UNIVERSITY OF GHANA

A COMPARATIVE ANALYSIS OF FORECAST PERFORMANCE BETWEEN SARIMA AND SETAR MODELS USING MACROECONOMIC VARIABLES IN GHANA

BY

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DECLARATION

CANDIDATE'S DECLARATION

This is to certify that this thesis is the result of my own extensive research and reading, under the guidance of my supervisors and that no part of it has been presented for another degree in this university or elsewhere.

SIGNATURE:............................ DATE:..........................

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SUPERVISORS' DECLARATION

We hereby certify that this thesis was prepared from the candidate's own work and supervised in accordance with guidelines on supervision of thesis laid down by the University of Ghana.

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DEDICATION

This work is dedicated to my beloved parents Hajia and Mr. Issah Bawourun Ahmed and also to my dear cousin Mr. Mohammed Nazif Senje.

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ABSTRACT

Most macroeconomic variables such as; inflation, GDP and others have been described by most financial and economics time series analysts to exhibit nonlinear behaviour. Therefore, to cater for this behaviour, the nonlinear class of models have been largely adopted to model and forecast such time series. In this study, the Keenan and Tsay tests for linearity showed inflation and CIC rates follow threshold nonlinear processes. Hence, the two-regime SETAR model was adopted to accommodate these nonlinearities in the datasets. Using the linear SARIMA model as a benchmark for comparative analysis. Results from both in-sample and out- of- sample forecast performance using MAE and RMSE measures revealed that, the nonlinear SETAR model outperformed the linear SARIMA model for inflation. This was however different for CIC rates, since the Linear SARIMA model turned to outperform the nonlinear SETAR model. Further analysis of forecast accuracy using the Diebold-Mariano test showed there was no significant difference between the two models for inflation but, there was significant difference between both models for CIC rates. Nevertheless, it is recommended that, continuous monitoring of these models, review market conditions and necessary adjustments are vital to make realistic use of these models.

CONTENTS

LIST OF ABBREVIATION

AIC .... Akaike Information Criterion

AICc .... Akaike Information Criterion Corrected

ACF .... Autocorrelation Function

ADF .... Augmented Dickey-Fuller

ANN .... Artificial Neural Network

AR .... Autoregressive

ARCH .... Autoregressive Conditional Heteroscedasticity

ARCH-LM .... Autoregressive Conditional Heteroscedasticity Lagrange Multiplier

ARIMA .... Autoregressive Integrated Moving Average

ARMA .... Autoregressive Moving Average

ASEAN .... Association of Southeast Asian Nations

BIC .... Bayesian Information Criterion

BIVARMA .... Bilinear Vector Autoregressive Moving Average

BoG .... Bank of Ghana

CIC .... Currency in Circulation

CNB .... Czech National Bank

DF .... Dickey-Fuller

DGP .... Data Generating Process

ECB .... European Central Bank

EGARCH . . . . Exponential Generalised ARCH

GARCH . . . . Generalised ARCH

GDP . . . . Gross Domestic Product

GNP . . . . Gross Net Product

KPSS . . . . Kwiatkowski-Phillips-Schmidt-Shin

LCL . . . . Lower Confidence Limit

LSTAR . . . . Linear Smooth Transition Autoregressive

MA . . . . Moving Average

MAE . . . . Mean Absolute Error

MAPE . . . . Mean Absolute Percent Error

Max . . . . Maximum

Min . . . . Minimum

MPE . . . . Mean Percent Error

MSE . . . . Mean Square Error

OLS . . . . Ordinary Least Square

PACF . . . . Partial Autocorrelation Function

PP . . . . Phillips -Perron

RESET . . . . Ramsey Regression Equation Specification Error Test

RMSE . . . . Mean Square Error

RMSPE . . . . Root Mean Square Percentage Error

SARIMA . . . . Seasonal Autoregressive Integrated Moving Average

SETAR .... Self-Excited Threshold Autoregressive

SSR .... Sum of Squared Error

STAR .... Smooth Transition Autoregressive

Std .... Standard Deviation

TAR .... Threshold Autoregressive

UCL .... Upper Confidence Limit

UK .... United Kingdom

VAR .... Vector Autoregressive

VARMA .... Vector Autoregressive Moving Average

ZA .... Zivot Andrews

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CHAPTER 1

INTRODUCTION

1.1 Background of study

The simple linear model gained popularity in modeling and forecasting most macroeconomic variables until the 1980's. Tong (1978) and Tong and Lim (1980) first proposed the threshold concept of nonlinear class of models after a challenge was thrown to a group of time series analysts in 1977 during an ordinary Royal Statistical Society meeting.

For the purpose of this study, macroeconomics is defined as a branch of economics which deals with the study of the whole economy characterized by variables. Inflation, Gross Domestic Product (GDP) and unemployment rates are major examples of macroeconomic variables that have received great attention in most economic and statistical researches worldwide with Ghana not being an exception. This is so because macroeconomic variables are the major indicators of an economy. For this reason, governments, policy makers and investors have developed keen interests in researches on macroeconomic variables. It is therefore not surprising to hear governments boasting of attaining single digit inflation, creating jobs especially for the youth and stabilizing an economy during its regime. Also, it is no lame reason that per Ghanaian setting, governments appoint a statistician whose sole responsibility is to constructively advise government on strategies to keep the economy in check and balance.

According to Marcellino (2007), the World War II crisis socially, economically and politically had an influence on modelling of macroeconomic variables worldwide. In this, many literatures have suggested time varying and non-linear class of models to be better off in dealing with such macroeconomic variables as compared to linear specifications in modelling and forecasting.

In a typical Ghanaian market, the dynamics in the business cycle associated with events such as economic expansion and recession results in discontinuities in a time series either as a result of fiscal or monetary policies by governments. Researchers such as Williams and Miller (1990) and Carreno and Madinaveita (1990) proposed some modifications to deal with these discontinuities. Other researchers such as Rosas and Guerrero (1994) attempted to cater for discontinuities in a time series data by examining an exponential smoothing forecast subject to a number of constraints taking into consideration seasonal variations.

Tsay and Tiao (1984), Campbell and Shiller (2001), Clements and Jeremy (2001) and Aidoo (2011) have also described most macroeconomic variables as nonlinear and hence propose the nonlinear class of models as best for forecasting such variables. However, other researchers such as Gooijer and Kumar (1992) are of divergent view that there is no clear evidence either in favour of linear or nonlinear models in terms of their forecast performance.

The fact that time series generated from macroeconomic variables appear irregular by nature, it is imperative to study, model and detect this asymmetric behaviour. Again, because of this irregular behaviour of these variables, the simple linear models cannot be used in their modelling and forecasting to produce accurate results.

Recently, quite a number of nonlinear class of models such as the Artificial Neural Networks (ANN) and the Threshold Autoregressive (TAR) models, have been proposed and extensively used for modelling and forecasting time series that appear not to be linear. These nonlinear models are classified as regime switching models but are further divided into "Threshold models" and "Markov Switching models" depending on the state process evolution between the two models.

The nonlinear models, "threshold models" introduced by Tong (1978) and later reviewed by Potter (1999) triggered by regime shifts in relation to an unknown threshold are considered to capture irregularities in the data generation process of a time series to be used for modelling and forecasting. The Self- Exciting Threshold Autoregressive (SETAR) models, a special case of the TAR model is accommodating in terms of structural changes in regimes or discontinuities in the data generation process. For this reason, the SETAR model is considered in this study to model some macroeconomic variables in Ghana. Again, according to Marcellino (2007) the nonlinear ANN model is a very powerful tool for any kind of nonlinear behavior but they are quite difficult to interpret from an economic perspective. Likewise, seasonal ARIMA (SARIMA) model is proposed as a benchmark for the study since it is comparatively easier to model and use for forecasting trended time series at regular intervals. Researchers such as Swanson and White (1995) have described the in-sample forecast of the SETAR models to outperform their linear counterpart but cannot vouch for their out of sample forecast. It is therefore imperative to consider both the in sample and the out of sample performance of the two models.

1.2 Problem Statement

The Linear class of time series models have received much attention over the years in both theory and applied research. Although, the linear class of models has widely been used in several areas of study, they usually do not capture vital features of most macroeconomic and financial data. Since the business cycle for most economies is bound to go through either an expansion or recession, data from these variables such as inflation, unemployment rate and GDP go through a system of structural breaks and behavioural changes. It is therefore reasonable to assume that each kind of data set may require a different type of time series model to explain and model it at different times.

In detecting and modelling the nonlinear behaviour of a time series, it seems a natural concept is to consider the nonlinear class of models as proposed by researchers like Watier and Richardson (1995), Camphell and Shiller (2001), Hsu et al. (2010) and Gharleghi et al. (2014) to effectively explain irregular behaviourial changes in macroeconomic and financial time series.

Models that assume the dynamic changes or discontinuities in time series are usually termed as Autoregressive models. Examples of such models include threshold autoregressive model, self-exciting threshold autoregressive (SETAR) and smooth transition autoregressive (STAR) models. The simple autoregressive models are arguably most frequently used time series models that can be easily estimated and interpreted. An extension of an AR process to cater for the nonlinear behaviour of time series makes the entire process nonlinear thereby making the nonlinear models easy to understand and explain.

Recently, Gharleghi et al. (2014) found that the nonlinear SETAR model outperformed their linear counterpart, the ARIMA model. According to Aidoo (2011), little of research has been done in comparing the forecast performance between SARIMA and SETAR models in an application to Ghana's inflation rate. It is in this similar spirit that this research will consider a case study of macroeconomic variables in Ghana. This research will produce multiple models which will be more accurate in choosing the best performing model since macroeconomic variables operate under different conditions though interrelated.

1.3 Objectives of the study

1.3.1 General objective

To identify macroeconomic variables that satisfy the conditions of SARIMA and SETAR models and compare their in-sample and out-of-sample forecast performance.

1.3.2 Specific Objectives

i. Identify appropriate linear and nonlinear models for forecasting macroeconomic variables.

ii. To examine the in-sample forecast performance of both models.

iii. To examine the out- of- sample forecast performance of both models.

1.4 Significance of study

This study will contribute to existing literature for researchers who wish to work on similar studies in the near future. The study will also identify appropriate models that will be useful to policy makers at all relevant levels, such as investors, governmental and non-governmental organisations, in the effective design of economic strategies to combat expected and unexpected changes in business cycles and also influence their financial planning.

1.5 Research Questions

i. What is the behavior of these macroeconomic variables?

ii. Which linear SARIMA model is appropriate for modelling each macroeconomic variable?

ii. Which nonlinear SETAR model is appropriate for modelling each macroeconomic variable?

ii. Which of the two models performs better in forecasting?

1.6 Organisation of Study

The structure of the thesis is as follows: Chapter one comprises the introduction and it caters for the following: background, problem statement, research questions, objectives of study and organisation of study. Chapter two reviews literature, which comprises: introduction, brief introductions of SARIMA and SETAR models and examines existing literature on the performance of both models and other linear and nonlinear models. It goes further to show what this study seeks to achieve. Chapter three focuses on the methodology of the study, whereas chapter four deals with data analysis and results and chapter five deals with conclusions and recommendations of the study.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter deals with brief introduction to SARIMA and SETAR models, related studies on forecast performance of SARIMA model, related studies on forecast performance of SETAR model and lastly other related studies on comparative forecast performance between linear and nonlinear models.

2.2 Brief introduction to SARIMA model

The simple linear model Autoregressive Integrated Moving Averages (ARIMA) has received much attention in modelling and forecasting time series data ranging from economics, health, agriculture and finance just to mention a few. The ARIMA model is considered to be a special kind of a regression model which comprises an Autoregressive (AR) and a Moving Average (MA) component that are dependent on past values and past errors respectively. A mixture of the AR and the MA models produces a mixed model called the ARMA model.

The ARIMA model was popularized by Box and Jenkins (1976), the "I" represents a differencing or an integration of a time series to become stationary. ARIMA models are comparatively good in modelling and forecasting as compared to other linear time series models. Researchers such as Meese and Geweke (1984), Marcellino (2007), Stock and Watson (2003) have confirmed this in their works.

However, these models are not capable of capturing both seasonal and non-seasonal patterns in a time series. An extension of these models to capture seasonality is a Seasonal Autoregressive Integrated moving Average (SARIMA) model. A time series is described as exhibiting seasonality if there exist a regular pattern in changes that repeats over a number of time intervals until the patterns repeat again. For monthly data, even changes usually occurs in specific months in the time series data. SARIMA models are well known for their statistical modelling and forecasting but are however unable to extract nonlinear relationships within the time series data.

2.3 Related studies on forecast performance of SARIMA model

This section reviews existing literature from various fields where the SARIMA model has been used.

An economics related thesis by Aidoo (2010), revealed that $SARIMA(1,1,1)(0,0,1)_{12}$ best represent the behaviour of Ghana's inflation rate considering monthly data from July 1991 to December 2009 obtained from the Ghana statistical service. In the study, the best model based on various selection criteria was used to forecast Ghana's inflation rate for the next seven months. The forecasted values demonstrated a decrease in Ghana's inflation rate for the forecast period with a turning point in the month of July.

In a related study by Nasiru and Sarpong (2012), $SARIMA(3,1,3)(2,1,1)_{12}$ was found to be the most appropriate modelling approach for monthly inflation in Ghana using Box-Jenkins methodology for the period between January 1990 to January 2012. Fannoh et al. (2014) further revealed $SARIMA(0,1,0)(2,0,0)_{12}$ to best examine short term data on inflation from January 2006 to December 2006 for Liberia while considering seasonality.

Moreover, Otu et al. (2014) developed a model for forecasting Nigeria's inflation rate. In their study, Seasonal ARIMA model was applied to short term monthly inflation rate collected from November 2003 to October 2013. $SARIMA(1,1,1)(0,0,1)_{12}$ was adjudged the best model to forecast inflation rates for Nigeria after meeting all model assumptions. Forecasted inflationary rates for November 2013 to November 2014 displayed a downwards trend for the first quarter of 2014 but made a U-turn at the beginning of the second quarter in the same year. Additionally, there was an upwards trend in inflation values till the month of September. They posited that volatility in Nigeria's inflation rates is attributed to many economic factors.

Again, Nasiru et al. (2013) compared the efficiency of ARIMAX and SARIMA models in predicting monthly currency in circulation for Ghana. Their predicted results indicated that, both models were good for prediction. Further diagnostic checks revealed both models were free from conditional heteroscedasticity and high-level autocorrelations. However, they recommended continuous evaluation of these models to improve forecasting performance.

Luguterah et al. (2013) proposed $SARIMA(0,1,1)(0,1,1)_{12}$ as the best model for predicting currency in circulation for Ghana. They employed Seasonal ARIMA in their model building and forecasting for Monthly data obtained from the Bank of Ghana from January 2000 to December 2011. From their results, the model with minimum values of AIC, BIC and AICc was chosen. Diagnostic checks of the model residuals with Ljung-Box and ARCH-LM tests showed the model was free from higher level autocorrelation and conditional heteroscedasticity.

In Agriculture, Akter and Rahman (2010) generated forecasts for milk supply of dairy cooperative in UK applying seasonal Holt winter's model and seasonal ARIMA model. They revealed that the forecasts produced less than three-percent errors and hence proposed both techniques to be effective in forecasting.

A health-related study by Martinez and da Silva (2011), resulted in developing the most appropriate prediction model for incidence of dengue cases in Campinas, southeast Brazil using SARIMA model in application to reported monthly dengue data. From their results, $SARIMA(2,1,3)(1,1,1)_{12}$ was proposed to be the best model fit in forecasting dengue cases in Brazil after satisfying all model developing assumptions. By their study, researchers were advised to take into account climate changes when dealing with data on dengue cases for Brazil in future.

Yet again, a health monitored study by Zhang (2013) predicted the incidence of typhoid fever in China using SARIMA and three different inspired neural network models. They applied the various models to data obtained from the Chinese center for disease control and prevention from 2005 to 2010. Their results showed that the SARIMA model outperformed the three other models.

An investigative research in the energy sector by Oduro-Gyimah et al. (2012) on microwave transmission of the Yeji- Salaga link, collated receive signal level data between January 2006 to December 2010. They found $SARIMA(1,1,1)(0,2,2)_{12}$ as the best model to predict microwave transmission pattern for 24-months. They also reported that the SARIMA model was the best model to use for time series with seasonal complexity.

Another energy sector study by Abledu (2013), pointed $SARIMA(1,1,1)(0,1,2)_{12}$ as the best fit model to explain the pattern of short-term energy data collected within the time frame 2010–2011. The developed model was then used to make predictions for the year 2013.

In addition, Asamoah-Boaheng (2014) determined an efficient model using seasonal ARIMA model to forecast average temperature for the Ashanti Region of Ghana with data between 1980 to 2013 from the meteorology and climate department. Out of the three models built in their study, the $SARIMA(2,0,2)(2,1,1)_{12}$ model had the least values of AICs and BICs and so was chosen as most adequate for predicting average temperatures. There was neither a decreasing nor increasing pattern in the average temperatures but there existed some amount of seasonality in the data after decomposition of the time series.

While examining the rainfall pattern by Abdul-Aziz et al. (2013) in the Ashanti region of Ghana, the study revealed rainfall patterns changes over time using data from 1974 to 2010. Their results further showed a slight decrease in rainfall figures from August to December months and also that, there was likely to be an increase in rainfall figures from February to march months. Afrifa-Yamoah et al. (2016) too, examined the rainfall pattern for Brong Ahafo Region of Ghana and proposed $SARIMA(0,0,0)(1,1,1)_{12}$ as the best model to forecast rainfall patterns for the region. Their results revealed increases in rainfall figures for the months of September and October and minimum figures for January, February and December months for the region using data from 1975-2009.

Recently, Dritsaki (2016) examined the precision of forecasting of Greece's unemployment rate following procedures of Box-Jenkins methodology and demonstrated that $SARIMA(0,2,1)(1,2,1)_{12}$ had the best predictive power and hence was used to forecast unemployment rates for Greece. Correspondingly, Mahipan et al. (2013) compared SARIMA and ANN models and found $SARIMA(0,1,1)_{12}$ to be the best predictive model for unemployment rates in Thailand after satisfying all model assumptions. They suggested that the ANN model did not fit the unemployment data adequately hence the choice of the SARIMA model for them.

2.4 Brief introduction to SETAR model

Tong in an editorial report, recounts an ordinary royal statistical society meeting in 1977 presided over by the then David Cox. He explained the need to revisit all the lynx data models widely researched by Mr. Campbell, Dr. Tong and Professor Walker since they are time reversible. In addition, Dr. Granville Tunnicliffe asked, “would we not prefer a model which would exhibit stable periodic deterministic behaviour a limit cycle?” such limit cycles cannot arise from linear models. Long before this meeting, there hardly existed the concept of nonlinearity in time series since time series at that time operated on the assumption of linearity.

As events unfolded during the meeting, a challenge was thrown to all time series analysts to go find a practical explanation to the chaotic nature of time series like data for lynx which appeared not to be linear. This task was indeed a daunting one, since the concept was new and therefore needed much experimentation.

Tong determined on raveling the mystery of this “non-linearity”, focused on data of animal population and river flow. He observed two advantages for these data sets. Firstly, a better insight to the dynamics underlying the data. Which indeed was a revelation to Dr. Granville’s comments on “limit cycles”. Secondly, specific sets of data were needed to constantly check the methodology if it headed in the right direction. Keen on his quest to finding a better explanation to nonlinearity, between the years 1977 -78, asked his research student, Lim to carry out a task using simulation but she misunderstood and showed the results obtained by recursion of a SETAR model. This actually was a blessing in disguise as it offered Tong the opportunity to have a glimpse of the SETAR model for the first time. By the later part of 1980, after reading widely and huge experimentation Tong wrote a paper entitled “Threshold auto regression, limit cycles and cyclical data” to a UK journal but unfortunately for him it was rejected. Though saddened, he tried the platform of the Royal Statistical Society and fortunately for him the paper was accepted. Although accepted, there was quite a lot to be done to smoothen the rough edges like "how to choose the threshold variable" in a time series. As a matter of fact, this threshold concept was a very broad one and therefore needed further research to explain adequately the behaviour of a time series. This therefore gave other researchers like Watier and Richardson (1995) and Clements and Smith (1999) to research further on nonlinear models.

Narrowing the concept of this threshold Autoregression to internal changes within a time series brought about "SETAR". The SETAR model actually is a typical example of the threshold Autoregressive (TAR) model. The threshold model deals with general discontinuities in a time series unlike the SETAR model, internal discontinuities in a time series derives the term "self- exciting". A set of different linear AR process characterized by regime switches according the value of their threshold makes the entire process a nonlinear process.

2.5 Related studies on forecast performance of SETAR models

Quiet a number of literature exists in various areas where SETAR models have been applied. This section makes a casual review of such literature ranging from health, agriculture to economics just to mention a few.

Investigating several approaches to obtain multi-step forecast ahead for SETAR models, Clements and Smith (1997) based their study on existing literature of h-step ahead forecast values. In their study, they compared the forecast performance of SETAR models to AR models using various methods of simulation for US Gross National Product (GNP). Their results expressed some doubt on the robustness of SETAR models over two sample periods and on the size of the forecast accuracy there was no doubt SETAR models could deliver. They therefore concluded that improvements for forecast performance of linear AR model depended largely on historical events. They also concluded that, the type of nonlinearity in a time series contributes to the out of sample forecast performance.

Further investigations by Feng and Liu (2002) revealed features of nonlinearity in Canadian GDP. In their study, they compared the out of sample forecast performance of non-linear SETAR model and a standard linear ARIMA model. Employing techniques for forecasting like the1- step forecast ahead and multi-step forecast ahead, they revealed that both models appears satisfactory in terms of their MAPEs and RSMPEs. As a result, they proposed both models for predicting the Canadian GDP. They however are of the view that, it is more practical to use the multi-step technique in real life situations.

For more than forty years of existence and application of nonlinear models, Clements et al. (2004) reviewed the current state -of- art of these models ranging from estimation, evaluation and selection of best forecasting models among others for some economic and financial time series. They argued that, evidence of non-linearity in these time series are patchy and hence proposed a variety of future topics for upcoming researchers.

In an economics related study, Christos (2005) evaluated the performance of competing models for UK's unemployment rate from January 1971 to December 2002. Firstly, it was found that, $MA(4)-ARCH(1)$ provided better forecasts for unemployment rate. On the other hand, two forecasting samples pointed that $MA(4)$ model does well, whereas both $MA(1)$ and $AR(4)$ proved to be the best forecasting models for the other two forecasting periods. Hence, an empirical evidence suggests that there exist a close relationship between forecasting theory and labour market conditions. Further findings bring to the conclusion that, forecasting methods are nearer to the realities of UK labour market.

Another economics related study by Marcellino (2007), evaluated the relative performance of a bootstrap algorithm to standard linear models in an application to bootstrap series of GDP and inflation. Considering a variety of evaluation criteria, the standard linear models was suggested to be hardly beaten in prediction if only they are carefully specified.

Similar economics study by Sjoberg (2010), compared the forecast performance of ARIMA, LSTAR and SETAR models using monthly Swedish Industrial production from 28 branches over the period January1990 to December 2008. In terms of their mean square error of prediction, the ARIMA model was adjudged the best model for majority of the industrial branches. For some industrial branches and in a few branches, the LSTAR and SETAR models also outperformed the ARIMA model.

Equally, Gharleghi et al. (2014) evaluated the prediction performance of SETAR-EGARCH, ARIMA-EGARCH and AR-EGARCH models by employing data from three ASEAN currencies, namely; Indonesian Rupiah, Malaysian Ringgit and Thai Baht. They revealed that, the non-linear SETAR–EGARCH model fits better than the other two models considering factors like the data generation process and therefore concluded that, the data generation process of the variables is nonlinear.

van Ruth (2014), further applied several linear and non-linear models to monthly inflation rates for Netherlands over the period 1970-2009. In an empirical approach, the nonlinear STAR model was revealed to outperform all other linear and nonlinear approaches to explain the dynamics of inflation in the Netherlands.

A comparative study by Gibson and Nur (2011) investigated the performance of a two regime AR and STAR models. From their results they revealed that, the in-sample forecast of the STAR remains problematic since it properly does not replicate the observed behaviour of the time series like the SETAR model does. They further revealed that, the SETAR model outperforms the STAR model in terms of their out- of -sample forecast.

Another comparative study by Akeyede et al. (2015), compared different forecasting methods proposed in existing literature to obtain h-step ahead forecast. They further compared the multi- step forecast performance of linear and nonlinear time series via method of simulation. In conclusion, they proposed AR model to be best for forecasting AR functions and that LSTAR model supersedes all other non-linear forecasting methods except for polynomials. SETAR models beats all other models in terms of their prediction ability.

Likewise, Badawi (2016) in an unpublished master's thesis, compared the forecast performance between SARIMA and SETAR models using data on pneumonia cases in the northern region of Ghana. The data was obtained from the Tamale Teaching Hospital over the period January, 2000 to October, 2015 based on the forecast values from the two models, the SETAR model was adjudged the best in terms of their MSE, RMSE and also other forms of criterion selection processes.

Again, Karlsson and Karlsson (2016) modelled Sweden's unemployment rate by exploiting model efficiency of SARIMA, SETAR and VAR models. As a measure of goodness of fit test, they concluded that, the SARIMA and SETAR models performs better than the VAR model in terms of their out of sample forecast using quarterly unemployment data within the life span 1983 to 2010 from OECD. They further revealed that, short term forecasts are better off than longer term forecasts.

Analysing structural changes in the Taiwan stock market, Hsu et al. (2010) found the SETAR model to better examine the out of sample forecast of the non-linear time series. Employing monthly data from January 2005 to December 2009, the best model was chosen by performing a unit root test and comparing the out of sample forecast between the standard linear ARIMA model and the nonlinear SETAR model. A graphical view of their results showed that the structural change in the time series occurred in the month of June 2008. Thereby they built a 2–regime SETAR model. In conclusion the nonlinear SETAR model was found to be superior in forecasting to the linear ARIMA model in the Taiwan stock market.

Contrary to the norm of comparing the performance of models and prediction, Öznur and GÜNERİ§ (2015) developed a hybrid models whose components are either parametric or non- parametric models. They used the technique of back fitting algorithm based on smoothing spline to producer price index in Turkey. Their results showed that, the hybrid model AAR and SETAR performs best amongst all other hybrid models.

Recently, Zak (2017) analysed the forecast performance of nonlinear models in application to Czech Republic's exchange rate against EUR. Secondary data from three sources namely; Czech National Bank (CNB) Board decisions minutes, Statistical Data Warehouse of European Central Bank (ECB) website and from the press releases of Governing Council of ECB was obtained within the periods 1999 to 2016 for the study. Employing SETAR models and simple random walk they showed that the simple random walk model out performs the complex SETAR model. This further supports the assertion exchange rates are generally difficult to forecast.

Again, Fırat (2017) applied SETAR modelling process to explain the non-linear pattern in modelling currencies in EUR/USD, EUR/ TRY and USD/TRY parities. The study compared the forecast performance of the observed values for both linear and nonlinear models. It was concluded that, the non-linear SETAR model, outperformed the other forms of nonlinear and linear models.

2.6 Related studies on comparative studies of other linear and nonlinear models

In the field of engineering, Rodas et al. (2017) compared forecast performance between linear and nonlinear models in application to a DC motor. In their study, they adopted two criteria for measuring the performance of both models namely; tracking error and the control effort. Their results indicated that, the linear controller presents a significant advantage over their nonlinear counterpart when experimenting at operation speeds very close to the operation argument.

A medical evaluation by Murphy and Dietrich (2006), assessed two classes of control loop algorithms namely; the linear adaptive filter and the adaptive nonlinear neural networks for highly irregular patient behaviours. From their discussion, they explained the nonlinear neural networks demonstrated robust adaptability to all of the observed breathing patterns of patients but their linear counterpart failed.

Motivated by the need to establish a Vector Autoregressive Moving averages (VARMA) model, Iwok and Etuk (2009) developed a general bivariate Vector Autoregressive Moving Averages (BIVARMA) model. In their model development, they graphically compared the two competing models based on their estimates and residual variances. Furtherly, they established the fact that, the BIVARMA model performed better than the VARMA model.

Other aspects of human life are not left out in the application of linear and nonlinear models as demonstrated by Isaac et al. (2016) in modelling road accidents in Ghana. They used count regression models in their study as they compared several models like Poisson, Negative Binomial and Conway- Maxwell-Poisson to fit count data expected in the field of transportation. The best models were then chosen using Akaike Information Criterion and Bayesian Information Criterion. From their results, the Negative Binomial model appeared to perform better than the other models.

A number of peer review researches showed the existence of rich literature on macroeconomic variables. For instance, van Gysen et al. (2013) comprehensively compared the predictive accuracy of linear and non-linear models in forecasting financial returns. In their study, they considered varieties of macroeconomic variables from Johannesburg's stock exchange and their results showed that markov switching models provided the best in- sample fit. Nevertheless, they are of the view that, the out- of-sample forecast of the mixed models, EGARCH linear models and 2-state dynamic regression models outperform their alternative models.

As well, Cuestas et al. (2011) considered inflation for a study by applying non-linear and fractional integration as a hypothesis of empirical inflation for a pool of African countries. Their discussions revealed that, most African countries do not support the hypothesis. They therefore proposed that, the LSTAR models are more stable.

Furthermore, Hadrat et al. (2015) compared performance between Artificial Neural Networks (ANN) and the traditional Autoregressive and Vector Autoregressive in application to Ghana's inflation rate. Their results indicated that, ANNs had less errors than their traditional counter parts. In conclusion, the ANNs models were suggested to accurately forecast Ghana's inflation.

Several inflationary studies in Ghana has found nonlinear models to be the best model but Antwi (2017) is of a divergent view. Firstly, the existence of nonlinear pattern in monthly inflation for Ghana using nonlinear models was applied. Secondly, the threshold models; SETAR and LSTAR were applied on the monthly inflation data obtained from the Ghana statistical service within the period January 1981 to August 2016.

Finally, the performance of the threshold models was compared to $AR(1)$ and $AR(2)$ models using minimum values of AIC and BIC. In conclusion, both SETAR and LSTAR models fitted the data best but in terms of their forecasting ability the simple AR models outperformed their non-linear alternatives. Also, it was revealed that Ghana's inflation was likely to experience double digit in the year 2017.

Lastly, investigations by Nchor et al. (2016) established a linear and nonlinear relationship between macroeconomic variables and oil prices in Ghana. They considered six variables namely; real industry value added, real government expenditure, the real effective exchange rate, real oil price, real imports and inflation. Vector Auto regression (VAR) and Vector Error Correction Model (VECM) used in the study pointed that, irrespective of a positive or a negative change in oil prices, there definitely would be an impact on macroeconomic variables.

2.7 Chapter Conclusion

This chapter has comprehensively dealt with review of literature from other works relevant to the study. Review of these literatures offered the opportunity to encounter diverse methodology employed by researchers to forecast and predict various forms of macroeconomic variables, health related cases, climate related cases and more. However, in most of the studies, various forms of linear and nonlinear models such as ARIMA and SETAR, SARIMA and ANN's have extensively been used. In the case of Ghana, little has been done in relation to applying macroeconomic variables to compare their forecast performance. Based on this research gap identified, this study employed the Seasonal Autoregressive Integrated Moving Average (SARIMA) models and Self Excited Threshold Autoregressive (SETAR) models in predicting inflation and Currency in circulation (CIC) of Ghana

CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter presents the methods and techniques used to achieve the objectives of the study. It contains the statistical techniques, analytical procedures and theories that were employed to achieve the stated objectives of the study. Similarly, the chapter will be sectioned into source of data, unit root test, trend analysis, $AR(p)$ model, $MA(q)$, mixed $ARMA(p,q)$ model, $ARIMA(p,d,q)$ model, $SARIMA(p,d,q)(P,D,Q)_s$ model, $SETAR(2;p,d)$ model, linearity test, model estimation, model diagnosis and Diebold-Mariano test.

3.2 Research Design

This study takes the form of an experimental approach since this concept is imperative to forecasting phenomenon and research design to explain better the possible trends in the datasets to be considered.

3.3 Source of Data

The study obtained secondary datasets of two macroeconomic variables; namely inflation and currency in circulation rates spanning from January 1990 to August 2016, from the website of the Bank of Ghana (BoG). An analytical method was employed for the description of these macroeconomic variables and also for comparative performance of the nonlinear SETAR and linear SARIMA models using R statistical software.

3.4 Trend Analysis

Generally, the long- term change in time series levels over time longer than a year tends to reflect a trend in the time series. Linear, exponential, nonlinear and quadratic trends are some types of trends exhibited by the time series. This study will evaluate the performance of the aforementioned trend models in forecasting inflation and currency in circulation. These models are however, presented either as additive or multiplicative. The linear trend in the additive model does not depend on the trend unlike the multiplicative model which intensifies in its variation as the trend increases.

If the trend in the time series is a linear function of time then,

$$Z(t) = \beta_0 + \beta_1 t + \varepsilon_t, \qquad t = 1, 2, \ldots, n. \tag{3.1}$$

The logarithmic representation of the linear trend is

$$\log Z(t) = \beta_0 + \beta_1 t + \varepsilon_t. \tag{3.2}$$

And also, for an exponential representation of the series

$$Z(t) = e^{\beta_0 + \beta_1 t + \varepsilon_t}. \tag{3.3}$$

where $Z(t)$ represents observations of the time series, time (t), $(t = 1, 2, \cdots, n-1, n)$ and ε_t is the error term, whereas β_0 and β_1 are constants.

If the series shows a parabola shape then it exhibits a quadratic trend represented as

$$Z(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \varepsilon_t. \tag{3.4}$$

If $\beta_1 < 0$, then a plot of the series eventually turns to negative. But if $\beta_2 > 0$, then a plot of the series sky-rockets up.

For a polynomial of order k,

$$Z(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \cdots + \beta_k t^k \varepsilon_t. \tag{3.5}$$

Equations (3.1) to (3.5) were obtained by the application of the concept of ordinary least squares (OLS).

3.5 Unit Root Test

As described by Aidoo (2010) and Nasiru et al. (2013), most time series of macroeconomic variables exhibit trending behaviour in their mean, autocorrelation and variance through graphical plots of their series. By this, the time series turns to be non-stationary if the coefficients of the estimated model are in absolute value less than one. This means that, if the roots of the characteristic equation lie outside the unit circle, then the series is considered stationary. Principal examples of such macroeconomic variables include; inflation, exchange rates, stock prices and currency in circulation.

An important task to dealing with such macroeconomic variables is to first check for stationarity in the dataset, this idea is what is called a unit root. The unit root test is usually performed to check for the existence of a stochastic or a deterministic trend in the series. In practice the ARMA model is non-stationarity, but attains stationarity if transformed and differenced. This process brings about the integration factor (I) in the term ARIMA model, this in reality makes the model stationary and effective for any forecast to produce accurate results.

A good number of time series related research have employed graphical and statistical tests in checking for the presence of unit root in a series. Dickey-Fuller and Augmented Dickey-Fuller (DF-ADF), Kwiatkowski, Philips, Schmidt and Shin (KPSS), Zivot

Andrews (ZA), Phillips-Perron tests are typically used examples for statistical tests. In the graphical approach an Autocorrelation Function (ACF) plot is observed and if there exists a strong and slow decaying ACF, then it suggests a deviation from stationarity. Hence, the series must be transformed and differenced to attain stationarity. However, one needs to be cautious when differencing a non-stationary series since over differencing may exclude relevant information to building an appropriate model for any meaningful predictions. To determine seasonal and non-seasonal behaviours in a series, various forms of tests must be conducted under both seasonal and non-seasonal parts each using their ACF and PACF plots. Relevant tests to this study will be discussed below.

3.5.1 Phillips-Perron (PP) Test

The Phillips- Perron test has received much attention in analysis of most financial time series. This unit root test is quite different from the Augmented Dickey Fuller (ADF) test in terms of their approach in dealing with errors arising from serial correlation and heteroskedasticity. Also, this test unlike the ADF test, makes use of non-parametric procedures in their test regression. By so doing, they correct any form of serial correlation in the test regression by modifying the Dickey-Fuller test statistic. The test regression for the PP tests can be written as follows:

$$\Delta y_t = \beta D_t + \pi y_{t-1} + \mu_t \tag{3.6}$$

where Δ is a difference operator, β is a constant, $D_{(t)}$ is a time trend, π is the dickey fuller test statistic and μ_t is an error term.

Under the null hypothesis $\pi = 0$, this means non-stationarity.

By modifying the Dickey-Fuller test statistic $t_{\pi=0}$ and T_{π}, the statistics z_t and z_{π} are formed. Just like the ADF test statistic, these modified statistics are asymptotically distributed and normalized bias with a limiting distribution.

$$Z_t = \left(\frac{\hat{\sigma}^2}{\hat{\lambda}^2}\right)^{\frac{1}{2}} \cdot t_{\pi=0} - \frac{1}{2}\left(\frac{\hat{\lambda}^2 - \hat{\sigma}^2}{\hat{\lambda}^2}\right) \cdot \left(\frac{TSE(\hat{\lambda}^2)}{\hat{\sigma}^2}\right) \tag{3.7}$$

$$Z_\pi = T\hat{\pi} - \frac{1}{2}\frac{T^2 SE(\hat{\pi})}{\hat{\sigma}^2}(\hat{\lambda^2} - \hat{\sigma}^2) \tag{3.8}$$

with parameters $\hat{\lambda}^2$ and $\hat{\sigma}^2$ as consistent estimates of sample and long-run variances respectively.

$$\sigma^2 = \lim_{T \longrightarrow \infty} T^{-1} \sum_{t=1}^{T} E[\mu_t^2] \tag{3.9}$$

$$\lambda^2 = \lim_{T \longrightarrow \infty} \sum_{t=1}^{T} E[T^{-1} S_T^2] \tag{3.10}$$

$$\tag{3.11}$$

where $S_T = \sum_{t=1}^{T} \mu_t$. The estimators $\hat{\sigma}^2$ and $\hat{\lambda}^2$, are consistent residual estimates (μ_t) of the sample and the long-run variances respectively.

Although, the ADF has some advantages over the PP test, the PP tests are relatively good in dealing with large samples because of their asymptotic assumption, their robustness to any form of heteroskedasticity in error terms makes it more advantageous than the ADF test. Furthermore, there are no mandatory specification for lagged values as in the case of the ADF.

3.5.2 Kwiatkowski-Philips-Schmidt-Shin (KPSS) Test

The KPSS test is a complementary unit root test for testing a null hypothesis whose observable time series is trend-stationary against an alternative hypothesis of a unit root, this however is the opposite hypothesis for most unit root tests. In addition, the existence of a unit root does not guarantee stationarity but, by design, trend stationary.

Thus, it has become imperative to distinguish between some behaviour of time series which may be non-stationary, non-unit root yet trend- stationary. If the data generating process assumes no linear trend, then

$$Y_t = z_t + \varepsilon_t \tag{3.12}$$

where z_t is a random walk i.e. $z_t = z_{t-1} + \mu_t$, and μ_t is a random error term $\sim (0, \sigma_v^2)$ assumed to be i.i.d and ε is also a stationary error.

The hypothesis to tested is as follows;

$$\begin{aligned} H_0 &: \sigma_v^2 = 0 \\ H_1 &: \sigma_v^2 > 0. \end{aligned}$$

If H_0 is true, then it means that, Y_t becomes a stationary process since it has components of stationary ε_t and constant terms. The test statistics proposed by Kwiatkowski et al. (1992) is given below:

$$KPSS = \frac{1}{T^2} \sum_{t=1}^{T} \frac{s_t^2}{\hat{\sigma}_\infty^2}$$

where T is the number of observations, $s_t = \sum_{i=1}^{t} \hat{W}_i$ with $\hat{W}_i = Y_i - \hat{Y}_i$ and $\hat{\sigma}_\infty^2$ is an estimator of $\sigma_\infty^2 = T^{-1} Var(\sum_{t=1}^{T} \varepsilon_t)$ is a long-run variance estimator of the ε_t process. The observable time series y_t attains stationary with an integration of order one $(I(1))$ and the denominator quantity in KPSS statistic is an estimator of its variance, which has a deterministic limit. The term in the denominator ensures that overall; the limiting distribution is free of unknown nuisance parameters. If, however, it is integrated of order one $(I(1))$, the numerator will grow without bounds, causing the statistic to become large for large sample sizes. The null hypothesis of stationarity is rejected for large values of KPSS.

3.6 Autoregressive AR(p)Model

A time series y_t is said to be an autoregressive model if future values of the series depend on past values. Where there exists some amount of correlation between preceded and succeeded values of the time series, then the model becomes more effective for prediction. The word "auto" from the term "autoregressive" meaning "self" only considers past values of a time series to model its behaviour. The AR model is measured by a simple linear regression model which usually comprises a time series regression of past values plus an error term. The order of AR model is paramount to model building since it measures the immediate preceding values in the time series to be used to predict the succeeding value. In that case, if the preceding model is of first-order or order one, then it is written as $AR(1)$ and represented as:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \varepsilon_t. \tag{3.13}$$

For second order, $AR(2)$

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t. \tag{3.14}$$

More generally, for order p, $AR(p)$

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t. \tag{3.15}$$

where observations of the time series are denoted by the y's, ε_t is the random error at time period t, the terms $t-j$ and ϕ_j are the AR parameters to be estimated with $j = 1,2,\cdots,p$ as AR orders. For simplicity, the backward shift operator B, such that $By_t = y_{t-1}$, $B^2 y_t = y_{t-2}$ and so on is used to simply the above general model of order p into:

$$\phi(B)y_t = \varepsilon_t$$

where $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$ is a polynomial of order p.

For a stationary $AR(p)$ model,

$$y_t = \phi^{-1}(B)\varepsilon_t$$

converges to zero.

3.7 Moving Averages (MA(q)) Model

A time series y_t is said to be a moving average process if present values of the series depends on past errors or white noise values. Usually this model is classified as a "naive" model due to its ability to stabilise datasets by creating another dataset with less variations and also it does not show traces of seasonal trend in the dataset. For this reason, they are designed to provide smoothing effect within a time series which may be relevant or not to future estimations of a variable. But more importantly, it provides a "tidier" dataset for estimation.

The general form of the MA model is in the form:

$$y_t = \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \cdots + \theta_q\varepsilon_{t-q} + \varepsilon_t. \tag{3.16}$$

where θ_j are the model parameters to be estimated, with $j = (1, 2, \cdots, q)$, q is the order of the model and ε is the error term assumed to be i.i.d with mean zero and constant variance σ^2. No matter the independent average values of the error terms, the MA process always assumes stationarity.

Equivalently, using the backward shift operator (B), this can be written as:

$$y_t = \theta(B)\varepsilon_t$$

where $\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q$ is a polynomial in B of order q, and as usual ε_t is the normally distributed error term with mean zero and a constant variance σ^2. If the characteristic roots of the polynomial $m^q + \theta_1 m^{q-1} + \theta_2 m^{q-2} + \cdots + \theta_q$ is an invertible $MA(q)$ process equals to zero, then the absolute terms are less than one.

3.8 Autoregressive Moving Averages (ARMA (p,q))

The mixed model $ARMA(p,q)$ is a linear combination of the Autoregressive (AR) and Moving Average (MA) model time series models, where p and q are orders of AR and MA processes respectively. Mostly the $ARMA(p,q)$ model is given as:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}$$

where ϕ_i and θ_j are the respective AR and MA parameters to be estimated with $i = 1, 2, \cdots, p$ and $j = 1, 2, \cdots, q$, ε_t's are the i.i.d random errors with mean zero and a constant variance σ^2.

For simplicity, the backward shift operator (B) is used to express the $ARMA(p,q)$ model as:

$$\phi(B) y_t = \theta(B) \varepsilon_t$$

with $\phi(B)$ and $\theta(B)$ as polynomials in B of orders p and q. The $ARMA(p,q)$ process is said to be stationary if the roots of the AR component of the characteristic function converges to zero and also, if the polynomial of the MA component is less than one in absolute terms, then the process is described to be invertible.

3.9 Autoregressive Integrated Moving Averages (ARIMA)

The ARIMA model has been extensively used in theory and practice to study and forecast the behaviour of most time series variables with macroeconomic variables not being an exception. In general, they are assumed to be the stationary form of ARMA models by transforming and differencing the time series. However, when differencing the model to become stationary, care should be taken so as not to over difference the series otherwise the data might not fit well into the model and hence results in inaccurate predictions. What the ARIMA model actually does is to remove any form of "noise" from both the AR and MA components of the model to make it stationary and then predicts future values with the transformed model. The non-seasonal ARIMA model is basically denoted as $ARIMA(p,d,q)$ model, where the parameters p,d,q are the AR, differenced and MA terms greater than or equal to zero respectively.

The general form of the ARIMA model using the backward shift operator is:

$$\phi(B)(1-B)^d y_t = \theta(B)\varepsilon_t.$$

3.10 Seasonal Autoregressive Integrated Moving Averages (SARIMA) model

A time series model is said to be a seasonal ARIMA model if it has the structure of an ARIMA model with two components to capture seasonal and non-seasonal parts of the model. When a time series is found to exhibit seasonal patterns, it is usually advisable to carefully identify models that are relevant to capture this behaviour. The Seasonal ARIMA model is usually denoted in the form $ARIMA(p,d,q)*(P,D,Q)s$ where;

p is the number of non-seasonal AR component

q is the number of non-seasonal MA component

d is the order of non-seasonal differencing

P is the number of seasonal AR component

Q is the number of seasonal MA component

D is order of seasonal differencing

s can either be represented monthly as, $s = 12$, or on quarterly basis, $s = 4$ and so on.

Using the backward shift operator (B), the Seasonal ARIMA is given as:

$$\phi(B)\Phi(B)^s.$$

When a time series data exhibit seasonal behaviour, the ARIMA model is usually not able to capture the behaviour along the seasonal part of the series, hence, the tendency for wrong order selection for the non-seasonal component. Identification of relevant models and inclusion of suitable seasonal variables are therefore necessary when a time series data exhibit periodic patterns. The SARIMA model therefore has the advantage of capturing both seasonal and non seasonal components. The general SARIMA model can be expressed using the backward shift operator (B) as:

$$\phi(B)\Phi(B)^s(1-B)^d(1-B)^s y_t = \theta(B)\Theta(B^s)\varepsilon_t.$$

where

$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p,$

$\Phi(B^s) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - \cdots - \phi_P B^{Ps},$

$\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q,$

$\theta(B^s) = 1 + \theta_1 B^s + \theta_2 B^{2s} + \cdots + \theta_Q B^{Qs},$

and

y_t is the time series observation at time period (t),

B is the backward shift operator,

ε_t is a sequence of error term with mean zero and constant variance (σ^2),

$\phi_(i)$ and Φ_j are the non- seasonal and seasonal AR components respectively,

θ_i and Θ_j are the non- seasonal and seasonal MA components respectively.

3.11 Self Exciting Threshold Autoregressive (SETAR) model

An extension of an AR model results in a special case of the general Threshold Autoregressive (TAR) model called the SETAR model. This model was first introduced by Howell Tong in 1977 through a seminal paper and later by Tong and Lim (1980). Also, this model explains better the data generation process of a time series especially where changes in behaviour occur from one regime to another. Typically, these models are employed to understand and predict regime switching behaviours for parameters of higher orders with flexibility. The switch from one regime to another depends on the past values of the time series (hence the Self-Exciting portion of the name). This model is usually presented in the form (k,p) where k indicates the different regime changes, p also indicates the order of AR. Furthermore, this model can be thought of as a set of different AR models changing according to their lagged values or threshold variables triggered by changes in a delay parameter d. The two regime SETAR model of order p as given by Boero and Marrocu (2004) can be presented as follows:

$$y_t = \begin{cases} \phi_0^{(1)} + \sum_{i=1}^{p(1)} \phi_1^{(i)} y_{t-i} + \varepsilon_t^{(1)} & \text{if} \quad y_{t-d} \leq \tau \\ \phi_0^{(2)} + \sum_{i=1}^{p(2)} \phi_1^{(i)} y_{t-i} + \varepsilon_t^{(2)} & \text{if} \quad y_{t-d} > \tau \end{cases} \tag{3.17}$$

where $\phi_i^{(1)}$ and $\phi_i^{(2)}$ are coefficients in lower and higher regimes respectively needed to be estimated, τ is the threshold value, d is the delay parameter, y_{t-d} is the threshold variable which describes the switch from one regime to another regime, $p^{(1)}$ and $p^{(2)}$ are orders of AR in lower and higher regimes respectively and ε_t's are random error terms which are i.i.d with mean zero and constant variance σ^2. One major advantage of the nonlinear SETAR model is its ability to capture the irregular behaviour of a time series which cannot be captured by the "naïve" linear models. In this study, the two-regime SETAR model of the simplest form $(2; p; d)$ will be considered and also the procedure discussed by Franses and van Dijk (2000) will be used for effective model selection, furthermore the basic approach and tests for modelling SETAR model will be discussed below.

3.12 Linearity Test

For an observable time series y_t, the SETAR model can only be applied if the series under consideration is found to be nonlinear or irregular in nature under the hypothesis:

$$
\begin{aligned}
H_0 &: \text{ linearity exists} \\
H_1 &: \text{ nonlinearity exists.}
\end{aligned}
$$

Sometimes, in testing for a certain nonlinear feature in the Data Generating Process (DGP) of a series, the H_1 is specifically pre-chosen but in some cases, the H_1 is relatively general.

In order to apply the SETAR model, one must first check for the existence of nonlinear behaviour in the series, if there exists any, then an appropriate linear $AR(p)$ model is specified to test for nonlinearity in the series. A discussion paper by Franses and van Djik revealed that, the maximum lag order is based on the choice of the minimum AIC value in an $AR(p)$ model. After careful determination of the linear $AR(p)$, then various forms of linearity tests such as; Tsay F-test and Keenan test, a special case of the Ramsey Regression Equation Specification Error Test (RESET) test will be used to test for linearity. Below are brief discussions of the tests to be used in this study.

3.13 Ramsey Regression Equation Specification Error Test (RESET)

Ramsey (1969) proposed linear least square regression analysis for specific tests based on the view that, nonlinearity occurs in the diagnostics of a fitted model only if the linear model residuals are correlated in some power terms. Thus, this test focuses on specific errors in the linear regression including those from unmodeled nonlinearity and its readily applicable to linear $AR(p)$ models.

Now consider a linear AR(p) model:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t.$$

In the first stage in a RESET test, the least square estimate $\hat{\phi}$, the error term which is denoted by $\varepsilon_t = y_t - \hat{y}_t$ and sum of square residual given by $SSR_0 = \sum_{t=1+p}^{n} \hat{\varepsilon}_t^2$ are obtained.

Also, in the Second stage, we consider the linear regression

$$\varepsilon_t = X'_{t-1} b + Y'_{t-1} d + V_t$$

where $X_{t-1} = (1, X_{t-1}, \cdots, X_{t-p})$ and $Y_{t-1} = (X_t^2, \cdots, X_t^{s+1})$ for $s \geq 1$.

Then the least square residuals are computed as follows;

$$\hat{V}_t = \hat{\varepsilon}_t - X'_{t-1}\hat{b} - Y'_{t-1}\hat{d}.$$

Thirdly, SSR_1 is computed as:

$$SSR_1 = \sum_{t=1+p}^{n} \hat{V}_t^2$$

For an adequate $AR(p)$, b and d parameters must be zero. This can be tested in the final stage by the usual F statistic given by:

$$F = \frac{(SSR_0 - SSR_1)/g}{(SSR_1)/(n-p-g)} \quad \text{with} \quad g = s+p+1$$

which under normality and linearity has $F_{g,n-p-g}$.

3.14 Keenan-test

Considering an irregularly behaved observation, Keenan (1985) proposed nonlinearity test for such time series. This test can be well- thought as a special case of the Ramsey **Regression Equation Specification Error Test (RESET)** as discussed above. Precisely, a modification of the second stage of the RESET test using X_t^2 avoids any form of multicollinearity between X_t^2 and X_{t-1}. Particularly, Keenan assumes that the series can be approximated by the Volterra expansion as follows:

$$X_t = \mu + \sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \theta_u \varepsilon_{t-u} + \sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \theta_{uv} \varepsilon_{t-u} \varepsilon_{t-v}$$

where μ is the mean level of the nonlinear observation X_t, with the error terms ε_{t-u} and ε_{t-v}. The idea of the Keenan's test is of a similar view as the F-test and so clearly from linear approximation $\sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \theta_u \varepsilon_{t-u} = 0$. Similarly, the Keenan test follows the same procedure used in the RESET test above to obtain fitted estimate $(\hat{X}_t)$, error terms $(\hat{\varepsilon}_t)$, the residual set $(\xi_t)$and SSR. And lastly, we calculate

$$\eta = \frac{\sum_{t=p+1}^{n} \hat{\varepsilon}_t \hat{\xi}_t}{\sum_{i=p+1}^{n} \hat{\xi}_t^2}.$$

The test statistic of the Keenan test is:

$$\hat{F} = \eta^2 \frac{(n-2p-2)}{SSR - \eta^2}$$

where;

P = lag order of the autoregressive process

n = Sample size considered

SSR = Sum of Square Residual of the AR(p) process

and the null hypothesis;

$$H_0 : \sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \theta_u \varepsilon_{t-u} = 0$$

with assumption that ε_t's has zero mean and constant variance.

When the null hypothesis is satisfied, $\hat{F}$ is approximately F-distributed with 1 and $n-2p-2$ degrees of freedom. The H_0 of linearity is rejected if the p – value associated with $\hat{F}$ is small (p-value $< \alpha$) or when the value of $\hat{F}$ is greater than the selected critical value of the F- distribution with 1 and $n-2p-2$ degrees of freedom.

3.15 Tsay's F -test

Tsay (1989) introduced the Tsay test for detecting nonlinearity in an observable time series after improving on the power of the Keenan (1985) test in 1986. He generalized the Keenan test by explicitly checking for quadratic serial dependence in the series, by doing so, he catered for the disaggregated nonlinear variables. The first stage of this test is the same as the first stage of the Keenan test. Unlike the second stage in the Keenan test, the second stage of this test involves regression of the products $X_{t-i}X_{t-j}, i = j = 1, \ldots, p$ on $(1, X_{t-1}, X_{t-2}, \ldots, X_{t-p})$. Hence, its corresponding test statistic F is asymptotically distributed as $F_{m,n-m-p-1}$, where $m = \frac{p(p-1)}{2}$, with the hypothesis as;

$$\begin{aligned} H_0 &: \text{linear } AR(p) \text{ model} \\ H_1 &: \text{non-linear threshold model} \end{aligned}$$

3.16 Criterion for Model Selection

In order to select tentative models that fits the datasets well, it is imperative to consider various model selection criteria. The Akaike Information Criterion (AIC), the Akaike Information Criterion corrected (AICc) and the Bayesian Information Criterion (BIC) are examples of model selection criteria frequently used in most time series research. In this study too, the AIC and other model selection criteria are used. To select appropriate models from competing models, models with minimum or least values of AIC are considered for further validation. This is because these least values have been described as closest to truth. Again, this study considers models with more parameters as tentative models since it improves goodness of fit test.

The AIC model can be given as:

$$AIC = 2k - 2\log(L)$$

where

K denotes the number of parameters in the model,

L is the maximized value of the likelihood function.

In the case of the SETAR model, the AIC model is given as follows:

$$AIC\,(p1, p2) = n_1 \ln \hat{\sigma}_1^2 + n_2 \ln \hat{\sigma}_2^2 + 2\,(p_1 + 1) + 2(p_2 + 1)$$

where

n_1 and n_2 are the observations in regimes1 and 2.

$\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ represents the variance of residuals in regimes 1 and 2.

$p_1$and p_2 denotes the minimized information criterion selected lag orders in regimes 1 and 2.

3.17 Model Diagnostics

After carefully selecting tentative models to be used for forecasting, we check the residuals of the models to ensure that, their time series satisfy all model assumptions. This step is paramount to making any meaningful inferences with the models. So, we employ the Ljung-Box and ARCH-LM tests on the model residuals as discussed below:

3.17.1 Ljung-Box Test

Ljung and Box (1978), described this test as a diagnostic tool used to check for the presence or absence of serial correlations in the residuals of a fitted model. Thus, a time series with any specified lag orders, say order m examines autocorrelations in the residuals. Instead of testing randomness at each distinct lag, usually it tests the "overall" randomness based on a number of specified lags, and so for this reason it is referred as a portmanteau test. The test procedure is given as follows;
The hypothesis to be tested is;

$$H_0 : \text{Residuals are uncorrelated up to order m}$$
$$H_1 : \text{Residuals are uncorrelated up to order m}$$

The test statistic is

$$Q_m = n(n+2)\sum_{k=1}^{m}\frac{\hat{\rho}_k^2}{n-k}$$

where

$\hat{\rho}_k^2$ represents residual autocorrelations of the series at lag m

m is the number lags being tested,

n is the number of residuals.

We reject the null hypothesis if the value of Q_m is greater than the chi-square table value. Hence, residuals are correlated. However, if the value of Q_m is less than the chi-square table value, we do not reject the null hypothesis. So, we conclude that the model residuals are uncorrelated and therefore the model fits the dataset well. Otherwise the whole estimation process has to be repeated again in order to get the most adequate model.

3.17.2 ARCH-LM Test

For residuals of a fitted model to become adequate enough to produce meaningful forecast values, the model must satisfy all model assumptions. Thus, the residuals should have constant variance. Hence, the reason why Engle (1982) proposed the ARCH-LM test to cater for issues of conditional heteroscedasticity in squared residuals. The test procedure is as follows;
The hypothesis is;

$$H_0 \;:\; \text{There is no heteroscedasticity in the model residuals}$$

$$H_1 \;:\; \text{There is heteroscedasticity in the model residuals.}$$

and the test statistic is

$$LM = nR^2 \tag{3.18}$$

where n is the number of observations and R^2 is the coefficient of determination of auxiliary residual regression.

$$e_t^2 = \beta_0 + \beta_1 e_{t-1}^2 + \beta_2 e_{t-2}^2 + \cdots + \beta_q e_{t-q}^2 + \varepsilon_t$$

where e_t is the residual and q is the length of ARCH lags.

We reject the null hypothesis if P-values are less than 5% level of significance, and hence we conclude the existence of heteroscedasticity in the model residuals. Also, p-values greater than 5% significance level indicates there is no heteroscedasticity. Hence, we conclude the model has constant variance.

3.18 Diebold-Mariano Test

Forecasting has been described as the last step to modelling procedures following the approach of Box-Jenkins. To ensure competing models are statistically significant, Diebold and Mariano (1995) proposed a test popularly known as the Diebold-Mariano test. The purpose of this test is to verify forecast accuracy between competing models. In this test, we assess forecasts between two competing models by considering their mean squared errors for statistical significance. Nevertheless, lower values of mean square errors of one forecast in comparison to the other do not necessarily translate into superiority of that forecast.

The hypothesis is given as;

H_0 : There is no significant difference between the two models

H_1 : There is significant difference between the two models.

Following a standard normal distribution, the test statistic is given as;

$$DM = \left[\hat{V}(\hat{d})\right]^{-\frac{1}{2}}\hat{d} \tag{3.19}$$

where;

$d_t = e_{t1}^2 - e_{t2}^2$ is the difference between the sets of squared error forecasts from the two competing models.

And $\hat{d}$=$\frac{1}{T}\sum_{t=1}^{T} d_t$ is the mean, T= no of residuals.

3.19 Markov Chain

The Markov chain is the smallest form of a Markov model. This Chain models the state of a system with changing random variable throughout time. This model operates on the Markov property which suggests that, the distribution of a current variable depends on previous distribution states. The Markov Chain Monte Carlo, an application of the Markov Chain uses the Markov property to prove that a particular method for performing a random walk will sample from its joint distribution. The model has the ability to model the behaviour of a system by constructing consistent block diagrams, fault trees and more. This model too can handle errors and recovery at detailed levels in modelling. Despite its advantages, it is cumbersome to handle since it requires large number of states for analysis. It is also quite difficult to construct and interpret though it models types of complexity solution techniques that are feasible but for only small models. In the study of a particular system, it does not offer the required speed and accuracy for the study.

3.20 Jarque-Bera Test

The Jarque-Bera test, a type of Lagrange multiplier named after Carlos Jarque and Anil K. Bera is used for checking normality in a data. Normality is one of the assumptions used in most statistical tests, just like the t or F tests. In order to check normality in a data, it is prudent to consider values of Skewness, (i.e. perfect symmetry around the mean) and Kurtosis (i.e. how much tailed a data is and how peaked the data is). However, it is needless to know the mean or standard deviation for the data before performing the test.

The test statistic is defined as;

$$JB = n\left[\frac{b_1}{6} + \frac{(b_2-3)^2}{24}\right]$$

where;

n is the sample size

b_1 is the sample skewness

b_2 is the sample Kurtosis

with the hypothesis is given as;

H_0 : The data is normally distributed

H_1 : The data does not come from a normal distribution.

Generally, the null hypothesis is rejected for smaller p-values less than 5% level of significance. Hence, the data under consideration is said not to be normally distributed.

3.21 Chapter Conclusion

The chapter has dealt with statistical techniques and methods which include; the unit root tests, linearity tests and Box-Jenkins methodology for both linear and nonlinear models. All these techniques are appropriate for the study due to their advantages over other time series.

Due to the cumbersome nature of large data, the unit root test is preferred for checking stationarity as compared to the graphical approach. The Box-Jenkins methodology is also most appropriate in terms of forecasting for shorter periods of time as compared to other time series like Markov chain which is difficult to interpret. Again, the normality test is an appropriate approach to understand the distribution of most data rather than assuming all datasets follow normal distribution.

CHAPTER 4

DATA ANALYSIS AND DISCUSSIONS

4.1 Introduction

This chapter deals extensively with the results and discussions obtained from the study. It includes preliminary and further analyses of datasets, the results and discussions. In the preliminary analysis, the source from which data was obtained and general characteristics of the time series datasets is explained. Also, in the further analysis, the various models and techniques employed in derivation of the best model(s) is explained. Furthermore, in the results and discussion, results obtained from analysis using the R statistical software are discussed. The chapter ends with the conclusion, which summarises the entire chapter.

4.2 Preliminary Analysis

Monthly data of Ghana's inflation and CIC rates were obtained from the website of the Bank of Ghana ranging from January 1990 to August 2016. The datasets each of which consists of 320 observations, were divided into two parts namely; the train and test datasets. A casual view of time series plots of these datasets appeared seasonally adjusted and asymmetric in nature. Following the Box and Jenkins (1976) procedure for forecasting, the first 296 points which starts from January 1990 to August 2015 represents the train dataset used to build the model whiles the remaining points of 12 months from September 2015 to August 2016 represented the test dataset retained for out of sample performance assessment for each variable.

4.2.1 Preliminary Analysis of Inflation rate

In this section, descriptive statistics of monthly inflation rates is discussed.

By inspection, Table 4.1 recorded a minimum value of 7.3 and a maximum value of 70.8 for inflation rate in March 1992 and December 1995 respectively. The sample moments of inflation for the time period under consideration appeared positively skewed and leptokurtic in nature about their mean values. The leptokurtic nature of the data means that, the inflation values were more peaked about the mean value of the inflation rate. The Jarque-Bera test for normality was used to confirm the asymmetric nature of the inflation rate. At 5% significance level, the test confirmed that the test was not normally distributed as its p-value, $2.2e-16$, was less that the significant level.

Table 4.1: Descriptive Statistics of Inflation Rates

Mean	Median	Std	Min	Max	Skewness	Kurtosis	Jarque-Bera (P-value)
21.16	17.30	12.88	7.30	70.80	1.86	3.83	2.2e-16

Figure 4.1, shows both increasing and decreasing pattern in inflation rate overtime. These fluctuations in the rate could be as a result of unstable market conditions. Somewhere around 1991, the figure shows volatility in the time series plot. Volatility in this plot could be as a result of some monetary factors such as poor agricultural production and world economic crisis as described by Aidoo (2011) and Ocran (2007). The years 2004 to 2008 saw consistent decline and pattern in inflation rate. This may have resulted due to the implementation of policies like reduced taxes leading to the reduction in price of goods and services. Yet again, inflation rates appeared stable from somewhere around the year 2011 to 2013. A trendy nature in the plot shows periodic pattern over time in regular intervals.

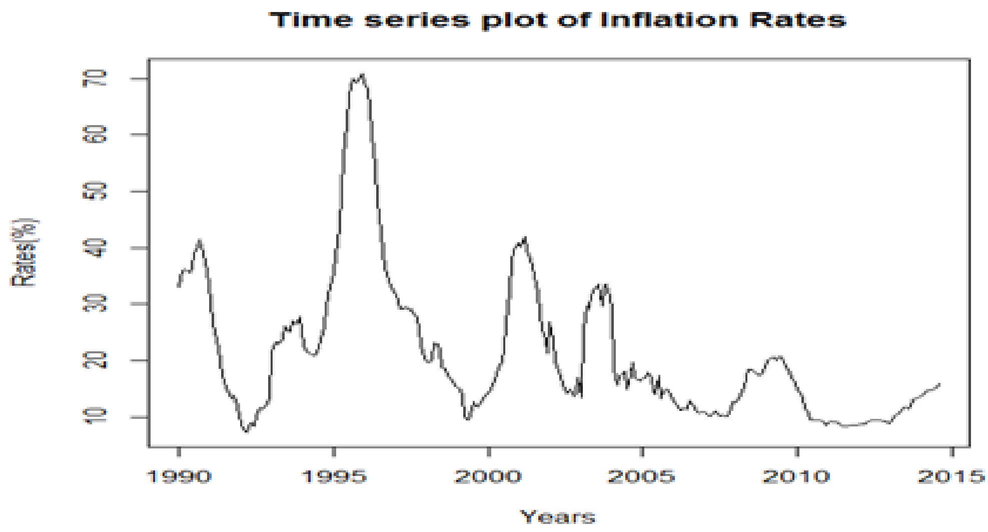

Figure 4.1: Time series plot of inflation

Figure 4.2 confirms Ghana's inflation is non-stationary, since there exists slow decay in the series for all lag values in the ACF plot. Likewise, the PACF plot shows a significant spike in lag 1 with marginal spikes in some few lags of the series.

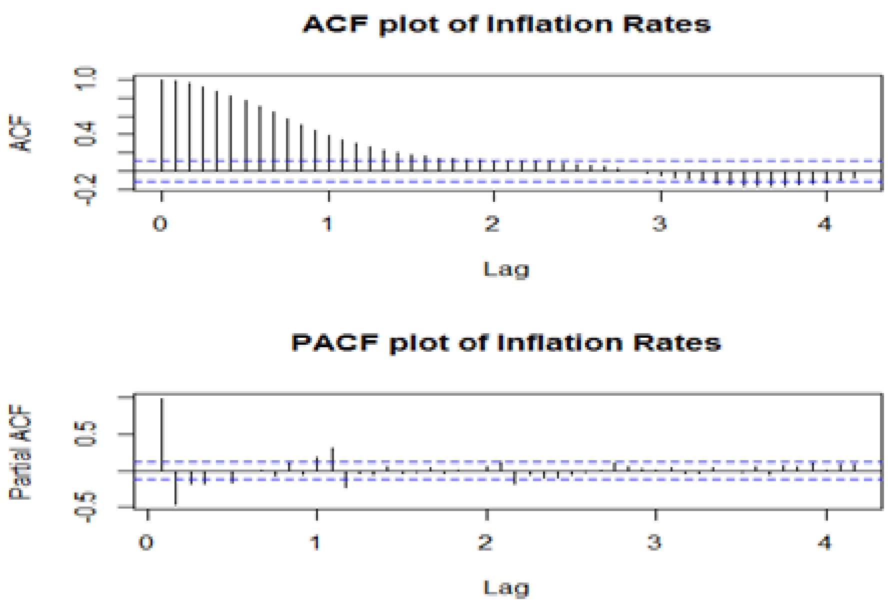

Figure 4.2: ACF and PACF plots of inflation rates

After Figure 4.2 confirmed the series was nonstationary, the data was logarithmically transformed and differenced to attain stationarity in the series as shown in Figure 4.3. Significant spikes for both ACF and PACF plots appear to revolve about fixed points, this affirms stationarity. Moreover, these plots demonstrate a sine wave-like pattern. This confirms Ghana's inflation has both seasonal and non-seasonal components. In this figure too, the PACF plot shows significant spikes in lags 12, 24 and 36 and at lag 12 for the ACF plot. This affirms the trendy pattern in the series at regular intervals. For further analysis, the transformed and differenced series is used.

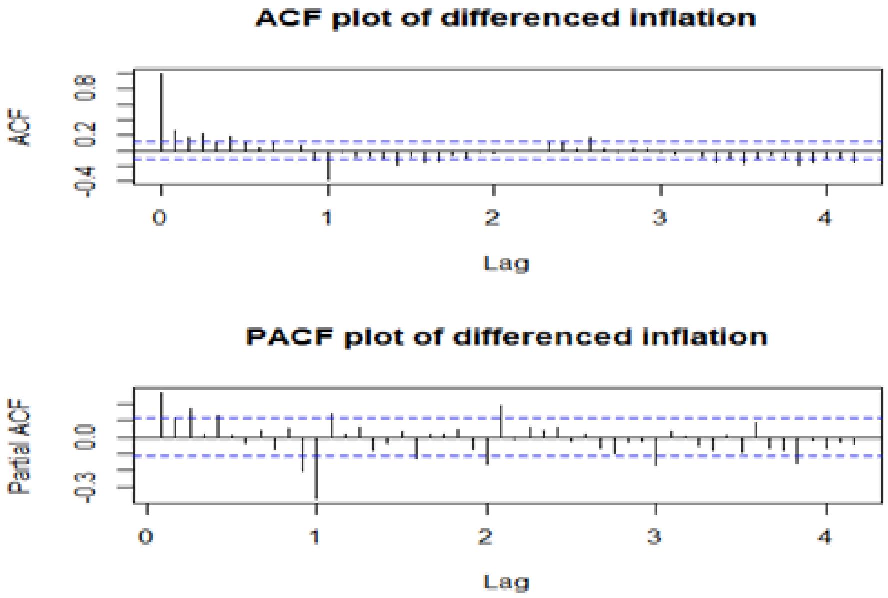

Figure 4.3: ACF and PACF plots of differenced inflation

4.2.2 Preliminary Analysis for CIC rate

An examination of descriptive statistics for the CIC data in Table 4.2, revealed the value 53.1 as the minimum CIC rate recorded in March 1996 and the value 9536.5 as the maximum CIC rate recorded in December 2015. The sample moments for the CIC rate under study appeared positively skewed and platykurtic in nature about the mean value. The platykurtic nature of the data indicates the CIC values are broadly spread about the mean value of the CIC rate. Just like the inflation, the test for normality in the CIC rates confirms the data is far from a normal distribution. So, we conclude the CIC rate is asymmetric in nature.

Table 4.2: Descriptive Statistics of CIC Rates

Mean	Median	Std	Min	Max	Skewness	Kurtosis	Jarque-Bera (P-value)
1729.6	730.5	2273.82	53.1	9536.5	1.68	1.75	2.2e-16

Although, the years 1996 and 1997 experienced steep decline in CIC rates, the years 1990 to 1993 recorded constant values for the CIC rate. In later years, CIC rates picked up until somewhere around the year 2000. From the year 2000 onwards, a gradual increase occurred in CIC rate at regular intervals. This indicates that as years go by the total supply of money in Ghana increases. Volatility explains better the consistent peaks in the time series plot of the CIC rates as shown in Figure 4.4. This volatility, may be due to instability in prices and market interest rates.

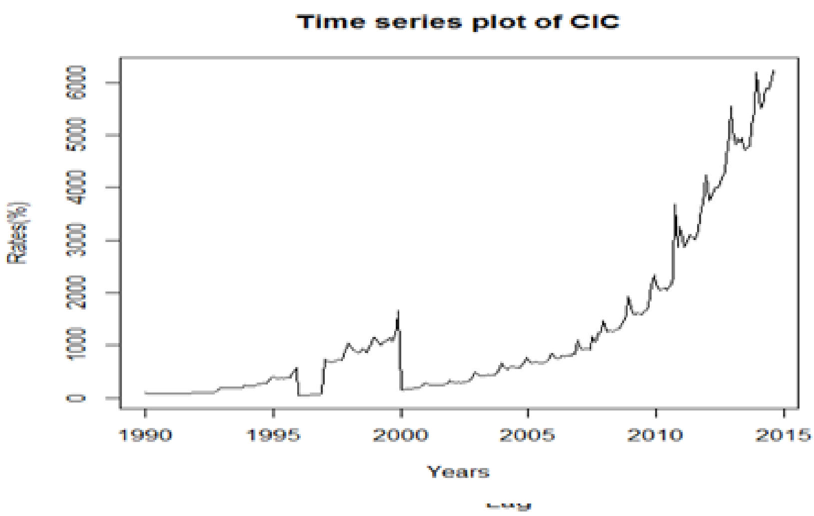

Figure 4.4: Time Series plot of CIC

A graphical view of the CIC rate presented in Figure 4.5 revealed that, Ghana's CIC rate is non- stationary. Since, the graphs do not revolve around a zero mean and a constant variance. In addition, the plot of ACF shows a slow decay in its series. But then, the PACF plot displays a single significant spike at lag 1 with several marginal spikes across all other lags.

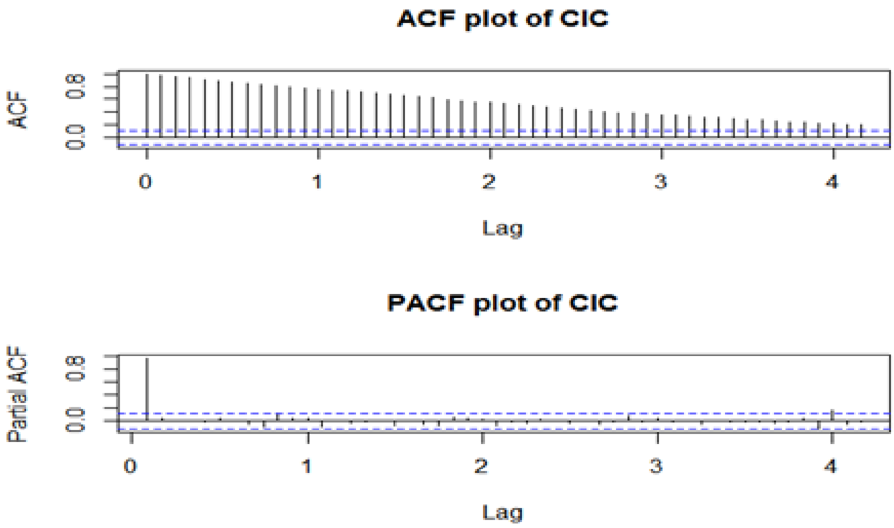

Figure 4.5: ACF and PACF plots of CIC

In Figure 4.6, the spikes in plots of ACF and PACF of CIC rate decay rapidly, this indicates the series is stationary. Stationarity of the series is again confirmed as the plots are perceived to revolve around fixed points.

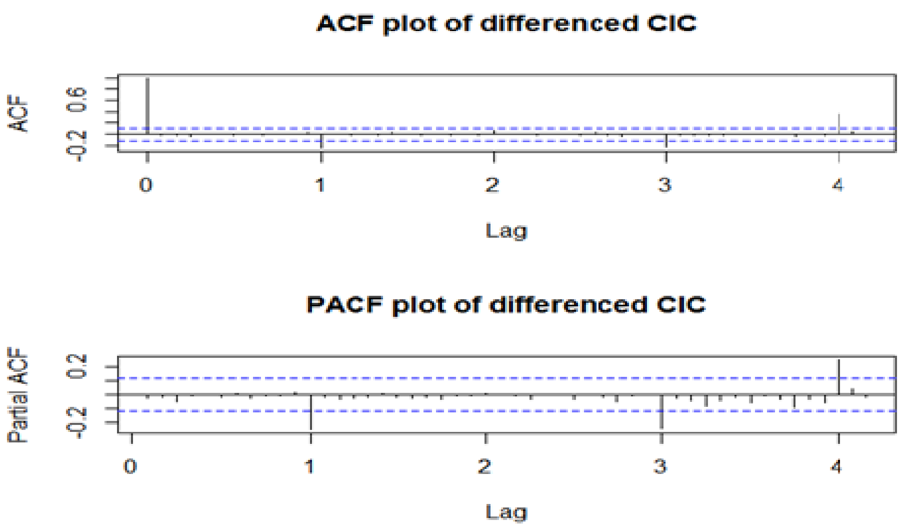

Figure 4.6: ACF and PACF plots of differenced CIC

4.3 Further Analysis

To explain the procedures used for model building, evaluating and forecasting in this study, we derived appropriate model(s) by employing SARIMA and SETAR modelling procedures for both inflation and CIC rates. This section too represents and explains actual statements and observations including tables and figures. Sufficient details of the study as well are discussed to serve as reference to other related studies.

4.3.1 SARIMA modelling

In this section, the seasonal ARIMA approach is followed to fit appropriate models to forecast Ghana's inflation and CIC rates. In this modelling cycle, stationarity is first checked in the original datasets for both rates using Phillips-Perron and KPSS unit root tests. Later, we follow the procedures described by Box and Jenkins in model building and forecasting.

SARIMA Modelling for inflation

To model inflation, we first checked for stationarity in the data using the KPSS and PP tests for unit root testing. For the PP-test, the null hypothesis of non-stationarity was not rejected because the resulting P-value was greater than 5% level of significance. So, we conclude the raw data is not stationary. Hence, the data was logarithmically transformed and differenced resulting in a P-value smaller than the 5% level of significance. This indicates the data is now stationary.

On the other hand, the alternative unit root test, the KPSS test was employed to cater for shocks within the series under the null hypothesis of stationarity. The P-Value that resulted from this test, suggests the raw data is not stationary. Just as in the PP-test, the raw data is transformed to attain stationarity. This concept of stationarity is important to most time series analysts. Since, it enables them to make meaningful predictions. Table 4.3 summarises the results from the unit root tests performed for inflation.

Table 4.3: Unit root test for inflation

Unit root test	**Raw Data**	**Transformed Data**
Phillips-Perron (p-value)	0.29	0.01
KPSS (p-value)	0.01	0.1

Now that our data is stationary, our next task is to build appropriate models that fit the data well. We do so, by further examination of Figure 4.3. Suggested AR and MA orders for the models under construction are chosen from Figure 4.3. While we selected AR orders from significant spikes of lag orders in the differenced PACF plot, MA orders were selected from the differenced ACF plot with significant spike of lag orders to make up tentative models. In this approach of selecting model components, we considered all spikes above zero as non-seasonal components and below zero as the seasonal components. By inspection of the differenced PACF plot in Figure 4.3, lag orders 1 and 3 were chosen as non-seasonal components and lag orders 1, 2 and 3 as seasonal components of the *AR* order to be added to the model. In the differenced ACF plot too, lag orders 1,2 and 3 were chosen as non-seasonal components with lag order 1 as the seasonal MA component to be added to the model.

The various model combinations are evaluated by considering models with least values of AIC, RMSE, MAE and MAPE. These are shown in Table 4.4.

Table 4.4: AIC, RMSE, MAE and MAPE of candidate seasonal ARIMA Models for Inflation

Model	**AIC**	**RMSE**	**MAE**	**MAPE**
$ARIMA(1,1,1)(1,0,1)_{12}$	-696.55	0.07550	0.05060	1.7985
$ARIMA(3,1,1)(1,0,1)_{12}$	-693.47	0.07540	0.05043	1.78758
$ARIMA(1,1,1)(2,0,1)_{12}$	-695.00	0.07543	0.05032	1.78517
$ARIMA(3,1,1)(2,0,1)_{12}$	-691.82	0.07534	0.05019	1.78018
$ARIMA(1,1,2)(1,0,1)_{12}$	-694.68	0.07549	0.05050	1.78991
$ARIMA(1,1,2)(2,0,1)_{12}$	-693.69	0.07542	0.05022	1.78136
$ARIMA(3,1,2)(1,0,1)_{12}$	-692.64	0.07524	0.05046	1.78775
$ARIMA(3,1,2)(2,0,1)_{12}$	-691.02	0.07518	0.05021	1.77997
$ARIMA(1,1,3)(1,0,1)_{12}$	-693.75	0.07537	0.05038	1.78537
$ARIMA(1,1,3)(2,0,1)_{12}$	-692.07	0.07535	0.05016	1.77862
$\mathbf{ARIMA(3,1,3)(1,0,1)_{12}}$	**-698.18**	**0.07377**	**0.04940**	**1.74989*****
$ARIMA(3,1,3)(2,0,1)_{12}$	-694.36	0.07411	0.04974	1.76357

*** means best based on the selection criteria.

Carefully scrutinizing the twelve tentative models in Table 4.4 for inflation,

$ARIMA(3,1,3)(1,0,1)_{12}$ could be adjudged the best model in terms of its minimum values of AIC, RMSE, MAE and MAPE for inflation rate.

In line with a said objective of the study, we sought to make an out of sample forecast from the best model. Hence, tentative models with the most minimum values were selected and assessed using the forecast accuracy test. In this test, the method of maximum likelihood was used to evaluate the values of parameter estimates of the models with their corresponding standard errors as shown in Table 4.5. Model variables with either corresponding positive or negative values for parameter estimates and standard errors are considered significant to the model. However, the greater the number of variables resulting from either a positive standard error and a positive estimate and vice-versa, the more accurate the model.

By the inspection of Table 4.5, seven parameter variables demonstrated to be more significant to the model ARIMA$(3,1,3)(1,0,1)_{12}$. This means that the variables AR(1), AR(2), AR(3), MA(1), MA(3), SAR(1) and SMA(1) were added to the model, since they proved to be significant.

Table 4.5: Estimated Parameters of $ARIMA(3,1,3)(1,0,1)_{12}$ for inflation

Variable	**Estimate**	**Standard Error**	**95% confidence Interval**	
			Lower limit	Upper Limit
$^*AR(1)$	-0.8740	0.0651	-1.0016	-0.7464
*AR (2)	0.5570	0.1067	0.3478	0.7662
$^*AR(3)$	0.8286	0.0599	0.7112	0.9459
$^*MA(1)$	1.1159	0.1238	0.8732	1.3585
$MA(2)$	-0.2394	0.1679	-0.5684	0.0897
$^*MA(3)$	-0.6872	0.0886	-0.8610	-0.5135
$^*SAR(1)$	0.1830	0.0821	0.0221	0.3439
$^*SMA\ (1)$	-0.8027	0.0592	-0.9186	-0.6867
				$\hat{\sigma}^2 = 0.00546$

* means significant at 5% significant level.

Table 4.6 revealed just *SAR (1)* model variable as the significant parameter to the model ARIMA$(1,1,2)(1,0,1)_{12}$ and hence, this model was considered in the model.

Table 4.6: Estimate Parameters of $ARIMA(1,1,2)(1,0,1)_{12}$ for inflation

Variable	Estimate	Standard Error	95% confidence Interval	
			Lower limit	Upper Limit
$AR(1)$	-0.8708	0.0620	0.7493	0.9923
$^*MA(1)$	-0.6669	0.0844	-0.8323	-0.5016
$MA(2)$	-0.0231	0.0642	-0.1490	0.1028
$SAR(1)$	0.1562	0.0872	-0.0147	0.3270
$^*SMA\ (1)$	-0.7591	0.0650	-0.8864	-0.6317
				$\hat{\sigma}^2 = 005717$

* means significant at 5% significant level.

In terms of the number of significant variables to the model, the model $ARIMA(3,1,3)(2,0,1)_{12}$ displayed six significant parameter variables as shown in Table 4.7. This indicates the model is significant. Hence, we consider the model for estimation.

Table 4.7: Estimate Parameters of $ARIMA(3,1,3)(2,0,1)_{12}$ for inflation

Variable	Estimate	Standard Error	95% confidence Interval	
			Lower limit	Upper Limit
$AR(1)$	0.0647	0.0644	-0.0616	0.1910
$^*AR(2)$	-0.3129	0.0508	-0.4124	-0.2135
$^*AR(3)$	0.8572	0.0631	0.7335	0.9810
$MA(1)$	0.1283	0.0962	-0.0602	0.3169
$^*MA(2)$	0.4864	0.0784	0.3328	0.6401
$^*MA(3)$	-0.6552	0.0916	-0.8347	-0.4756
$SAR(1)$	0.1664	0.0976	-0.0250	0.3577
$SAR(2)$	0.0795	0.0795	-0.0763	0.2353
$^*SMA(1)$	-0.7934	0.0791	-0.9484	-0.6385
				$\hat{\sigma}^2 = 0.005511$

* means significant at 5% significant level.

Obtaining parameter estimates of tentative models is not enough to conclude that, a particular model best fits a data. An important task therefore is to check if all assumptions of the SARIMA model are satisfied. This was done by performing diagnostic checks on model residuals as shown in Table 4.8. Thus, we checked if residuals have zero mean, constant variance and are also uncorrelated, that is if residuals follow a white noise process.

The Ljung- Box test, a portmanteau test, was performed on the residuals of the model under the null hypothesis of residuals being random, P-values resulting from this test revealed the residuals were random. This was so, because P-values obtained from the test were greater than 5% level of significance. Equally, the ARCH-LM test was performed on the model residuals under the null hypothesis of there is no ARCH effect. This test was included in the study due to its ability to test for variations in model variances. The resulting P-values greater than the 5% level of significance indicated, there were no ARCH effect in the model residuals. This therefore, enables us conclude the presence of constant variance with zero mean in all the tentative models.

Table 4.8: Residual diagnostic tests for Seasonal ARIMA models for inflation

Model	**P-values**	
	ARCH-LM test	Ljung-test
$ARIMA(3,1,3)(1,0,1)_{12}$	0.9111	0.8702
$ARIMA(1,1,2)(1,0,1)_{12}$	0.7321	0.4823
$ARIMA(3,1,3)(2,0,1)_{12}$	0.6512	0.7015

The ACF plots of the residual models was observed to be zero, depicting residuals are uncorrelated. Also, it was observed that all spikes of the plots in Figure 4.7 are within the confidence boundaries. This means the models fits best the data under study. Clearly, all these tentative models appear to be satisfactory of all SARIMA model assumptions, thereby we consider all these models for further valuation.

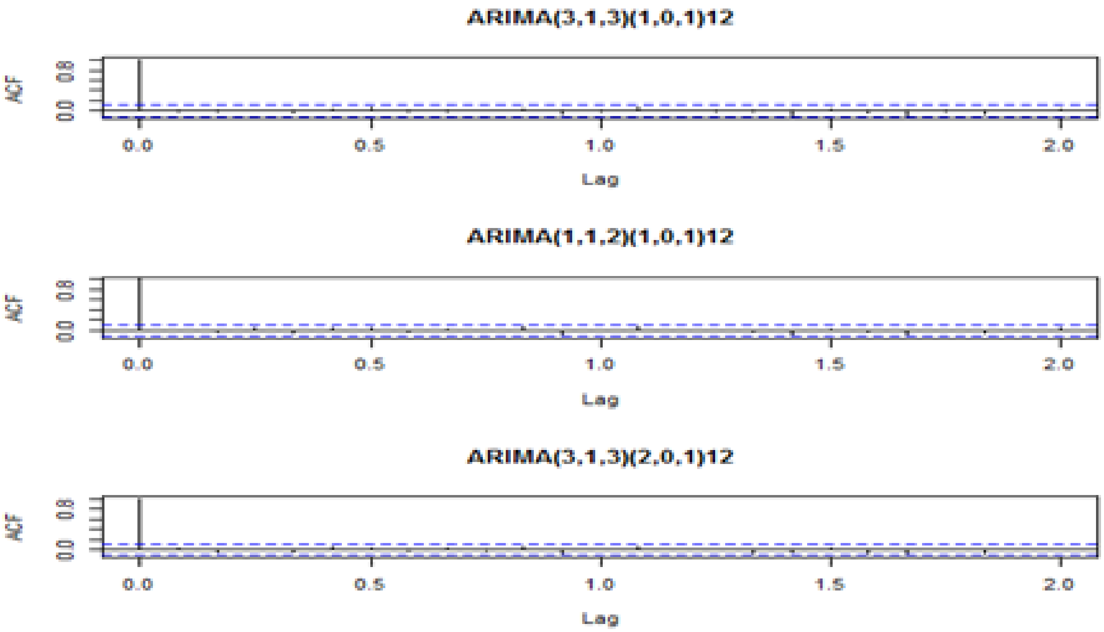

Figure 4.7: ACF plots of residuals for inflation

SARIMA Modelling for CIC

As shown in Table 4.9, the resulting P-values from the raw data for CIC was not stationary for both PP and KPSS tests under their respective null hypotheses. Statistically, if a P-value is greater than 5% significance level, the null hypothesis is not rejected and vice- versa. Contrarily to the PP test, the KPSS test produced a P-value of 0.01, which is less than a 5% significance level. This affirms non-stationarity in the raw data considering both unit root tests. As usual, the non-stationary data was transformed to attain stationarity in the data.

Table 4.9: Unit Root test for CIC rate

Unit root test	**Raw Data**	**Transformed Data**
Phillips-Perron (p-value)	0.99	0.01
KPSS (p-value)	0.01	0.1

After attaining stationarity in the data, the next task was to select model constituents to enable us derive appropriate model(s) for further analysis. Just as it was done earlier for inflation, the same procedure was followed to select our seasonal and non-seasonal components of the model(s) to be derived by examining Figure 4.6. On successfully deriving our models with their respective values of AIC, RMSE, MAE and MAPE, models with minimum values were considered as the selection criterion for the tentative models for further assessment. Table 4.10 below provides models with their respective measure of values.

Table 4.10: AIC, RMSE, MAE and MAPE of candidate seasonal ARIMA Models for CIC

Model	AIC	RMSE	MAE	MAPE
$ARIMA(0,1,0)(1,0,1)_{12}$	-16.49	0.23235	0.07934	1.32993
$ARIMA(0,1,0)(1,0,3)_{12}$	-27.72	0.22494	0.08272	1.36114
$ARIMA(0,1,0)(3,0,1)_{12}$	-26.01	0.22675	0.08745	1.44632
$\mathbf{ARIMA(0,1,0)(3,0,3)_{12}}$	**-52.20**	**0.20165**	**0.07955**	**1.30287*****

$***$ Means minimum values of the selection criterion.

Later, the estimated parameters were of the tentative models were checked for their significance with the least values of selection criterion. By this, the forecast accuracy was employed as discussed earlier. Results from Tables 4.11, 4.12 and 4.13 indicates all estimates of the model components are relevant. Here too, variable parameters with resulting positive values for both estimates and standard errors or vice-versa were considered as relevant to the model under construction.

Table 4.11: Estimated parameters of ARIMA (0,1,0) $(3,0,3)_{12}$ for CIC

Variable	Estimate	Standard Error	95% confidence Interval	
			Lower limit	Upper Limit
$^*SAR(1)$	-1.0917	0.1203	-1.3274	-0.8559
$^*SAR(2)$	-0.7941	0.1551	-1.0981	-0.4790
$^*SAR(3)$	-0.2543	0.1076	-0.4652	-0.0435
$^*SMA(1)$	1.0499	0.1509	0.7542	1.3456
$^*SMA(2)$	0.8201	0.2140	0.4007	1.2394
$SMA(3)$	-0.1499	0.1316	-0.4079	0.1081
				$\hat{\sigma}^2 = 0.04079$

* means significant at 5% significant level.

Table 4.12: Estimated parameters of $ARIMA(0,1,0)(1,0,3)_{12}$ for CIC

Variable	Estimate	Standard Error	95% confidence Interval	
			Lower limit	Upper Limit
$^*SAR(1)$	-0.5825	0.0798	-0.7390	-0.4261
$^*SMA(1)$	0.5118	0.0781	0.3587	0.6650
$SMA(2)$	0.0455	0.0618	-0.0757	0.1667
$^*SMA(3)$	-0.2883	0.0815	-0.4481	-0.1285
				$\hat{\sigma}^2 = 0.05076$

* means significant at 5% significant level.

Table 4.13: Estimated parameters of $ARIMA(0,1,0)(1,0,1)_{12}$ for CIC

Variable	Estimate	Standard Error	95% confidence Interval	
			Lower limit	Upper Limit
$SAR(1)$	-0.7978	0.0821	-0.9587	-0.6369
$SMA(1)$	0.6319	0.1059	0.4243	0.8394
				$\hat{\sigma}^2 = 0.05416$

* means significant at 5% significant level.

To certify the models are appropriate SARIMA models, all the model assumptions of constant variance, zero mean and autocorrelation were checked for residuals of the models by employing the Ljung- Box and ARCH- LM tests under their respective null hypothesis. Resulting P-values from Table 4.14 indicates, all models have zero means and constant variances.

Table 4.14: Residual diagnostic tests for Seasonal ARIMA models for CIC

Model	P-values	
	ARCH-LM test	Ljung-test
$ARIMA(0,1,0)(3,0,1)_{12}$	0.9997	0.9636
$ARIMA(0,1,0)(1,0,3)_{12}$	0.9094	0.8556
$ARIMA(0,1,0)(3,0,3)_{12}$	0.9996	0.9006

Lastly, the assumption of autocorrelation were checked. Figure 4.8 revealed the ACF plots of residuals to be zero, describing residuals are uncorrelated. A supplementary observation showed except for lag zero that all spikes in all other lags are within the confidence boundaries. This means that, all the three models fit CIC data well. Hence, we consider all the models for further analysis.

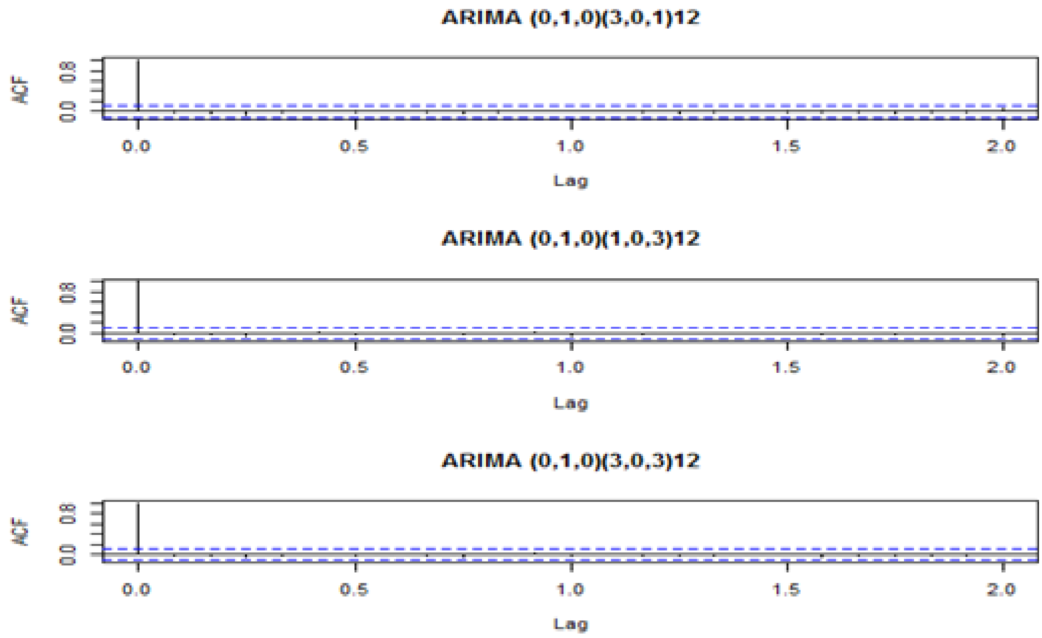

Figure 4.8: ACF plots of residuals for CIC

4.3.2 SETAR Modelling

In this section, the SETAR modelling procedure proposed by Franses and Van Dijk (2000) is followed to model and forecast Ghana's inflation and CIC rates. The section includes the two- regime SETAR model and forecast approach for the two rates under study.

SETAR modelling for inflation rate

In order to model inflation rate with the SETAR model, its time series must first satisfy the condition of nonlinearity. So, the nonlinearity was checked in this series by specifying the order of the linear $AR(p)$ model. The order of the linear $AR(p)$ is chosen as the $AR(p)$ model based on the maximum lag order with the least value of AIC. Using the Keenan and Tsay tests of linearity, $AR(14)$ was found to be the linear AR order with the least AIC value. In addition, P-values from these tests were both less than the 5% significant level, suggesting that Ghana's inflation rate follow threshold nonlinear. While considering the null hypothesis of linearity for Keenan-1-degree test and the alternative hypothesis of threshold-type nonlinearity in the Tsay F-test. Employing regime switching models can best account for these discontinuities and switching of inflation as compared to the simple linear model. Table 4.15 displays a summary of the linearity tests performed on inflation rate.

Table 4.15: Linearity test for inflation rate

Test	Test statistic	P- value	Order	Decision
Keenan-1-degree	4.2232	0.0041	14	Reject Linearity
Tsay	3.034	1.936e-11	14	No threshold nonlinearity rejected

Following that, the time series of inflation is confirmed to be nonlinear, the appropriate SETAR model identified best fit the dataset. In doing so, the lag order p in each regime and the threshold variable y_{t-d}, here d represents the delay parameter were determined. After performing grid search of tentative model combinations for SETAR models that best fit the dataset, lag orders with minimum values of AIC and MAPE were chosen as the tentative model(s). Tables 4.16 below provides a summary of competing models to be used in forecasting the two rates. The table suggest that SETAR (2;14,4) with threshold variable y_{t-4}, SETAR (2;14,5) with threshold variable y_{t-5} and SETAR (2;14,13) with threshold y_{t-13} was the appropriate models in explaining nonlinearity in Ghana's inflation rate. Hence, the three models were considered for further assessment.

Table 4.16: AIC and MAPE values of tentative SETAR models

Model	AIC	MAPE
SETAR (2;14,4)	-1541.411	1.5680
SETAR (2;14,5)	-1549.031	1.7785
SETAR (2;14,13)	-1538.501	1.7000

In accordance with Franses and Dijk's (2000) approach, the method of conditional least squares is used to estimate parameters of the candidate models as performed in Tables 4.17, 4.18 and 4.19 with their corresponding threshold values.

Resulting proportions from Table 4.17 with delay parameter of four and threshold value of 0.03944, can attribute structural breaks in inflation to data emanating from the low regime rather than the high regime as displayed.

Table 4.17: Estimate Parameters of SETAR (2;14,4) for inflation

Coefficient	**Low Regime**			**High Regime**		
	Estimate	**Std Error**	**t- value**	**Estimate**	**Std Error**	**t-value**
Constant	-0.0055	0.0063	-0.8821	0.0232	0.0171	1.3566
ϕ_1	0.2311	0.0629	3.6766	-0.1208	0.1617	-0.7467
ϕ_2	-0.0584	0.0648	-0.9018	0.7412	0.1738	4.2647
ϕ_3	0.0559	0.0709	0.7890	-0.0279	0.1055	-0.2640
ϕ_4	0.0544	0.0664	0.8197	0.0228	0.1055	0.2157
ϕ_5	0.1563	0.0908	1.7218	-0.0773	0.1097	-0.7050
ϕ_6	-0.0669	0.0789	-0.8479	0.0437	0.0880	0.4965
ϕ_7	0.0549	0.0688	0.7978	-0.0933	0.1081	-0.8625
ϕ_8	0.0568	0.0683	0.8306	0.0364	0.0963	0.3776
ϕ_9	-0.0840	0.0681	-1.2336	0.1066	0.0933	1.1422
ϕ_{10}	0.1396	0.0644	2.1678	-0.0387	0.1044	-0.3704
ϕ_{11}	-0.0563	0.0617	-0.9115	-0.0807	0.1222	-0.6606
ϕ_{12}	-0.3874	0.0628	-6.1712	-0.3996	0.1161	-3.441
ϕ_{13}	0.1143	0.0801	1.4268	0.0248	0.1104	0.2249
ϕ_{14}	-0.0341	0.0666	-0.5115	0.1700	0.1435	1.1851
		Threshold value =		0.03944		
Proportion	75.09%			24.91%		

In Table 4.18 too, more than half the data recorded switches and discontinues in the low regime. This indicates the earlier years of the inflation data demonstrates discontinuities in the dataset given a delay parameter of five with threshold value of 0.01342.

Table 4.18: Estimate Parameters of SETAR (2;14,5) for inflation

Coefficient	**Low Regime**			**High Regime**		
	Estimate	**Std Error**	**t- value**	**Estimate**	**Std Error**	**t-value**
Constant	-0.0137	0.0078	-1.7433	0.0225	0.0111	2.0377
ϕ_1	0.3381	0.0789	4.2820	0.1130	0.0856	1.3205
ϕ_2	-0.0597	0.0686	-0.8706	0.3225	0.1202	2.6834
ϕ_3	0.0873	0.0737	1.1841	0.0315	0.0864	0.3648
ϕ_4	0.1221	0.0776	1.5737	-0.0060	0.0772	-0.0775
ϕ_5	-0.0176	0.0699	-0.2516	0.0979	0.0944	1.0371
ϕ_6	-0.0927	0.0989	-0.9375	-0.0280	0.0918	-0.3054
ϕ_7	0.0680	0.0808	0.8408	-0.0519	0.0769	-0.6746
ϕ_8	0.0155	0.0712	0.2182	0.1419	0.0912	1.5565
ϕ_9	-0.0293	0.0755	-0.3884	0.0038	0.0779	0.0490
ϕ_{10}	0.2872	0.0820	3.5007	-0.0726	0.0723	-1.0046
ϕ_{11}	-0.1464	0.0678	-2.1583	-0.0382	0.0892	-0.4283
ϕ_{12}	-0.3134	0.0739	-4.2402	-0.5414	0.0792	-6.8377
ϕ_{13}	0.0797	0.0780	1.0217	0.2289	0.0926	2.4729
ϕ_{14}	-0.0064	0.0792	-0.0808	0.0311	0.0914	0.3403
			Threshold value =	0.01342		
Proportion	61.77%			38.23%		

Unlike Tables 4.17 and 4.18, Table 4.19 explains that, more than 80% of the inflation data experienced switching regimes and discontinuities in the higher regimes as compared to the low regime.

Similarly, in SETAR modelling we check models to confirm if they satisfy all model assumptions as mentioned earlier. Thus, we check the model residuals for no auto correlation, zero mean and constant variance of residuals. Also, we perform the ARCH-LM and Ljung -Box tests on the residuals under their specific hypotheses as discussed earlier. The results from Tables 4.20 confirms non-existence of autocorrelation and random residuals in all tentative models under the null hypothesis of residuals being random and residuals being non-correlated. A critical look at the table again, reveals $SETAR(2;14,4)$ failed its ARCH-test for lag order 12, this indicates the possibility of autocorrelation, nevertheless we will still consider this model for further assessment.

Table 4.19: Estimate Parameters of SETAR (2;14,13) for inflation

Coefficient	**Low Regime**			**High Regime**		
	Estimate	**Std Error**	**t- value**	**Estimate**	**Std Error**	**t-value**
Constant	-0.0412	0.0326	-1.2643	-0.0032	0.0053	-0.5957
ϕ_1	-0.0105	0.1322	-0.0792	0.3050	0.0673	4.5348
ϕ_2	0.1790	0.0937	1.9099	0.0405	0.0825	0.4916
ϕ_3	0.2177	0.0862	2.5269	0.0254	0.0740	0.3428
ϕ_4	-0.0780	0.1049	-0.7434	0.1051	0.0715	1.4697
ϕ_5	-0.1625	0.1468	-1.1070	0.0750	0.0635	1.1805
ϕ_6	0.2319	0.1053	2.2024	0.0294	0.0644	0.4567
ϕ_7	-0.1882	0.0957	-1.9670	0.0611	0.0687	0.8895
ϕ_8	-0.1880	0.1425	-1.3193	0.0794	0.0612	1.2972
ϕ_9	-0.0910	0.1105	-0.8232	0.0033	0.0617	0.0532
ϕ_{10}	0.0822	0.0967	0.8496	0.1159	0.0682	1.6990
ϕ_{11}	-0.1485	0.1287	-1.1543	-0.0950	0.0638	-1.4897
ϕ_{12}	-0.3674	0.1453	-2.5280	-0.3932	0.0632	-6.2224
ϕ_{13}	0.1225	0.1223	1.0013	0.0935	0.0761	1.2296
ϕ_{14}	-0.1817	0.1972	-0.9214	0.0223	0.0804	0.2779
			Threshold value =	-0.07091		
Proportion	17.41%			82.59%		

Table 4.20: Residual diagnostics of tentative SETAR models for inflation

Model	**Ljung-Box test**	**ARCH-LM test**
SETAR (2;14,4)	0.8371	0.0030
SETAR (2;14,5)	0.8667	0.4480
SETAR (2;14,13)	0.9776	0.5915

SETAR modelling for CIC rate

We follow the same procedure for testing linearity as we did earlier. In this section, we found *AR (13)* to be our *AR(p)* model order with the least AIC while employing the Keenan and Tsay tests for linearity. The P-values from Table 4.21 indicates Ghana's CIC rate follow Threshold nonlinear under the null hypotheses of Keenan and Tsay tests as mentioned earlier. Again, P-values from the Keenan test indicates nonlinearity in the data as well as the presence of threshold from the Tsay test. Equally, regime switching models best caters for these discontinuities and switching of inflation as compared to the simple linear model.

Table 4.21: Linearity test for CIC rate

Test	Test statistic	P- value	Order	Decision
Keenan-1-degree	29.405	1.236e-07	13	Reject Linearity
Tsay	3.905	5.066e-16	13	No threshold nonlinearity rejected

After confirming the CIC rate is nonlinear, we identified an appropriate SETAR model that best fits the data as shown in Table 4.22. Interestingly, a grid search of tentative model combination for SETAR model revealed only one model as the tentative model with close AIC and MAPE values for the CIC rate. Hence, we consider this model for further assessment.

Table 4.22: AIC and MAPE candidate models for CIC

Model	AIC	MAPE
SETAR (2;13,9)	3148.395	0.1662

Looking at Table 4.23, it can be inferred early years of CIC values resulted in more than 80% of structural breaks in the data. The standard errors from the table are relatively low in the low regime as compared to the high standard error in the high regime. The t- values in the table was obtained by dividing the estimate of each constant by its corresponding standard error.

Again, we check the model if it satisfies the model assumptions of autocorrelation, zero mean and constant variance. The ARCH -LM and Ljung-Box tests on the model residual confirms the constructed model satisfied model assumptions after lag 6. Also, the ACF plot of the model residual is not correlated. Hence, we conclude the model fit the data well.

Table 4.23: Estimate Parameters of $SETAR(2;13,9)$ for CIC

Coefficient	Low Regime			High Regime		
	Estimate	Std Error	t- value	Estimate	Std Error	t-value
Constant	13.6062	15.8325	0.8594	318.4621	94.3598	3.3750
ϕ_1	0.6990	0.0640	10.9183	0.8922	0.1096	8.1432
ϕ_2	0.2298	0.0788	2.9180	-0.0661	0.1198	-0.5520
ϕ_3	-0.0254	0.0801	-0.3164	-0.0229	0.1190	-0.1924
ϕ_4	0.0106	0.0799	0.1327	-0.0808	0.1187	-0.6807
ϕ_5	0.0337	0.0800	0.4209	0.1599	0.1177	1.3590
ϕ_6	0.0317	0.0800	0.3956	-0.2129	0.1198	-1.7772
ϕ_7	0.0214	0.0804	0.2665	0.1279	0.1269	1.0074
ϕ_8	-0.0501	0.0832	-0.6020	0.0769	0.1198	0.6416
ϕ_9	0.0580	0.1015	0.5708	-0.2244	0.0971	-2.3109
ϕ_{10}	0.0297	0.1076	0.2761	0.0083	0.0939	0.0884
ϕ_{11}	0.0166	0.1072	0.1547	0.2542	0.0919	2.7654
ϕ_{12}	0.0367	0.1072	0.3422	0.5410	0.0915	5.9121
ϕ_{13}	-0.0922	0.0829	-1.1126	-0.4658	0.0967	-4.8164
			Threshold value =	2159		
Proportion	82.71%			17.29%		

Table 4.24: Residual Diagnostics of suggested SETAR models for CIC

Model	Ljung-Box test	Arch-test
SETAR (2;13,9)	0.9163	0.4476

4.3.3 Comparative Analysis of forecast between SARIMA and SETAR models

As described by most researchers, forecasting is an important phenomenon to effective decision making and policy planning. It is therefore necessary to obtain good models that provide best forecast values. The main task in this research is to compare the forecasts performance between the linear SARIMA model and the non-linear SETAR

Model for both inflation and CIC rates. This is core to the policy maker and the planner for effective economic growth.

Inflation rate

The tentative models for inflation have shown to adequately fit the inflation data well and satisfied all model assumptions. Confidently, we use these models to forecast inflation rate over 12 months period from 2015(8) to 2016(9). Thus, we compare forecast performance between the tentative models in terms of their RMSE and MAE. As shown in Table 4.25, none of these models recorded consistent least values from both in sample and out of sample forecasts. Inconsistently, the model $ARIMA(3,1,3)(1,0,1)_{12}$ recorded the most least values of RMSE and MAE for the in-sample forecasts but failed to record same for the out of sample forecasts. The $ARIMA(1,1,2)(1,0,1)_{12}$ model also recorded the most least values for the out of sample forecasts but recorded otherwise for the in-sample forecasts. This makes it quite difficult to select any of the models to represent inflation in terms of the most least error term measures. Hence, the maximum P-values resulting from the Ljung-Box and ARCH tests in Table 4.20 were relied on to make a choice on the most appropriate linear SARIMA model to consider for comparison with the nonlinear SETAR model. Finally, we consider $ARIMA(3,1,3)(1,0,1)_{12}$ as most appropriate for comparison because it produced the most maximum P-values from table 4.8. This is because the greater the P-value from 5%, the better the model. The chosen model as well satisfies all SARIMA modelling assumptions.

Table 4.25: Forecast results of competing SARIMA models for inflation

Model	**Out of sample**		**In-sample**	
	RMSE	**MAE**	**RMSE**	**MAE**
$\mathbf{ARIMA(3,1,3)(1,0,1)_{12}}$	**2.7711**	**2.7709**	**0.0738**	**0.0494**
$ARIMA(1,1,2)(1,0,1)_{12}$	2.7686	2.7683	0.0755	0.0506
$ARIMA(1,1,1)(2,0,1)_{12}$	2.7834	2.7832	0.0754	0.0503

As explained earlier for Table 4.25, Table 4.26 demonstrates similar results. Yet again, no particular model shows consistent values for both in sample and out of sample forecasts. Also, the P-values from Table 4.20 with the largest values from the Ljung-Box and ARCH -LM tests were relied on to make a choice on the most appropriate model to consider for valuation. The model *SETAR* (2;14,13) is chosen to be the most appropriate model. This is because, it displayed to have the largest P- values.

Table 4.26: Forecast results of competing SETAR models for inflation

Model	Out of sample		In-sample	
	RMSE	MAE	RMSE	MAE
SETAR (2;14,4)	0.0015	0.0350	0.0058	0.0528
SETAR (2;14,5)	0.0021	0.0394	0.0055	0.0514
SETAR (2;14,13)	**0.0017**	**0.0353**	**0.0057**	**0.0514**

The selected models from the two modelling approaches have shown to fit the inflation data well and satisfied all model assumptions. Although, the nonlinear SETAR model outperformed the linear SARIMA model for inflation rates by their least error terms, the two models were compared to see which of the models is more significant. This comparison is done using the Diebold - Mariano test under the null hypothesis of the existence of significant difference between the two models. Results from Table 4.27 indicates all the P-values from the five different forecast horizons was rejected. Hence, it was concluded that the linear SARIMA and nonlinear SETAR models are statistically significant to forecast inflation.

Table 4.27: Forecast accuracy test for inflation

Forecast Horizon	DM statistic	P-value
1	-0.2488	0.8037
2	-0.2397	0.8107
3	-0.2443	0.8071
4	-0.2767	0.7822
5	-0.2958	0.7676

Currency in Circulation

To choose the most appropriate linear SARIMA model for comparison as we did for the inflation rate, the three competing SARIMA models shown to satisfy all model assumptions are examined. In Table 4.28 below, no model seems to have consistent least values of RMSE and MAE for their out of sample and in sample forecasts over our sample forecast period from 2015(8) to 2016(9). So, we inferred from the results of Table 4.14 and adjudged $ARIMA(0,1,0)(3,0,1)_{12}$ as our best model. We did this, by choosing the model with the most maximum P-values resulting from the Ljung-Box and ARCH -LM tests.

Table 4.28: Forecast results of competing SARIMA models for CIC

Model	**Out of sample**		**In-sample**	
	RMSE	**MAE**	**RMSE**	**MAE**
$\mathbf{ARIMA(0,1,0)(3,0,1)_{12}}$	**8.8428**	**8.8428**	**0.2268**	**0.0874**
$ARIMA(0,1,0)(1,0,3)_{12}$	8.8700	8.8699	0.2249	0.0827
$ARIMA(0,1,0)(3,0,3)_{12}$	8.9813	8.9812	0.2016	0.0795

Interestingly, the SETAR model for the CIC rate is just one and once it satisfies all SETAR modelling assumptions, therefore the model was chosen from Table 4.29 for further investigation.

Table 4.29: Forecast result of SETAR model for CIC

Model	**Out of sample**		**In-sample**	
	RMSE	**MAE**	**RMSE**	**MAE**
SETAR (2;13,9)	191086.54	280.35587	23697.61	74.85154

The Diebold - Mariano test was employed to test the forecast accuracy of the two selected models for SARIMA and SETAR models. The test was performed under the null hypothesis of the two models being significant. From Table 4.30, five forecasts were considered to test the statistical significance of the models. The results from the test indicates only one model is significant to forecast inflation, indicating that the null hypothesis can be rejected. However, by comparing the error terms from both models, out rightly it can be concluded that the linear SARIMA model outperforms the nonlinear SETAR model.

Table 4.30: Forecast accuracy test for CIC

Forecast Horizon	DM statistic	P-value
1	-3.0383	0.0026
2	-2.6836	0.0077
3	-2.6397	0.0087
4	-2.6492	0.0085
5	-2.6525	0.0084

4.4 Results and Discussion

An observation of datasets for inflation and CIC rates did not appear to follow a normal distribution, this was confirmed using the Jarque -Bera test. While inflation rate had a broader surface due to its positively skewed value greater than three, CIC rate showed a positively skewed value less than three indicating CIC values are peaked about the mean value as compared to a normal distribution. The series for both rates were observed to be nonstationary through their time series plots. These series were therefore logarithmically transformed and differenced to attain stationarity. Additionally, the differenced series showed a sine- wave like structure in the ACF's of the datasets, indicating the presence of both seasonal and non-seasonal behaviour in the series. However, after the first non-seasonal differencing the series became stationary, hence there was no need seasonally differencing the series. These transformed and differenced series was then used to build the appropriate models following procedures of Box-Jenkins. All chosen tentative models for both inflation and CIC performed well in terms of their ACF plots of residuals indicating that any of these models can serve as a good model for forecasting. This was confirmed through diagnostic checks on residuals of the tentative models. Further results from the ARCH-LM and Ljung-Box tests depict all chosen models are good for forecasting since they were above 5% level of significance and hence, satisfy all assumptions of the SARIMA model.

Before fitting the SETAR model, the datasets for both inflation and CIC rates must be nonlinear. So, a linearity test was performed on both rates using the Keenan and Tsay tests for linearity under their respective null hypothesis. The resulting P-values were less than 5%, implying the datasets follow threshold nonlinear. In like manner as in the SARIMA modelling, a diagnostic check was performed on the residuals of the fitted model(s) after performing a grid search of tentative models. Again, results from the ARCH and Ljung -Box tests showed P-values satisfying the assumptions of SETAR model.

The fear of the "unknown" makes the policy maker and the planner disorganized, since it becomes difficult to plan into the future. However, as discussed by Box and Jenkins (1976), forecasting provides the basis for effective economic planning, optimization of industrial activities and improved inventory and production control. Therefore, obtaining good models to provide accurate forecasts is core to the researcher, policy makers, investors and governments. Inflation and CIC rates are some examples of the major drivers of an economy of which Ghana is not left out. Hence, forecasting of these variables will serve as a guiding tool to financial and economic analysts to effectively make decisions and advise key stakeholders to formulate important policies to reduce inflation rates and regulate CIC rates. As a result, the study adjudges $ARIMA(3,1,3)(1,0,1)_{12}$ and $SETAR$ $(2;14,13)$ as best models in predicting inflation rates. Concurrently, $ARIMA(0,1,0)(3,0,1)_{12}$ and $SETAR(2;13,9)$are best models to predict CIC rates in Ghana.

Further checks on the chosen models in terms of their forecast accuracy using the Diebold-Mariano test revealed that both SARIMA and SETAR models can be used to forecast inflation. Since, the test proved there was no significant difference between the two models. Divergently, there was significant difference between the SARIMA and

SETAR models used in predicting the CIC rate. This could have resulted due to vast difference in the criterion used to measure forecast accuracy. Therefore, we predict inflation using both models for a twelve months period. However, for CIC rates we prefer to use the SARIMA model for predictions.

4.5 Chapter Conclusion

This chapter has comprehensively dealt with analyses and discussions from the study. It also represented concise and precise major findings of the study. The chapter also showed that the two rates considered increased in regular intervals, implying the datasets are seasonal by nature. Further test using the Jarque-Bera test revealed the datasets do not follow a normal distribution. In affirmation, the linearity test proposed by Keenan and Tsay proved the datasets are threshold nonlinear. Additionally, forecast performance indicates that the behaviour of a dataset does not necessarily inform the best model fit. This was confirmed in the analysis for the CIC rate. Even though, the dataset appeared to be nonlinear, the linear model fitted and forecasted the rate better. Hence, it can be concluded that the SARIMA model is best for forecasting CIC rates.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

This chapter represents conclusion and recommendations of the study.

5.2 Conclusion

In the study, SARIMA and SETAR models were fitted to time series datasets on monthly inflation and currency in circulation (CIC) rates obtained from the Bank of Ghana. Observation of the ACF and PACF of the datasets showed that the data were not stationary. This was confirmed by the KPSS and PP unit root tests on the datasets. The datasets were logarithmically transformed and differenced non-seasonally to obtain a stationary time series. In the linear SARIMA modelling, the Box-Jenkins approach was used and identified twelve and four tentative models for inflation and CIC rates respectively. Amongst the tentative models, $ARIMA(3,1,3)(1,0,1)_{12}$ and $ARIMA(0,1,0)(3,0,1)_{12}$ were respectively identified as best models for inflation and CIC rates in the sense that, it gave least values of AIC, MAPE, RMSE and MAE. Further diagnosis proved that all SARIMA model assumptions have been satisfied. Thus, the models are adequate.

In the nonlinear SETAR modelling, Keenan and Tsay-F tests showed the datasets were threshold nonlinear, so a two regime SETAR model was applied. A grid search of tentative models revealed three and one SETAR models for inflation and CIC, respectively. SETAR (2;14,13) and SETAR (2;13,9) were identified amongst the tentative models as the best models. Again, further diagnosis showed the datasets satisfied all model assumptions. After modelling both rates, the performance between the SARIMA and SETAR models were compared for inflation and CIC rates by employing forecast measures, Root Mean Square Error (RMSE) and Mean Absolute Error. Based on forecast results for the period of twelve months, the nonlinear SETAR model outperformed the linear SARIMA model for inflation. However, the case was different for CIC rates, the Linear SARIMA model outperformed the nonlinear SETAR model. This confirms the stand of Gooijer and Kumar (1992) that there is no clear evidence either in favour of linear or nonlinear models in terms of their forecast performance.

Although, the results of forecast performance between the nonlinear SETAR model and the linear SARIMA model showed the SETAR model has superiority over the SARIMA model, the Diebold-Mariano test suggests both models are significant for forecasting Inflation rates.

Finally, the results for forecast performance between models showed the linear SARIMA model is superior to the nonlinear SETAR model. Hence, we conclude that the SARIMA model is best to model and forecast inflation. This is confirmed from the Diebold-Mariano test for different forecast horizons.

5.3 Recommendations

The research compared the linear SARIMA and nonlinear SETAR models. Since models approximate reality, it is recommended that more research should be done considering other linear and nonlinear models.

The results from the research revealed that the SARIMA model is better off in predicting CIC rates as compared to the SETAR model; it is therefore recommended close monitoring of financial markets should be done on regular basis in order to regulate the total amount of money supply in the economy.

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