

Time-varying dependence dynamics between international commodity prices and Australian industry stock returns: a Perspective for portfolio diversification

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ABSTRACT

This paper investigates the time-varying dependence dynamics between the international commodity prices of Brent crude oil, natural gas, cocoa, and Australia's sectoral stock returns, using daily prices ranging from 1st December 2011 to 23rd April 2020. The considered sector stocks include Technology, Telecom, Real Estate, Energy, Basic Materials, Utilities, Industrials, Financials and Health Care. We employ the time-varying Markov-switching copulas to investigate the dynamic dependence between those international commodity prices and the Australian industry stocks returns, as well as using various portfolio techniques including the minimum variance portfolio, the minimum correlation portfolio and the recently developed minimum connectedness portfolio to test portfolio performance. We establish that the dependence between the commodities and sectoral stock returns is Markov switching and time varying. Our results further show that Brent crude oil is a good hedge for the financial sector, natural gas is a good hedge for all sectors except the real estate, and cocoa co-crashes with the technology, industrials and real estate sectoral returns. Based on the hedging effectiveness of the competing portfolios, we show that bivariate and multivariate portfolios significantly reduce the risk of investing in a single asset. In particular, the results indicate the significant role for commodities, with weights ranging from approximately 25% to 50%, depending on the portfolio construction of choice. Australian stocks typically have the lowest portfolio weights under all competing portfolios, thus confirming their hedging role to a commodity investment portfolio. Our analysis contributes unique evidence to the literature on the role of commodity assets as a complement to mainstream investment.

1. Introduction

The correlation between international commodity prices and global stock markets and the role of commodity markets in influencing financial markets have been a popular topic among academics and practitioners, owing to its important implications for asset allocation, policy making, and risk management (e.g., Baur and Lucey, 2009; Baur and Smales, 2018; Flavin et al., 2014; Tiwari et al., 2020; Arouri et al., 2011; Azimli, 2020; Cevik et al., 2020; Cunado and Perez de Gracia, 2014; Lin and Su, 2020; Shahbaz et al., 2021; Le et al., 2020). Both developing and developed economies of the world are significantly impacted by changes

in commodity prices. In particular, the aftermath of the repeated episodes of economic crises around the world at different times such as the global financial crisis of 2008, the European debt crisis of 2011, the Brexit of 2016 and the COVID-19 pandemic in 2020, investors are now keen to know the potential effect of fluctuating commodity prices generally on the global financial markets, with particular emphasis on the stock market returns (Shahzad et al., 2019).

Most researchers assert that shocks in commodities (e.g., oil) exert a considerable impact, indirectly or directly on stock markets (Miller and Ratti, 2009; Arouri et al., 2011; Broadstock and Filis, 2014). Such studies suggest that changes in oil prices can directly impact stock prices

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by affecting the current and future cash flows, or indirectly, by influencing the interest rates used to discount future cash flows of the corporate world (Jones and Kaul, 1996; Arouri et al., 2011, 2012). Stock prices in the long-run equilibrium should equal the sum of discounted values of expected future cash flows at different investment horizons (Basher et al., 2018). On one hand, another way of looking at the relationship between commodity prices and stock markets is through financialization (Zhang, 2018). As a consequence of the financialization process, the inter-connectedness between commodities and stock markets has been further strengthened (Mensi et al., 2017). Ji and Zhang (2019) conclude that the increasing number of oil-related financial derivatives has sped up the process of financialization in the international commodity markets, leading to closer linkages between commodities and stock markets.

Investors and portfolio managers in the pursuit of higher level of diversification rely on alternative asset classes, like commodities. The justification for the diversification value of commodity markets from the economic standpoint is simple. The physical features of commodity markets are still important diversification considerations (Domanski and Heath, 2007) since commodity markets experience business cycles that are different from stock markets (Roll, 2013). In addition, the macro-financial fundamental factors that parade low connectedness to stocks are not necessarily the chief drivers of commodity asset prices. In particular, stock and commodity futures markets are driven by different economic and financial fundamentals (Gorton and Rouwenhorst, 2006). Furthermore, commodity markets are increasingly becoming like financial markets, and financial investors are increasingly becoming active in commodity markets (Domanski and Heath, 2007). The increased financialization of commodity markets has increased investment or hedging demands in order to reduce the adverse effect of the physical characteristics of commodities markets and the lower liquidity as well as the higher reallocation cost of capital (Hong and Yogo, 2012).

The existing literature, however, provides mixed empirical evidence on the diversification value of commodity markets (Greer, 2000; Hong and Yogo, 2010; Chong and Miffre, 2010; Buyuksahin and Harris, 2011; Silvennoinen and Thorp, 2013; Delatte and Lopez, 2013). Overall, achieving portfolio diversification has become a more difficult job for investors and portfolio managers. For example, linkages between international commodity prices and stock markets vary with respect to the nature of the country. Several studies show that the responses of stock markets to oil shocks depend on the net position of the country in the global oil market and the driving forces of the oil price shocks. For instance, a positive relationship is unanimously observed in oil-exporting economies (Kilian and Park, 2009). On the other hand, oil-importing economies show an inconsistent direction of oil-stock nexus (Badeeb and Lean, 2018). Hence, further country level investigation is needed to develop a better understanding of commodity and stock market linkages, which can generate clear benefits for financial market investors and risk management managers.

Most studies on the linkages between commodity markets like gold and oil and international stock markets focus mostly on developed equity markets around the world, with most studies focusing on the G7 economies (Nguyen and Liu, 2017; Lucey and Li, 2015; Conover et al., 2009; Shahzad et al., 2019; Junttila et al., 2018; Tiwari et al., 2020), emerging economies (Basher and Sadorsky, 2016; Maghyreh et al., 2017; Mensi et al., 2016) with only few studies considering the stock market of Australia and its diverse commodities (Hillier et al., 2006; Beckmann et al., 2015; Lucey and Tully, 2003).

Against this backdrop, in this article, we investigate the nature of dependence between international commodity indices: crude oil, natural gas and cocoa and Australia stock market. In particular, we examine the relationship between international prices crude oil, natural gas and cocoa and industry stock returns of Australia including Technology, Telecom, Real Estate, Energy, Basic Materials, Utilities, Industrials, Financials and Health Care, using the time-varying Markov-switching copula approach. We focus on Australia due to its richness in diverse

resources suitable for investment diversification. The significance of the commodity industry for the overall economic performance in Australia is well known and long standing since they play a significant role on the Australian stock market, representing about 30% of market capitalization (Heaton et al., 2011). Even though, several commodities produced domestically are traded on the Sydney Futures Exchange, major commodity prices are set in large international markets. Interestingly, only a few studies investigate the impact of globally determined commodity prices on the performance of firms listed on the Australia stock market (ASX).

In the context of Australia, Heaton et al. (2011) investigated the relationship between energy, metals and agricultural commodities and the Australian stock market. Additionally, it has been shown that oil (Chaudhuri and Smiles, 2004) and gold (Chan and Faff, 1998) significantly affect the Australian stock market. A notable study close to our paper is that of McSweeney and Worthington (2008) who provided empirical evidence on the impact of oil shocks on Australia's industry stock returns.

In this article, we extend the literature on the commodity-stock market nexus by testing the dependence structure and the dynamics between international prices of crude oil, natural gas and cocoa and industry stock returns in Australia since most studies employ aggregate stock market indices and not sectoral indices in the energy finance literature. We focus on industry returns because according to Baker and Wurgler (2006), sector stocks that exhibit less stable cash flows experience swings in sentiments more often than those traditional and matured sectors. Furthermore, as volatility is generally viewed as a key force in hedging possibilities, our goal is to explore dependence between Australia stock market and commodity markets at the sectoral level from the new angles of portfolio implications.

Concerning our choice of the three commodities used in this paper, we selected crude oil because Australia produces a significant volume of this fuel. Interestingly, most of this oil is exported, with the largest export destinations being Indonesia and Singapore. Despite this, Australia is a net importer of crude oil. Thus, Australia acts as both an oil exporting and importing country. As a results, further studies testing the impact of oil shocks on industry stock indices and not aggregate stock index are required to establish whether the nature of the dependence between the crude oil-industry stock returns is either positive or negative for investment and hedging strategies. Additionally, Australia aside oil is a producer of other commodities including metals, natural gas, coal, cocoa among others. However, in this paper, we strategically test the effects of changes in international price of cocoa on industry stocks because Australia in recent times started growing cocoa near Darwin in the Northern Territory. The cocoa industry in Australia is relatively new, however, an eight-year feasibility study showed that the value of the raw cocoa product to the region would be around AU\$10–12 million, suggesting that cocoa will be a profitable activity that will contribute to the broader economy in the distant future (*Commercialising Cocoa Growing in North Queensland*, 2013). Lastly, our quest to test the impact of international prices of natural gas is driven by the fact that oil accounted for the largest share of Australia's primary energy mix in 2017–2018, standing at 39%, followed by coal (30%) and natural gas (25%). Additionally, natural gas consumption grew by 4% in 2017–2018. This raises an important question: How do the volatilities of the international crude oil which Australia produces in large quantities and natural gas and cocoa which Australia produces in lesser quantities impact the Australian stock markets? We do not focus on the impact of gold since several studies have documented the relationship between gold and international stock markets (Shahzad et al., 2019; Wen and Cheng, 2018; Shahzad et al., 2019 etc), with the results showing the safe-haven properties of gold to stocks. To the best of our knowledge, this is the foremost comprehensive study to have investigated the time-varying dependence and correlation between international prices of crude oil, natural gas and cocoa and industry stock returns in Australia using copulas. This is the major contribution to the literature.

This paper contributes to the literature in the following ways. (i) It is a pioneering effort to investigate the tail dependence dynamics between oil, natural gas, cocoa and sectoral stock returns in Australia. (ii) It employs the novel regime-switching copula model that allows for a regime change in the copula parameter in order to assess the time-varying dependence structure between oil, natural gas, cocoa and sectoral stocks returns in Australia. The main advantage of this model is that it does not require an ad hoc determination of change points in the dependence structure (da Silva Filho et al., 2012; Boubaker and Sghaier, 2016). (iii) It examines the time-varying dependence between important commodities and the Australian stock market at the disaggregated sector level instead of the aggregate market level. This approach reveals a more detailed look at markets by considering the industrial linkages and differences. The aggregate indices may mask the heterogeneity at the sector level. Such heterogeneity is important, given that international portfolio diversification strategies and risk management analyses are sometimes performed at the sector level rather than the aggregate level. (iv) Following that achieving higher diversification is an important strategic goal for investors and portfolio managers, we provide analysis on the portfolio implications of our results given the increasing difficulties investors face in achieving portfolio diversification.

Our main findings reveal a negative dependence between natural gas and all the considered equity sectors except the real estate sector. This means that natural gas is a good hedge against all the sectors except the real estate sector, thus revealing the diversification benefits of including natural gas in portfolios. Second, we document a negative dependence structure between the returns series of cocoa and the telecom, energy, basic materials, financials and health care sectors. This indicates that cocoa is a good hedge to these assets for investors in Australia since both cocoa and these equity assets move in opposite directions. However, for cocoa and the technology, industrials and real estate sectors, we find a positive dependence structure which means these assets co-move in same direction. Lastly, we find that Brent crude oil may potentially decrease investors' benefits for portfolios composed of Brent crude oil and technology, telecom, basic materials, utilities, industrials and health care stocks as oil co-crashes with these sectoral stocks. Finally, from the portfolio analysis, we find the weights for our risk factors for the health care and technology sectors reflect high returns recorded from the efficient frontier analysis.

The remainder of the paper is structured as follows. The second section briefly reviews the existing literature by first reviewing the channels through which commodities and stocks markets are intertwined followed by a discussion on findings from prior studies. The third section presents the methodological framework adopted in the study. We then analyze the findings and conclude the paper in the fourth and the fifth sections, respectively.

1.1. Channels linking international commodity and stock markets

A significant question that requires answering is related to the economic mechanisms at play that explain the impact oil, natural gas, cocoa on sectoral stock returns in Australia and the different channels by means of which these commodities' price fluctuations directly or indirectly (i.e., through responses by monetary and/or fiscal authorities) affect the sectoral stock returns in Australia. Choi and Hammoudeh (2010) surmise that portfolio investors in commodity assets continually track price changes in commodities and stocks in their quest to make optimal portfolio choices. Consequently, there exists a portfolio rebalancing or a substitution mechanism among commodity assets (including crude oil, gold, silver, natural gas, wheat, and cocoa) and financial assets (including stock, exchange rate etc.), following the viability of the former as suitable assets for hedging and safe-haven purposes (Jain and Biswal, 2016). On the mechanism of transmission, the literature suggests diverse channels through which commodity and financial assets correlate. It is noteworthy to mention that these channels, to a large extent, explain the linkages across the markets mostly in a

split, thereby connoting that channels that connect all markets together in one attempt are rare.

The notable channel appears to be inflation mechanism. Economic theory reveals that any real asset price should be determined by its expected discounted cash flows (Williams, 1938; Fisher, 1930). This connotes that any factor that can alter the discounted cash flows should have significant effects on the asset price (Filis et al., 2011). In effect, an increase in commodity prices such as oil can lead to a reduction in products production, as inputs become more expensive and contribute directly to the level of inflation. This promotes a decrease in investors' expected earnings realized from stock markets (Sadorsky, 1999; Aroui and Nguyen, 2010). Thus, an increase in the prices of commodities (e.g., oil) should be accompanied with a decline in stock prices. Taking oil prices as an example, the big question is that should the response of stock prices to an oil price increase be similar for both oil-importing and oil-exporting economies? Prior studies suggest that commodity assets impact stock markets indirectly through macroeconomic fundamental factors such as inflation and economic growth. Increases in oil prices have been found to impact stock markets positively in an oil-exporting country as the income of the country increases (Bjornland, 2009; Jiménez-Rodríguez and Sánchez, 2005). Subsequently, the increase in the country's income levels generates a rise in investments and expenditures, which in turn, leads to greater productivity (Filis et al., 2011). In the case of oil-importing country, Hooker (1996) finds that an increase in oil prices have an opposite effect. This is because an increase in commodity prices such as oil which is a major factor of production will lead to an increase in costs of production. Consequently, the increase in cost of production will affect consumers' behaviour, which will in turn decrease their spending as a result of higher consumer prices (Bernanke, 2006; Abel and Bernanke, 2001). The decrease in consumption in effect would lead to a decrease in production, and in return causing an increase in the unemployment level (Lardic and Mignon, 2006; Brown and Yücel, 2002). In this case, the stock markets decline as documented by Sadorsky (1999) and Jones and Kaul (1996). Thus, increases in commodity prices such as the oil price positively or negatively impact stock markets across diverse countries (i.e., oil exporting or oil importing). However, the effects of other commodities aside from oil on stock markets are largely unexplored.

Another prominent transmission channel takes its stand from the unified currency measurement (U.S. dollar) of most globally traded assets. Assuming the dollar appreciates, the purchasing power of countries other than the U.S. diminishes, thereby reducing their demand for all assets denominated in this common currency. Hence, their prices jointly fall (see Adekoya and Oliyide, 2020). The interest rate channel explains how investment in stocks become discouraged when interest rates are low. This could make them consider other high interest-yielding assets in order to gain higher returns. Moreover, we should not ignore the impact of commodity price shocks on stock markets due to the uncertainty they create for the financial world, depending on the forces pushing up commodity prices (demand-side or supply-side). In fact, stock markets are expected to respond positively to increases in commodity price shocks originating from an increase in global demand, and negatively if the shock originates from the supply-side (Filis et al., 2011; Kilian and Park, 2009). The last but not the least channel which appears to be the most all-encompassing as it brings more markets together is the macroeconomic factors-based channel. This is also the channel through which we derive the objective of this study. It explains that different global markets (oil, metals and financial) are connected via common economic factors, especially global economic events. For example, Zhang (2017), for example, shows that oil price changes have been increasingly linked to stock markets since the 2008 global financial crisis.

1.2. Literature review

Testing empirically for the dynamic interactions between financial

assets and commodity markets has been quite represented in the literature. This literature has often concentrated on gold as commodity and its relationship with portfolios consisting majorly of bonds and stocks and other investment dimensions (Beckmann et al., 2015; Hood and Malik, 2013; Lucey and Tully, 2006). Beckmann et al. (2019) analyzed the relationship between gold quotations on the Shanghai Gold Exchange and sectorial stocks in China and show that adding gold to Chinese stock portfolios can assist in the reduction of overall portfolio risks. Prior studies had equally underscored the diversification potentials of including gold in a portfolio. These studies found significant relationships between gold and stocks, thereby exhibiting negative correlations and lower risk additions from gold to the whole portfolio, which aids in effective diversification (Choudhry et al., 2015; Ham-moudeh et al., 2013; Kumar, 2014; Malliaris and Malliaris, 2015).

Other studies have studied the relationship between gold and other extractive resources, gold and macroeconomic factors such as inflation (Le Long et al., 2013; Worthington and Pahlavani, 2007), crude oil and macroeconomic factors and their efficiency as a hedge against inflation, and exchange rate risks and political instability (Aggarwal et al., 1999; Capie et al., 2005; Ghosh et al., 2004). As mentioned in the previous section, a number of studies have also indicated the use of gold as a hedge or a safe haven for erratic oil prices and stock prices (Baur and Lucey, 2009; Baur and Smales, 2018; Flavin et al., 2014; Tiwari et al., 2020).

Recent literature on crude oil and stock market behaviour indicates that commodity futures investment has the potential to increase capital inflows into the commodity markets (Ahmed and Huo, 2020; Hamdi et al., 2019). Oil prices have been seen in the literature to influence other commodities and financial assets (Arouri et al., 2011; Azimli, 2020; Cevik et al., 2020; Cunado and Perez de Gracia, 2014; Lin and Su, 2020). Hamdi et al. (2019) study the relationship between the oil price volatility and sectoral stock markets in oil-exporting economies and indicate that all the equity sectors were interdependent of oil price volatility but are insusceptible to oil price volatility in both the low and high quantiles. These studies are however unclear on the impact of oil price movements on the prices and returns of other commodities and financial stocks or sectors (Faff and Brailsford, 1999). Intuitively, despite commodities' diversification potential of investors' portfolios, hikes in crude oil prices affect the income of crude dependent production assets adversely. By this, there seems to be a trade-off as increases in crude prices erode the profits or returns of other assets included in the portfolio (Huang et al., 1996a).

Beckmann et al. (2019) analyzed the relationship between gold quotations and sectorial stocks in China using different copulas. Nguyen et al. (2017) use a mixed copula method and affirm the hedge and the safe haven potential of gold. Using an ADCC GARCH model to estimate time-varying correlations, Maheshwari et al. (2018) assess the sector diversification of the international stock indices of Australia, India, and China and suggest the presence of diversification benefits. Applying the Pedroni panel cointegration analysis and the full modified ordinary least square method (FMOLS) and using different precious metals, Tuna (2019) finds that gold, silver, platinum and palladium are effective portfolio diversification tools for developed Islamic stock markets as Abdulkarim et al. (2019) have done by studying stock markets in Africa. Akkoc and Civir (2019) study the dynamic linkages between commodities and stock markets in Turkey using an SVAR-DCC-GARCH model and confirm the presence of time-varying co-movements as well as volatility spillovers onto stocks. Ahmed and Huo (2020) use a multivariate volatility model (the VAR-BEKK-GARCH model) in assessing the dynamic relationship between the Chinese stock market, commodity markets and global oil prices and find a significant unidirectional return spillover effect from the oil market to the stock market, thereby suggesting a stock market dependence on crude oil as a safe haven during periods of uncertainty.

From the foregoing, the portfolio benefits of mixing commodities with other assets via diversification cannot be vouched for in all

circumstances. Some studies (Gorton and Rouwenhorst, 2006) find that commodity futures is negatively correlated with stocks and bonds, therefore making diversification plausible. On the contrary, some studies do not find diversification benefits, which places some doubt on the general view of the diversification potentials of commodities (Daskalaki and Skiadopoulos, 2011). Silvennoinen and Thorp (2013) report evidence of decreased diversification benefits for portfolio investors with portfolios comprising of commodities, stocks and bonds.

The burgeoning literature have focused on the dynamic interdependencies between major trade commodities-gold and crude oil and financial assets-equities and bonds. For instance, Hasan (2017) studies the transmission of energy price shocks to the Australian stock market and observes a significant energy price volatility on equities. Our study fills a gap in the literature by being the first study to provide a comprehensive analysis on the time-varying tail dependence structure between global commodity prices [oil, natural gas and cocoa] and sectoral stock returns of the Australian stock market by considering nine equity sector indices.

2. Econometric methodology

This section establishes the methods adopted to test the presence of regime change in the dynamic or time-varying dependence structure between the commodities and the nine Australian sectoral stock returns. The bivariate copulas are first presented and then we discuss the Markov-switching time-varying copula functions.

2.1. Bivariate copulas

A function is termed a copula when it permits the joining several univariate distributions and forms a valid multivariate distribution while preserving all relevant information in the multivariate distribution. Schweizer and Sklar (1983) and da Silva Filho et al. (2012), suggest that a copula with n dimensions $C = (u_1, \dots, u_n)$ is a multivariate distribution function in $[0, 1]^n$ with uniform marginal distributions in the $[0, 1]$ interval. Following Sklar (1959) assume that H is the joint distribution function with n dimensions and that marginal distribution function of various single dimension variable is F_1, F_2, \dots, F_n respectively, and there will be a copula function C to establish Eq. (1)

$$H(x_1, \dots, x_n) = C(F_1(x_1), \dots, F_n(x_n)) \quad (1)$$

where, x_1, \dots, x_n are real numbers and H denotes an n -dimensional distribution function with margins (F_1, \dots, F_n) . The theorem also states that the copula $C(u_1, \dots, u_n)$ associated with H is unique if marginals F_1, \dots, F_n are strictly increasing continuous and can be obtained by

$$C(u_1, \dots, u_n) = H(F_1^{-1}(u_1), \dots, F_n^{-1}(u_n)) \quad (2)$$

where $u_i = F_i(x_i)$, $i = 1, \dots, n$ i.e., $F_i^{-1}(u_i)$ are the inverse distribution functions of the marginals. The density function related to the joint distribution can be obtained by differentiating Eq. (1) due to the n -differentiable characteristic of marginals F_1, \dots, F_n and C . Therefore, given a situation of a bivariate, the density function is given by

$$h(x_1, x_2) = c(F_1(x_1), F_2(x_2)) \prod_{i=1}^2 f_i(x_i) \quad (3)$$

where h is the density function associated with H , f_i is the density function for each marginal and the copula density c is obtained by differentiating Eq. (1), and can be written as.

The copula functions are associated with a quadratic form of the correlation between the marginals, and thus, they have elliptical distributions (da Silva Filho et al., 2012). For instance, the dependence structure of both the Normal copula and the Student-t copula is the linear correlation coefficient which belongs to the $[-1, 1]$ interval with

symmetric distribution functions.

Again, the tail dependence measures aid in determining the model that best produces the empirical facts about commodity markets. Also, the tail dependence is fully defined to be the dependence structure and thus not affected by variations in the marginal distributions. The probability of interdependencies of extreme occurrences between events in different markets can be assessed with the tail dependence measures. The Gumbel copula has mostly been used in extreme value theory (EVT) and allows only positive (upper tail) dependence structures. The Gumbel parameter belongs to the interval $[1, +\infty]$. Despite the diversities per-

$$\begin{aligned}
 l(\theta|X_t) &= \sum_{i=1}^T \log \left(c_{\theta_{ct}} \left(F_1(X_{1t}|\mu_1, h_{1t}, \theta_1), F_2(X_{2t}|\mu_2, h_{2t}, \theta_2) \mid \theta_{c_{\theta_{ct}}, S_t} \right) \times \prod_{i=1}^2 f_{it}(X_{it}|\mu_i, h_{it}, \theta_i) \right) \\
 &= \sum_{i=1}^T \log f_{1t}(X_{1t}|\mu_1, h_{1t}; \theta_1) + \sum_{i=1}^T \log f_{2t}(X_{2t}|\mu_2, h_{2t}; \theta_2) + \sum_{i=1}^T \log c_{\theta_{ct}}(\mu_1, \mu_2 \mid \mu_1, \mu_2, h_{1t}, h_{2t}; \theta_{c_{\theta_{ct}}, S_t}), l(\theta|X_t) = \ell_{f_1}(\theta_1) + \ell_{f_2}(\theta_2) + \ell_c(\theta_{c_{\theta_{ct}}, S_t}) \tag{5}
 \end{aligned}$$

taining to various copula functions with specific dependence structures tail dependencies make comparisons possible.

The tail dependence is defined as if the limit $D\lim_{\epsilon \rightarrow 0} \Pr[U_1 \leq \epsilon \mid U_2 \leq \epsilon] = \lim_{\epsilon \rightarrow 0} \Pr[U_2 \leq \epsilon \mid U_1 \leq \epsilon] = \lim_{\epsilon \rightarrow 0} C(\epsilon, \epsilon)/\epsilon = \tau^L$ exists, then copula C has a lower tail dependence, if $\tau^L \in [0, 1]$. Otherwise, C has no lower tail dependence.

if the limit $\lim_{\delta \rightarrow 1} \Pr[U_1 > \delta \mid U_2 > \delta] = \lim_{\delta \rightarrow 1} \Pr[U_2 > \delta \mid U_1 > \delta] = \frac{\lim_{\delta \rightarrow 1} (1 - 2\delta + C(\delta, \delta))}{1} - \delta = \tau^U$ exists, then copula C has an upper tail dependence, if $\tau^U \in [0, 1]$. Otherwise, C has no upper tail dependence.

Our choice of the tail dependence allows for the determination of the model that best reproduces empirical facts about the Australian commodity market and other assets. The probability of interdependencies of extreme occurrences between events in different markets can be assessed with the tail dependence measures. Again, the tail dependence is fully defined by the dependence structures and not affected by variations in the marginal distributions. Several time-varying copulas that capture different patterns of dependence. This study uses the Normal or Gaussian copula, Clayton copula, Rotated Clayton copula, Plackett Copula, Frank copula, Gumble copula, Rotated Gumble copula, Student-t copula and the Symmetrized Joe-Clayton copula. The characteristics of the copula functions used and others in the literature are presented in [Table 1A](#) in the appendix.

2.2. Estimation

Following [da Silva Filho et al. \(2012\)](#), the log-likelihood for the problem is specified as:

$$l(\theta|X_t) = \sum_{i=1}^T \log \left(c_{\theta_{ct}} \left(F_1(X_{1t}|\theta_1), F_2(X_{2t}|\theta_2) \mid \theta_{c_{\theta_{ct}}, S_t} \right) \times \prod_{i=1}^2 f_{it}(X_{it}|\theta_i) \right) \tag{4}$$

where $\theta_i = \mu_i, h_{it}, i = 1, 2$ and θ is a vector with all the model parameters. It may be computationally intensive to evaluate the likelihood in Eq. (4) due to the existence of the unobserved processes such as h_{1t}, h_{2t} and S_t . Nevertheless, a two-step maximum likelihood estimation procedure could be adopted because Eq. (4) is a separable function. This implies that the inference function for margins (IFM) as proposed by [Joe and Xu \(1996\)](#) can be used.

The IFM method consists of two steps. First, the parameters of the univariate marginal distributions are estimated, and then use these estimates to assess the dependence parameters in the second step. Following [da Silva Filho et al. \(2012\)](#) and [Tiwari et al. \(2020\)](#), the

marginal distributions are modelled as univariate GARCH processes, and then the dependence parameters are specified by the copula function choice. This procedure can be alluded to as the Copula-GARCH. The marginal distributions follow the traditional approach for the GARCH models, which is deemed straight forward unlike the dependence parameter estimation through copulas. This is because θ_{ct} depends on a non-observable discrete variable S_t that follows a Markov chain process.

2.2.1. Copula estimation

The log-likelihood in Eq. (4) is rewritten as

where $\ell_{f_1}(\theta_1) = \sum_{t=1}^T \log f_{1t}(X_{1t}|\mu_1, h_{1t}; \theta_1)$, $\ell_{f_2}(\theta_2) = \sum_{t=1}^T \log f_{2t}(X_{2t}|\mu_2, h_{2t}; \theta_2)$ and $\ell_c(\theta_{c_{\theta_{ct}}, S_t}) = \sum_{t=1}^T \log c_t(\mu_{1t}, \mu_{2t}|\mu_1, \mu_2, h_{1t}, h_{2t}; \theta_{ct}, S_t)$; $\ell_{f_1}(\theta_1)$ and $\ell_{f_2}(\theta_2)$ are the log-likelihood functions used to estimate parameters of the marginal distributions in the first step. Note $\ell_c(\theta_{c_{\theta_{ct}}, S_t})$ can be rewritten, taking into account the non-observable variables and the decomposing of C_t as:

$$\ell_c = \sum_{t=1}^T \log \left(\sum_{S_t=0}^1 c_{\theta_{ct}}(u_1, u_2 | S_t, w_{t-1}) Pr[S_t | w_{t-1}] \right) \tag{6}$$

The weights $Pr[S_t | w_{t-1}]$ for $S_t = 0$ and $S_t = 1$ are constructed in order to evaluate the log-likelihood in Eq. (6) since the states S_t are not observable. Applying [Kim and Nelson's \(1999\)](#) filter of estimation which produces the following algorithm, iterated through the sample $t = 1, \dots, T$.

(a) Prediction of S_t

$$Pr[S_t = l | w_{t-1}] = \sum_{k=0}^1 p_{kl}^{t-1} Pr[S_{t-1} = k | w_{t-1}] \tag{7}$$

for $l = 0, 1$ and $p_{kl}^{t-1} = \Pr(S_t = l | S_{t-1} = k, w_{t-1})$, the transition probabilities between the states k and l .

(b) Filtering of S_t

$$Pr(S_t = l | w_t) = \frac{c_{\theta_{ct}}(u_1, u_2 | S_t = l, w_{t-1}) Pr[S_t = l | w_{t-1}]}{\sum_{k=0}^1 c_{\theta_{ct}}(u_1, u_2 | S_t = k, w_{t-1}) Pr[S_t = k | w_{t-1}]} \tag{8}$$

where $w_t = [w_{t-1}, u_1, u_2]$. In $t = 1$, the filter is initialized using stationary probabilities of S_t for $Pr(S_0 = k | w_0)$.

The above filter aids in obtaining the probability distribution of S_t given the information set by t . It is useful to know the distribution of S_t given the full sample information set, that is obtaining better estimates using all T observations (opposed to a prediction and filtering process) since information extraction about the past from the future is possible in a times series context.

Thus, the smoothed probabilities regarding $S_t, Pr(S_t = l | w_T) = \sum_{k=0}^1 Pr(S_t = l, S_t = k | w_T)$ can be constructed recursively from the filtered probabilities. This process operates like a backward-smoothing algorithm as follows:

1. With the above filtering process, $Pr(S_t = l | w_T)$ can be obtained for $l = 0, 1$ and $t = 1, \dots, T$.
2. Then, we begin the smoothing algorithm in $t = T$ and go backwards recursively with $Pr(S_t = l | w_T)$ being equal to the filtered probability in $t = T$.
3. For each $t = T - 1, T - 2, \dots, 1$, the smoothed probability distribution $Pr(S_t = l | w_T)$ is given by

$$Pr(S_t = l | w_T) = \frac{\sum_{k=0}^1 p_{lk}(t) Pr(S_t = l | w_t) Pr(S_{t+1} = k | w_T)}{\sum_{j=0}^1 p_{jk}(t) Pr(S_t = j | w_t)} \quad (9)$$

where $p_{lk}(t) = Pr(S_{t+1} = k | S_t = l, w_t)$ are the transmission probabilities between the states l and k .

At $t = 0$, the smoothing algorithm results in $Pr(S_0 = l | w_T)$ which can be considered as the initial value in the filtering algorithm. Therefore, the forward-filtering-backward smoothing algorithm can be completed and then be used to maximize Eq. (7) directly in a numerical approach in relation to the model.

Computing standard errors by block bootstrap

We use the block bootstrap approach to construct standard errors of our model for estimating the covariance matrix. We follow Politis and White (2004), and Politis (2007) to construct the optimal block length. That is, we use the following procedure:

1. The IFM estimator for $\theta, \hat{\theta}$, was obtained by solving $\left(\frac{\partial \ell_{f1}}{\partial \theta_1}, \frac{\partial \ell_{f2}}{\partial \theta_2}, \frac{\partial \ell_c}{\partial \theta_{\alpha, S_t}}\right)' = 0$ separately, that is marginal distribution parameters in the first step dependence parameters in the second step.
2. Sample n/l sub-samples (with replacement) from the observed data and generating a set of time series with size n , where l is the block size.
3. Re-estimating parameters using the generated time series.
4. Repeating steps (2) and (3) R times.
5. Constructing the standard errors for the parameters using the covariance matrix $R^{-1} \sum_{r=1}^R (\hat{\Omega}(r) - \hat{\Omega})(\hat{\Omega}(r) - \hat{\Omega})'$, where $\hat{\Omega}(r)$ is the estimated parameter vector for each replication r and $\hat{\Omega}$ is the parameter vector obtained in (1).

2.3. Portfolio back testing-model

To proffer more insights on the diversification potential of international prices of crude oil, natural gas and cocoa against the sectoral stocks in Australia, we use a portfolio back-testing approach to examine the investment performance of the assets under examination. We used three estimation approaches used in the portfolio construction comprising of conventional approaches and some recent connectedness-oriented portfolios. In particular, we use the conventional Minimum Variance Portfolio (MVP) and the recent Minimum Correlation Portfolio (MCP) and the Minimum Connectedness Portfolio (MCoP). A number of assumptions underline our portfolio analysis. These include: the index can be purchased by the investor directly, market participants are interested only in international prices of crude oil, cocoa and natural gas and the Australian industry stock investment, and are open to investing in both commodities and sector stocks. Even though these assumptions are narrow, we argue that they are sufficient in illustrating our objective.

Outlined below is a summary of the portfolio estimation approaches adopted.

2.3.1. Minimum variance portfolio (MVP)

A commonly used approach in portfolio analysis is the MVP method which attempts to create the portfolio with the least volatility founded on multiple assets as documented by Markovitz (1959). The portfolio weight is estimated using the formula below:

$$w_{Ht} = \frac{H_t^{-1} I}{I H_t^{-1} I} \quad (10)$$

where w_{Ht} denotes the $m \times 1$ dimensional portfolio weight vector, I represents the m -dimensional vector of ones and H_t depicts the $m \times m$ dimensional conditional variance-covariance matrix in period t .

2.3.2. Minimum correlation portfolio (MCP)

In recent times, another emerging procedure in the construction of portfolios is the approach put forward by Christoffersen et al. (2014). In this procedure, portfolio weights are obtained by employing the conditional correlation matrix and not the conditional covariance matrix. Described below is the conditional correlation employed in the construction of MCP.

$$R_t = \text{diag}(H_t)^{-0.5} H_t \text{diag}(H_t)^{-0.5} \quad (11)$$

In Eq. (5) above, R_t denotes the $m \times m$ dimensional matrix. Following this, we estimate the weights for MCP as follows.

$$w_{Rt} = \frac{R_t^{-1} I}{I R_t^{-1} I} \quad (12)$$

2.3.3. Minimum connectedness portfolio (MCoP)

Following the construction of the MVP and MCP portfolio techniques, we next generate MCoP by employing all the pairwise connectedness indices rather than the correlation or variance matrix. The minimization of the interconnections among series and their spillovers proffers a portfolio that is not affected heavily by, or more irrepressible to, network shocks. Thus, variables (investment assets) that are not able to influence other assets and are not influenced by other assets are allocated with a higher weight in the constructed portfolio. This is expressed as shown below.

$$w_{Rt} = \frac{PCI_t^{-1} I}{I PCI_t^{-1} I} \quad (13)$$

PCI_t denotes the pairwise connectedness index matrix, while the identity matrix is represented by I .

2.3.4. Strategies for hedging and hedging effectiveness

To ascertain the performance of the portfolios, we rely on the Hedging Effectiveness (HE) index to evaluate the performance of several optimal hedge ratios (OHRs) obtained from using varied versions of GARCH models following Chang et al. (2011) and Ku et al. (2007).

$$HE = \frac{\text{var}_{unhedged} - \text{var}_{hedged}}{\text{var}_{unhedged}} \quad (14)$$

where var_{hedged} denotes the portfolio returns variance, $\text{var}_{unhedged}$ represents the unhedged asset variance. The HE index shows the percentage of reduction in the variance of the unhedged position. A higher HE index denotes a larger risk reduction or shows that the hedging effectiveness is higher.

3. Data, descriptive statistics, and time-trends

To investigate the dependence between the international prices of

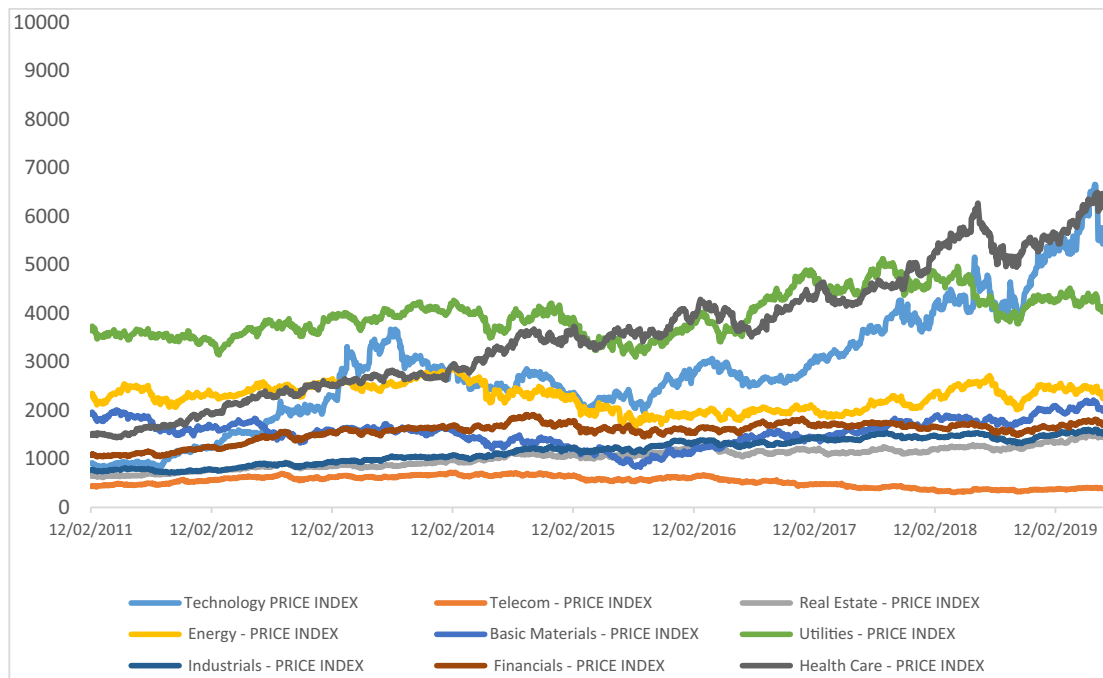


Fig 1. Time series plots of daily price sectoral stocks.

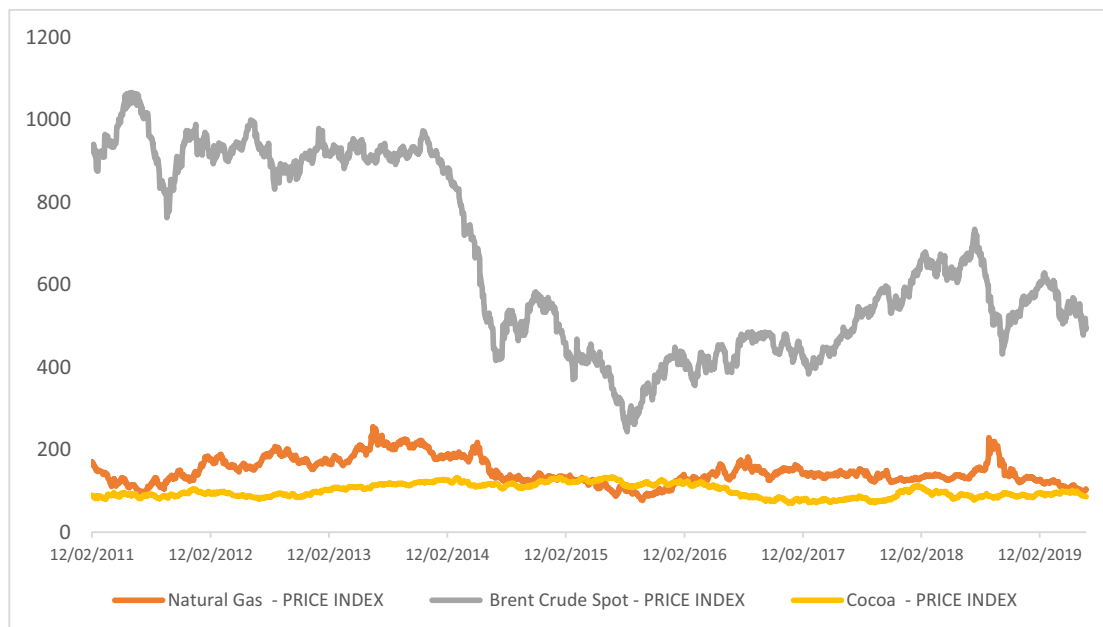


Fig. 2. Plots of price series for gold, oil, natural gas and cocoa. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

oil, natural gas and cocoa and Australia's nine sectoral stocks, we obtain daily spot price indices of global oil, natural gas and cocoa and the sectoral price indices of Australia's stock market sourced from DataStream for the period from 1st December 2011 to 23rd April 2020, yielding a total of 2191 daily observations. The nine sectoral stocks considered are: Technology, Telecom, Real Estate, Energy, Basic Materials, Utilities, Industrials, Financials and Health Care. We estimate the daily returns of all series as $\ln(P_t) - \ln(P_{t-1})$, the logarithmic difference between the closing price on day t and the closing price of day $t - 1$.

Fig. 1 illustrates the time series plots of all the nine sectoral stock price indices. We observe that the utilities sector even though started on

a high note, it recorded marginal price fluctuations throughout the period to end the period falling below the price levels of Health Care and Technology sectors. The Telecom sector emerged to be the most underperformed sector for the entire period by recording the least price levels. The Health Care sector even though started the period on a low side from 1st December 2011, the sector recorded sharp increases throughout to emerge as the most performed sector followed by the Technology sector. The energy sector also recorded a continual decline in the price level around 2014 up until the end of December 2018 but declined again in 2019. Sectors such as real estate, financial and industrials regarded as major contributors to the broader economy

Table 1
Descriptive Statistics of returns series.

	Sectoral Stocks									International Commodity Prices		
	Technology	Telecom	Real Estate	Energy	Basic Materials	Utilities	Industrials	Financials	Health Care	Natural Gas	Brent Crude	Cocoa
Mean	0.067	-0.013	0.015	-0.020	-0.017	-0.005	0.014	0.002	0.071	-0.036	-0.093	-0.009
Arithmetic Mean	0.067	-0.013	0.015	-0.020	-0.017	-0.005	0.014	0.002	0.071	-0.036	-0.093	-0.009
Geometric Mean	0.749	0.571	0.439	0.570	0.681	0.502	0.396	0.428	0.535	1.310	0.785	0.807
Harmonic Mean	0.144	0.216	0.151	0.112	0.211	0.106	0.117	0.130	0.186	0.647	0.321	0.385
Median	0.132	0.036	0.050	0.047	0.008	0.045	0.064	0.060	0.074	-0.057	0.019	-0.032
Std. Dev	1.740	1.298	1.154	1.430	1.465	1.085	1.000	1.097	1.223	2.643	2.236	1.629
Kurtosis	5.823	7.263	35.894	19.997	4.173	4.862	26.834	20.849	13.722	3.356	22.160	0.957
Skewness	-0.012	-0.866	-2.631	-1.721	-0.180	-0.791	-1.940	-1.287	-0.074	-0.043	-1.066	0.058
Min	-10.526	-11.183	-15.512	-18.432	-10.515	-8.034	-13.909	-11.435	-11.121	-19.183	-26.825	-6.492
Max	14.200	7.395	7.770	5.934	9.655	4.303	6.693	8.233	11.054	16.643	19.080	7.241
Obs.	2012	2012	2012	2012	2012	2012	2012	2012	2012	2012	2012	2012
Jarque-Bera Test	2825.00*	4647.00*	1100.00*	3400.00*	1461.00*	2180.00*	6100.00*	3700.00*	1600.00*	938.60*	4100.00*	76.99*
ADF	-22.654*	-21.483*	-22.735*	-23.438*	-23.441*	-20.677*	-22.259*	-25.026*	-23.907*	-24.697*	-25.038*	-24.090*
PP	-37.755*	-40.273*	-40.497*	-44.854*	-41.338*	-42.336*	-43.326*	-45.427*	-48.515*	-44.835*	-44.745*	-40.198*
Ljung Box LM Test	15.599	7.511	32.82*	20.777**	19.342**	7.707	35.924*	68.423*	80.966*	6.084	17.820***	6.558
C-H Test	20.829**	11.004*	55.275*	24.633*	25.688**	11.602	53.845*	92.185*	92.347*	8.176	31.771*	8.968
ARCH LM (1) Test	8.65*	6.70*	8.22*	8.16*	6.89*	4.32*	7.33*	8.20*	6.01*	7.94*	17.22*	4.06*

Notes: ADF and PP test the estimates of the Augmented Dikey Fuller (1979) and the Phillips and Perron (1988) unit roots tests, respectively. The Ljung Box test is the test for serial correlation in all the series, while the C-H test is the Cumby-Huizinga test for autocorrelation for the return series. JB denotes the Jarque-Bera test for the normality ARCH is the Lagrange multiplier test for autoregressive conditional heteroscedasticity of order 1. *, ** and *** denotes significance at the 1%, 5% and 10% levels, respectively.

recorded marginal fluctuations in the price levels.

In Fig. 2, we present the time series plot of oil, natural gas and cocoa price series. We note the extreme sharp declines in the price of oil around 2015, with a marginal increase following the aftermath of the 2015 oil crisis. The price levels of natural gas and cocoa for the entire period witnessed less price fluctuations as shown in Fig. 2. Comparing the time series plots reported in Fig. 1 and Fig. 2, they seem to portray that there could be a linkage between oil, gold, cocoa, natural gas and the sectoral stocks in Australia's stock markets even though this will be confirmed from the application of copula models. Daily returns of all series under consideration are in log form.

Table 1 reports the statistical properties of returns for the nine sectoral stock price indices and the three commodity price indices of oil, natural gas and cocoa under consideration in the study. Comparing the mean returns of the nine sectoral stocks, we find that the daily mean returns of the technology, real estate, industrials, financials and health care sectors are positive, with the health care sector recording the highest mean return, which is equal to 0.071, followed by the technology sector (0.067) as displayed in Fig. 1. The least mean return which is equal to -0.020 was recorded by the energy sector. We note that the average returns of all the three commodities price indices are negative with Brent Crude recording the least mean returns of -0.093.

From the standard deviation of the sectoral stocks and the commodity price indices, even though the extent of volatility is low for all the series, the results show a considerable number of fluctuations in all series under examination. On skewness, we observe a negative skewness

for all the series except that of cocoa, which is mostly affected by weather conditions. Negative (positive) skewness connotes the tendency of higher negative (positive) returns without matching the tendency of positive (negative) returns. For kurtosis, all the series examined in this study except that of cocoa recorded a kurtosis which exceeds the threshold of 3, which surmises that the return series for the period have flatter tails, compared to what would be anticipated from a normally distributed series. We use the Jarque-Bera (JB) statistic to test for the normality assumption in the series. We reject the null hypothesis of normality for all the series under examination at the 1% level.

We adopt the ADF test of Dickey and Fuller (1979), and the PP test of Philips and Perron to test for the stationarity of all the variables examined. Our results show that the series are stationary in the first difference, which is confirmed by the ADF and PP unit root tests. We test for the presence of autocorrelation using the Ljung-Box test and the Cumby-Huizinga test with the results confirming the presence of autocorrelation for all the series. We also test for the presence of ARCH effects using the ARCH-LM test of Engle (1982) and the results suggest the presence of the ARCH effects in all the series. Table 2 presents Spearman's correlation coefficient across all the series examined, with the results showing a strong correlation across the sectoral stocks, which suggests the possibility of diversification benefits. On the correlation between the nine sectoral stocks and oil, cocoa and natural gas, we find weak correlations. Fig. 3 illustrates both the price index and return series for all the nine sectoral stocks and the three global commodity price indices. We observe that all the return series exhibit volatility clustering

Table 2
Spearman's Correlation.

	Technology	Telecom	Real Estate	Energy	Basic Materials	Utilities	Industrials	Financials	Health Care	Natural Gas	Brent Crude
Technology	1										
Telecom	0.472	1									
Real Estate	0.359	0.397	1								
Energy	0.393	0.354	0.486	1							
Basic Materials	0.340	0.322	0.359	0.679	1						
Utilities	0.358	0.394	0.532	0.693	0.532	1					
Industrials	0.489	0.437	0.711	0.664	0.548	0.634	1				
Financials	0.433	0.446	0.735	0.640	0.557	0.611	0.780	1			
Health Care	0.389	0.348	0.447	0.470	0.353	0.465	0.627	0.570	1		
Natural Gas	0.027	0.028	-0.009	-0.016	-0.021	0.003	-0.019	-0.014	-0.021	1	
Brent Crude	0.164	0.147	0.046	0.213	0.138	0.148	0.134	0.117	0.111	0.141	1
Cocoa	0.114	0.080	0.040	0.059	0.049	0.055	0.052	0.063	0.013	0.060	0.133

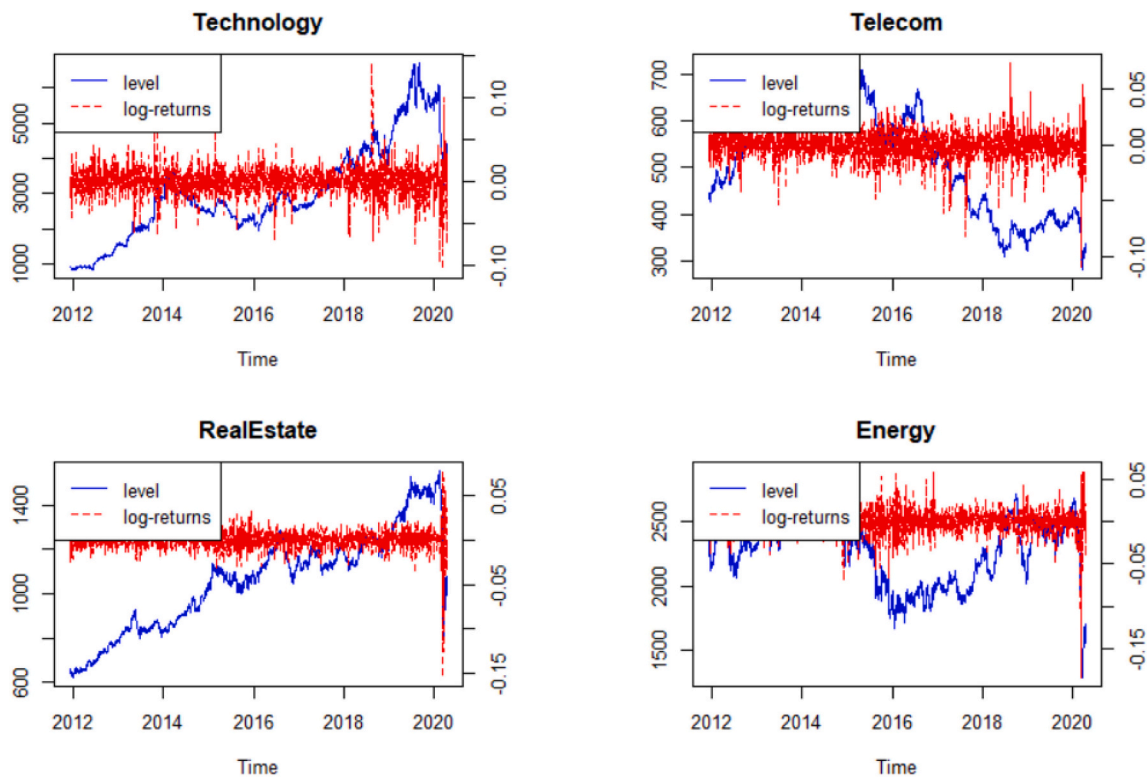


Fig 3. Plots of price series and returns for sectoral stocks and commodity indices.

throughout the period, which surmises that the use of a GARCH-type model may be useful.

4. Empirical results

In this section, we first discuss the marginal distributions followed by the tail dependence structure results obtained from the copula model. Finally, we examine the diversification benefits from a portfolio perspective to ensure that our findings benefit practitioners.

4.1. Marginal distribution results

To obtain the residuals of each series for the copula estimation, we first use the ARMA filters to examine the features of each series returns to ensure that the residuals are free from autocorrelation. Next, we test the fitted series for the ARCH effects using the ARCH-LM test, with results showing evidence of heteroscedasticity for each of the series. We determine the optimal lag length for each univariate GARCH and fit the various specifications to the second moments. In Table 3, we report the estimates of the ARMA-GARCH models for the series under examination where we select the best fitting models based on the Akaike Information Criterion (AIC) of all order combinations from (0,0) to (6,6). Using the AIC, we find the best fitting models are ARMA (1,0)-GJR-GARCH (1,1) for all the nine sectors including technology, telecom, real estate, energy, basic materials, utilities, industrials, financials, health care. For the global commodities examined, the best fit model for natural gas and cocoa is ARMA (1,0)-GJR-GARCH (1,1), while the best fit model for Brent crude is ARMA (1,0)-GJR-GARCH (1,1).

From the estimates of the mean equation, we find the conditional mean is positive and significant for technology and financials, while we note a negative significance for health care and Brent crude. From the variance equation, we find that the ARCH-effects denoted by alpha 1 in Table 3 are positive and significant as anticipated for series except for the utilities, industrials and financial sectors. Likewise, we note that the

GARCH component (beta 1) is statistically significant and positive for all the series. This suggests that all the series are characterized by significant GARCH effects. It is interesting to note that most of the series in our sample are symmetric in their own variance component. After obtaining the optimal models using the marginal specifications, we next employ the empirical distribution function (ecdfs) to transform the standardized residuals into uniform margins, we carry out the goodness of fit tests for the marginal models by employing the ARCH-LM and Ljung-Box tests to the probability integral transform (PIT) residuals of the underlying error terms for each of the ARMA-GARCH (p,q) processes. From Panel C of Table 3, the results suggest the absence of serial correlation in the series, which justifies the appropriateness of the marginal models.

4.2. Copula dependence structure results

The dependence structures between natural gas, cocoa, oil and the sectoral stocks of Australia's stock market, using the time-invariant, traditional time-varying and time-varying Markov copulas, are discussed in this section. We present estimates for the returns and compare the log likelihood (LL), BIC and AIC values to select the best fit copula model.¹ It is noteworthy to mention that some studies report all values for LL, AIC and BIC (Ji et al., 2018), while other studies report only the LL values (Tiwari et al., 2020; Tiwari et al., 2021; Abakah et al., 2021a, 2021b). In this paper, we report only the LL estimates for the best copula fit model.

¹ We only report the LL values for the best copula fit model following Tiwari et al. (2020) and Abakah et al. (2021a, 2021b). Those criteria yielded the same best results.

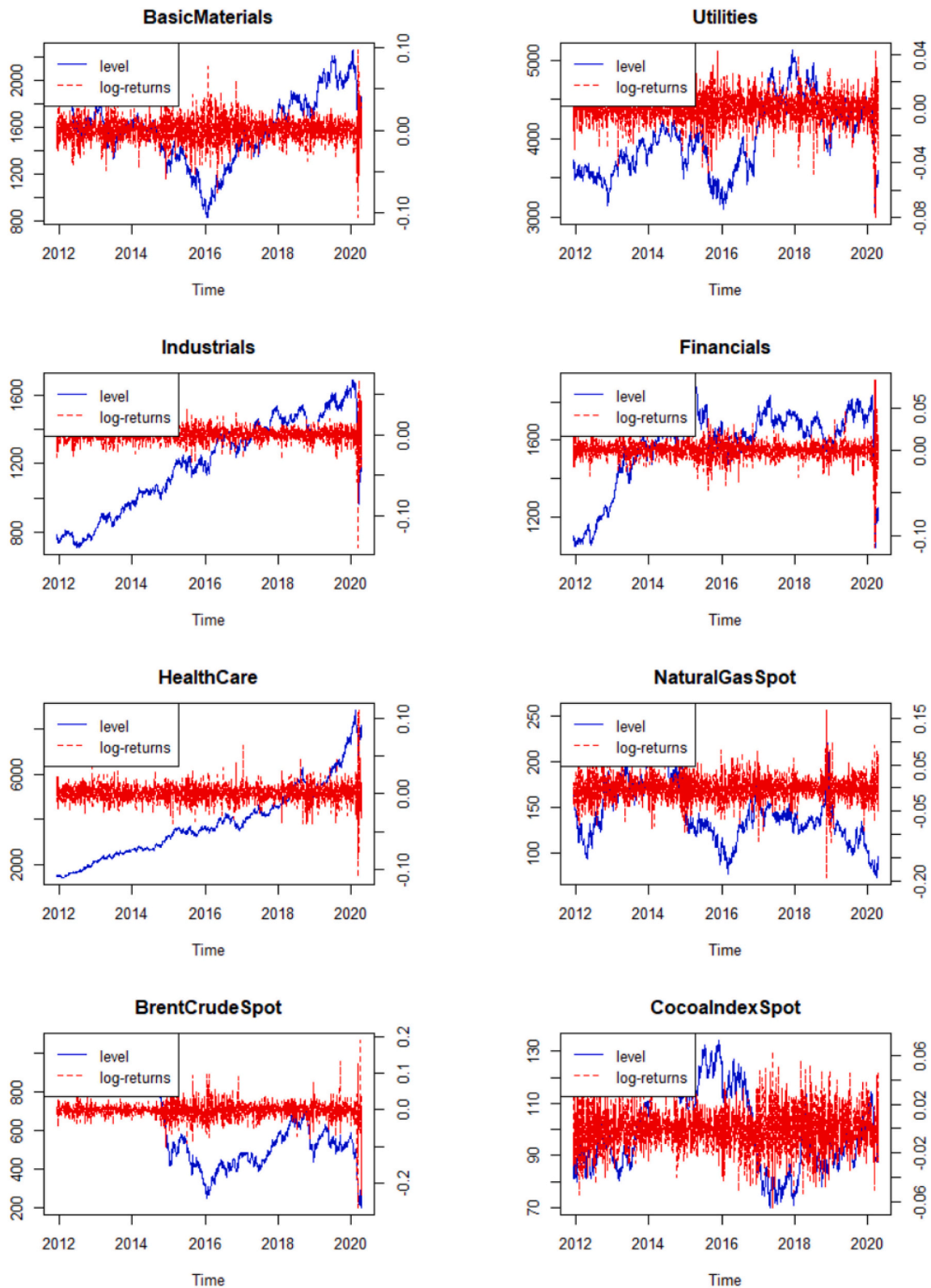


Fig 3. (continued).

Table 3
Estimate of ARMA-GARCH models.

	Technology	Telecom	Real Estate	Energy	Basic Materials	Utilities	Industrials	Financials	Health Care	Natural Gas	Brent Crude	Cocoa
Panel A: Mean Equation												
c	0.060***	0.009	0.024	-0.006	-0.023	-0.003	0.020	0.004	0.072*	0.006	-0.060**	0.013
ϕ_1	0.057**	-0.025	-0.026	0.032	0.020	-0.005	-0.025	0.063**	-0.041***	-0.002	-0.047**	-0.006
Panel B Variance Equation												
ω	0.225***	0.064**	0.039**	0.022**	0.019**	0.016	0.027*	0.035**	0.099**	0.116**	0.011***	0.008
α_1	0.131*	0.038**	0.029***	-0.010*	0.014***	0.014	0.013	0.007	0.044***	0.090*	0.118**	0.021**
α_2											-0.102**	
β_1	0.792*	0.904*	0.880*	0.945*	0.952*	0.953*	0.893*	0.869*	0.830*	0.911*	0.955*	0.965
θ_1	0.005	0.031	0.088*	0.096*	0.051*	0.037**	0.101*	0.157*	0.082**	-0.032*	0.061	0.022***
θ_2											-0.005	
Asymmetry	-0.086*	-0.083	-0.106*	-0.138*	-0.103*	-0.139*	-0.161*	-0.152*	-0.042	0.025	-0.121*	-0.002
Tail	5.734*	5.539	8.378*	7.171*	11.567*	9.193*	10.334*	8.642*	7.323*	12.444*	5.097*	13.829*
Log-Likelihood	-3717.96	-3167.65	-2582.05	-3152.08	-3414.23	-2845.49	-2374.53	-2472.88	-2926.18	-4639.43	-3879.16	-3719.96
AIC	3.704	3.157	2.574	3.141	3.402	2.836	2.368	2.466	2.916	4.619	3.865	3.706
Panel C: Diagnostic Test – Goodness of Fit Test												
Q(10)	8.643	2.890	11.382	15.019	7.447	12.014	9.189	9.885	6.899	10.517	14.723	2.589
	[0.471]	[0.968]	[0.250]	[0.090]	[0.591]	[0.212]	[0.420]	0.359]	[0.647]	[0.829]	[0.098]	[0.978]
Q ² (10)	3.995	18.557	11.470	6.869	27.034	15.821	4.218	14.168	6.962	8.861	6.992	6.469
	[0.857]	[0.017]	[0.176]	[0.551]	[0.001]	[0.045]	[0.836]	[0.077]	[0.541]	[0.354]	[0.429]	[0.594]
ARCH-LM Test	0.395	2.014	1.172	0.671	2.736	1.467	0.445	1.347	0.679	0.8775	0.700	0.612
	[0.949]	[0.276]	[0.305]	[0.751]	[0.002]	[0.146]	[0.925]	[0.199]	[0.744]	[0.0.553]	[0.725]	[0.804]

Notes: The table shows the marginal model estimates for the bond yield for each series over the period. In Panel A, we report the estimates for the conditional mean, modelled using the ARMA (p, q) model. In Panel B, we present the parameter estimates from GARCH (p, q) models of the conditional variance. Panel C presents estimates and p -values for the test of serial correlation in the standardized residuals of the returns. Q(10) and Q2(10) are the Ljung-Box statistics for serial correlation in the model residuals and squared residuals, respectively, computed with 10 lags. ARCH is the Engle LM test for the ARCH effect in the residuals up to the 10th order. The p -values in the square brackets () below. *, ** and *** denotes the 1%, 5% and 10% significance levels.

Table 4
 Estimation of the dependence-switching copula model for Natural Gas and Sectoral Stocks.

	Technology	Telecom	Real Estate	Energy	Basic Materials	Utilities	Industrials	Financials	Health Care
Panel A: Parameter Estimates for time-varying Markov copula									
Time-varying Normal Markov copula									
ω_c^0N	0.0477	-0.0044	-0.0254	-0.0101	-0.0481	0.01718	-0.0924	-0.07279	-0.2256
ω_c^1N	-1.6106	-2.0640	0.5767	4.8874	-4.0569	0.01445	0.2418	1.24491	5.9985
β_cN	-2.0088	-1.4748	1.1547	1.2493	-1.4399	-1.89358	-1.2757	-1.26996	-2.0090
α_cN	0.1296	-0.4241	-0.1415	-0.0362	-0.1223	-0.31796	-0.2501	-0.14127	-0.3235
ρ	0.6066	0.6600	0.6173	0.6372	0.6296	0.55069	0.6339	0.61886	0.6197
q	0.3934	0.3400	0.3827	0.3628	0.3704	0.44931	0.3661	0.38114	0.3803
Log-like	-0.8489	-1.8448	-2.9042	0.3018	-0.8825	-1.21000	-1.3812	-0.7497	-4.2818
Time-varying Clayton Markov copulas									
ω_c^0C	-0.2577	0.0412	-0.2087	-0.1741	-0.0001	0.05157	0.1114	-0.0002	2.59E-06
ω_c^1C	0.0638	-0.7020	-0.2087	-0.1741	-0.0001	0.05164	0.0491	-0.0002	-1.7157
β_cC	0.8630	1.0099	1.5698	1.1182	1.4571	0.95820	3.0523	1.6044	-0.0348
α_cC	0.3812	-0.1201	0.7310	0.7020	-1.94E-05	-0.67001	-0.1384	9.31E-06	-8.78E-07
ρ	0.5098	0.3616	0.5000	0.5000	0.5000	0.50000	0.5046	0.5000	0.6106
q	0.4902	0.6384	0.5000	0.5000	0.5000	0.50000	0.4954	0.5000	0.3894
Log-like	-1.5479	-1.1793	-0.4015	-0.4357	0.0044	-0.73360	-0.0982	0.0031	-1.0248
Time-varying Rotated Clayton Markov copulas									
ω_c^0RC	-0.0001	-0.0002	0.0652	-0.0001	-0.4334	0.37575	-0.3567	-0.3534	0.0700
ω_c^1RC	-0.0001	-0.0002	-1.6010	-0.0001	-0.4334	-0.07957	-1.5676	-1.2773	-1.8712
β_cRC	1.4766	1.5843	0.1598	1.4058	1.5305	-0.39304	1.1719	-0.2508	-0.1289
α_cRC	0.0000	0.0001	-0.2441	8.14E-06	1.1519	-1.19206	1.1385	1.2957	-0.1723
ρ	0.5000	0.5000	0.6094	0.5000	0.5000	0.61060	0.6651	0.6419	0.5555
q	0.5000	0.5000	0.3906	0.5000	0.5000	0.38940	0.3349	0.3581	0.4445
Log-like	0.0048	0.0060	-0.162	0.003	-0.5823	-0.17440	-0.8491	-0.504	-0.2642
Time-varying Gumbel Markov copulas									
ω_c^0G	-1.27E-08	0.1506	0.3061	0.7470	-2.6839	0.56441	-1.8190	0.2814	9.40E-07
ω_c^1G	-1.58E-08	0.1506	0.3061	0.7470	-2.6839	0.56441	-1.8086	0.2814	9.38E-07
β_cG	2.95E-08	-0.0688	-0.3437	-0.6168	2.8644	-0.16791	1.5947	-0.5149	-9.48E-07
α_cG	-5.01E-08	-0.2907	0.1442	-0.4732	-0.8499	-1.20999	0.7067	0.8710	2.74E-08
ρ	0.5000	0.5000	0.5000	0.5000	0.5000	0.50000	0.5014	0.5000	0.5000
q	0.5000	0.5000	0.5000	0.5000	0.5000	0.50000	0.4986	0.5000	0.5000
Log-like	0.0006	0.1460	0.1422	0.0433	-0.4702	-0.36390	-0.78	-0.2823	0.0296
Time-varying Rotated Gumbel Markov copulas									
ω_c^0RG	-0.9776	1.0369	1.8075	-2.0280	7.03E-08	-5.33403	7.47E-07	2.90E-06	1.33E-06
ω_c^1RG	-0.9777	0.7807	1.8075	-2.0280	1.81E-08	-5.43473	7.47E-07	2.95E-06	1.33E-06
β_cRG	0.9788	-0.7103	-1.6887	2.1541	-1.05E-07	5.48059	-7.63E-07	-2.88E-06	-1.22E-06
α_cRG	-0.3138	-0.8311	-0.4678	-0.5188	1.52E-07	-0.65213	1.21E-08	-8.23E-08	-4.84E-07
ρ	0.5000	0.3982	0.5000	0.5000	0.5000	0.53474	0.5000	0.5000	0.5000
q	0.5000	0.6018	0.5000	0.5000	0.5000	0.46526	0.5000	0.5000	0.5000
Log-like	-0.2723	-1.8828	-0.4226	0.039	0.2182	-1.77860	0.1965	0.2027	0.108
Time-varying SJC Markov copulas									
ω_c^0U	-7.8659	-10.4983	3.8713	-9.5766	-16.8401	4.11029	4.7299	4.2692	1.0667
ω_c^1U	-23.8331	-23.3704	-8.7442	-17.4208	-14.7100	-6.25636	-5.4540	-8.9062	5.9791
β_cU	-17.7878	-17.5297	-20.1695	-18.5737	-17.8108	-20.64138	-19.2894	-19.6618	-20.4492
α_cU	-18.4560	-18.8484	-20.2889	-19.3627	-17.5760	-22.85819	-22.9995	-19.8083	-22.0030
ρ	-6.2232	-6.0603	0.5481	-6.2089	-6.3337	-3.38902	-2.4393	-5.9187	-6.1083
q	-5.8775	-6.3098	-6.6972	-6.1265	-6.4427	-4.61560	-5.8347	-6.2841	-2.2050
Log-like	4.9333	6.7834	9.8917	7.6603	8.0789	4.89340	9.2371	9.2520	11.6194
Time-varying T Markov copulas									
ω_c^0T	0.0626	-0.0031	-0.0249	-0.0738	0.4316	0.01857	0.2402	-0.6561	-0.1848
ω_c^1T	0.0626	-0.0031	-0.0249	-0.0738	-0.4760	0.01857	-0.8228	0.2944	-0.1848
β_cT	-1.6905	-1.4798	1.1563	-1.9878	-1.3101	-1.89344	-1.2407	-1.2454	-1.9135
α_cT	-0.0329	-0.4165	-0.1403	-0.1450	-0.0530	-0.30234	-0.4315	-0.3389	-0.3526
ρ	-0.0009	0.0007	0.0197	-4.43E-07	-1.1843	-2.62E-07	1.5868	2.3804	-9.36E-07
q	-0.0009	0.0007	0.0194	2.61E-07	-1.4313	-4.83E-07	2.4732	1.8015	9.60E-07
Log-like	0.04833	-1.4282	-2.7616	-1.1068	-0.9055	-1.30270	-3.1477	-1.5523	-3.2136

NB: This table contains the ML estimates for the Markov switching copulas. We report the ML estimates for the different static and time-variant bivariate copula models for the natural gas-sectoral stock return in Table 2A in the appendix. All coefficients are significantly different from zero. The loglikelihood value (value in bold) indicates the best copula fit.

4.2.1. Dependence-switching copula model for natural gas and sectoral stocks

Table 4² presents the estimated results for time-varying dependence between natural gas and the nine sectoral stocks under study. We note that TVP Clayton copula is the best fit for natural gas and technology with the nature of the dependence emerging to be negative. This suggests that natural gas can act as a good hedge against the technology stocks in Australia since both move in opposite directions. For natural gas and telecom, we find the time-varying Rotated Gumbel Markov copula is the best fit, using the LL estimate with a significant negative dependence structure. Furthermore, we find that the time-varying Normal Markov copula gives the better fit for natural gas and the following sectors: real estate, basic materials, financials and health care. Consequently, a look at the probabilities ρ and q for the dependence between natural gas and the following five sectors telecom, real estate, basic materials, financials and health care reveals that the dependence structure between natural gas and each of these sectors is Markov-switching time-varying, which suggests that using a constant and traditional time-varying copula to test the dependence structure may not be able to capture the relationship adequately. This result indicates that the symmetric nonzero tail dependence is a feature of the dependence structure between natural gas and the five stock sector indices.

Focusing on the estimates of β ³ for the raw returns, we find they are significant, which further suggests that the relationships between natural gas and the five sectors are time-varying. We follow the sign of β to establish the nature of the dependence. From Table 4, we find a negative dependence structure for the return series between natural gas and the telecom, basic materials, financials and health care sectors. This connotes that these sectors move in an opposite direction to natural gas, thus investors could gain for holding a portfolio with such assets. However, we find a positive dependence between natural gas and the real estate sector, thus suggesting that natural gas is not a good hedge for the real estate sector since both move in the same direction. For the dependence between NG and the energy sector, the Student *t* copula gives the best fit. The Student-*t* copula, which is a better model when the datasets have a tail dependence, shows series of symmetric upper and lower tail dependence. This suggests a co-movement between natural gas and the energy sector in both negative and positive extreme events. For the dependence between natural gas and utilities and natural gas and industrials, we find the best fit model to be the TVP Rotated Gumbel copula and the TVP Gumbel copula, respectively, with a negative dependence structure. These Archimedean copulas (i.e., Gumbel) suggest a presence of a greater upper tail dependence than the lower tail, signifying the presence of asymmetry in the bivariate relationships. This shows that the relationship between natural gas and the utilities sector stock and natural gas and the industrials sector stock is asymmetric.

Overall, we find a negative dependence between natural gas and all the stock sectors except the real estate sector. This means that natural gas can act as a good hedge for all the sectors except the real estate sector, thus revealing the diversification benefits of natural gas to portfolio investors.

To provide further evidence on the time-varying dependence between natural gas and industry stocks of Australia's stock markets reported in Table 4 above, we next report the dependence parameter plots between natural gas and the Australian industry stocks in a time-varying framework in Fig. 4 and compare the dependence structure in normal times and crisis periods. For the crisis period, we consider the European debt crisis of 2011, the Brexit of 2016 and the COVID-19 pandemic of 2020. We note that the dependence structure is time varying in both

normal and crisis periods. We find from Fig. 4 that the dependence structure varies for both the high and low dependence regimes and that the dependence is positive in both regimes.

4.2.2. Dependence-switching copula model for cocoa and sectoral stocks

Table 5 reports the dependence structures between cocoa and all nine sectors using different copulas. We find strong evidence that the time-varying Rotated Gumbel Markov copula is the best fit model for the dependence between cocoa and the following sectors: telecom, real estate, energy and basic materials. Thus, these sectors exhibit an asymmetric time-varying dependence, which means there is a stronger dependence in the lower tails instead of the upper tails. The implication of this is that the returns series have a negative tail asymmetric dependence. While for the case of relationship between cocoa and the industrials and health care sectors, we find the Time-varying Rotated Clayton Markov copulas to be the best specification model. Time-varying SJC Markov copulas give the best fit for the dependence between cocoa and financials. The Cocoa and the technology sector are best modelled using the time-varying Clayton Markov copula. The estimated results of the dependence structure between cocoa and the technology, telecom, real estate, energy, basic materials, industrials, financials and health care sectors is Markov-switching time varying, which means that using constant and conventional time-varying copula models will not be appropriate to estimate their dependence structure. For the dependence between cocoa and the eight sectors (cocoa, technology, telecom, real estate, energy, basic materials, industrials, financials and health), we observe that the estimates of β are significant, suggesting that the dependence between cocoa and these sectors is time varying in nature.

We next examine the nature of dependence between cocoa and these eight sectors (cocoa, technology, telecom, real estate, energy, basic materials, industrials, financials and health) by relying on the sign of the estimated β . The estimates reported in Table 4 show a negative dependence between the returns series of cocoa and the telecom, energy, basic materials, financials and health care sectors. This indicates that cocoa can act as a good hedge to these assets for investors in Australia since both cocoa and these assets move in opposite directions. However, for cocoa and the technology, industrials and real estate sectors, we find a positive dependence structure which means these assets co-move in the same direction. Thus, cocoa can serve as a good hedge for the technology, industrials and real estate assets. The results obtain for industrials is not surprising since some industries rely on cocoa as a raw material for production. Finally, we examine the structure between cocoa and utilities sector and find that the TVP Rotated Gumbel copula is the best specification model, with the nature of dependence emerging to be positive. Thus, cocoa can't serve as a good hedge for the utility stocks.

Overall, the findings in Table 5 show that cocoa and the majority of sectoral stocks in Australia have a negative dependence structure. This suggests that extreme events on cocoa and sectoral stocks in Australia are negatively related. This finding is not surprising since the contribution of Australia to the world's cocoa market is marginal. These findings reveal the diversification potential of our results to domestic and international portfolio investors. It is anticipated that the negative dependence between cocoa and the Australian stock market may change in future once Australia's contribution to the world cocoa markets rises since the cocoa markets will become more integrated with Australian stock markets.

Similar to the conclusion drawn Fig. 4, we note from Fig. 5 that the dependence dynamics between cocoa and Australia's sectoral stocks are time varying and time dependent on the prevailing markets' conditions.

4.2.3. Dependence-switching copula model for Brent crude oil and sectoral stocks

In this section, we investigate whether Brent crude oil is a good hedge for sectoral stocks in Australia by examining the nature dependence structure between Brent crude and industry stock returns using

² Tables 4, 5 and 6 only report the Markov switching copulas. The time invariant and variant copulas are reported in the appendix.

³ For the results reported in Tables 4, 5 and 6, we do not present the standard errors, AIC and BIC for simplicity reasons. However, they are readily available upon request.

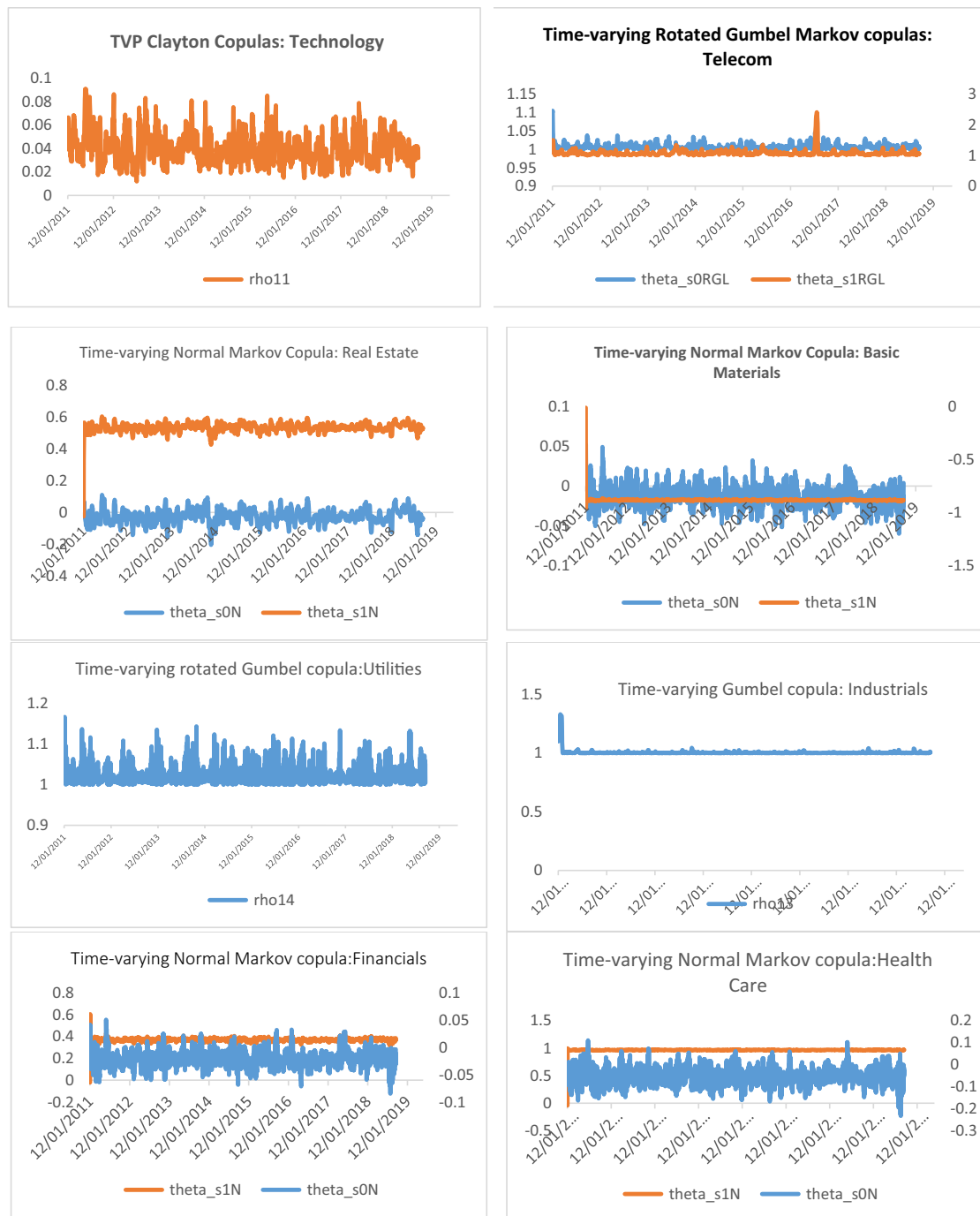


Fig 4. Dependence parameter plot between natural gas and sectoral stocks.

several copulas. We present the empirical results for the relationship between Brent crude oil and the Australian sectoral stocks in Table 6. We observe from the results that the time-varying Rotated Gumbel Markov copula is the best fit for Brent crude and the telecom, basic materials and utilities sectors. The Time-varying Clayton Markov copulas estimates emerged to be the specification model for the tail dependence between Brent crude oil and the technology, industrials and health care sectors, suggesting an asymmetric tail dependence structure. For the dependence structure between Brent crude oil and the financial sector, we again find that the time-varying Clayton Markov copula gives a better fit.

From the discussion above backed by the estimates of the probabilities measured by p and q that are persistent, we conclude that the

dynamic dependence structure between Brent crude and the following six sectors telecom, basic materials and utilities technology, industrials and health care, financials is Markov switching time varying. Hence, adopting the constant and conventional time-varying copula models to study their dependence will be inadequate. After establishing that there is a dependence between Brent crude and these above sectors, we next examine the sign of the dependence for the benefits of portfolio diversification using the sign of the β estimates. The results from Table 6 confirm that the nature of dependence between Brent crude and the technology, telecom, basic materials, utilities, industrials and health care sectors is positive in nature. Thus, Brent crude is not a good hedge for these assets because oil cannot protect investors against losses that may arise from oil price fluctuations as Brent crude oil co-moves in the

Table 5
Estimation of the dependence-switching copula model for Cocoa and Sectoral Stocks.

	Technology	Telecom	Real Estate	Energy	Basic Materials	Utilities	Industrials	Financials	Health Care
Panel A: Parameter Estimates for time-varying Markov copula									
Time-varying Normal Markov copula									
ω_c^0N	0.2696	0.3315	0.0011	0.0217	0.0283	0.1772	0.1159	0.0286	0.0667
ω_c^1N	3.9090	3.4707	3.5651	-2.4837	4.1422	6.1762	6.6814	20.3407	2.4017
β_cN	-1.6159	-2.0173	1.8198	1.4995	1.2188	-2.0079	-1.7915	1.3623	-1.7822
α_cN	-0.0447	0.2031	0.0404	0.0452	0.0687	0.1154	-0.1188	0.0886	-0.1464
ρ	-0.4512	0.6168	0.6035	0.6761	0.6204	0.6212	0.6286	0.6843	0.6197
q	1.4512	0.3832	0.3965	0.3239	0.3796	0.3788	0.3714	0.3157	0.3803
Log-like	-13.7045	-10.453	-3.9645	-3.6948	-1.9447	-5.552	-1.8425	-7.361	-0.5629
Time-varying Clayton Markov copulas									
ω_c^0C	-9.22E-01	-7.11E-01	-5.51E-01	-4.31E-01	-5.49E-01	-3.94E-01	3.81E-01	-4.54E-01	-0.1133
ω_c^1C	-0.7296	-1.223	-1.1878	-1.7772	-8.0273	-1.2113	-0.4013	-1.1773	-0.3498
β_cC	2.0692	1.381	1.3856	0.5634	0.1212	1.1481	0.8087	1.2861	-0.9027
α_cC	1.17E+00	5.63E-01	1.18E+00	7.10E-01	9.50E-01	3.86E-01	-1.09E+00	8.74E-02	0.3266
ρ	0.4936	-0.002	0.3602	-0.2743	0.5819	0.2658	0.3968	0.3236	0.8591
q	0.5064	1.002	0.6398	1.2743	0.4181	0.7342	0.6032	0.6764	0.1409
Log-like	-16.4614	-7.0982	-7.8487	-2.2298	-4.5966	-2.2836	-7.1365	-6.4281	-3.1288
Time-varying Rotated Clayton Markov copulas									
ω_c^0RC	-0.5318	0.2624	0.2320	-0.0715	-0.1811	0.4312	0.2526	0.6181	0.5932
ω_c^1RC	-0.5118	-0.325	0.2320	-0.5173	-0.4730	-0.1228	-0.6306	0.1311	0.7934
β_cRC	1.2631	1.286	2.6110	1.2921	0.8315	0.9566	0.8721	0.7406	-0.7084
α_cRC	0.4009	-0.718	-0.9151	0.1450	0.9678	-1.6321	-0.7439	-1.7030	-1.8924
ρ	0.5000	0.500	0.4699	0.4452	0.5646	0.4259	0.4339	0.4396	0.5197
q	0.5000	0.500	0.5301	0.5548	0.4354	0.5741	0.5661	0.5604	0.4803
Log-like	-6.1373	-6.2649	-0.0732	-1.8061	-1.4884	-3.3684	-0.4169	-2.0221	-4.0336
Time-varying Gumbel Markov copulas									
ω_c^0G	1.76E+00	-7.44E-01	-2.07E+00	1.18E+00	2.37E+00	-6.23E-01	1.15E+00	7.34E-01	0.0266
ω_c^1G	1.30E+00	-1.32E+00	-1.98E+00	1.39E+00	2.91E+00	-1.21E+00	1.01E+00	8.53E-01	-0.0780
β_cG	-1.15E+00	8.15E-01	2.00E+00	-1.46E+00	-2.17E+00	9.94E-01	-1.29E+00	-1.10E+00	-0.2802
α_cG	-7.58E-01	-1.38E-01	2.23E-01	5.25E-01	-5.12E-01	-1.32E+00	4.11E-01	7.67E-01	0.6404
ρ	0.4808	0.430	0.4782	0.5250	0.3639	0.3487	0.4621	0.8176	0.4719
q	0.5192	0.570	0.5218	0.4750	0.6361	0.6513	0.5379	0.1824	0.5281
Log-like	-7.8751	-6.9651	-0.4292	-5.8398	-3.5302	-4.5371	-0.4723	-3.0088	-1.1118
Time-varying Rotated Gumbel Markov copulas									
ω_c^0RG	2.09E+00	1.21E+00	-8.33E-01	1.96E+00	8.27E-01	1.21E+00	1.44E-01	-1.04E-01	1.1773
ω_c^1RG	1.62E+00	6.18E-01	-1.37E+00	1.96E+00	6.92E-01	1.05E+00	-1.06E-01	-3.22E-01	1.0303
β_cRG	-1.23E+00	-9.70E-01	7.00E-01	-2.11E+00	-9.69E-01	-1.29E+00	-2.78E-01	-1.74E-01	-1.2587
α_cRG	-9.40E-01	8.36E-01	1.32E-01	1.06E-01	2.02E-01	2.08E-01	5.79E-02	4.29E-01	0.1254
ρ	0.3679	0.521	0.3520	0.5012	0.4230	0.4680	0.3774	0.3885	0.3833
q	0.6321	0.479	0.6480	0.4988	0.5770	0.5320	0.6226	0.6115	0.6167
Log-like	-15.2766	-15.2721	-8.3071	-5.1654	-4.9418	-5.799	-5.9963	-8.9901	-3.609
Time-varying SJC Markov copulas									
ω_c^0U	-8.0110	-15.173	-11.3127	-3.5900	-11.6748	-9.0294	-16.1978	-12.3842	-16.7620
ω_c^1U	1.9268	-2.839	-2.1505	3.7588	1.0913	0.8117	-5.4065	-3.6328	-0.9487
β_cU	-19.7711	-19.760	-21.3747	-21.6880	-13.8966	-19.9822	-15.1516	-20.2684	-17.0347
α_cU	-20.4539	-21.973	-22.7840	-20.3421	-15.3489	-20.0519	-21.8267	-22.5061	-13.9479
ρ	-6.7533	-7.770	-7.7832	-6.5825	-6.8099	-6.4528	-12.4504	-7.8789	-8.8904
q	-15.7789	-6.722	5.9214	-2.4104	-3.6703	-4.4572	-1.9541	-2.5309	-3.2659
Log-like	-15.3069	-12.9229	-2.443	-2.7126	-1.4761	-3.2157	-1.6976	-10.5514	2.4765
Time-varying T Markov copulas									
ω_c^0T	0.2575	0.333	-0.7475	1.1414	-0.5281	0.9132	-0.1505	-0.0850	0.0761
ω_c^1T	0.6924	0.315	1.1605	-0.4682	0.8745	-0.3416	0.1843	0.1822	-0.0397
β_cT	-1.9071	-2.016	-1.6867	-2.0343	-1.4834	-2.0219	1.3243	1.4442	1.6296
α_cT	0.0737	0.156	0.4295	0.2749	-0.3135	0.1039	0.0921	0.0724	0.0347
ρ	-2.67E+00	-1.41E+00	-4.14E+00	-4.52E-01	2.48E+00	-1.03E+00	-2.10E+00	-2.83E+00	-0.4150
q	-4.21E-01	-1.09E+00	-8.86E-01	-1.95E+00	2.51E+00	-3.54E+00	-1.87E+00	-1.15E+00	-3.0007
Log-like	-13.3089	-12.6242	-9.026	-7.5165	-4.5124	-8.3897	-7.9085	-10.218	-3.2831

NB: This table contains the ML estimates for the Markov switching copulas. We report the ML estimates for the different static and time-variant bivariate copula models for the cocoa-sectoral stock return in Table 3A in the appendix. All coefficients are significantly different from zero. The loglikelihood value (value in bold) indicates the best copula fit.

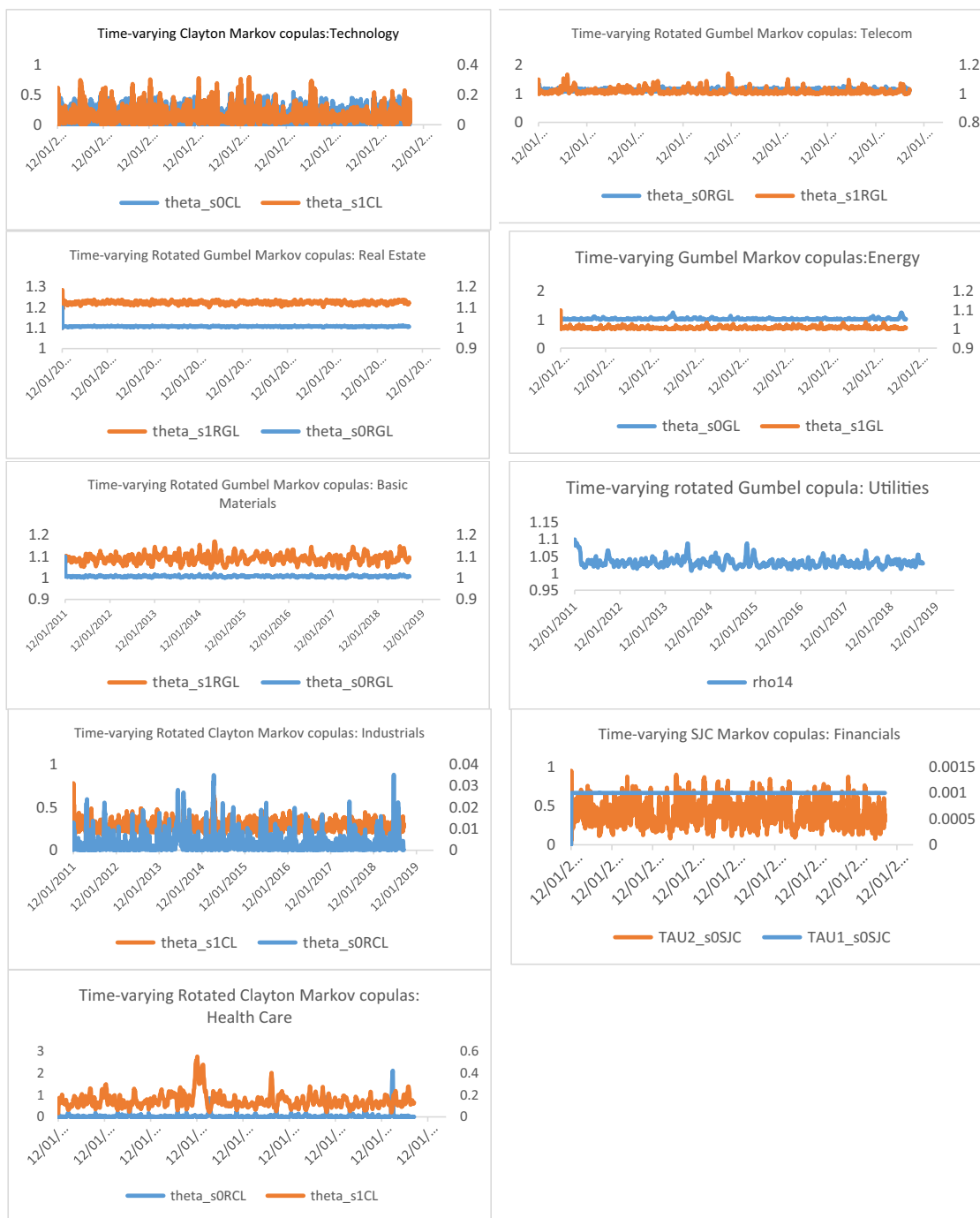


Fig. 5. Dependence Parameter Plot between Cocoa and sectoral stocks.

same direction with Australia's sectoral stocks. We however find a negative dependence structure between Brent crude oil and the financial sector, thus suggesting that investors may reap diversification benefits generated by a portfolio made of Brent crude oil and financial stocks in Australia. For the case of the real estate and energy sectors, we note the dependence structure between Brent crude and real estate is characterized by the TVP Clayton copula, while with the energy sector, the best fit model is the TVP SJC copula. We find that Brent crude co-moves in the same direction with the real estate stocks and the energy stocks, and as a result can't server as a good hedge for these sector stocks in Australia.

Overall, our findings reveal that Brent crude oil may potentially decrease investors' benefits for a portfolio composed of Brent crude oil

and technology, telecom, basic materials, utilities, industrials and health care stocks as oil co-crashes with these sectoral stocks. Interestingly the case of oil and financial stocks is different following their negative dependence structure. The finding is not surprising since the activities of financial stocks do not relate with oil in any way. Thus, they are unrelated assets.

The dependence parameter plots between Brent crude oil and the sectoral stocks are displayed in Fig. 6, with the results further showing the dependence dynamic is time varying for Brent crude and telecom, basic materials and utilities technology, industrials and health care, financials as revealed in Table 6. A critical observation further reveals that the dependence between Brent crude and sector stocks in Australia is time dependent. Thus, distortions in the price level of Brent crude can

Table 6
 Estimation of the dependence-switching copula model for Brent Crude and Sectoral Stocks.

	Technology	Telecom	Real Estate	Energy	Basic Materials	Utilities	Industrials	Financials	Health Care
Panel C: Parameter Estimates for time-varying Markov copula									
Time-varying Normal Markov copula									
ω_c^0N	0.1107	0.0012	0.0907	0.3349	0.3225	0.3799	0.2465	0.0311	0.1390
ω_c^1N	4.0115	17.0949	1.7049	2.7692	4.4426	4.6592	4.1399	0.7633	-3.6209
β_cN	1.0577	1.9902	-1.0934	-0.4014	-1.7114	-1.8983	-1.8475	1.4824	-1.8636
α_cN	0.1420	0.0140	0.2124	-0.2486	0.3735	0.2687	0.5004	0.0942	0.4329
ρ	0.6192	0.6237	0.6191	0.1671	0.6199	0.6302	0.6236	0.6218	0.6005
q	0.3808	0.3763	0.3809	0.8329	0.3801	0.3698	0.3764	0.3782	0.3995
Log-like	-22.2197	-20.3110	-2.0561	-22.5864	-12.8277	-12.9551	-10.1691	-9.7668	-4.7182
Time-varying Clayton Markov copulas									
ω_c^0C	1.0377	0.2383	-0.7786	0.2207	-0.2728	-0.5169	-0.5164	0.0463	-0.4892
ω_c^1C	0.2603	-0.8690	-1.1509	-0.5542	-0.6872	-1.2276	-1.0927	-0.5740	-1.4354
β_cC	-0.3374	0.8298	1.4284	-0.5429	1.5710	0.9758	1.0224	-0.6143	0.7364
α_cC	-1.9091	-0.0880	1.3217	0.6965	-0.6517	1.4794	1.2948	0.9548	1.5728
ρ	0.4637	0.4574	0.2062	0.4550	0.3479	0.5025	0.5037	0.4622	0.4391
q	0.5363	0.5426	0.7938	0.5450	0.6521	0.4975	0.4963	0.5378	0.5609
Log-like	-23.7448	-21.9241	-5.2141	-27.8633	-13.6314	-23.1066	-18.5368	-20.4409	-11.4192
Time-varying Rotated Clayton Markov copulas									
ω_c^0RC	-0.3039	-0.2436	-0.1972	0.4178	-0.8831	-0.2080	-0.5228	-0.6598	-0.4141
ω_c^1RC	-0.9068	-1.1444	-0.1974	-2.9716	-1.2512	-0.9638	-1.2035	-1.7178	-2.7291
β_cRC	1.3741	1.1753	-1.1397	0.2303	1.5431	1.2533	1.2938	0.7544	0.3703
α_cRC	-0.5665	-0.2338	0.5054	-0.4574	1.5504	-0.5828	1.2665	1.7939	0.9865
ρ	0.2709	0.1761	0.4866	1.1604	0.3790	0.0873	0.1727	0.2070	-0.1264
q	0.7291	0.8239	0.5134	-0.1604	0.6210	0.9127	0.8273	0.7930	1.1264
Log-like	-6.6377	-8.9388	-0.0187	-11.1643	-6.1614	0.8704	-1.0363	-0.0025	-3.1590
Time-varying Gumbel Markov copulas									
ω_c^0G	-0.4119	-0.3806	1.9117	0.0780	0.7507	-0.6402	0.3131	0.5017	-0.7539
ω_c^1G	-0.8374	-0.7857	1.9117	-0.4747	0.6806	-2.4661	0.4423	0.2609	-0.7539
β_cG	0.5641	0.5586	-2.1544	0.5593	-1.0169	0.4085	-0.8044	-0.7736	0.2767
α_cG	0.5759	0.6520	0.6764	-1.5738	0.4448	0.6802	0.9691	1.1122	1.1334
ρ	0.5314	0.6085	0.5000	0.4606	0.4599	1.6511	0.5542	0.4494	0.5000
q	0.4686	0.3915	0.5000	0.5394	0.5401	-0.6511	0.4458	0.5506	0.5000
Log-like	-15.3933	-13.6214	-0.2081	-17.4851	-7.1673	-8.6933	-4.4211	-5.6766	-2.3483
Time-varying Rotated Gumbel Markov copulas									
ω_c^0RG	-0.3601	-0.4679	-1.6200	-0.6597	-0.7168	-0.7030	-0.4244	-0.4347	-0.3613
ω_c^1RG	-0.4276	-0.5916	-1.6200	-0.6597	-1.0214	-1.1853	-0.5214	-0.8451	-0.9472
β_cRG	0.6542	0.7927	0.9182	0.9632	1.0298	1.0154	0.8019	0.8484	0.7625
α_cRG	-0.0738	-0.1346	1.6569	-0.3088	-0.3455	-0.3400	-0.4545	-0.6797	-1.2922
ρ	0.5344	0.5364	0.5000	0.5000	0.4908	0.5113	0.5447	0.5471	0.3999
q	0.4656	0.4636	0.5000	0.5000	0.5092	0.4887	0.4553	0.4529	0.6001
Log-like	-21.7331	-22.1712	-6.7264	-29.0552	-18.1715	-23.1772	-15.6708	-17.1684	-8.9469
Time-varying SJC Markov copulas									
ω_c^0U	-10.1747	-5.4602	-16.4081	-9.7490	-15.8296	-14.0273	-15.5040	-15.8207	-15.0167
ω_c^1U	-1.9233	0.5292	2.2561	-1.4510	2.4001	-1.9096	2.7880	2.3398	-2.2959
β_cU	-17.8883	-23.0461	-18.7230	-14.3514	-20.1986	-20.6785	-19.1898	-18.9980	-18.2200
α_cU	-19.2238	-23.2941	-21.5489	-20.1927	-22.0474	-21.9207	-21.0416	-20.9669	-23.0940
ρ	-5.9614	9.3005	-6.9945	-6.0562	-6.8707	-7.5837	-6.6069	-6.7587	-7.5613
q	-4.9444	-1.0793	-3.3220	4.6700	-6.2123	5.3015	-3.3537	-0.6596	-6.6188
Log-like	-23.0972	-21.4114	-1.3796	-30.6670	-17.4462	-21.5884	-14.4734	-15.4076	-5.9453
Time-varying T Markov copulas									
ω_c^0T	0.1071	-0.0027	0.0896	0.3539	1.0727	0.3633	0.2441	1.0523	0.2537
ω_c^1T	0.1071	0.0302	0.0896	0.7581	-0.4352	0.4095	0.2442	-0.6180	0.2585
β_cT	1.0840	1.9856	-1.1274	-2.0192	-1.8359	-2.0449	-1.8523	-2.0311	-2.0270
α_cT	0.1136	0.0152	0.2017	0.0489	0.4036	0.1921	0.4585	0.3092	0.4847
ρ	-3.32E-06	1.2865	-0.0012	-4.5041	-0.5138	-1.0612	-3.64E-06	-3.1515	-0.6043
q	-2.92E-06	1.8350	-0.0012	-4.5959	-0.4845	0.0385	3.11E-06	-2.2101	0.2213
Log-like	-23.3783	-20.3183	-1.6788	-24.1631	-15.0703	-16.3878	-10.2259	-10.6145	-7.5493

NB: This table contains the ML estimates for the Markov switching copulas. We report the ML estimates for the different static and time-variant bivariate copula models for the crude oil-sectoral stock return in Table 4A in the appendix. All coefficients are significantly different from zero. The loglikelihood value (value in bold) indicates the best copula fit.

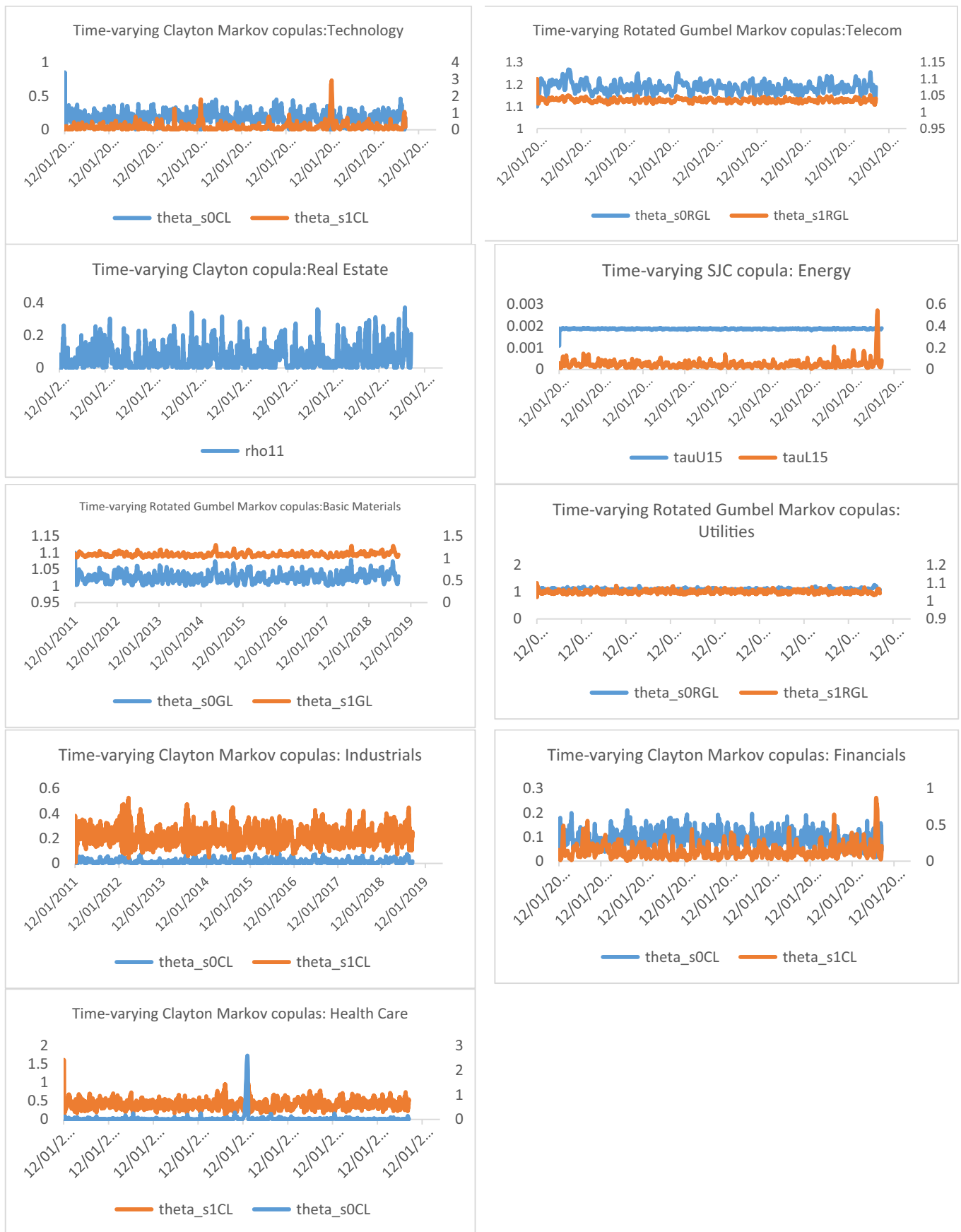


Fig. 6. Dependence Parameter Plot between Brent Crude and sectoral stocks.

Table 7
Bilateral Hedge ratios.

	Mean	Std. Dev.	5%	95%	HE	p-value
Telecom/Technology	0.35	0.10	0.21	0.51	0.23	0.00
RealEstate/Technology	0.18	0.07	0.10	0.29	0.12	0.00
Energy/Technology	0.29	0.13	0.15	0.54	0.14	0.00
BasicMaterials/Technology	0.29	0.10	0.16	0.49	0.12	0.00
Utilities/Technology	0.21	0.06	0.12	0.31	0.13	0.00
Industrials/Technology	0.24	0.07	0.15	0.36	0.23	0.00
Financials/Technology	0.22	0.08	0.13	0.38	0.18	0.00
HealthCare/Technology	0.25	0.07	0.15	0.35	0.15	0.00
NaturalGasSpot/Technology	0.02	0.05	-0.06	0.09	0.00	0.95
BrentCrudeSpot/Technology	0.18	0.11	0.06	0.40	0.02	0.61
CocoaIndexSpot/Technology	0.11	0.04	0.05	0.18	0.02	0.63
Technology/Telecom	0.59	0.15	0.40	0.84	0.24	0.00
RealEstate/Telecom	0.29	0.10	0.15	0.41	0.14	0.00
Energy/Telecom	0.31	0.13	0.16	0.56	0.10	0.02
BasicMaterials/Telecom	0.32	0.10	0.17	0.52	0.10	0.02
Utilities/Telecom	0.29	0.07	0.19	0.40	0.15	0.00
Industrials/Telecom	0.27	0.09	0.14	0.39	0.17	0.00
Financials/Telecom	0.31	0.10	0.16	0.46	0.18	0.00
HealthCare/Telecom	0.25	0.08	0.13	0.38	0.12	0.00
NaturalGasSpot/Telecom	-0.01	0.06	-0.12	0.07	0.00	0.99
BrentCrudeSpot/Telecom	0.19	0.11	0.07	0.41	0.02	0.67
CocoaIndexSpot/Telecom	0.11	0.06	0.04	0.23	0.01	0.85
Technology/RealEstate	0.58	0.16	0.38	0.86	0.22	0.00
Telecom/RealEstate	0.53	0.12	0.35	0.72	0.22	0.00
Energy/RealEstate	0.52	0.17	0.32	0.85	0.25	0.00
BasicMaterials/RealEstate	0.50	0.15	0.29	0.78	0.19	0.00
Utilities/RealEstate	0.56	0.11	0.41	0.75	0.36	0.00
Industrials/RealEstate	0.55	0.10	0.39	0.71	0.50	0.00
Financials/RealEstate	0.65	0.14	0.47	0.93	0.53	0.00
HealthCare/RealEstate	0.55	0.11	0.38	0.73	0.18	0.00
NaturalGasSpot/RealEstate	-0.08	0.09	-0.25	0.02	0.00	0.94
BrentCrudeSpot/RealEstate	0.12	0.08	0.04	0.24	-0.03	0.57
CocoaIndexSpot/RealEstate	0.03	0.06	-0.09	0.13	0.02	0.62
Technology/Energy	0.48	0.17	0.27	0.72	0.21	0.00
Telecom/Energy	0.31	0.09	0.19	0.46	0.15	0.00
RealEstate/Energy	0.27	0.08	0.16	0.40	0.24	0.00
BasicMaterials/Energy	0.70	0.15	0.48	0.97	0.47	0.00
Utilities/Energy	0.54	0.11	0.35	0.72	0.51	0.00
Industrials/Energy	0.39	0.10	0.23	0.57	0.42	0.00
Financials/Energy	0.38	0.09	0.24	0.54	0.39	0.00
HealthCare/Energy	0.36	0.10	0.20	0.54	0.22	0.00
NaturalGasSpot/Energy	-0.06	0.08	-0.19	0.05	-0.01	0.88
BrentCrudeSpot/Energy	0.27	0.11	0.15	0.45	0.03	0.50
CocoaIndexSpot/Energy	0.06	0.04	0.00	0.15	0.02	0.72
Technology/BasicMaterials	0.41	0.13	0.26	0.64	0.13	0.00
Telecom/BasicMaterials	0.26	0.05	0.18	0.35	0.12	0.01
RealEstate/BasicMaterials	0.22	0.08	0.13	0.34	0.14	0.00
Energy/BasicMaterials	0.58	0.14	0.40	0.80	0.44	0.00
Utilities/BasicMaterials	0.37	0.07	0.26	0.48	0.29	0.00
Industrials/BasicMaterials	0.32	0.09	0.19	0.46	0.30	0.00
Financials/BasicMaterials	0.32	0.09	0.21	0.47	0.29	0.00
HealthCare/BasicMaterials	0.24	0.06	0.15	0.34	0.12	0.00
NaturalGasSpot/BasicMaterials	-0.05	0.08	-0.15	0.05	0.00	0.98
BrentCrudeSpot/BasicMaterials	0.16	0.12	0.07	0.33	0.02	0.73
0.04	0.04	-0.01	0.11	0.01	0.87	

Table 7 (continued)

	Mean	Std. Dev.	5%	95%	HE	p-value
CocoaIndexSpot/BasicMaterials						
Technology/Utilities	0.52	0.13	0.34	0.77	0.15	0.00
Telecom/Utilities	0.42	0.08	0.30	0.55	0.17	0.00
RealEstate/Utilities	0.43	0.12	0.30	0.59	0.29	0.00
Energy/Utilities	0.79	0.23	0.50	1.26	0.46	0.00
BasicMaterials/Utilities	0.66	0.17	0.44	0.97	0.29	0.00
Industrials/Utilities	0.48	0.12	0.33	0.64	0.39	0.00
Financials/Utilities	0.48	0.14	0.32	0.69	0.36	0.00
HealthCare/Utilities	0.45	0.10	0.32	0.61	0.22	0.00
NaturalGasSpot/Utilities	0.00	0.07	-0.11	0.11	0.00	0.96
BrentCrudeSpot/Utilities	0.26	0.16	0.12	0.49	0.02	0.64
CocoaIndexSpot/Utilities	0.08	0.06	0.00	0.18	0.01	0.86
Technology/Industrials	0.95	0.27	0.60	1.36	0.33	0.00
Telecom/Industrials	0.61	0.14	0.42	0.85	0.25	0.00
RealEstate/Industrials	0.66	0.12	0.50	0.87	0.52	0.00
Energy/Industrials	0.90	0.21	0.63	1.33	0.46	0.00
BasicMaterials/Industrials	0.88	0.20	0.59	1.27	0.36	0.00
Utilities/Industrials	0.75	0.14	0.55	0.98	0.49	0.00
Financials/Industrials	0.77	0.14	0.60	1.01	0.60	0.00
HealthCare/Industrials	0.83	0.15	0.61	1.07	0.37	0.00
NaturalGasSpot/Industrials	-0.11	0.14	-0.30	0.05	0.00	0.97
BrentCrudeSpot/Industrials	0.24	0.12	0.09	0.47	0.01	0.78
CocoaIndexSpot/Industrials	0.04	0.06	-0.07	0.14	0.03	0.48
Technology/Financials	0.78	0.21	0.46	1.15	0.28	0.00
Telecom/Financials	0.61	0.14	0.41	0.84	0.25	0.00
RealEstate/Financials	0.70	0.12	0.50	0.92	0.53	0.00
Energy/Financials	0.78	0.20	0.54	1.16	0.42	0.00
BasicMaterials/Financials	0.79	0.19	0.51	1.15	0.42	0.00
Utilities/Financials	0.67	0.13	0.47	0.89	0.48	0.00
Industrials/Financials	0.68	0.11	0.49	0.85	0.61	0.00
HealthCare/Financials	0.69	0.14	0.47	0.89	0.32	0.00
NaturalGasSpot/Financials	-0.11	0.12	-0.30	0.03	0.00	0.99
BrentCrudeSpot/Financials	0.21	0.11	0.08	0.42	0.00	0.97
CocoaIndexSpot/Financials	0.09	0.06	-0.01	0.20	0.02	0.58
Technology/HealthCare	0.57	0.14	0.38	0.81	0.18	0.00
Telecom/HealthCare	0.33	0.09	0.20	0.49	0.17	0.00
RealEstate/HealthCare	0.39	0.11	0.26	0.58	0.19	0.00
Energy/HealthCare	0.49	0.17	0.29	0.83	0.22	0.00
BasicMaterials/HealthCare	0.39	0.11	0.23	0.60	0.17	0.00
Utilities/HealthCare	0.41	0.09	0.29	0.56	0.24	0.00
Industrials/HealthCare	0.48	0.11	0.34	0.67	0.38	0.00
Financials/HealthCare	0.45	0.12	0.30	0.70	0.32	0.00
NaturalGasSpot/HealthCare	-0.11	0.11	-0.33	0.00	0.00	0.98
BrentCrudeSpot/HealthCare	0.11	0.09	-0.01	0.28	0.02	0.64
CocoaIndexSpot/HealthCare	0.01	0.05	-0.08	0.09	0.01	0.87
Technology/NaturalGasSpot	0.01	0.02	-0.02	0.05	0.00	1.00
Telecom/NaturalGasSpot	0.00	0.01	-0.03	0.02	0.00	0.95
RealEstate/NaturalGasSpot	-0.01	0.01	-0.03	0.00	0.00	0.93
Energy/NaturalGasSpot	-0.01	0.02	-0.04	0.02	0.00	1.00
BasicMaterials/NaturalGasSpot	-0.01	0.02	-0.04	0.01	0.00	1.00
Utilities/NaturalGasSpot	0.00	0.01	-0.02	0.02	0.00	0.97
Industrials/NaturalGasSpot	-0.01	0.01	-0.03	0.01	0.00	0.99
-0.01	0.01	-0.03	0.00	0.00	0.99	

(continued on next page)

Table 7 (continued)

	Mean	Std. Dev.	5%	95%	HE	p-value
Financials/						
NaturalGasSpot						
HealthCare/						
NaturalGasSpot	-0.02	0.01	-0.04	0.00	0.00	0.99
BrentCrudeSpot/						
NaturalGasSpot	0.10	0.06	0.03	0.21	0.02	0.71
CocoaIndexSpot/						
NaturalGasSpot	0.04	0.02	0.02	0.08	0.00	0.97
Technology/						
BrentCrudeSpot	0.14	0.06	0.06	0.24	0.04	0.40
Telecom/						
BrentCrudeSpot	0.09	0.04	0.04	0.18	0.03	0.53
RealEstate/						
BrentCrudeSpot	0.03	0.02	0.01	0.06	0.00	0.98
Energy/BrentCrudeSpot	0.13	0.05	0.07	0.23	0.05	0.27
BasicMaterials/						
BrentCrudeSpot	0.09	0.04	0.04	0.18	0.03	0.54
Utilities/						
BrentCrudeSpot	0.08	0.03	0.04	0.13	0.02	0.68
Industrials/						
BrentCrudeSpot	0.05	0.03	0.02	0.11	0.02	0.70
Financials/						
BrentCrudeSpot	0.05	0.03	0.02	0.11	0.01	0.77
HealthCare/						
BrentCrudeSpot	0.04	0.03	0.00	0.08	0.02	0.66
NaturalGasSpot/						
BrentCrudeSpot	0.17	0.07	0.08	0.29	0.03	0.51
CocoaIndexSpot/						
BrentCrudeSpot	0.12	0.06	0.04	0.25	0.03	0.56
Technology/						
CocoaIndexSpot	0.12	0.07	0.05	0.23	0.01	0.83
Telecom/						
CocoaIndexSpot	0.08	0.05	0.02	0.16	0.01	0.85
RealEstate/						
CocoaIndexSpot	0.02	0.07	-0.02	0.05	0.00	0.93
Energy/CocoaIndexSpot	0.04	0.05	0.00	0.12	0.00	0.99
BasicMaterials/						
CocoaIndexSpot	0.04	0.05	-0.01	0.11	0.00	0.99
Utilities/						
CocoaIndexSpot	0.03	0.03	0.00	0.08	0.00	0.95
Industrials/						
CocoaIndexSpot	0.02	0.05	-0.01	0.05	0.00	0.97
Financials/						
CocoaIndexSpot	0.03	0.06	0.00	0.07	0.00	0.98
HealthCare/						
CocoaIndexSpot	0.01	0.04	-0.03	0.06	0.00	0.99
NaturalGasSpot/						
CocoaIndexSpot	0.11	0.05	0.04	0.21	0.00	0.98
BrentCrudeSpot/						
CocoaIndexSpot	0.16	0.11	0.07	0.37	0.02	0.60

Notes: Results are based on Kroner and Sultan (1993).

impact industry stock returns of Australia.

4.3. Portfolio and hedging strategies for portfolio diversification

4.3.1. Bilateral hedge ratios and portfolio weights

In our quest to shed further insights on the diversification potential between the international prices of crude oil, natural gas and cocoa and the Australian industry stock returns, we investigate the bilateral hedge ratios and the portfolio weights in this subsection. Table 7 outlines the summary statistics of the hedge ratios and the hedging effectiveness (HE) between the first asset to the second asset following Kroner and Sultan (1993). The table reports that a \$1 long position in the first asset can be hedged with the average value of hedge ratio percentage of a short position in the second assets.

For example, the mean estimates of the hedge ratio in the case of Brent crude/Real Estate which is \$0.12 reveal that a \$1 long position in Brent crude can be hedged by \$0.12 investment in real estate stocks in Australia. A critical look at Table 7 shows that the hedge ratios mean values between natural gas and Australia's sectoral stocks are negative

Table 8

Bilateral Portfolio weights.

	Mean	Std. Dev.	5%	95%	HE	p-value
Technology/Telecom	0.29	0.16	0.05	0.59	0.51	0.00
Technology/RealEstate	0.16	0.12	0.02	0.37	0.65	0.00
Technology/Energy	0.33	0.21	0.07	0.75	0.48	0.00
Technology/						
BasicMaterials	0.39	0.19	0.10	0.72	0.49	0.00
Technology/Utilities	0.22	0.12	0.04	0.44	0.64	0.00
Technology/Industrials	0.06	0.10	0.00	0.25	0.68	0.00
Technology/Financials	0.12	0.13	0.00	0.38	0.65	0.00
Technology/HealthCare	0.22	0.12	0.05	0.44	0.57	0.00
Technology/						
NaturalGasSpot	0.68	0.14	0.44	0.88	0.37	0.00
Technology/						
BrentCrudeSpot	0.55	0.21	0.20	0.88	0.37	0.00
Technology/						
CocoaIndexSpot	0.48	0.17	0.19	0.73	0.54	0.00
Telecom/Technology	0.71	0.16	0.41	0.95	0.12	0.01
Telecom/RealEstate	0.28	0.14	0.08	0.51	0.44	0.00
Telecom/Energy	0.50	0.19	0.22	0.83	0.28	0.00
Telecom/BasicMaterials	0.56	0.14	0.33	0.80	0.26	0.00
Telecom/Utilities	0.37	0.12	0.17	0.56	0.40	0.00
Telecom/Industrials	0.21	0.14	0.04	0.45	0.50	0.00
Telecom/Financials	0.24	0.15	0.04	0.51	0.44	0.00
Telecom/HealthCare	0.41	0.13	0.20	0.62	0.38	0.00
Telecom/NaturalGasSpot	0.78	0.11	0.57	0.91	0.24	0.00
Telecom/BrentCrudeSpot	0.67	0.17	0.39	0.92	0.21	0.00
Telecom/CocoaIndexSpot	0.61	0.14	0.36	0.81	0.38	0.00
RealEstate/Technology	0.84	0.12	0.63	0.98	0.20	0.00
RealEstate/Telecom	0.72	0.14	0.49	0.92	0.29	0.00
RealEstate/Energy	0.71	0.15	0.44	0.96	0.15	0.00
RealEstate/BasicMaterials	0.76	0.13	0.52	0.95	0.28	0.00
RealEstate/Utilities	0.62	0.17	0.34	0.86	0.38	0.00
RealEstate/Industrials	0.39	0.19	0.09	0.71	0.28	0.00
RealEstate/Financials	0.45	0.24	0.08	0.93	0.17	0.00
RealEstate/HealthCare	0.65	0.16	0.35	0.87	0.23	0.00
RealEstate/						
NaturalGasSpot	0.86	0.08	0.75	0.95	0.36	0.00
RealEstate/						
BrentCrudeSpot	0.79	0.11	0.58	0.94	0.23	0.00
RealEstate/						
CocoaIndexSpot	0.73	0.15	0.47	0.90	0.52	0.00
Energy/Technology	0.67	0.21	0.25	0.93	0.23	0.00
Energy/Telecom	0.50	0.19	0.17	0.78	0.40	0.00
Energy/RealEstate	0.29	0.15	0.04	0.56	0.45	0.00
Energy/BasicMaterials	0.61	0.24	0.20	0.98	0.14	0.00
Energy/Utilities	0.31	0.24	0.00	0.73	0.46	0.00
Energy/Industrials	0.14	0.14	0.00	0.43	0.50	0.00
Energy/Financials	0.20	0.16	0.00	0.49	0.41	0.00
Energy/HealthCare	0.40	0.20	0.06	0.72	0.38	0.00
Energy/NaturalGasSpot	0.78	0.12	0.57	0.92	0.29	0.00
Energy/BrentCrudeSpot	0.70	0.14	0.45	0.89	0.15	0.00
Energy/CocoaIndexSpot	0.61	0.19	0.23	0.85	0.52	0.00
BasicMaterials/						
Technology	0.61	0.19	0.28	0.90	0.27	0.00
BasicMaterials/Telecom	0.44	0.14	0.20	0.67	0.42	0.00
BasicMaterials/RealEstate	0.24	0.13	0.05	0.48	0.55	0.00
BasicMaterials/Energy	0.39	0.24	0.02	0.80	0.19	0.00
BasicMaterials/Utilities	0.26	0.16	0.02	0.52	0.48	0.00
BasicMaterials/Industrials	0.10	0.12	0.00	0.36	0.56	0.00
BasicMaterials/Financials	0.15	0.15	0.00	0.42	0.51	0.00
BasicMaterials/						
HealthCare	0.35	0.15	0.11	0.61	0.46	0.00
BasicMaterials/						
NaturalGasSpot	0.75	0.09	0.59	0.90	0.28	0.00
BasicMaterials/						
BrentCrudeSpot	0.64	0.16	0.34	0.87	0.25	0.00
BasicMaterials/						
CocoaIndexSpot	0.56	0.17	0.27	0.80	0.50	0.00
Utilities/Technology	0.78	0.12	0.56	0.96	0.08	0.06
Utilities/Telecom	0.63	0.12	0.44	0.83	0.15	0.00
Utilities/RealEstate	0.38	0.17	0.14	0.66	0.30	0.00
Utilities/Energy	0.69	0.24	0.27	1.00	0.06	0.16
Utilities/BasicMaterials	0.74	0.16	0.48	0.98	0.05	0.21
Utilities/Industrials	0.26	0.19	0.01	0.57	0.35	0.00

(continued on next page)

Table 8 (continued)

	Mean	Std. Dev.	5%	95%	HE	p-value
Utilities/Financials	0.33	0.20	0.06	0.69	0.29	0.00
Utilities/HealthCare	0.53	0.15	0.29	0.78	0.21	0.00
Utilities/NaturalGasSpot	0.84	0.08	0.67	0.94	0.18	0.00
Utilities/BrentCrudeSpot	0.76	0.13	0.53	0.95	0.13	0.00
Utilities/CocoaIndexSpot	0.69	0.14	0.43	0.87	0.34	0.00
Industrials/Technology	0.94	0.10	0.75	1.00	0.03	0.54
Industrials/Telecom	0.79	0.14	0.55	0.96	0.15	0.00
Industrials/RealEstate	0.61	0.19	0.29	0.91	0.05	0.29
Industrials/Energy	0.86	0.14	0.57	1.00	-0.02	0.62
Industrials/BasicMaterials	0.90	0.12	0.64	1.00	0.06	0.17
Industrials/Utilities	0.74	0.19	0.43	0.99	0.24	0.00
Industrials/Financials	0.58	0.23	0.22	1.00	0.05	0.29
Industrials/HealthCare	0.80	0.17	0.46	1.00	0.02	0.67
Industrials/ NaturalGasSpot	0.88	0.07	0.78	0.96	0.26	0.00
Industrials/ BrentCrudeSpot	0.83	0.11	0.59	0.97	0.10	0.01
Industrials/ CocoaIndexSpot	0.77	0.14	0.53	0.92	0.43	0.00
Financials/Technology	0.88	0.13	0.62	1.00	0.11	0.01
Financials/Telecom	0.76	0.15	0.49	0.96	0.22	0.00
Financials/RealEstate	0.55	0.24	0.07	0.92	0.09	0.05
Financials/Energy	0.80	0.16	0.51	1.00	-0.01	0.90
Financials/BasicMaterials	0.85	0.15	0.58	1.00	0.13	0.00
Financials/Utilities	0.67	0.20	0.31	0.94	0.30	0.00
Financials/Industrials	0.42	0.23	0.00	0.78	0.21	0.00
Financials/HealthCare	0.70	0.20	0.27	0.95	0.13	0.00
Financials/ NaturalGasSpot	0.87	0.09	0.74	0.96	0.32	0.00
Financials/ BrentCrudeSpot	0.81	0.11	0.58	0.95	0.15	0.00
Financials/ CocoaIndexSpot	0.75	0.16	0.48	0.92	0.48	0.00
HealthCare/Technology	0.78	0.12	0.56	0.95	0.13	0.00
HealthCare/Telecom	0.59	0.13	0.38	0.80	0.30	0.00
HealthCare/RealEstate	0.35	0.16	0.13	0.65	0.31	0.00
HealthCare/Energy	0.60	0.20	0.28	0.94	0.16	0.00
HealthCare/ BasicMaterials	0.65	0.15	0.39	0.89	0.23	0.00
HealthCare/Utilities	0.47	0.15	0.22	0.71	0.38	0.00
HealthCare/Industrials	0.20	0.17	0.00	0.54	0.34	0.00
HealthCare/Financials	0.30	0.20	0.05	0.73	0.30	0.00
HealthCare/ NaturalGasSpot	0.82	0.09	0.67	0.92	0.29	0.00
HealthCare/ BrentCrudeSpot	0.72	0.14	0.47	0.93	0.18	0.00
HealthCare/ CocoaIndexSpot	0.67	0.13	0.44	0.84	0.47	0.00
NaturalGasSpot/ Technology	0.32	0.14	0.12	0.56	0.72	0.00
NaturalGasSpot/Telecom	0.22	0.11	0.09	0.43	0.82	0.00
NaturalGasSpot/ RealEstate	0.14	0.08	0.05	0.25	0.88	0.00
NaturalGasSpot/Energy	0.22	0.12	0.08	0.43	0.79	0.00
NaturalGasSpot/ BasicMaterials	0.25	0.09	0.10	0.41	0.78	0.00
NaturalGasSpot/Utilities	0.16	0.08	0.06	0.33	0.86	0.00
NaturalGasSpot/ Industrials	0.12	0.07	0.04	0.22	0.89	0.00
NaturalGasSpot/ Financials	0.13	0.09	0.04	0.26	0.88	0.00
NaturalGasSpot/ HealthCare	0.18	0.09	0.08	0.33	0.85	0.00
NaturalGasSpot/ BrentCrudeSpot	0.35	0.18	0.11	0.68	0.60	0.00
NaturalGasSpot/ CocoaIndexSpot	0.29	0.14	0.11	0.60	0.73	0.00
BrentCrudeSpot/ Technology	0.45	0.21	0.12	0.80	0.62	0.00
BrentCrudeSpot/Telecom	0.33	0.17	0.08	0.61	0.74	0.00
BrentCrudeSpot/ RealEstate	0.21	0.11	0.06	0.42	0.80	0.00
BrentCrudeSpot/Energy	0.30	0.14	0.11	0.55	0.65	0.00
BrentCrudeSpot/ BasicMaterials	0.36	0.16	0.13	0.66	0.68	0.00

Table 8 (continued)

	Mean	Std. Dev.	5%	95%	HE	p-value
BrentCrudeSpot/Utilities	0.24	0.13	0.05	0.47	0.80	0.00
BrentCrudeSpot/ Industrials	0.17	0.11	0.03	0.41	0.82	0.00
BrentCrudeSpot/ Financials	0.19	0.11	0.05	0.42	0.79	0.00
BrentCrudeSpot/ HealthCare	0.28	0.14	0.07	0.53	0.75	0.00
BrentCrudeSpot/ NaturalGasSpot	0.65	0.18	0.32	0.89	0.44	0.00
BrentCrudeSpot/ CocoaIndexSpot	0.45	0.21	0.09	0.74	0.69	0.00
CocoaIndexSpot/ Technology	0.52	0.17	0.27	0.81	0.47	0.00
CocoaIndexSpot/Telecom	0.39	0.14	0.19	0.64	0.61	0.00
CocoaIndexSpot/ RealEstate	0.27	0.15	0.10	0.53	0.76	0.00
CocoaIndexSpot/Energy	0.39	0.19	0.15	0.77	0.63	0.00
CocoaIndexSpot/ BasicMaterials	0.44	0.17	0.20	0.73	0.59	0.00
CocoaIndexSpot/Utilities	0.31	0.14	0.13	0.57	0.71	0.00
CocoaIndexSpot/ Industrials	0.23	0.14	0.08	0.47	0.79	0.00
CocoaIndexSpot/ Financials	0.25	0.16	0.08	0.52	0.76	0.00
CocoaIndexSpot/ HealthCare	0.33	0.13	0.16	0.56	0.70	0.00
CocoaIndexSpot/ NaturalGasSpot	0.71	0.14	0.40	0.89	0.30	0.00
CocoaIndexSpot/ BrentCrudeSpot	0.55	0.21	0.26	0.91	0.42	0.00

Notes: Results are based on Kroner and Ng (1998).

and vice versa in most cases. For instance, the mean value of the relation between Natural Gas/financials is -0.11, while financials/natural gas is -0.01; Natural gas/energy is -0.06, while energy/natural gas is -0.01. The negative mean values arise when the asset pairs are negatively correlated. This connotes that natural gas can act as a good hedge against Australian industry stocks, similar to what we documented from the copula results.

Next, we examine the hedging effectiveness (HE) of the asset pairs. We observe from Table 8 that, except for Brent crude/ real estate (HE = -0.03) and Natural gas/energy (HE = -0.01), investment in all asset pairs will reduce assets' volatility as evidenced by their positive hedging effectiveness values. Analysing the significance of the HE values indicates that these reductions in volatility are statistically significant at the 1% level of significance, thus suggesting that the reductions in volatility for investing the asset pairs with a positive HE are financially meaningful. The bilateral hedge ratios are insignificant in few cases including the natural gas/industrials; cocoa/health care; crude oil/basic materials pairs etc.

In Table 8, we follow Kroner and Ng (1998) and present the summary statistics of the bilateral portfolio weights and hedging effectiveness across the asset pairs to deepen our understanding of the investment implications of our results. It can be observed from Table 8 that with the exception of the bilateral portfolio weights between utilities/energy; utilities/real estate; industrials/technology; industrials/real estate; industrials/energy; industrials/basic material; industrials/financial; industrial/health care; and financials/energy pairs, investment in the portfolio weights of all paired assets reduces volatility, followed the HE index level of significance. It is noteworthy to mention that the international commodity prices including crude oil, natural gas and cocoa can significantly reduce investment risk when paired with sectoral stocks in Australia.

4.3.2. Multivariate portfolio

In this section, we explore the implications of our findings for portfolio and risk management purposes. we in particular compare and

Table 9
Dynamic Multivariate Portfolio Weights: Results are based on the time-varying variance-covariance retrieved from the TVP-VAR (0.99,0.99) with one lag.

	Mean	Std.Dev.	5%	95%	HE	p-value
Panel A: Minimum Variance Portfolio (MVP)						
Technology	0	0.01	0	0.03	0.84	0.00
Telecom	0.06	0.04	0	0.13	0.72	0.00
Real Estate	0.15	0.11	0	0.33	0.64	0.00
Energy	0.03	0.05	0	0.15	0.77	0.00
Basic Materials	0.03	0.04	0	0.13	0.78	0.00
Utilities	0.06	0.07	0	0.2	0.6	0.00
Industrials	0.23	0.16	0	0.51	0.52	0.00
Financials	0.12	0.12	0	0.36	0.6	0.00
HealthCare	0.07	0.06	0	0.18	0.68	0.00
Natural Gas Spot	0.05	0.02	0.02	0.09	0.93	0.00
Brent Crude Spot	0.07	0.06	0	0.18	0.9	0.00
Cocoa Index Spot	0.12	0.08	0.03	0.26	0.82	0.00
Panel B: Minimum Correlation Portfolio (MCP)						
Technology	0.05	0.02	0.01	0.08	0.73	0.00
Telecom	0.08	0.02	0.05	0.11	0.52	0.00
Real Estate	0.11	0.01	0.08	0.13	0.4	0.00
Energy	0.03	0.01	0	0.05	0.61	0.00
Basic Materials	0.1	0.02	0.08	0.13	0.63	0.00
Utilities	0.02	0.01	0	0.04	0.32	0.00
Industrials	0	0	0	0	0.2	0.98
Financials	0	0	0	0	0.33	0.09
HealthCare	0.12	0.01	0.1	0.15	0.46	0.00
Natural Gas Spot	0.2	0.01	0.18	0.21	0.88	0.00
Brent Crude Spot	0.13	0.01	0.11	0.15	0.84	0.00
Cocoa Index Spot	0.17	0.01	0.15	0.19	0.70	0.00
Panel C: Minimum Connectedness Portfolio (MCoP)						
Technology	0.09	0.02	0.03	0.12	0.75	0.00
Telecom	0.09	0.03	0.05	0.13	0.55	0.00
Real Estate	0.1	0.03	0.05	0.14	0.43	0.00
Energy	0.01	0.02	0	0.06	0.63	0.00
Basic Materials	0.1	0.01	0.08	0.13	0.65	0.00
Utilities	0.07	0.02	0.02	0.09	0.36	0.00
Industrials	0	0	0	0.01	0.25	0.04
Financials	0.01	0.02	0	0.05	0.37	0.00
HealthCare	0.11	0.02	0.06	0.14	0.5	0.00
Natural Gas Spot	0.16	0.02	0.13	0.19	0.89	0.00
Brent Crude Spot	0.13	0.02	0.11	0.16	0.85	0.00
Cocoa Index Spot	0.15	0.02	0.13	0.18	0.72	0.00

Notes: Results of MVP, MCP and MCoP are based on [Markovitz \(1959\)](#), [Christoffersen et al. \(2014\)](#) and [Broadstock et al. \(2020\)](#), respectively.

contrast the conventional portfolio analysis approaches including the minimum variance portfolio (MVP) and the minimum correlation portfolio (MCP) as well as recent approaches such as the minimum connectedness portfolio (MCoP) by exploring the hedging effectiveness reported in [Table 9](#) for MVP, MCP and MCoP in Panels A, B and C, respectively. The results of MVP, MCP and MCoP are based on [Markovitz \(1959\)](#), [Christoffersen et al. \(2014\)](#) and [Broadstock et al. \(2020\)](#), respectively. It is worth reminding ourselves with regards to the competing portfolio construction techniques. The MVP procedure aims to minimize the portfolio volatility by definition. On the other hand, The MCP techniques center on reducing correlations across the assets. Finally, the MCoP procedure minimizes pairwise connectedness or the bilateral return spillovers across assets.

Before we discuss the hedging effectiveness ratios, we first briefly examine the average portfolio allocations for each asset across the competing portfolios. From the average weights reported in [Table 9](#), we observe that the international commodity prices of crude oil, natural gas and cocoa contribute significantly to a portfolio containing the Australian industry stocks. We make this conclusion because the portfolio weights for the commodity prices account for about 50% of the allocated portfolio in MCP and 44% of allocated portfolio in MCoP and 24% in MVP. Focusing on the specifics, it can be seen that natural gas contributes significantly in a portfolio of stock comprising of Australian sectors

stock since the portfolio weights of natural gas account for about 20% of the portfolio under MCP and 16% under MCoP. In the case of MVP, cocoa is seen to be the asset with the largest portfolio weight (12%) among the commodities. In the case of sectoral stocks, we document several differences. In the case of MVP, industrials emerged as the stock with largest portfolio weight (23%). For MCP and MCoP, health care attained the largest portfolio weight of 12% and 11%, respectively. We thus confirm the trivial contribution of sectoral stocks since the industry stocks record the least portfolio weights.

Focusing on the hedging effectiveness ratios in [Table 9](#), we document several interesting findings. From Panel A which document the MVP procedure, we find that if we invested 5% in natural gas, 7% in Brent crude, 12% in cocoa along with the sector stocks, then these asset volatility in the portfolio would be reduced by 93% for natural gas, 90% for Brent crude and 82% for cocoa. On the other hand, investment in Australian sector stocks would reduce asset volatility of more than 50% in all cases. It is worth mentioning that these reductions in volatility are statistically significant at the 1% level, confirming the reductions are meaningful from a financial perspective. Next, we focus on the portfolios from the MCP procedure. Findings suggest that investing 20% in natural gas, 13% in Brent crude oil and 17% in cocoa would reduce volatility of each asset by 88%, 84% and 70%, respectively. The same is seen for investment in the sector stocks. Interestingly, all these reductions in each asset's volatility are seen to be significant at the 1% level. Finally, from the MCoP procedure, we observe that investment in commodities and the sector stocks reduces assets' volatility as evidenced by the significance of the hedging portfolios. For instance, investing 16% in natural gas, 13% in crude oil, 15% in cocoa, 11% in health care reduces asset volatility as evidenced by the positive HE value of 89%, 85%, 72% and 50%, respectively. Comparing the weights under the three alternative portfolios show that the portfolio weights vary across MVP, MCP and MCoP. In all, we provide evidence through the portfolio analysis demonstrating the existence of some level of dynamic dependence, which permits for diversification benefits.⁴

Additionally, we next estimate the cumulative portfolio returns for both the static portfolio strategies as illustrated in [Fig. 7](#). We find that the cumulative portfolio returns from the minimum variance portfolio outperform the returns using the minimum correlation portfolio strategy throughout the period and follow a similar pattern, and with a dip in 2016 and 2020 a period that witnessed the Brexit vote and the outbreak of COVID-19. Lastly, we examine the asset performance from the efficient frontier considering two scenarios with result reported in the appendix.

5. Conclusion

In this paper, we have examined the dependence structure of the returns of oil, natural gas, cocoa and nine sectoral stocks in Australia, using time invariant, traditional time-varying and time-varying Markov copulas. Using daily prices spanning from 1st December 2011 to 23rd April 2020, our results show that the dependence between those three global commodity prices and the nine sectoral stock returns is Markov switching, time varying and can be symmetric or asymmetric, depending on the sector considered. We note that with exception of the financial sector, the nature of the dependence between oil and the other sectors is positive, thus suggesting the prices of assets co-move in the same direction. For the case of natural gas, the finding show that this energy commodity is a safe haven for all sectors except the real estate sector. Concerning cocoa, we find this commodity to be a safe haven for the telecom, energy, basic materials, financials and health care sectors. Next, we investigate the robustness of our results from the portfolio perspective to ascertain how investors could benefit in their portfolio

⁴ We also report the results for the risk parity portfolio in [Table 5A](#) in the appendix.

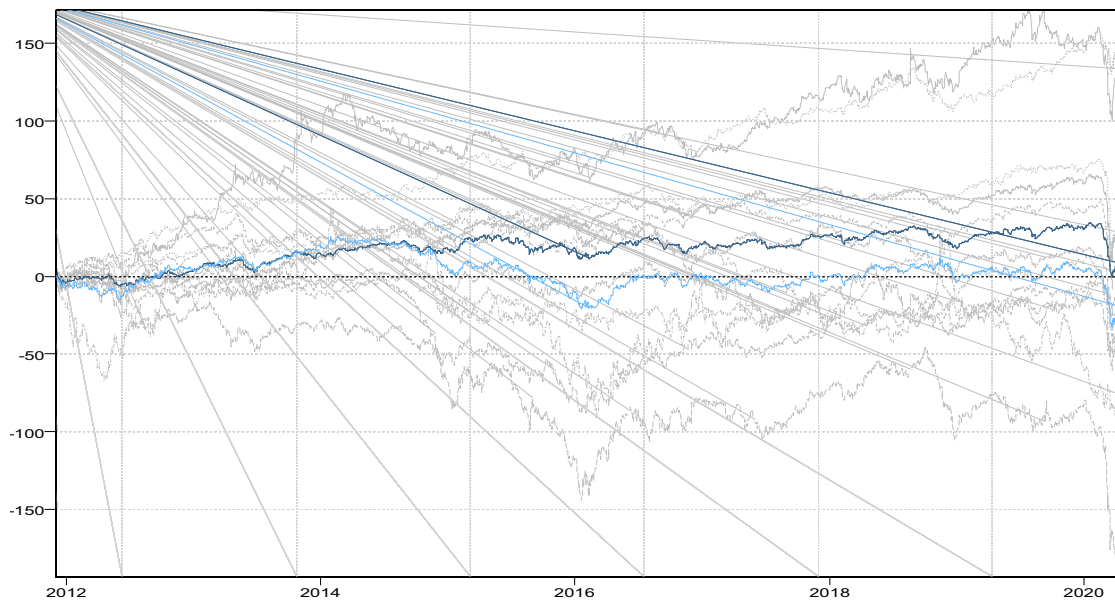


Fig. 7. Plot for cumulative portfolio returns. The coloured lines are for minimum variance portfolio (dark blue) and minimum correlation portfolio (sea blue). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

formulation strategies using competing portfolio approaches.

The findings have important implications. From a practical perspective, the time-varying dependence structure results documented in this paper can help market participants with different investment targets and horizons adopt better hedging strategies and portfolio diversification to aid optimal policy measures. For short term investors and portfolio managers, constructing a well-diversified portfolio comprising of commodities used in this study is a complicated task, especially in times of financial crises. For the case of Australia, in particular, we show that crude oil is a safe haven for the financial sector, natural gas is a safe haven for all sectors except real estate, and cocoa co-crashes with technology, industrials and real estate sectoral returns. Thus, combining large proportions of crude oil and related equities in a portfolio is inadvisable.

From an academic standpoint, an analysis based on time invariant or simple regime-switching models will not capture well the best features of the dependence structure dynamics between global prices of oil, natural gas, cocoa and Australian sectoral stocks returns, compared to the Markov switching copulas. In addition, the assumption that market participants and economic agents are homogeneous is not empirically documented. Hence, it is essential that any analysis of the relationship between commodities (oil, natural gas and cocoa) and the aggregate sectoral stock indices take into account the premise that economic agents are homogeneous.

From the policy perspective, policy makers' understanding and knowledge on whether a strong dependence structure exists between the global prices of oil, natural gas, cocoa and equity sector indices will help

guide decisions about whether specific policies are needed to protect investors from the short-term and long-term impacts of fluctuations in the oil, natural gas and cocoa prices on stock markets in Australia. The key implication of our study is that portfolio managers and investors should spend a greater proportion of their investments on the health care and technology sectors if they seek to create a portfolio that includes commodities such as Brent crude and natural gas.

Author contribution statement

Aviral Kumar Tiwari proposed the methodological approach and conducted the estimation of empirical results. Emmanuel Joel Aikins Abakah proposed the idea/concept and contributed to the literature review, the empirical analysis, the results interpretations, the conclusion, and the abstract. Nana Kwasi Karikari completed the introduction, the literature, and methodology sections. Shawkat Hammoudeh contributed to the introduction, the literature review, the results interpretations, and the conclusion.

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Appendix A. Appendix

Table 1A
Characteristics of bivariate copulas.

Copula Name	Formula	Parameter	Tail Dependence
Gaussian Copula	$\Phi_2 \{ \Phi^{-1}(u_1), \Phi^{-1}(u_2); \theta \}$	$\theta = \rho$	$\tau(\theta) = \frac{2}{\pi \arcsin(\theta)}$
Clayton Copula	$[\max\{u_1^{-\theta} + u_2^{-\theta} - 1; 0\}]^{-1/\theta}$	$\theta \in [(-1, \infty) \setminus \{0\}]$	$\tau(\theta) = \frac{\theta}{\theta + 2}$
Rotated Clayton Copula	$u - [\max\{u_1^{-\theta} + u_2^{-\theta} - 1; 0\}]^{-1/\theta} (u, 1 - v; -\theta)$	$\theta < 0$	Asymmetric tail dependence
Plackett Copula	$\frac{1}{2(\theta-1)} (1 + (\theta-1)(u+v) - \sqrt{(1+(\theta-1)(u+v))^2 - 4\theta(\theta-1)uv})$	θ	Zero tail dependence
Frank Copula	$-\frac{1}{\theta} \log \left[1 + \frac{(\exp(-\theta u_1) - 1)(\exp(-\theta u_2) - 1)}{\exp(-\theta) - 1} \right]$	$\theta \in [-1, 1]$	$\tau(\theta) = 1 - \frac{4}{\theta} \left(1 - \frac{1}{\theta} \int_0^\theta \frac{a}{e^a - 1} da \right)$
Gumbel Copula	$\exp \left[- \left((-\log(u_1))^\theta + (-\log(u_2))^\theta \right) \right]^{1/\theta}$	$\theta \in [1, \infty]$	$\tau(\theta) = 1 - \theta^{-1}$
Rotated Gumbel Copula	$u + v - 1 + \left(\exp \left[- \left((-\log(u_1))^\theta + (-\log(u_2))^\theta \right) \right]^{1/\theta} \right) (1 - u, 1 - v; \theta)$		Upper tail dependence and lower tail dependence.
Student-t Copula	$T(t_{\nu}^{-1}(u_1), \dots, t_{\nu}^{-1}(u_d))$	$\theta = \rho$	Symmetric tail dependence : $\lambda_U = \lambda_L = 2t_{\nu+1}(-\sqrt{\nu+1} \sqrt{1-\rho} / \sqrt{1+\rho}) > 0$
Survival Joe Copula	$1 - [(1-u)^\theta + (1-v)^\theta - (1-u)^\theta(1-v)^\theta]$	$\theta \in [1, \infty]$	$\tau(\theta) = 1 + \frac{4}{\theta^2} \int_0^1 x \log(x) (1-x)^{2(1-\theta)/\theta} dx$
Joe Copula	$1 - [(1-u)^\theta + (1-v)^\theta - (1-u)^\theta(1-v)^\theta]$	$\theta \in [1, \infty]$	$\tau(\theta) = 1 + \frac{4}{\theta^2} \int_0^1 x \log(x) (1-x)^{2(1-\theta)/\theta} dx$
Survival Clayton Copula	$[\max\{u_1^{-\theta} + u_2^{-\theta} - 1; 0\}]^{-1/\theta}$	$\theta \in [(-1, \infty) \setminus \{0\}]$	$\tau(\theta) = \frac{\theta}{\theta + 2}$
Survival Gumbel Copula	$\exp \left[- \left((-\log(u_1))^\theta + (-\log(u_2))^\theta \right) \right]^{1/\theta}$	$\theta \in [1, \infty]$	$\tau(\theta) = 1 - \theta^{-1}$
Tawn Copula	$\exp(\log u_1 u_2) A \left(\frac{\log u_2}{\log(u_1, u_2)} \right)$	$A \in [0, 1]$	$\tau(A) = \int_0^1 \frac{t(1-t)}{A(t)} dA'(t)$

Notes: $\tau(\bullet)$ denotes tail dependence. For Gaussian copula, $\Phi^{-1}(\bullet)$ is the standard normal quantile function and Φ is the bivariate standard normal cumulative distribution function with correlation ρ . For the Student-t copula, $t_{\nu}^{-1}(\bullet)$ is the quantile function for the univariate Student-t distribution with (\bullet) is the degree-of-freedom parameter and ρ as the correlation.

Table 2A
Estimation of the dependence-switching copula model for Natural Gas and Sectoral Stocks.

	Technology	Telecom	Real Estate	Energy	Basic Materials	Utilities	Industrials	Financials	Health Care
Panel A: Parameter Estimates for time-invariant copulas									
Normal Copula									
ρ	0.0169	0.0006	-0.0244	-0.0139	-0.0137	0.0061	-0.0236	-0.0211	-0.0408
Log-like	-0.2887	-0.0004	-0.6000	-0.1953	-0.1895	-0.0371	-0.5613	-0.4461	-1.6792
Clayton's copula									
α	0.0392	0.0211	1.00E-04	0.0001	0.0001	0.0258	0.0002	1.00E-04	1.00E-04
Log-like	-1.397	-0.4157	0.0029	0.0004	0.0044	-0.6127	0.0000	0.0031	0.0014
Rotated Clayton copula									
α	1.00E-04	1.00E-04	1.00E-04	1.00E-04	0.0001	0.0001	0.0001	1.00E-04	1.00E-04
Log-like	0.0048	0.0060	0.0057	0.003	0.0004	0.0026	0.0063	0.0036	0.0124
Plackett copula									
δ	1.0511	1.0195	0.9573	0.9779	1.0042	1.0311	0.9522	0.9617	0.9230
Log-like	-0.2867	-0.0427	-0.2146	-0.0557	-0.0019	-0.1034	-0.2619	-0.1679	-0.7081
Frank Copula									
δ	0.1015	0.0392	1.00E-04	0.0001	0.0083	0.0607	0.0001	1.00E-04	1.00E-04
Log-like	-0.2917	-0.0432	0.0005	0.0002	-0.0019	-0.1025	0.0005	0.0004	0.0009
Gumble Copula									
δ	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1
Log-like	27.6878	30.4772	32.9793	28.8513	23.3911	24.8276	33.3272	28.8435	39.1070
Rotated Gumble Copula									
δ	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1
Log-like	18.7018	19.3576	29.8523	26.3392	29.7082	18.3757	28.5900	31.1109	31.4344
Student's t copula									
ρ	0.0175	0.0001	-0.0243	-0.0133	-0.0123	0.0067	-0.0223	-0.0203	-0.0402
ν	100.000	100.00	100	40.6558	61.2199	68.5241	28.9514	61.1592	93.8305
Log-like	0.4276	0.2649	-0.5447	-0.7784	-0.4096	-0.2153	-1.6424	-0.6603	-1.7749
SJC Copula									
λ_U	1.53E-05	1.53E-05	1.53E-05	1.53E-05	1.53E-05	1.53E-05	1.53E-05	1.53E-05	1.53E-05
λ_L	1.53E-05	1.53E-05	1.53E-05	1.53E-05	1.53E-05	1.53E-05	1.53E-05	1.53E-05	1.53E-05
Log-like	1.6806	2.9427	4.9658	3.4258	3.771	1.7253	4.4076	4.4938	6.5426

(continued on next page)

Table 2A (continued)

	Technology	Telecom	Real Estate	Energy	Basic Materials	Utilities	Industrials	Financials	Health Care
Panel B: Parameter Estimates of time-varying copulas									
TVP Normal copula									
ψ_0	0.0082	-0.0050	-0.0253	-0.0101	-0.04939	0.0035	-0.0923	-0.0726	-0.1893
ψ_1	-0.0116	-0.4223	-0.1413	-0.0361	-0.11752	-0.0357	-0.2504	-0.1421	-0.3517
ψ_2	1.5265	-1.4676	1.1550	1.2499	-1.43624	1.5306	-1.2756	-1.2710	-1.9122
Log-like	-0.3274	-1.7420	-2.8086	-0.3914	-0.3546	-0.3756	-1.3791	-0.7121	-3.0975
TVP Clayton copula									
ψ_0	-0.0551	-0.0250	0.2082	0.1736	-0.00165	0.6667	-0.0736	0.0004	-0.0001
ψ_1	-0.2785	-2.0386	-1.5717	-1.1213	-0.11162	-0.1297	-0.6415	-0.5349	0.5428
ψ_2	-0.3953	-0.1987	-0.7296	-0.7006	0.00396	-1.9837	0.0911	0.0003	0.0003
Log-like	-1.6164	-0.6712	-0.4015	-0.4357	0.0044	-0.9546	-0.0037	0.0031	0.0014
TVP Rotated Clayton copula									
ψ_0	0.0027	0.2509	0.2873	-0.0001	0.22059	0.3748	0.3468	0.2490	0.0002
ψ_1	0.3106	-1.3054	-1.4985	-0.2474	2.25077	-0.3926	-1.2057	-1.0505	-0.0659
ψ_2	-0.0066	-0.8238	-0.9094	0.0006	-1.00674	-1.1901	-1.1062	-0.9226	-0.0007
Log-like	0.0048	-0.1077	-0.5548	0.003	-0.2442	-0.1407	-0.8250	-0.4166	0.0124
TVP Gumble copula									
Ω	0.6308	0.1271	0.6232	0.5326	-2.68250	0.5647	1.9715	-0.2818	0.6451
β	-0.6324	-0.0467	-0.6576	-0.4010	2.86320	-0.1677	-1.7694	0.5155	-0.6461
α	0.0039	-0.2858	0.1321	-0.4761	-0.85036	-1.2114	-0.6325	-0.8718	0.0038
Log-like	0.0019	0.1460	0.1424	0.0441	-0.4702	-0.3639	-2.2304	-0.2823	0.0305
TVP Rotated Gumble copula									
ω_U	-0.9815	1.7814	1.8072	-2.0180	-0.00710	-3.6856	-0.0117	-0.0038	-0.0040
α_U	0.9826	-1.7387	-1.6883	2.1438	0.00762	3.7495	0.0119	0.0039	0.0040
β_U	-0.3138	-0.3507	-0.4681	-0.5179	-0.00128	-0.8647	-0.0008	-0.0001	-0.0001
Log-like	-0.2723	-1.7973	-0.4226	0.039	0.2182	-3.299	0.1965	0.2027	0.108
TVP SJC copula									
ω_U	-24.6412	-23.3079	-18.8663	-18.9432	-17.39565	-22.3314	-20.3960	-18.1276	-21.1748
α_U	-4.8312	-3.8870	-2.4507	-2.5996	-1.99210	-3.6363	-2.8950	-2.1387	-3.2401
β_U	-0.0137	-0.0124	-0.0079	-0.0080	-0.00639	-0.0114	-0.0094	-0.0071	-0.0102
ω_L	-16.8061	-17.0321	-17.7874	-17.7022	-19.3371	-17.3080	-17.8284	-18.3630	-17.6163
α_L	-1.6955	-1.9780	-2.0884	-2.0457	-2.9813	-2.0271	-2.3609	-2.4537	-2.1679
β_L	-0.0057	-0.0060	-0.0068	-0.0067	-0.0083	-0.0062	-0.0067	-0.0074	-0.0066
Log-like	4.9333	6.7834	10.0357	7.6603	8.0789	4.9179	9.274	9.3431	12.5534
TVP Student t copula									
ψ_0	0.0127	0.0045	-0.0130	-0.0156	0.0048	0.0236	-0.0813	-0.0286	-0.1281
ψ_1	0.0110	-0.0478	-0.0657	-0.0178	-0.0817	0.0080	-0.1140	-0.0022	-0.1736
ψ_2	1.2136	1.4401	1.3858	-0.0190	-1.5443	-0.0225	-1.5873	-0.0223	-1.9272
ν	5.0000	0.0045	5.0000	5.0000	5.000	5.0000	5.0000	5.0000	5.000
Log-like	37.9849	29.6808	24.775	21.7870	21.722	22.871	17.2141	21.5787	21.098

NB:

Table 3A
Estimation of the dependence-switching copula model for Cocoa and Sectoral Stocks.

	Technology	Telecom	Real Estate	Energy	Basic Materials	Utilities	Industrials	Financials	Health Care
Panel A: Parameter Estimates for time-invariant copulas									
Normal Copula									
ρ	0.1081	0.0918	0.0169	0.0495	0.0412	0.0485	0.0288	0.0532	0.014
Log-like	-11.8218	-8.507	-0.2887	-2.4628	-1.7072	-2.3698	-0.8341	-2.8453	-0.2038
Clayton's copula									
α	1.26E-01	1.12E-01	5.42E-02	5.72E-02	5.80E-02	6.12E-02	6.26E-02	8.30E-02	0.034
Log-like	-12.5824	-10.1058	-3.052	-3.0017	-3.1325	-3.4526	-3.7154	-6.0663	-1.1399
Rotated Clayton copula									
α	8.79E-02	7.49E-02	1.00E-04	4.38E-02	2.35E-02	4.13E-02	1.00E-04	2.51E-02	0.003
Log-like	-6.0369	-4.6401	0.0008	-1.6841	-0.4818	-1.5418	0.001	-0.5265	-0.009
Plackett copula									
δ	1.4073	1.3292	1.0338	1.1461	1.1236	1.1460	1.1057	1.2094	1.048
Log-like	-12.6701	-8.6028	-0.116	-1.9839	-1.4545	-2.0047	-1.0852	-3.7288	-0.2407
Frank Copula									
δ	6.75E-01	5.54E-01	6.38E-02	2.66E-01	2.28E-01	2.67E-01	1.96E-01	3.63E-01	0.093
Log-like	-12.4497	-8.3405	-0.1117	-1.9309	-1.4187	-1.9652	-1.0617	-3.5564	-0.2388
Gumble Copula									
δ	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
Log-like	-2.4711	0.7811	21.293	10.0942	16.0804	10.2483	21.0158	12.2	18.8736
Rotated Gumble Copula									
δ	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
Log-like	-11.0645	-8.4449	9.587	7.2633	7.6151	6.0743	7.439	-0.0042	15.9982
Student's t copula									
ρ	0.1097	0.0929	0.0145	0.0489	0.0409	0.0478	0.0298	0.0563	0.0140
ν	28.7078	18.3624	14.2926	24.5766	33.0182	20.1707	24.5762	16.6290	27.2261
Log-like	-12.9733	-10.974	-4.51	-3.8615	-2.4485	-4.5202	-2.2465	-5.7122	-1.4247

(continued on next page)

Table 3A (continued)

	Technology	Telecom	Real Estate	Energy	Basic Materials	Utilities	Industrials	Financials	Health Care
SJC Copula									
λ_U	9.70E-05	1.32E-06	9.53E-10	1.53E-05	2.98E-08	2.79E-08	3.73E-09	2.79E-09	1.53E-05
λ_L	2.46E-02	2.35E-02	1.63E-03	1.26E-04	1.23E-03	1.84E-03	3.25E-03	1.09E-02	1.53E-05
Log-like	-13.7095	-11.2188	-2.4139	-3.3369	-2.9065	-3.9045	-2.8435	-6.0737	0.1316
Panel B: Parameter Estimates of time-varying copulas									
TVP Normal copula									
ψ_o	0.2168	0.1268	0.0010	0.0217	0.0284	0.0139	0.0058	0.0286	0.0026
ψ_1	0.0256	0.0274	0.0403	0.0452	0.0688	0.0408	0.0507	0.0886	0.0465
ψ_2	-0.0070	0.6036	1.8212	1.4998	1.2176	1.6348	1.6979	1.3624	1.6266
Log-like	-11.8399	-8.5542	-2.9604	-3.1595	-2.6379	-3.4598	-2.9545	-4.9172	-1.4491
TVP Clayton copula									
ψ_o	1.2719	-0.3448	-0.3216	-0.2669	-0.3813	-0.3451	-0.2522	-0.3932	-0.2070
ψ_1	-0.5863	-0.7686	-1.1811	-1.3171	-0.7502	-1.0699	-1.2788	-0.8788	-1.5930
ψ_2	-2.9050	0.3093	0.4979	0.3566	0.5947	0.5540	0.2742	0.6146	0.2306
Log-like	-13.715	-10.6654	-8.0443	-4.1954	-4.7266	-4.9663	-4.7809	-8.325	-2.7032
TVP Rotated Clayton copula									
ψ_o	0.2404	-0.2419	0.2319	-0.2428	-0.3495	-0.0044	-0.3236	-0.4505	0.2228
ψ_1	-0.8580	-1.1774	2.6062	-1.4158	1.3552	1.8608	-1.6305	-0.9653	-1.5493
ψ_2	-1.0938	0.1880	-0.9150	0.3098	0.5048	-0.9879	0.9516	1.1097	-0.7742
Log-like	-3.8119	-4.931	-0.0732	-2.1639	-0.5535	-2.3624	-0.1408	-1.2958	-1.1348
TVP Gamble copula									
Ω	2.1526	-2.7745	2.6939	1.0370	-3.7205	-2.4283	1.2843	0.8282	-0.1809
β	-1.6889	2.0693	-2.6076	-1.3489	3.2916	2.4212	-1.5977	-1.1785	-0.1588
α	-0.4788	1.2527	-0.3312	0.6008	0.8405	-0.7574	0.8314	0.7702	0.7667
Log-like	-7.2673	-9.2443	-0.1911	-2.8612	-0.7969	-2.7597	-0.6555	-3.1918	-1.105
TVP Rotated Gumble copula									
ω_U	2.4149	-1.3165	1.4185	1.5653	1.0309	1.8434	1.9717	0.9851	1.8536
α_U	-1.8276	1.5093	-1.6608	-1.7572	-1.2604	-2.0027	-2.1143	-1.2646	-2.0258
β_U	-0.6779	-0.1249	0.4179	0.2293	0.3006	0.1487	0.1144	0.4446	0.2303
Log-like	-14.835	-12.5042	-8.1368	-4.9085	-4.6308	-6.2281	-5.6961	-9.286	-3.363
TVP SJC copula									
ω_U	-11.4404	-13.8406	-20.7714	-15.7581	-17.3306	-17.3946	-19.4084	-19.6963	-20.3321
α_U	-0.6810	-0.1191	-3.17E-05	-1.6533	-0.0006	-0.0004	-0.0001	-0.0002	-3.2457
β_U	-0.0024	-0.0003	-3.20E-07	-0.0048	-1.75E-06	-1.41E-06	0.0000	-1.74E-07	-0.0094
ω_L	2.4198	-2.3387	-13.2186	-25	-12.3105	-14.3531	-10.8590	-0.7809	-16.7024
α_L	-21.8599	-6.1945	-3.6086	-6.5301	-3.2621	-3.5310	-2.5325	-14.4400	-2.1874
β_L	-7.9219	11.7431	-0.0142	-0.0190	-0.0109	-0.0182	-0.0146	0.0556	-0.0058
Log-like	-15.3736	-11.6071	0.9606	-2.6983	-1.078	-3.1559	0.5653	-4.4409	2.4754
TVP Student t copula									
ψ_o	0.3551	0.0689	0.0012	0.1272	0.0774	0.0118	0.0075	0.0248	0.0756
ψ_1	-0.0401	-0.0159	0.0154	-0.0457	0.1376	0.0047	0.0188	0.0178	-0.1636
ψ_2	-1.3123	1.2561	1.784192	-0.8427	-0.4139	1.7076	1.7142	1.5452	-1.7103
ν	5.000	5.0000	5.0000	5.000	5.000	5	5	5	5
Log-like	5.7881	2.8477	6.0756	12.47	14.9811	10.9287	13.8691	6.2994	15.967

NB:

Table 4A
Estimation of the dependence-switching copula model for Brent Crude and Sectoral Stocks.

	Technology	Telecom	Real Estate	Energy	Basic Materials	Utilities	Industrials	Financials	Health Care
Panel A: Parameter Estimates for time-invariant copulas									
Normal Copula									
ρ	0.135	0.1227	0.0356	0.1410	0.0999	0.1073	0.0793	0.0777	0.0442
Log-like	-18.5209	-15.2625	-1.2733	-20.2018	-10.0883	-11.6425	-6.3508	-6.0871	-1.9651
Clayton's copula									
α	0.166	0.1560	0.0723	0.1746	0.1234	0.1547	0.1259	0.1272	0.0883
Log-like	-22.3305	-18.3409	-5.1099	-24.6262	-13.1726	-20.3170	-14.1345	-14.7477	-7.2591
Rotated Clayton copula									
α	0.111	0.0890	0.0001	0.1166	0.0787	0.0616	0.0246	0.0262	0.0010
Log-like	-9.1806	-6.1171	0.0025	-10.7535	-4.8728	-3.0601	-0.5022	-0.5835	-0.0008
Plackett copula									
δ	1.463	1.4480	1.0624	1.4430	1.3344	1.3195	1.2230	1.2104	1.1182
Log-like	-15.8111	-15.1170	-0.4157	-14.5933	-8.8667	-8.2406	-4.4466	-3.9635	-1.3073
Frank Copula									
δ	0.754	0.7386	0.1223	0.7232	0.5624	0.5422	0.3992	0.3764	0.2148
Log-like	-15.5713	-15.0054	-0.4197	-14.2957	-8.6143	-8.0233	-4.4021	-3.9014	-1.2587
Gumble Copula									
δ	1.100	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000
Log-like	-11.7785	-5.3078	24.2884	-12.1252	-0.2428	1.3923	11.0592	9.6320	16.3631
Rotated Gumble Copula									
δ	1.100	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000
Log-like	-20.894	-17.7296	8.9415	-26.9377	-11.8849	-20.1513	-7.6744	-8.4220	1.8207

(continued on next page)

Table 4A (continued)

	Technology	Telecom	Real Estate	Energy	Basic Materials	Utilities	Industrials	Financials	Health Care
Student's t copula									
ρ	0.133	0.1244	0.0347	0.1356	0.0983	0.1032	0.0775	0.0734	0.0422
ν	18.784	32.1736	99.5373	18.6590	18.8352	20.5131	37.0468	21.2587	22.5408
Log-like	-21.2088	-16.1529	-0.9741	-22.7736	-12.5556	-13.7073	-7.0157	-13.8380	-3.4638
SJC Copula									
λ_U	0.001	9.54E-07	1.16E-10	0.0011	0.0000	1.79E-06	1.50E-05	0.0734	5.96E-08
λ_L	0.046	0.0525	0.0043	0.0575	0.0302	0.0520	0.026813	21.2587	0.0108
Log-like	-23.0065	-18.6146	-3.7420	-27.5046	-14.2065	-20.8968	-11.9278	-13.8380	-5.6807
Panel B: Parameter Estimates of time-varying copulas									
TVP Normal copula									
ψ_o	0.1119	0.0012	0.0226	0.0914	0.0967	0.0433	0.2466	0.0310	0.0331
ψ_1	0.1432	0.0140	0.0658	0.0344	0.1477	0.0310	0.5005	0.0942	0.1320
ψ_2	1.0484	1.9903	1.2202	1.3320	0.8503	1.5723	-1.8476	1.4829	1.0698
Log-like	-21.3078	-17.8737	-2.1529	-20.4590	-12.4136	-12.1143	-9.9910	-9.7264	-4.4070
TVP Clayton copula									
ψ_o	0.7850	-0.4116	-1.1160	-0.4106	-0.9544	-0.3366	0.8871	1.1619	0.2151
ψ_1	0.8099	-0.5227	0.7272	-0.7162	0.7258	1.3756	-0.9986	0.2074	-1.0408
ψ_2	-3.3683	0.3173	2.6178	0.4101	1.7280	-0.7136	-1.2787	-3.4264	-0.8745
Log-like	-10.0292	-18.8363	-8.2130	-27.1639	-15.9312	-22.3399	-16.5079	-14.3202	-4.1962
TVP Rotated Clayton copula									
ψ_o	-0.3111	-0.3890	-0.1975	-0.3795	-0.2723	-0.2840	-0.4583	0.4265	-0.5492
ψ_1	-0.7659	1.6134	-1.1407	-0.7738	-1.1350	0.6444	-1.0065	1.2525	0.3462
ψ_2	0.2075	-0.2055	0.5060	0.4240	0.2705	-0.0110	1.0879	-1.4448	1.2900
Log-like	-9.5676	-6.2377	-0.0187	-11.4778	-5.3682	-3.0602	-2.8428	-1.2937	-1.0599
TVP Gamble copula									
Ω	-0.6495	-1.2278	1.9103	2.4611	-2.5687	-3.3152	0.7587	0.4078	-0.7568
β	0.8981	1.4263	-2.1532	-1.6775	1.9404	2.5977	-1.1541	-0.9366	0.2796
α	-0.1450	-0.1386	0.6767	-1.3341	0.9569	1.4937	0.9447	1.6315	1.1328
Log-like	-13.4624	-9.0996	-0.2081	-15.3545	-7.5922	-6.3158	-4.2498	-5.2224	-2.3483
TVP Rotated Gumble copula									
ω_U	-0.6210	0.4255	-1.6198	-0.6599	2.0053	-1.0534	1.7869	-0.9078	-1.4681
α_U	0.8697	-0.0048	0.9181	0.9634	-1.3110	1.2857	-1.1478	1.1889	1.1663
β_U	-0.1077	-0.4427	1.6567	-0.3086	-1.1731	-0.1850	-1.0934	-0.3837	0.1675
Log-like	-21.6300	-18.9712	-6.7264	-29.0552	-16.9907	-23.0689	-14.5106	-15.8953	-6.7614
TVP SJC copula									
ω_U	-8.6059	-13.9089	-22.874	-6.9655	-14.2475	-13.6295	-16.2550	-21.4876	-16.6359
α_U	-3.4391	-0.0152	-5.58E-05	-0.2979	-0.0411	-0.1189	-1.7782	-2.56E-05	-0.0002
β_U	-0.0023	-4.53E-05	7.04E-07	-0.0013	-0.0001	-0.0004	-0.0052	-6.75E-07	0.0000
ω_L	-2.4811	-0.4275	1.9255	-1.4377	3.5246	-1.6934	2.3399	2.6627	-5.3481
α_L	-3.5300	-8.9714	-24.6073	-6.3699	-25.0000	-6.2233	-20.9030	-25.0000	-0.2676
β_L	9.0226	-1.2360	-1.9313	4.6825	-13.9087	7.1640	-5.8591	-0.0695	-0.0080
Log-like				-30.7074	-16.5101	-21.4456	-12.4535	-13.8793	-2.5264
TVP Student t copula									
ψ_o	0.3720	0.2794	0.0296	0.1090	0.0642	0.3375	0.2068	0.1066	0.0130
ψ_1	0.1697	0.1458	0.0550	0.0141	0.0478	0.0875	0.1981	0.1064	0.0338
ψ_2	-1.3975	-0.6178	0.1776	1.0588	1.1738	-2.0189	-1.9147	-0.1004	1.5460
ν	5.0000	5.000	5.000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
Log-like	-6.9887	2.8714	30.2150	-8.4152	0.4776	0.2509	11.6373	6.8007	10.0848

NB:

Table 5A
Risk Parity Portfolios.

	Mean	Std. Dev.	5%	95%	HE	p-value
Technology	0.06	0.01	0.04	0.08	0.81	0.00
Telecom	0.08	0.01	0.06	0.10	0.67	0.00
RealEstate	0.10	0.02	0.08	0.13	0.58	0.00
Energy	0.07	0.01	0.05	0.09	0.72	0.00
BasicMaterials	0.07	0.01	0.05	0.09	0.74	0.00
Utilities	0.08	0.01	0.06	0.10	0.52	0.00
Industrials	0.09	0.01	0.07	0.12	0.44	0.00
Financials	0.09	0.01	0.07	0.11	0.53	0.00
HealthCare	0.09	0.01	0.07	0.11	0.62	0.00
NaturalGasSpot	0.08	0.02	0.05	0.11	0.92	0.00
BrentCrudeSpot	0.08	0.02	0.04	0.12	0.89	0.00
CocoaIndexSpot	0.11	0.03	0.07	0.17	0.79	0.00

Notes: Results are based on [Spinu \(2013\)](#).

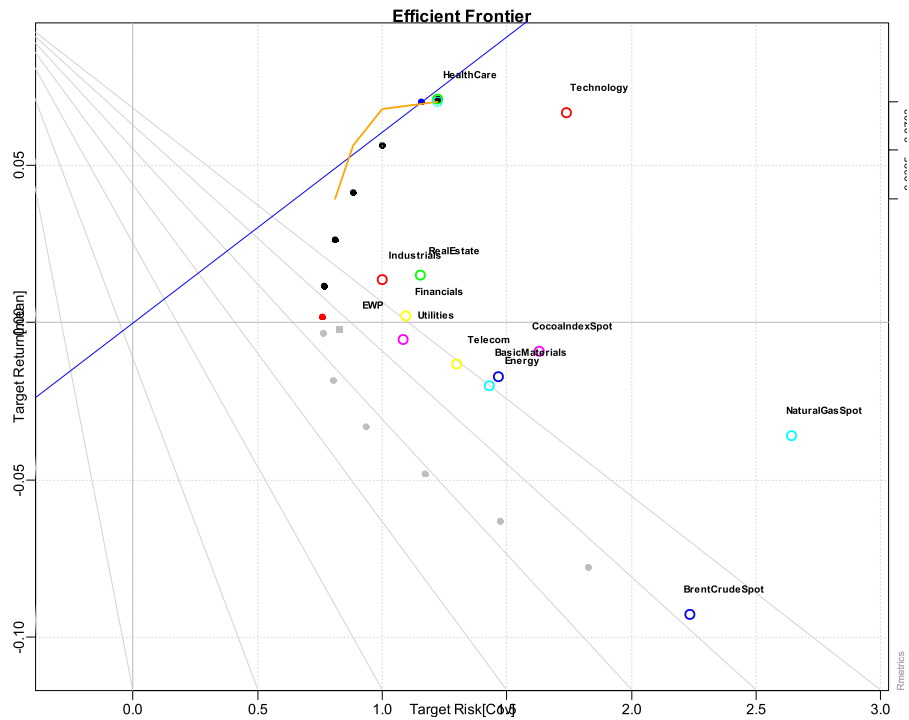


Fig. 1A. Efficient frontier (constraints “Long Only”).

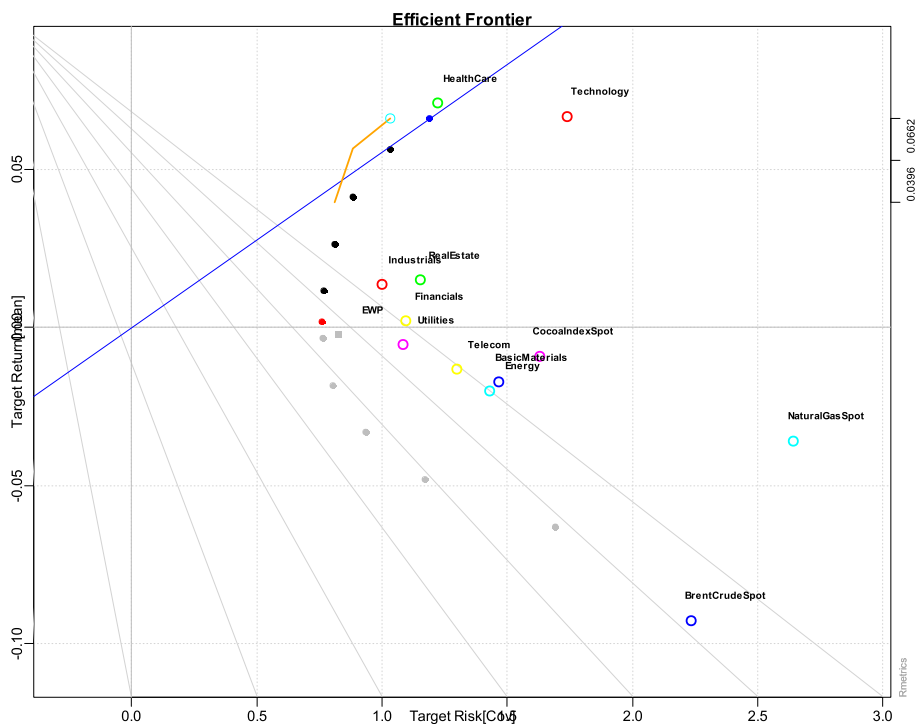


Fig. 2A. Efficient frontier (constraints: “Long Only” and 0% minimum, 50% maximum allocation).

Appendix B. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.eneco.2022.105891>.

References

- Abakah, E.J.A., Addo Jr., E., Gil-Alana, L.A., Tiwari, A.K., 2021a. Re-examination of international bond market dependence: evidence from a pair copula approach. *Int. Rev. Financ. Anal.* 74, 101678.
- Abakah, E.J.A., Tiwari, A.K., Alagidede, I.P., Gil-Alana, L.A., 2021b. Re-examination of risk-return dynamics in international equity markets and the role of policy uncertainty, geopolitical risk and VIX: evidence using Markov-switching copulas. *Financ. Res. Lett.* 102535.
- Abdulkarim, F.M., Akinlaso, M.I., Hamid, B.A., Ali, H.S., 2019. The nexus between oil price and Islamic stock markets in Africa: a wavelet and multivariate-GARCH approach. *Borsa Istanbul Rev.* 20 (2), 108–120. <https://doi.org/10.1016/j.bir.2019.11.001>.
- Abel, A.B., Bernanke, B.S., 2001. *Macroeconomics*, 4. bs. Addison Willey Longman inc., Boston, San Francisco, New York.
- Adekoya, O.B., Oliyide, J.A., 2020. The hedging effectiveness of industrial metals against different oil shocks: evidence from the four newly developed oil shocks datasets. *Res. Policy* 69, 101831.
- Aggarwal, R., Inchan, C., Leal, R., 1999. Volatility in emerging markets. *J. Financ. Quant. Anal.* 34, 33–55.
- Ahmed, A.D., Huo, R., 2020. Volatility transmissions across international oil market, commodity futures and stock markets: empirical evidence from China. *Energy Econ.* xxx <https://doi.org/10.1016/j.eneco.2020.104741>.
- Akkoc, U., Cevcir, I., 2019. Dynamic linkages between strategic commodities and stock market in Turkey: evidence from SVAR-DCC-GARCH model. *Res. Policy* 62 (January), 231–239. <https://doi.org/10.1016/j.resourpol.2019.03.017>.
- Arouri, M.E.H., Nguyen, D.K., 2010. Oil prices, stock markets and portfolio investment: evidence from sector analysis in Europe over the last decade. *Energy Policy* 38 (8), 4528–4539.
- Arouri, M.E.H., Jouini, J., Nguyen, D.K., 2011. Volatility spillovers between oil prices and stock sector returns: implications for portfolio management. *J. Int. Money Financ.* 30 (7), 1387–1405.
- Arouri, M.E.H., Youssef, A.B., M'henni, H., & Rault, C., 2012. Energy consumption, economic growth and CO2 emissions in Middle East and North African countries. *Energy Policy* 45, 342–349.
- Azimli, A., 2020. The oil price risk and global stock returns. *Energy* 198, 117320. <https://doi.org/10.1016/j.energy.2020.117320>.
- Badeeb, R.A., Lean, H.H., 2018. Asymmetric impact of oil price on Islamic sectoral stocks. *Energy Econ.* 71, 128–139.
- Baker, M., Wurgler, J., 2006. Investor sentiment and the cross-section of stock returns. *J. Financ.* 61 (4), 1645–1680.
- Basher, S.A., Sadorsky, P., 2016. Hedging emerging market stock prices with oil, gold, VIX, and bonds: a comparison between DCC, ADCC and GO-GARCH. *Energy Econ.* 54, 235–247.
- Basher, S.A., Haug, A.A., Sadorsky, P., 2018. The impact of oil-market shocks on stock returns in major oil-exporting countries. *J. Int. Money Financ.* 86, 264–280.
- Baur, D.G., Lucey, B.M., 2009. Flights and contagion—an empirical analysis of stock-bond correlations. *J. Financ. Stab.* 5 (4), 339–352. <https://doi.org/10.1016/j.jfs.2008.08.001>.
- Baur, D.G., Smales, L., 2018. Gold and geopolitical risk. In: Baur, Dirk G., Smales, Lee A. (Eds.), *Gold and Geopolitical Risk* (January 24, 2018) doi:10.2139/ssrn.3109136 SSRN Working Paper No. 3109136.
- Beckmann, J., Berger, T., Czudaj, R., 2015. Does gold act as a hedge or a safe haven for stocks? A smooth transition approach. *Econ. Model.* 48, 16–24.
- Beckmann, J., Berger, T., Czudaj, R., Hoang, T.H.V., 2019. Tail dependence between gold and sectoral stocks in China: perspectives for portfolio diversification. *Empir. Econ.* 56 (3), 1117–1144. <https://doi.org/10.1007/s00181-017-1381-8>.
- Bernanke, B.S., 2006. *Energy and the Economy* (No. 221).
- Bjørnland, H.C., 2009. Oil price shocks and stock market booms in an oil exporting country. *Scottish J. Politic. Econ.* 56 (2), 232–254.
- Boubaker, H., Sghaier, N., 2016. Markov-switching time-varying copula modelling of dependence structure between oil and GCC stock markets. *Open J. Stat.* 6 (4), 565–589. <https://doi.org/10.4236/ojs.2016.64048>.
- Broadstock, D.C., Filis, G., 2014. Oil price shocks and stock market returns: new evidence from the United States and China. *J. Int. Financ. Mark. Inst. Money* 33, 417–433.
- Broadstock, D.C., Chatziantoniou, I., Gabauer, D., 2020. Minimum connectedness portfolios and the market for green bonds: Advocating socially responsible investment (SRI) activity. Available at SSRN 3793771.
- Brown, S.P., Yücel, M.K., 2002. Energy prices and aggregate economic activity: an interpretative survey. *Quart. Rev. Econ. Fin.* 42 (2), 193–208.
- Buyuksahin, B., Harris, J.H., 2011. Do speculators drive crude oil futures prices? *Energy J.* 32 (2).
- Capie, F., Mills, T.C., Wood, G., 2005. Gold as a hedge against the dollar. *Journal of. Cevik, N.K., Cevik, E.I., Dibooglu, S., 2020. Oil prices, stock market returns and volatility spillovers: evidence from Turkey. J. Policy Model* 1–18. <https://doi.org/10.1016/j.jpmod.2020.01.006>.
- Chan, H., Faff, R., 1998. The sensitivity of Australian industry equity returns to a gold price factor. *Acc. Finc.* 38 (2), 223–244.
- Chang, C.L., McAleer, M., Tansuchat, R., 2011. Crude oil hedging strategies using dynamic multivariate GARCH. *Energy Econ.* 33 (5), 912–923.
- Chaudhuri, K., Smiles, S., 2004. Stock market and aggregate economic activity: evidence from Australia. *Appl. Financ. Econ.* 14 (2), 121–129.
- Choi, K., Hammoudeh, S., 2010. Volatility behavior of oil, industrial commodity and stock markets in a regime-switching environment. *Energy Policy* 38 (8), 4388–4399.
- Chong, J., Miffre, J., 2010. Conditional return correlations between commodity futures and traditional assets. *J. Altern. Invest.* 12 (3), 61–75.
- Choudhry, T., Hassan, S.S., Shabi, S., 2015. Relationship between gold and stock markets during the global financial crisis: evidence from nonlinear causality tests. *Int. Rev. Financ. Anal.* 41, 247–256.
- Christoffersen, P., Errunza, V., Jacobs, K., Jin, X., 2014. Correlation dynamics and international diversification benefits. *Int. J. Forecast.* 30 (3), 807–824.
- Commercialising Cocoa Growing in North Queensland, 2013. RIRDC Publication.
- Conover, C.M., Jensen, G.R., Johnson, R.R., Mercer, J.M., 2009. Can precious metals make your portfolio shine? *J. Invest.* 18 (1), 75–86.
- Cunado, J., Perez de Gracia, F., 2014. Oil price shocks and stock market returns: evidence for some European countries. *Energy Econ.* 42, 365–377.
- da Silva Filho, O.C., Ziegelmann, F.A., Dueker, M.J., 2012. Modeling dependence dynamics through copulas with regime switching. *Insurance: Math. Econ.* 50 (3), 346–356.
- Daskalaki, C., Skiadopoulos, G., 2011. Should investors include commodities in their portfolios after all? New evidence. *J. Bank. Financ.* 35 (10), 2606–2626.
- Delatte, A.L., Lopez, C., 2013. Commodity and equity markets: some stylized facts from a copula approach. *J. Bank. Financ.* 37 (12), 5346–5356.
- Domanski, D., Heath, A., 2007. *Financial Investors and Commodity Markets*. BIS quarterly review. March.
- Engle, R.F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometr. J. Econometr. Soc.* 987–1007.
- Faff, R.W., Brailsford, T.J., 1999. Oil price risk and the Australian stock market. *J. Energy Fin. Dev.* 4 (1), 69–87. [https://doi.org/10.1016/s1085-7443\(99\)00005-8](https://doi.org/10.1016/s1085-7443(99)00005-8).
- Filis, G., Degiannakis, S., Floros, C., 2011. Dynamic correlation between stock market and oil prices: the case of oil-importing and oil-exporting countries. *Int. Rev. Financ. Anal.* 20 (3), 152–164.
- Fisher, I., 1930. *Theory of Interest: As Determined by Impatience to Spend Income and Opportunity to Invest it*. Augustum Kelly Publishers, Clifton.
- Flavin, T.J., Morley, C.E., Panopoulou, E., 2014. Identifying safe haven assets for equity investors through an analysis of the stability of shock transmission. *J. Int. Financ. Mark. Inst. Money* 33, 137–154.
- Ghosh, D., Levin, E.J., Macmillan, P., Wright, R.E., 2004. Gold as an inflation hedge? *Stud. Econ. Financ.* 22, 1–25.
- Gorton, G., Rouwenhorst, K.G., 2006. Facts and fantasies about commodity futures. *Financ. Anal. J.* 62 (2), 47–68.
- Greer, R.J., 2000. The nature of commodity index returns. *J. Altern. Invest.* 3 (1), 45–52.
- Hamdi, B., Aloui, M., Alqahtani, F., Tiwari, A., 2019. Relationship between the oil price volatility and sectoral stock markets in oil-exporting economies: evidence from wavelet nonlinear denoised based quantile and granger-causality analysis. *Energy Econ.* 80, 536–552. <https://doi.org/10.1016/j.eneco.2018.12.021>.
- Hammoudeh, S., Santos, P.A., Al-Hassan, A., 2013. Downside risk management and VaR-based optimal portfolios for precious metals, oil and stocks. *North Am. J. Econ. Fin.* 25, 318–334.
- Hasan, M.Z., 2017. Transmission of international energy price shocks to Australian stock market and its implications for portfolio formation. *Asian Econ. Fin. Rev.* 7 (4), 393–412. <https://doi.org/10.18488/journal.aefr/2017.7.4/102.4.393.412>.
- Heaton, C., Milunovich, G., Passé-de Silva, A., 2011. International commodity prices and the Australian stock market. *Econ. Rec.* 87 (276), 37–44.
- Hillier, D., Draper, P., Faff, R., 2006. Do precious metals shine? An investment perspective. *Financ. Anal. J.* 62 (2), 98–106.
- Hong, H., Yogo, M., 2010. *Commodity Market Capital Flow and Asset Return Predictability*. Work. Pap., Princet. Univ./Fed. Reserve Bank, Minneap.
- Hong, H., Yogo, M., 2012. What does futures market interest tell us about the macroeconomy and asset prices? *J. Financ. Econ.* 105 (3), 473–490.
- Hood, M., Malik, F., 2013. Is gold the best hedge and a safe haven under changing stock market volatility? *Rev. Financ. Econ.* 22 (2), 47–52.
- Hooker, M.A., 1996. What happened to the oil price-macroeconomy relationship? *J. Monet. Econ.* 38 (2), 195–213.
- Huang, R.D., Masulis, R.W., Stoll, H.R., 1996a. Energy shocks and financial markets. *J. Futur. Mark.* 16 (1), 1–27.
- Jain, A., Biswal, P.C., 2016. Dynamic linkages among oil price, gold price, exchange rate, and stock market in India. *Res. Policy* 49, 179–185.
- Ji, Q., Zhang, D., 2019. China's crude oil futures: Introduction and some stylized facts. *Financ. Res. Lett.* 28, 376–380.
- Ji, Q., Bouri, E., Gupta, R., Roubaud, D., 2018. Network causality structures among Bitcoin and other financial assets: A directed acyclic graph approach. *Quart. Rev. Econ. Fin.* 70, 203–213.
- Jiménez-Rodríguez, R., Sánchez, M., 2005. Oil price shocks and real GDP growth: empirical evidence for some OECD countries. *Appl. Econ.* 37 (2), 201–228.
- Joe, H., Xu, J.J., 1996. *The Estimation Method of Inference Functions for Margins for Multivariate Models*. Technical Report No.166. Department of Statistics. University of British Columbia.
- Jones, C.M., Kaul, G., 1996. Oil and the stock markets. *J. Financ.* 51 (2), 463–491.
- Junttila, J., Pesonen, J., Raatikainen, J., 2018. Commodity market based hedging against stock market risk in times of financial crisis: The case of crude oil and gold. *J. Int. Financ. Mark. Inst. Money* 56, 255–280.
- Kilian, L., Park, C., 2009. The impact of oil price shocks on the US stock market. *Int. Econ. Rev.* 50 (4), 1267–1287.
- Kim, C.J., Nelson, C.R., 1999. Friedman's plucking model of business fluctuations: tests and estimates of permanent and transitory components. *J. Money Credit Bank.* 317–334.
- Kroner, K.F., Ng, V.K., 1998. Modeling asymmetric movements of asset prices. *Rev. Financ. Stud.* 11 (04), 817–844.
- Kroner, K.F., Sultan, J., 1993. Time-varying distributions and dynamic hedging with foreign currency futures. *J. Financ. Quant. Anal.* 28 (04), 535–551.

- Ku, Y.H.H., Chen, H.C., Chen, K.H., 2007. On the application of the dynamic conditional correlation model in estimating optimal time-varying hedge ratios. *Appl. Econ. Lett.* 14 (7), 503–509.
- Kumar, D., 2014. Return and volatility transmission between gold and stock sectors: application of portfolio management and hedging effectiveness. *IIMB Manag. Rev.* 26, 5–16.
- Lardic, S., Mignon, V., 2006. The impact of oil prices on GDP in European countries: an empirical investigation based on asymmetric cointegration. *Energy Policy* 34 (18), 3910–3915.
- Le, T., Abakah, E.J.A., Tiwari, A.K., 2020. Time and frequency domain connectedness and spill-over among fintech, green bonds and cryptocurrencies in the age of the fourth industrial revolution. *Technol. Forecast. Social Change* 162. <https://doi.org/10.1016/j.techfore.2020.120382>.
- Le Long, H., De Ceuster, M.J., Annaert, J., Amonhaemanon, D., 2013. Gold as a hedge against inflation: the Vietnamese case. *Proced. Econ. Finan.* 5, 502–511.
- Lin, B., Su, T., 2020. Mapping the oil price-stock market nexus researches: a scientometric review. *Int. Rev. Econ. Financ.* 67 (December 2019), 133–147. <https://doi.org/10.1016/j.iref.2020.01.007>.
- Lucey, B.M., Li, S., 2015. What precious metals act as safe havens, and when? Some US evidence. *Appl. Econ. Lett.* 22 (1), 35–45.
- Lucey, B.M., Tully, E., 2003. International portfolio formation, skewness and the role of gold. In: *Skewness and the Role of Gold* (September 2003).
- Lucey, B.M., Tully, E., 2006. Seasonality, risk and return in daily COMEX gold and silver data 1982–2002. *Appl. Financ. Econ.* 16 (4), 319–333.
- Maghyereh, A.I., Awartani, B., Tziogkidis, P., 2017. Volatility spillovers and cross-hedging between gold, oil and equities: Evidence from the Gulf Cooperation Council countries. *Energy Econ.* 68, 440–453.
- Maheshwari, S., Gupta, R., Li, J., 2018. A comparative analysis of sector diversification in Australia, India and China. *Fin. Plan. Res. J.* 1, 76–100.
- Malliaris, A.G., Malliaris, M., 2015. What drives gold returns? A decision tree analysis. *Financ. Res. Lett.* 13, 45–53.
- Markovitz, H., 1959. *Portfolio Selection: Efficient Diversification Of Investments*. John Wiley, NY.
- McSweeney, E.J., Worthington, A.C., 2008. A comparative analysis of oil as a risk factor in Australian industry stock returns, 1980–2006. *Studies in economics and finance*.
- Mensi, W., Hammoudeh, S., Nguyen, D.K., Kang, S.H., 2016. Global financial crisis and spillover effects among the US and BRICS stock markets. *Int. Rev. Econ. Financ.* 42, 257–276.
- Mensi, W., Hammoudeh, S., Shahzad, S.J.H., Shahbaz, M., 2017. Modeling systemic risk and dependence structure between oil and stock markets using a variational mode decomposition-based copula method. *J. Bank. Financ.* 75, 258–279.
- Miller, J.I., Ratti, R.A., 2009. Crude oil and stock markets: Stability, instability, and bubbles. *Energy Econ.* 31 (4), 559–568.
- Nguyen, P., Liu, W.H., 2017. Time-varying linkage of possible safe haven assets: a cross-market and cross-asset analysis. *Int. Rev. Financ.* 17 (1), 43–76.
- Phillips, P.C., Perron, P., 1988. Testing for a unit root in time series regression. *Biometrika* 75 (2), 335–346.
- Politis, D.N., 2007. Model-free versus model-based volatility prediction. *J. Financ. Econ.* 5 (3), 358–359.
- Politis, D.N., White, H., 2004. Automatic block-length selection for the dependent bootstrap. *Econ. Rev.* 23 (1), 53–70.
- Roll, R., 2013. Volatility, correlation, and diversification in a multi-factor world. *J. Portfolio Manag.* 39 (2), 11–18.
- Sadorsky, P., 1999. Oil price shocks and stock market activity. *Energy Econ.* 21 (5), 449–469.
- Schweizer, B., Sklar, A., 1983. *Probabilistic Metric Spaces*. Elsevier Science.
- Shahbaz, M., Trabelsi, N., Tiwari, A.K., Abakah, E.J.A., Jiao, Z., 2021. Relationship between green investments, energy markets, and stock markets in the aftermath of the global financial crisis. *Energy Econ.* 105, 655.
- Shahzad, S.J.H., Bouri, E., Roubaud, D., Kristoufek, L., Lucey, B., 2019. Is Bitcoin a better safe-haven investment than gold and commodities? *Int. Rev. Financ. Anal.* 63, 322–330.
- Silvennoinen, A., Thorp, S., 2013. Financialization, crisis and commodity correlation dynamics. *J. Int. Financ. Mark. Inst. Money* 24, 42–65.
- Tiwari, A.K., Aye, G.C., Gupta, R., Gkillas, K., 2020. Gold-oil dependence dynamics and the role of geopolitical risks: Evidence from a Markov-switching time-varying copula model. *Energy Econ.* 88 (104), 748. <https://doi.org/10.1016/j.eneco.2020.104748>.
- Sklar, M., 1959. Fonctions de repartition an dimensions et leurs marges, 8. *Publ. Inst. Statist. Univ. Paris*, pp. 229–231.
- Spinu, F., 2013. An Algorithm for Computing Risk Parity Weights. SSRN 2297383.
- Tiwari, A.K., Abakah, E.J.A., Le, T.L., Leyva-de la Hiz, D.I., 2021. Markov-switching dependence between artificial intelligence and carbon price: The role of policy uncertainty in the era of the 4th industrial revolution and the effect of COVID-19 pandemic. *Technol. Forecast. Soc. Chang.* 163, 120434.
- Tuna, G., 2019. Interaction between precious metals price and Islamic stock markets. *Int. J. Islam. Middle East. Financ. Manag.* 12 (1), 96–114. <https://doi.org/10.1108/IMEFM-06-2017-0143>.
- Wen, X., Cheng, H., 2018. Which is the safe haven for emerging stock markets, gold or the US dollar? *Emerg. Mark. Rev.* 35, 69–90.
- Williams, J.B., 1938. *The Theory of Investment Value*. Harvard University Press, Cambridge.
- Worthington, A.C., Pahlavani, M., 2007. Gold investment as an inflationary hedge: Cointegration evidence with allowance for endogenous structural breaks. *Appl. Financ. Econ. Lett.* 3 (4), 259–262. <https://doi.org/10.1080/17446540601118301>.
- Zhang, D., 2017. Oil shocks and stock markets revisited: Measuring connectedness from a global perspective. *Energy Econ.* 62, 323–333.
- Zhang, Y., 2018. Investigating dependencies among oil price and tanker market variables by copula-based multivariate models. *Energy* 161, 435–446.
- Dickey, D.A., Fuller, W.A., 1979. Distribution of the estimators for autoregressive time series with a unit root. *J. Am. Stat. Assoc.* 74 (366a), 427–431.