Retrieved from http://www2.math.su.se/matstat/reports/master/2011/rep8/report.pdf

Fellingham, G. W., Dennis Tolley, H., & Herzog, T. N. (2005). Comparing Credibility Estimates of Health Insurance Claims Costs. *North American Actuarial Journal*, *9*(1), 1–12. https://doi.org/10.1080/10920277.2005.10596181

Frees, E. W., & Wang, P. (2005). Credibility Using Copulas. *North American Actuarial Journal*, *9*(2), 31–48. https://doi.org/10.1080/10920277.2005.10596196

Gerber, H., & Jones, D. (1975). Credibility formulas of the updating type. *Transactions of the Society of Actuaries*, *27*, 31–52.

Hachemeister, C. (1975). Credibility for Regression Models with Application to Trend. Kahn, P. (Ed.), Credibility Theory and Applications. *Academic Press, New York.*

Haiss, P. R. (2006). The Relationship of Insurance and Economic Growth – A Theoretical and Empirical Analysis The Relationship of Insurance and Economic Growth. *World Applied Sciences Journal*.

IIM. (2019). How Insurance Works. Retrieved May 22, 2019, from http://www.iiminfo.org/CONSUMERS/HowInsuranceWorks/tabid/1714/Default.aspx

Jewell, W. S. (1973). Multidimensional credibility. . . *Actuarial Research Clearing House*, *4*.

Jewell, W. S. (1975). The use of collateral data in credibility theory: A hierarchical model. Giornale d e l l ’ Istituto degli Attuari Italiani, *38*, 1-

16. 18.

Kagen, J. (2018). Insurance Claim. Retrieved January 4, 2018, from www.investopedia.com/terms/i/insurance_claim.asp

King, R. O. (2008). Ocean piracy and its impact on insurance. *Congressional Research Service,* 1–7. https://doi.org/10.1083/jcb.200603039

Kyriaki, N. (2007). *The Principle of Indemnity in Marine Insurance Contracts*. New York: Springer-Verlag Berlin Heidelberg.

Marrewijk, C. V. (2002). International Trade and the world economy. *New York Uninversity*.

Mazviona, B. W., Dube, M., & Sakahuhwa, T. (2017). An Analysis of Factors Affecting the Performance of Insurance Companies in Zimbabwe, *6*(1), 11–30.

Meulbroek, L. K. (2000). The Efficiency of Equity-Linked Compensation: Understanding the Full Cost of Awarding Executive Stock Options. *Ssrn*. https://doi.org/10.2139/ssrn.215530

Mishra. (2004). *Modern Concepts of Insurance*. New Delhi: Sultan Chand Publication.

Mowbray, A. H. (1914). How extensive a payroll exposure is necessary to give a dependable pure premium? *Proceedings of Casualty Actuarial Society*, 1, 24–30.

NIC. (2015). *National Insurance Commission Annual Report*.

Niehaus, H. (2003). *Risk Management and Insurance* (2nd ed.). New York: Mc Graw Hill.

Petersson, G. J. (2010). *Swedish Marine Insurance between the World Wars*.

Quaye, A. (2014). "Loss Reserve Variability and Loss Reserve Errors: An

Empirical Analysis of the Ghanaian Property and Liability Insurance Industry,." *Journal of Risk Finance*, *15*(3), 26–32.

Ravichandran, R. (2001). The Maritime Commons ; Digital Repository of the World Emerging trends in marine insurance : is there a trend towards demutualisation of mutual clubs ? *The World Maritime University World*, 2–6.

Shapland, M. R. (2007). Loss Reserve Estimates: A Statistical Approach for Determining "Reasonableness." *Variance: Advancing The Science of Risk*, *1*(1), 120–148.

Stuart A.Klugman, Harry H.Panjer, G. E. W. (1949). *Loss Models From Data to Decisions* (2nd ed.). A John Willeyand Sons, Inc., Publication.

Thiery, Y., & Schoubroeck, C. Van. (2018). Fairness and Equality in Insurance Classification, The Geneva Papers on Risk and Insurance . Issues and Practice , Vol . 31 , No . 2 , Special Issue on Law and Economics and International Liab, *31*(2), 190–211.

Verall, N. (1997). Credibility Theory and Generalized Linear Models, *27*.

Viaene, S., Ayuso, M., Guillen, M., Van Gheel, D., & Dedene, G. (2007). Strategies for detecting fraudulent claims in the automobile insurance industry. *European Journal of Operational Research*, *176*(1), 565–583. https://doi.org/10.1016/j.ejor.2005.08.005

Waters H. R. (1994). An Introduction to Credibility Theory, London and Edinburgh. *Institute of Actuaries and Faculty of Actuaries*.

Whitney, A. W. (1918). The Theory of Experience Rating. *Proceedings of the Casualty Actuarial Society*, *2*, 274–292.

Winter, W. (1919). *Marine Insurance; Its Principles and Practice*. London:

McGraw-Hill Book Company, Inc.

Zhang Z. (2004). A Bayesian loss reserving model. *Alberta: The University of Calgary*.

Zogbenu, M. (2018). Ghana's marine insurance regime - Time to deliberate. Retrieved March 14, 2019, from https://www.myjoyonline.com/business/2018/june-7th/ghanas-marine-insurance-regime-time-to-deliberate.php

Appendix

PROOF OF BUHLMANNS-STRAUB CREDIBILITY ESTIMATOR

To find the credibility estimator, $m_{n+1}(\Theta_i)$ which is a linear function of the observations $Y_{i1}, Y_{i2}, \ldots, Y_{in}$ with minimum mean square error from the pure premium; there is the need to choose $a_{i0}, a_{i1}, \ldots, a_{in}$ that will optimize the estimator of $m_{n+1}(\Theta_i)$ given by

$$m_{n+1}(\Theta_i) = a_{i0} + a_{i1}Y_{i1} + a_{i2}Y_{i2} + \cdots + a_{in}Y_{in} = a_{i0} + \sum_{j}^{n} a_{ij}Y_{ij} . \qquad (3.1)$$

Lemma 3.1 With the set-up and notation of Bühlmann and Straub's credibility model given the company or risk i and time periods $j = 1, 2, \ldots\ldots..,$ n we have

(i) $E[Y_{ij}] = E[\bar{Y}_i / \Theta_i] = E[m(\Theta)]$

(ii) $E[Y_{ij}\, m(\Theta_i)] = E[m^2(\Theta_i)]$

(iii) $E[Y_{ij}\, Y_{ik}] = E[m^2(\Theta)]$ *For* $j \neq k$;

(iv) $E[Y_{ij}^2] = \dfrac{1}{W_{ij}} E[s^2(\Theta)] + E[m^2(\Theta)].$

In the case of Bühlmann Straub's model, the constants $a_{i1}, a_{i2}, \ldots, a_{in}$ are not equal because Y_{ij} ($j = 1, 2, \ldots\ldots$, n) are not identically distributed.

Theorem 3.1 Let $Y_1, Y_2, \ldots, Y_n$ be a sequence of random variables, each of whose distribution depends on a risk parameter Θ, and which given Θ are independent with $W_{ij}\, Var[Y_{ij} \mid \Theta_i] = s^2(\Theta_i),\ j = 1, \ldots, n$.

Then the mean square error of the estimator, $a_{i0} + \sum_{j=1}^{n} a_{ij}Y_{ij}$ from $m(\Theta_i)$ given by

$$E\left[\left(m(\Theta_i)-\mathrm{m}_{n+1}(\Theta_i)\right)^2\right]=E\left[\left(m(\Theta_i)-a_{i0}-\sum_{j=1}^{n}a_{ij}Y_{ij}\right)^2\right] \quad (3.2)$$

and is minimized and given by

$$m_{n+1}(\Theta_i)=(1-Z_i)E[m(\Theta)+Z_i\bar{Y}_i \quad (3.3)$$

where

$\bar{Y}_i =$ The average claim per claim count for company i

$$\bar{Y}_i = m(\Theta_i)=\frac{\sum_{j=1}^{n}W_{ij}Y_{ij}}{\sum_{j=1}^{n}W_{ij}} \quad (3.4)$$

$Z_i =$ Credibility factor of claim history for company i

$$Z_i = \frac{\sum_{j=1}^{n}W_{ij}}{\sum_{j=1}^{n}W_{ij}+\dfrac{E[s^2(\Theta)]}{\mathrm{var}[\mathrm{m}(\Theta)]}} \quad (3.5)$$

$Z_i = [0,1]$

To obtain the values of a_0 and a_{ij}, ($j=1$, 2, ….., n) that minimize the estimate in equation (3.1), there is the need to take partial derivative of equation (3.2) and equating them to zero and solving for a_0 and a_{ij} simultaneously by applying **lemma (3.1)** and **theorem (3.1)** to obtain equation (3.3).

The estimate $m_{n+1}(\Theta_i)$ in equation (3.14) is the credibility premium or claim cost for company i for the next immediate period $n+1$

Now considering how to estimate the three structural parameters $\mathrm{E}[\mathrm{m}(\Theta)]$, $\mathrm{E}[\mathrm{s}^2(\Theta)]$ and $\mathrm{var}[\mathrm{m}(\Theta)]$ using data from a collective of *I* (fixed) comparable risks or companies.

Given that,

$$m_{n+1}(\Theta_i) = a_{i\mathbf{0}} + a_{i1}Y_{i\mathbf{1}} + a_{i2}Y_{i\mathbf{2}} + \cdots + a_{in}Y_{in} = a_{i0} + \sum_{j}^{n} a_{ij}Y_{ij} \text{.} \qquad (3.1)$$

The mean square error of the estimator, $m_{n+1}(\Theta_i) = a_{i0} + \sum_{j=1}^{n} a_{ij}Y_{ij}$ from $m(\Theta_i)$ given by

$$E\left[\left(m(\Theta_i) - \mathrm{m}_{n+1}(\Theta_i)\right)^2\right] = Q = E\left[\left(m(\Theta_i) - a_{i0} - \sum_{j=1}^{n} a_{ij}Y_{ij}\right)^2\right] \qquad (3.2)$$

Estimating the coefficients, $a_{i\mathbf{0}}, a_{i\mathbf{1}}, \ldots, a_{in}$ in equation (3.12) that minimize MSE (Q) in equation (3.13) by taking the derivative of Q with respect to $a_{i\mathbf{0}}, a_{i\mathbf{1}}, \ldots, a_{in}$

$$\partial Q / \partial a_{i0} = -2E\left[\left(m(\Theta_i) - a_{i0} - \sum_{j=1}^{n} a_{ij}Y_{ij}\right)\right]$$

$$\partial Q / \partial a_{i0} = 0$$

$$-2E\left[\left(m(\Theta_i) - a_{i0} - \sum_{j=1}^{n} a_{ij}Y_{ij}\right)\right] = 0$$

$$E[m(\Theta_i)] - a_{i0} - \sum_{j=1}^{n} a_j \, \mathrm{E}[Y_{ij}] = 0$$

Solving for a_{i0} we have, and applying lemma 3.1 we have

$$E[m(\Theta)] - a_{i0} - \sum_{j=1}^{n} a_j E[m(\Theta)] = 0$$

$$a_{i0} = (1 - \sum_{j=1}^{n} a_j) E[m(\Theta)] \qquad (3.6)$$

Now solving for any of the coefficients, say a_{ik} .in equation (3.2)

Equation (3.13) can be written to include a_{ik} as,

$$E\left[\left(m(\Theta_i)-\mathrm{m}_{n+1}(\Theta_i)\right)^2\right]=Q=E\left[\left(m(\Theta_i)-a_{i0}-a_{ik}Y_{ik}-\sum_{\substack{j=1\\ j\neq k}}^{n}a_{ij}Y_{ij}\right)^2\right] \quad (3.2a)$$

Differentiate with respect to a_{ik}

$$\partial Q \Big/ a_{ik} = 0 = -2Y_{ik}E\left[\left(m(\Theta_i)-a_{i0}-a_{ik}Y_{ik}-\sum_{\substack{j=1\\ j\neq k}}^{n}a_{ij}Y_{ij}\right)\right]$$

$$E[m(\Theta_i)\,\mathrm{Y}_{ik}]-E[a_{i0}Y_{ik}[-E[a_{ik}Y_{ik}{}^2]-\sum_{\substack{j=1\\ j\neq k}}^{n}E[a_{ij}Y_{ij}Y_{ik}]=0$$

$$E[m^2(\Theta)]-a_{i0}E[m(\Theta)]-\frac{a_{ik}}{W_{ik}}E[s^2(\Theta)]-a_{ik}[m^2(\Theta)]-\sum_{\substack{j=1\\ j\neq k}}^{n}a_{ij}E[m^2(\Theta)]=0$$

NB that, $a_{ik}+\sum_{\substack{j=1\\ j\neq k}}^{n}a_{ij}=\sum_{j=1}^{n}a_{ij}$

$$E[m^2(\Theta)]=a_{i0}E[m(\Theta)]-\frac{a_{ik}}{W_{ik}}E[s^2(\Theta)]-(a_{ik}+\sum_{\substack{j=1\\ j\neq k}}^{n}a_{ij})E[m^2(\Theta)]$$

$$E[m^2(\Theta)]=a_{i0}E[m(\Theta)]+\sum_{j=1}^{n}a_{ij}E[m^2(\Theta)]+\frac{a_{ik}}{W_{ik}}E[s^2(\Theta)]$$

Substituting a_{i0} from equation (3.6)

$$E[m^2(\Theta)]=(1-\sum_{j=1}^{n}a_{ij})E[m(\Theta)]^2+\sum_{j=1}^{n}a_{ij}E[m^2(\Theta)]+\frac{a_{ik}}{W_{ik}}E[s^2(\Theta)]$$

$$E[m^2(\Theta)]-E[m(\Theta)]^2+\sum_{j=1}^{n}a_{ij}E[m^2(\Theta)]-\sum_{j=1}^{n}a_{ij}E[m(\Theta)]^2=\frac{a_{ik}}{W_{ik}}E[s^2(\Theta)]$$

$$(1-\sum_{j=1}^{n}a_{ij})\,\mathrm{var}[m(\Theta)]=\frac{a_{ik}}{W_{ik}}E[s^2(\Theta)]$$

Solving for a_{ik}

$$a_{ik} = (1-\sum_{j=1}^{n} a_{ij})\frac{\text{var}[m(\Theta)]W_{ik}}{E[s^2(\Theta)]}$$

Summing over k

$$\sum_{k=1}^{n} a_{ik} = (1-\sum_{j=1}^{n} a_{ij})\frac{\text{var}[m(\Theta)]}{E[s^2(\Theta)]}\sum_{k=1}^{n} W_{ik} \tag{3.7}$$

Though a_{ik} and a_{ij} may not be the same we assume their respective sums over n are equal. Thus,

$\sum_{k=1}^{n} a_{ik} = \sum_{j=1}^{n} a_{ij}$, solve for either in equation (3.7) here, let solve for $\sum_{j=1}^{n} a_{ij}$

NB because of $\sum_{j=1}^{n} a_{ij}$, we write $\sum_{k=1}^{n} W_{ik}$ in terms of j as $\sum_{j=1}^{n} W_{ij}$

$$\sum_{j=1}^{n} a_{ij} E[s^2(\Theta)] = \text{var}[m(\Theta)]\sum_{j=1}^{n} W_{ij} - \sum_{j=1}^{n} a_{ij}\ \text{var}[m(\Theta)]\sum_{j=1}^{n} W_{ij}$$

Make $\sum_{j=1}^{n} a_j$ the subject

$$\sum_{j=1}^{n} a_{ij} E[s^2(\Theta)] + \sum_{j=1}^{n} a_{ij}\ \text{var}[m(\Theta)]\sum_{j=1}^{n} W_{ij} = \text{var}[m(\Theta)]\sum_{j=1}^{n} W_{ij}$$

$$\sum_{j=1}^{n} a_{ij}(E[s^2(\Theta)] + \text{var}[m(\Theta)]\sum_{j=1}^{n} W_{ij}) = \text{var}[m(\Theta)]\sum_{j=1}^{n} W_{ij}$$

$$\sum_{j=1}^{n} a_{ij} = \frac{\text{var}[m(\Theta)]\sum_{j=1}^{n} W_{ij}}{E[s^2(\Theta)] + \text{var}[m(\Theta)]\sum_{j=1}^{n} W_{ij}}$$

$$\sum_{j=1}^{n} a_{ij} = \frac{\sum_{j=1}^{n} W_{ij}}{\sum_{j=1}^{n} W_{ij} + \frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}} \tag{3.8}$$

For individual coefficient in (3.1) we have equation (3.9)

$$a_{ij} = \frac{W_{ij}}{\sum_{j=1}^{n} W_{ij} + \frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}} \tag{3.9}$$

Putting the result of equation (3.8) into (3.6) knowing that $\sum_{k=1}^{n} a_{ik} = \sum_{j=1}^{n} a_{ij}$

$$a_{i0} = (1 - \frac{\sum_{k=1}^{n} W_{ij}}{\sum_{j=1}^{n} W_{ij} + \frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}}) E[m(\Theta)$$

$$a_{i0} = \frac{\frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}}{\sum_{j=1}^{n} W_{ij} + \frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}} E[m(\Theta) \qquad (3.20)$$

Putting the estimates a_{i0} and a_{ik} in equation (3.9) and (3.20) into equation (3.1)

$$m_{n+1}(\Theta_i) = a_{i0} + \sum_{j}^{n} a_{ij} Y_{ij}$$

$$m_{n+1}(\Theta_i) = \frac{\frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}}{\sum_{j=1}^{n} W_{ij} + \frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}} E[m(\Theta) + \frac{\sum_{j=1}^{n} W_{ij} Y_{ij}}{\sum_{j=1}^{n} W_{ij} + \frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}}$$

$$m_{n+1}(\Theta_i) = \frac{\frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}}{\sum_{j=1}^{n} W_{ij} + \frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}} E[m(\Theta) + \frac{\frac{\sum_{j=1}^{n} W_{ij}}{\sum_{j=1}^{n} W_{ij}} \sum_{k=1}^{n} W_{ij} Y_{ij}}{\sum_{j=1}^{n} W_{ij} + \frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}}$$

But $\frac{\sum_{j=1}^{n} W_{ij} Y_{ij}}{\sum_{j=1}^{n} W_{ij}} = \bar{Y}_i$

$$m_{n+1}(\Theta_i) = \frac{\frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}}{\sum_{j=1}^{n} W_{ij} + \frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}} E[m(\Theta) + \frac{\bar{Y}_i \sum_{j=1}^{n} W_{ij}}{\sum_{j=1}^{n} W_{ij} + \frac{E[s^2(\Theta)]}{\text{var}[m(\Theta)]}}$$

let

$$Z_i = \frac{\sum_{j=1}^{n} W_{ij}}{\sum_{j=1}^{n} W_{ij} + \frac{E[s^2(\Theta)]}{\mathrm{var}[m(\Theta)]}}$$

Hence

$$m_{n+1}(\Theta_i) = (1 - Z_i)E[m(\Theta) + Z_i \overline{Y_i}$$

DATA

Gross claim Years

CMPANY NO.	2013	2014	2015	2016	2017	2018
1	582619	333889	436666	121728	410270	529932
2	44592	1829787	34959	24704	74846	86143
3	1158	139653	87283	55279	32002	96006
4	374831	257995	303461	212357	221100	309540
5	206921	241132	213724	370707	393340	386347
6	717007	25818	141315	33000	84419	127082
7	123154	30414	55887	595226	250405	265134
8	10525	2002	133006	119029	29126	58252
9	52212	40081	8693	9120	9220	13488
10	3543	3000	12766	75561	14477	10053
11	6597	65871	20000	65972	17915	60143
12	24221	40558	921565	810125	767119	789681
13	4437325	311473	883583	671039	2183900	2404226
14	2023	135991	312545	163771	423596	427874
15	29928	9131	10803	12527	89320	79917

Policy count Years

COMPANY NO.	2013	2014	2015	2016	2017	2018
1	115	158	125	87	72	93
2	154	38	30	45	53	61
3	14	48	30	19	11	33
4	1620	2702	1163	1581	1175	1645
5	154	54	154	190	225	221
6	163	163	163	93	93	140
7	200	203	207	43	51	54
8	13	24	33	54	16	32
9	202	160	78	68	54	79
10	35	82	24	34	36	25
11	3	30	15	30	14	47
12	145	14	11	20	68	70
13	1608	1596	1896	790	902	993
14	219	222	85	140	198	200
15	24	66	82	91	114	102

Claim count Years

COMPANY NO.	2013	2014	2015	2016	2017	2018
1	25	6	1	12	8	2
2	1	2	1	1	1	2
3	1	2	1	2	1	2
4	149	134	145	125	154	121
5	1	11	8	8	11	10
6	4	2	4	3	3	4
7	5	5	4	4	8	7
8	2	2	3	3	4	2
9	2	1	1	1	5	4
10	1	1	1	2	3	2
11	1	2	2	1	4	3
12	1	1	13	11	18	4
13	117	33	5	3	4	8
14	1	2	4	4	4	4
15	1	1	1	2	4	2

RCODES

```
#######################################################
#  CREDIBILITY CLAIM COST OR PREMIUM DETERMINATION
#######################################################
# import grossclaim first from excelwith no header command
# import gross claim as grossclaim
a<-data.frame(grossclaim)
colnames(a)<-NULL
a
# list of companies used in the analysis
a4<-c("Activa Int.","Allianz Ins.",   "Donewell","Enterprise Ins.",
"Ghana Union Ass", "Glico General",  "Hollard Ins.","NSIA Ghana Ins.",
 "Phoenix Insuranc","Provident Ins.",  "Quality Ins.","RegencyNem Ins.",
  "SIC.Ins.","Star Ass.", "Unique Ins.")
gclm<-data.frame(a)
A<-as.matrix(sapply(gclm, as.numeric)) # coverting the dataframe to matrix,
# descriptive statistics of the grossclaims
A1<-apply(A,1,mean)[1:15],A2<-apply(A,1,max)[1:15], min1<-
apply(A,1,min)[1:15]
A3<-apply(A,1,sd)[1:15], A4<-apply(A,1,skewness)[1:15], A5<-
apply(A,1,kurtosis)[1:15]
A6<-round(cbind(A1,A2,min1, A3,A4,A5),5)
A7<-matrix(A6,ncol = 6 )
colnames(A7)<-c("Mean","Max", "Min","Std.dev","sk'nes","kurt")
# DO SAME FOR POLICYCOUNT AS MATRIX B
b<-data.frame(policycount)
colnames(b)<-NULL, plc<-data.frame(b)
```

```
B<-as.matrix(sapply(plc, as.numeric))

# determing claim frequency for each company for each year

annual.cost<-A/B, a5<-data.frame(a4,annual.cost)

colnames(a5)<-c("company","2013","2014","2015","2016","2017","2018")

B1<-apply(B,1,mean)[1:15], B2<-apply(B,1,max)[1:15], min2<-
apply(B,1,min)[1:15]

B3<-apply(B,1,sd)[1:15], B4<-apply(B,1,skewness)[1:15], B5<-
apply(B,1,kurtosis)[1:15]

B6<-round(cbind(B1,B2,min2,B3,B4,B5),5), B7<-matrix(B6,ncol = 6 )

colnames(B7)<-c("Mean","Max", "Min","Std.dev","sk'nes","kurt")

a1<-rowSums(A) # sum of grossclaim

b1<-rowSums(B) # sum of policycount

# importing claimcounts

c<-data.frame(claimcount)

colnames(clc)<-NULL, C<-as.matrix(sapply(clc, as.numeric))

C1<-apply(C,1,mean)[1:15], C2<-apply(C,1,max)[1:15], min3<-
apply(C,1,min)[1:15]

C3<-apply(C,1,sd)[1:15], C4<-apply(C,1,skewness)[1:15], C5<-
apply(C,1,kurtosis)[1:15]

C6<-round(cbind(C1,C2,min3,C3,C4,C5),5), C7<-matrix(C6,ncol = 6 )

colnames(C7)<-c("Mean","Max", "Min","Std.dev","sk'nes","kurt")

# xi-bars(xi) (individual means, claim size per policy for every individual risk)

xi_bar<-a1/b1

totclaim<-sum(A)  #total claim size(grossclaim)

totpolicy<-sum(B) # total policy count

xbar<-totclaim/totpolicy

Average.Claim.Cost<-xi_bar, xi5<-data.frame(a4, Average.Claim.Cost)

colnames(xi5)<-c("company","Average.Claim.Cost")
```

```
xi5 <- melt(xi5, id = "company")

p<-ggplot() + geom_bar(data = xi5, aes(x = company, y = value,fill=variable),

                              position = "dodge", stat = "identity")

pp<-p + xlab("company") + ylab("Claim Cost") + theme_bw()

p1<-pp+theme(axis.text.x = element_text(angle = 45,hjust = 1,size = 10),

   axis.title=element_text(size=10),  legend.text = element_text(size = 10))

collective.Risk.mean<-xbar

pp1<-p1 + geom_hline(aes(yintercept=collective.mean,

colour="collective.Risk.mean"),size=0.55) + scale_color_manual(values =
"black")

pp1 + scale_fill_manual( values = c("Average.Claim.Cost"="orange"),

         name="Estimated \npure Premium

pp1 + scale_fill_manual( values = c("Average.Claim.Cost"="orange"),

                   name="Estimates" )

xij<-A/B

apply(xij,1,max)[1:15]

x5<-data.frame(a4,xij)

colnames(x5)<-c("company","2013","2014","2015","2016","2017","2018")

x5 <- melt(x5, id = "company")

p<-ggplot() + geom_bar(data = x5, aes(x = company, y = value, fill =
variable),

position = "dodge", stat = "identity")

pr<-p + theme_bw() + geom_hline(aes(yintercept=xbar,color =
"collective.mean"),

size=0.9)

prr<-pr + xlab("companies") + ylab("Annual Claim Cost")  +

                    scale_color_manual(values = "black")

pprr<-prr + theme(axis.text.x = element_text(angle = 45, hjust = 1,size = 12,
```

```
              face = "bold" ), axis.title=element_text(size=13,face="bold"),
              legend.text = element_text(size = 12,face = "bold"))
pprr + scale_fill_discrete(name="YEARS") + ylim(0,60000)
dev.xij<-(xij-xi_bar)^2, d<-B*dev.xij, si2<-rowSums(d)/(ncol(d)-1)
xxi<-data.frame(a4,xi_bar, si2 )
colnames(xxi)<-c("Marine Insurance company","Risk mean","Risk Variance")
length(si2), mpv<-sum(si2)/length(si2)
v2ij<-(xij-xbar)^2, v2ijb<-B*v2ij, length(v2ijb), sum(v2ijb)
varx<-sum(v2ijb)/(length(v2ijb)-1)
p*=(1/length(B))*sum(rowsums(B)*(1-(rowsums(B)/totpolicy)))
        rowsums<-rowSums(B)
prop<-rowsums/totpolicy, q<-1-prop
bq<-rowSums(B)*q, p<-sum(bq)/(length(B)-1)
px <-(1/(length(B)-1))*(sum(rowSums(B)*(1-(rowSums(B)/(totpolicy)))))
# finally we can calculate variace of the harmonic means, vhm or tau^2
vhm<-(varx-mpv)/p, k<-mpv/vhm, parameter.estx<-data.frame(xbar,mpv,vhm)
z<-rowSums(B)/(rowSums(B)+k), cred.pr<-z*xi_bar+(1-z)*xbar
credib.claim.cost<-cred.pr, zz<-data.frame(rw,b1,z,1-z, xi_bar,cred.pr)
colnames(zz)<-c("company","Policycount","Zi","1-
Zi","xi_bar","Est.Cred.Prem.")
dat<-data.frame(a4,xi_bar,cred.pr)
colnames(dat)<-c("company","Average.claim.cost","Credib.claim.cost")
dat <- melt(dat, id = "company",variable.name = "Estimates")
gp<-ggplot() + geom_bar(data = dat, aes(x = company, y = value,
        fill = Estimates), position = "dodge", stat = "identity")
gr<-gp + theme_bw() + geom_hline(aes(yintercept=xbar,
          color = "Collective.Risk.mean"),size=0.9)
```

```
grr<-gr + xlab("companies") + ylab("Claim Cost") +
                  scale_color_manual(values = "black")
grr1<-grr + theme(axis.text.x = element_text(angle = 45, hjust = 1,size = 10))
grr2<-grr1 + theme(axis.title=element_text(size=10),
         legend.text = element_text(size = 10))
grr2 + scale_fill_manual(name= "Estimates",  values = c( "orange", "blue"))
# PREMIUM BASED ON FREQUENCY AND SEVERITY
gclm<-data.frame(a)
A<-as.matrix(sapply(gclm, as.numeric))
# DO SAME FOR CLAIM COUNT AS MATRIX C
clc<-data.frame(c), C<-as.matrix(sapply(clc, as.numeric))
a1<-rowSums(A), c1<-rowSums(C), yi<-a1/c1, yyi<-data.frame(a4,yi)
colnames(yyi)<-c("company","severity"), totclaim<-sum(A)
totclaimcount<-sum(C), ybar<-totclaim/totclaimcount, yij<-A/C
dev.yij<-(yij-yi)^2, d1<-C*dev.yij, syi2<-rowSums(d1)/(ncol(d1)-1),
length(syi2)
mpv1<-sum(syi2)/length(syi2), v2yij<-(yij-ybar)^2, v2yijc<-C*v2yij,
length(v2yijc)
vary<-sum(v2yijc)/(length(v2yijc)-1)
N*=(1/length(C))*sum(rowsums(C)*(1-(rowsums(C)/totclaimcount)))
rowsums1<-rowSums(C), prob1<-rowsums1/totclaimcount, q1<-1-prob1
bq1<-rowSums(C)*q1, N<-sum(bq1)/(length(C)-1)
Nx<-(1/(length(C)-1))*(sum(rowSums(C)*(1-(rowSums(C)/(totclaimcount)))))
vhm1<-(vary-mpv1)/N, parameter.est<-data.frame(ybar,mpv1,vhm1)
k1<-mpv1/vhm1, z1<-rowSums(C)/(rowSums(C)+k1)
cred.sev<-z1*yi+(1-z)*ybar, cred.sev1<-data.frame(a4,cred.sev)
colnames(cred.sev1)<-c("company","sev.cred.")
```

```
zz1<-data.frame(a4,c1 ,yi,z1,1-z1,cred.sev)

colnames(zz1)<-c("company","claim.count","weight","Z","1-
Z","Cred.Severity")

# Calculating claim frequency

fibar<-rowSums(C)/rowSums(B)

lambda<-sum(C)/sum(B)        ### lambda

fi_bar<-fibar*rowSums(B) , f_bar<-mean(fibar)

var_theta<-(1/length(fibar))*sum((fibar-f_bar)^2)

v.hm<-(lambda^2)*var_theta

k<-lambda/v.hm, cred.fact<-(rowSums(B))/(rowSums(B)+k)

 cred.freq.est<-cred.fact*fibar + (1-cred.fact)*lambda

 data.frame(rowSums(B), fibar, cred.fact,cred.freq.est)

 credib.F.S<-cred.freq.est*cred.sev

dat1<-data.frame(a4,xi_bar, cred.pr,credib.F.S)

colnames(dat1)<-c("company","Average.claim.cost","Cred.claim.cost",

                    "Cred.F.S.claim cost")

dat1 <- melt(dat1, id = "company")

d1<-ggplot() + geom_bar(data = dat1, aes(x = company, y = value, fill =
variable),

 position = "dodge", stat = "identity")

d2<-d1 + theme_bw() + theme(axis.text.x = element_text(angle = 45,

                  hjust = 1, size = 10))

d3<-d2 + theme(axis.text.x = element_text(angle = 45, hjust = 1, size = 10),

      axis.title=element_text(size=10),legend.text = element_text(size = 10))

d4<-d3 + geom_hline(aes(yintercept=collective.mean,color =
"Collective.mean"),

     size=0.6) + scale_color_manual(values = "black")
```

```
d5<-d3 + scale_fill_manual(name="Estimates", values = c("orange",
"blue","green"))

d5 + xlab("Companies") + ylab("Claim Cost")

dat12<-data.frame( a4,cred.pr,credib.F.S)

colnames(dat12)<-c("company", "Cred.claim.cost","Cred.F.S.claim cost")

dat12 <- melt(dat12, id = "company")

d12<-ggplot() + geom_bar(data = dat12, aes(x = company, y = value, fill =
variable),

#### test by which claim frequency follows no distribution ###

colnames(cc)<-NULL, cc1<-as.matrix(sapply(cc, as.numeric))

colnames(B)<-NULL, annual.f<-cc1/B, a5<-data.frame(rw,annual.f)

colnames(a5)<-c("company","2013","2014","2015","2016","2017","2018")

a1<-rowSums(cc1), b1<-rowSums(B), xi_bar<-a1/b1, totcc1<-sum(cc1)

totpolicy<-sum(B), xbar<-totcc1/totpolicy, Average.Claim.F<-xi_bar

xi5<-data.frame(a4, Average.Claim.F)

colnames(xi5)<-c("company","Average.Claim.F")

xij<-cc1/B, # individual claim Freq per unit of policy

x5<-data.frame(a4,xij)

colnames(x5)<-c("company","2013","2014","2015","2016","2017","2018")

x5 <- melt(x5, id = "company")

p<-ggplot() + geom_bar(data = x5, aes(x = company, y = value, fill =
variable),

position = "dodge", stat = "identity")

pr<-p + theme_bw() + geom_hline(aes(yintercept=xbar,

                    color = "collective.Claim.F"), size=0.9)

prr<-pr + xlab("companies") + ylab("Annual Claim F")  +

                  scale_color_manual(values = "black")

pprr<-prr + theme(axis.text.x = element_text(angle = 45,
```

```
         hjust = 1,size = 12,face = "bold" ),
        axis.title=element_text(size=13,face="bold"),
legend.text = element_text(size = 12,face = "bold"))
dev.xij<-(xij-xi_bar)^2, d<-B*dev.xij, si2<-rowSums(d)/(ncol(d)-1)
xxi<-data.frame(a4,xi_bar, si2 )
colnames(xxi)<-c("Marine Insurance company","Risk mean","Risk Variance")
# mean of the process variance
length(si2), mpv<-sum(si2)/length(si2)
v2ij<-(xij-xbar)^2, v2ijb<-B*v2ij
varx<-sum(v2ijb)/(length(v2ijb)-1)
# variance of the harmonic mean(tau^2) or vhm # (total variance-mpv)/p*

k<-mpv/vhm, parameter.estx<-data.frame(xbar,mpv,vhm)
z<-rowSums(B)/(rowSums(B)+k)
cred.F<-z*xi_bar+(1-z)*xbar, credib.claim.F<-cred.F
zz<-data.frame(data3,b1,z,1-z, xi_bar,cred.F)
colnames(zz)<-c("company", "Policycount","Zi","1-Zi","xi_bar","Est.Cred.F")
dat<-data.frame(a4,xi_bar,cred.F)
colnames(dat)<-c("company","Average.claim.cost","Credib.claim.F")
######################################################
# now calculating for (credibility frequency severity) credib.F.S1 claim
Estimate
#####################################
credib.F.S1<-cred.F*cred.sev
data.frame(credib.F.S1)
dat31<-data.frame(data3,Average.Claim.Cost,credib.claim.cost ,
credib.F.S,credib.F.S1)
```

```
colnames(dat31)<-c("company","Average.claim.cost","Cred.claim.cost",
"credib.F.S","Cred.F.S1")
dat31 <- melt(dat31, id = "company")
d31<-ggplot() + geom_bar(data = dat31, aes(x = company, y = value,
 fill = variable),  position = "dodge", stat = "identity")
d32<-d31 + theme_bw() + theme(axis.text.x = element_text(angle = 45,
            hjust = 1, size = 10))
d33<-d32+theme_bw() +
geom_hline(aes(yintercept=collective.Risk.mean,color
  = "collective.Risk.Mean"),size=0.9)+ scale_color_manual(values = "black")
d34<-d33 + theme(axis.text.x = element_text(angle = 45, hjust = 1, size = 10),
              axis.title=element_text(size=10),legend.text = element_text(size =
10))
d35<-d34 + scale_fill_manual(name="Estimates",
      values = c("orange", "blue","green","yellow"))
d35 + xlab("Companies") + ylab("Claim Cost")
collective.Risk.mean
######### cred clm cost and crd fs
dat12<-data.frame( data3, credib.claim.cost,credib.F.S,credib.F.S1)
colnames(dat12)<-c("company", "Cred.claim.cost","Cred.F.S","credib.F.S1")
# use melt on the dataframe to show or identify variables
# on x-axis. so that the remaining as values on y-axis
dat12 <- melt(dat12, id = "company")
d12<-ggplot() + geom_bar(data = dat12, aes(x = company,
 y = value, fill = variable), position = "dodge", stat = "identity")
d22<-d12 + theme_bw() + theme(axis.text.x = element_text
(angle = 45,  hjust = 1, size = 10))
```

```
d32<-d22 + theme(axis.text.x = element_text(angle = 45,
 hjust = 1, size = 10),axis.title=element_text(size=10),
              legend.text = element_text(size = 10))
d52<-d32 + scale_fill_manual(name="Estimates",
          values = c("blue","green","yellow"))
d52 + xlab("Companies") + ylab("Claim Cost")
```