

**SIMULATION OF GROUNDWATER FLOW IN THE VOLTAIAN
(AROUND TAMALE) USING CARBON-14**

DEPARTMENT OF NUCLEAR ENGINEERING
SCHOOL OF NUCLEAR AND ALLIED SCIENCES
UNIVERSITY OF GHANA

BY



In partial fulfillment of the requirements for the award of

MASTER OF PHILOSOPHY DEGREE

IN

COMPUTATIONAL NUCLEAR SCIENCES AND ENGINEERING

JULY, 2013

DECLARATION

This is to certify that this thesis is the result of research undertaken by Isaac Nti-Agyemang towards the award of the Master of Philosophy Degree in Computational Nuclear Sciences and Engineering in the Department of Nuclear Engineering, University of Ghana.

..... Date.....
Isaac Nti-Agyemang
(Student)

I hereby declare that the preparation of this project was supervised in accordance with the guidelines of the supervision of thesis work laid down by the University of Ghana.

.....
Date.....
Dr. T. T. Akiti
(Supervisor)

.....
Date.....
Dr. Adomako Dickson
(Co-Supervisor)

DEDICATION

To my wife and children

Mrs. Gloria Nti-Agyemang, Osei Kwaku Nti-Agyemang and Kofi Opoku Nti-Agyemang.



ACKNOWLEDGEMENT

I would like to thank first and foremost the Almighty God for his protection and care that has brought me this far.

I wish to thank, my supervisor, Dr T.T Akiti, for his dedication, insightful discussion, suggestions and criticism that really helped in bringing this work to a successful end.

I would also like to express my sincere thanks and appreciation to Dr. Adomako Dickson for his role as co-supervisor, who gave me advice and support in carrying out this research.

My special thanks go to all the lecturers in the Department of Nuclear Engineering especially Prof. Ayensu for the knowledge imparted on me and Dr H. Obiri for his help in using the Comsol Multiphysics software. I would also like to express my appreciation to my course mates and pioneers particularly Christopher Bansa for their advice and support.

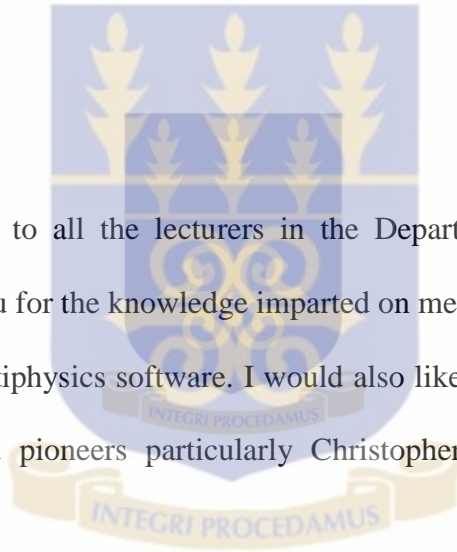


TABLE OF CONTENTS

DECLARATION	ii
DEDICATION	iii
ACKNOWLEDGEMENT	iv
TABLE OF CONTENTS	v
LIST OF TABLES	viii
LIST OF FIGURES	viii
SYMBOLS DEFINITION	xi
ABSTRACT	xii
CHAPTER ONE	1
1.1 Introduction	1
1.2 Problem Statement	3
1.3 Research Objectives	4
1.4 Scope of Work	4
1.5 Structure	5
1.6 Background of Study the Area	6
1.6.1 Location of the study area	6
1.6.2 Climate and vegetation of the study area	8
1.6.3 Geology and Hydrogeology of the study area	8
1.6.4 Hydrology of the study area	10
1.6.5 Water supply	10

CHAPTER TWO	12
LITERATURE REVIEW.....	12
2.1 Groundwater Resource.....	12
2.2 Groundwater Flow Systems.....	13
2.3 Fluxes Affecting Groundwater in the Study Area	15
2.4 Water Movement in Aquifers	18
2.5 Carbon-14	19
2.5.1 Naturally Occuring Carbon-14	19
2.5.2 Carbon-14 Introduced by Man.....	20
2.6 Carbon-14 Studies in the Voltaian.....	21
2.7. Carbon-14 Analysis in the Study of Groundwater.....	21
2.8 General background of Comsol Multiphysics.....	24
2.8.1 Modeling with Comsol Multiphysics.....	25
CHAPTER THREE	28
METHODOLOGY	28
3.1 Solute Transport in Subsurface Water	28
3.1.1 Advection.....	28
3.1.2 Diffusion	29
3.2 Conceptual Model.....	30
3.2.1 Model Definition.....	32
3.3 Mathematical Model Formulation	34
3.3.1 Governing equations	39
3.3.2 Mass Transport Modeling (Advection and Dispersion).....	41

3.4 Model Assumptions.....	42
3.5 Model Simulation Using Comsol Graphic User Interface.....	43
3.5.1 Boundary conditions.....	44
3.5.1 Model parameters.....	47
3.6 Calculation of Groundwater Flow Velocity.....	48
3.6.1 Determination of groundwater age using Carbon-14 Date.....	49
3.6.2 Calculation of groundwater flow rate using Carbon-14 Date.....	50
3.7 Model Calibration.....	51
CHAPTER FOUR.....	52
RESULTS AND DISCUSSIONS.....	52
4.1 RESULTS.....	52
4.1.1 Determining Rate of Flow from Flow Net.....	53
4.1.2 Rate of Flow Using Carbon -14 Date	54
4.2 DISCUSSIONS.....	56
4.2.1 Pattern of Groundwater Flow in the Study Area.....	56
CHAPTER FIVE	58
CONCLUSIONS AND RECOMMENDATIONS.....	58
5.1 Conclusions.....	58
5.2 Implications for Groundwater Management.....	59
5.3 Recommendations.....	60
REFERENCES.....	61
APPENDIX A Algorithm for Simulation using Comsol Graphical User Interface	67

LIST OF TABLES

Table 2.1:	Recharge estimates from some previous studies in Northern Ghana	16
Table 3.1:	Model input parameters	47
Table 3.2:	Carbon-14 sampling in the wells	49

LIST OF FIGURES

Figure 1.1:	Location map of Northern Region showing the study area	7
Figure 2.1:	Subsurface moisture zones	13
Figure 2.2:	Hydrologic section showing local, intermediate and regional groundwater flow systems.	15
Figure 2.3:	Schematic representation of the hydrologic cycle	18
Figure 3.1:	Elevation profile of the study area	30
Figure 3.2:	Model framework	31
Figure 3.3:	Hydrologic setting for groundwater flow	32
Figure 3.4:	Problem definition for the transport of solute	33
Figure 3.5:	Diagram for mathematical modeling	34
Figure 3.6:	Geometric set up of the elevation cross section of the study area using the Comsol Graphical User Interface	43
Figure 3.7:	Model in boundary mode	46

Figure 3.8:	Calculating groundwater flow rate using age of groundwater determined in two different boreholes	48
Figure 4.1:	Hydraulic head as surface plot of groundwater at Kanshegu, Yipeligu and Nawuni	52
Figure 4.2:	Flow lines and equipotential lines (Flow Net) across Kanshegu to Nawuni	53
Figure 4.3:	horizontal Flow direction using groundwater age at three locations (Kanshegu ,yipeligu and Nawuni)	55

SYMBOLS DEFINITION

Symbol	Definition
R	Vertical Recharge (m/s)
\bar{D}	Dispersion tensor (m ² /s)
$D_{ii}, D_{jj}, D_{ij}, D_{ji}$	Components of the dispersion tensor (m ² /s)
K	Hydraulic conductivity (m/s)
α_L	Longitudinal dispersivity (m ² /s)
α_T	Transverse dispersivity (m ² /s)
H	Hydraulic head (m)
hp	Pressure head (m)
$t_{\frac{1}{2}}$	Carbon-14 half-life (s)
k	Radioactive decay coefficient (s ⁻¹)
t	Time (s)
u	Darcy's velocity (m/s)
\bar{F}_a	Advective flux (Kg/m ² s)
C	Carbon-14 concentration (pmC)
\bar{K}	Hydraulic conductivity tensor
∇h	Hydraulic head gradient
\bar{q}	Specific discharge
Pe	Peclet number
D'_d	Effective molecular diffusion coefficient (m ² /s)
x, y, z	Spatial distance

\overline{F}_d	Diffusive flux (Kg/m ² s)
∇C	Concentration gradient
∇^2	Laplacian
S_s	Specific storage (m ⁻¹)
ρ	fluid density (kg/m ³)
κ	permeability (m ²)
η	fluid viscosity.

ABSTRACT

Studying the process of groundwater flow in subsurface systems using numerical simulation has been widely practiced. The purpose of this study was to establish a 2D groundwater flow model for evaluating groundwater resources of the Voltaian Basin (around Tamale) in the Northern Region of Ghana. To understand the rate of abstraction of groundwater in the study area, a finite-element, steady-state groundwater flow model was used to simulate groundwater flow in the aquifer. COMSOL Multiphysics' (FEMLAB) Earth Science Module (ESM) package which is finite element analysis and solver software was used. The radioisotope used in the study was Carbon-14. Three wells were sampled for Carbon-14 concentration and used for the model verification, based on elevation.

From the results, groundwater in the study area moves generally from higher to lower hydraulic head along paths perpendicular to the equipotential lines. The groundwater flow paths in the aquifer in the study area indicated that flow is predominantly regional.

There was a regional groundwater flow from Kanshegu to Nawuni. Kanshegu appears to be recharge area and Nawuni as discharge area. The flow rate obtained using Carbon-14 date was 2.86×10^{-7} m/s. The overall flow rate obtained from the model simulations was 2.66×10^{-7} m/s with an error margin of 6%.

CHAPTER ONE

1.1 INTRODUCTION

Water is a precious natural resource, without which there would be no life on Earth. Human beings are composed of two-thirds water by bodyweight (Wikipedia- composition of the Human body). Our daily activities depend on the availability of inexpensive, clean water and safe ways to dispose of it after use. Water is also essential in supporting food production and industrial activity. The demand for freshwater has increased worldwide due to rapid population growth. A UNEP (2002) study observes that, with the current projected human population growth, industrial development and expansion of irrigated agriculture in the next two decades will increase water demand to levels that will make the task of providing water for human sustenance more difficult. Africa in general and West Africa in particular will be among the regions that are most likely to be affected (UNEP, 2002).

Statistical analysis of long-term (1951-1991) observed rainfall and river discharge data in the Volta Basin of West Africa show significant reductions in both rainfall and river discharge in most areas of the Basin (Opoku-Ankomah and Amisigo, 1996; Opoku-Ankomah, 2000; Gyau-Boakye and Tumbulto, 2006). Similar analysis of observed rainfall from the 1960s and 1970s in other areas in West Africa found significant reductions in rainfall amounts and river discharges (Servat *et al.*, 1997, Lebarbé and Lebel, 1997; Amani, 2001). Observations made

on major rivers in the West African region since the early 1970s show a mean decline of 40-60 % in discharge (Niasse *et al.*, 2004). Recharge to groundwater aquifers in the region have also decreased noticeably. This is attributed to the decline in rainfall and runoff (Niasse *et al.*, 2004). As a result, surface water supplies will be increasingly unreliable and insufficient to meet the water demands for socio-economic development in many places in the basin.

Groundwater modeling is a tool that can help analyze many groundwater problems. It is good as a management tool and also useful for investigation studies to determine the groundwater flow conditions. In order to understand the groundwater flow conditions in the Voltaian sediments, it is necessary to construct a mathematical model to define groundwater flow systems and also use isotope data to validate the model.

1.2 PROBLEM STATEMENT:

The Northern Region of Ghana is a particularly difficult place to find groundwater. The region is largely underlain by sedimentary rocks of the Voltaian Basin, which vary from sandstone, to mudstone with interlayered siltstone (MacDonald and Davies, 2010). Unsuccessful boreholes have been drilled throughout the region, but are particularly common in areas that are underlain by mudstones, which are generally poorly fractured. The rocks have low permeability (MacDonald and Davies, 2010). The porosity is not widely studied but is expected to be low, secondary porosities such as fractures have not been studied in the past to allow for hydrogeologic understanding of the Voltaian System. The rate of movement of groundwater within the Voltaian sediments is not well studied.

Selected academic theses detailing hydrogeological research related to the Northern Region are available, including Teeuw (1995), Pelig-Ba (2004) and Ewusi (2006). Other research has looked in more detail at similar Voltaian Basin sedimentary rocks to the south (e.g. Acheampong and Hess 1998). This work is an attempt to use nuclear techniques and modeling to study the groundwater flow rate in the study area.

1.3 RESEARCH OBJECTIVES:

The general objectives are to use mathematical model and Carbon-14 to study the groundwater flow rate in the Volta Basin in Northern Region Ghana.

The specific objectives of this thesis are to:

1. Evaluate the use of carbon-14 data to study the groundwater flow rate of large aquifers,
2. To solve the transport equation analytically and
3. To determine groundwater flow rate from Carbon-14 data,

1.4 SCOPE OF WORK

The thesis was conducted in the Northern Region of Ghana in the area around Tamale which forms part of the Volta Basin. The Carbon-14 isotope is used in this study due to its long half-life. The study considered groundwater flow only in the saturated zone. Three boreholes were used to construct the line of section for the model flow through the aquifer. The software used for the simulation was COMSOL Multiphysics (3.4) and the Earth Science Module (ESM) was used.

1.5 STRUCTURE

This study is divided into five chapters. Chapter one introduces the importance of water and the significance of the study per the problem statement and the main objectives for this thesis, and finally outlines some background information relating to the location, climate, hydrogeology, topography and water supply of the study area. Chapter two contains the literature review. This chapter discusses groundwater flow systems, some of the relevant references in groundwater studies and isotope hydrology from literature as well as similar work previously done elsewhere in Ghana. Chapter three outlines the description of the procedures followed to accomplish the modeling task. Detail Mathematical equations governing the transport of Carbon-14 in subsurface water are discussed. Additionally procedures used to obtain results are presented. Chapter four gives information on the main results obtained. Finally, chapter five summarizes the results obtained and gives general conclusions and recommendations.

1.6 BACKGROUND OF THE STUDY

1.6.1 Location of Study Area

The study area is a small part of the White Volta Basin located around Tamale in the Northern Region of Ghana. It is located between longitudes $0^{\circ}33'30''$ W and $1^{\circ}40'W$ and latitude $9^{\circ}7'30''N$ and $9^{\circ}38'0''$ and covers an area of 3.99×10^3 square Kilometers. Figure 1.2 shows the location map of Northern Region with the study area marked in red.

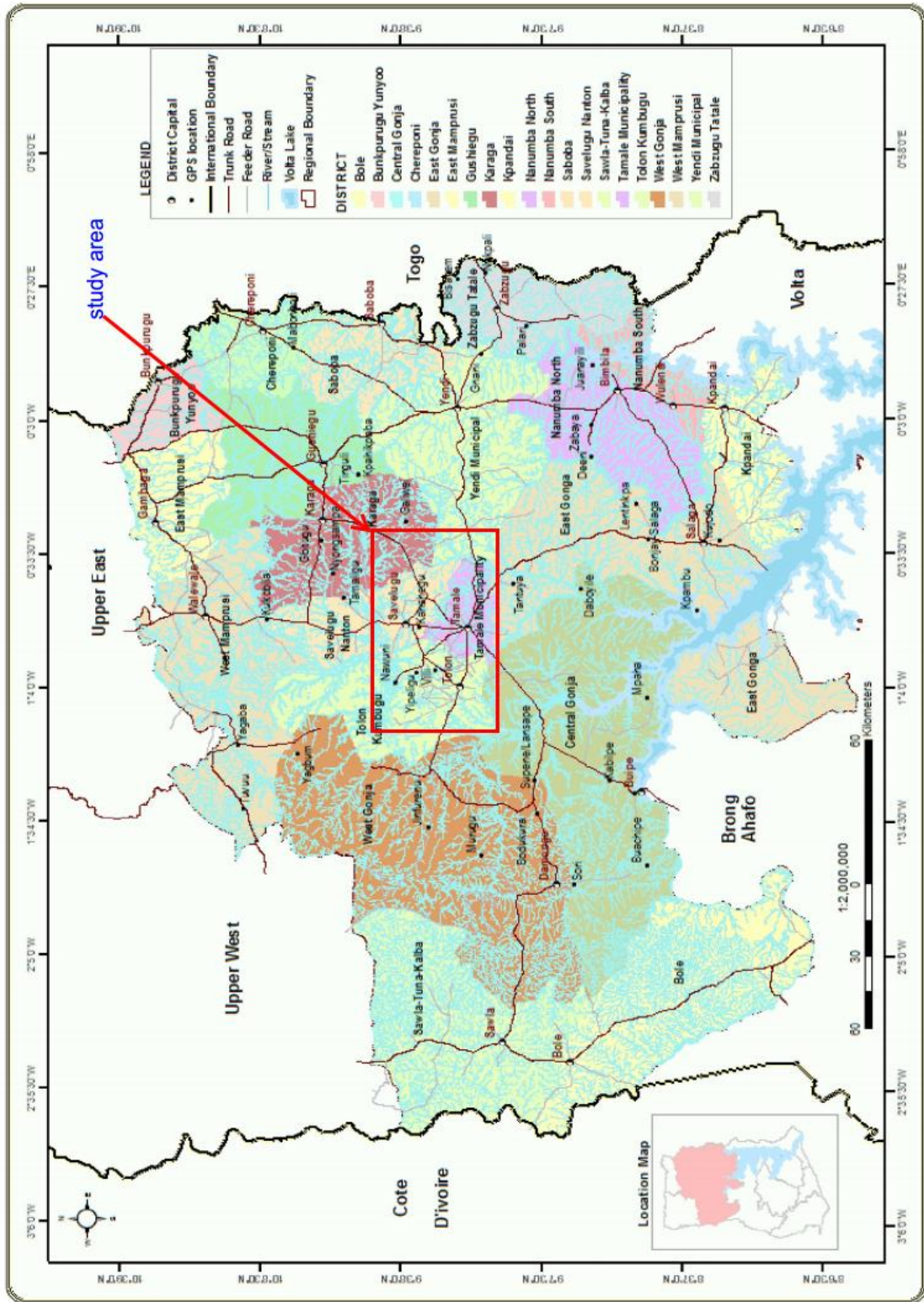


Figure 1.1 location map of Northern Region showing the study area.

1.6.2 Climate and Vegetation of the Study Area

The climate of the study area is tropical and semi-arid. The mean annual rainfall is 1050 mm, while potential evapotranspiration is about 1770 mm (Pelig-Ba, 2000). There are only about 4 to 6 months of rainfall during the year, from May to October. The rest, from November to May, is relatively dry and hot (Pelig-Ba, 2000). After the raining period, the Northeast Trade Winds, locally termed the harmattan, prevails up to February/March. During the period, the weather becomes very dry and hot in the day but cools to less than 20°C in the night. The area is basically guinea savannah woodland with the main tree species being acacia, baobab, and shea nut trees and it is thickly grassed (Unihydro, 2002).

1.6.3 Geology and Hydrogeology of the Study Area.

The area is underlain by consolidated sedimentary rocks locally referred to as the Voltaian formation. The Voltaian system is highly heterogeneous consisting of mudstones, sandstones, conglomerate, sandy and pebbly beds and limestones (Dapaah-Siakwan and Gyau Boakye, 2000). The groundwater fortunes of this topography have broadly been studied by Junner and Hirst (1946), Gill (1968), Buckley (1986), Bannerman (1988), Minor *et al* (1995) and Acheampong (1996). The rocks of the basement complex and the Voltaian formation have little or no

primary porosity. Groundwater occurrence is thus associated with the development of secondary porosity as a result of jointing, shearing, fracturing and weathering (WRC).

This has given rise to two main types of aquifers; the weathered zone aquifer and the fractured zone aquifers (WRC). These aquifers are either phreatic semi-confined or confined depending on the clay and mica content of the upper weathered layer (Kortatsi, 1997). The fractured zone aquifers usually occur at some depth beneath the weathered zone. Both type of aquifer are normally discontinuous and limited in area. Due to the sandy clay nature of the weathered overburden, the groundwater occurs mostly under semi-confined or leaky conditions (Ministry of Works and Housing, 1998).

Groundwater occurrence in the rock formation in the Voltaian system is linked with the development of secondary porosity (WARM, 1998). Fractured rock aquifer of the Voltaian province generally have a low to moderate transmissivity ranging from $0.3 \text{ m}^2/\text{d}$ to $267 \text{ m}^2/\text{d}$ with an average of $11.9 \text{ m}^2/\text{d}$ (Darko and Krasny, 2003). The existing data reveals that borehole yields range from $0.3 \text{ m}^3/\text{h}$ to $72 \text{ m}^3/\text{h}$ with a mean yield of $7.3 \text{ m}^3/\text{h}$ (Acheampong and Hess, 1998).

1.6.4 Hydrology of the Study Area

The area is essentially flat. Ground elevations range between 120 m and 180 m above mean sea level. The White Volta basin drains the area. Apart from the White Volta, some of the rivers that drain the area are only seasonal and become ponded or dry up completely during the long dry season. There are two main mechanisms of groundwater recharge, generally percolation of soil water, which moves past the plants root zone and percolation from the beds of the rivers in the area.

1.6.5 Water Supply

The high temperatures resulting in high evapotranspiration make surface water body sources not reliable in the area (Gyau-Boakye and Dapaah-Siakwan, 1999). Most surface water sources dry up during the long dry season periods that are normally associated with high temperatures. The Nawuni River, a tributary of the White Volta the main source of drinking water for the Tamale Metropolis in the Northern Region of Ghana is under serious threat. Its depth has reduced drastically due to siltation which has undermined its water holding capacity. This has resulted in repeated flooding along its banks during the rainy season which sometimes lead to loss of life and property in the area and also reduces the amount of water the river feeds into the Akosombo Dam, the country's major supplier of electricity. The siltation of the river as a result of sand winning

activities along its banks poses a great danger to the metropolis, threatening the river's future capacity for supplying the required volume of water to the metropolis.

The additional treatment cost of surface water bodies also makes groundwater more favorable in the area. The efficient utilization of groundwater and efficient management of aquifers, hand-dug wells and boreholes therefore appear to be the solution to problems of water supply systems (Gyau-Boakye and Dapaah-Siakwan, 1999).

CHAPTER TWO

LITERATURE REVIEW

2.1 GROUNDWATER RESOURCE

Groundwater is a term used to denote water found beneath ground surface, as part of the hydrological cycle. However, often this definition is reserved for the term subsurface water, while the term groundwater is associated primarily with that part of the water in the hydrological cycle that occurs only in the zone of saturation. Most groundwater hydrologists employ the latter definition, although water in the unsaturated zone is an important part of the hydrological cycle. Subsurface water is the sum of soil moisture (above the water table) and groundwater (below it). The water table is that subsurface horizon upon which the pore water pressure at any given point in time exactly equals atmospheric pressure.

Most groundwater occurs in the small openings in pores or fractures. Todd (1959) traces the term 'aquifer' to its Latin origin: *aqui* comes from *aqua*, meaning "water", and *fer*, from *ferre*, meaning "to bear". Thus, an aquifer is a term used for a porous geological formation that (1) contains water at full saturation (i.e., the entire interconnected void space is filled with water), and (2) permits water to move through it under ordinary field conditions. This means that whether a geological formation can be designated as an aquifer, or not, depends on its ability to store and transmit water relative to other formations in the vicinity.

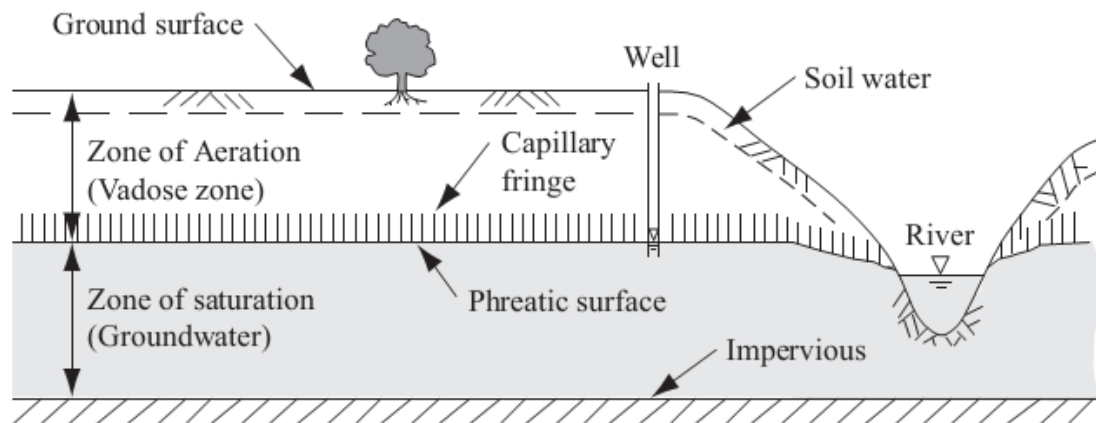


Figure 2.1: Subsurface moisture zones (after Bear, 1979).

Groundwater can be extracted through a well drilled into the aquifer. Groundwater supplies are replenished, or recharged, by precipitation. Sometimes groundwater is polluted by human activities. In areas where material above the aquifer is permeable, pollutants can readily sink into groundwater supplies. Groundwater can be polluted by landfills, septic tanks, leaky underground gas tanks, and from overuse of fertilizers and pesticides.

2.2 GROUNDWATER FLOW SYSTEMS

The direction which groundwater takes to a discharge point is known as a *flow line*. A set of flow lines with common recharge and discharge areas is termed a groundwater flow system. Groundwater possesses energy mainly by virtue of its elevation (gravitational head) and its pressure (pressure head) (Jacob Bear and Alexander Cheng, 2009). Groundwater can also possess kinetic energy by virtue of its movement. The kinetic energy due to groundwater is negligible because of groundwater's low velocities (Jacob Bear and Alexander Cheng, 2009). Groundwater moves from regions of higher energy to regions of lower energy. A measure of

groundwater's energy is the level at which the water stands in a borehole drilled into an aquifer and measured with reference to an (arbitrary) reference level. This height at which water stands above a reference datum is called hydraulic head. The hydraulic head is composed of the sum of the pressure head and gravitational or elevation head. Both of these component forms of energy are known as potential energy (Kevin, 2005).

The change in hydraulic head over a distance along the groundwater flow path is called hydraulic gradient and constitutes the driving force for groundwater movement (Hubert 1940). Darcy's law describes the flow of groundwater through an aquifer. The groundwater flow rate is directly proportional to the cross sectional area through which flow is occurring and directly proportional to the hydraulic gradient (Davis and Dewiest, 1966). Naturally groundwater moves down gradient until it reaches the land surface through a spring or the root zone where it is evapotranspired to the atmosphere. A groundwater flow pattern is controlled by the configuration of the water table and by the distribution of hydraulic conductivity in the rocks. The water table in turn is affected by the topography. The flow pattern is a function of *topography, geology and climate*. These three parameters have been termed the *hydrogeologic environment*. Three distinct types of flow system have been recognized. (Toth,1962,1963).

1. A local system that has its recharge area at a topographic high and its discharge area at the immediately adjacent topographic low.
2. An intermediate system, characterized by one or more topographic highs and lows located between the recharge and discharge areas.

3. A regional system that has its recharge area at the major topographic high and its discharge area at the bottom of the basin. Regional flow systems are at the top of this hierarchical organization. All other flow systems are nested within them.

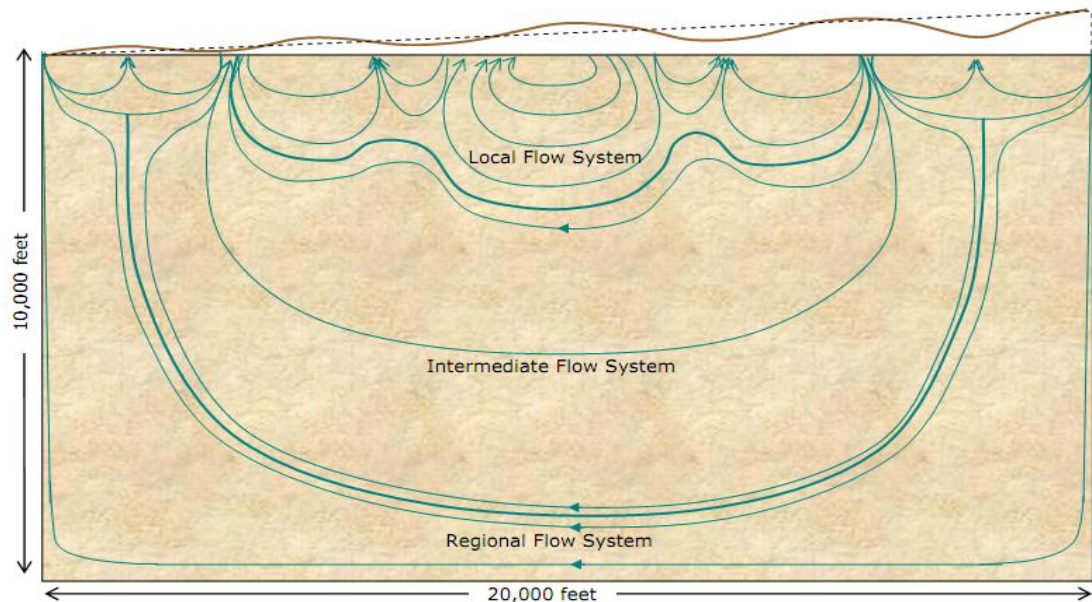


Figure 2.2 Hydrologic section showing local, intermediate and regional groundwater flow systems (Toth, 1962, 1963).

2.3 FLUXES AFFECTING GROUNDWATER IN THE STUDY AREA.

Most of the water reaching the Earth's surface that does not evaporate will return to the ocean directly through surface water runoff. The portion of water remaining will infiltrate the subsurface where it may evaporate, become temporarily stored in the soil, flow to streams prior to reaching the water table (a process termed "interflow"), or eventually recharge the groundwater reservoir by reaching the water table (Fitts, 2002), (Figure 2.2). Natural recharge occurs without influence or enhancement by humans, while artificial recharge occurs as the result of deliberate human activity, such as the direct injection of water into the subsurface or irrigation (Fitts, 2002).

In arid- and semi-arid- regions of the world, which include the study area, recharge by downward flow of water through the unsaturated zone is generally the most important mode of recharge (Xu and Beekman, 2003). Hence, groundwater recharge as used in this study refers to the downward flow of water reaching the water table from the unsaturated zone (Freeze and Cherry, 1979; Lerner *et al.*, 1990). Recharge may occur naturally from rainfall, rivers, and man-induced through activities of irrigation and urbanization (Lerner *et al.*, 1990). Recharge from rainfall, is the most important category of recharge in the study area (Martin, 2005; Kortatsi, 1994). The recharge to groundwater is controlled by factors such as rainfall amount or irrigation, the antecedent moisture condition of the soil profile, geology, soil properties, the depth to water table and aquifer properties, vegetation and land use, topography and landform (Lerner *et al.*, 1990). Recharge can be expressed as a percentage of the annual precipitation or as an average rate of water in millimeters per year.

Table 2.1. Represent recharge estimates from some previous studies in north of Ghana.

Region	Annual rainfall(mm)	Estimated Recharge (% of rainfall)	Reference
UER	1232	8	Acheampong (1988)
UER	910 - 1138	4 - 13	Martin (2005); Martin and van de Giesen (2005)
Northern Ghana	1002	5	Friesen <i>et al.</i> (2005)
UER	824 - 1294	7 - 8	Obuobie (2008)
Northern Ghana	990	1.5 - 15.9	Carrier (2008)

Discharge represents the upward outflow of groundwater from the subsurface that occurs naturally or as the result of human activity, notably well pumping. Groundwater discharges naturally from the subsurface to lakes, rivers, and other wetlands. Groundwater may also return to the atmosphere directly by evaporation within the soil and by transpiration through vegetation, but these processes are not formally considered as discharge. Without human intervention discharge rates often approximate to recharge rates, and hence a balance or equilibrium is achieved between influxes and out fluxes to the subsurface water budget. However, “overdrafts” resulting from human exploitation of groundwater resources and long period of drought can lead to declining water levels (thereby raising pumping costs), hydrostatic pressure reductions, and other related problems such as land subsidence.

The hydrological cycle can be quantified in its simplest form using the following mass balance expression:

$$\text{Rainfall amount} = \text{Potential Evapotranspiration} + \text{Runoff} + \text{Infiltration} \quad (2.1)$$

This is a steady state equation that implies no water is held in storage on the Earth’s surface, and that the influx is equivalent to the out flux. Therefore, runoff and infiltration are interlinked and are often not independent variables. Evapotranspiration rates act as a major control upon runoff and infiltration, and ultimately upon recharge.

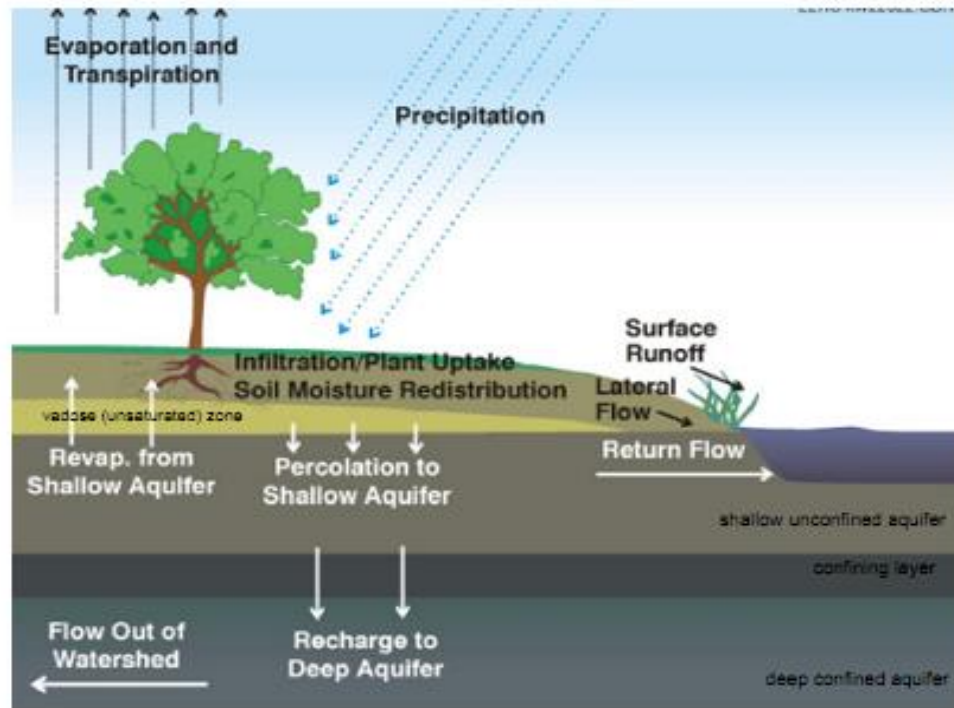


Figure 2.3 Schematic representation of the hydrologic cycle (EERC-University of North Dakota, 2008, modified from Neitsch *et al.*, 2005).

2.4 WATER MOVEMENT IN AQUIFERS:

The ease with which water can flow through a ground formation under a hydraulic head is the hydraulic permeability which is expressed as the velocity of flow of water through the ground per unit of hydraulic gradient, e.g. mm/sec, m/day. Permeability is a measure of the degree to which the pore spaces are interconnected, and the size of the interconnections. Low porosity usually results in low permeability, but high porosity does not necessarily imply high permeability. It is possible to have a highly porous rock with little or no interconnections between pores. Permeability depends on the porosity, the average pore size and the distribution of the fissures. Ground layers with a very low hydraulic permeability (less than 10^{-6} mm/sec) are said to be impermeable and those with higher hydraulic permeability are regarded as permeable (Fitts, 2002).

In some permeable materials groundwater may move several meters in a day; in other places, it moves only a few centimeters in a century. Groundwater moves very slowly through relatively impermeable materials such as clay and shale. After entering the saturated zone, water moves slowly toward lower lying places and eventually is discharged from the saturated zone from springs, seeps into streams, or is withdrawn from the ground by wells. Groundwater in aquifers between layers of poorly permeable rock, such as clay or shale, may be confined under pressure. If such a confined aquifer is tapped by a well, water will rise above the top of the aquifer and may even flow from the well onto the land surface. Water confined in this way is said to be under artesian pressure, and the aquifer is called an artesian aquifer.

2.5 CARBON-14

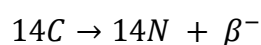
2.5.1 NATURALLY OCCURRING CARBON-14

^{14}C is produced naturally from atmospheric nitrogen according to the following equation:



(Mackay, 1961; Mak *et al.*, 1999) where the ^{14}C oxidizes to ^{14}CO and then $^{14}\text{CO}_2$ and mixes with the atmosphere. $^{14}\text{CO}_2$ is absorbed by plants during photosynthesis and becomes incorporated into the Earth's biological and hydrological cycles. The atmospheric mixing ratio of ^{14}CO is balanced by the rate of cosmogenic production, uptake by the various carbon reservoirs on Earth (atmosphere, oceans, marine organisms, plants) and radioactive decay.

Carbon-14 decays according to:



with a generally accepted half-life of $5,730 \pm 40$ years (Godwin, 1962). (Chiu *et al.*, 2007) review the various measurements of the ^{14}C half-life and suggest that the modern half-life may be underestimated by approximately 300 years.

Unlike tritium, ^{14}C concentration does not show any trend with latitude. However, the ^{14}C concentration of the Southern Hemisphere is 4 to 5% less than the Northern Hemisphere. This is due to the CO_2 exchange with the larger ocean surface in the Southern Hemisphere. Over time, equilibrium has been reached between ^{14}C production and ^{14}C depletion due to radioactive decay. The equilibrium concentration of ^{14}C has been defined as 100% modern carbon (pmc). This equilibrium represents about 13 disintegrations per minute per gram of Carbon and is also defined as the activity of ^{14}C before the industrial revolution (Libby 1955).

2.5.2 CARBON-14 INTRODUCED BY MAN

The ^{14}C concentration in the atmosphere has both been increased and decreased due to human activities (Mayo, 1991). During the Industrial Revolution the burning of fossil fuels caused the ^{14}C concentration in the atmosphere to decrease. The ^{14}C content of fossil fuels is essentially zero (Mayo, 1991). When humans began large scale burning of fossil fuels, the amount of carbon in the atmosphere was increased (Mayo, 1991). However, the carbon introduced at this time into the atmosphere contained no ^{14}C . Therefore, the percent of ^{14}C in the atmosphere decreased (Mayo, 1991).

2.6 CARBON- 14 STUDIES IN THE VOLTAIAN

Various studies on isotopic application in West Africa, in particular, include Edmunds & Wright (1979), Sonntag *et al.* (1979) Edmund &Walton (1980), Fontes *et al.* (1986), Gasse *et al.* (1987) and many others. Pioneering work in Ghana on isotope hydrology is attributed to Akiti (1980). Akiti (1987) also applied environmental isotopes such as ^2H , ^{18}O , ^3H (tritium) and ^{14}C to study groundwater in the foothills of the Accra Plains, Upper Regions and the Ketu Basin. Pelig-Ba *et al.* (1990) used ^2H and ^{18}O for waters in the Upper East and Upper West Regions and the Accra Plains while Kortatsi and Sekpe (1994) did a similar study using the same stable isotopes for groundwater in the two Upper Regions of Ghana.

2.7. CARBON-14 ANALYSIS IN THE STUDY OF GROUNDWATER.

The application of conventional radiocarbon dating using the inorganic carbon contained in speleothems was first proposed by Franke (1951). Soon afterwards Deevey *et al.* (1954) reported a significant reduction in the ^{14}C specific activity of the dissolved inorganic carbon (DIC) compounds in lake water due to the participation of geologically old carbonate where this occurs within the catchment—the so-called “hard water effect” or “reservoir effect” as was later coined by Olsson (1980).The direct dating of groundwater based on the measured ^{14}C activity was introduced by Muennich (1957, 1968). However, it soon became evident that the initial ^{14}C activity (C_0) imparted during groundwater recharge is determined in large part by the ongoing hydrogeochemical mass balance reactions towards establishment of the carbonate/ CO_2 equilibrium state. In general, therefore, the appropriate (C_0) value used to determine a ^{14}C age for the DIC tends to be set at less than 100% modern carbon

(pMC) i.e., the theoretical equivalent to time zero on the conventional ^{14}C time scale. By international convention the ^{14}C age (t) in years BP (before AD 1950) of groundwater DIC is calculated from its measured ^{14}C activity (C) and according to the relationship

$$t = \frac{\tau}{\ln\left(\frac{C_0}{C}\right)} \quad (2.3)$$

Where τ is set at either the Libby half-life of ^{14}C (5570 yrs.) or the physical half-life of 5730 yrs. In addition to any initial uncertainty over the actual ^{14}C activity (C_0), the apparent decrease in ^{14}C activity due to radioactive decay during subsequent storage in the aquifer can also be distorted by the superimposed influence of hydrochemical reactions, physical processes, and geo-hydraulic mixing. Consequently, the great challenge in applied groundwater dating is to resolve and quantify the true age controlled effect of radioactive decay from other possible influences on the measured ^{14}C activity.

The simplest radioactive groundwater dating model assumes an ideal tracer travelling through the aquifer in a piston flow (convective) fashion. The basic equation describing the rate at which a radioactive substance decays is given by:

$$\frac{dN}{dt} = -\lambda N \quad (2.4)$$

where N is number of atoms of radioactive element, λ is decay constant, t is the time.

Integrating equation (2.4) subject to the initial condition $N = N_0$ at $t = 0$ and rearranging the parameters, yields

$$N = N_0 e^{-\lambda t} \quad (2.5)$$

Equation (2.5) may be expressed in terms of concentration rather than number of atoms by

$$C = C_0 e^{-\lambda t} \quad (2.6)$$

where C is concentration (mass per volume) and Co initial concentration.

Equation 2.6 can also be expressed in terms of activity by

$$A = A_0 e^{-\lambda t} \quad (2.7)$$

where A and Ao are initial and final activities (disintegrations per time per unit total mass).

The solution of equation (2.7) is given by

$$t = -\frac{1}{\lambda} \ln \left(\frac{A}{A_0} \right) \quad (2.8)$$

Subject to the above assumptions, the elapsed time is equivalent to the time since recharge occurred. This is termed the groundwater age.

Equation (2.8) can be expressed in terms of concentrations as:

$$t(\text{age}) = -\frac{1}{\lambda} \ln \left(\frac{C}{C_0} \right) \quad (2.9)$$

Hence the groundwater age using carbon -14 can be written as:

$$t(\text{age}) = -\frac{5730}{\ln(2)} \times \ln \left(\frac{C}{C_0} \right) \quad (2.10)$$

where :

t – time

5730 – Half – life of carbon – 14

C – current carbon – 14 activity or concentration

C_0 – initial carbon – 14 activity or concentration at $t = 1$

The current ^{14}C activity is measured, the initial ^{14}C activity is estimated, and then the time is computed. The initial ^{14}C is assumed to be 110 pmC due to the complete transformation of soil CO_2 into total dissolved inorganic carbon (TDIC). Such a process was proposed to explain values for ^{13}C and ^{14}C of TDIC in groundwater from Northern Ghana (Akiti, 1980).

\

2.8 GENERAL BACKGROUND OF COMSOL MULTIPHYSICS.

This section provides an introduction to the software program used for simulating the groundwater flow for the study area and an overview of its capabilities. COMSOL MultiPhysics, formerly known as FEMLAB, is a computer software that makes it possible to numerically solve partial differential equations. The numerical solution relies on the Finite Element Method (FEM), in which the geometry to be studied is divided into a finite element mesh. Thus, instead of trying to solve a highly non-linear problem on the entire geometry, an approximate solution is sought in each element. If this element is considerably small the physical problem is assumed to vary linearly (FEMLAB, 2004)

COMSOL Multiphysics offer a complete environment that allows one to perform all the steps in the modeling process. Its graphical user interface handles CAD (Computer-aided design) modeling, the import of drawings and images, physics and equation definition, mesh generation, equation solving, visualization, and post processing. COMSOL has developed a module (i.e, the Earth Science Module) that is

helpful for fluid flow applications. The Earth Science Module is a package of COMSOL Multiphysics for simulation of subsurface flow and other applications in earth science. The fluid-flow equations in the earth science application mode represent a wide range of possibilities which includes:

- Navier-Stokes equations: for surface and other free flows.
- Richards' equation: describes nonlinear flow in variably saturated porous media.
- Darcy's law: for steady flow in, saturated porous media
- The Brinkman equations: describe fast flow in porous media where shear is non negligible.

COMSOL provides generic partial differential equation (PDE) solvers that are robust in handling coupled equations. This group of application modes estimates the hydraulic head flowing in free systems and within the interstices of a porous medium (COMSOL Multiphysics, 2008).

2.8.1 MODELING WITH COMSOL MULTIPHYSICS

From a methodological point of view groundwater modeling can be subdivided into flow and transport modeling. Flow modeling deals with the movement of water, whereas transport modeling is concerned with the distribution and migration of solutes or heat in the subsurface. Flow modeling is relevant for the management of water supply systems and supervision of hydraulic works. Transport modeling is mainly applied to problems in which groundwater quality is of concern. Advanced

features of groundwater modeling are reactive transport modeling and the consideration of density-driven flow due to changes of salinity and temperature.

The Multiphysics concept, on which the software is built, is best suited for the modeling of hydrogeological systems, in which flow and transport are connected or coupled. The relevant differential equations, implemented in the COMSOL Earth Science Module (ESM), are the following:

$$\nabla \left(\frac{K}{\rho g} \nabla (p + \rho g z) \right) = Q \quad (2.11)$$

Equation (2.11) is a differential equation in 1D, 2D or 3D. K denotes hydraulic conductivity, ρ fluid density, g the gravitational constant, and z the coordinate in vertical direction. Q is a general source and sink-term with the physical unit [1/s].

An alternative formulation offered by COMSOL ESM is given using permeability κ (physical unit [m^2]) instead of conductivity:

$$\nabla \left(\frac{\kappa}{\eta} \nabla (p + \rho g z) \right) = Q \quad (2.12)$$

where η denotes fluid viscosity.

An important variable in both formulations (2.11) and (2.12) is hydraulic head \mathbf{h} , defined by:

$$h = \frac{p}{\rho g} + z \quad (2.13)$$

In terms of h the differential equation (2.11) can also be re-written in the compact form:

$$\nabla(K\nabla h) = Q \quad (2.14)$$

A special case is given for 1D and 2D models in a horizontal cross-section, for which the differential equation (3.29) transforms to the formulation:

$$\nabla(KH\nabla h) = Q \quad (2.15)$$

with variable aquifer depth H . Q has the unit [m/s].

CHAPTER THREE

METHODOLOGY

3.1 SOLUTE TRANSPORT IN SUBSURFACE WATER

The mass conservation statement for dissolved species in groundwater system is defined by (Freeze and Cherry, 1979) as:

$$\left[\begin{array}{c} \text{Net rate of change} \\ \text{of tracer} \\ \text{mass within} \\ \text{the system} \end{array} \right] = \left[\begin{array}{c} \text{flux of tracer} \\ \text{out of the system} \end{array} \right] - \left[\begin{array}{c} \text{flux of tracer} \\ \text{into the system} \end{array} \right] + \left[\begin{array}{c} \text{loss or gain} \\ \text{of tracer} \\ \text{mass due} \\ \text{to reaction} \end{array} \right] \quad (3.1)$$

The physical processes that control the tracer flux into and out of the groundwater system are advection, mechanical dispersion and molecular diffusion. Loss and gain of tracer mass in the system are attributed to the chemical, biological and radiological reactions. The basic concepts related to advection, mechanical dispersion, and molecular diffusion are presented in the following sections.

3.1.1 ADVECTION

Advection is the mechanical transport of solutes along with the bulk flux of the water. It is driven by the total mechanical energy of the solution, just as the water flux is driven. Advective fluxes are simply the product of the water flux based on Darcy's law (1856) with the solute concentration:

$$\overline{F}_a = C\overline{K}\nabla h = C\overline{q} \quad (3.2)$$

where \overline{F}_a is the advective mass flux, C the volume concentration, \overline{K} the hydraulic conductivity tensor, ∇h the hydraulic head gradient, and \overline{q} the specific discharge.

Advection transports all solutes at the same rate.

3.1.2 DIFFUSION

Diffusion differs fundamentally from advection in that it is entropy driven, following the principle that solutes within a system will redistribute themselves so as to maximize the entropy of the system. Diffusive transport is therefore, governed by concentration gradients:

$$\overline{F}_d = D'_d \nabla C \quad (3.3)$$

Where \overline{F}_d is the diffusive flux, D'_d the effective diffusion coefficient (accounting for the effects of porosity and tortuosity), and ∇C the concentration gradient. Since the concentration distributions of various solutes in a single system need not be identical, different solutes generally have different diffusive fluxes. The relative importance of advection and diffusion is of fundamental significance for understanding solute transport in various settings. This can be assessed using the **Peclet** number, as described by (Perkins and Johnston, 1963) as;

$$P_e = \frac{q_s d_m}{D'_d} \quad (3.4)$$

Where q_s is the average linear velocity of the water, d_m the mean grain diameter, and D'_d the effective diffusion coefficient. For values of P_e less than 1, diffusion dominates, but above this value advection dominates (Perkins and Johnston, 1963). Application of this equation to the various flow systems described above indicates that for typical local or aquifer-scale systems, advection dominates decisively over diffusion. However, at the very slow flow velocities, characteristic of the deeper portions of regional flow systems, diffusion may become dominant. Diffusion may also dominate in shallower flow systems where permeabilities are very low or in

unusual circumstances where hydraulic gradients are negligible. In general, diffusion is an effective mechanism for redistributing solutes over short distances under most circumstances, but is a significant transport mechanism at aquifer or regional scales only when the time for transport is long.

3.2 THE CONCEPTUAL MODEL

An elevation profile of the study area was drawn using the topographic maps obtained from the Survey Department of Ghana as shown in figure 3.1

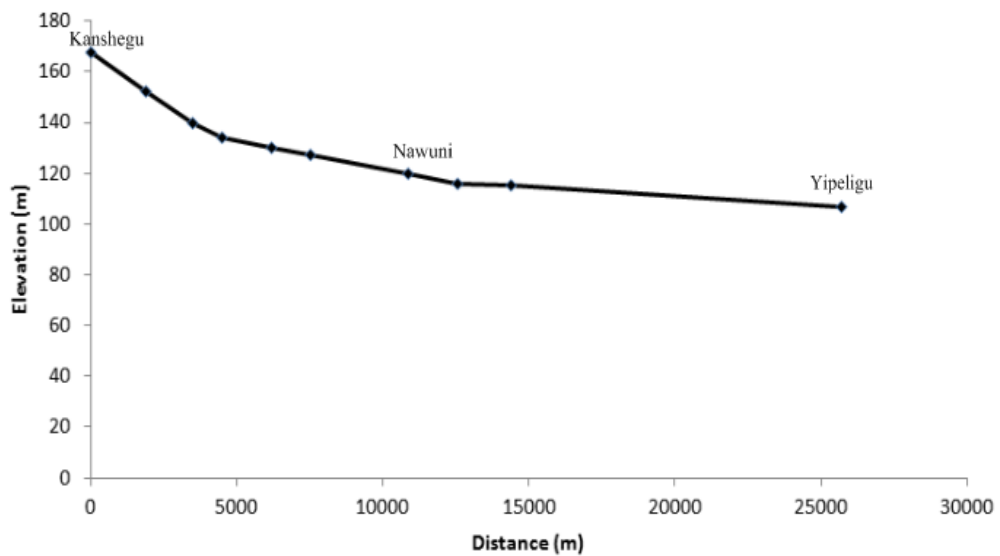


Figure 3.1 Elevation profile of the study area

The system is simplified into a two-dimensional conceptual model as shown in Figure 3.2. Groundwater enters the upper surface of the saturated zone to the outlet. The total length of the cross section is 25700 m and the maximum elevation is 167.6 m.

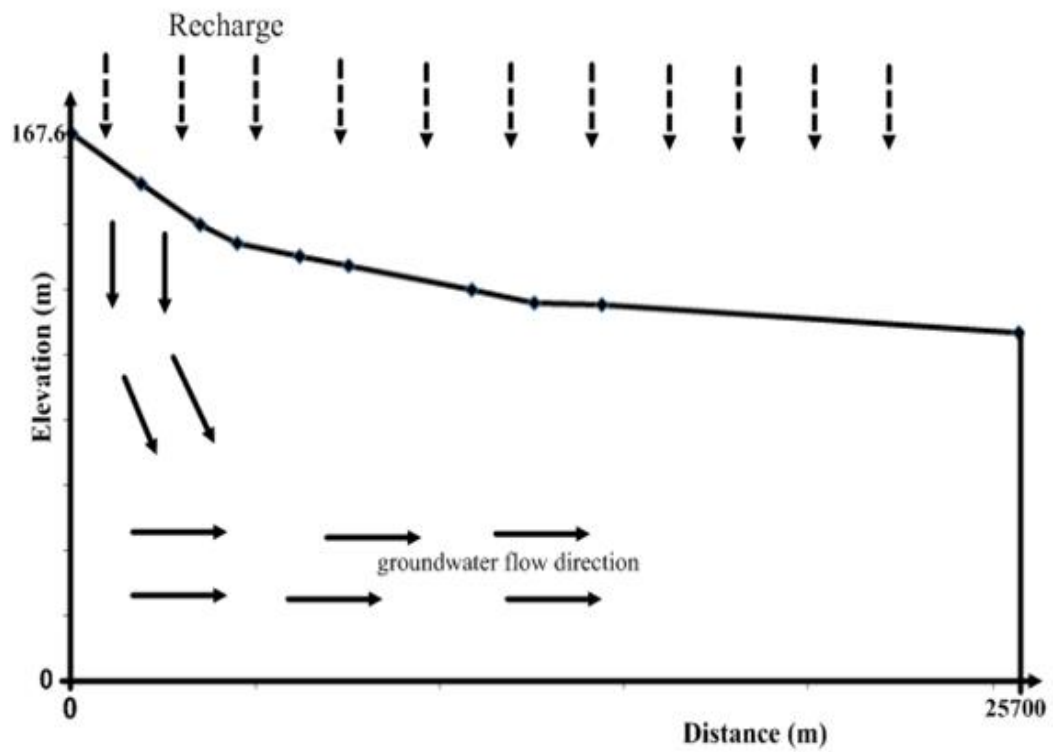


Figure 3.2 Model framework.

3.2.1 MODEL DEFINITION

The hydrologic setting for this problem is described in Figure 3.3 for groundwater flow at steady state. The aquifer is composed largely of sandstone of hydraulic conductivity $K = 10^{-3}$ m/s. Groundwater moves from the upper surface of the saturated zone, the water table, to the outlet at $x = 25700$ m. The water table is a free surface, that is, fluid pressure equals zero, across which there is vertical recharge, denoted by R , 2.5×10^{-5} m/s according to Carrier (2008). The groundwater divide, a line of symmetry, occurs at $x = 0$. The base of the aquifer is impermeable.

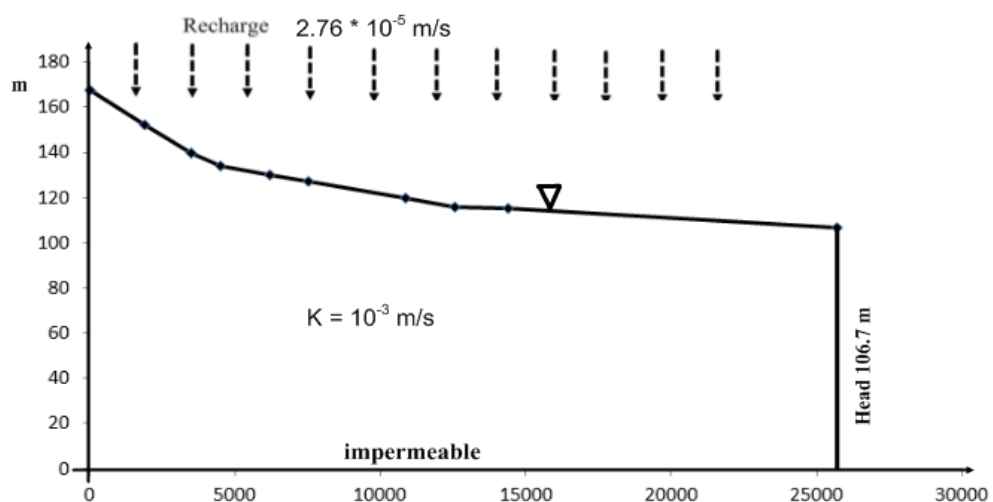


Figure 3.3 Hydrologic setting for groundwater flow.

Figure 3.4 shows conditions related to solute transport. The aquifer is initially assumed to have Carbon-14 concentrations equal to zero. For the first five years, a relative concentration of 105.6 pmC is loaded over the interval $0 \text{ m} < x < 1900 \text{ m}$ at the water table. The solute migrates within the aquifer by advection, dispersion and decay. Throughout the domain, porosity of sandstone, denoted by n , is 0.25, the longitudinal dispersivity, α_L , and transverse vertical dispersivity, α_T , are 0.5 m and 0.005 m, respectively, and the effective molecular diffusion coefficient for radiocarbon, D_m , according to Sonntag (1979) is $3.8 \times 10^{-9} \text{ m}^2 / \text{a}$, equivalent to $1.22 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$.

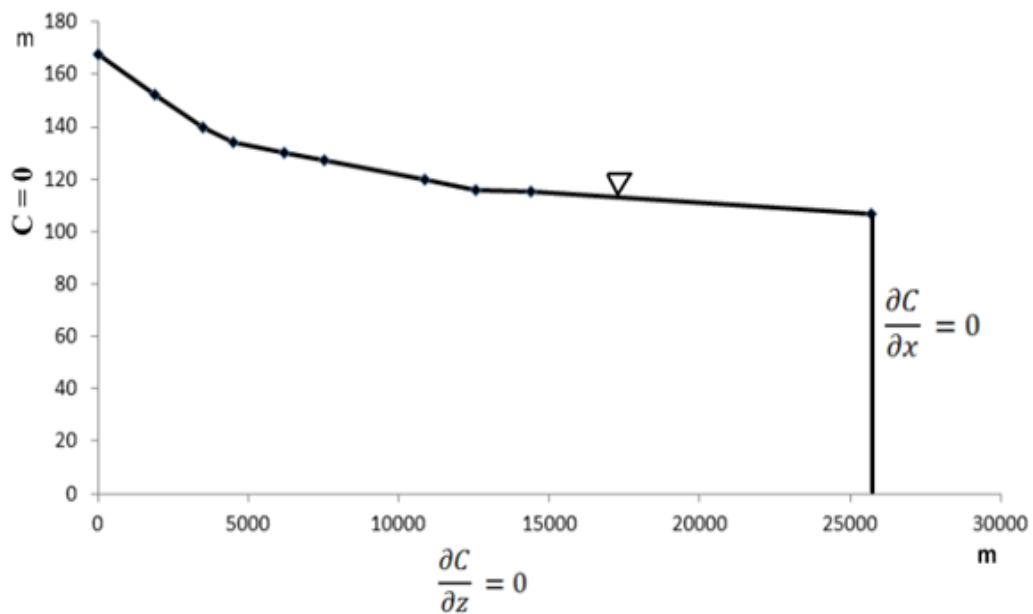


Figure 3.4 Problem definition for the transport of solute.

3.3 MATHEMATICAL MODEL FORMULATION

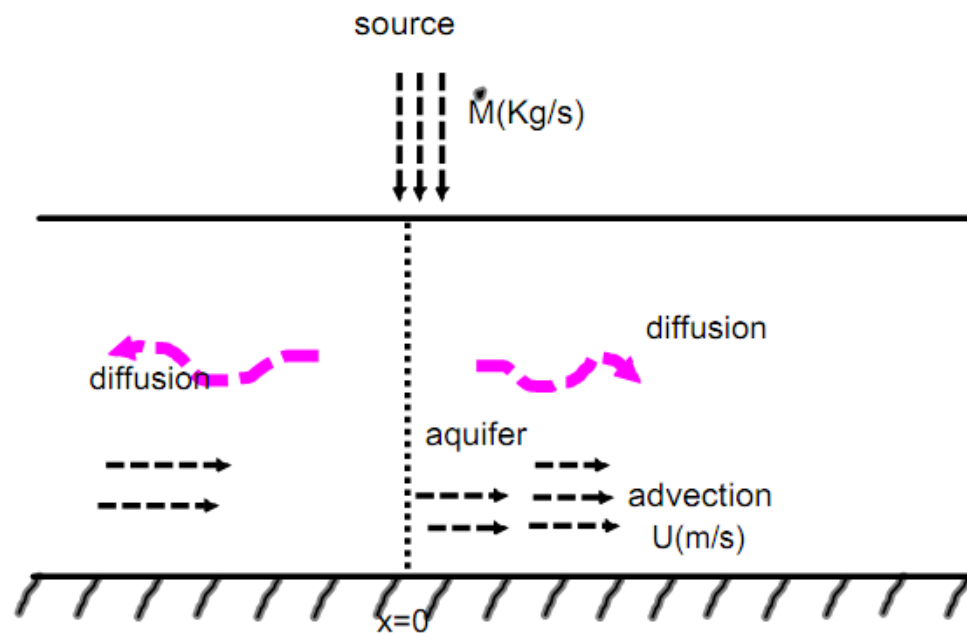


Figure 3.5 Diagram for mathematical modeling.

Figure 3.5 Considers recharge which enters the aquifer through the upper surface and it is transported by the process of advection and diffusion in the aquifer. The driving force being gravity and flux due to Carbon-14 concentration.

Process of solute migration

Advection is movement of solute with the bulk fluid where it moves with the average velocity of the water. The **advective flux is the** velocity of water multiplied by the concentration of solute.

$$F_{adv} = uC \quad (3.5)$$

Where C is concentration of Carbon-14 in water and u is velocity of water.

Dispersion (hydrodynamic dispersion). The diffusive flux in porous media is given by Fick's first law:

$$F_{diff} = -D \frac{dc}{dx} \quad (3.6)$$

where D is diffusion coefficient in porous media (L^2/T).

From the mass conservation statement for dissolved species in groundwater system in equation (3.1), we have:

$$\equiv \frac{\partial C}{\partial t} = \frac{\partial F}{\partial x} + (reaction) \quad (3.7)$$

where F is the total flux $= uC - D \frac{dC}{dx}$

and $reaction = radioactive\ decay = -kC$,

$$k = decay\ constant\ (T^{-1}).$$

Therefore equation (3.7) becomes:

$$\begin{aligned} \frac{\partial C}{\partial t} &= -\frac{\partial}{\partial x} \left(uC - D \frac{dC}{dx} \right) - kC \\ \frac{\partial C}{\partial t} &= -u \frac{\partial C}{\partial x} + D \frac{\partial^2 C}{\partial x^2} - kC \end{aligned} \quad (3.8)$$

Equation (3.8) represents the one dimension advection diffusion reaction equation.

Equation (3.8) can be written in 1D, 2D and 3D as:

$$\frac{\partial C}{\partial t} = \bar{D} \nabla^2 C - u \cdot \nabla C - kC \quad (3.9)$$

For steady state $\frac{\partial C}{\partial t} = 0$

Equation (3.8) becomes

$$-u \frac{\partial C}{\partial x} + D \frac{\partial^2 C}{\partial x^2} - kC = 0, \quad u > 0 \quad (3.10)$$

We assume a solution of the form:

$$C = Ae^{\lambda x}$$

Differentiating, we have:

$$\frac{\partial C}{\partial x} = A\lambda e^{\lambda x} \quad \text{and} \quad \frac{\partial^2 C}{\partial x^2} = A\lambda^2 e^{\lambda x}$$

where λ is a constant.

Substituting in equation (3.10) we have

$$DA\lambda e^{\lambda x} - uA\lambda e^{\lambda x} - KAe^{\lambda x} \quad (3.11)$$

Dividing through equation (3.11) by $Ae^{\lambda x}$ we have:

$$D\lambda^2 - u\lambda - K \quad (3.12)$$

Equation (3.12) has two roots:

$$\lambda_+ = \frac{u + \sqrt{u^2 + 4Dk}}{2D}$$

and

$$\lambda_- = \frac{u - \sqrt{u^2 + 4Dk}}{2D}$$

The first root is always positive whilst the second root is always negative.

$$\text{Hence the solution is: } C(x) = Ae^{\lambda_+ x} = Ae^{\left(\frac{u + \sqrt{u^2 + 4Dk}}{2D}\right)x}, x < 0 \quad (3.13)$$

and

$$C(x) = Ae^{\lambda_- x} = Ae^{\left(\frac{u - \sqrt{u^2 + 4Dk}}{2D}\right)x}, x > 0 \quad (3.14)$$

The constant A of integration is the same in each case to ensure continuity of the concentration at $x = 0$.

To find the constant A, we balance the flux at the vicinity of the source.

$$\left[\begin{array}{c} \text{Flux coming} \\ \text{in} \end{array} \right] + [\text{source}] = \left[\begin{array}{c} \text{flux going} \\ \text{out} \end{array} \right] \quad (3.15)$$

$$F_- + \dot{M} = F_+,$$

$$\text{But } F = uC - D \frac{dC}{dx}, \quad \text{and} \quad C = Ae^{\lambda x}$$

Equation (3.15) becomes

$$uAe^{\lambda_+ x} - DAe^{\lambda_+ x} \cdot \lambda_+ + \dot{M} = uAe^{\lambda_- x} - DAe^{\lambda_- x} \cdot \lambda_- \quad (3.16)$$

At the source $x=0$, equation (3.16) becomes:

$$uA - DA \cdot \lambda_+ + \dot{M} = uA - DA \cdot \lambda_- \quad (3.17)$$

$$A = \frac{\dot{M}}{(\lambda_+ - \lambda_-)D} = \frac{\dot{M}}{\sqrt{u^2 + 4DK}}$$

The solution of equation (3.8) is given as:

$$C(x) = \frac{\dot{M}e^{\lambda_+x}}{\sqrt{u^2 + 4DK}} \text{ for } x \leq 0 \text{ and } C(x) = \frac{\dot{M}e^{\lambda_-x}}{\sqrt{u^2 + 4DK}} \text{ for } x \geq 0$$

Substituting the values of λ_+ and λ_- into the relations we have:

$$C(x) = \frac{\dot{M}e^{\left(\frac{u+\sqrt{u^2+4DK}}{2D}\right)x}}{\sqrt{u^2 + 4DK}} \quad (3.18)$$

and

$$C(x) = \frac{\dot{M}e^{\left(\frac{u-\sqrt{u^2+4DK}}{2D}\right)x}}{\sqrt{u^2 + 4DK}} \quad (3.19)$$

3.3.1 GOVERNING EQUATIONS

Mathematical equations describing groundwater flow through a porous medium are based mainly on Darcy's law and the continuity equation. According to Darcy's law, the average flow velocity is proportional to the hydraulic gradient (Freeze and Cherry 1979) and the effective porosity. The proportionality factor is determined as the hydraulic conductivity of the rock.

$$u_x = \frac{K}{n} \frac{dh}{dx} \quad (3.20)$$

where x is the distance in x -direction [L], u_x is the average flow velocity in the x -direction [L/T], n is the porosity, K is the hydraulic conductivity [L/T], h is the hydraulic head [L], and the component dh/dx is the hydraulic gradient in the x -direction [L/L].

The mass balance principle requires that the rate of change in mass storage of an elemental volume with time be equal to the mass inflow rate minus the mass outflow rate. There is no change in head with time in steady state conditions, so time is not one of the independent variables, and steady state flow can be described by the following three-dimensional partial differential equation:

$$\frac{\partial}{\partial x} \left(K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial h}{\partial z} \right) = 0 \quad (3.21)$$

Where x , y , and z are the Cartesian coordinates [L], K_x , K_y , and K_z are the hydraulic conductivity components, and h represents the hydraulic head with the unit of [L]. Hydraulic head is a function of pressure and gravitational potential and it is defined as:

$$h = h_p + z \quad (3.22)$$

where h_p is the pressure head (m) and z is the elevation (m). If the medium is homogeneous and isotropic, $K_x = K_y = K_z$, then Equation (3.21) can be expressed by Laplace's equation as follows:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (3.23)$$

In transient conditions, the law of conservation of mass and the storage parameters of the porous medium are to be taken into account. The flow equation for a transient flow through a saturated anisotropic porous medium is given as:

$$\frac{\partial}{\partial x} \left(K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial h}{\partial z} \right) = S_s \frac{\partial h}{\partial t} \quad (3.24)$$

Where S_s represents the specific storage [L^{-1}] and t is the time [T]. The continuity equation for the groundwater system is developed to solve the values of the hydraulic head when the other aquifer parameters are known.

$$\frac{\partial}{\partial x} \left(K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial h}{\partial z} \right) = S_s \frac{\partial h}{\partial t} + W \quad (3.25)$$

where W represents the source/sink term given in [L^3/T].

3.3.2 MASS TRANSPORT MODELING (ADVECTION AND DISPERSION)

The transport equation is:

$$\frac{\partial}{\partial t}(\theta c) = \nabla(D\nabla c - vc) \quad (3.26)$$

It is a differential equation with the unknown concentration C , porosity θ , dispersion tensor D and velocity vector \mathbf{v} . \mathbf{V} denotes the Darcy velocity, also called filtration velocity or seepage velocity. \mathbf{D} is defined by:

$$D = (D_{ij}) = \left((\tau D_m + \alpha_L v) \delta_{ij} \right) + (\alpha_L - \alpha_T) \frac{v_i v_j}{v} \quad (3.27)$$

with longitudinal dispersivity α_L , transversal dispersivity α_T , tortuosity factor τ and molecular diffusivity D_m . The subscripts i and j denote indices related to the directions of the coordinate system and δ_{ij} is the Kronecker symbol. If additionally degradation is taken into account, the differential equation (3.26) has to be extended to:

$$\frac{\partial}{\partial t}(\theta c) = \nabla \cdot (D\nabla c - vc) - \theta \lambda_f c \quad (3.28)$$

with decay constants λ_f for the fluid phase.

3.4 MODEL ASSUMPTIONS

1. The model uses the Cartesian coordinate system (x,y,z) as a frame of reference.
2. A two dimensional flow shall be considered.
3. The aquifer is modeled as an isotropic, homogeneous porous media of infinite lateral extent and finite thickness. The hydraulic conductivity, K is constant in all directions.
4. Transport of Carbon-14 is only considered in the saturated zone.
5. Transport is limited to only Carbon-14 that may decay as a function of time.
6. The Carbon-14 is assumed to move as a dissolved substance. Transport in vapor phase is not considered.
7. Atmospheric production rate of (Carbon-14) is assumed constant. (Constant input rate of tracer).
8. The groundwater flow is assumed to be under steady state conditions. The velocity of flow is considered not to change with time since groundwater flow is a slow process.
9. The density and temperature of water is assumed constant.

3.5 MODEL SIMULATION USING COMSOL GRAPHIC USER INTERFACE

The model is two dimensional and examines condition in which flow and transport are linked. The model simulation begins with the geometry set up of the elevational cross section of the study area (Figure 3.1) using the graphical user interface. This was followed by setting up the physics in the ESM mode with the model input parameters and constants. An algorithm for the simulation using the graphic user interface is provided in Appendix A.

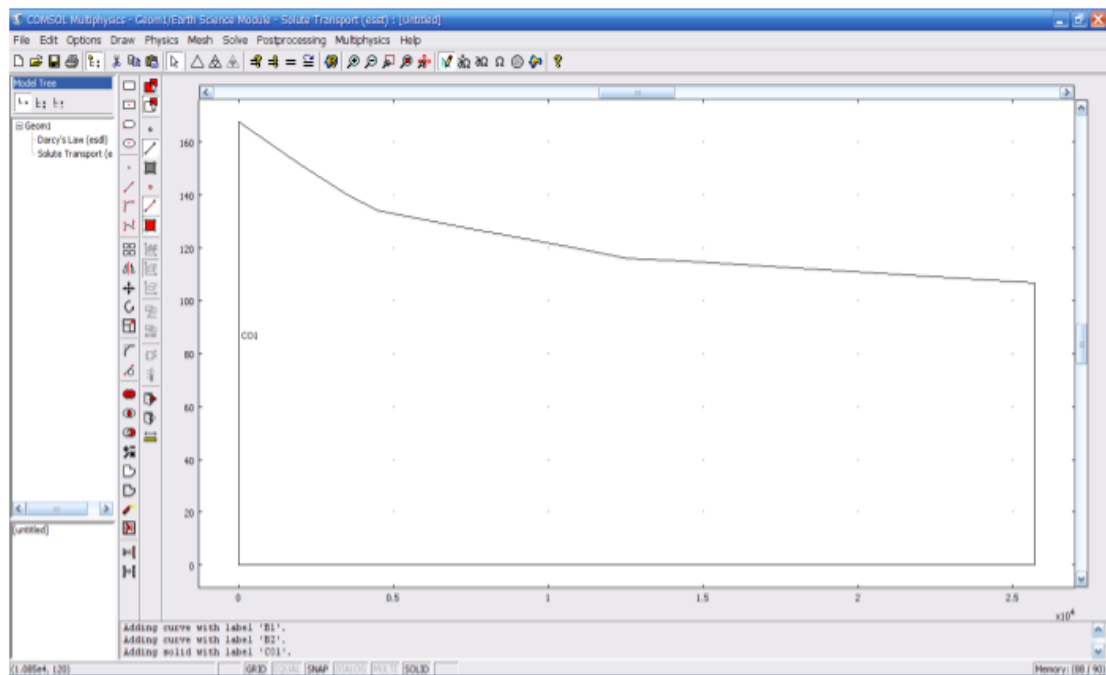


Figure 3.6: Geometric set up of the elevational cross section of the study area using the Comsol Graphical user interface.

3.5.1 BOUNDARY CONDITIONS

A unique solution to the governing statements requires boundary conditions for all models. The Darcy's law application mode of the Earth Science Module requires a number of boundary conditions. This was specified for unique conditions by entering expressions in the boundary settings dialog boxes or by altering the boundary mechanics in the equation systems dialog boxes. Three types of boundary conditions for the Darcy's law application mode are described below:

1. Dirichlet Condition

The Dirichlet condition is the boundary condition where it can be assumed no flow is considered through the boundary to or from the system under consideration. It is also referred to as boundary type 1 (Reilly, 2001). It is given as: $H = H_0$, Where H_0 is a known head given as a number, a distribution, or an expression.

2. The Zero Flux Condition

This boundary condition is also called the Neumann condition. In various literatures this is referred to as boundary type 2. It is a specified flow boundary and it is given as:

$$K\nabla H = 0$$

Where, K is the hydraulic conductivity and H is the head. While this Neumann condition specifies zero flow across the boundary, it allows for movement along it. In this way the equation for the zero flux condition also describes symmetry about an axis or a flow divide (FEMLAB, 2004).

3. Constant Flux (Inward Flux)

With the inward flux boundary condition, positive values correspond to flow into the model domain whereas negative values correspond to flow out of the domain. This is expressed as: $K\nabla H = N$, where N is a value or expression for the flux.

4 Mixed Boundary Condition

This type of boundary condition is used when there is the need to specify the flux into the flow domain combined with information about the hydraulic potential at some finite distance. The model domain might connect to a larger body of water through a semi-pervious layer. You can represent this condition with the mixed boundary expression. The equation for the mixed boundary condition is expressed in COMSOL Multiphysics as:

$$n \cdot \frac{\kappa}{\eta} (\nabla p + \rho_f g \nabla D) = N_0 + R_b [(p_b - p) + \rho_f g (D_b - D)]$$

Where p_b and D_b are the pressure and elevation of the distant fluid source; and R_b is the conductance of materials between the source and the model domain. $R_b = K'/B'$ where K' is the permeability of the thin layer and B' is its thickness. The stated flux N_0 can assume a value or can be set to zero.

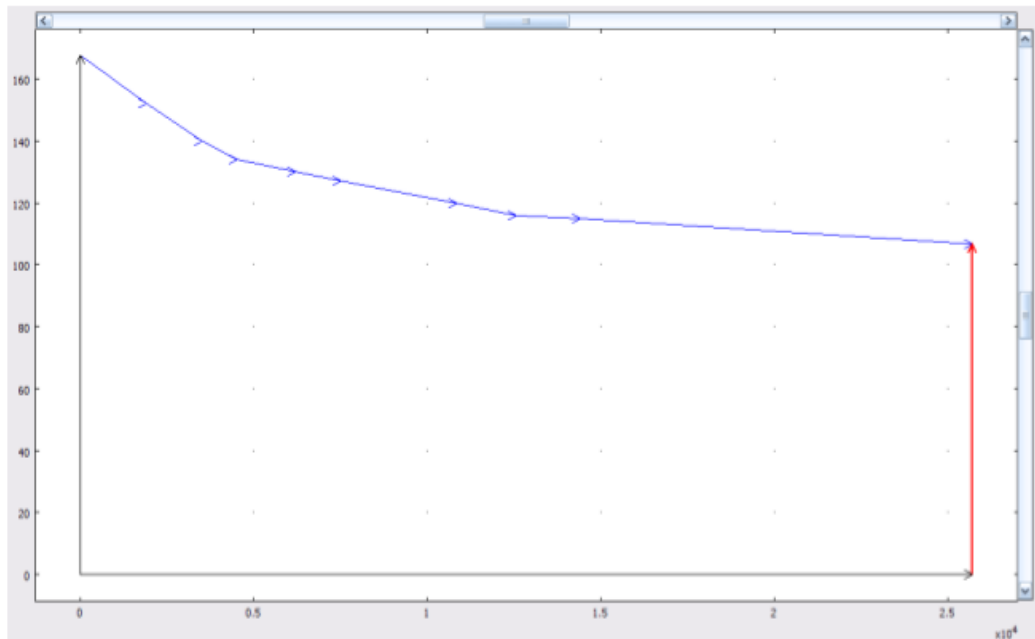


Figure 3.7 Model in boundary mode.

The steady state flow field was modeled using Darcy's law, there are no flows (Neumann) condition and given head (Dirichlet) conditions at different parts of the boundary. After the flow model has been set up the transport model is added with the following input parameters; initial concentration, longitudinal and transverse dispersivities, molecular diffusivity and porosity.

3.5.2 MODEL PARAMETERS

Table 3.1: Model input parameter.

Parameter	Value / Unit	Parameter	Value / Unit
Porosity(sandstone)	0.25	Decay rate of carbon-14 in fluid	$3.84 \times 10^{-12} \text{ s}^{-1}$
Input concentration	105.6 pmC	Model length	25700 m
recharge	$2.76 \times 10^{-5} \text{ m/s}$	Model height	167.6 m
Hydraulic conductivity (K)	10^{-3} m/s	Transverse dispersivity	0.005
Bulk density	1000 kg/m^3	Longitudinal dispersion	0.5
Diffusive constant (D)	$1.22 \times 10^{-9} \text{ m}^2/\text{s}$		

The problem is treated in two steps. It is first solved for flow, in the steady state mode as the flow rate was assumed to be constant. The second step requires a solution for the transport only which was done in the transient solver mode. In order to perform these two steps correctly the solver parameters had to be set.

3.6. CALCULATION OF THE GROUNDWATER FLOW VELOCITY

As shown in Figure 3.8, the velocity of groundwater flow can be calculated if we calculate the age of groundwater at two separate boreholes along a particular line of section.

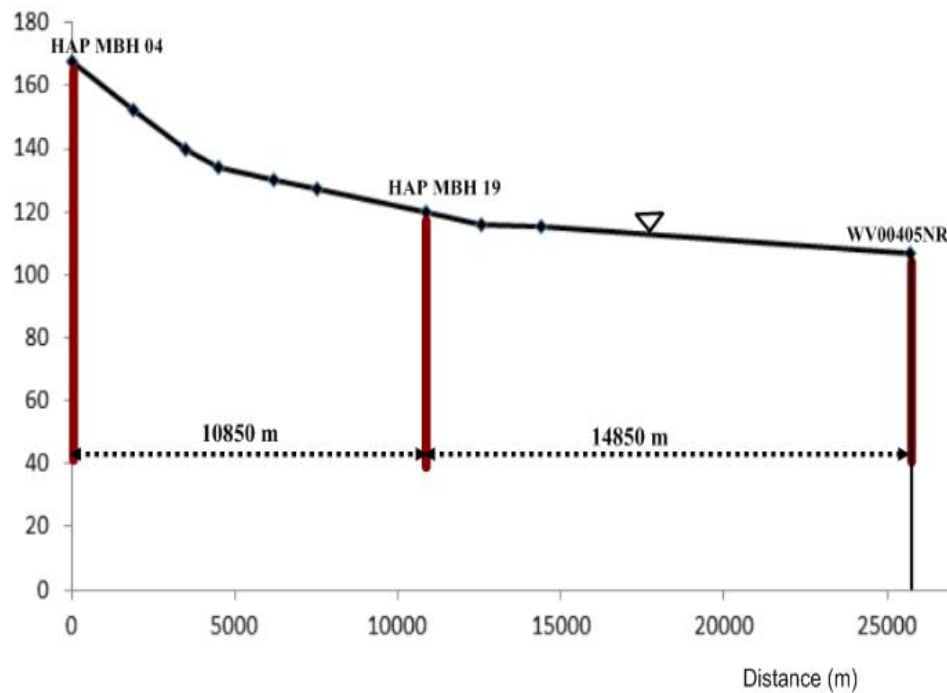


Figure 3.8 Calculating groundwater flow rate using age of groundwater determined in two different boreholes.

3.6.1. DETERMINATION OF GROUNDWATER AGE USING CARBON-14 DATE

Dating groundwater with ^{14}C is similar to dating organic material. The basic equation is as shown in Equation (2.10). Using the data in Table 3.2; the ages of the groundwater in the wells were computed.

Table 3.2 Carbon-14 Sampling in the wells

HydroID	Original Well ID	WellType	Longitude	Latitude	Community	District
9868	HAP MBH 04	Monitoring	-0.836860	9.574130	Kanshegu	Savelugu Nanton
25704	HAP MBH 19	Monitoring	-1.049080	9.659060	Nawuni	Tolon Kumbungu
7728	WV00405NR	Domestic	-1.008940	9.586800	Yipeligu	Tolon Kumbungu

Table 3.2 (continue)

Temp ($^{\circ}\text{C}$)	pH	Eh (mV)	DO (mg/l)	TDS (mg/l)	Ec($\mu\text{S}/\text{cm}$)	pmC	$\delta^{13}\text{C}$	dpm/gC
31.0	7.8	-63.4	0.19	354.0	708.0	105.6	-18	14.3
30.2	7.2	-41.9	0.01	1428.0	2860.0	74	-16.4	12.3
30.5	7.9	-83.4	0.16	661.0	1322.0	91.2	-19	10

At Kanshegu, the age of the groundwater in the borehole (HAP MBH 04) is given by:

$$t = -\frac{5730}{\ln(2)} \times \ln\left(\frac{105}{110}\right) = 385 \text{ y.}$$

At Nawuni, the age of the groundwater in the borehole (HAP MBH 19) is given by:

$$t = -\frac{5730}{\ln(2)} \times \ln\left(\frac{74}{110}\right) = 3277 \text{ y.}$$

At Yipeligu, the age of the groundwater in the borehole (WV00405NR) is given by:

$$t = -\frac{5730}{\ln(2)} \times \ln\left(\frac{91}{110}\right) = 1567 \text{ y}$$

3.6.2 CALCULATION OF GROUNDWATER FLOW RATE USING CARBON-14 DATE.

The velocity of groundwater flow can be calculated and the direction of groundwater flow determined if, groundwater age at two separate boreholes along a particular line of section is determined.

The groundwater flow rate in the aquifer is calculated using the wells HAP MBH 04 and HAP MBH 19 as:

$$\text{flow rate} = \frac{\text{seperation between boreholes(HAP MBH 04 and HAP MBH 19)}}{\text{time}} \quad (4.1)$$

Also the groundwater flow rate in the aquifer is calculated using the wells HAP MBH 19 and WV00405NR as:

$$\text{flow rate} = \frac{\text{seperation between boreholes(HAP MBH 19 and WV00405NR)}}{\text{time}} \quad (4.2)$$

Also the groundwater flow rate in the aquifer is calculated using the wells

HAP MBH 04 and WV00405NR as:

$$\text{flow rate} = \frac{\text{seperation between boreholes (HAP MBH 04 and WV00405NR)}}{\text{time}} \quad (4.3)$$

3.7 MODEL CALIBRATION

Model calibration consist the process of adjusting model parameters to reduce the difference, known as residuals, between simulated and observed values. It involves changing values of model input parameters in an attempt to match field conditions within some acceptable criteria. The hydraulic conductivity of the aquifer was manually adjusted to achieve a best fit between simulated and observed values of groundwater flow rate. The model calibration process for the Voltaian sandstone aquifer included comparisons between model simulated conditions and field conditions for the Carbon-14 data, groundwater flow direction and groundwater flow rate.

The calibration process resulted in a hydraulic conductivity of 10^{-3} m/s which was within the reasonable range of previously reported Sandstone aquifer Properties, which ranges from 10^{-5} to 10^{-1} m/s (Anderson and Woessner, 1992). (Istok, 1989) found that the hydraulic conductivity of sandstone ranges from 10^{-6} to 10^{-10} m/s.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1 RESULTS

The graphical output of the simulation is shown in Figure (4.1) and (4.2) which shows hydraulic head as a surface plot, streamline flow of the flow pattern and contour plots of equipotential lines of the aquifer in the study area respectfully.

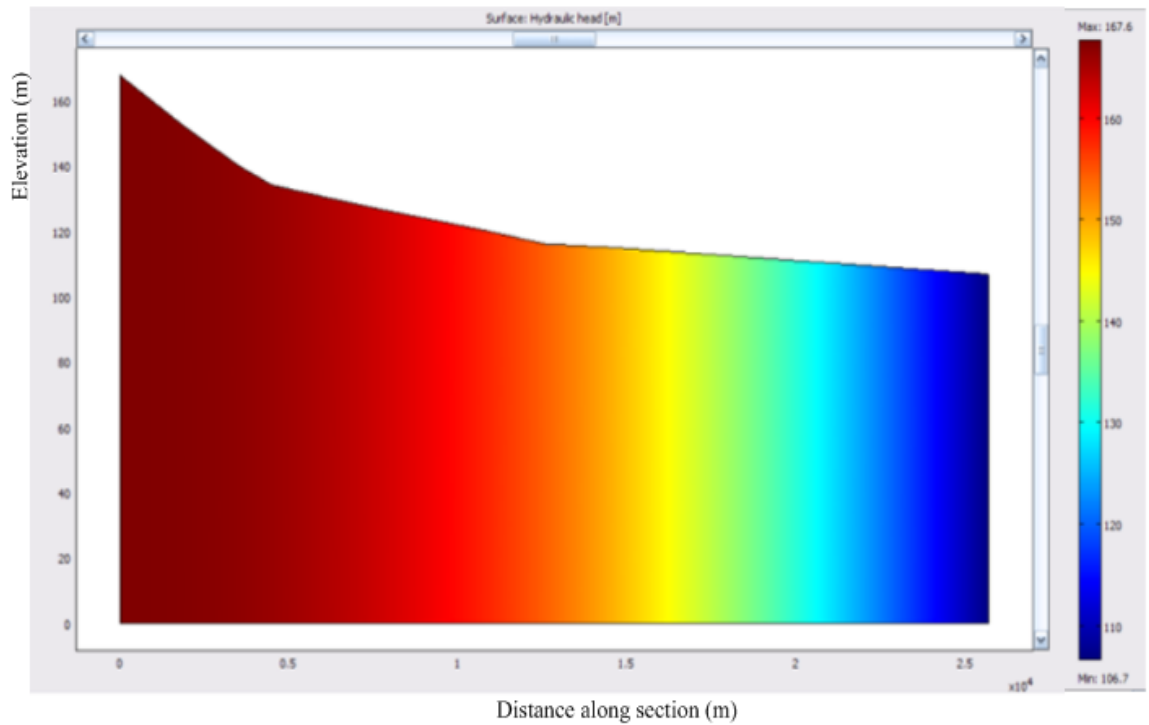


Figure 4.1 Hydraulic head as surface plot of groundwater at Kanshegu, Yipeligu and Nawuni

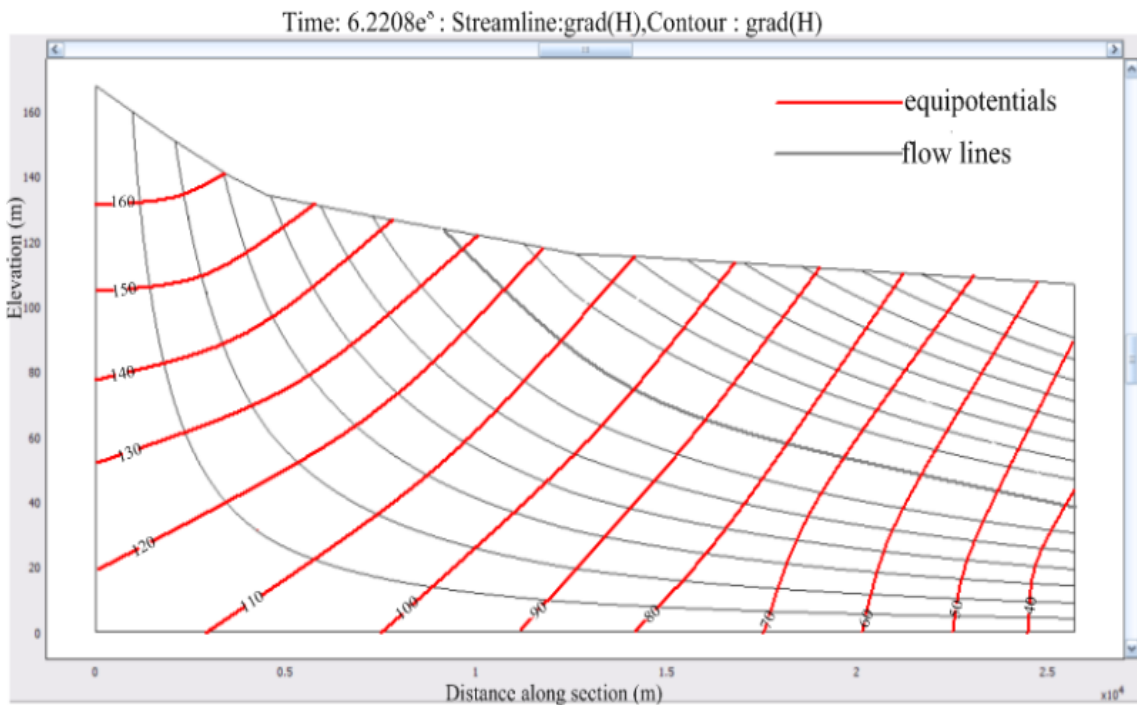


Figure 4.2 Flow lines and equipotential lines (Flow Net) across Kanshegu to Nawuni.

4.1.1 DETERMINING RATE OF FLOW FROM FLOW NET.

The rate of flow can be determined from flow nets if the hydraulic conductivity is known. The total volumetric rate of flow (Q) can be calculated by.

$$Q = N_f K \Delta H = \frac{N_f}{N_e} K H \quad (4.1)$$

Where

N_f = number of flow channels

N_e = number of equipotential drops

H = total head drop ($H = N_e \times \Delta h$)

From Figure 4.3 and Equation 4.1

$$N_f = 14, \quad N_e = 120, \quad K = 10^{-3} \text{ ms}^{-1}, \quad \Delta h = 60.9 \text{ m}$$

$$Q = 14 \times 10^{-3} \times 60.9 = 0.8526 \text{ m}^3\text{s}^{-1}$$

$$\text{But flow rate (velocity)} = \frac{\text{Volumetric flow rate}}{\text{total area of aquifer}}$$

$$\text{flow rate (v)} = \frac{0.8526}{3.2 \times 10^6} = 2.66 \times 10^{-7} \text{ ms}^{-1}$$

4.1.2 RATE OF FLOW USING CARBON-14 DATE

The groundwater flow rate calculated using Equation (4.1) between the wells HAP MBH 04 and HAP MBH 19:

$$\text{flow rate} = \frac{10850 \text{ m}}{(1567 - 385) \text{ y}} = \frac{9.18 \text{ m}}{\text{y}} = 2.95 \times 10^{-7} \text{ m/s}$$

Also the groundwater flow rate calculated using Equation (4.2) between the wells HAP MBH 19 and WV00405NR:

$$\text{flow rate} = \frac{14850 \text{ m}}{(3277 - 1567) \text{ yrs}} = \frac{8.68 \text{ m}}{\text{y}} = 2.79 \times 10^{-7} \text{ m/s}$$

Also the groundwater flow rate calculated using Equation (4.3) between the wells HAP MBH 04 and WV00405NR:

$$\text{flow rate} = \frac{25700 \text{ m}}{(3277 - 385) \text{ yrs}} = \frac{8.87 \text{ m}}{\text{yr}} = 2.85 \times 10^{-7} \text{ m/s}$$

The average flow rate in the aquifer is 2.86×10^{-7} m/s on the basis of Carbon-14 analysis.

Figure 4.3, provides a summary of the groundwater ages and flow rate determined from the wells at Kanshegu, Yipeligu and Nawuni respectively.

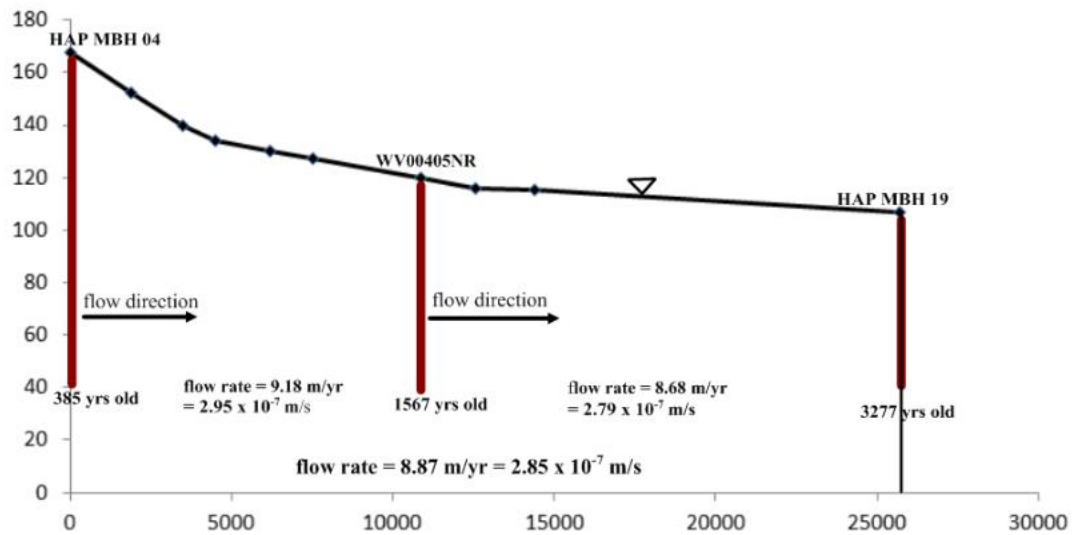


Figure 4.3. Horizontal flow direction using groundwater age at three locations (Kanshegu, Yipeligu, Nawuni). Groundwater age increases along the flow line.

4.2 DISCUSSIONS

4.2.1 PATTERN OF GROUNDWATER FLOW IN THE STUDY AREA

Figure 4.2 represents a potentiometric map of the study area, which shows a family of equipotentials, with flow lines. The groundwater flow paths in the aquifer in the study area shown in figure 4.2 are long which indicates that flow in the aquifer is predominantly regional. Horizontal flow dominates in regional-scale aquifers, except in close proximity to recharge areas and discharge zones; hence most flow is indeed horizontal. Depending on where infiltration takes place, these flow paths may reach different depths, and impact extremely different travel times to the outlet, affecting water chemistry and mineralization accordingly. Regional groundwater flow originates at the Kanshegu area and discharges at the Nawuni area.

The groundwater flow pattern in the aquifer in the study area differs from the simplified conceptual models presented in Figure 2.1 considering the water table of the aquifer in the study area because the groundwater flow pattern is complex in the horizontal as well as the vertical plane. As in most natural groundwater flow systems, flow in the aquifer is three-dimensional and not easily displayed in two dimensions. Also aquifer heterogeneities might alter this pattern to some level.

The groundwater age determined from the wells at Kanshegu (HAP MBH 04), Yipeligu (WV00405NR) and Nawuni (HAP MBH 19) using Equation 2.10 and the Carbon-14 data in Table 3.2, were 385 y., 1567 y. and 3277 y. Old respectfully. Groundwater age increases along the flow line.

The TDS of the groundwater sample in the well at Kanshegu is very low, 354 mg/l as well as the residence time of 385 y. which suggest a zone of recharge in the aquifer. Also the residence time and the TDS of the groundwater in the well at Yipeligu, which were 1567 y. and 661 mg/l respectively, are high compared to that at Kanshegu. This suggests a groundwater flow direction from Kanshegu to Yipeligu. Hence Yipeligu showing a zone of discharge in the aquifer. This flow is an intermediate flow on the basis of distance travelled. Furthermore the resident time of 3277 y. and TDS of 1428 (mg/l) observed at the well at Nawuni, which were higher than that of the wells at Kanshegu and Yipeligu indicate a zone of discharge in the aquifer. A short resident time value at Kanshegu and a very long resident time at Nawuni suggest a regional flow system in the aquifer. Hence groundwater flow in the aquifer is continuous.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 CONCLUSIONS

The two dimensional model developed for the Voltaian sandstone aquifer fulfills its basic goals. That is, it demonstrates that the geometric model provides a basis for constructing a flow model for the area and it helps in understanding the flow dynamics of the aquifer.

The results of this study led to the following conclusions regarding groundwater flow and recharge in the aquifer of the study area. The main conclusions were as follows:

The aquifer system of the study area can be said to have a regional flow on the basis of topography, water-table configuration and the flow lines obtained through the simulation and also long flow paths.

On the basis of Carbon-14 date, the minimum age, of groundwater in the aquifer in the study area is 385 years occurring in the well at Kanshegu which suggest a recharge zone in the aquifer. The oldest groundwater age is 3277 years occurring in the well at Nawuni. Which suggest a zone of groundwater discharge in the aquifer.

The flow rate obtained using Carbon-14 date was 8.91 m/y. equivalent to 2.86×10^{-7} m/s. The overall flow rate obtained from the model simulations was $2.66 \times 10^{-7} \text{ ms}^{-1}$. The hydraulic conductivity which gave the best fit of the observed and calculated head was found to be 10^{-3} m/s.

5.2 IMPLICATIONS FOR GROUNDWATER MANAGEMENT

Information contained in the flow net (Figure 4.2) can be an important management tool. For example, priority areas can be identified for protection of groundwater quality in the aquifer. It is more important to protect recharge areas than transitional or discharge areas because groundwater in recharge areas moves deeper into the flow system where, if contaminated, it might affect water-supply wells. Accurate information about the groundwater flow path is needed in many water resources projects such as in construction of dams (determining different routes that water stored in the dam lake may escape), movement of plumes, mixing between different quality groundwater, and study of surface water–groundwater interaction.

5.3 RECOMMENDATIONS

The model, just as many mathematical models of this kind, is of a primary nature. The following approach is recommended to help improve the model.

- All hydrogeologic features of the basin, such as springs, wetlands, streams, and major pumping centers should be mapped.
- A water-table map from water-level data (well data) and transmissivity map using geologic maps and available information such as pumping-test data, specific-capacity data should be constructed.
- A recharge and discharge map of the basin should be generated using a two-dimensional flow model with the water-table map and transmissivity map providing the necessary input data.
- Protection of the recharge area is recommended.

REFERENCES

- Acheampong, P. K., (1988) Water balance analysis for Ghana. *Geography* 73, 125-131.
- Acheampong, S., Hess, J., (2000) Origin of the shallow groundwater system in the Southern Voltaian sedimentary basin of Ghana: an isotopic approach. *Journal of Hydrology* 233 (1-4), 37-53.
- African Journal of Environmental Science and Technology* Vol. 3 (10), pp. 341-352, October,(2009) Available online at <http://www.academicjournals.org/AJEST> ISSN 1991-637X © 2009 Academic Journals.
- Anderson, M.P., and Woessner, W.W. (1992), *Applied Groundwater modeling: Simulation of Flow and Advective Transport*: San Diego, Academic Press, Inc.
- Akiti T. T. (1980). *Etudé géochimique et isotopique de quelques aquifers du Ghana*. (Thesis) Univeristé Paris-Sud. 232 pp.
- Akiti T. T. (1987). Environmental isotope study of groundwater in crystalline rocks of the Accra Plains, Ghana. *Proceedings of the 4th Working Meeting, Isotopes in Nature*, Leipzig, September 1986.
- Amani A (2001) Assessment of precipitation and resources variability across the Sahelian region. In: Gash JHC, Odada EO, Oyebande L and Schulz RE (eds) *Report of the International Workshop on Africa's Water Resources*.
- Amisigo B.A (1996) *Modeling river flow in the Volta Basin of West Africa: A data-driven framework*. PhD Thesis. Ecology and Development Series No. 34.ZEF Bonn.CuvillierVerlag, Göttingen.
- Andah EI, van de Giesen N, Biney CA (2003) *Water, climate, food, and environment in the Volta Basin.Adaptation strategies to changing environments*. Contribution to the ADAPT project: <http://www.weap21.org/downloads/ADAPTVolta.pdf>.

- Arnell NW (1999) Climate change and global water resources, *Glob. Environ. Change - Human Policy Dimens*, 9, S31-S49.
- Back, W., Rosenshein, J.S., and Seaber, P.R, 1988, *Hydrogeology. The Geology of North America*, Vol. O-2. Geological Society of America, Boulder, CO.
- Charles R. Fitts, (2002) *Groundwater science*, p.22-28, p.54-56.
- Carrier, M.A., (2008) *Hydrogeological synthesis of Northern Ghana. A MS thesis. Avant-Garde University, Canada.*
- Chand R, Hodlur GK, Prakash MR, Mondal NC, Singh VS (2005) Reliable natural recharge estimates in granitic terrain. *Current science*, 88 (5).
- Clark, I. and P. Fritz. (1997). *Environmental Isotopes in Hydrogeology*. Lewis Publishers. New York. 328 p.
- COMSOL Multiphysics, (2008) *User's Guide version 3.4, COMSOLAB.*
- Dapaah-Siakwan, S., Gyau-Boakye, P., (2000) Hydrogeologic framework and borehole yield in Ghana. *Hydrogeology Journal* 8, 405-416.
- Darko, P.K., Krasny, J., (2003). Regional transmissivity distribution and groundwater potential in hard rocks of Ghana. *Proceedings of the IAH International Conference on Groundwater in fractured rocks, Czech Republic, Sept. 2003*, p. 45-46
- Davis and Dewiest, (1966) *Hydrogeology*; John Willey and sons, New York p.181.
- Edmunds W. M and. Walton N. R. C. (1980). A geochemical and isotopic approach to recharge evaluation in semi-arid zones. *Proceedings of the Advisory Group Meeting on the Application of Isotope Techniques in Arid Zone Hydrology, Vienna, (1978).* International Atomic Energy Agency (IAEA), Vienna.
- Edmunds W. M. and Wright E. P. (1979). Groundwater recharge and paleo-climate in the Sirte and Kufra Basin, Libya. *J. Hydrol.* 40: 215–235.
- Ewusi A. (2006) *Groundwater exploration and management using Geophysics: Northern Region of Ghana. PhD Thesis, Brandenburg Technical University of Cottbus*

- FAO (1997) Irrigation potential in Africa: A basin approach. FAO Land and Water Bulletin 4
- FEMLAB, (2004) User's Guide version 3.1, COMSOLAB.
- Fontes J. C., Yousfi M. and Allison G. B. (1986) Estimation of long-term diffuse groundwater discharge in the northern Sahara using stable isotope profiles in soil water, *J. Hydro.* 86: 315.
- Foster SSD (1988) Quantification of groundwater recharge in arid regions: a practical view for resource development and management. In: Simmers I (Ed) Estimation of natural groundwater recharge. Reidel publishing Company, Dordrecht, The Netherlands pp323-338.
- Freeze RA, Cherry JA (1979) *Groundwater*. Prentice Hall, Englewood, New Jersey.
- Gasse F., Fontes J. Ch., Carbonel F., Plaziat J. C., Soulie_Marsche I., Kaczmarcska I. and Dupeuble P. A. (1987). Biological remains, geochemistry and stable isotope for the reconstruction of environmental and hydrological changes in the Holocene lakes from North Sahara, *Palaeo* 3(60) 1–46.
- Gonfiantini R, Frölich K, Araguás-Araguás L, Rozanski K (1998) Isotopes in groundwater hydrology. p. 203–246. In: C. Kendall and J. McDonnell (ed.) *Isotope tracers in catchment hydrology*. Elsevier Sci., Amsterdam.
- Gyau-Boakye, P. (2001) Sources of rural water supply in Ghana. *Water International* 26 (1), 96–104.
- Gyau-Boakye, P., Tumbulto, J., (2000) The Volta Lake and declining rainfall and stream flows in the Volta river basin. *Environment, Development, and Sustainability* 2 (1), 1–11.
- Gyau-Boakye P, Tumbulto JW (2006) Comparison of rainfall and runoff in the humid southwestern and the semiarid northern savannah zone in Ghana. *African Journal of Science and Technology. Science and Engineering Series* Vol. 7, No. 1, pp. 64-72

- Hubert, M K (1940) the theory of groundwater motion: J. Geol, 48(7), part 1, pp.785-944.
- IPCC (2001) Climate Change (2001): The scientific basis. Eds. J. T. Houghton, Y. Ding, M. Nogua, D. Griggs, P. Vander Linden, K. Maskell, Cambridge Univ. Press., Cambridge, U.K.
- Istok, J., 1989, Groundwater Modeling by the Finite Element Method: Washington DC, American Geophysical Union, 495 p.
- Jacob Bear, A.H.-D. Cheng (2009) Modeling Groundwater Flow and Contaminant Transport, Theory and Applications of Transport in Porous Media 23
- Joubert AM Hewitson BC (1997) Simulating present and future climates of southern Africa using general circulation models. ProgPhysGeog 21:51–78.
- Kevin M. Hiscock, (2005) Hydrogeology principles and practice, 58pp.
- Kortatsi BK (1994) Groundwater utilization in Ghana, in: Future Groundwater Resources at Risk (Proceedings of the Helsinki Conference) IAHS Publ. No. 222, 149-156, IAHS Press, and Wallingford.
- Kortatsi B. K.and Sekpey N. K. (1994). Chemical and isotopic techniques for the origin of groundwater in the crystalline basement complex of the Upper Region of Ghana. Regional trends in Geolgy in African Geology. Proceedings of the 9th International Geological Conference, Accra, 2nd–7th November, (1992).Geological Society of Africa.
- Kunstmann H, Jung G (2003) Investigation of feedback mechanisms between soil moisture, land use and precipitation in West Africa; IAHS publications no. 280; Water Resources Systems - Water Availability and Global Change.
- Lerner DN, Issar AS, Simmers I (1990) Groundwater recharge. A guide to understanding and estimating natural recharge. International contributions to hydrogeology. Verlag Heinz Heise.
- Martin N (2005) Development of a water balance for the Atankwidi catchment, West Africa - A case study of groundwater recharge in a semi-arid climate. Doctoral thesis. University of Gottingen.

- MacDonald A. M and Davies J. (2010) BGS Consultancy: UNICEF IWASH Project, Northern Ghana. Groundwater science programme. Visit Report 25/12/2012. BGS Internal Report IR/10/.
- Ministry of Works and Housing (1998) “Water Resources Management Study, Information ‘Building Block’ Study. Part II, Volta Basin System, Groundwater Resources.” Ministry of Works and Housing, Accra.
- Nelson RG, Ascough II JC, Langemeier MR (2005) Environmental and economic analysis of switchgrass production for water quality improvement in northeast Kansas. *J. Environ. Mgmt.* 79(4): 336-347.
- Niasse M, Afouda A, Amani A (Eds) (2004) Reducing West Africa’s Vulnerability to Climate Impacts on Water Resources, Wetlands and Desertification. Elements for a Regional Strategy for Preparedness and Adaptation. IUCN, Gland (Switzerland) and Cambridge (UK).
- Obuobie, E., 2008. Estimation of groundwater recharge in the context of future climate change in the White Volta River Basin. Doctoral thesis. West Africa.
- Odada EO (2006) Freshwater resources of Africa: major issues and priorities. *Global Water News*, No. 3. http://www.gwsp.org/downloads/GWSP_NL3_Internet.pdf.
- Ofori S. 2005. The Hydrogeology of the Voltaian Formation of the Northern Region of Ghana. M.Sc. Thesis. University of Reno, Nevada, U.S.A
- Oguntunde PG (2004) Evapotranspiration and complementarity relations in the water balance of the Volta Basin: field measurements and GIS-based regional estimates. *Ecology and development series No. 22*. ZEF Bonn. Cuvillier Verlag Göttingen.
- Opoku-Ankomah Y, Dembélé Y, Ampomah BY, Somé L (2006) Hydro-political assessment of water governance from the top-down and review of literature on local level institutions and practices in the Volta Basin. Colombo, Sri Lanka. International Water Management Institute. 36p. IWMI Working Paper 111. www.iwmi.cgiar.org/pubs/working/WOR111.pdf

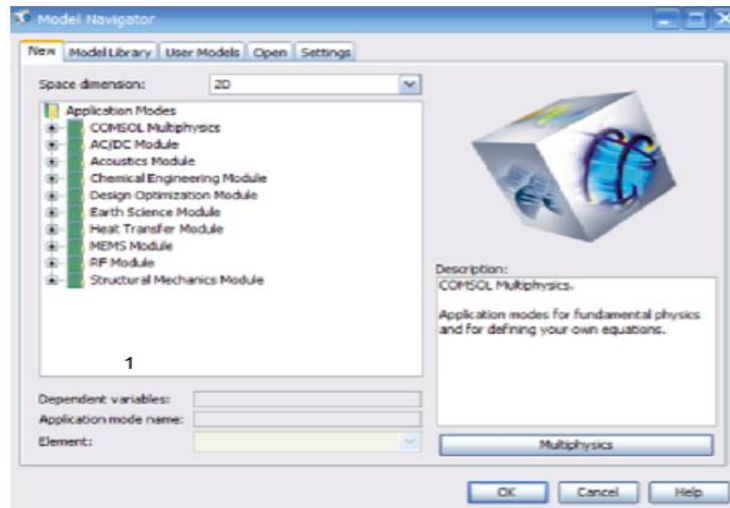
- Opoku-Ankomah Y (2000) Impacts of Potential Climate Change on River Discharge in Climate Change Vulnerability and Adaptation Assessment on Water Resources of Ghana. Water Research Institute (CSIR), Accra. Ghana.
- Pelig-Ba K. B., Kortatsi B. K. and Edmunds W. M. (1990). Application of isotope techniques in Groundwater studies in the Upper Regions of Ghana. Paper presented at the Regional Seminar on Isotope in Hydrology for Developing Countries in Africa from 15th to 19th October 1990 in Vienna, Austria.
- Perkins T. K. and Johnston O. C (1963) A review of diffusion and dispersion in porous media. Soc. Petrol. Eng. J. 3, 70 – 83.
- Reilly, T.E., (2001) System and Boundary Conceptualization in Groundwater Flow Simulation: Techniques of Water-Resources Investigations of the U.S Geological Survey, Book 3, Chapter B8, 30p.
- Sanford W (2002) Recharge and groundwater models: An overview. Hydrogeol. J., 10: 110-120.
- Scanlon BR, Healy RW, Cook PG (2002) Choosing appropriate techniques for quantifying groundwater recharge. Hydrogeology Journal (2002) 10:18–39. DOI 10.1007/s10040-0010176-2. <http://dx.doi.org/10.1007/s10040-001-0176-2>.
- Servat E, Paturol JE, Lubès H, Kouamé B, Ouedraogo M, Masson JM (1997) Climatic variability in humid Africa along the Gulf of Guinea, Part I: Detailed analysis of the phenomenon in Côte d'Ivoire, Journal of Hydrology, 191, 1-15.
- Sonntag C., Klitsch E., Lohnert E. P., El Shzly E. M., Munnich K. O., Junghans C., Thornweithe V., Weistroffer K. and Swalem F. M. (1979). Palaeoclimatic information from deuterium and oxygen-18 in carbon-14-dated north Saharan groundwaters: Groundwater formation in the past, Isotope Hydrology. Proc. Symp. Neuharberg (1978). IAEA, Vienna. 569 pp.
- Sophocleous MA, Schloss JA (2000) Estimated Annual Groundwater Recharge. <http://www.kgs.ukans.edu/HighPlains/atlas/atrch.htm>, Lawrence, Kansas.

- Todd, D.K. Annotated Bibliography on Artificial Recharge of Ground Water through 1954. U.S. Geological Survey Water-Supply Paper 1477, 115 p.1959.
- Todd, D.K. Groundwater Hydrology, 2nd ed. Wiley, 535 p., 1980.
- Tóth J.A (1963). Theoretical analysis of groundwater flow in small drainage basins. Journal of Geophysical Research 68, 4795–4812. [A classic article delineating groundwater flow systems.]
- UNEP (2002) State of the environment and policy retrospective: 1972-2002. In: Global environment outlook 3: past, present and future retrospectives. Accessed on December 17, 2012, at <http://www.grid.unep.ch/geo/press.htm>
- Van de Giesen NC, Andreini M, van Edig A, Vlek PLG (2001) Competition for water resources of the Volta basin. Regional Management of Water Resources. Proceedings of a symposium held during the Sixth IAHS. Scientific Assembly at Maastricht, the Netherlands, July 2001. IAHS Publ. no. 268. 2001.
- VBRP (2002) Volta Basin Research Project water resources management GIS maps on the Volta Basin. Accra, Ghana.
- WARM, (1998) Water resources management study, information ‘building block’ study. Part II, Volta basin system, groundwater resources. Accra: Ministry of Works and Housing.
- WRC-Groundwater Management: Accessed on June 15, 2013, wrc-gh.org/en/water-resources-mgt/ground-water-mgt.
- WRI-Water Resources Research Institute / DANIDA (1993) “Rural Drinking Water Supply and Sanitation Project in the Volta Region – Final Report on the Inventory and Assessment of Potential for Hand Dug Wells in the Volta Region, Vol I – Main Report.” WRI, Accra.
- Xu Y, Beekman HE (Eds) (2003) Groundwater recharge estimation in Southern Africa. UNESCO IHP Series No. 64, UNESCO Paris. ISBN 92-9220-000-3.

APPENDIX A

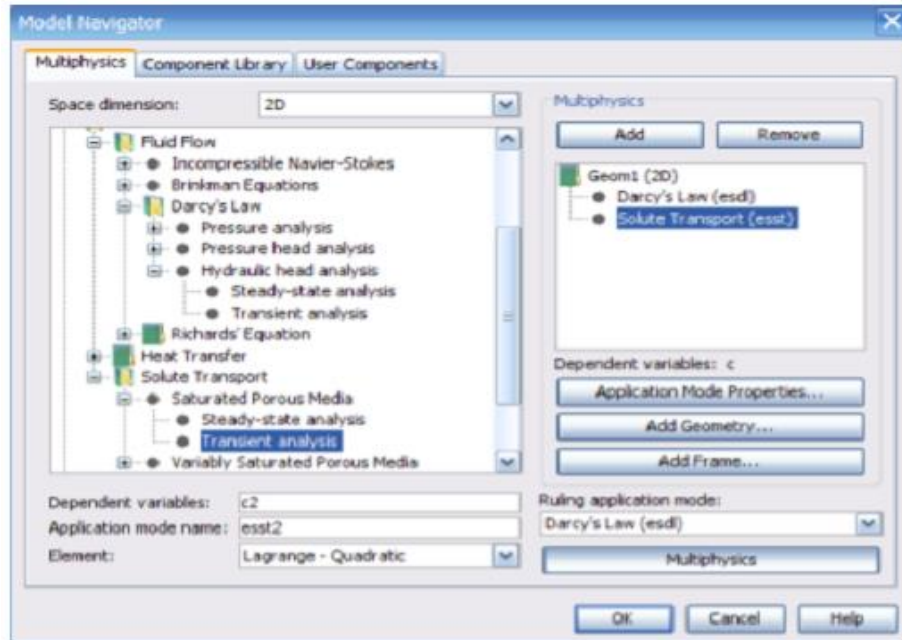
ALGORITHM FOR SIMULATION USING COMSOL GRAPHICAL USER INTERFACE:

1. The Model Navigator was opened from COMSOL Multiphysics shortcut from the Desktop.

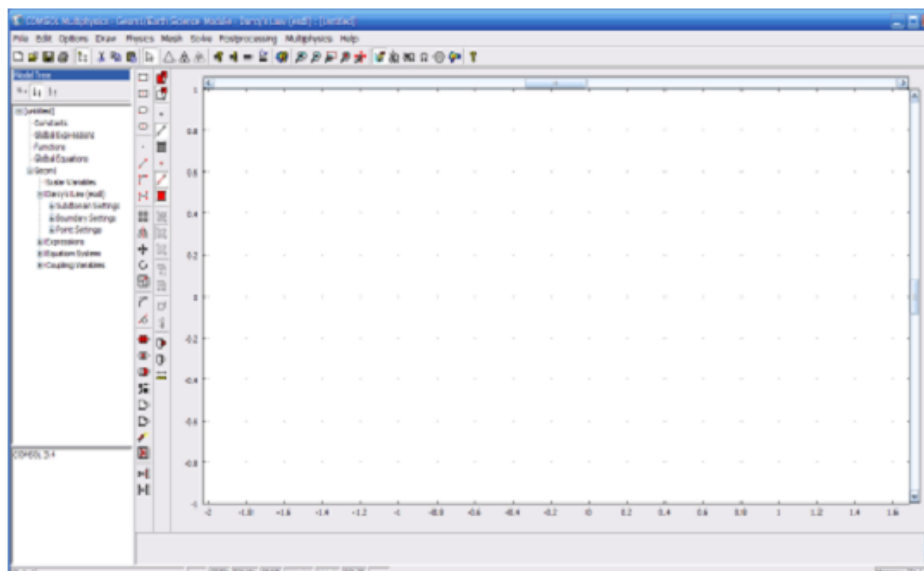


MODELING FLOW

2. The New tab was chosen, and the space dimensions set to 2-D.
3. In the Application Mode → the Steady State analysis was reached through the Earth Science Module → Fluid Flow → Darcy's Law →Hydraulic Head →Add.
4. The application mode, Earth Science Module→Solute Transport→Saturated Porous Media→Transient analysis was selected and added.
Darcy's Law (**esdl**) and Solute Transport (**esst**) is now obtained in the folder under Geom1 in the upper right of the dialog box.

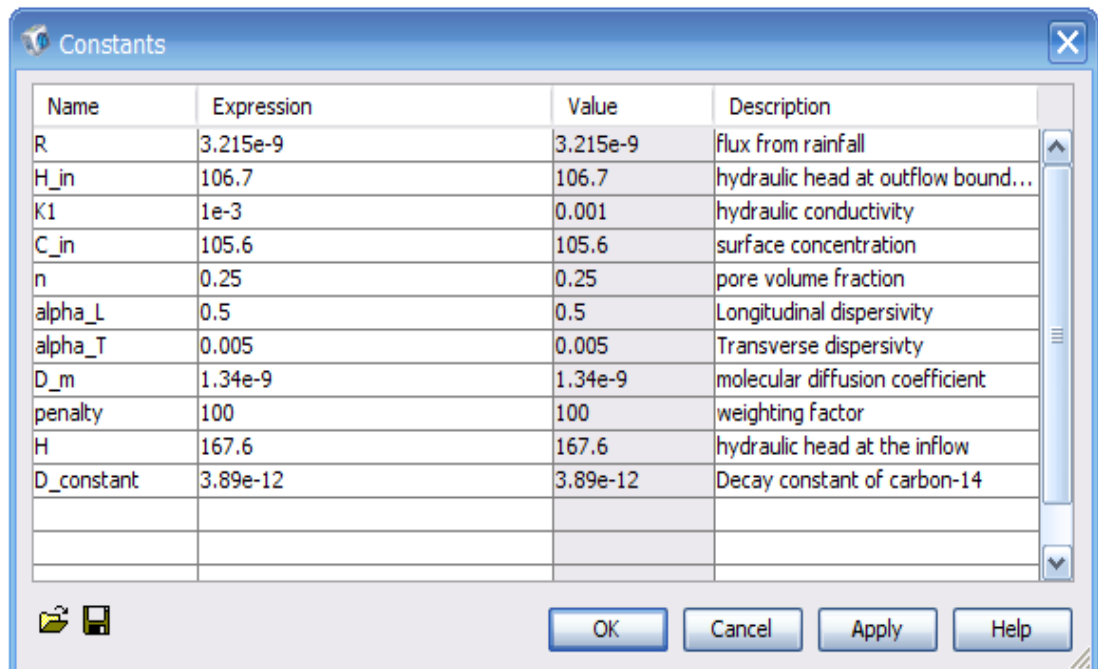


5. Before clicking OK, the Solute Transport (esst) was selected, and in the Application Mode Properties button, the Material was changed to Liquid.
6. The ruling application mode was changed to Solute Transport (esst); and OK selected to open the default Drawing Environment.



OPTIONS AND SETTINGS

7. The model constants were defined by entering their names, expressions and (optionally) descriptions from the Options menu. OK

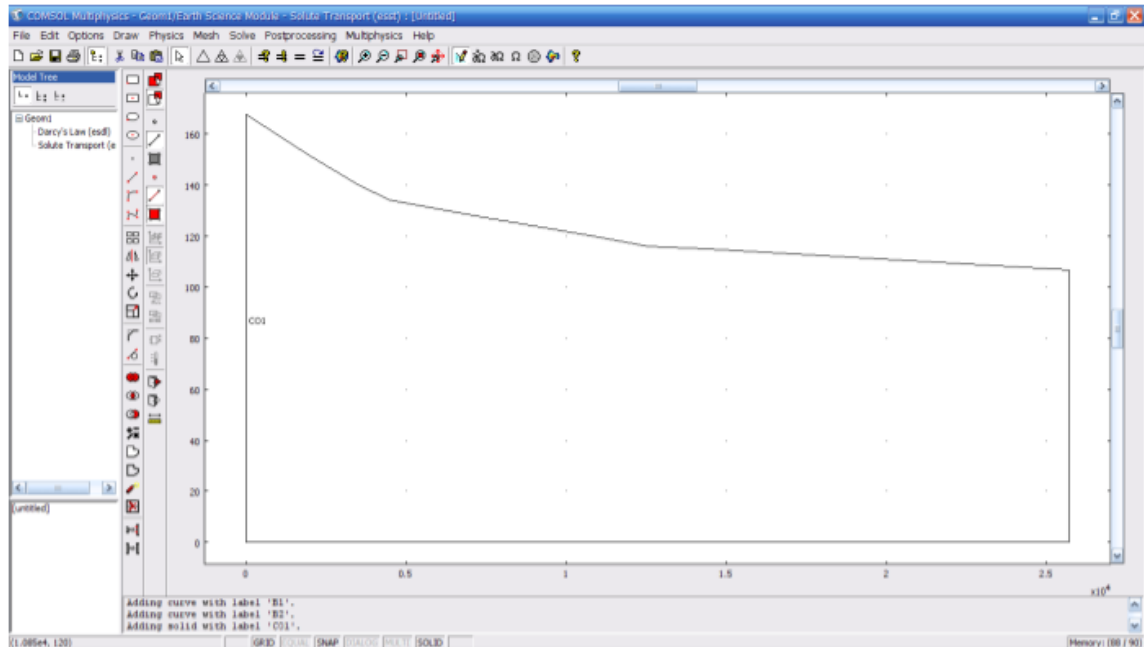


GEOMETRY MODELING

The flow domain was created in the drawing area by the following actions.

8. Shift-click the Line button on the Draw toolbar. In the Line dialog box that opened, the following data were entered: OK

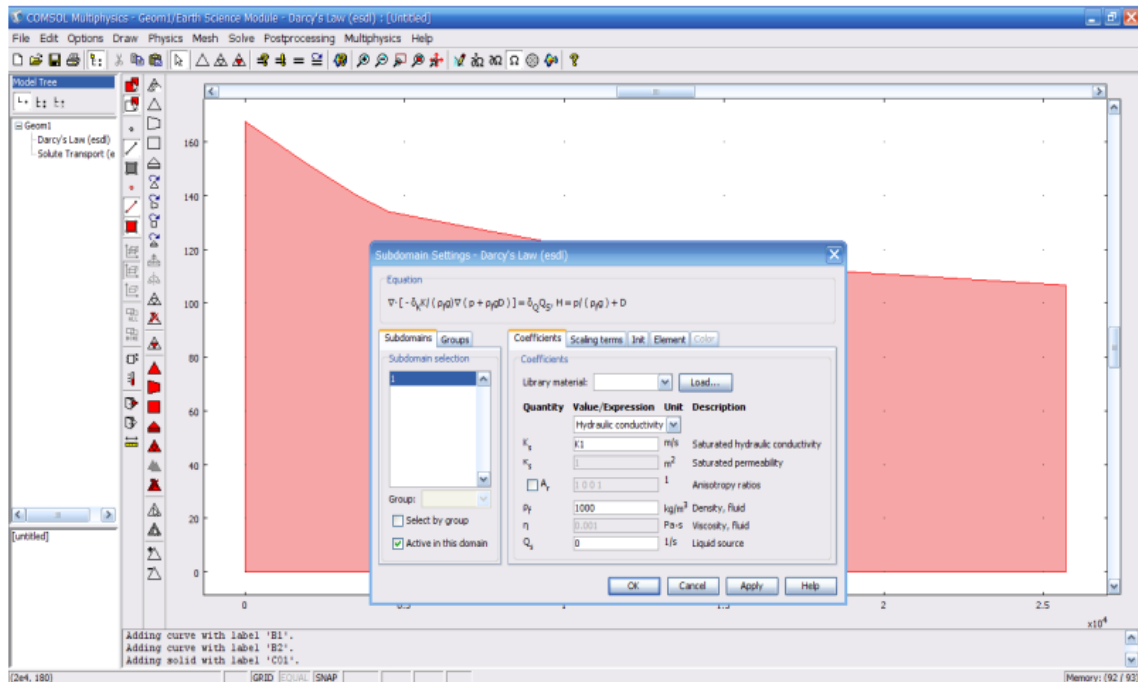
x	0, 25700, 25700, 14400, 12550, 10850, 7500, 6200, 4550, 3500, 1900, 0.
y	0, 0, 106.7, 115, 116, 120, 127, 130, 134, 140, 152, 167.6
style	Closed polyline (solid)



PHYSICS SETTINGS

Subdomain Settings—Darcy's Law

9. Subdomain Settings dialog box for the Darcy's Law application mode was reached under the menu item, Multiphysics→1 Darcy's Law (**esdl**) and then Physics→Subdomain Settings. A material property for K_s was entered, all other settings kept at their default values.



BOUNDARY CONDITIONS—DARCY'S LAW

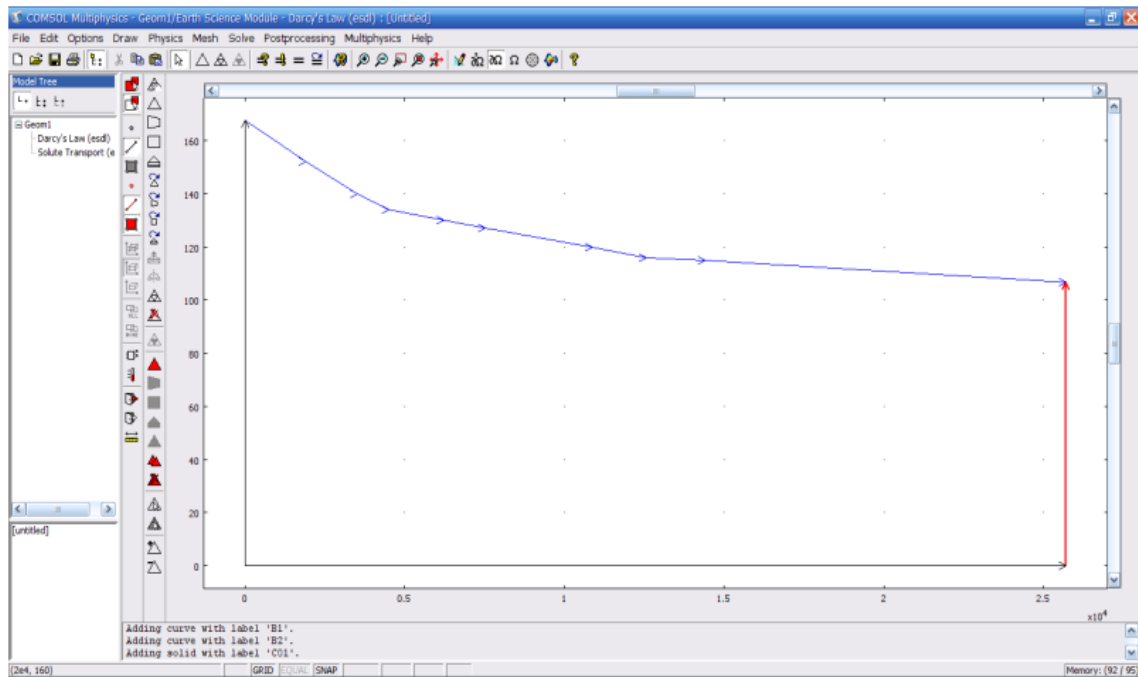
The boundary conditions for the Darcy's Law (**esdl**) were set by the following actions:

10. Model Tree → Darcy's Law (**esdl**) → Boundary Settings.

11. The boundary conditions were entered according to the following table. OK.

SETTINGS	BOUNDARIES 1,2	BOUNDARY 3-11	BOUNDARIES 12
Boundary condition	Zero flux/Symmetry	Inward flux	Hydraulic head
H_0			H_{in}
N_0		R	

It was noticed that COMSOL color's the no-flow boundaries black, and the head boundaries in colour once the boundary conditions were set.

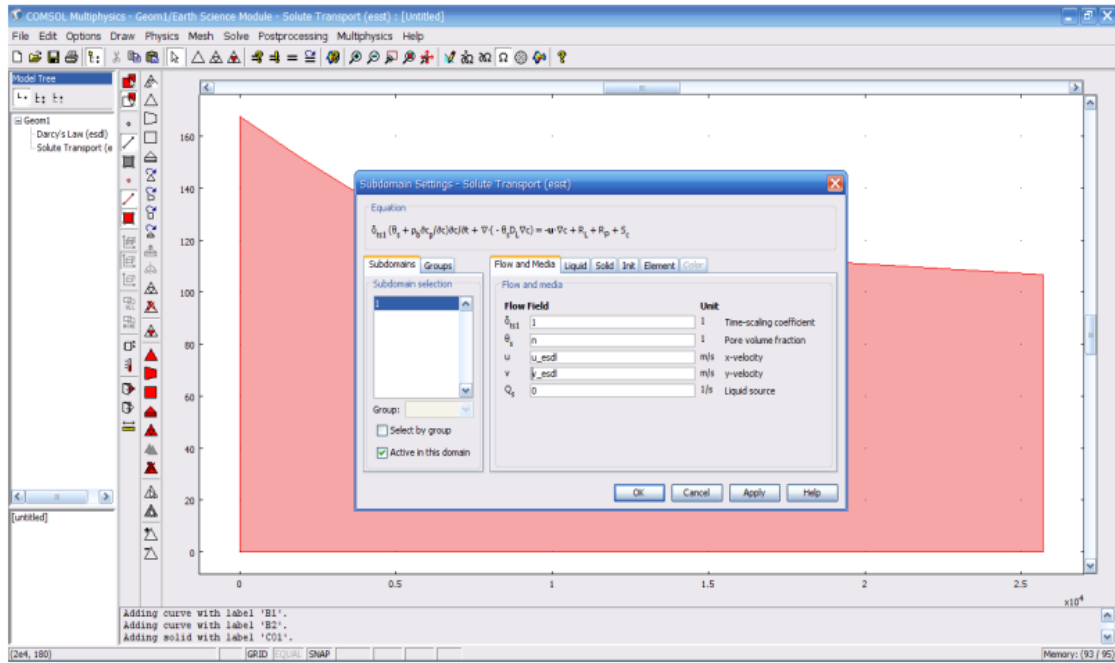


SUBDOMAIN SETTINGS—SOLUTE TRANSPORT

12. The Solute Transport (esst) → Subdomain Settings dialog was reached in the Model Tree.
13. On the Flow and Media page, the following material properties were entered:

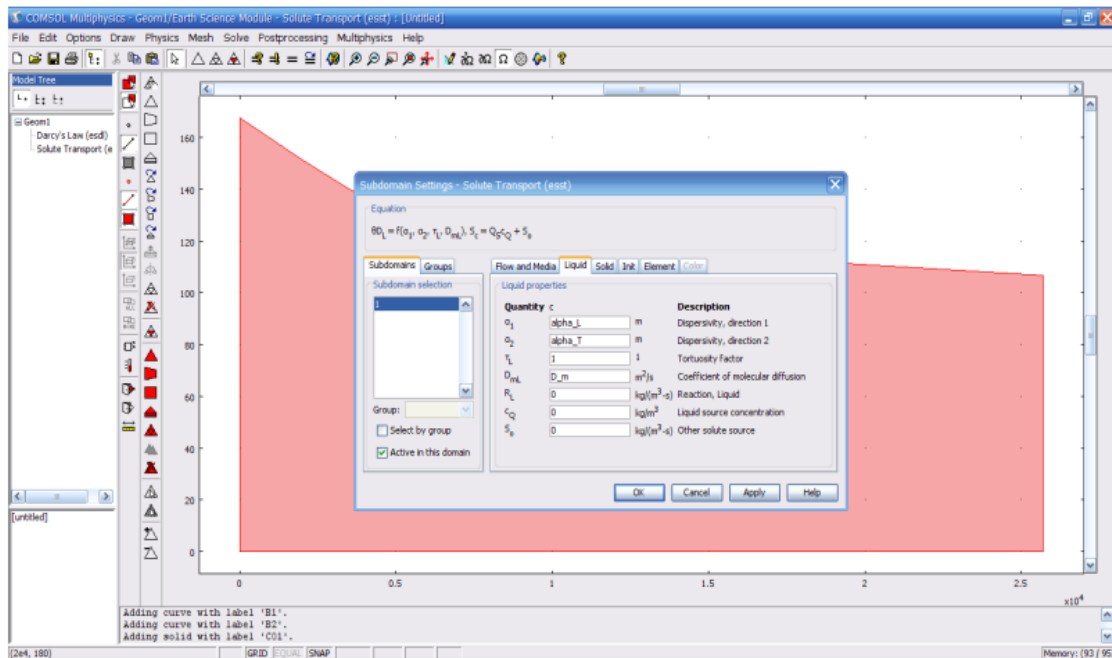
SETTINGS	SUBDOMAIN 1
θ_s	n
u	u_esdl
v	v_esdl

By setting the solute velocity components to **u_esdl** and **v_esdl**, the advective term in the equation is linked to the flow result governed by the Darcy flow equations.



14. On the Solute page, the following material properties were entered:

SETTINGS	SUBDOMAIN 1
α_1	alpha_L
α_2	alpha_T
D_{mL}	D_m



15. The action was completed by Clicking Ok.

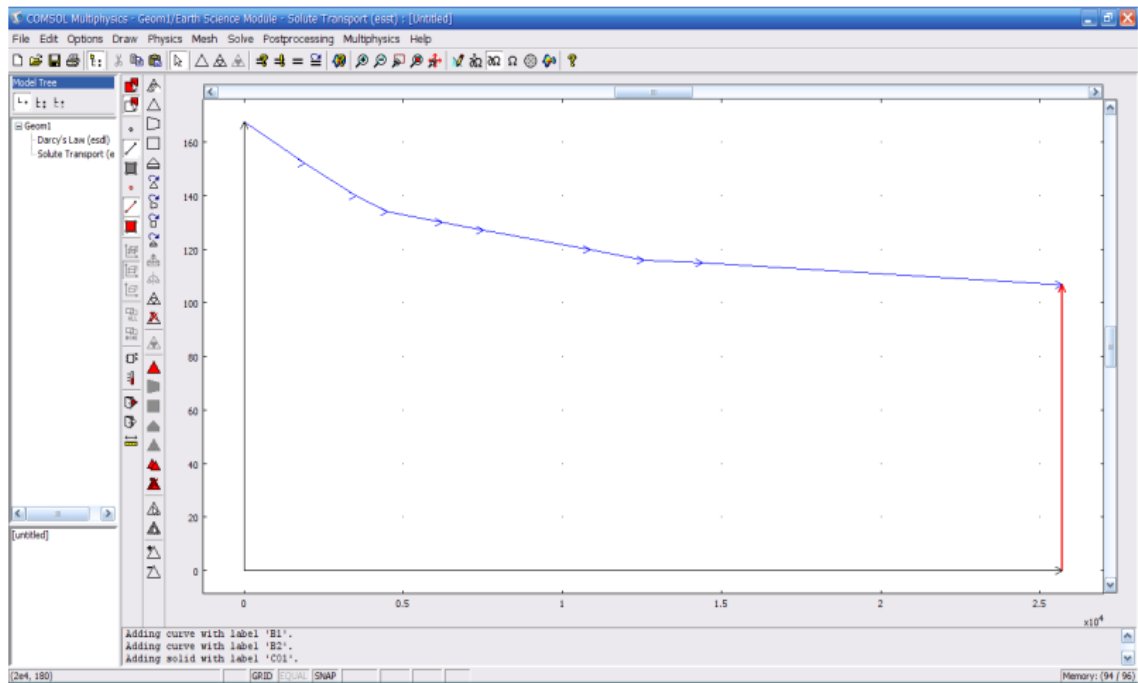
BOUNDARY CONDITIONS—SOLUTE TRANSPORT

The boundary conditions for the Solute Transport (**esst**) were set by the following actions:

16. Model Tree → Solute Transport (**esst**) → Boundary Settings.

17. The boundary conditions were entered according to the following table.

Settings	Boundary 1	Boundary 2	Boundaries 3 - 11	Boundary 12
Boundary condition	Concentration	No flux/ Symmetry	Flux	Advective flux
C_o	0			
N_o			$C_{in} * R * (1500 \leq x) * (x \leq 12550) * (t \leq 20 * 360 * 86400)$	

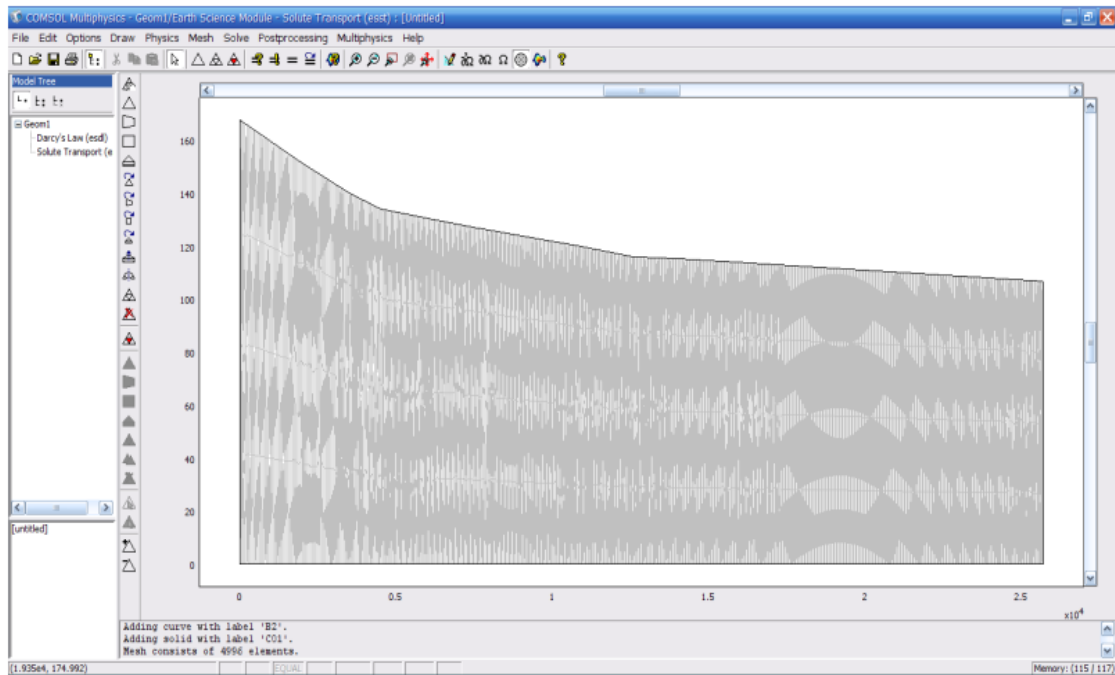


18. The action was completed by Clicking Ok.

The inward flux value at the top boundary (Boundary 3-11) is a Boolean expression that deposited a flux of $C_{in} \cdot R$ along the top surface from $x = 1500$ m to 12550 m and from $t = 0$ until 20 years.

CREATE INITIAL MESH

19. The Free Mesh Parameters was chosen from the Mesh menu.
20. On the Global page, the Custom mesh size was chosen.
21. The Resolution of narrow regions was set to 4. This number specifies the minimum number of mesh layers in narrow regions of the model geometry.



22. The Remesh button was selected to complete the meshing.

COMPUTING THE SOLUTION

23. The Solver was set to Stationary from the Solver Parameters dialog box.
24. Click OK.
25. In the Solver Manager, the Solve For tab, Darcy's Law module for (pressure) was selected.
26. The Solve button was selected to compute the steady-state solution.
27. The solute transport (**esst**) was selected.
28. The Solver Parameters dialog box was opened and Time Dependent selected from the solver list.
29. 0:360*86400:20*360*86400 was entered in the times edit field.
30. The restart button was selected to compute a new solution.

POSTPROCESSING AND VISUALIZATION

31. In the **Postprocessing** menu, **Plot Parameters** was chosen.
32. In the **Plot Parameters** dialog box, the **Surface** tab was selected.
33. From the Predefined quantities list, Solute Transport (**esst**) → Concentration, **c** was selected.
34. From the Streamline tab, the Streamline plot check box was selected.
35. Under Streamline data, hydraulic head gradient in the Predefined quantities list is selected.

- 36.** In the Specify start point coordinates button, the following values were entered.

x - 1500 3000 4500 6000 7500 9000 13000 14500 16000 17500 19000 20500 22000 23500
y - 150 150 150 150 150 150 150 150 150 150 150 150 150

- 37.** The Color legend check box was selected, and then applied.
- 38.** The Plot Parameters dialog box was opened again.
- 39.** On the Surface page, concentration, c as Surface data was selected.
- 40.** On the Contour page, c as Contour data was select.
- 41.** the vector with isolevels button was opened and the following data were entered in the edit field: 160,150,140,130,120,110,100,90,80.70,60,50,40,20
- 42.** On the General page, the Contour check box was selected and a new solution computed.