

## Determination of the series resistance of a solar cell through its maximum power point

H. A. Koffi, A. A. Yankson, A. F. Hughes, K. Ampomah-Benefo & J. K. A. Amuzu

To cite this article: H. A. Koffi, A. A. Yankson, A. F. Hughes, K. Ampomah-Benefo & J. K. A. Amuzu (2020): Determination of the series resistance of a solar cell through its maximum power point, African Journal of Science, Technology, Innovation and Development, DOI: [10.1080/20421338.2020.1731073](https://doi.org/10.1080/20421338.2020.1731073)

To link to this article: <https://doi.org/10.1080/20421338.2020.1731073>



Published online: 23 Mar 2020.



Submit your article to this journal [↗](#)



Article views: 23




View related articles [↗](#)



View Crossmark data [↗](#)

---

## Determination of the series resistance of a solar cell through its maximum power point

H. A. Koffi <sup>\*</sup>, A. A. Yankson, A. F. Hughes, K. Ampomah-Benefo and J. K. A. Amuzu

Department of Physics, University of Ghana, Accra, Ghana

\*Corresponding author email: hakoffi@ug.edu.gh, Drazokoff@gmail.com

A simple analytical approach has been developed to determine the series resistance,  $R_s$ , of a solar cell. The method adopted here depends only on the knowledge of the open-circuit voltage,  $V_{oc}$ , and the current and voltage at the maximum power point,  $I_{sc}$  and  $V_{mp}$  respectively. This approach, based on a knowledge of these operating output parameters of the cell, provides a theoretical framework for an existing computer simulated approach which has been widely used in industries.

**Keywords:** solar cell, five-parameter model, series resistance, light generated current, shunt resistance, short-circuit current

### Introduction

Modelling a solar cell is a basic means for extracting the effective operating values of the parameters governing the behaviour of the device. The most widely considered among existing models is the five-parameter model. In that model, the equation governing the behaviour of the cell is formulated as a transcendental exponential equation involving five parameters namely the light generated current ( $I_L$ ), the reverse saturated current ( $I_s$ ), the operating temperature ( $T$ ), the shunt resistance ( $R_{sh}$ ), and the series resistance ( $R_s$ ).

The most investigated parameters out of the five is the series resistance which represents the sum of the contact resistance in the metal–semiconductor contacts, the ohmic resistance in the metal contacts, and the ohmic resistance in the semiconductor material (Goetzberger, Knobloch, and Voss 1998, 79). This is due to the fact that the series resistance affects the output power of the solar cell more than any other parameter (Goetzberger, Knobloch, and Voss 1998). Also, to determine one or more of these parameters, different approaches have been developed although some are somehow cumbersome (Koffi 2013). In this paper, an analytical approach has been developed to underpin an existing computer-based solution employed in determining the series resistance of a solar cell. To do so, an alternative formulation of the five-parameter model equation of a solar cell has been used with the view of establishing a simple method to determine the series resistance of a solar cell. Indeed, this method helps to determine in a simple way the series resistance of any operating solar cell by knowing its open circuit voltage and the voltage and current at its maximum power point which are three parameters easily accessible from any  $I$ - $V$  curve or manufacturer datasheet.

### Background

In the five-parameter model, a solar cell is represented by the equivalent circuit shown in Figure 1: The circuit consists of:

- (i) a light-based current generator representing the source of current production when the inherent voltage across the junction is connected to an external load;

- (ii) a diode expressing the requirement of a threshold energy level for the photons to trigger a significant production and circulation of paired electrons and holes across the junction;
- (iii) a series resistance representing the metal–semiconductor contact resistance, the ohmic resistance in the metal contacts, and the ohmic resistance in the semiconductor material;
- (iv) a shunt resistance representing the leaking currents along the edges of the solar cell.

From Figure 1, the current produced by the solar cell is equal to that produced by the current source,  $I_L$ , minus that which flows through the diode,  $I_D$ , minus that which flows through the shunt resistor,  $I_{sh}$ , so that the equation governing the current flowing across the load is given by equation 1:

$$I = I_L - I_s \left[ e^{\left[ \frac{q(V + IR_s)}{Ak_B T} \right]} - 1 \right] - \frac{V + IR_s}{R_{sh}} \quad (1)$$

where  $I$  is the output current,  $T$  the operating temperature,  $I_s$  the reverse saturation current,  $q$  the unit electric charge,  $K_B$  the Boltzmann constant and  $A$  the ideality factor, a constant that measures how closely the basic Shockley equation describes the ideal diode that is set to unity in the five-parameter model. The relevant five parameters of the model are  $T$ ,  $I_L$ ,  $I_s$ ,  $R_s$ ,  $R_{sh}$ .

Several approaches, ranging from analytical to computer-based solutions, exist to determine the series resistance of a solar cell and one of the earliest approaches to the solution of equation 1 is experiment-based and involves the determination of  $R_s$  through the measurement of the output current of the cell which is exposed to light of known intensity (Rauschenbach and Wolf 1963). However, analytical approaches providing more accurate results have been widely used (Sharma et al. 1993). In some cases, three characteristic points, namely the short-circuit-current ( $I_{sc}$ , 0), the open-circuit voltage (0,  $V_{oc}$ ) and the maximum power point ( $I_{mp}$ ,  $V_{mp}$ ) derived from the current–voltage ( $I$ - $V$ ) curve, are then used to express the current as follows (Lasnier and

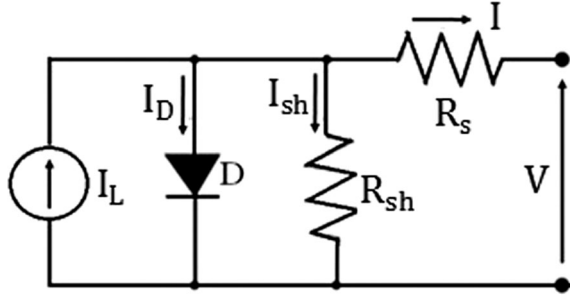


Figure 1. Diagram of the five-parameter model of a solar cell.

Sivoththaman 1990):

$$I = I_{SC} \left[ 1 - C_1 e^{\left[ \frac{V}{C_2 V_{oc}} - 1 \right]} \right] \quad (2)$$

where  $I_{SC}$  is the short-circuit current,

$$C_1 = \left( 1 - \frac{I_m}{I_{sc}} \right) e^{\left[ \frac{-V_m}{C_2 V_{oc}} + 1 \right]} \quad (3)$$

And

$$C_2 = \frac{\left[ \frac{V_m}{V_{oc}} - 1 \right]}{\log \left[ 1 - \frac{I_m}{I_{sc}} \right]} \quad (4)$$

Concurrently with analytical solutions, special functions-based forms of solutions also exist. Among them, the Lambert W-function is used to determine the series resistance of the solar cell (Jain and Kapoor 2004). Also, the Special Trans Function Theory (STFT) is used to determine the series resistance of the solar cell. (Singh, Jain, and Kapoor 2009).

On the other hand, digital forms or computer-based solutions of the equations governing the output of solar devices are most often used when the behaviour of the device is simulated either in a laboratory or with a computer. Among them are the Sandia National Labs Model (King, Boyson, and Kratochvill 2004) and the Agilent Technology method to determine the series resistance and shunt resistances. The Agilent Technology method is an empirical computer-based analytical determination of  $R_s$ . In this method (Agilent 2020), a graphical approach (Figure 2) and a set of equations (equations 5, 6, 7 and 8) are used to determine the series resistance.

This approach yields the following results:

$$V = \frac{V_{oc} \times \ln \left[ 2 - \left( \frac{I}{I_{sc}} \right)^N \right] - R_s \times (I - I_{sc})}{1 + \frac{R_s \times I_{sc}}{V_{oc}}} \quad (5)$$

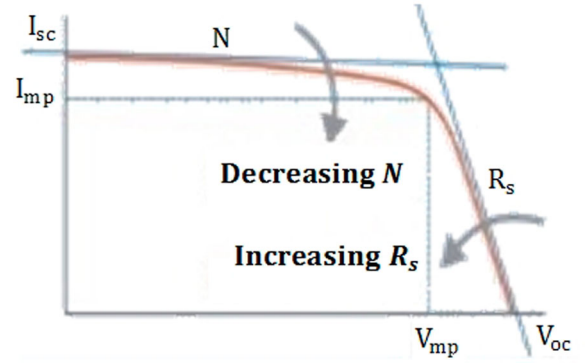


Figure 2: The Agilent method to determine the series and the shunt resistances (Agilent 2020).

$$R_s = \frac{V_{oc} - V_{mp}}{I_{mp}} \quad (6)$$

$$N = \frac{\ln(2 - 2^a)}{\ln\left(\frac{I_{mp}}{I_{sc}}\right)} \quad (7)$$

$$a = \frac{V_{mp} \times \left( 1 + R_s \times \frac{I_{sc}}{V_{oc}} \right) + R_s \times (I_{mp} - I_{sc})}{V_{oc}} \quad (8)$$

where  $V$  is the output voltage,  $V_{mp}$  and  $I_{mp}$  the voltage and current at the maximum power point respectively, and  $a$  and  $N$  are inbuilt algorithm-based parameters with  $N$  strongly related to the shunt resistance.

#### Determination of the series resistance

The analytical approach to the determination of  $R_s$  discussed in this work is based on the fact that our previous work has established that equation 1 can be re-arranged as follows (Koffi et al. 2015):

$$I = I_{SC} - I_{SR} \left[ e^{\left[ \frac{q(V + IR_s)}{Ak_B T} \right]} - 1 \right] - \frac{V}{R_s + R_{sh}} \quad (9)$$

with

$$I_{SC} = \frac{I_L}{\tau} \quad (10)$$

$$I_{SR} = \frac{I_S}{\tau} \quad (11)$$

$$\tau = 1 + \frac{R_s}{R_{sh}} \quad (12)$$

and  $A$ ,  $k_B$  and  $T$  have their usual meanings.

Since the reverse saturation current is as follows (Sze 1981, 796–804):

$$I_s = I_L \left[ e^{\left[ -\frac{q(V_{oc})}{Ak_B T} \right]} \right] \quad (13)$$

then

$$I_{SR} = I_{SC} \left[ e^{\left[ -\frac{q(V_{oc})}{Ak_B T} \right]} \right] \quad (14)$$

so that equation 1 becomes

$$I = I_{SC} \left\{ 1 - \left[ e^{\left[ \frac{q(V + IR_s - V_{oc})}{Ak_B T} \right]} - e^{\left[ \frac{-qV_{oc}}{Ak_B T} \right]} \right] \right\} - \frac{V}{R_s + R_{sh}} \quad (15)$$

The  $I$ - $V$  characteristics describing equation 9, including the effect of variation in the series resistance, are presented in Figure 3.

For usual operating temperatures in the range  $300 \text{ K} \leq T \leq 350 \text{ K}$  and  $A=1$ , one can estimate that  $e^{\left[ \frac{-q}{Ak_B T} \right]} \equiv e^{[-34]} \cong 1.7 \times 10^{-15}$ . Therefore, in equation 15, for any solar device for which  $V_{oc} \geq 1 \text{ v}$ , the second exponential term,  $e^{\left[ \frac{-qV_{oc}}{Ak_B T} \right]}$  can be neglected for any value of operating temperature in the usual range.

Also, Figure 3 shows that the effect of increasing series resistance is to reduce the area under the curve which gives the maximum power generated by the cell (Worldscibooks 2020). It also shows that the influence of the series resistance is more significant after the maximum power point where the exponential factor controls the shape of the  $I$ - $V$  curve.

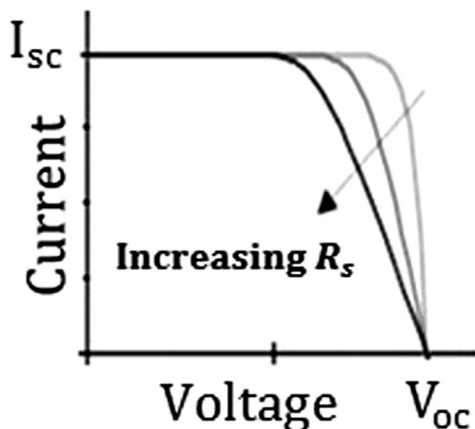


Figure 3:  $I$ - $V$  characteristic with effect of increasing series resistance (Worldscibooks 2020).

Secondly, for any value of  $R_{sh}$  and since  $R_{sh} \rightarrow \infty$  for an ideal cell,  $e^{\left[ \frac{q(V + IR_s)}{Ak_B T} \right]} \gg \frac{V}{R_s + R_{sh}}$  and the contribution of the linear term in equation 15 can also be neglected so that at optimal conditions, equation 15 becomes

$$I = I_{sc} \left[ 1 - e^{\left[ \frac{q(V + IR_s - V_{oc})}{Ak_B T} \right]} \right] \quad (16)$$

and the output power given by

$$P = |VI| = \left| VI_{sc} \left[ 1 - e^{\left[ \frac{q(V + IR_s - V_{oc})}{Ak_B T} \right]} \right] \right| \quad (17)$$

At the maximum output power,  $\frac{dP}{dV} = 0$  and since  $\frac{dP}{dV} = I \frac{\partial V}{\partial V} + V \frac{\partial I}{\partial V} = I + V \frac{\partial I}{\partial V}$ , then

$$\begin{aligned} \frac{dP}{dV} &= I_{sc} \left[ 1 - e^{\left[ \frac{q(V + IR_s - V_{oc})}{Ak_B T} \right]} \right] + VI_{sc} \\ &\times \left[ -\frac{q}{Ak_B T} \left( 1 + R_s \frac{\partial I}{\partial V} \right) \right] e^{\left[ \frac{q(V + IR_s - V_{oc})}{Ak_B T} \right]} \end{aligned} \quad (18)$$

$$\frac{dP}{dV} = I_{sc} \left\{ 1 - \left[ 1 + \frac{qV}{Ak_B T} \left( 1 + R_s \frac{\partial I}{\partial V} \right) \right] e^{\left[ \frac{q(V + IR_s - V_{oc})}{Ak_B T} \right]} \right\} \quad (19)$$

Previous work (Rodriguez and Amaratunga 2007) has established that

$$\left. \frac{\partial I}{\partial V} \right|_{V_{oc}} = -\frac{1}{R_s} \quad (20)$$

and Koffi (2013) has shown that equation 20 is equally applicable at the maximum power point:

$$\left. \frac{\partial I}{\partial V} \right|_{V_{mp}} = -\frac{1}{R_s} \quad (21)$$

so that for equation 19, at the maximum power point,

$$\frac{qV}{Ak_B T} \left( 1 + R_s \frac{\partial I}{\partial V} \right)_{V_{mp}} = 0 \quad (22)$$

leading to

$$\left. \frac{dP}{dV} \right|_{V_{mp}} = I_{SC} \left\{ 1 - e^{\frac{q[V_{mp} + (I_{mp} \times R_s) - V_{oc}]}{Ak_B T}} \right\} \quad (23)$$

Clearly, therefore, for  $I_{SC} \neq 0$ ,  $\frac{\partial P}{\partial V} = 0$  implies

$$e^{\frac{q[V_{mp} + (I_{mp} \times R_s) - V_{oc}]}{Ak_B T}} = 1 \quad (24)$$

Consequently, for any given value of  $T \neq 0$ ,

$$V_{mp} + (I_{mp} \times R_s) - V_{oc} = 0 \quad (25)$$

Hence the series resistance,  $R_s$ , is found to be

$$R_s = \frac{V_{oc} - V_{mp}}{I_{mp}} \quad (26)$$

Equation 26 is identical to the result obtained by the empirical computer-based analytical approach to the determination of the series resistance of a solar cell (equation 6).

### Conclusion

This work has formulated a simple approach to the determination of the series resistance of a solar cell. Indeed, it has shown that the series resistance is fully determined only by the knowledge of the open-circuit voltage and the current and voltage at the maximum power point. This result, based on a new formulation of the general equation guiding the behaviour of a solar cell, provides an analytical basis approach to underpin an existing computer simulated solution to the series resistance of a solar cell. This is shown by the identical formulation of our result (equation 26) to that of the Agilent Technology method (equation 6).

### ORCID

H. A. Koffi  <http://orcid.org/0000-0003-4783-5999>

### References

- Agilent Technology. 2020. "Generating I-V Curves with the Agilent E4360A Solar Array Simulator Using the Parameters Voc, Isc, N, and Rs." Accessed January 2, 2020. [www.agilenttechnology.com](http://www.agilenttechnology.com).
- Goetzberger, A., J. Knobloch, and B. Voss. 1998. *Crystalline Silicon Solar Cells*. Chichester, UK: John Wiley & Sons. ISBN 0-471-9744-8.
- Jain, A., and A. Kapoor. 2004. "Exact Analytical Solutions of the Parameters of Real Solar Cells Using Lambert W-Function." *Solar Energy Materials and Solar Cells* 81: 269–277.
- King, D., W. Boyson, and J. Kratochvill. 2004. *Photovoltaic Array Performance Model, SAND2004-3535*. Albuquerque, NM: Sandia National Laboratories.
- Koffi, H. A. 2013. "Influence of Climatic Parameters on the Performance of Silicon Solar Panels in Southern Ghana." PhD thesis, Department of Physics, University of Ghana.
- Koffi, H. A., A. Kuditcher, V. C. K. Kakane, E. A. Armah, A. A. Yankson, and J. K. A. Amuzu. 2015. "The Shockley Five-Parameter Model of a Solar Cell: A Short Note." *African Journal of Science, Technology, Innovation and Development* 7 (6): 491–494.
- Lasnier, F., and S. Sivonthaman. 1990. "Photovoltaic System Sizing and Performance by the Comparison of Demand and Expected Radiations." *International Journal of Solar Energy* 9: 65–76.
- Rauschenbach, H., and M. Wolf. 1963. "Series Resistance Effects on Solar Cell Measurements." *Advanced Energy Conversion* 3: 455–479.
- Rodriguez, C., and G. Amaratunga. 2007. "Analytic Solution to the Photovoltaic Maximum Power Point Problem." *IEEE Transactions on Circuits and Systems* 54 (9): 2054–2060.
- Sharma, S. K., D. Pavithra, N. Srinivasamurthy, and B. L. Agrawal. 1993. "Determination of Solar Cell Parameters: An Analytical Approach." *Journal of Physics D: Applied Physics* 26: 1130–1133.
- Singh, S. N., A. Jain, and A. Kapoor. 2009. "Determination of the Solar Cell Junction Ideality Factor Using Special Trans Function Theory (STFT)." *Solar Energy Materials and Solar Cells* 93: 1423–1426.
- Sze, S. M. 1981. *Physics of Semiconductor Devices*. 2nd ed. New York: Wiley & Sons.
- Worldscibooks. 2020. *The Physics of Solar Cells*. World Sciences Books. pp. 14. Accessed January 2, 2020. [www.Worldscibooks-comp.com/pdf](http://www.Worldscibooks-comp.com/pdf).