UNIVERSITY OF GHANA, LEGON

DETERMINING PREMIUM IN AN EXCESS-OF-LOSS REINSURANCE CONTRACT -AN EXTREME VALUE APPROACH

BY

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DECLARATION

Candidate's declaration:

I hereby declare that this thesis is my own work towards the award of the Master of Philosophy degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

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Supervisors' declaration:

"We hereby declare that this thesis was supervised in accordance with the laid down rules and procedures by the Graduate School of the University of Ghana."

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ABSTRACT

Statistics of extremes deals with the estimation of rare events that may have catastrophic effects on life, environment, among others. Since the introduction of Extreme value theory (EVT), it has been used in modelling various extreme events in fields such as finance, insurance, transportation, etc. In this thesis, the EVT is applied to model two claims datasets from the Ghanaian insurance industry. To do this, we employ the Peak Over Threshold (POT) method using the splicing Generalized Pareto Distribution (GPD) in modelling the tails of the underlying distributions. The primordial parameter in the estimation of extreme events is the tail index or Extreme Value Index (EVI). The EVI enables the classification of the underlying distribution of a dataset into three family of distributions that have short, light, or heavy tails. Thereafter, any of the parameters of extremes such as extreme quantiles, small exceedance probabilities, right endpoints and return periods can be estimated. Excess Loss Premium (XLP), Expected Shortfall (ES) and Value at Risk (VaR) as risk measures were thereafter calculated through the splicing method. The impact of the extreme value index (EVI) on these risk measures for the two datasets are discussed and suggestions made on how these could help the primary insurer in limiting the danger of large claims on the solvency of these companies. Based on this, the insurance companies can assess the risk associated with large claims and transfer some of these risks to reinsurance companies given their retention level. This study recommends that the splicing method should be used in fitting insurance data which behaves differently at various intervals of claims amount.

DEDICATION

This study is graciously dedicated to my beloved parents, Samuel and Rosina Adams who have been my source of motivation and continually given me their spiritual, moral, emotional and financial support.

To my husband, Michael Filson for all your love and encouragement in this period. You have been my best cheerleader.

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List of Abbreviation

AMS Annual Maximum Series

EVT Extreme Value Theory

POT Peaks-over-Threshold

BMM Block Maxima Method

EPI Estimated Premium Income

ES Expected Shortfall

CTE Conditional Tail Expectation

TVaR Tail Value-at-Risk

CVaR Conditional Value-at-Risk

VaR Value-at-Risk

XLP Excess-of-Loss Premium

EVI Extreme Value Index

EVA Extreme Value Analysis

GPD Generalized Pareto Distribution

GEV Generalized Extreme Value Distribution

MCMC Markov Chain Monte Carl

ME Mixed Erlang

MLE Maximum Likelihood Estimator

PWM Probability Weighted Moment

ROL Rate on Line

Chapter 1

Introduction

1.1 Background of the Study

Insurance is a core financial product that is key in hedging against risk of financial losses (Hurley, 2020). The insured pays a certain amount of premium to the insurer to be able to transfer the risk of losses to the insurance company (insurer). After expansive risk pooling, the insurer reduces their risk by diversification and other hedging strategies. An insurer might suddenly collapse in the event of extreme risks because of the rareness and severity of such incidents. According to insurance specialists in the United States for instance, reported sums insured are sometimes just 60% of the actual insured value; other countries are in a similar scenario. In Germany, a specialist in the computation of sums insured stated that the average degree of under-insurance was around 20%, with a broad range in certain circumstances (Ronken & Re, 2020). To deal with these types of risks, an insurer must be well-prepared ahead of time, with the information and tools needed to accurately calculate the size of potential losses and put appropriate safeguards in place if they occur (Chukwudum, 2018). One possible option for diversification for the underwriters of both non-life and life Insurance is to employ reinsurance as one of the main risk mitigation and risk transfer measures (Salaudeen et al., 2021).

Insurers frequently use reinsurance contracts to protect themselves against portfolio contamination caused by extreme claims (Beirlant et al., 2001). Reinsurance is classified into two main types: treaty and facultative (Insurance Information

Institute, 2020). Treaties are agreements that cover large groupings of policies, such as the whole auto business of a primary insurer. Facultative reinsurance addresses specific individual hazards that are typically high-value or dangerous, such as a hospital, and would not be recognized under a treaty reinsurance . In most treaties, once the contract terms including the categories of risks covered are defined, any policies that fit within those parameters – in many cases both old and current businesses are covered automatically, until the agreement is revoked. Because reinsurance firms often reinsure an insurance company's whole portfolio of insurance policies (save those reinsured facultatively), rather than individual insurance policies, they are frequently unaware of all possible liability from any one individual policyholder. The cedant company is not required to provide all claims details to the reinsurer in the case of non-proportional reinsurance, in which the reinsurer compensates for excesses above certain threshold, say T, except when the claims amount is greater than T (Albrecher et al., 2017).

The 2017-2019 National Insurance Commission (NIC) Annual Reports show that the reinsurance sector in Ghana has experienced a significant growth of 30% from 2016-2019 representing an increase from GHS 0.59bn to GHS 0.78bn over the last three years which shows the increase demand for reinsurance by ceding companies. Also, on average, the non-Life sector ceded about 60% of premiums written to reinsurers in the year 2019. This continuous growth of the reinsurance sector in Ghana shows the need for Extreme Value Theory (EVT) techniques to be applied to distributions in order to accurately determine how much premium the ceding company should pay a reinsurer to ensure continuous solvency in this sector.

Reinsurers, like insurance companies, want to know where their risks will come from and how much they would lose. The reinsurer pays for claims that exceed the retention amount or level in an excess-of-loss reinsurance contract. An apt description of the upper tail of the claim size distribution is critical for a reinsurer to set competitive prices. This brings to light that determining the retention level and premium a ceding company should pay a reinsurer is a very crucial point for the reinsurer. This is because if the premium is priced too high, reinsurers stand high chance of losing market. And also, if the pricing of premium is too low, they expose themselves to a high risk of great financial loss.

Extreme Value Theory (EVT) is a statistical discipline with the objective of analyzing and forecasting the occurrence of unusually large or small events. There has been rapid development over the last few decades in this field both in its theory and applications. The EVT techniques are also becoming increasingly popular in many other disciplines such as finance; for risk assessment on financial market, telecommunications; for traffic prediction, hydrology; for estimation of the probability of an unusually large flooding event and also, in insurance; for portfolio adjustment (Coles, 2001). Until recently, EVT was more important in hydrology and climatology research than in insurance (de Haan 1990; Smith 1989).

Several authors have recently reported that EVT is as crucial to the modeling of extreme insurance losses as it is to the modeling of high river levels or temperatures (Beirlant & Teugels, 1992; Embrechts & Klüppelberg, 1993).

EVT is quite different from other statistical approaches because EVT fits a distribution to a subset of the accessible data, thus, it allows you to focus on the part of the claims distribution of interest whiles other approaches fit a chosen distribution to the entire data set. In an excess-of-loss reinsurance contract, the issue of pricing is particularly relevant to the reinsurer when pricing heavy-tailed losses or high-excess layer, thus, losses of which extreme values occurs with high probability.

There are two main approaches in determining extremities, namely; Block Maxima Method (BMM), the oldest group which is used for the largest observations collected from large samples of observations equally distributed and the Peaks-over-threshold Method (POT), which is a model for extreme samples that goes beyond a certain threshold.

This study applies the extreme value approach based on the methodology described in Beirlant et al. (2001) to help estimate the ideal premium a ceding company pays given its retention level/ limit using motor insurance claims data from insurance companies in Ghana. The study focuses on the Peaks-Over-Threshold (POT) Method and the use of the Generalized Pareto Distribution. The drawback of the BMM method is the lack of sufficient use of data since there is clustering of all extremes in one block which gives rise to the Pickands-Balkema-de Haan theorem (1974,1975) culminating in the fit of the GPD through the POT method which also provides a simple tool for estimating measures of tail risk including; Value-at-Risk (VaR), Expected Shortfall (ES), and also the Return period (McNeil,1999).

1.2 Research Problem

When reviewing the claim data reports of insured firms that have experienced serious losses, the reported sum insured is sometimes less than the actual sum insured (Ronken & Re, 2020). In the event of a claim, knowing the exact sum insured allows the claim to be settled quickly and appropriately, reducing the risk of a loss. Although the ceding firm is frequently given free reign over the amount required, the reinsurer should have a stake in the accuracy of the sum insured. To begin with, the insurer estimates a risk premium that is too low, resulting in a lack of premiums proportional to the risk to pay a claim. In addition to the excessively low estimate of exposure, the insurer may consider the predicted loss profile unjustified due to an insufficient evaluation of the maximum loss due to excessively low quantities covered. As a result, both can have an impact on the reinsurance purchase and structure, possibly forcing the insurer to bear a portion of the loss themselves in the event of an incorrect assessment.

A comparison is made between the sum of all quoted loss amounts for a specific operation and the sum of all stated sums insured. Excessively low insurance amounts typically result in coverage gaps and the danger of insufficient compensation being provided in the event of a loss. To reduce the danger of underinsurance and the inability of the cedant companies to pay out claims given their retention level, the insurance company should estimate the premium using the Extreme Value Theory. The extremes of the data are often fitted with a thin tail distribution such as the Pareto or Normal distribution in data analysis of insured losses. The tails of these standard distributions, however, are too thin to address extreme values in an excess of loss reinsurance which may cause a bias in the estimation of the reinsurance premium. In an excess-of-loss reinsurance contract, the focus is solely on large claims in the data, thus, the tail of the claims distribution (McNeil, 1997). There is therefore the need to employ the Peaks-Over-Threshold (POT) Model and the use the Generalized Pareto Distribution (GPD) to help estimate the reinsurance premium more efficiently. This gap motivates the study.

1.3 Research Objectives

The main objective of this study is to estimate the premium a ceding company will pay a reinsurer given its retention level in order for the reinsurer to be able to efficiently compensate the ceding company in case of any loss. The objectives of the study are to:

1. Determine the domain of attraction of the underlining distribution.

2. Estimate the tail index.

3. Determine the risk measures (Return period, Value-at-Risk, Expected Short-

fall, and the excess-of-loss premium).

1.4 Research Questions

1. What is the domain of attraction or tail characteristics of the claims amount distribution?

2. What distribution is suitable for fitting the tail of the distribution?

3. How do we estimate the risk measures?

1.5 Significance of the Study

The relevance of reinsurance to an insurance company's long-term viability cannot be overstated, considering its role in reducing risk exposures. Reinsurance is a huge concern that has prompted a whole slew of research projects (Mayers & Smith, 1990; Adams et al., 2012; Werner, 2014; Brown, 2017). Mayers and Smith (1990) used empirical evidence to test the theoretical hypotheses that support reinsurance demand. For the primary insurer to mitigate the problem of large losses, it should accurately estimate tail risk measures including; Return period, Value-at-Risk (VaR), Expected Shortfall (ES), and Excess-of-Loss Premium (XLP). The relevance of this study is to be able to help the reinsurance sector especially in Ghana to accurately determine the reinsurance premium in such a way that takes into account large claims of the data given the ceding company's retention level. The findings of the study have implications for insurance practitioners, academics, and in particular regulators and government as it could play a paramount role in promoting evidence-informed reinsurance policy.

1.6 Scope of the Study

The scope of this study covers datasets of motor insurance claims from two unnamed insurance companies (due to confidentiality issues) in Ghana from the period 2011 - 2021 .

1.7 Organization of the Study

The final report on this study is presented in five chapters. Chapter One presents the background to the study; followed by a problem statement and continues with research purpose, the scope of study, and the relevance of the study. Chapter Two elaborates extensively on reinsurance and reviews the existing extreme value approaches used by various researchers and their contributions to the field of study and its applications. Chapter Three, which is the methodology, gives a brief description of the BMM method which shows how GEV is used to fit the maxima of each block and compute some tail risk measures and later focuses on a detailed description of the POT method which is used to determine the exceedances in the data to fit the splicing GPD (which is a mixture of Mixed Erlang and Generalized Pareto Distributions) which estimates the quantiles, thus, tail risk measures (Return period, VaR and ES) and also the reinsurance premium assuming a retention level. Chapter Four presents the actual data analysis; and Chapter Five summarizes the work in terms of findings, conclusions, and recommendations.

Chapter 2

Literature Review

2.1 Introduction

This chapter presents a review of literature on the topic of reinsurance and determination of premiums in excess-of-loss contracts using Extreme Value Theory. It provides literature explaining the concept of reinsurance, the forms, and types. It also covers the excess-of-loss concept and reinsurance pricing techniques and then narrowing down to focus on the extreme value analysis. Some empirical works in line with the objectives of the study are discussed focusing on the methods applied and the salient findings.

2.2 Definition of Reinsurance

Reinsurance has been defined in various ways by many researchers across the world. Rodermund (1965) defined reinsurance as insurance for insurance companies. Similarly, Patrik (2006) describes reinsurance as a legal insurance contract in which the reinsurer agrees to compensate the cedant (insurance company) for a specific proportion of specified types of insurance claims paid by the ceding company for a single policy or a group of policies. A proportional share (a percentage of each claim) or an excess-of-loss basis (the portion of each claim, or aggregation of claims, above a specified amount) can be used to describe the cession, or share of claims to be paid by the reinsurer. The essence and goal of reinsurance is to limit the variability of financial expenses incurred by insurance firms as a result of certain insurance claims occurring (Cummins et al., 2021). As a result, it boosts insurance market innovation, competition, and efficiency (Patrik, 2006). Additionally, Patrik (2006) explains that the coverage specified in a reinsurance contract is frequently a one-of-a-kind agreement between the two parties. It's challenging to make accurate generalizations about reinsurance because there are so many special cases and exceptions. Some legal jurisdictions do not regulate the form and wording of reinsurance contracts as strictly as they do for insurance contracts. The reinsurance between private companies is not regulated at all (Patrik, 2006). Simply put, reinsurance can be described as a mechanism for spreading risk, in which the traditional trade-off between the risk retained and the premium paid to the reinsurer is preserved (You et al., 2021; Butun, 2013).

Abramovsky (2008 p.345), on the other hand describes reinsurance as a source of independent and often unchecked contractual influence on insurer activity, and as a possible cause of interference with regulatory proposals. Abramovsky (2008) notes that even though reinsurance is initiated by private contract, those contracts have the potential for regulatory effect, accordingly, he describes reinsurance as a silent regulator.

Reinsurance differs from primary insurance in that it is much more directly tailored to the reinsured (Clark, 1996). According to Clark (1996), this creates the pricing paradox: "if you can price a contract precisely, the ceding company will refuse to buy it". Thus, if the recent records are capable enough to provide data for an accurate average loss estimate, the reinsured should be ready to take on the risk. This leads to any pricing tool usually only used as a starting point for determining a reasonable premium. When the rate-making process' assumptions aren't met, the actuary and underwriter must be able to agree on adjusting the premium.

2.3 Purpose of Reinsurance

Berger et al. (1992) claims that reinsurance contracts take different forms, which serve correspondingly diverse purposes. The purpose of reinsurance is to facilitate optimal diversification to enable insurers to participate fully in markets for risk coverages. Basically, reinsurance is designed to support direct insurers in two ways in terms of providing additional risk capacity and helping to spread risks across time, different types of businesses and industries, and geography (Haueter, 2021).

The most common type of liability insurance contract is an excess-of-loss contract, where the reinsurance company accepts to cover losses in excess of a retention level (Patrik, 1990). This type of reinsurance is a critical risk diversification method in the insurance market because it safeguards the ceding company against large losses and possible insolvency (Cummins et al., 2021).

2.4 Functions of Reinsurance

The primary nature of an insurance coverage is not altered by reinsurance. On a long-term basis, it is impossible to turn a bad business into a good one. However, it does provide the cedant with the following direct assistance. Reinsurance functions have been discussed in general under a range of concepts including capacity, stabilization, financial results management, and management advice (Ceci et al., 2022; Wang & Bølviken, 2022).

2.4.1 Capacity

If a cedant has reinsurance, larger policy limits can be written while keeping the risk level manageable. The net retained loss exposure for every policy or in total can be kept in check with the cedant's surplus by ceding shares of all policies or just larger policies. As a result, smaller insurance companies can compete with larger insurance companies, and policies that are larger than any single insurer's capacity can be written (Gutierrez & Anderson, 2018; Upreti & Adams, 2015). For example, Upreti and Adams (2015) provide empirical evidence to show that reinsurance helps to improve insurers' product market share in the United Kingdom, while Gutierrez and Anderson (2018) find that reinsurance facilitated the expansion of local insurance capacity in Spain and Brown (2017) for Ghana.

The term -"capacity" is also sometimes used to refer to the total volume of business. This component of capacity is best thought of as part of the larger category of financial results management.

2.4.2 Financial Results Management (Surplus Relief)

During periods of rapid premium growth, reinsurance relieves the strain on the reinsured's surplus. Reinsurance can change the income timing, boost statutory and/or financial surplus, and enhance a range of economic ratios used to assess insurers. An insurer with a growing book of business that is straining its surplus can cede a portion of their liability to a reinsurance company and use the reinsurer's surplus to supplement their own. This is primarily a surplus loan from the reinsurer to the ceding insurance company, which lasts until the cedant's surplus is sufficient to sustain the new business. This aspect of reinsurance, according to Patrik (2001), has led to some abuses in its application in free markets. This suggests that reinsurance provides an opportunity for the maximization of utility in terms of risk reduction. Examples of this could be seen in the the use of securitization and derivatives (Cummins & Barrieu, 2013).

2.4.3 Stabilization

Reinsurance is said to have a direct stabilizing effect on the insurance industry and the overall economy of a country (Haueter, 2021). It aids in the consistency of the reinsured's overall operating performance. Reinsurance can also aid a ceding company's underwriting and financial results stay consistent over the period, as well as safeguard the cedant's surplus from large, unexpected losses (Ceci et al., 2022). This is in line with Patrik's (2006) assertion that reinsurance helps to decrease the variability of financial costs incurred by stakeholders as a result of specific events. Reinsurance is typically written so that the cedant keeps the smaller, more predictable claims while sharing the larger, less predictable claims. As a result, the financial and underwriting implications of large accumulations of claims can be distributed evenly over time. Cummins et al. (2021) also observed that reinsurance helps firms to manage their risks through key activities, including reducing financial constraints and stabilizing the funding sources, which have the tendency to improve the firm's competitive advantage.

2.4.4 Management Advice

Reinsurance provides a source of underwriting data to the reinsured when introducing a new product and/or entering a new line of insurance or market. A large number of reinsurers have the understanding and skills to offer informal advisory services to their ceding companies. This service can assist with underwriting, marketing, pricing, loss prevention, claims handling, reserving, actuarial, investment, and personnel issues among others. The reinsurer's self-interest motivates him to examine the cedant's operations critically and, as a result, to offer advice.

In most cases, the reinsurer has more practical experience pricing high-limit policies and dealing with large, unusual claims. By contacting a number of similar cedant companies, the reinsurer may also be able to offer an overview of risk arising. Market Entrance (assisting the reinsured spread the risk on new lines of business until premium volume reaches a certain point of maturity; can add confidence when in unfamiliar coverage areas); and Market Withdrawal (providing a means for the reinsured to withdraw from a line of business, geographic area, or production) are some of the other functions of reinsurance (Butun, 2013).

In the discussion of the reinsurance, however, it is important to appreciate its global scope, which stood at nearly US 650 billion according to Aon (2021), which is even expected to increase in the years of ahead because of increasing uncertainty of global economy. Furthermore, the reinsurance industry is expected to play a significant role beyond the economy to such areas as climate change, natural catastrophes, and state reinsurance (Cummins et al., 2021; Surminski et al., 2022).

2.5 Forms of Reinsurance

Reinsurance contracts come in a number of different forms. Each type of reinsurance contract specifies the rules for ceding risk to the reinsurer. Treaty, facultative, and facultative Obligatory Treaty are the three types of reinsurance (Butun, 2013).

2.5.1 Treaty Reinsurance

In a treaty reinsurance, there is an agreement between the reinsurer and reinsured on a particular size and class of the reinsurance contract. For a specified coverage period, a treaty reinsures a portion of the loss exposure for a group of insurance policies (Patrik, 2001). The claims covered by an ongoing treaty may be those that occur during the term of the treaty or those that occur on policies written during the term.

The term "occurring" in the context of claims-made coverage refers to claims being made to the ceding company during the term. The premium for the treaty is based on policies of the specified type that were in effect or written during the treaty's term. The Annual Statement line of business, or some variants or subsets thereof, is usually used to describe the subject exposure. Since an ongoing treaty relationship includes a close sharing of much of the insurance exposure, the parties can form a close working partnership; the cedant can use the reinsurer's or broker's expertise and services. This is especially true when it comes to direct-written treaties or when a strong reinsurer is in charge of a brokered treaty.

2.5.2 Facultative Reinsurance

This is a reinsurance risk that is placed through a separate contract agreement rather than being ceded under a reinsurance treaty (Brockett et al., 1991). A facultative certificate only reinsures a single primary policy. Its primary purpose is to offer extra capacity. It's used to protect a portion of certain large, especially harmful, or unexpected exposures to cap their possible effect on the cedant's net results.

The reinsurer underwrites and accepts every certificate separately, similar to how primary insurance underwrites and accepts individual risks. Because facultative reinsurance often covers higher-risk or unexpected risks, the reinsurer must be made aware of the possibility for anti-selection (adverse selection) within and among insured classes. When property certificate coverage is written proportionally, the reinsurer compensates a fixed percentage of each claim on the subject policy. The majority of casualty certificate coverage is written on a surplus basis, which means the reinsurer pays a portion of each claim that exceeds the subject policy's fixed value attachment point.

2.5.3 Facultative Obligatory Treaty Reinsurance

The insured chooses the particular risk to cede to the reinsurer in this type of reinsurance contract. All cessions made by the reassured must be accepted by the reinsurer as long as they are within the scope of the treaty. Many primary policies of a specific type are reinsured by a facultative automatic agreement.

The exposure is uniform because these policies are typically very similar. Its primary goal is to increase capacity, but it also provides some stability because it covers so many policies. It's a group of facultative certificates all underwritten at the same time. It can provide proportionate or excess coverage. It is generally written to cover new or unique programs advertised by the cedant, and the reinsurer may work hand-in-hand with the cedant to design the main underwriting and pricing guidelines. For instance, a facultative automatic agreement could cover 90% of the cedant's private umbrella business, in which case the reinsurer would almost definitely offer expert advice and closely monitor the cedant's underwriting and pricing (Patrik, 2006).

Automatic facultative agreements are typically written at a fixed cost, with no retrospective premium adjustments or variable ceding commissions like treaties. Non-obligatory agreements exist in which the cedant is not obligated to cede and the reinsurer is not required to assume all policies of the specified type.

2.6 Types of Reinsurance

There are four main types of reinsurance contracts; they are: Excess of loss, Stop loss, Quota Share and Surplus. The rules and processes used in determining the premiums are peculiar to the types of reinsurance contracts.

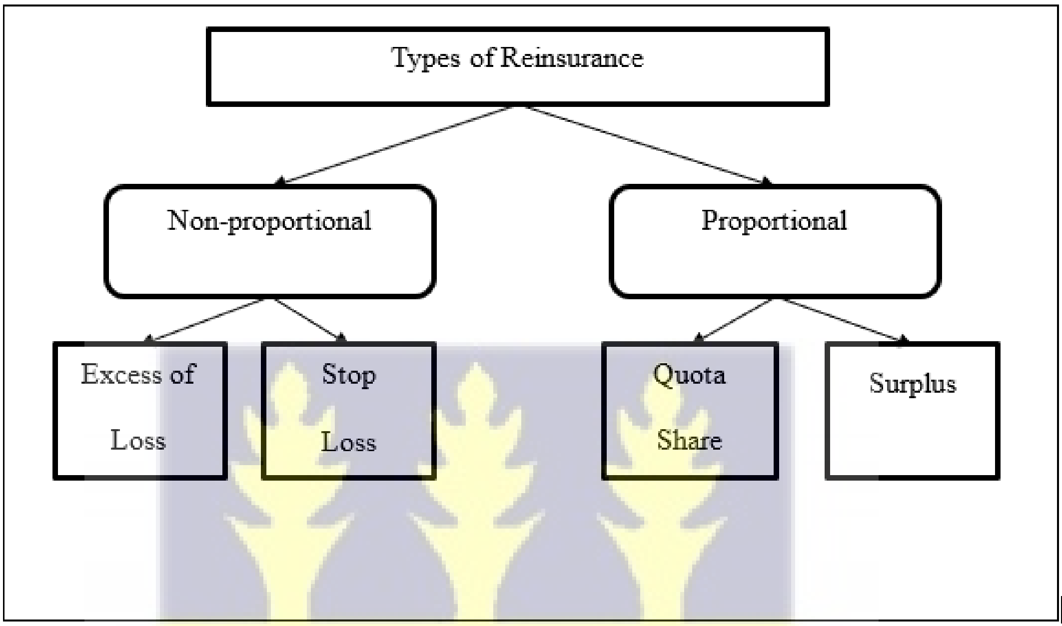


Figure 2.1: Types of Reinsurance

2.6.1 Non-Proportional Reinsurance

Each type of reinsurance contract can be classified into four categories: Quota share, Surplus, Excess of loss, and Stop loss. Each specific type of the reinsurance contract has its own set of rules and processes for determining how much premium the reinsured must pay the reinsurance company and how much recoveries the reinsurer should pay the reinsured.

Excess-of-Loss: It is a form of non-proportional reinsurance that covers stipulated losses suffered by the reinsured that exceed a specified retention level.

Stop Loss: It is also known as excess-of-loss ratio reinsurance. It stipulates that the reinsurer will make either a full or partial payment of the reinsured's losses that exceed a given percentage of the reinsured's premium to the reinsured's entire account. This entire account is mostly subject to the overall policy limit.

2.6.2 Proportional Reinsurance

It is also a reinsurance type wherein the reinsurance company splits the premiums received and the claims paid by the reinsured, as well as some associated costs. Proportional reinsurance takes the form of quota share and surplus line treaties.

Quota Share - It requires that the reinsured must transfer to the reinsurer a fixed percentage of all premiums received in respect of a particular section or all of its underwriting account for a specific period in exchange for the reinsurer paying the same percentage of any claims and stipulated expenses arising on the reinsured account for the same period.

Surplus - The cedant's line is a type of reinsurance in which bands of coverage known as lines are granted above a given retention. Each line is the same size, and the treaty's capacity is expressed as a multiple of the cedant's line, so a three-line treaty with a retention of ¢2 million would provide reinsurance coverage of ¢6 million in excess of ¢2 million. The reinsurer is paid an equal share of the total risk premium. A surplus treaty is a type of reinsurance that is proportional to the risk. The focus of this research is on excess-of-loss reinsurance.

2.6.3 Features of Excess of Loss Reinsurance

Excess and Limit - The attachment point or excess point, as well as the contract's limit, are the most important aspects of an excess-of-loss reinsurance contract. A ¢5m x ¢5m excess layer, for instance, implies the contract will ensure that the reinsured is safeguarded for any loss in excess of ¢5m but less than ¢10m. The formula below is used to calculate the recoveries:

$$Recoveries = Min[Max(X - E, 0), L]$$

Where X: Loss amount of a single chain: E: Excess point; L: Limit

ROL and Return Period - The term ROL (Rate On Line) or return period is frequently used to price a XoL layer. A 10% ROL for a ¢5m x ¢5m layer for instance means that the reinsured should pay a premium of ¢500,000.00 to the reinsurer. Below is the definition of the return period which is given as:

$$ReturnPeriod = 1/RoL$$

It simply implies that for a 10% ROL, the layer will be completely burned once every ten years. In general, a ROL can be as high as 45% but in practice, a reinsured would be willing to pay no more than 35%.

Adjustment Rate and M & D - A reinsurance contract cover guards against losses resulting from the reinsured's underlying book. Because the reinsurance contract covers the coming year and is purchased before the reinsured begins writing any business, the price is determined solely by the reinsured's Estimated Premium Income (EPI). The reinsured may initially write more than he stated. It means that the reinsured is exposed to a significantly larger amount of risk, making it riskier for the reinsurer.

As a result, when the reinsurance contract begins, the reinsured pays a non-refundable M&D (Minimum and Deposit) to the reinsurer depending on the EPI given by the reinsured. If the reinsured has written more business after the contract expires, the reinsurer will be paid an additional premium. The additional premium is calculated as :

$$AdditionalPremium = AdjustmentRate * (WrittenPremiumIncome - EPI)$$

where Adjustment Rate=(M&D)/EPI

The M&D premium is non-refundable if the reinsured wrote less business than expected.

The rationale for the assumptions is simple since there is just one assumption, namely claim sizes that reflect a portfolio of equal risks. Using the assumptions, a more easy and simplified modeling is created, implying that insurance firms should divide their exposure into homogeneous portfolios in order to precisely monitor the risk.

The independence of claim sizes and claim arrivals simplifies computation, but it also indicates that a period of numerous claim occurrences is unlikely to coincide with a period of greater claim sizes. These assumptions are not required to be satisfied. In order to comprehend reinsurance and insurance policies, the next part will offer an explanation of a broad collective risk model. Using the previously mentioned assumptions, the model for claims processing is specified as a counting process.

2.7 Reinsurance and Claim Sizes

Some insurance portfolios are too volatile $S(t)$ (total loss processes) to be handled properly. In order to meet this volatility, insurance companies have mutual agreements to share the risk and the premium of the portfolio (Mikosch, 2009). Sharing of risk is not much different from a community founding its own insurance company to share the risks among the insured who are also the owners. Such companies are usually referred to as mutual and have a long history in Europe.

There are many different types of re-insurance agreements who all are designed to share the risk between insurance companies in their own way and many of the usual types of agreements aim to modify the total loss process S(t) in some kind of way. Mikosch (2009) describes the three common treaties of types of reinsurance.

The Extreme Value Theory field was pioneered by Leonard Tippett and R.A. Fisher in 1928. Tippet was working with the British Cotton Industry Research Association where he was assigned to focus on making the cotton thread stronger. In his research, he noticed that the weakest fibers determine the strength of a thread. R.A. Fisher assisted Tippet to obtain the three asymptotic limits that describe the distributions of the extremes assuming independent variables.

The book published by Emil Julius Gumbel (Statistics of Extremes, 1958) codified this theory and has for many years been considered as the main reference point for the practice of extreme value theory especially in engineering. The major starting point for theoretical developments in EVT was L. de Haan's doctoral dissertation on Regular Variation and its Applications to the Weak Convergence of Sample Extremes, published in 1970 (Beirlant et al., 2001). Theoretical developments from that period till date has increased significantly in all fields of study.

McNeil (1997) applied parametric curve-fitting methods for modelling extreme historical losses under generalised Pareto distribution and extreme value theory. This research focused on showing that fitting GPD to insurance losses which exceed high thresholds is a useful method for estimating the tails of loss severity distributions. This modelling approach was developed in Davison (1984) and Davison and Smith (1990). McNeil (1997) established that the tail of the claim distribution may be sensitive to the choice of the threshold.

Diagnostic tests such as the Q-Q plot were used to assess if the distribution of the data is heavy tailed. With over 2000 data points used, it was concluded that the tail of the data is heavy tailed since it was compared to the exponential distribution which is medium-sized tailed.

The author also developed a software written in Splus to fit the GPD to exceedances of high thresholds and to produce the kinds of graphical output shown in their findings. His findings buttress the fact that shape and scale parameters for the tails of loss severity distributions are very important for pricing the high excess loss layers in reinsurance.

In the Insurance sector, Beirlant et al. (2001) proposed a parametric model, termed generalized Burr-gamma distribution. They illustrated these models using the Norwegian Fire claims portfolio. Models in EVT methods are generally non-parametric and semi-parametric in nature which only helps to assess a portfolio risk in the tail of the claim. However, Beirlant et al. (2001) shared that for reinsurance rating, there is a need for a fully parametric model to assess a portfolio risk in both tails and more central parts of the distribution in different layers. This paper used all three models, thus, non-parametric approach, semi-parametric approach (specifically Pareto-type distribution which is deduced from the limit theory) and finally the proposed parametric approach (generalized Burr-gamma distribution) to efficiently estimate the retention level, R and the reinsurance premium. The inspection of the tail behavior, which is a requirement for accurate premium estimations, especially with reinsurance layers that cover the highest risks, is a recurring theme throughout this approach.

In the finance sector, the research work done by Gilli and Këllezi (2006) focuses on the use of EVT to compute the tail risk measures such as Value-At-Risk (VaR), Expected Shortfall (ES), and return period and the related confidence intervals which was applied to several major stock market indices. Both EVT Methods

(Block Maxima Method (BMM) and Peak Over Threshold (POT)) were used in demonstrating that EVT is very useful for assessing the size of extreme events. The BMM method divided the large observations into equally distributed blocks to obtain the block maxima data. Using the Maximum Likelihood estimator (MLE), the block maxima were fitted with the Generalized Extreme Value Distribution (GEV) (which is a combination of three main families of distributions namely: Fréchet, Gumbel and Weibull Distributions) to estimate the shape parameter, ξ after which the tail risk measures were computed.

The POT method also focused on the exceedances, thus, realizations beyond a given threshold, u. Using the MLE, they fitted the exceedances with the Generalized Pareto Distribution to obtain the estimates ξ (shape parameter) and σ (scale parameter). After which the tail estimator was constructed and tail risk measures were also computed using this approach.

The authors show that the POT method was better than the BMM method in assessing the size of extreme events because the POT better exploits the information in the data sample for the desired results.

In addition, Adebayo and Sunday (2017) focused on the modelling of oil and gas insurance claims for five Nigerian insurance companies to estimate their Value-at-Risk (VaR) using different statistical approaches. Under normal or exceptional market conditions, the VaR measures the worst or minimal expected loss over a period for a given probability. VaR is a good risk indicator because of its ability to compress all market factors into a single number.

Diagnostic tests such as the Mean-excess-plot and Q-Q plot were used to determine the threshold and also assess the shape (thus, the tail index) of the distribution of the tail. The VaR was estimated using different approaches, namely; Historical approach (non-parametric), Generalized Pareto Distribution approach (GPD) under the POT method based on EVT (semi-parametric method) and also the Gaussian approach which is parametric in nature.

The detection of the extreme values in the claims distributions was carried out using Grubb' Test which showed the presence of extreme values in the dataset. Based on the various approaches used, it was established that the GPD-EVT under the POT method is the most suitable method to estimate the VaR. The EVT Method of calculating the VaR is also suitable due to the fact this method estimates the VaR outside the sampling interval and also its ability to model the tail area of the distribution very well.

Prepic (2018) also fitted a flexible extreme value mixture model consisting of a non-parametric bulk part and a parametric tail part to Danish fire insurance data using Bayesian methods and an estimation with a Markov chain Monte Carl (MCMC) algorithm. The posterior distributions of the mixture model parameters were estimated together with the threshold which divides the data into the bulk part and the tail part. A further application on Excess-of-Loss reinsurance contract price estimation was done. The model gave a large variance non-symmetric distribution to all the parameters making the model difficult to use in pricing of reinsurance contract based on the results.

Oyewole et al. (2018) also employed EVT on historical claims data of four Nigerian insurance companies to estimate the minimum expected claims for the marine and aviation insurance class of business in Nigeria, thus, the tail risk measure VaR.

The marine and aviation industry is one of Nigeria's most important sectors, contributing significantly to the economy by facilitating trade and commerce, transporting goods and people, and creating jobs, among others. The insurer is responsible for providing insurance against risks such as loss of marine vessels and aviation hulls, marine and air cargoes, freights, collision, and passenger liability. This highlights the need for the insurer to calculate the VaR, or the minimum expected loss over a given time period with a given probability. In this study, three outlined techniques for estimating the VaR namely: Gaussian approach(Normal Distribution) , Historical approach(non-parametric) and GPD (semi-parametric) are estimated and compared. The Grubb's test and quartile test were used in finding extreme values or outliers in the data.

Mean-excess plot was once again used to help determine the threshold in order to obtain exceedances to fit GPD model with Maximum Likelihood Estimator based on EVT. Also, the linear Q-Q and diagnostic plots employed showed that the parametric model fits the data well. This paper also revealed that the extreme value method using the Maximum Likelihood estimate estimate on the GPD outperforms other methods of estimating the VaR as a result of the EVT method having the ability to model the tail area of the distribution much better.

In the research field of hydrology, Tanaka and Takara (2002) in their study used both the Annual Maximum Series (AMS) and Peaks-Over-Threshold (POT) in the frequency analysis of the extreme values of floods and storms. It was shown that the POT procedure is better than the AMS method in the case of short records.

This paper focused more on the methods used in determining the optimal threshold which is very crucial in the POT analysis. The theoretical distributions namely; exponential and GPD were used for samples of mean areal rainfalls which were evaluated at two sites; Shimono ($3534km^2$) and Ryouu Ridge ($6519km^2$) in the Mogami River Basin, Japan. These samples have no clear pre-determined physical threshold. The parameters of the theoretical distributions used were estimated using the L-moment method.

In order to determine the optimal number of upper extremes, they compared the fluctuations of six indices namely; (I) Quantile corresponding to exceedance probability of 1/100 (ii) Parameters of distribution (iii) Standardized Least Squares Criterion (SLSC) (iv) Jack-knife error (v) Automatic choice using shape parameter (vi) Sample Mean Excess Function (SMEF).

Their comparisons showed that the parameters of the theoretical distributions perform very well in determining the optimal threshold and using some of these indices is efficient in accepting or rejecting a threshold.

Kong and Lü (2021) investigated the problem of excess-of-loss reinsurance and investment for an insurer seeking to maximize the terminal wealth's expected exponential utility. A Brownian motion with drift was used to describe the insurer's surplus process, while a stochastic factor influenced the claim arrival process, insurance and reinsurance premiums.

The risky asset in the financial market was also assumed to have time-varying and random coefficients. Both the value function and the corresponding optimal strategies were obtained and characterized using the Hamilton- Jacobi-Bellman (HJB) equation approach under various premium calculation principles. The empirical review revealed that various modeling techniques can be applied to test for extreme values in data sets for financial services, particularly insurance, and across diverse industries. The review reveals that The Extreme Value Theory technique of computing VaR surpasses other methods of estimation since it is renowned for its superior ability to predict the distribution's tail region. They further show that other estimation methods perform well with high-profile data, but VaR estimated using the EVT method still performed better regardless of the amount of data sets.

Most of the reviewed researches fitted GPD to only the tail of the distribution but in this thesis, we focus on using the splicing GPD which consists of Mixed Erlang (ME) Distribution and the Generalized Pareto Distribution in fitting the whole of our data where the ME will be used to fit the part of the data below the tail while the GPD will be used to fit the tail of the distribution.

Chapter 3

Methodology

3.1 Introduction

Extreme Value Theory (EVT) is a statistical approach that enables the modelling of the occurrence of extreme events (McNeil, 1999). Gilli and Këllezi (2006) demonstrated that EVT is used to model the maxima of a random variable in the same way that the central limit theorem is used to model the sums of independent random variables. The theory informs us of what the limiting distributions are in both instances. However, it must be noted that, this can be extended to the sample minimum and the reader is referred to Coles (2001). This chapter takes a look at the EVT methods; Peaks-Over-Threshold (POT) and the Block Maxima Method(BMM) used in Extreme Value Analysis. In addition, the estimation of the tail index, theoretical setting of GPD, and risk measures adopted by practitioners in estimating insurance risk.

3.2 EVT Methods

There are two main methods in identifying extremes in a sample of the data.They are:

1. **Block Maxima Method (BMM)**

   The Block Maxima Method is a traditional approach which is influenced by the limit behavior of the normalized maximum of a random sample

(Davinson et al., 1990). It involves grouping the data into blocks of equal length and find the maxima of each block (Özari et al., 2019). This approach proposes the model, Generalized Extreme Value distribution (GEV) to be fitted to the maxima of each block and also to estimate the risk measures.

2. **Peaks Over Threshold (POT)** The POT method gives a more natural approach in assessing whether an observation is extreme or not. All observations exceeding a certain value, thus, threshold are considered as an extreme value. As a result, the POT Method appears to be the preferred method in current applications because it makes efficient use of data.

   In the next subsections we describe these methods in details.

3.3 Block Maxima Method (BMM)

The block maxima approach of fitting extreme value distributions involves partitioning all observations of the data into different groups of the same length where the greatest or the maximum observations for every group are taken into consideration (Özari et al., 2019; Ferreira & De Haan, 2015). This section covers the distribution of the Maxima, limiting distribution of the maxima, The Generalized Extreme Value (GEV) Distribution

3.3.1 Distribution of the Maxima

Suppose the claims amount denoted by the sequence of random variables ($r.vs$) $X_1, X_2, \ldots, X_n$ are independent and identically distributed ($i.i.d$), then the distribution function (d.f) of a sequence of maxima (M_n) is defined as:

$$M_n = max\{X_1, X_2, \ldots, X_n\} \tag{3.1}$$

which is given as:

$$P(M_n \leq x) = F^n(x) \tag{3.2}$$

where $F(x)$ is the marginal distribution function of the sequence $X_1, X_2, \ldots, X_n$. Let M_n be a sequence of maxima obtained from the sample $(X_1, X_2, \ldots, X_n)$, that is,

$$M_n = max\{X_1, X_2, \ldots, X_n\}$$

Since $X_1, X_2, \ldots X_n$ are $r.vs$, it implies that M_n is also a $r.v$. By definition of the $d.f$ of a $r.v$, we have that the $d.f$ of M_n is $P(M_n \leq x)$.
Therefore $(M_n \leq x) \implies (X_1 \leq x, X_2 \leq x, \ldots, X_n \leq x)$, therefore,

$$P(M_n \leq x) = P(X_1 \leq x, X_2 \leq x, \ldots, X_n \leq x). \tag{3.3}$$

Since $(X_1, X_2, \ldots, X_n)$. are independent, we have that:

$$P(X_1 \leq x, X_2 \leq x, \ldots, X_n \leq x) = P(X_1 \leq x) \cdot P(X_2 \leq x) \cdots P(X_n \leq x). \tag{3.4}$$

Also, $(X_1, X_2, \ldots, X_n)$ are $i.i.d$ with distribution function $F(x)$,

$$\prod_{i=1}^{n} P(X_i \leq x)) = F(x) \cdot F(x) \cdots F(x) \qquad (n\ times) \tag{3.5}$$

$$= F^n(x)$$

$$\therefore\ P(M_n \leq x) = F^n(x).$$

However, in this thesis we are interested in determining the limiting distribution of the maxima as the sample size increases. The theorem 1 gives the limiting distribution of the maxima.

3.3.2 Limiting Distribution of the Maxima

Theorem 1. *Let $F(x)$ be the underlying distribution function of a sequence of random variables and x^F is it's right endpoint that is $x^F = \sup\{x : F(x) < 1\}$, which may be infinite. Then*

$$\lim_{n\to\infty} M_n \xrightarrow{p} x^F \tag{3.6}$$

where $\xrightarrow{p}$ means convergence in probability.

By definition, a sequence of random variables $Y_1, Y_2, \ldots, Y_n, \ldots$ converges in probability to X, iff $\forall\epsilon > 0$,

$$\lim_{n\to\infty} P\left(|Y_n - X| \geq \epsilon\right) = 0. \tag{3.7}$$

Now, $|M_n - x^F| \geq \epsilon$ can be expressed as a union of two mutually exclusive intervals. That is,

$$|M_n - x^F| \geq \epsilon = M_n \geq \epsilon + x^F \cup M_n \leq x^F - \epsilon. \tag{3.8}$$

Hence,

$$\begin{aligned} P(|M_n - x^F| \geq \epsilon) &= P(M_n \geq \epsilon + x^F \cup M_n \leq x^F - \epsilon) \\ &= P(M_n \geq \epsilon + x^F) + P(M_n \leq x^F - \epsilon). \end{aligned}$$

Since $x^F = \sup\{x : F(x) < 1\}$, we have that,

$$P(M_n \geq \epsilon + x^F) = 0.$$

Therefore,

$$P(|M_n - x^F| \geq \epsilon) = P(M_n \leq x^F - \epsilon). \tag{3.9}$$

However, the distribution function of the maxima is given as $F^n(x)$*, hence*

$$P(M_n \leq x^F - \epsilon) = F^n(x^F - \epsilon) \tag{3.10}$$

Substituting (3.10) *into* (3.9) *we have that,*

$$P(|M_n - x^F| \geq \epsilon) = F^n(x^F - \epsilon). \tag{3.11}$$

Taking the limit as $n \to \infty$,

$$\lim_{n\to\infty} P(|M_n - x^F| \geq \epsilon) = \lim_{n\to\infty} F^n(x^F - \epsilon).$$

Since $F^n(x^F - \epsilon) < 1$ *by definition of* x^F *we have that,*

$$\lim_{n\to\infty} F^n(x^F - \epsilon) = 0.$$

Therefore,

$$\lim_{n\to\infty} P(|M_n - x^F| \geq \epsilon) = 0. \tag{3.12}$$

Theorem 1 suggests that, the limiting distribution of the sample maximum converges in probability to a degenerate distribution defined as;

$$\lim_{n\to\infty} F^n(x) = \begin{cases} 0, \text{ if } x < x^F \\ 1, \text{ if } x \geq x^F. \end{cases} \tag{3.13}$$

To make meaningful inference, a non-degenerate distribution is required. A concept similar to the central limit theorem is required to normalize the degenerate function in (3.13) to a more suitable non-degenerate distribution function.

Theorem 2. ***(Fisher and Tippett(1928),Gnedenko (1943)):*** *Let* X_n *be a sequence of independent and identically distributed (i.i.d) random variables (rvs),* *where $n = 1, 2, 3, \ldots$. If there exist constants $c_n > 0$, $d_n \in \mathbb{R}$ and some non-degenerate distribution function H such that"*

$$\frac{M_n - d_n}{c_n} \to H,$$

then H belongs to one of the three standard extreme value distributions.

$$Frechet : \Phi_\alpha(x) = \begin{cases} 0, & x \leq 0 \quad \alpha > 0 \\ e^{-x^{-\alpha}}, & x > 0 \end{cases} \tag{3.14}$$

$$Weibull : \Psi_\alpha(x) = \begin{cases} e^{-(-x)^\alpha}, & x \leq 0 \quad \alpha > 0 \\ 0, & x > 0 \end{cases} \tag{3.15}$$

$$Gumbell : \Lambda(x) = e^{-e^{-x}}, x \in \mathbb{R} \tag{3.16}$$

Theorem 2 suggests that M_n (the maxima order statistic) can be normalized by selecting suitable constants d_n and c_n. However, it is not practicable to adopt the limiting distribution of just one of the three standard extreme value distributions whiles ignoring the rest.

To resolve this challenge, a universal extreme value distribution that comprises of all the three standard extreme value distributions is deemed necessary.

3.3.3 Generalized Extreme Value Distribution(GEV)

Jenkinson(1955) and Von Mises (1954) suggested that the three standard extreme value distributions belong to a one parameter family of distribution given as:

$$H_\xi(x) = \begin{cases} e^{-(1+\xi x)^{-1/\xi}}, & \text{if } \xi \neq 0 \\ e^{-e^{-x}}, & \text{if } \xi = 0, \end{cases} \tag{3.17}$$

such that values of x satisfies $1 + \xi x > 0$. This representation is referred to as the generalized extreme value (GEV) distribution.

To obtain the distributions of the standard extreme value distribution, the following conditions on ξ must be satisfied:

i. for $\xi = \alpha^{-1} \Rightarrow$ Frechet distribution.

ii. for $\xi = -\alpha^{-1} \Rightarrow$ Weibull distribution.

iii. for $\xi = 0 \Rightarrow$ Gumbell distribution.

The GEV distribution is particularly useful since the limiting distribution of the maxima is not pre-determined. It is also helpful in finding the maximum likelihood estimate for the unknown parameters.

3.3.4 Maximum Likelihood estimates for the Distributional Parameters

Suppose $X_1, X_2, \ldots, X_n$ are $i.i.d$ random variables following $GEV(\mu, \sigma, \xi)$. Using the maximum likelihood estimation criteria, estimates for the unknown parameters μ, σ, ξ are found by maximizing the likelihood function. Proof of the maximum likelihood estimates of the parameters (μ, σ, ξ) is given below. The likelihood function which is a product of the marginal density functions of the random variables X_i is given as:

$$
\begin{aligned}
L(\mu, \sigma, \xi) &= \prod_{i=1}^{n} \frac{1}{\sigma} \left(1 + \xi \frac{x_i - \mu}{\sigma}\right)^{-\frac{1}{\xi} - 1} \cdot \exp\left[-\left(1 + \xi \frac{x_i - \mu}{\sigma}\right)^{-\frac{1}{\xi}}\right] \\
&= \frac{1}{\sigma^n} \left[\prod_{i=1}^{n} \left(1 + \xi \frac{x_i - \mu}{\sigma}\right)\right]^{-\frac{1}{\xi} - 1} \cdot \exp\left[-\sum_{i=1}^{n} \left(1 + \xi \frac{x_i - \mu}{\sigma}\right)^{-\frac{1}{\xi}}\right]
\end{aligned}
$$

Taking the logarithm of both sides gives:

$$\log(L(\mu,\sigma,\xi)) = \log\left(\frac{1}{\sigma^n}\left[\prod_{i=1}^{n}\left(1+\xi\frac{x_i-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}-1}\cdot\exp\left[-\sum_{i=1}^{n}\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right]\right)$$
$$= -n\log(\sigma) - \left(\frac{1}{\xi}+1\right)\sum_{i=1}^{n}\log\left(1+\xi\frac{x_i-\mu}{\sigma}\right) - \sum_{i=1}^{n}\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}.$$

Taking the partial derivative with respect to the parameters yields the following three equations:

$$\frac{\partial\log L}{\partial\mu} = \frac{1+\xi}{\sigma}\sum_{i=1}^{n}\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-1} - \frac{1}{\sigma}\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}-1}, \qquad (3.18)$$

$$\frac{\partial\log L}{\partial\sigma} = -\frac{n}{\sigma}+\frac{1+\xi}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-1}-\frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}-1}, \qquad (3.19)$$

$$\frac{\partial\log L}{\partial\xi} = \frac{1}{\xi^2}\sum_{i=1}^{n}\log\left(1+\xi\frac{x_i-\mu}{\sigma}\right) - \frac{1+\xi}{\xi\cdot\sigma}\sum_{i=1}^{n}(x_i-\mu)\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-1}$$
$$-\frac{1}{\xi^2}\sum_{i=1}^{n}\log\left(1+\xi\frac{x_i-\mu}{\sigma}\right)\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}}+\frac{1}{\xi\cdot\sigma}\sum_{i=1}^{n}(x_i-\mu)\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-\frac{1}{\xi}-1}. \qquad (3.20)$$

Solving equation (3.18), (3.19), and (3.20) simultaneously gives the maximum likelihood estimates $\hat{\mu},\hat{\sigma},\hat{\xi}$ for the parameters μ,σ,ξ respectively. However, there is no closed analytical solution for this simultaneously equation unless a numerical approach is used. These estimators are assumed to be unbiased and asymptotically normal (Ferreira & De Haan, 2015).

3.3.5 Quantile Estimation

Extreme quantile estimation at a level p can be obtained by finding x_p such that:

$$P(X \leq x_p) = p. \tag{3.21}$$

The *pth* quantile of the GEV distribution is derived by converting (3.17)

$$x_p = \mu + \frac{\sigma}{\xi}\left[(-\log p)^{-\xi} - 1\right]. \tag{3.22}$$

Let $H_{\xi,\sigma,\mu}(x)$ be the distribution function of the three parameter family of distribution for the generalized extreme value. Then by equation (3.21),

$$H_{\xi,\sigma,\mu}(x_p) = p. \tag{3.23}$$

However, $H_{\xi,\sigma,\mu}(x) = e^{-\left(1+\xi\frac{x_p-\mu}{\sigma}\right)^{-1/\xi}}$, hence from equation (3.23) we have,

$$H_{\xi,\sigma,\mu}(x_p) = e^{-\left(1+\xi\frac{x_p-\mu}{\sigma}\right)^{-1/\xi}} = p. \tag{3.24}$$

3.3.6 Estimation of Risk Measures

Quantile of the Generalized Extreme Value Distribution

Taking the logarithm of both sides of equation (3.24) and simplifying we have,

$$
\begin{aligned}
-\left(1+\xi\frac{x_p-\mu}{\sigma}\right)^{-1/\xi} &= \log p \\
1+\xi\frac{x_p-\mu}{\sigma} &= (-\log p)^{-\xi} \\
\xi\frac{x_p-\mu}{\sigma} &= (-\log p)^{-\xi}-1 \\
\therefore x_p &= \mu+\frac{\sigma}{\xi}\left[(-\log p)^{-\xi}-1\right]. \qquad (3.25)
\end{aligned}
$$

The pth quantile of the GEV distribution is $x_p = \mu+\frac{\sigma}{\xi}\left[(-\log p)^{-\xi}-1\right]$.

(3.25) can be used to compute the risk measures such as Value at Risk, Expected Shortfall, among others.

Return period of the GEV Distribution

A return period with period T (in years) is defined as the level expected to be exceeded on average once in every T years. The return period can be expressed mathematically as:

$$x_T = \mu+\frac{\sigma}{\xi}\left[\left(-\log\left(1-\frac{1}{T}\right)\right)^{-\xi}-1\right] \qquad (3.26)$$

Suppose x_T represents the return period with period T. Then,

$$P\left(X > x_T\right) = \frac{1}{T}.$$

Applying the axioms of probability we have that,

$$P\left(X \leq x_T\right) = 1 - \frac{1}{T}. \tag{3.27}$$

The distribution function of $P\left(X \leq x_T\right) = H_{\xi,\sigma,\mu}(x)$, hence substituting in equation (3.27) gives,

$$P\left(X \leq x_T\right) = H_{\xi,\sigma,\mu}(x) = 1 - \frac{1}{T}.$$

However, $H_{\xi,\sigma,\mu}(x) = e^{-\left(1+\xi\frac{x_p-\mu}{\sigma}\right)^{-1/\xi}}$, therefore equation (3.27) becomes,

$$H_{\xi,\sigma,\mu}(x_T) = e^{-\left(1+\xi\frac{x_T-\mu}{\sigma}\right)^{-1/\xi}} = 1 - \frac{1}{T}. \tag{3.28}$$

$$\begin{aligned} -\left(1+\xi\frac{x_T-\mu}{\sigma}\right)^{-1/\xi} &= \log 1 - \frac{1}{T} \\ 1+\xi\frac{x_T-\mu}{\sigma} &= \left(-\log 1 - \frac{1}{T}\right)^{-\xi} \\ \xi\frac{x_T-\mu}{\sigma} &= \left(-\log 1 - \frac{1}{T}\right)^{-\xi} - 1 \\ \therefore x_T &= \mu + \frac{\sigma}{\xi}\left[\left(-\log 1 - \frac{1}{T}\right)^{-\xi} - 1\right] \end{aligned}$$

The return period can be expressed as:

$$x_T = \mu + \frac{\sigma}{\xi}\left[\left(-\log\left(1 - \frac{1}{T}\right)\right)^{-\xi} - 1\right].$$

3.4 Peaks-Over Threshold Approach

This method concentrates on and selects just those observations from the entire sample that exceed a certain high and defined threshold, and then fits the suitable parametric model to the excesses above that threshold.

3.4.1 Generalised Pareto Distribution

A random variable X follows the Generalized Pareto if its distribution function is given as:

$$GP_{\xi,\sigma}(y) = \begin{cases} \left(1+\xi[\frac{y-u}{\sigma}]\right)^{-\frac{1}{\xi}}, & \text{if } \xi \neq 0 \\ e^{-\frac{y-u}{\sigma}}, & \text{if } \xi = 0, \end{cases} \tag{3.29}$$

where $y \in (u, \infty)$ if $\xi \geq 0$ and $y \in \left(u, u - \frac{\sigma}{\xi}\right)$ if $\xi < 0$.

3.4.2 Maximum Likelihood Estimation of GPD

Although there are several methods of finding the parameters of the GPD, such as the Maximum Likelihood Estimates (MLE), the Probability Weighted Methods (PMW), and Method of Moments (MM), among others, the most popular is the MLE. The maximum likelihood estimation is easy to understand and generally has estimators that are consistent and efficient in most cases (Brenner & Miller, 2001)

After the threshold has be obtained through either the use of the mean residual life plot (mrl) or any other suitable method, the maximum likelihood can be used in estimating the parameters, ξ and σ (Coles, 2001).

As pointed out by Coles (2001), if the u is large enough, then conditioned on $X > u$, the distribution of $X - u$ could be written as:

$$GP_{\xi,\sigma}(y) = 1 - \left(1 + \frac{\xi y}{\sigma}\right)^{-\frac{1}{\xi}}, \tag{3.30}$$

and any distribution that follows (3.30) is defined as the Generalized Pareto Distribution.

Hence the standard distribution function of the GPD can be written as:

$$GP_{\xi,\sigma}(y) = \begin{cases} 1 - \left(1 + \frac{\xi y}{\sigma}\right)^{-\frac{1}{\xi}}, & if \;\; \xi \neq 0 \\ 1 - e^{-\frac{y}{\sigma}}, & if \;\; \xi = 0 \end{cases} \tag{3.31}$$

Differentiating the standard CDF of the GPD in (3.31) one obtained the probability density function:

$$f(\xi, \sigma) = \begin{cases} \frac{1}{\sigma}\left(1 + \frac{\xi y}{\sigma}\right)^{-\frac{1+\xi}{\xi}}, & if \;\; \xi \neq 0 \\ \frac{1}{\sigma} e^{-\frac{y}{\sigma}}, & if \;\; \xi = 0 \end{cases} \tag{3.32}$$

Assuming that the values $y_1, y_2, ...y_k$ are the k exceedances of threshold, u.Taking the case where $\xi \neq 0$, the maximum likelihood can be established as:

$$L(\xi, \sigma) = \prod_{i=1}^{k} \frac{1}{\sigma}\left(1 + \frac{\xi y_i}{\sigma}\right)^{-\frac{1+\xi}{\xi}}, \tag{3.33}$$

taking the natural log of (3.33), we obtain the log-likelihood function, $\ell(\xi, \sigma)$

$$= \sum_{i=1}^{k} \log \frac{1}{\sigma} + \sum_{i=1}^{k} \log \left(1 + \frac{\xi y_i}{\sigma}\right)^{-\frac{1+\xi}{\xi}}$$

$$\ell(\xi, \sigma) = -k \log \sigma - \frac{1+\xi}{\xi} \sum_{i=1}^{k} \log \left(1 + \frac{\xi y_i}{\sigma}\right). \tag{3.34}$$

(3.34) holds if $1 + \frac{\xi y_i}{\sigma}$ is positive for $i = 1, 2, ..., k$ partially differentiating $\ell(\xi, \sigma)$ with respect to the ξ and σ respectively, we have (3.35) and (3.36) as:

$$\frac{\partial \ell(\xi, \sigma)}{\partial \xi} = -\frac{1+\xi}{\xi \sigma} \sum_{i=1}^{k} y_i (1 + \frac{\xi y_i}{\sigma})^{-1} + \frac{1}{\xi^2} \sum_{1=1}^{k} \log \left(1 + \frac{\xi y_i}{\sigma}\right), \tag{3.35}$$

Equation (3.35) and (3.36) are solved numerically to obtained the estimators $\hat{\xi}$

and $\hat{\sigma}$

$$\frac{\partial \ell(\xi,\sigma)}{\partial \sigma} = -\frac{k}{\sigma} + \frac{1+\xi}{\sigma^2}\sum_{i=1}^{k} y_i(1+\frac{\xi y_i}{\sigma})^{-1}, \tag{3.36}$$

Taking the case where $\xi = 0$ for the pdf of the GPD, the maximum likelihood function is:

$$L(\sigma) = \prod_{i=1}^{k} \frac{1}{\sigma} e^{-\frac{y_i}{\sigma}}, \tag{3.37}$$

Taking the natural log of both side, we obtain the log likelihood function, $\ell(\sigma)$:

$$\ell(\sigma) = \sum_{i=1}^{k} \log \frac{1}{\sigma} + \sum_{i=1}^{k} \log e^{-\frac{y_i}{\sigma}},$$

$$\ell(\sigma) = -k \log \sigma - \frac{1}{\sigma}\sum_{i=1}^{k} y_i, \tag{3.38}$$

taking the derivative of $\ell(\sigma)$ with respect to σ, we have;

$$\frac{d\ell(\sigma)}{\sigma} = -\frac{k}{\sigma} + \frac{1}{\sigma^2}\sum_{1=1}^{k} y_i. \tag{3.39}$$

(3.39) is also solved numerically to obtain the estimator, $\hat{\sigma}$ in the case where $\xi = 0$.

3.4.3 Estimation of Risk Measures

Quantile of The Generalized Pareto Distribution

A quantile y_p of a random variable X is defined by:

$$P\left(X \leq y_p\right) = p. \tag{3.40}$$

The pth quantile of the GPD distribution is mathematically expressed as $y_p = u + \frac{\sigma}{\xi}\left\{\left[\frac{1-p}{P(X>u)}\right]^{-\xi} - 1\right\}$. Let $GP_{\xi,\sigma}(y)$ be the distribution function for the Generalized Pareto Distribution. Then by equation (3.40),

$$GP_{\xi,\sigma}(y_p) = p. \tag{3.41}$$

However, $GP_{\xi,\sigma}(y) = 1 - P(X > u)\left(1 + \xi\frac{y-u}{\sigma}\right)^{-1/\xi}$, hence from equation (3.41) we have,

$$GP_{\xi,\sigma}(y_p) = 1 - P(X > u)\left(1 + \xi\frac{y_p - u}{\sigma}\right)^{-1/\xi} = p. \tag{3.42}$$

Simplifying equation (3.42) gives,

$$\begin{aligned}
\left(1 + \xi\frac{y_p - u}{\sigma}\right)^{-1/\xi} &= \frac{1-p}{P(X > u)} \\
\left(1 + \xi\frac{y_p - u}{\sigma}\right) &= \left[\frac{1-p}{P(X > u)}\right]^{-\xi} \\
\therefore x_p = u + \frac{\sigma}{\xi}&\left\{\left[\frac{1-p}{P(X > u)}\right]^{-\xi} - 1\right\}
\end{aligned}$$

The pth quantile of the $GP_{\xi,\sigma}(y)$ distribution is $x_p = u + \frac{\sigma}{\xi}\left\{\left[\frac{1-p}{P(X>u)}\right]^{-\xi} - 1\right\}$.

Return period of The GPD

A return period with period T (in years) is defined as the level expected to be exceeded on average once in every T years (Gilli & Këllezi, 2006). The return period can be expressed mathematically as:

$$y_T = u + \frac{\sigma}{\xi}\left\{\left[\frac{1}{mTP(X > u)}\right]^{-\xi} - 1\right\}. \tag{3.43}$$

Suppose y_T represents the return period with period T. Then,

$$P(Y > y_T) = \frac{1}{mT},$$

where m denotes the average number of extremes per year. Applying the axioms of probability we have that,

$$P(Y \leq y_T) = 1 - \frac{1}{mT}. \tag{3.44}$$

The distribution function of $P(Y \leq y_T) = GP_{\xi,\sigma}(y)$, hence substituting in equation (3.44) gives,

$$P(Y \leq y_T) = GP_{\xi,\sigma}(y) = 1 - \frac{1}{mT}.$$

However, $GP_{\xi,\sigma}(y_T) = 1 - P(X > u)\left(1 + \xi\frac{y_T - u}{\sigma}\right)^{-1/\xi}$, therefore equation (3.44) becomes,

$$GP_{\xi,\sigma}(y_T) = 1 - P(X > u)\left(1 + \xi\frac{y_T - u}{\sigma}\right)^{-1/\xi} = 1 - \frac{1}{mT}. \tag{3.45}$$

Simplifying equation (3.45) gives,

$$y_T = u + \frac{\sigma}{\xi}\left\{\left[\frac{1}{mTP(X > u)}\right]^{-\xi} - 1\right\}.$$

The return period can be expressed as:

$$y_T = u + \frac{\sigma}{\xi}\left\{\left[\frac{1}{mTP(X > u)}\right]^{-\xi} - 1\right\}.$$

Value at Risk (VaR) of the GPD

The VaR as a measure of risk is used to quantify the loss within an insurance firm at a small probability or risk level thus, it is used to estimate the future loss of the insurance company. It estimates the effect of an adverse event that could be more devastating than what has already been observed. The VaR of a claim paid is the minimum value of the claim amount distribution such that the probability of the future loss larger than this value is not beyond a given probability (Tse, 2009). The VaR could be used to assess three major quantities: the size of the future loss to the insurance company, the probability of futuere loss, and the time period. Assuming the CDF of the GPD is F, then the VaR is defined as the qth quantile of F. VaR could be calculated by solving (3.39)

$$F(\text{VaR}_p) = q \tag{3.46}$$

which can be written as:

$$\text{VaR}_p = F^{-1}(q) \tag{3.47}$$

where F^{-1} is the inverse of the CDF, F and from (3.3) the CDF of a GP(σ, ξ) distribution has been shown as:

$$GP_{\xi,\sigma}(y_p) = 1 - P\left(X > u\right)\left(1 + \xi \frac{y_p - u}{\sigma}\right)^{-1/\xi} = p. \tag{3.48}$$

This implies that if $p = 1 - q$

$$1 - q = 1 - P\left(X > u\right)\left(1 + \xi \frac{y_p - u}{\sigma}\right)^{-1/\xi} \tag{3.49}$$

but $P\left(X > u\right)$ is normally estimated by $(\frac{N_u}{n})$ where n is the total number of observations and N_u is the number of observations beyond the threshold u, and hence,

$$1 - q = 1 - \frac{N_u}{n}\left(1 + \xi \frac{y_p - u}{\sigma}\right)^{-1/\xi} \tag{3.50}$$

simplifying (3.47) we have

$$y_p = u + \frac{\sigma}{\xi}\left[\left(\frac{nq}{N_u}\right)^{-\xi} - 1\right]. \tag{3.51}$$

Therefore, since $\text{VaR}_p = y_p$, we have

$$VaR_p = u + \frac{\sigma}{\xi}\Big[\Big(\frac{nq}{N_u}\Big)^{-\xi} - 1\Big]. \tag{3.52}$$

Expected Shortfall (ES) of the GPD

Besides the VaR, another powerful measure of risk is the Expected Shortfall (ES). The expected shortfall is used to determine the amount of future loss that is beyond the VaR. It is seen as more coherent than the VaR (Artzner et al., 1999 as cited in Gilli & Këllezi, 2006). The limitation of the VaR is that, it is able to make use of only the thresholds associated with the probability value and does not incorporate any other information about the tail of the distribution into the measure beyond the probability thresholds given (Gilli & Këllezi, 2006). The ES caters for this limitation and therefore could be called Tail Value-at-Risk (TVaR) or Conditional Tail Expectation (CTE) for a continuous random variable which can also be defined as a high quantile of the conditional distribution of the potential loss of a system proxy such as a market index (or a financial institution) conditional on the event that one institution (or the system) is in distress (Nolde & Zhou, 2022). Simply put, the ES is the average of the quantiles in excess of the VaR (Tse, 2009). That is;

$$\text{ES}_p = \text{E}(X|X > \text{VaR}_p) \tag{3.53}$$

The ES is also called Conditional VaR (CoVaR) or Conditional Tail Expectation if the random variable X is continuous (as in our case). (3.53) can be rewritten as;

$$\text{ES}_p = \widehat{\text{VaR}}_p + \text{E}(X - \widehat{\text{VaR}}_\text{p}|X > \widehat{\text{VaR}}_p) \tag{3.54}$$

Given the linear function of the mean excess plot for the GP distribution as;

$$e(h) = \mathrm{E}(X - h \mid X > h) = \frac{\sigma + \xi h}{1 - \xi} \tag{3.55}$$

where h is the threshold and $\xi < 1$, and $\sigma + \xi h > 0$. The mean excess function produces the expected values of exceedances of X over different values of the threshold, h. Using (3.54) and (3.55) and defining $h = \mathrm{VaR} - u$, the expected shortfall can be rewritten as;

$$\begin{aligned}
\mathrm{ES}_p &= \widehat{\mathrm{VaR}}_\mathrm{p} + \frac{\hat{\sigma} + \hat{\xi}(\widehat{\mathrm{VaR}}_\mathrm{p} - u)}{1 - \hat{\xi}} \qquad (3.56)\\
&= \widehat{\mathrm{VaR}}_\mathrm{p} + \frac{\hat{\sigma}}{1 - \hat{\xi}} + \frac{\widehat{\mathrm{VaR}}_\mathrm{p}\hat{\xi}}{1 - \hat{\xi}} + \frac{-u\hat{\xi}}{1 - \hat{\xi}}\\
&= \widehat{\mathrm{VaR}}_\mathrm{p} + \frac{\widehat{\mathrm{VaR}}_\mathrm{p}\hat{\xi}}{1 - \hat{\xi}} + \frac{\hat{\sigma}}{1 - \hat{\xi}} + \frac{-u\hat{\xi}}{1 - \hat{\xi}}\\
&= \frac{\widehat{\mathrm{VaR}}_\mathrm{p} + \widehat{\mathrm{VaR}}_\mathrm{p}\hat{\xi} - \widehat{\mathrm{VaR}}_\mathrm{p}\xi}{1 - \hat{\xi}} + \frac{\hat{\sigma} - u\hat{\xi}}{1 - \hat{\xi}}
\end{aligned}$$

Therefore,

$$\mathrm{ES}_p = \frac{\widehat{\mathrm{VaR}}_\mathrm{p}}{1 - \hat{\xi}} + \frac{\hat{\sigma} - u\hat{\xi}}{1 - \hat{\xi}}. \tag{3.57}$$

Excess Loss Premium-XLP

The Excess-of-loss premium is associated with the excess-of-loss reinsurance contract. In this type of insurance, a primary insurance company will have its claim beyond a certain limit paid by a reinsurer. This premium paid by the reinsurer is called excess loss premium. This clearly means that, this is the premium that is paid when a certain limit of loss is exceeded. For any excess loss (XL) reinsurance contract, the excess loss premium (XLP) is given by the fundamental formular;

$$\Pi(R) = e(R)\bar{F}(R), \tag{3.58}$$

where $\bar{F}$ is the survival function of the GP model.

From (3.49), it can be inferred that

$$\bar{F}(R) = F(u)\left(1+\xi\frac{R-u}{\sigma}\right)^{-1/\xi} \tag{3.59}$$

Where $F(u) = \frac{N_u}{n} = \frac{k}{n}$, where N_u is the number of exceedances, or rather k is the number of extreme order statistics.

$$\bar{F}(R) = \frac{k}{n}\left(1+\xi\frac{R-u}{\sigma}\right)^{-1/\xi} \tag{3.60}$$

Similary from (3.55)

$$e(R) = \mathrm{E}(X-R \mid X>R) = \frac{\sigma+\xi h}{1-\xi} = \frac{\sigma}{1-\xi}(1+\xi R)$$

but $h = R-u$

$$\begin{aligned} e(R) = \mathrm{E}(X-R \mid X>R) &= \frac{\sigma+\xi(R-u)}{1-\xi} \\ &= \frac{\sigma}{1-\xi}\left(1+\xi\frac{R-u}{\sigma}\right) \end{aligned} \tag{3.61}$$

combining (3.59) and (3.61), the XLP for the GP model is;

$$\begin{aligned} \Pi(R) &= \frac{\sigma}{1-\xi}\left(1+\xi\frac{R-u}{\sigma}\right) \times \frac{k}{n}\left(1+\xi\frac{R-u}{\sigma}\right)^{-1/\xi} \\ &= \frac{k}{n}\frac{\sigma}{1-\xi}\left(1+\xi\frac{R-u}{\sigma}\right)^{1-\frac{1}{\xi}}. \end{aligned} \tag{3.62}$$

For instances where the EVI, ξ is greater than 1, the risk measures above cannot be computed hence, the need for the splicing method which uses the interval between 0 and 1, thus, the upper limit of the EVI will be strictly less than 1.

3.5 The Splicing Method of EVT

Proposed by Albrecher et al. (2017), the splicing method is used to fit the claims amount data accross the range of the small claims to the very few large claims. The splicing method involves dividing the data in to some m components where various distribution will be used to fit a particular component. The idea of the splicing method is a mixture of two or models which comes as a remedy to where Pareto-type distributions are not able to fit the left or below the tail part. The splicing method is important also in situations where an insurance claims data displays different type of behaviour within certain intervals of claim amount.

As given in Albrecher et al. (2017), the m-component spliced distribution has a density function:

$$f(y) = \begin{cases} \pi_1 \frac{f_1(y)}{F_1(l_1)-F_1(l_0)}, & l_0 < x \leq l_1, \\ \pi_2 \frac{f_2}{F_2(l_2)-F_2(l_1)}, & l_1 < x \leq l_2 \\ ... & \\ \pi_m \frac{f_m(y)}{F_m(l_m)-F_m(l_{m-1})}, & l_{m-1} < x \leq l_m \end{cases} \tag{3.63}$$

with $\pi_j > 0$ and $\sum_{j=1}^{m} \pi_j = 1$ where f_j and $F_j, (j = 1, ..., m)$ are the probability density functions and cummulative density functions respectively of the random variable Y. Sometimes differentiability and continuity of the density function is required for the points $l_1, l_2, ..., l_{m-1}$. Even though there are several splicing methods with mixture of two distributions (that is $m = 2$) such as composite exponential Pareto model proposed by Beirlant et al. (2004), composite log-normal Pareto model as proposed by Cooray and Ananda (2005) as cited in Albrecher et al. (2017), but in our analysis we adopted the two component mixture model of the Mixed Erlang distribution and Generalized Pareto in fitting.

The CDF of the mixture of the splicing of the Generalized Pareto Distribution with a Mixed Erlang distribution for fitting the body of the data.

$$1 - F(y) = \begin{cases} 1 - F_1(y), & y \leq u \\ (1 - F_1(u))\left(1 + \frac{\xi}{\sigma}(y-u)\right)^{-\frac{1}{\xi}}, & y > u. \end{cases} \quad (3.64)$$

F_1 is the CDF of the Mixed Erlang Distribution. Accordingly, the PDF, $f(.)$ of the Splicing Generalized Distributon is,

$$f(y) = \begin{cases} f_1(y), & y \leq u \\ (1 - F_1(u))\frac{1}{\sigma}\left(1 + \frac{\xi}{\sigma}(y-u)\right)^{-\frac{1+\xi}{\xi}}, & y > u. \end{cases} \quad (3.65)$$

The parameters σ and ξ are estimated using the MLE (Albrecher et al., 2017).

Chapter 4

Results and Analyses

In this chapter, we present the Extreme Value Analysis of two datasets on insurance claims from two Insurance companies in Ghana. This chapter is organized into three sections; Section 4.1 uses descriptive statistics such as the boxplot, histogram, mean excess plot and QQ plot to show the presence of extremes. Section 4.2 shows preliminary analysis such as the estimation of the Extreme Value Index (EVI) and also the goodness of fit. The quantiles and tail risk measures such as the Return Period, Value-at-Risk, Expected Shortfall, and Excess-of-Loss Premium are shown in section 4.3.

4.1 Descriptive Statistics

This section gives a detailed description of the claims distribution of both Company A and Company B.

Insurance Company A dataset consists of claims amount that has a sample size of 2085. The least claim paid by company A within the coverage period was GHC26.10 and the largest loss recorded was 15,346,800.00. From Table 4.1,

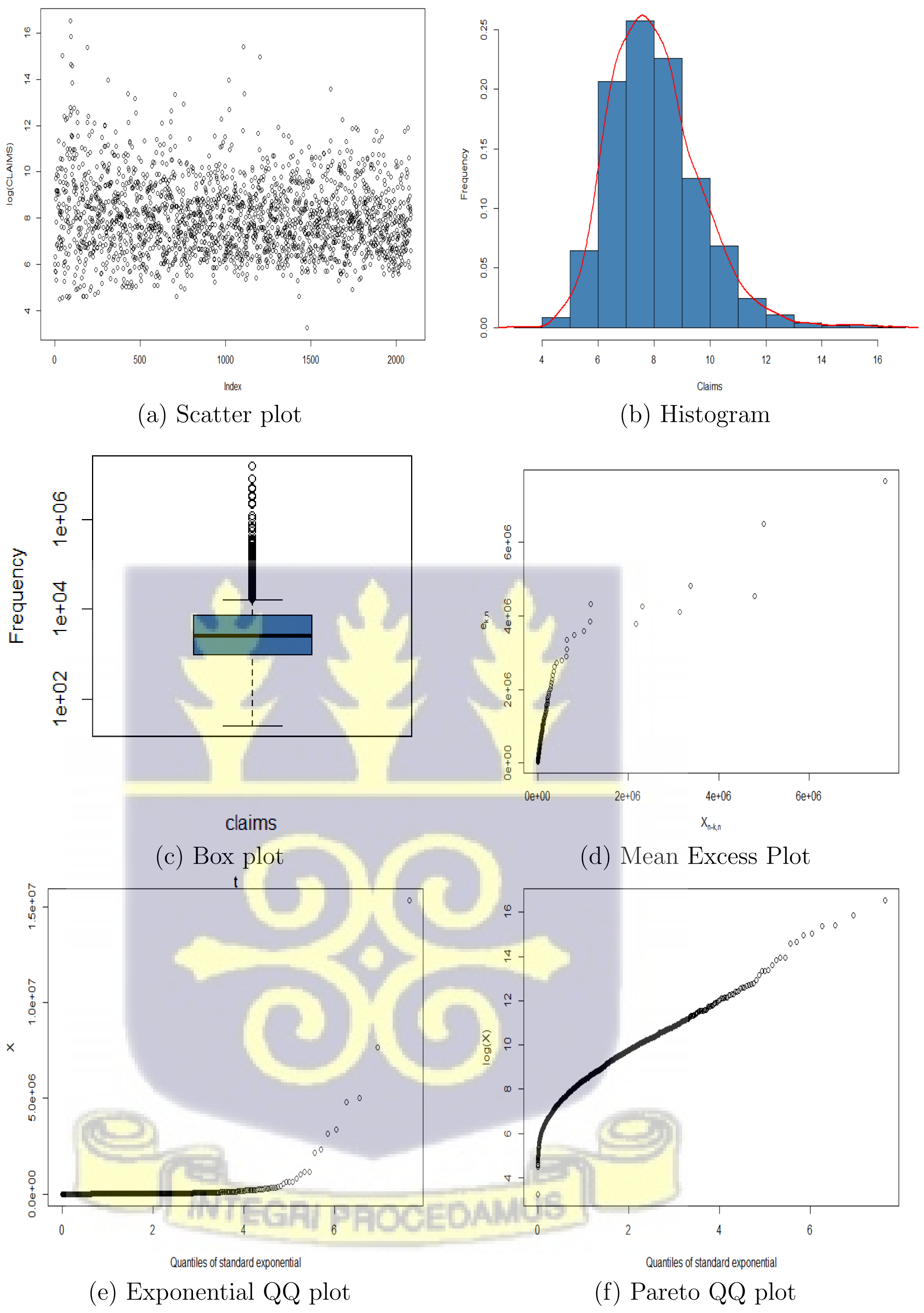
Table 4.1: Summary Statistics for Company A and Company B

Summary statistics	**Company A**	**Company B**
Minimum	26.10	180.00
Maximum	15,346,800.00	775,125.00
Mean	35,028.20	1,871.70
Standard dev.	426,593.20	43,162.32
Skewness	26.93	13.08
Kurtosis	862.49	216.65

the average claims the Insurance company A paid out was GHc 35,028.20. The standard deviation recorded was GHc 426,593.20 which is very far from an average claims of GHc 35,028.20 and shows that larger amount of claims are paid out. The dataset gave a positive skewness of 26.93 which shows the data is right-tailed as shown in the histogram. The data has a huge kurtosis of 862.49 which is greater than three and is termed as leptokurtic which shows heavy tailedness which does not depict normality. The skewness and the kurtosis signify that Company A recorded large claims.

From Table 4.1, it can be seen that, within the insurance period, a minimum claim amount of GHC180.00 was paid by Company B while the highest amount of claims paid was GHC774,945.00. The standard deviation of GHC43,162.32 is an indication of how distant the individual claims paid are from the average claim paid of GHC1,871.70. The skewness and the kurtosis of the data shows that the claims do not follow a normal distribution. The positive skewness and kurtosis are indications that the data is right skewed and heavy tailed. Practically, this implies that, there are few large claims. This is a common occurrence in the insurance sector where claims hardly occur but in the event of occurrence, the magnitude is huge.

From Figure 4.1(a), the scatter plot shows that most of the claims amount are within 1 million cedis to 10 million cedis. Figure 4.1(b) which is the Histogram and the density plots reveal that the distribution of the data is right tailed. The boxplot in Figure 4.1(c) clearly shows there are outliers in the Company A data set. The mean excess plot in Figure 4.1 (d) which shows the values of the mean excesses plotted against the exceedances increases linearly after which there were some distortions suggesting extreme values in the data. This behaviour of the mean excess plot is an indication that, the Pareto-type distribution will be a good fit for the data. This is confirmed also by the convex nature of the exponential QQ plot (Figure 4.1 (e)). The convex behaviour of the exponential QQ plot implies



(a) Scatter plot (b) Histogram

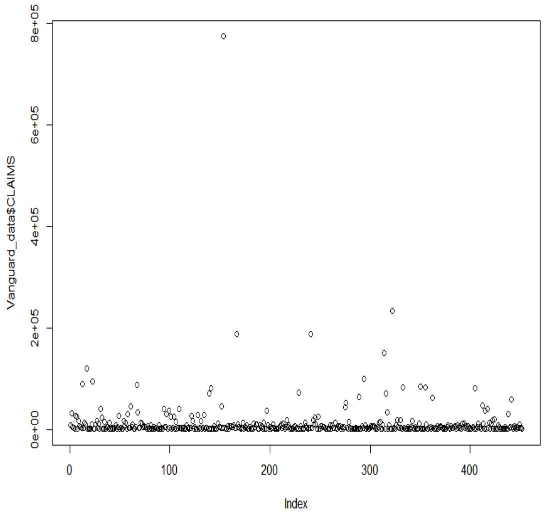
(c) Box plot (d) Mean Excess Plot

(e) Exponential QQ plot (f) Pareto QQ plot

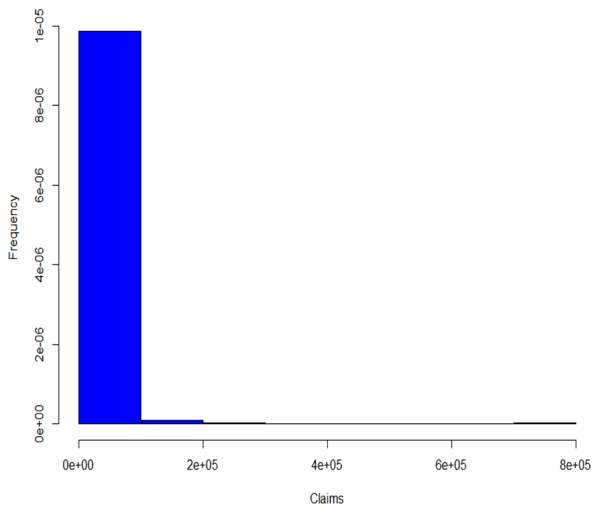
Figure 4.1: Descriptive Statistics for company A.

that the distribution of the data is heavier than the exponential distribution. The Pareto QQ plot (Figure 4.1(f)) looks linear which also means that the distribution of the data comes from the Pareto family.

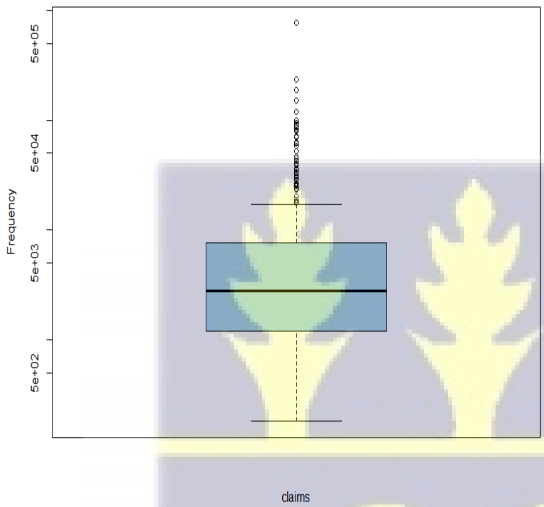
For Company B, the scatter plot in Figure 4.2(a) shows that, there is no clear relationship or pattern among the claims paid. However, it can be seen that most of the claims are clustered around GHC200.00 to GHC200,000.00. Few claims are above GHC200,000.00. Figure 4.2(b) reveals that most of the claims occurs below GHC100,000.00 and it can be seen hat, there are times where large claims occur but not often. The histogram confirms that, the dataset is right tailed, adding to the fact that there are extreme values in the data. The extreme nature of this data could also be substantiated by the portion of the histogram with 500,000.00 cedis and 600,000 cedis while there are no bars within 300,000.00 cedis and 500,000.00 cedis. Figure 4.2(c) is a representation of the presence of extreme values (outliers)in the dataset. The mean excess plot in Figure 4.2 (d) behaves linearly which indicates that, the distribution of the data is from the Pareto-family of distribution. This is also re-affirm by the convex behaviour of the exponential QQ plot. The Pareto QQ plot also looks linear which shows that the distribution of data comes from the pareto-type distribution.



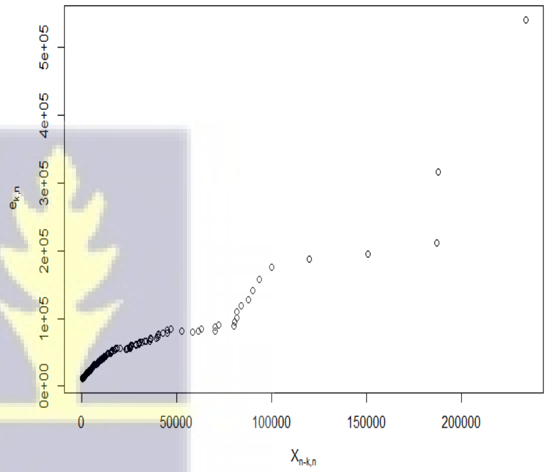
(a) Scatter plot



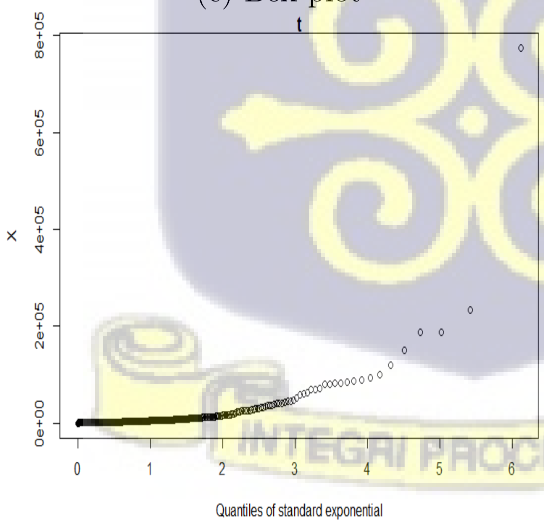
(b) Histogram



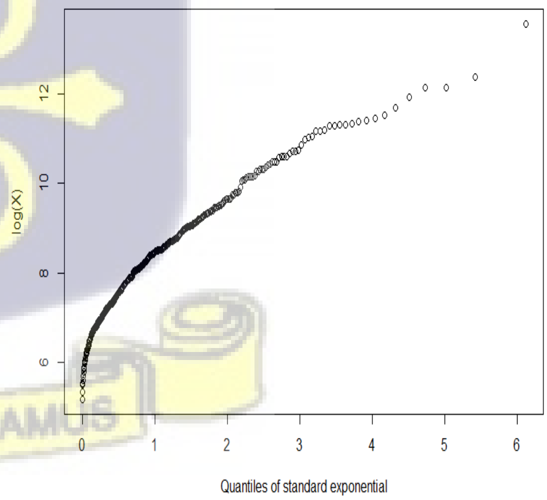
(c) Box plot



(d) Mean Excess Plot



(e) Exponential QQ plot



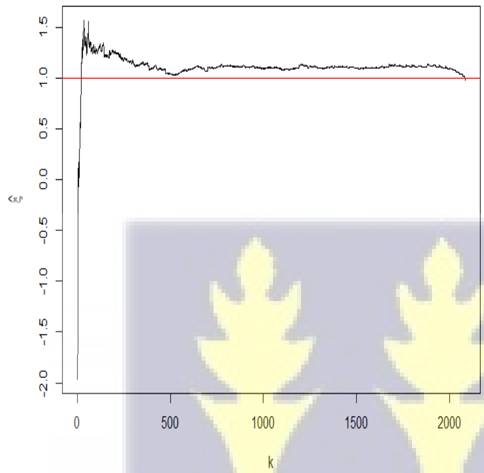
(f) Pareto QQ plot

Figure 4.2: Descriptive Statistics for company B.

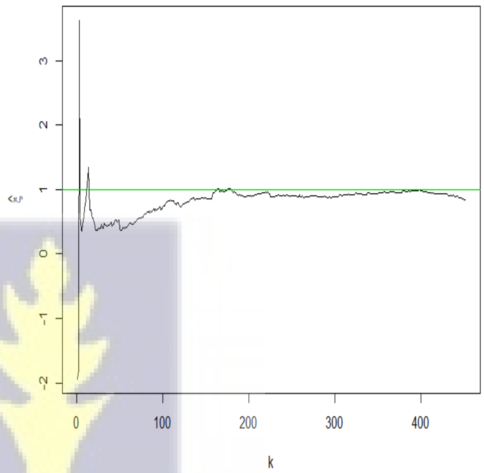
4.2 Preliminary Analysis

In the next two subsections, the results of the parameter estimation and diagnostics test were presented.

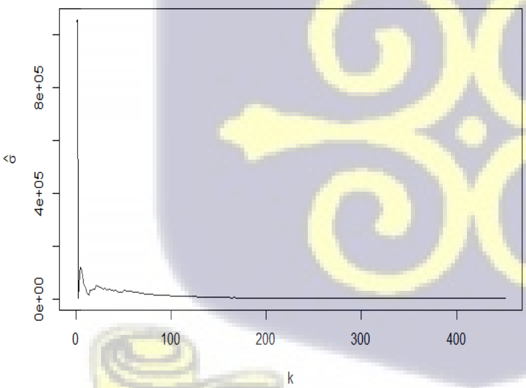
4.2.1 Parameter Estimation of the Splicing GPD



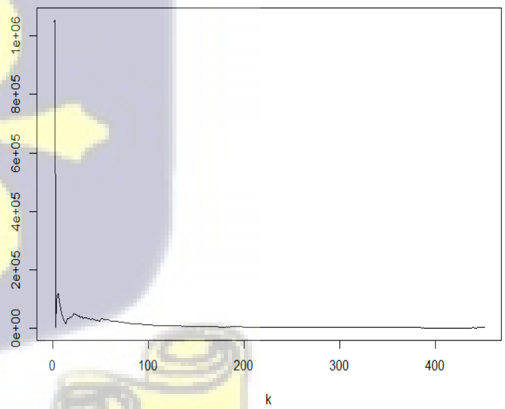
(a) EVI for Company A



(b) EVI for Company B



(c) scale parameter for Company A



(d) scale parameter for Company B

Figure 4.3: The Plots of Maximum Likelihood Estimators $\hat{\xi}$ and $\hat{\sigma}$ at each k

Figure 4.3 (a) and 4.3 (b) show the shape parameter, ξ as a function of k for Company A and Company B respectively. Figure 4.3(a) shows that, the extreme value index for Company A is greater than 1 for all values of k while that of

Table 4.2: Estimated parameters of ME distribution for Company A and Company B

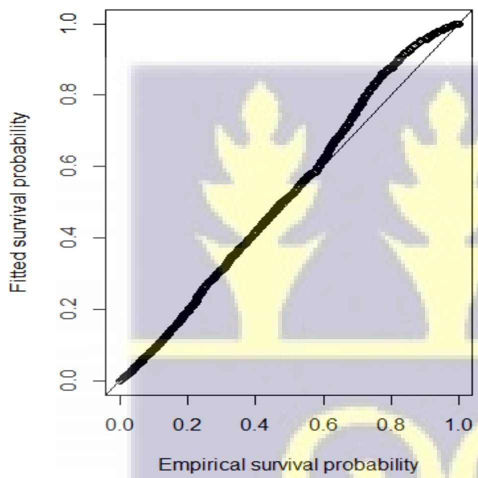
Estimated Parameter	**Company A**	**Company B**
Shape	3 - 9	3
Scale	261.2347	487.1354

company A is less than 1 for almost all values of k. At the stable part of the EVI plots, we have $\xi = 1.11$ and $\xi = 0.88$ for Company A and Company B respectively. Since $\xi > 0$ for both Company A and Company B, both datasets have heavy-tailed distributions.

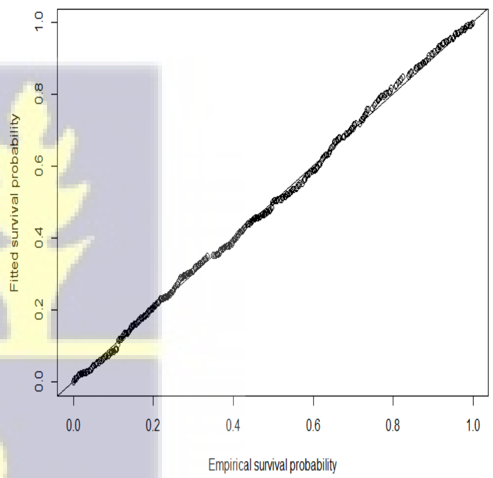
Figure 4.3 (c) and Figure 4.3 (d) show the estimated scale parameter, $\hat{\sigma}$ which was plotted against the k values for both Company A and Company B respectively. Table 4.2 also shows the estimated scale and shape parameters for the Mixed Erlang Distribution.

4.2.2 Goodness of Fit or Diagnostic Test

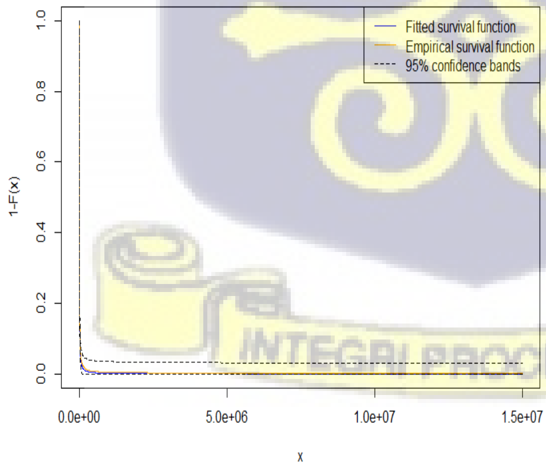
Figure 4.4 presents the goodness of fit and predictive accuracy of the splicing GPD Model. Figure 4.4(a) plots the probabilities of the theoretical or the fitted distribution against those of the empirical distribution and compares the probability values of the empirical data with those obtained from the fitted model to evaluate whether they agree. If the PP plots deviate a lot from the straight line, it shows that the theoretical distribution does not fit the data. From Figure 4.4(a), the PP plot fit the data quite well for Company A. But Figure 4.4(a) shows that, the splicing GPD fits well for Company B than company A datasets.



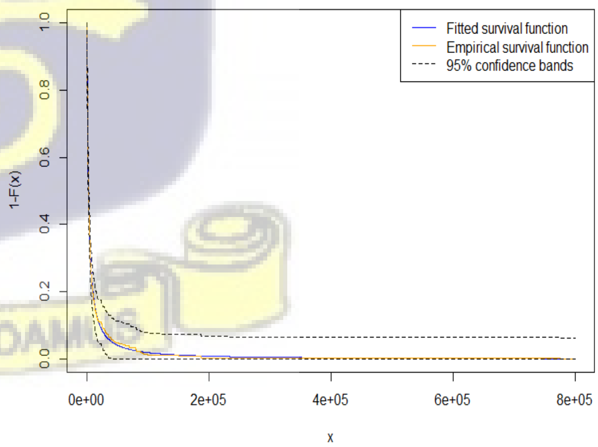
(a) PP plot for Company A



(b) PP plot for Company B



(c) Survival Plot for Company A



(d) Survival Plot for Company B

Figure 4.4: Diagnostics Tests

Figure 4.4(c) and 4.4 (d) describe the survival functions of the empirical distributions to the survival functions of the fitted or theoretical distributions. This function produces the survival probabilities at each point of observation. That is, the y-axis is the survival probabilities while the x-axis is the number of observations. It could be observed from Figure 4.4(c) and 4.4(d) that the empirical survival functions and the theoretical survival functions both decrease as usual when the amount of observations (claims) increases. From Figure 4.4(c) and 4.4 (d) the fitted survival plot (blue) agree mostly with the empirical survival plot (yellow). The 95% confidence interval (dashed line) indicates that the fitted model are within the bounds.

4.3 Tail Risk Measures

The risk measures obtained from fitting the mixed distribution on the Company are presented in this section. The return period, Value at Risk, Expected Shortfall, and Excess-of-loss premiums were calculated through the splicing GPD where the Mixed Erlang (ME) distribution was used to fit the body of the data while the GPD was used to fit the tail of the data which was done.

4.3.1 Return period as a Risk Measure

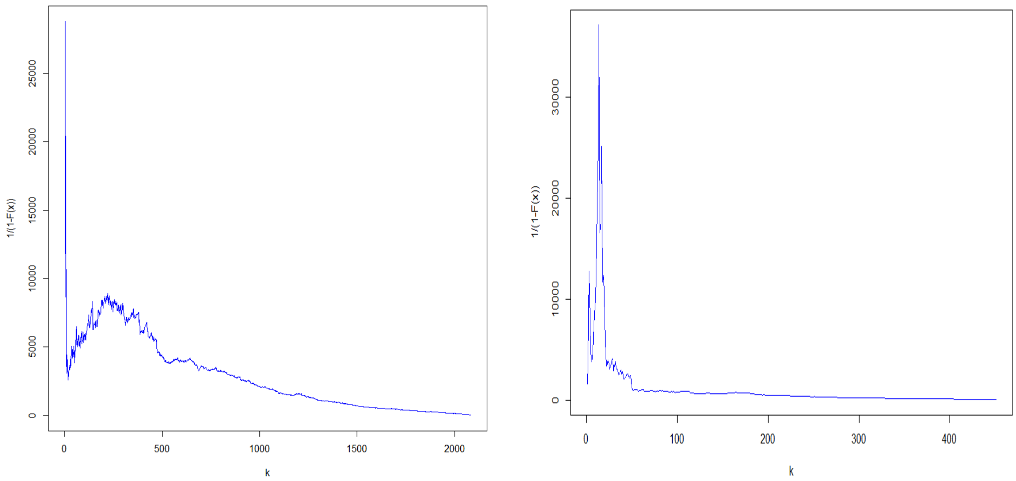
Table 4.3: Return period at different k values for Company A

k	20	500	1000	1500	2000
Return period	2,664.03	3,798.64	1,970.65	709.98	142.67

Table 4.4: Return period at different k values for Company B

k	5	100	200	300	400
Return period	4,091.02	835.17	346.79	102.61	265.40

The return period tells the number of claims that will be paid out before the possibility of a particular claims amount re-occuring. In thesis, the focus is



(a) Company A (b) Company B

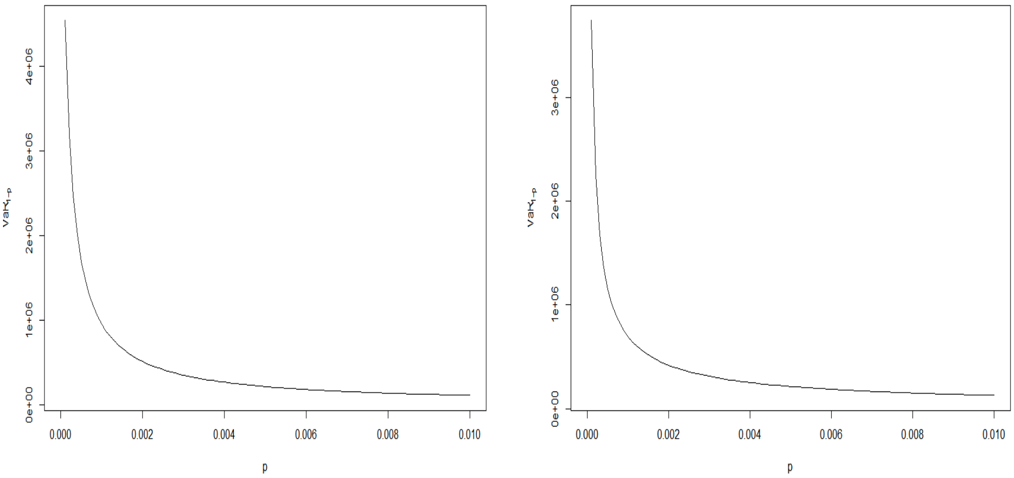
Figure 4.5: Return period Plots

thrice the maximum claims amount re-ocurring . As the exceedances decrease the return period decreases. From Figure 4.5 (a), the k value within the range of 500 - 700 gives a corresponding claims count of 3,800 - 4,000 claims as the number of claims that will occur before the possibility of thrice the maximum claims amount occurring again for Company A.

From Figure 4.5(b), the k value within the range of 100 -200 gives a corresponding claims count of 300 - 800 claims as the number of claims that will occur before the possibility of thrice the maximum claims amount occurring again for Company B.

4.3.2 Value-at-Risk(VaR)

The VaR quantifies the level above which future claim amount will increase with only a small probability. Practically, the VaR tells us the sufficient amount of capital to put down to cover claims that should not go beyond a certain probability. From Figure 4.6, it could be observed that, as the probability (p) increases, the VaR decreases.



(a) Company A (b) Company B

Figure 4.6: VaR Plots

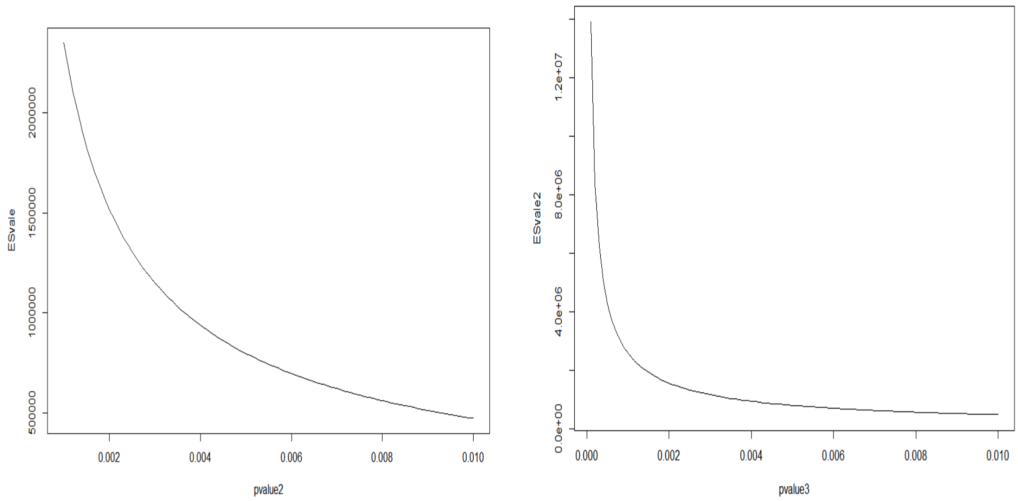
Table 4.5: VaR at Various Probabilities and Quantiles

p	0.002	0.004	0.006	0.008	0.01
$p\%$	0.2	0.4	0.6	0.8	1
$quantile(1-p)\%$	99.8	99.6	99.4	99.2	99
VaR Company A	515,912.30	273,139.90	166,320.00	137,187.30	108,054.60
VaR Company B	420,764.90	250,403.70	184,152.10	148,681.00	139,076.70

Table 4.5 shows the insurance company the worst loss to expect at different probability levels for Company A and Company B.

4.3.3 Expected Shortfall (ES)/Conditional Tail Expectation (CTE)

Figure 4.7 shows the values of the ES risk measure (which is the expected financial loss that exceeds a given VaR) plotted against the various probabilities values or risk levels for both Companies A and B. The ES plot at Figure 4.7 shows the various amount of losses given the VaR and the probabilities. Also, Table 4.6 shows that as the risk levels or probabilities increases, the ES decreases. One observation that can be made between the VaR Plot and the ES plot is that, the ES values are bigger than the VaR.



(a) Company A (b) Company B

Figure 4.7: Expected Shortfall Plots

Table 4.6: Expected Shortfall at Various Probabilities and Quantiles

p	0.002	0.004	0.006	0.008	0.01
$p\%$	0.2	0.4	0.6	0.8	1
$quantile(1-p)\%$	99.8	99.6	99.4	99.2	99
ES Company A	1,520,635.00	937,317.70	694,953.40	555,285.80	473,128.40
ES Company B	1,508,478.2	978,772.1	713,919	537,350.3	478,494.1

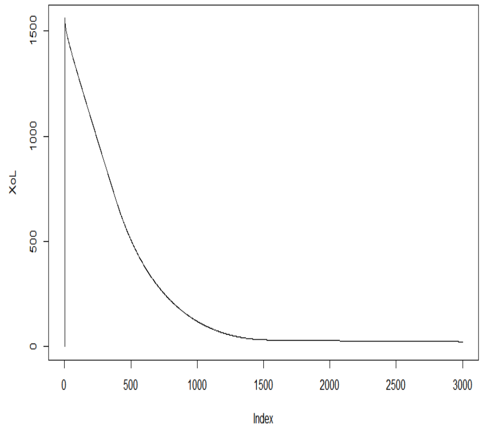
4.3.4 Excess-of-Loss Premium

The plot showed in Figure 4.8 illustrate the various amount of intervention from the reinsurance company A and Company B to the insurer when the retention level is thrice the maximum claim amount for both companies. It plots the excess loss premium (XLP) as against the ordered statistic. It tells us the amount of money the reinsurance company is supposed to pay on behalf of the insurer at various level of the ordered statistic.

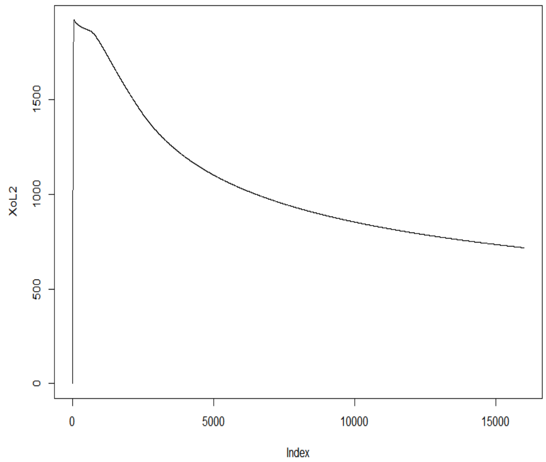
Table 4.7: Excess Loss Premium at different k values

k	500	1000	1500	2000
XOL Company A	482.04	122.92	37.42	30.58
XOL Company B	471.9795	363.3255	317.3564	283.9244

From Table 4.7, it can be seen that, as the ordered statistics (k values) increases (or the exceedances decreases), the excess of loss premium (XOL) decreases. This



(a) Company A



(b) Company B

Figure 4.8: XoL Plots

suggests that, the fewer the large claims the primary insurance has, the lower the amount to be reinsured and hence the lower the premium they have to pay to the reinsurance company.

Chapter 5

Conclusion and Recommendations

This chapter summarizes the results obtained from the analysis of the two claim datasets. Conclusion was also drawn in line with the objective of the study and recommendations were made for further research. Suggestions are made for the considerations of policymakers and industry players regarding insurance and reinsurance.

5.1 Summary of Findings

Company A dataset contains claims paid to 2085 policyholders within the period 2011 - 2020. The maximum amount of claims paid was GHC15,346,800.00 and the least amount of claim paid was GHC26.10. Skewness and kurtosis of 26.93 and 862.49 respectively, are indications of a right-tailed and heavy-tailed distribution. The mean excess plot of Company A revealed that the claim amount distribution shows heavy-tailedness.

The tail index obtained using the Maximum Likelihood Estimation of the GPD was 1.11. However, the splicing method was used to obtain a minimum tail index at 0.6931472 (since the estimation of the risk measures restrict the tail index to less than 1).

Risk measures such as return period, Value-at-Risk, Expected Shortfall, and Excess-of-Loss Premiums were estimated using the estimators obtained from the splicing method of the GPD. The return period of company A indicates that, the company is supposed to pay out about 3,800 to 4,000 claims before a claim amount of thrice the maximum value reoccurs. The VaR which is the worst loss a firm should expect at a given probability was GHC515,912.30 at 99.8% and GHC108,054.60 at 99%. The expected loss at 0.2% was GHC1,520,635.00 and GHC473,128.40 at 1%. These expected losses are monies the company need to put down at these risk probabilities to cover future losses given the Value at Risk.

The expected loss premium which is one of the practical risk measures used in reinsurance contract is used to determine the premium an insurance company is supposed to pay the reinsurance company in the event of a certain limit of claims being exceeded. These excess loss premiums for the company was GHC482.04 when there are 1500 exceedances, and GHC30.58 at less than 100 exceedances. These excess loss premiums were calculated at a retention level of thrice the previous losses or claims paid.

For the Company B dataset, there were 452 policyholders whose claims have been paid. The minimum claims amount paid was GHC18.00 and the maximum was GHC775,12.00. The skewness and kurtosis of 862.49 and 216.65 respectively, revealed that the claims amount distribution is right-tailed and heavy-tailed. When the MLE of the GPD was used to obtain a tail index of 0.88 it falls in the interval of the tail index given by the splicing method.

Using the estimators produced by the splicing method of the GPD we estimate the following risk measures: VaR, return period, Expected Shortfall, and Excess-of-loss Premium.

It was found out that, VaR at 98% was GHC420,764.90 and at 99% was GHC139,076.70. This implies that the insurance firm has to make these monies available at the respective risk probabilities to cover the future financial losses. The value beyond the VaR for company B was found to be GHC1,508,478.2 at 99.8% and GHC478,494.1 at 99%. The results also showed that, the insurance firm is supposed to put a maximum amount of GHC471.98 as a premium to be paid to the reinsurance company given a retention level that is thrice the claim amounts.

The results also reveal that, the claims amount distribution of Company A was more heavy-tailed than the claims amount distribution of Company B. This is explained by the tail index of Company A being greater than 1 and that of Company B being less than 1. The behaviour of the Pareto QQ plots curving outward initially before increasing linearly for both datasets is a good reason why the GPD splicing method is a good candidate for modelling the datasets. This is in line with the view of Albrecher et al. (2017) that, the splicing method is also important in the situation where an insurance claims data displays different types of behaviour within certain intervals of claims amount.

5.2 Conclusions

Based on the following results, it can be concluded that, the risk measures depend on the claims amount and hence the tail index obtained. It was realized that the tail index beyond 1 couldn't be used to estimate the risk measures until the mixed distribution under the splicing method was used. The following conclusions can be drawn:

1. The two datasets were right tailed and exhibited heavy tailedness which buttresses the point of using extreme value analysis.

2. The tail indexes of both companies were obtained. The two datasets tail indexes were different, with Company A's tail index being more heavy tailed than Company B, indicating that Company A paid out more large claims than Company B.

3. The risk measures were computed at different risk levels. Company A has to wait for a longer period before its retention reoccurs than company B.

Company A needs to allocate more funds to cover future financial losses than Company B. Hence need to pay more premiums to a reinsurance for future intervention.

5.3 Recommendations

At the back of these findings and the fact that this is a case study we recommend the following:

1. Such companies should take a greater look at how high their risk measures such as the VaR, ES, and predicted Excess loss premiums are so as to mitigate future risks.
2. For the consideration of further research, a regression model could be built with GPD so as examine the influence of the risk factors in the computation of the risk models.
3. The practitioner should adopt both internal and external measures in forestalling future financial risks.
4. The Mixed Distribution should be adopted in cases where there claims data behave differently at various intervals of claims amount. And other mixed distributions should be fitted and results compared.

REFERENCES

Abramovsky, A. (2008). Reinsurance: The silent regulator. Conn. Ins. LJ, 15, 345.

Adebayo, A. D., & Sunday, A. O. (2017). Modelling claims of Nigeria Oil & Gas Insurance Class of business with Extreme Value Theory. Annals. Computer Science Series, XV(2), 97–105.

Albrecher, H., Beirlant, J., & Teugels, J. L. (2017). Reinsurance: actuarial and statistical aspects. John Wiley & Sons.

Aon. (2021). Aon Global Market Insights Q1 2021 (foleon.com).

Beirlant, J., Matthys, G., & Dierckx, G. (2001). Heavy-tailed distributions and rating. Astin Bulletin, 31(I), 41–62.

Berger, L. A., Cummins, J. D., & Tennyson, S. (1992). Reinsurance and the liability insurance crisis. Journal of Risk and Uncertainty, 5(3), 253-272.

Brockett, P. L., Witt, R. C., & Aird, P. R. (1991). An overview of reinsurance and the reinsurance markets. Journal of Insurance Regulation, 9(3), 432-454.

Brown, B. (2017, July). Determinants of reinsurance demand in the Ghanaian insurance market [Master's thesis], University of Ghana, Accra, Ghana.

Butun, B. (2013). Property Excess of Loss Reinsurance Pricing in the Lloyd's market.

Ceci, C., Colaneri, K., & Cretarola, A. (2022). Optimal reinsurance and investment under common shock dependence between financial and actuarial markets. Insurance: Mathematics and Economics.

Chukwudum, Q. (2018). Reinsurance Pricing of Large Motor Insurance Claims in Nigeria: An Extreme Value Analysis. HAL Id: hal-01855973 https://hal.archives-ouvertes.fr/hal-01855973

Clark, D. R. (1996). Basics of reinsurance pricing. CAS Study Note, 41-43.

Coles, S. (2001). An Introduction to Statistical Modelling of Extreme Values. Springe

Cooray, K., & Ananda, M. M. (2005). Modeling actuarial data with a composite lognormal-Pareto model. Scandinavian Actuarial Journal, 2005(5), 321-334.

Cummins, J. D., Dionne, G., Gagné, R., & Nouira, A. (2021). The costs and benefits of reinsurance. The Geneva Papers on Risk and Insurance-Issues and Practice, 46(2), 177-199.

Cummins, J.D. and P. Barrieu, 2013. Innovations in insurance markets: Hybrid and securitized risk- transfer solutions. In G. Dionne (Ed.), Handbook of Insurance, Springer, 547-602.

Davison, A. C., & Smith, R. L. (1990). Models for exceedances over high thresholds. Journal of the Royal Statistical Society: Series B (Methodological), 52(3), 393-425.

Ferreira, A., & De Haan, L. (2015). On the block maxima method in extreme value theory: PWM estimators. The Annals of statistics, 43(1), 276-298.

Gilli, M., & Evis, K. (2006). An Application of Extreme Value Theory for Measuring Financial Risk. Computational Economics, 27(1), 1–23. https://doi.org/10.1007/s10614-006-9025-7

Haueter, N. V. (2021). The Impact of Reinsurance. In Oxford Research Encyclopedia of Business and Management.

Hurley, P. (2020). Re-engineering the Insurance Industry for Innovation. Journal of Applied Financial Research, 1, 60-71.

Insurance Information Institute. (2020). Background on : Reinsurance (p. 10).

Kong, X., & L¨u, Y. (2021). Optimal excess-of-loss reinsurance and investment with stochastic factor process. Communications in Statistics - Theory and Methods, 0(0), 1–24. https://doi.org/10.1080/03610926.2021.1904989

Laurin, I. (1930). An introduction into Lundberg ' s theory of risk. Scandinavian Actuarial Journal, 1930(1), 84–111. https://doi.org/10.1080/03461238.1930.10416888

McNeil, A. J. (1997). Estimating the tails of loss severity distributions using extreme value theory. Astin Bulletin, 27(1), 117–137.

McNeil, A. J. (1999). Extreme value theory for risk managers. Departement Mathematik ETH Zentrum, 12(5), 121-237.

Mikosch, T. (2009). Non-Life Insurance Mathematics : An Introduction with the Poisson Process. Springer.

Nolde, N., Zhou, C., & Zhou, M. (2022). An extreme value approach to CoVaR estimation. arXiv preprint arXiv:2201.00892.

Oyewole, O., Adesina, O. S., & Adekola, L. O. (2018). On Extreme Value Theory in Modelling Nigeria Marine and Aviation Insurance Class of Business. Assumption University-EJournal of Interdisciplinary Research (AU-EJIR), 3(1), 41–50.

Özari, Ç., Eren, Ö., & Saygin, H. (2019). A New Methodology for the Block Maxima Approach in Selecting the Optimal Block Size. Tehnički vjesnik, 26(5), 1292-1296.

Patrik, G.S. (2001) 'Reinsurance'. In: Causality Actuarial Society: Foundations of Casualty Actuarial Sciences. 4th ed. Arlington, Virginia: Causality Actuarial Society.343-484).

Patrik, G. (2006). Reinsurance. Encyclopedia of Actuarial Science, 3.

Prepic, A. (2018). An Application of A Bayesian Extreme Value Mixture Model to Reinsurance Excess-of-Loss Pricing and Extreme Value Threshold Selection.

Rodermund, M. (1965). Four Points of Confusion about Reinsurance: Comment.The Journal of Risk and Insurance, 32(1), 133-136.

Ronken, L., & Re, G. (2020). Avoiding Underinsurance – Why an Accurate Sum Insured Is Vital (but Often Neglected). January.

Rychlik, I., & Ryd´en, J. (2006). Probability and Risk Analysis An Introduction for Engineers. Springer Berlin Heidelberg.

Salaudeen, T. O., Salam, R. A., & Mudashiru, G. A. (2021). Hedging Risk through Reinsurance in the Nigerian Insurance Companies: Do the Benefits Outweigh the Costs?. Global Journal of Education, Humanities & Management Sciences, 3(1).

Surminski, S., Barnes, J., & Vincent, K. (2022). Can insurance catalyse government planning on climate? Emergent evidence from Sub-Saharan Africa. World Development, 153, 105830.

Upreti, V., & Adams, M. (2015). The strategic role of reinsurance in the United Kingdom's (UK) non-life insurance market. Journal of Banking & Finance, 61, 206-219.

Wang, Y., & Bølviken, E. (2022). How much is optimal reinsurance degraded by error?. North American Actuarial Journal, 26(2), 283-297.

You, H., & Zhou, X. (2021). The Pareto-Optimal Stop-Loss Reinsurance. Mathematical Problems in Engineering, 2021.

APPENDICES

Appendix A

Company A

```
library(ReIns)
library(POT)
library(psych)
library(MASS)
#Importing and viewing
CompA_data< read.csv("Company A.csv")
View(CompA_data)
#Descriptive Statistics describe(CompA_data)
summary(CompA_data)
describeData(CompA_data,head=4,tail =4)
#Plotting a histogram
par(mar=c(4,4,1,1))
hist(log(CompA_data$CLAIMS), xlab ="Claims" , ylab = "Frequency", main = "
",col = "steelblue",freq = F)
lines(density(log(CompA_data$CLAIMS)),col="red",lwd=2)
#Scatterplot
attach(CompA_data)
par(mar=c(4,4,1,1))
plot(log(CLAIMS))
```

```
#Constructing the boxplot
boxplot(CompA_data$CLAIMS, log =”y”, col=”steelblue”, main= ” ”, xlab =
”claims”,ylab = ”Frequency”)
#Plotting the mean excess plot
MeanExcess(CompA_data$CLAIMS, plot = TRUE, k = FALSE, main = ””)
#Exponential qq plot ExpQQ(CompA_data$CLAIMS, plot = TRUE, main = ”t”)
#Pareto QQ plot ParetoQQ(CompA_data$CLAIMS, plot = TRUE, main = ””)
#Using the ReIns package we have help.search(”ReIns”)
#Hill estimator helps you to know the behaviour of the data using GPD
#The Hill estimator to estimate the gamma values at each k
help(”Hill”)
par(mar=c(4,4,1,1))
Hillgpd<Hill(CompA_data$CLAIMS,k = TRUE, logk = FALSE, plot = TRUE, add
= FALSE, main = ’ ’ )
#Estimating the EVI AND scale parameter
par(mar=c(4,5,1,1))
plot(gpd_est$gamma, type =”l”,ylab=expression(hat(xi)), xlab=expression(k))
abline(h=1, col=2)
locator()
plot(gpd_est$sigma, type =”l”,ylab=expression(hat(sigma)), xlab= expression(k))
#Estimating Return Period
?Return
par(mar=c(4,4,1,1))
Returnqq<Return(CompA_data$CLAIMS,Hillgpd$gamma, q=45000000, plot=
TRUE, col= ”blue”,main = ” ”)
locator()
ME<MeanExcess(CompA_data$CLAIMS, main =” ”)
?abline
abline(v=quantile(CompA_data$CLAIMS,c(0.45,0.999996)))
#Fitting the model using the SpliceFitPareto()
??SpliceFiticPareto
```

```
FitSplice<SpliceFitGPD(CompA_data$CLAIMS,const = 0.45, M=3)
#Creating Mixed Erlang Distribution object mefit < MEfit(p=c(0.5626319, 0.4373681), shape=c(3,9), theta=261.2347,M=2)
evtfit < EVTfit(gamma=c(1.14,0.6931472), endpoint=c(7673400, Inf))
splicefit < SpliceFit(const=c(0.45,0.999996), trunclower=0, t=c(2082.5, 7673400.0),type=c("ME","TPa","Pa"),MEfit=mefit, EVTfit=evtfit)
#SEQUENCE OF VECTORS FOR OUR CLAIM AMOUNTS
xvector<seq(0, 15*106̂, 2000)
?SpliceFitPareto()
#SURVIVAL FUNCTION FOR THE SPLICED DISTRIBUTION
Activa_sv<SpliceECDF(xvector,CompA_data$CLAIMS, splicefit)
#PP PLOT FOR THE SPLICE DISTRIBUTION PPSPLICE<-SplicePP(CompA_data$CLAIMS,FitSplice, main = " ")
#diagnostics xvector<seq(0, 15*106̂, 2100)
GPDSPLICE<SpliceFitGPD(CompA_data$CLAIMS,const = 0.5, M=3)
par(mar=c(4,4,1,1)) QQGPDSPLICE<SpliceQQ(CompA_data$CLAIMS, GPDSPLICE)
QQGPDSPLICE<SplicePP(CompA_data$CLAIMS,GPDSPLICE,main = " ")
QQGPDSPLICE<SpliceECDF(xvector, CompA_data$CLAIMS, GPDSPLICE)
?SpliceFitGPD
mefit < MEfit(p=c(0.5974537, 0.4025463),shape=c(3,8), theta=256.6492,M=2)
evtfit < EVTfit(gamma=1.100392,sigma = 4923.127, endpoint= Inf)
splicefit1 < SpliceFit(const=0.5, trunclower=0, t=2557.8, type=c("ME","GPD"),MEfit=mefit, EVTfit=evtfit)
#VaR
pvalue<seq(0, 0.01, 0.0001)
VAR<VaR(p=pvalue, splicefit1)
plot(pvalue, VAR, xlab ="p",ylab=expression(VaR[1-p]), type="l")
#Expected Shortfall
?CTE
```

```
pvalue2<seq(0.001, 0.01, 0.0001)
ESvale<CTE(p=pvalue2,splicefit1)
plot(pvalue2, ESvale, type="l")
locator()
#Excess of loss premium
?ExcessSplice
R<seq(0,15000000,5000)
XoL<ExcessSplice(R, L=3*R,splicefit=splicefit1)
plot(XoL,type="l")
```

Appendix B

Company B

```
library(ReIns)
library(POT)
library(psych)
library(MASS)
#Importing and viewing Company B datasets
CompB_data <- read.csv("Company B.csv")
View(CompB_data)
#Using some descriptive statistics
describe(CompB_data)
summary(CompB_data)
describeData(CompB_data,head=4,tail =4)
#Plotting a histogram
par(mar=c(4,4,1,1))
hist(CompB_data$CLAIMS, xlab ="Claims" , ylab = "Frequency", main = " ", col
= "blue",freq = FALSE)
lines(density(log(CompB_data$CLAIMS)),col="red",lwd=2)
#Scatterplot
par(mar=c(4,4,1,1))
plot(CompB_data$CLAIMS)
#Constructing the boxplot
boxplot(CompB_data$CLAIMS, log ="y", col="lightblue", main= " ",xlab =
"claims",ylab = "Frequency" )
locator()
summary(CompB_data$CLAIMS)
#Plotting the mean excess plot
```

```
help.search("meplot")
par(mar=c(4,4,1,1))
meplot(CompB_data$CLAIMS, main =' ')
MeanExcess(CompB_data$CLAIMS, plot = TRUE, k = FALSE, main = "")
#Exponential qq plot
ExpQQ(CompB_data$CLAIMS, plot = TRUE, main = "t")
#Pareto QQ plot
ParetoQQ(CompB_data$CLAIMS, plot = TRUE, main = "")
#Using the ReIns package we have
help.search("ReIns")
#The Hill estimator to estimate the gamma values at each k
par(mar=c(4,4,1,1))
Hillgpd2<-Hill(CompB_data$CLAIMS,k = TRUE, logk = FALSE, plot = TRUE,
add = FALSE, main = ' ')
#ESTIMATING THE SHAPE AND SCALE PARAMETER
par(mar=c(4,5,1,1))
plot(gpd_est2$gamma, type ="l",ylab=expression(hat(xi)), xlab=expression(k))
abline(h=1, col=3)
plot(gpd_est2$sigma, type ="l",ylab=expression(hat(sigma)), xlab= expression(k))
#Return Period Returnqq2<-Return(CompB_data$CLAIMS, Hillgpd2$gamma,
q=2500000, plot= TRUE, col= "blue", main = " ")
#Mean excess plots to determine where to splice
ME2<-MeanExcess(CompB_data$CLAIMS, main = " ")
abline(v=quantile(CompB_data$CLAIMS,0.85))
#SURVIVAL FUNCTION FOR THE SPLICED DISTRIBUTION xvector2<-seq(0,
8*10^5, 100)
SpliceECDF(xvector2, CompB_data$CLAIMS,FitSplice2)
#PP PLOT FOR THE SPLICE DISTRIBUTION
PPSPLICE2<-SplicePP(CompB_data$CLAIMS,FitSplice2,main = ' ')
```

```
GPDSPLICE2<-SpliceFitGPD(CompB_data$CLAIMS,const = 0.5, M=3)
QQGPDSPLICE2<-SpliceQQ(CompB_data$CLAIMS, GPDSPLICE2,main = " ")
#Diagnostics
xvector2<-seq(0, 8*10^5, 100)
GPDSPLICE2<-SpliceFitGPD(CompB_data$CLAIMS, const = 0.55, M=3)
QQGPDSPLICE<-SpliceQQ(CompB_data$CLAIMS, GPDSPLICE2)
par(mar=c(4,4,1,1))
QQGPDSPLICE<-SplicePP(CompB_data$CLAIMS,GPDSPLICE2,main = " ")
QQGPDSPLICE<-SpliceECDF(xvector2, CompB_data$CLAIMS, GPDSPLICE)
FitSplice3<-SpliceFitPareto(CompB_data$CLAIMS,const = c(0.5,0.99), M=3)
GPDSPLICE2<-SpliceFitGPD(CompB_data$CLAIMS,const = 0.55, M=3)
mefit3 <- MEfit(p=c(0.2551152,0.7448848), shape=c(3,13), theta=441.3609, M=2)
#Creating Pareto Distribution object
evtfit3 <- EVTfit(gamma=0.9007509,endpoint=Inf)
splicefit3 <- SpliceFit(const=0.5486726,trunclower=0, t= 3331.8,
type=c("ME","GPD"), MEfit=mefit3, EVTfit=evtfit3)
pvalue 2<-seq(0, 0.01, 0.0001)
VAR2<-VaR(p=pvalue2, FitSplice2)
plot(pvalue2, VAR2, xlab ="p",ylab=expression(VaR[1-p]), type="l")
locator()
#Expected Shortfall
pvalue3<-seq(0.0001, 0.01, 0.0001)
ESvale2<-CTE(p=pvalue3,FitSplice2)
plot(pvalue3, ESvale2, type="l")
locator()
#Excess Loss Premium
R<-seq(0,800000,50)
XoL2<-ExcessSplice(R, splicefit=splicefit2)
plot(XoL2,type="l")
```