Title: Global Stein Theorem on Hardy Spaces

Abstract: Let f be an integrable function which has integral 0 on $\mathbb{R}^n$. What is the largest condition on $|f|$ that guarantees that f is in the Hardy space H1�1 ($\mathbb{R}^n$)? When f is compactly supported, it is well-known that the largest condition on $|f|$ is the fact that $|f| \in L \log L(\mathbb{R}^n)$. We consider the same kind of problem here, but without any condition on the support. We do so for H1�1 ($\mathbb{R}^n$), as well as for the Hardy space Hlog�log ($\mathbb{R}^n$) which appears in the study of pointwise products of functions in H1�1 ($\mathbb{R}^n$) and in its dual BMO..