Modelling and forecasting volatility of the Botswana and Namibia stock market returns: Evidence using GARCH models with different distribution densities

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Abstract: This paper estimates and compares alternative distribution density forecast methodology of three generalised autoregressive conditional heteroscedasticity (GARCH) models for Botswana and Namibia stock market returns. The symmetric GARCH and asymmetric Glosten Jagannathan and Runkle (GJR) version of GARCH (GJR-GARCH) and exponential GARCH methodology are employed to investigate the effect of stock return volatility in both stock markets using Gaussian, Student-t and generalised error distribution densities. The evidence reveals that the current shocks to the conditional variance will have less impact on future volatility in both markets. News impact is asymmetric in both stock markets leading to the existence of leverage effect in stock returns. Besides, both markets exhibit reverse volatility asymmetry, contradicting the widely accepted theory of volatility asymmetry. Regarding forecasting evaluation, the results reveal that the symmetric GARCH model coupled with fatter-tail distributions present a better out-of-sample forecast for both stock markets.

Keywords: leverage effect; GARCH; EGARCH; GJR-GARCH; forecasting volatility; conditional variance; distribution densities.

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Biographical notes: William Coffie has a cumulative work experience of 17 years in Ghana, the UK and Hong Kong, which straddles four very important sectors in the public and private sectors: the academia, local government, charity and industry. Currently, he is a senior member of faculty at the University of Ghana Business School and formerly a Senior Lecturer and Course Director for MSc Finance and Accounting and MSc International Banking and Finance at University of Wolverhampton Business School. He has previously worked as a Lecturer in Finance, Econometrics, Accounting and Business at Birmingham City University, Staffordshire University, City College Birmingham, Birmingham Metropolitan College and Kumasi Polytechnic, Ghana. He was a Visiting Professor in Corporate Finance at Hong Kong City University from 2008 to 2012. His research interest lies in asset pricing and portfolio theory, volatility forecasting and correlations, time series and cross-sectional analysis, stock market efficiency, Africa and emerging markets finance and cost of capital.

1 Introduction

A prominent characteristic of high-frequency financial time series of stock returns is that they are frequently characterised by fat-tailed distribution. It is established in the finance literature as a matter of fact that the kurtosis of most financial asset returns is greater than 3 (Simkowitz and Beedles, 1980; Kon, 1984). This suggests that extreme values are much more likely to be observed in stock market returns than the normal distribution. Whereas the high kurtosis of stock market returns is established by sound evidence in the finance literature, the state of the symmetrical distribution is still obscure.

Empirical work document by Hsu et al. (1974), Hagerman (1978); Lau et al. (1990), and Kim and Kon (1994), show that stock returns follow non-normal distribution density. Their studies show that the kurtosis of time series of stock returns is greater than normal, the distribution is either skewed to the left or to the right and the variance of the stock returns is heteroscedastic (i.e., non-constant variance). This heteroscedasticity in the error variance is described as uncertainty or risk by the financial analyst and it has become important in modern theory of finance. Engle (1982) applies an econometric technique known as autoregressive conditional heteroscedasticity (ARCH) to model the time varying variances of UK inflation. Many researchers have applied the linear ARCH technique to model economic and financial time series since then. However, the linear ARCH (q) model requires a long lag length of q in many of its applications. In an attempt to resolve this empirical weakness of the ARCH, Bollerslev (1986) introduce a more flexible lag structure of the ARCH known as the generalised autoregressive conditional heteroscedasticity (GARCH). Some empirical works have shown that the first order lag length of the GARCH is adequate to model the long memory processes of time varying variance (French et al., 1987; Franses and Van Dijk, 1996).

Researchers into economic and financial time series have long identified that stock returns exhibit heavy-tailed distribution probability. One major reason for this heavy-tailed feature is that the conditional variance may be non-constant. Although excess kurtosis of stock returns can successfully be removed by GARCH model, it cannot cope with the skewness of the distribution of stock market returns. Therefore, forecast estimates from GARCH can be expected to be biased for a skewed time series. Latest econometric studies have seen the development of alternative nonlinear models which can take into account the skewed distribution, for example, the exponential GARCH (EGARCH) model, introduced by Nelson (1991). Besides, recent econometric softwares have been embedded in them alternative distribution densities for GARCH models (i.e., normal vis-à-vis non-normal). Stock market returns distribution has tails that are heavier than implied by the GARCH process with Gaussian. Therefore, by modelling financial time series such as stock returns, one must assumed not only Gaussian white noise but also independently identical distribution (i.i.d) white noise process with a heavy-tailed distribution.

Against this backdrop, this study fills the gap by introducing alternative density distribution methodology of symmetric and asymmetric GARCH models for Botswana and Namibia stock returns. The performance of GARCH (1, 1), GJR-GARCH (1, 1) and EGARCH (1, 1) models are compared with the introduction of different distribution densities (Gaussian, Student-t and GED). The study is motivated by recognising the importance of accurate volatility measurement and forecast in a wide range of financial applications and the non-existence of empirical evidence available to date for Botswana and Namibia stock markets. Furthermore, the paper contributes to the academic literature in three ways. First, dataset from emerging African stock markets are used, where such kind of study has not been conducted previously. Second, both symmetric and asymmetric GARCH models (i.e., GARCH vis-à-vis GJR and EGARCH) are applied. The latter do not only capture the time series features of skewness, kurtosis and volatility clustering but also the leverage effect. Third, in line with Nor and Shamiri (2007) study, this study compares the performance of GARCH-type models with the introduction of three different distribution densities (i.e., Gaussian versus non-normal) for modelling and forecasting the stock returns volatility of Botswana and Namibia stock markets. This addresses the methodological issue as to which GARCH-type model couple with distribution density variant better estimates and forecasts stock returns volatility in these uncharted stock markets of emerging Africa.

The next section presents the review of empirical literature and the empirical models are described in Section 3. Data description and methodology used in this study are offered in Section 4. The fifth section presents the results and analyses and the conclusions are presented in the final section.

2 Review of literature

Black (1976) study shows that fluctuation in stock price has an asymmetric impact on volatility. This feature in financial time series is known as leverage effect (i.e., large negative returns appear to increase volatility more than do positive returns of the same magnitude). The standard GARCH is found inadequate to model the dynamics of this leverage effect. Furthermore, Nelson (1991) and Glosten et al. (1993) respectively introduce the EGARCH and threshold GARCH (also known as GJR after its proponents) to account for this asymmetric response of volatility.

French et al. (1987) find evidence that the expected market risk premium is positively related to unpredictable volatility of stock returns. They used several estimates of the relation between the expected risk premium and the predicted volatility of New York Stock Exchange common stocks over the 1928 to 1984 period. Chong et al. (1999) use the rate of returns from the daily stock market indices of the Kuala Lumpur stock exchange including composite index, tins index, plantations index, properties index and finance index to evaluate the following models: stationary GARCH, unconstrained GARCH, non-negative GARCH, GARCH-M, and EGARCH and integrated GARCH ability to forecast stock market volatility. Their results show that EGARCH though did not have the best of goodness of fit statistics proved most effective in forecasting stock market volatility.

Elyasiani and Mansur (1998) use the stocks of traded commercial banks on the American stock exchange from the period of the study from January 1970 to December 1992 to investigate the effect of interest rate and its volatility on the bank stock return using GARCH-M. Their evidence shows that long-term interest rate has a negative and significantly impact on the bank stock return. They also find that interest rate volatility is found to be an important determinant of the bank stock return volatility and bank stock risk premium. Tan et al. (2012) in their macro analysis of interest rate volatility on stock market return in which they applied two separate GARCH(1, 1) models and data from the FBM Kuala Lumpur Composite Index (KLCI) and three months deposit yields with similar data from Singaporean market , asserted that interest rate volatility has a strong positive relationship with stock market volatility. They further find that there exist an insignificant inverse relationship between interest rate volatility and stock market return.

Chiang and Doong (2001) investigates time series behaviour of stock returns by constructing a meta-analysis including Malaysia, Philippines, Singapore, South Korea, Thailand, USA and Japan using the daily stock price indexes for these participating countries stock markets spanning January 1988 to June 1988. Their results show that higher levels of volatility are significantly correlated with higher average returns. A further review of the results using a TAR-GARCH (1, 1) unearthed the fact that, the size and significance level of the GARCH effect becomes smaller in weekly returns. They also rejected the hypothesis of no asymmetric effect at high significance level.

Ortiz and Arjona (2001) examines the time series characteristics of six major Latin American markets including Argentina, Brazil, Chile, Colombia, Mexico, from 1989–1994. They employed different GARCH models to typify the conditional heteroscedasticity characteristics for each market. Their results show that all the six markets studied were time dependent, heteroscedastic, and asymmetric, with both right and left skewness obvious. They further find that no single GARCH model is able to describe the stock returns volatility in these markets. Instead, they report that ‘different GARCH models are more appropriate for each country’, and that ‘the best models seem adequate’.

Nor and Shamiri (2007) scrutinises the modelling of high-frequency by examining the evaluations of three GARCH(1, 1) models (i.e., GARCH, EGARCH and GJR-GARCH) using daily price data from Singapore’s Strait Times Index (STI) and Malaysia’s KLCI over a 14-year period, from 2 January 1991 and ending on 31 December 2004. This was based on using the Gaussian normal, Student-t and generalised error distributions. The study was premised on the shortcomings of traditional regression tools for modelling high-frequency (weekly, daily or intra-daily) data such as the assumptions that are usually detached from reality that only the mean response changes with covariates, etc. Their results show that the forecasting performance of asymmetric GJR-GARCH and EGARCH are better than symmetric GARCH. They further finds that the AR (1)-GJR model provides the best out-of-sample forecast for the Malaysian stock market, while AR(1)-EGARCH provides a better estimation for the Singaporean stock market.

Leon (2007) investigates the relationship between stock market returns and volatility on the Bourse Regionale des Valeurs Mobilieres (BRVM), which is the Regional Financial Exchange for the eight French-speaking West African countries that form the West African Economic and Monetary Union. He employed EGARCH-M methodology to model weekly returns from 1999 to 2005 and found positive but insignificant relationship between conditional market returns and conditional volatility. Coffie and Chukwulobelu (2014) investigate volatility persistence by comparing evidence from selected emerging African and Western developed markets, taking into account the rate of volatility decay. GARCH and GARCH-in-mean (GARCH-M) models are used to estimate volatility persistence and risk premium for these markets. Their results show that volatility persists in the four emerging African markets and the five developed markets. Furthermore, Coffie (2015) examines volatility persistence in Southern and East African stock markets taking into account the rate of volatility decay. GARCH and GARCH-M models were used to estimate volatility persistence and risk premium for these markets. The result presented suggests that there is volatility persistence in emerging Southern and East African stock markets. Further empirical estimates show that rate of volatility decay varies considerably among the markets, for example, volatility in Mauritius diminishes to half of its original size within seven hours, while it takes almost eight months for volatility in Zambia to taper off to half of its original size.

While there has been extensive research on symmetric and asymmetric GARCH models in the academic literature since the introduction of ARCH/GARCH, GJR-GARCH and EGARCH (Engle, 1982; Bollerslev, 1986; Glosten et al., 1993; Nelson, 1991), less effort has been made towards comparing alternative density forecast models. Although Nor and Shamiri (2007) compare alternative density forecast models in Malaysia and Singapore, nothing is found on Emerging African Stock markets.

3 Empirical models

Two moment (i.e., mean and variance) equations are used to define the ARCH/GARCH models. The return process, r_t, is captured by the mean equation which is made up of the conditional mean, μ, which might encompass terms of autoregressive (AR) and moving average (MA) and error term, ε_t, that follows a conditional normal distribution with mean of zero and variance, σ^2. Furthermore, the information set available to investors up to time $t - 1$ is represented by, Ω_{t-1}, thus,

$$r_t = \mu + \varepsilon_t \tag{1}$$

where

$$\varepsilon_t \left| \Omega_{t-1} \approx N\left(0, \sigma_t^2\right) \quad \sigma_t^2 = h_t \right. \tag{2}$$

The conditional variance h_t is modelled using symmetric and asymmetric GARCH models with the introduction of three different distribution densities (i.e., Gaussian, Student-t and GED).

3.1 ARCH model

Engle (1982) seminal work proposed to model time varying conditional heteroscedasticity using past error term to estimate the series variance as:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \tag{3}$$

3.2 GARCH model

Bollerslev (1986) proposed the GARCH model which suggest that time varying heteroscedasticity is a function of both past innovations and past conditional variance (i.e., past volatility).The GARCH model represents an infinite order ARCH model express as:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{4}$$

where α_0, α_i and β_j are non-negative constants.

3.3 EGARCH model

Nelson (1991) introduced the EGARCH model to capture the asymmetric (or 'directional') response of volatility. Nelson and Cao (1992) argue that the imposition of non-negativity constraints on the parameters; α_i and β_j in the linear GARCH model are too restrictive, while in the EGARCH model there is no such restriction. The conditional variance, ht, in the EGARCH model is an asymmetric function of lagged disturbances as follows.

$$\ln(h_t) = \alpha_0 + \alpha\left(|z_{t-1}| - E\left(|z_{t-1}|\right)\right) + \gamma z_{t-1} + \beta \ln(h_{t-1}) \tag{5}$$

Since the log of the conditional variance is modelled, the leverage effect is exponential, rather than quadratic and even if the parameters are negative, the conditional variance will be positive. For $\gamma < 0$ means that negative shocks will have a bigger impact on expected volatility than positive shocks of the same magnitude. This is often referred to in the literature as the leverage effect. The EGARCH model allows positive and negative shocks to have a distinct impact on volatility. It also allows large shocks to have a superior impact on volatility than the standard GARCH model.

3.4 The GJR-GARCH model

The GJR-GARCH model was introduced by Glosten et al. (1993). The GJR augments the standard GARCH with an additional ARCH term conditional on the sign of the past innovation and express as:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \left(\alpha_i \varepsilon_{t-i}^2 + \lambda_i \varepsilon_{t-i}^2 I_{t-1}\right) + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{6}$$

where λ_1 measures the asymmetric (or leverage) effect and I_t is a dummy variable which is equal to 1 when ε_t is negative. In the GJR (1, 1) model, good news, $\varepsilon_{t-1} > 0$ and bad news, $\varepsilon_{t-1} < 0$, possess differential effects on the conditional variance. Good news has an impact of α_1, while bad news has an impact of $\alpha_1 + \lambda_1$. If $\lambda_1 > 0$, bad news increases volatility and this in turn means that there is a leverage effect for the AR (1)-order. If $\lambda_1 \neq 0$, the news impact is asymmetric.

4 Data and methods

4.1 Data description

The daily stock price indices data used in this study are obtained from Standard & Poor/International Finance Corporation Emerging Market Database (S&P/IFC EMDB). This source is used largely because it is a very organised and comprehensive source of stock price data, providing readily accessible and reliable data on emerging equity markets than most other sources. For example, S&P/IFC EMDB was the first database, from 1975, to track comprehensive information and statistics on emerging stock market indices. The S&P/IFC Global indices, used in this study, do not impose restrictions on foreign ownership and include sufficient number of stocks in individual market indices without imposing float or artificial industry-composition models on markets. Besides, the S&P/IFC database is attractive because they have been adjusted for all capital changes as well as the effects of corporate restructuring such as merger, acquisition, and spin offs/demerger as well as being free from data backfilling and survivorship bias.

The daily return r_t consists of transformed daily closing index price P_t measured in local currency. Our measurements include the Botswana Stock Exchange's Domestic Companies Index (BSEI) and Namibia's Stock Exchange Overall Index (NSEI). The stock price indices are transformed into their returns in order to obtain stationary series as:

$$r_t = \ln\left(p_t \big/ p_{t-1}\right) * 100 \tag{7}$$

where r_t is the market return at time t, p_t and p_{t-1} are the contemporaneous and one period lagged equity price indices, respectively. Natural lognormal is preferred as it computes continuous compound returns.

Table 1 provides further details of the data used in this research including the types of the stock indices used, the time period of the data for each market (and hence sample observations), and currency of denomination. The indices used in this study are the benchmark indices in their respective stock markets.

Table 1 Stock market data profile

	Method of compiling data	*index name*	*Period of data*	*No. of obs.*	*Currency*	*Source of data*
Botswana	Weighted index market capitalisation	Domestic companies index (DCI)	1996–2013	4,588	Pula	S&P/IFC EMDB
Namibia	Weighted index market capitalisation	NSE overall index (NSEI)	2000–2013	3,523	Namibian dollar	S&P/IFC EMDB

Table 2 Descriptive statistics for daily returns

	Mean	*Std. dev.*	*Skewness*	*Kurtosis*	*J. Bera*	*Q-stat.*	*ACF(100)*	*PACF(100)*
Botswana	0.075	1.072	12.867	356	23,927,436***	434***	–0.004***	–0.000***
Namibia	0.042	0.6329	3.046	90	1,124,423***	124**	–0.003**	–0.003**

The descriptive statistics in Table 2 indicate that both markets produce positive mean returns. However, the mean returns for Botswana is slightly higher than that of Namibia. Furthermore, the non-conditional variance as measured by the standard deviation for Botswana is higher than that of Namibia. The differences in mean returns and standard deviation could be attributed to the time difference in data. The returns distribution for both indices is positively skewed. The null hypothesis for skewness that conforms to a normal distribution with coefficients of zero is rejected by both indices. The returns for both indices exhibit fat tail as seen in the significant kurtosis well above the normal value of 3. The high value of J. Bera test for normality decisively rejects the hypothesis of a normal distribution at 1% significance level. Ljung-Box Q test statistic (Q-Stat) rejects the null hypothesis of no autocorrelation at 1% and 5% levels for all numbers of lags (100) considered as shown by autocorrelation function (ACF) and partial autocorrelation function (PACF) results in Table 2. The preceding statistics legitimise the use of AR conditional heteroscedastic models.

The statistical results indicate that both indices display similar characteristics. For instance, they both have positive mean returns, are positively skewed, found to display non-normal distribution and exhibit autoregression. These stylised features are similar to the existing empirical literature from the developing markets (Kim, 2003; Ng, 2000) and developed markets (Fama, 1976; Kim and Kon, 1994). Further, as return series revealed high value of kurtosis, it can be expected that a fatter-tailed distribution density such as the Student-t or GED should provide a more accurate results than the Gaussian (Normal) distribution.

4.2 Methods

The GARCH models are estimated using maximum likelihood estimation (MLE) process. This allows the mean and variance processes to be jointly estimated. The MLE has numerous optimal properties in estimating parameters and these include sufficiency (i.e., complete information about the parameter of importance contained in its MLE estimator); consistency (true parameter value that generated the data recovered asymptotically, i.e., data of sufficiently large samples); efficiency (lowest possible variance of parameter estimates achieved asymptotically. Furthermore, many methods of inference in statistics and econometrics are developed based on MLE, such as chi-square test, modelling of random effects, inference with missing data and model selection criteria such as Akaike information criterion and Schwarz criterion.

The MLE estimation of the standard GARCH assumes that the error distribution is Gaussian; however, evidence shows that the error exhibits non-normal distribution densities, for example, Nelson (1991). The choice of the underlying distribution for the error term is crucial if the volatility model is used in risk modelling. As it is expected that the problems pose by skewness and kurtosis by the residuals of conditional heteroscedasticity models will be reduced when appropriate distribution density is used, this study considers and evaluate the three most commonly used densities, the Gaussian, Student-t and generalised error distribution (GED).

4.2.1 Gaussian

The Gaussian, also known as the normal distribution, is the widely used model when estimating GARCH models. For a stochastic process, the log-likelihood function for the normal distribution is calculated as:

$$L_{gaussian} = -\tfrac{1}{2}\sum_{t=1}^{T}\left(\ln[2\pi] + \ln\left[\sigma_t^2\right] + z_t\right) \tag{8}$$

where T is the number of observations.

4.2.2 Student's-t distribution

For a student-t distribution, the log-likelihood is computed as:

$$\begin{aligned} L_{stu-t} &= \ln\left(\Gamma\left[\frac{v+1}{2}\right]\right) - \ln\left(\Gamma\frac{v}{2}\right) - \tfrac{1}{2}\ln\left(\pi[v-2]\right) \\ &\quad -\tfrac{1}{2}\sum_{t=1}^{T}\left(\ln\sigma_t^2 + [1+v]\ln\left[1+\frac{z_t^2}{v-2}\right]\right) \end{aligned} \tag{9}$$

where v is degrees of freedom, $2 < v = \infty$ and $\Gamma(\cdot)$ is the gamma function.

4.2.3 Generalised error distribution

In applied finance, such as, asset pricing, option pricing, portfolio selection and VaR, skewness and kurtosis are very important. The GED is an error distribution that represents a generalised form of the Gaussian, possesses a natural multivariate form, has a parametric kurtosis that is unbounded above and has special cases that are identical to the normal and property which controls the skewness. Thus, choosing the appropriate distribution density that can model these two moments is important, hence, the GED log-likelihood function of a normalised random error is computed as:

$$L_{GED} = \sum_{t=1}^{T}\left(\ln\left[\tfrac{v}{\lambda_v}\right] - 0.5\left|\frac{z_t}{\lambda_v}\right|^{v} - [1+v^{-1}]\ln 2 - \ln\Gamma\left[\tfrac{1}{v}\right]0.5\ln\left[\sigma_t^2\right]\right) \tag{10}$$

where

$$\lambda_v = \sqrt{\frac{\Gamma\left(\tfrac{1}{v}2^{-2/v}\right)}{\Gamma\left(\tfrac{3}{v}\right)}} \tag{11}$$

The order of the GARCH process can be identified by computing Q-statistic from the squared residuals and the Engle (1982) LM test is used to test for the ARCH effect in the residuals. The GARCH models in this study are compared by using various goodness-of-fit diagnostics such as Akaike information criterion, Schwarz Bayesian information criterion and log-likelihood.

4.4 Forecast evaluation

The one-step-ahead forecast of the conditional variance for the GARCH, EGARCH and GJR is obtained by updating equations (4), (5) and (6) by one period as,

$$h_{t+1} = \alpha_0 + \alpha_1 \varepsilon_t^2 + \beta_1 h_t \tag{12}$$

$$\ln(h_{t+1}) = \alpha_0 + \alpha_1 g(Z_t) + \beta_1 \ln(h_t) \tag{13}$$

$$h_{t+1} = \alpha_0 + \alpha_1 \varepsilon_t^2 + \lambda_1 \varepsilon_t^2 I_t + \beta_1 h_t \tag{14}$$

Similarly, j-step-ahead forecast on the conditional variance can be obtained by updating equations (12), (13) and (14) by *j*-periods as,

$$h_{t+j} = \alpha_0 + \alpha_1 \varepsilon_{t+j-1}^2 + \beta_1 h_{t+j-1} \tag{15}$$

$$\ln(h_{t+j}) = \alpha_0 + \alpha_1 g(Z_{t+j-1}) + \beta_1 \ln(h_{t+j-1}) \tag{16}$$

$$h_{t+j} = \alpha_0 + \alpha_1 \varepsilon_{t+j-1}^2 + \lambda_1 \varepsilon_{t+j-1}^2 I_{t+j-1} + \beta_1 h_{t+j-1} \tag{17}$$

However, it is rather difficult to obtain the *j*-step-ahead forecasts than the one-period-ahead forecasts assumed in this study although it is possible to obtain the *j*-step-ahead forecasts of the conditional heteroscedasticity recursively.

In order to evaluate the forecasting performance of the GARCH, EGARCH and GJR models, forecasting tests encompassing different distribution densities are performed. The model that minimises the loss function under these evaluation criteria is preferred. To assess the performance of the asymmetric GARCH models in forecasting the conditional variance, we compute four statistical measures of fit as follows;

- Mean absolute error (MAE) – this is represented as:

  $$MAE = \frac{1}{h}\sum_{t=s}^{s+h} \left| \hat{\sigma}_t^2 - \sigma_t^2 \right| \tag{18}$$

  where *h* is the number of steps ahead (i.e., number of forecast data points), s the sample size, $\hat{\sigma}^2$ is the forecasted variance and σ^2 is the conditional variance computed from equations (4), (5) and (6).

- Root mean square error (RMSE) is represented as:

  $$RMSE = \sqrt{\frac{1}{h}\sum_{t=s}^{s+h} \left(\hat{\sigma}_t^2 - \sigma_t^2 \right)^2} \tag{19}$$

- The mean absolute percentage error (MAPE) is represented as:

  $$MAPE = \frac{1}{h}\sum_{t=s}^{s+h} \left| \left(\hat{\sigma}_t^2 - \sigma_t^2 \right) \Big/ \sigma_t^2 \right| \tag{20}$$

- Theil inequality coefficient (TIC) is represented as:

$$TIC = \frac{\sqrt{MSE}}{\sqrt{\frac{1}{h}\sum_{t=s}^{s+h}\sigma^2 + \frac{1}{h}\sum_{t=s}^{s+h}\hat{\sigma}^2}} \tag{21}$$

To calculate daily forecast and in order to evaluate the forecasting performance of each model, we simply split the respective time series in half between the in-sample period, $t = 1 \ldots, T$ and the out-of-sample period, $t = T \ldots, h$. We further estimate each model over the first part of the sample and then apply these results to forecast the conditional variance (volatility) over the second part of the sample period.

5 Empirical results and analyses

This section presents and analyse our results of the estimated models. Tables 3, 4 and 5 presents the results for the estimated parameters for GARCH, EGARCH and GJR models respectively, while some useful in-sample diagnostics statistics are reported in Tables 6, 7 and 8.

Table 3 Estimated statistics-comparative distribution density GARCH model

	Botswana			*Namibia*		
	Gaussian	*Student-t*	*GED*	*Gaussian*	*Student-t*	*GED*
μ	0.1256	0.0014	9.66e-07	0.0479	0.0163	–0.0088
	(7.9943)***	(0.0980)	(8.15e-05)	(4.6811)***	(3.6668)***	(–3.2546)***
α_0	0.7360	0.2968	0.3574	0.1555	0.1623	0.3799
	(1.6216)	(0.7746)	(0.7539)	(1.8554)*	(0.7899)	(1.3168)
α_1	–0.0014	–0.0002	–0.0003	–0.0059	–0.0020	–0.0094
	(–5.6043)***	(–37.5106)***	(–5.8944)***	(–7.7961)***	(–1.0993)	(–11.5462)***
β_1	0.5966	–0.1034	0.2420	0.7148	0.1466	0.5756
	(1.9908)**	(–0.0726)	(0.2405)	(3.8993)***	(0.2001)	(1.7894)*
$\alpha_1 + \beta_1$	0.5952	–0.1036	0.2417	0.7089	0.1446	0.5662

Table 4 Estimated statistics-comparative distribution density EGARCH model

	Botswana			*Namibia*		
	Gaussian	*Student-t*	*GED*	*Gaussian*	*Student-t*	*GED*
μ	4.60e-05	8.01e-08	–4.16e-06	0.0002	–2.04e-09	1.73e-14
	(1.1642)	(0.1877)	(–0.0024)	(2.0853)**	(0.8919)	(0.0003)
α_0	0.0361	–2.8206	–1.0253	–0.4698	–1.9201	–5.1453
	(0.4291)	(–0.4599)	(–11.0104)***	(–4.3795)***	(–1.8737)*	(–29.1693)***
α_1	–0.4423	0.0578	0.1130	–1.2447	–0.0932	0.2217
	(–4.3966)***	(0.2542)	(13.2862)***	(–10.9113)***	(–0.7650)	(11.3676)***
β_1	0.7360	0.2187	0.6377	0.3738	0.6082	–0.1937
	(20.1983)***	(6.4135)***	(19.0690)***	(10.4032)***	(30.7346)***	(–7.2266)***
γ	–0.2318	–0.1447	–0.0878	0.0591	0.0820	–0.2146
	(–4.2060)***	(–0.2543)	(–11.0296)***	(1.1811)	(0.7654)	(–11.3752)***

Table 5 Estimated statistics-comparative distribution density GJR-GARCH model

	Botswana			*Namibia*		
	Gaussian	*Student-t*	*GED*	*Gaussian*	*Student-t*	*GED*
μ	0.1059	0.0002	–0.2597	0.0574	0.0304	0.0442
	(5.5936)***	(0.1532)	(–6.1274)***	(5.3931)***	(0.7865)	(1.6057)
α_0	1.0776	0.4042	1.0124	0.3737	0.3239	0.3870
	(3.3381)***	(0.1619)	(3.5338)***	(3.1900)***	(2.7907)***	(2.3324)**
α_1	0.0466	0.0530	0.0520	0.0268	0.0423	0.0405
	(3.1877)***	(0.1614	(2.2521)**	(2.6865)***	(1.9724)**	(1.7730)**
β_1	0.3975	–0.0057	0.5496	0.3916	0.5078	0.5853
	(4.2715)***	(–3.2657)***	(4.3340)***	(2.4568)**	(2.9024)***	(3.3042)***
λ_1	–0.0585	–0.0557	–0.0679	–0.0345	–0.0507	–0.0524
	(–4.4001)***	(–0.1622)	(–2.9677)***	(–3.5913)***	(–2.3649)**	(–2.2918)**

The statistics reported in Tables 3, 4 and 5 show that the use of GJR with normal and non-normal distribution appears justified to model the asymmetric characteristics of both indices. All the asymmetric coefficients with the exception of student-t for Botswana are statistically significant at standard levels for both indices. The evidence also shows that news impact is asymmetric in both stock markets as the asymmetric coefficients for all densities are unequal to zero.

The sum of the lagged error and the lagged conditional variance of the symmetrical GARCH model for both indices are far from the expected value of 1 (i.e., unity) regardless of the distribution density. This implies that the current shocks to the conditional variance will have less impact on future volatility (see Coffie, 2015). In Botswana, the leverage effect term, *g* in the EGARCH has the correct sign for all distribution densities and is significant with Gaussian and GED. This means that in Botswana, negative shocks will have a greater impact on future volatility than positive shocks of the same magnitude, confirming the existence of leverage effect. Furthermore, in Namibia, as typical of EGARCH model, the leverage effect term has the correct sign and is statistically significant with GED, while the Gaussian and Student-t produce positive and insignificant coefficients. The GED-EGARCH model (Nelson, 1991) replaces the traditional use of conditionally normal error distribution assumption of GARCH models with the assumption of innovations that follow GED. Hence, the presence of leverage effect suggests that investors in these markets are to be rewarded for taking up additional leverage risks. Therefore, investors and fund managers should go beyond the simple mean-variance approach when allocating portfolios for these markets. Instead, they should explore information about volatility, information asymmetry, correlation, skewness and kurtosis. Required rate of return is expected to be high in these markets due to compensation for additional leverage risk which places additional burden on indigenous companies seeking to raise finance from the domestic capital markets.

For both markets, the coefficient estimates of the GJR are negative and statistically significant at standard levels, suggesting that positive instead of negative shocks imply a higher next period conditional variance of the same sign. This means that negative shocks would have no greater effects on volatility than positive shocks as expected. Instead positive shocks would have greater effect on volatility as the asymmetric term, λ, is less than zero for all density distributions. This evidence invalidate the GJR proposition that bad news has greater impact on volatility than good news. Therefore, like Wan et al. (2014), the evidence in Botswana and Namibia shows that both markets exhibit a reverse volatility asymmetry, controverting the widely accepted theory of volatility asymmetry (i.e., negative returns induce a higher return volatility than positive returns). In both markets, the evidence demonstrate that return volatilities react more intensely to positive returns than their reaction to negative returns. Mainly, this reverse volatility asymmetry is attributed to higher trading volume associated with momentum stocks (i.e., price rising stocks) as investors from Botswana and Namibia are known to rush for such stocks than their contrarian counterparts and this leads to the arousal of higher volatility for positive returns than negative returns. Hence, positive return-volatility correlation is observed in both stock markets.

The estimated parameters for both indices of the asymmetric GARCH model indicate that the ARCH (α_1) term is statistically significant at 1% level (with the exception of student-t for Namibia) for all distribution densities. However, the GARCH (β_1) estimates present weak statistical coefficients. For example, only the β_1 for the normal-density is significant at 5 for Botswana. In Namibia, the Gaussian and GED are significant at 1 and 10% levels respectively. The estimated coefficients for the asymmetric GARCH models produced statistically significant results (a justification of the use of asymmetric GARCH models). For example, the ARCH (α_1) in the EGARCH model is statistically significant at 1% level for Gaussian and GED densities for both markets, while the GARCH (β_1) is significant at 1% for all densities. Furthermore, the ARCH (α_1) estimates in the GJR for both markets are statistically significant at standard levels except student-t for Botswana, while the GARCH (β_1) is significant with all densities at standard levels for both markets.

Table 6 Diagnostics statistics-comparative distribution density GARCH model

	Botswana			*Namibia*		
	Gaussian	*Student-t*	*GED*	*Gaussian*	*Student-t*	*GED*
$Q^2(20)$	0.6357	0.6775	0.6798	7.6173	7.7312	7.8690
	(1.000)	(1.000)	(1.000)	(0.994)	(0.994)	(0.993)
ARCH(2)	0.0461	0.0693	0.0715	0.7503	0.7104	0.8321
	(0.9772)	(0.9659)	(0.9649)	(0.6872)	(0.7010)	(0.6596)
AIC	3.0684	0.6125	0.4196	1.9505	–2.2312	–0.6351
SBIC	3.0740	0.6195	0.4266	1.9575	–2.2225	–0.6264
Log-like	–7,035	–1,400	–958	–3,432	3,935	1,124

Notes: $Q^2(20)$ are the Ljung-Box statistic at lag 20 of the squared standardised residuals. ARCH (2) refers to the Engle (1982) LM test for the presence of ARCH effect at lag 2. P-values are given in parentheses. AIC, SBIC and Log-like are Akaike information criterion, Schwartz Bayesian information criterion and Log-likelihood value respectively.

Table 7 Diagnostics statistics-comparative distribution density EGARCH model

	Botswana			*Namibia*		
	Gaussian	*Student-t*	*GED*	*Gaussian*	*Student-t*	*GED*
$Q^2(20)$	1.0787	0.6727	0.6500	6.0575	215.83	8.1373
	(1.000)	(1.000)	(1.000)	(0.999)	(0.000)	(0.991)
ARCH(2)	0.4545	0.0706	0.0681	0.3473	0.2218	0.8027
	(0.7967)	(0.9653)	(0.9665)	(0.8406)	(0.8950)	(0.6694)
AIC	2.6532	–3.8547	–0.5846	1.7269	–14.8785	–18.2948
SBIC	2.6602	–3.8463	–0.5762	1.7356	–14.8680	–18.2843
Log-like	–6,081	8,849	1,347	–3,037	26,214	32,232

Table 8 Diagnostics statistics-comparative distribution density GJR-GARCH model

	Botswana			*Namibia*		
	Gaussian	*Student-t*	*GED*	*Gaussian*	*Student-t*	*GED*
$Q^2(20)$	0.6555	0.6792	1.3700	7.6733	7.7889	7.7087
	(1.000)	(1.000)	(1.000)	(0.994)	(0.993)	(0.994)
ARCH(2)	0.0597	0.0714	0.6329	0.7398	0.7614	0.7060
	(0.9706)	(0.9649)	(0.7287)	(0.6908)	(0.6834)	(0.7026)
AIC	3.0815	–2.5084	2.9568	2.0145	1.7048	2.1838
SBIC	3.0885	–2.5000	2.9652	2.0233	1.7153	2.1943
Log-like	–7,064	5,760	–6,777	–3,544	–2,997	–3,841

Turning to distribution densities (Tables 6, 7 and 8); the fatter tails (Student-t and GED) distributions clearly outperform the Gaussian. For instance, the log-likelihood function strongly increases when fatter tailed distribution densities are used for both indices. Furthermore, using the non-normal densities of Student-t and GED produces lower AIC and SBIC than the Gaussian. From the preceding evidence, all the three GARCH models perform well with non-normal distribution densities. All models appear effective by describing the dynamics of the series as shown by the Ljung-Box statistics for the squared standardised residuals with lag 20 which are all non-significant at 1% level for both indices. The LM test for the presence of ARCH at lag 2, indicate that conditional heteroscedasticity are removed for all three GARCH models regardless of the distribution density which are all non-significant at 1% level.

The comparison between models with each distribution density indicates that, giving the different measures used for modelling volatility, the EGARCH with Student-t distribution provides the best in-sample estimation for Botswana, clearly outperforms EGARCH with Gaussian and GED as well as GARCH and GJR models. Furthermore, from the results, GED-EGARCH provides a better in-sample estimation for Namibia than with Gaussian and Student-t and clearly outperforms symmetric GARCH and GJR models.

Table 9 Forecast performance-comparative distribution density

Model	Botswana			Namibia		
	GARCH	*GJR*	*EGARCH*	*GARCH*	*GJR*	*EGARCH*
	Gaussian					
MAE	0.292479	0.273953	0.174511	0.168829	0.177900	0.123308
RMSE	1.281024	1.280752	1.284478	0.755088	0.755586	0.754393
MAPE	4.554697	4.561448	4.597685	4.716367	4.743300	4.879925
TIC	0.908472	0.921161	0.999961	0.941179	0.930796	0.999770
Model	*Botswana*			*Namibia*		
	GARCH	*GJR*	*EGARCH*	*GARCH*	*GJR*	*EGARCH*
	Student-t					
MAE	0.292479	0.174657	0.174485	0.138682	0.152106	0.123145
RMSE	1.281024	1.284466	1.284480	0.754290	0.754483	0.754396
MAPE	4.554697	4.597632	4.597695	4.795569	4.721913	4.880817
TIC	0.908472	0.999832	0.999985	0.978720	0.961406	1.000000
Model	*Botswana*			*Namibia*		
	GARCH	*GJR*	*EGARCH*	*GARCH*	*GJR*	*EGARCH*
	GED					
MAE	0.174469	0.426068	0.174472	0.131480	0.165264	0.123145
RMSE	1.284482	1.329788	1.284482	0.754599	0.754925	0.754396
MAPE	4.597701	4.702422	4.597703	4.926768	4.711494	4.880817
TIC	0.999999	0.861133	0.999997	0.988757	0.945371	1.000000

Table 10 Ranking performance forecast

Model	Botswana			Namibia		
	GARCH	*GJR*	*EGARCH*	*GARCH*	*GJR*	*EGARCH*
	Gaussian					
MAE	3	2	1	2	3	1
RMSE	2	1	3	2	3	1
MAPE	1	2	3	1	2	3
TIC	1	2	3	2	1	3
Total	*7*	*7*	*10*	*7*	*9*	*8*
Model	*Botswana*			*Namibia*		
	GARCH	*GJR*	*EGARCH*	*GARCH*	*GJR*	*EGARCH*
	Student-t					
MAE	3	2	1	2	3	1
RMSE	1	2	3	1	3	2
MAPE	1	2	3	2	1	3
TIC	1	2	3	2	1	3
Total	*6*	*8*	*10*	*7*	*8*	*9*

Table 10 Ranking performance forecast (continued)

Model	*Botswana*			*Namibia*		
	GARCH	*GJR*	*EGARCH*	*GARCH*	*GJR*	*EGARCH*
	GED					
MAE	1	3	2	2	3	1
RMSE	1	3	1	2	3	1
MAPE	1	3	2	3	1	2
TIC	3	1	2	2	1	3
Total	*6*	*10*	*7*	*9*	*8*	*7*

Table 11 Summary of best performing model

	Botswana	*Namibia*
Gaussian	GARCH/EGARCH	GARCH
Student-t	GARCH	GARCH
GED	GARCH	EGARCH

Table 10 ranks the GARCH models when evaluated against each other with the introduction of the three different distribution densities for the error term. The evidence in Tables 9 and 10 indicates that the symmetric GARCH model clearly outperform the GJR and EGARCH in Botswana, while in the case of Namibia, no single model completely dominates the other. However, the symmetric GARCH slightly outperforms the asymmetric GARCH models in Namibia. Furthermore, Table 11 indicates that the symmetric GARCH model provides the best out-of-sample forecast followed by EGARCH for both stock markets. This contradicts the evidence found in Malaysia and Singapore where asymmetric GARCH models clearly outperform the symmetric GARCH (Nor and Shamiri, 2007). The findings also show that forecasting with heavy-tailed distribution densities yield no significant reduction of the forecast error than when normal distribution is assumed. However, it appears that the symmetric GARCH model with fatter-tailed distribution have a slight tendency over normal distribution to produce superior forecast.

6 Conclusions

Over the last three decades many academics and analysts have paid particular attention to stock market volatility since it can be used to measure and forecast in a wide range of financial applications including portfolio selection, value at risk, asset pricing, hedging strategies and option pricing. This paper aimed to model and forecast the performance of the symmetric GARCH model and asymmetric GARCH (i.e., GJR and EGARCH) models with the introduction of different distribution densities for Botswana and Namibia stock markets.

The statistical results point towards the fact that the current shocks to the conditional variance will have less impact on future volatility in both markets. Further, the results from EGARCH show that the leverage effect exists in Botswana regardless of the distribution density assumed, while EGARCH is efficient in estimating the leverage effect in Namibia when GED is assumed. However, the evidence from GJR for both markets reveal a reverse volatility asymmetry, contradicting the widely accepted observation of volatility asymmetry, where negative returns induce a higher return volatility than positive returns.

The comparison between models with each distribution density indicates that, giving the different measures used for modelling volatility, the EGARCH with Student-t distribution provides the best in-sample estimation for Botswana, while the GED-EGARCH provides a better in-sample estimation for Namibia. Regarding forecasting evaluation, the results reveal that the symmetric GARCH model coupled with fatter-tailed distribution presents a better out-of-sample forecast for both stock markets.

Finally, there are areas where further studies might be useful. For example, future research should focus on modelling and forecasting GARCH models with high frequency trading (i.e., intra-day) data. Further research may also consider exploring variety of models including other conditional variance models such as APARCH and long memory models such as FIEGARCH, FIAPARCH and CGARCH in order to allow a superior insight into the dynamics of these two markets. Lastly, similar study should be conducted in other African stock markets in order to provide a wider insight into how GARCH models are capable of modelling volatility in African stock markets.

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