UNIVERSITY OF GHANA

COLLEGE OF HUMANITIES

BOOTSTRAPPING AND ALLOCATION OF ASSETS ON STOCK MARKETS IN AFRICA

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THIS THESIS IS SUBMITTED TO THE UNIVERSITY OF GHANA, LEGON IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE AWARD OF MASTER OF PHILOSOPHY IN FINANCE DEGREE

DEPARTMENT OF FINANCE

JULY 2015

DECLARATION

I hereby declare that, this study is my original work and has not been submitted by anyone for an academic award at the University of Ghana or any other university.

I bear sole responsibility for any shortcomings.

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ESTHER ASSABEA MILLS DATE

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CERTIFICATION

This is to certify that this thesis has been supervised with the laid down principles for thesis writing at the University.

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DR. LORD MENSAH DATE

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(Co-Supervisor)

DEDICATION

This work is dedicated to:

My beloved parents, Mr. Ebenezer Agyakwa Mills and Mrs Felicity Mills as well as my colleagues and supervisors for their encouragement, support and assistance throughout the period of my studies.

ACKNOWLEDGEMENT

I thank the Almighty God for all His wisdom, strength, and mercies throughout the period of this study.

I would like to express my sincere appreciation to my supervisors Dr. Lord Mensah and Dr. Godfred Alufar Bokpin for their guidance and constructive criticisms which shaped and enriched the quality of this study.

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ASCE	-	Abuja Securities and Commodities Exchange
ASE	-	Algiers Stock Exchange
BVC	-	Bolsa de Valores de Cabo Verde
BVM	-	Bolsa de Valores de Mozambique
BSE	-	Botswana Stock Exchange
BVMT	-	Bourse de Tunis
BVRM	-	Bourse Regionale des Valeurs Mobilieres
CAL	-	Capital Allocation Line
CAPM	-	Capital Asset Pricing model
Casa SE	-	Casablanca Stock Exchange
CD	-	Cote D'Ivoire
DSE	-	Dar es Salaam Stock Exchange
DSX	-	Douala Stock Exchange
EGX	-	Egyptian Exchange
EQWP	-	Equally Weighted Portfolio
ER	-	Expected Return
GDP	-	Gross Domestic Product
GSE	-	Ghana Stock Exchange
JSE	-	Johannesburg Stock Exchange
KSE	-	Khartoum Stock Exchange
LSM	-	Libyan Stock Market
LuSE	-	Lusaka Stock Exchange
MSE	-	Malawi Stock Exchange
MVP	-	Minimum Variance Portfolio

MVS	-	Minimum variance set
NSE	-	Nairobi Securities Exchange
NSX	-	Namibia Stock Exchange
NSE	-	Nigerian Stock Exchange
PT	-	Portfolio Theory
RSE	-	Rwanda Stock Exchange
SA	-	South Africa
SD	-	Standard Deviation
SSE	-	Seychelles Securities Exchange
SSTE	-	Somalia Stock Exchange
SPDJ	-	S&P Dow Jones
SR	-	Sharpe ratio
SSTE	-	Somalia Stock Exchange
SEM	-	Stock Exchange of Mauritius
SSX	-	Swaziland Stock Exchange
TP	-	Tangency Portfolio
USE	-	Uganda Securities Exchange
UK	-	United Kingdom
US	-	United States of America
VAR	-	Value at risk
ZSE	-	Zimbabwe Stock Exchange

ABSTRACT

The purpose of this study is to construct a composite optimal risky portfolio across eleven African countries from which optimal portfolio decisions can be made by investors. This is done through a static model. The study further assesses how robust the optimal portfolio is to possible variations in economic conditions of a country through the use of a bootstrap algorithm. This, therefore, makes the optimal choices of this study reliable and robust to non-normality biases. The various optimal choices for various investors are also examined.

The study found that although, the individual African countries' portfolios are highly risky, a well-diversified portfolio can offer a better risk-return trade-off by reducing the risk and increasing the expected return. The outcomes of the study also indicated that possible variations that can affect macroeconomic variables, resulting in differences in returns can have a significant effect on optimal choices. Also, the percentage investors would apportion to portfolios of risky assets depends on their risk preferences.

CHAPTER ONE

INTRODUCTION

1.1 Research Background

In recent years, the power and advanced features of computers have made the application of the Portfolio Theory (PT) developed by Markowitz (1952) prominent in the modelling and allocating of assets (Wang & Forsyth, 2011; Huang & Lee, 2010; Huang, 2008; Elton & Gruber, 2000; Markowitz, 1999; Evans & Archer, 1968) in both academia and industry. The portfolio theory stipulates that investors seek to maximize their expected returns and minimize the risks associated with their portfolios. In effect the PT allows for the quantification of risks and expected returns and selections of optimal portfolios by investors in financial markets (Hibiki, 2006).

Portfolio selection can be seen as a 2-step approach. The first requires investors to have prior knowledge and insight about the behaviour and performance of stock returns. Based on this, investors would conclude this stage by predicting the performances of assets in advance. The second commences with the consideration of these crucial predictions in the first stage and concludes with portfolio choices by investors (Markowitz, 1952). The second stage which has been the focus of most studies presupposes that, investment choices depend on both the expected returns which investors consider as attractive and the variance which is unattractive (Sharpe, 1963). Therefore, the allocation of assets needs to be done in a strategic manner.

Strategic asset allocation is mostly the optimal allocation of risky assets (bonds, lease financing, stocks) and risk free (treasury bills, government bonds) assets to an investor over his or her investment horizon (Cesari & Cremonini, 2003). The popularly known strategies are the static, also known as the tactical asset allocation and the dynamic strategies. With the static or tactical asset allocation the investor defines the mean-variance strategy of optimization over a single period (Brennan, Schwartz, & Lagnado, 1997). However, the dynamic asset allocation assumes either a continuous or a multiple period model (Yao, Li, & Chen, 2014). This study employs a static asset allocation, thus the classic Markowitz mean-variance optimization technique and builds up on previous literature in the context of the study.

Several techniques have been proposed for handling the dynamic (multi-period) asset allocation. Techniques such as dynamic & stochastic programming, sample paths, decision trees with quadratic programming and an analytical optimal solution (Gulpinar & Rustem, 2007; Mulvey & Shetty, 2004; Li & Ng, 2000). However, most of these techniques have flaws and are usually not practical or result in problems when employed in real world numerical data. For instance, dynamic programming when applied to a multiple period asset allocation results in the curse of dimensionality and is not practical with actual world multiple periods asset allocation decisions made by investors (Calafiore, 2008). To the best of my knowledge most of the studies that have proposed a dynamic asset allocation did not apply it in a real world situation as a result of its complex nature (Liu, Zhang, & Xu, 2012; Calafiore, 2008). Also, the application of a dynamic model would require a huge data set, since the time period would be sub-divided into several time periods to obtain a multi-period situation (Mossin, 1968). This is difficult to attain and specifically in terms of Africa due to data unavailability. Hence, this study employs a static asset allocation.

The basic allocation problem is to decide which asset classes to include in a portfolio and in what proportions (Gratcheva & Falk, 2003). This is because the asset allocation decision has a cumulative influence on the portfolios overall performance than any other decision. This asset allocation decision may not necessarily imply diversification in one market, but cross country diversification may also apply. Cross border diversification of products and processes has the tendency of improving the risk-expected return performances of portfolios and securities. This is on the premise that, macroeconomic factors of the various countries do not cause stock returns to be strongly correlated (Obstfeld, 1994). However, investors prefer to invest locally or in countries close by or familiar, thus investors would prefer to invest in markets they are conversant with (Coval & Moskowitz, 1999). On the other hand, the literature has indicated that investing internationally increases the performance of a portfolio's return, specifically investing in developing economies can significantly contribute to the performance of an investor's portfolio since the returns of stocks are usually driven to a higher extent by internal factors (Jacobs, Muller, & Weber, 2014; Harvey, 1995).

Recent studies have indicated that frontier markets such as most markets in Africa are becoming a lucrative place to invest in. Over the years, these developing markets have illustrated rapid financial, economic and developmental growths (Groot, Pang, & Swinkels, 2012; Li, Sarkar, & Wang, 2003). Groot et al. (2012) found that investing in a frontier market tends to significantly extend the mean-variance efficient frontiers and provides investment opportunities which yield higher returns. Also, a cross border diversification of some African countries can serve as an ideal out of sample evidence, since most studies on cross country portfolio selection and diversification focused more on developed economies (Guidolin & Hyde, 2012; Horneff, Maurer, Mitchell, & Stamos, 2009). Despite the perceptions of most foreign investors about emerging and frontier markets to be a very highly risky environment (Bekaert & Harvey, 2003) due to the disasters these markets are characterized by such as wars and coup d'états. Studies by Groot et al. (2012) and Berger, Pukthuanthong and Yang, (2011) found that the returns of stocks across countries in frontier markets exhibited excess returns with the same factors that affect returns of developed markets, though they are disintegrated. This is to say that whether an investor invests in a developed or developing market, the returns tend to be somewhat affected by similar factors. Hence, an investment in Africa, combined with diversification and the risky nature of African stock markets, can yield higher returns since risks and expected returns of a portfolio are positively related (Black & Litterman, 1992; Jorion, 1992).

Klassen and Yoogalingam (2013) have also argued that to determine whether or not a technique, strategy or model employed in the allocation and selection of the optimal portfolio would be able to withstand any variations in macroeconomic variables, as well as any other idiosyncratic errors in measurement, the simulation process must be adopted. This is because the simulation process addresses the possible stochastic factors of the sampling process while simultaneously determining whether there exist significant differences between the simulated results to that of the original (Aslanidis & Casas, 2013; Adachi & Gupta, 2005). The bootstrapping technique would therefore be employed as a simulation technique. Studies that employed bootstrapping as a simulation technique used it mainly because of its independence to distribution of stock returns (Jacob et al., 2014). Bootstrapping can depict various and probable situations that could result in variations in stock returns in a given economy. That is, bootstrapping has the ability to provide various datasets (replicates) that can result from variations such as changes in business cycles, inflation and other diversifiable errors in the economy (Assaf, Barros, & Matousek, 2011; Simar &Wilson, 2007; Simar & Wilson, 2000; Simar &Wilson, 1998). Also the bootstrap is a more effective and efficient way of obtaining possible datasets that can result from variations in the economy and will be difficult to gather and collect for this purpose (Tortosa-Ausina, Armero, Conesa, & Grifell-Tatje, 2012).

The complexity of portfolio diversification and selection, and its continuing importance in investment decisions, makes it still relevant for academic cross-examination. Hence, this study employs the concept of the Markowitz portfolio selection model to obtain an optimal risky portfolio for a selected number of stock markets in Africa since it has become a hot spectrum for good returns on investments. Bootstrapping is used as a simulation approach to determine if the optimal risky portfolio is robust to possible variations that can occur in a developing economy.

1.2 Research Problem

Modern Portfolio Theory has had and continues to have a great impact in finance. This stems from the fact that it has resulted in the birth of various asset allocation strategies and techniques. These strategies have been employed in constructing and deriving efficient frontiers and optimal portfolios for investors (Fortin & Hlouskova, 2011; Zhu & Zhou, 2009; Detemple, Garcia, & Rindisbacher, 2005; Harlow, 1991; Elton & Gruber, 1978; Merton, 1969).

The benefits of diversification in portfolio selection have been indicated in several studies (Guidolin & Hyde, 2012; Horneff et al., 2009; De Roon & Nijman, 2001; Huberman & Kandel, 1987). However, studies have shown that investors tend to limit themselves to local investments, though most countries have withdrawn constraints on diversifying overseas (Driessen & Laevan, 2007). This is because investors prefer to invest within an accustomed spectrum. It has been found that diversifying internationally is very beneficial to investors, most especially those from developing economies (De Roon & Nijman, 2001). Let me emphasize, substantial studies have been done on the benefits of diversification. However, most of these studies focused on investment portfolios in one economy. For example, whereas Guidolin and Hyde (2012), as well as Horneff et al. (2009) focused on the United States, Mensah, Avuglah, and Dedu (2013) focused only on Ghana. Hence, the need for cross-country studies in this context. To the best of my knowledge, a few studies such as Chen, Silvapulle and Silvapulle (2014) and Driessen and Laevan (2007) have tried to bridge this gap by performing a cross-country study consisting of 8 countries and the other consisting of 52 countries respectively, spread across the globe. There is therefore the need for more cross-country studies to provide a better picture of the merits of international diversification in portfolio selection.

In portfolio selection, investment decisions are based on risks and expected returns. However, these risks and returns are subject to changes in economic factors like inflation, interest rates, exchange rates and other fiscal and monetary policies of governments (Rjoub, Tursoy, & Gunsel, 2009; Adjasi, 2009; Adjasi, Harvey, & Agyapong, 2008). There is therefore the need for any study on optimal portfolio selection to conduct a sensitivity analysis to assess the effect of any economic perturbation. This also provides insights on the robustness of the optimal choice. Research on portfolio selection has often employed simulation to address this condition (Lin & Lu, 2012; Marekwica, 2012). However, the few researches on cross-country portfolio optimization, or the same context as this study, have seldom performed simulation to determine if the optimization techniques employed can withstand the possible variations that may occur in an economy (see Chen et al., 2014; Driessen & Laevan, 2007). Therefore, this study employs the bootstrapping technique, to determine if the various resampling will yield similar and insignificant differences to that of the optimal portfolio and also complement existing literature on the benefits of international diversification.

Another important observation from review of literature on portfolio optimization and selection is that, these studies seem to be skewed primarily towards developed economies to the neglect of emerging ones. For example, Aslanidis and Casas (2013), Chu (2011), Cesari and Cremonini (2003) and Campbell, Huisman and Koedijk (2001) have all focused on developed economies. Even the few papers that conducted cross-country assessments used samples dominated by developed countries. For example, whereas Chen et al. (2014) only focused on 8 developed countries, for Driessen and Laevan (2007), of their sample of 52 countries, only 18 were developing economies of which 6 were African countries. To the best of our knowledge, only Mensah et al. (2013) have considered country specific optimized portfolio choice in Africa, and specifically Ghana. It is important for more insights on investment portfolios in developing economies, especially Africa. This is because Africa is gradually becoming a lucrative investment destination.

Forbes Report (2014) suggests that Africa is a two trillion dollar economy with about a third of its fifty-four countries having Gross Domestic Product (GDP) growth rates of at least 6% annually. Out of the top ten fastest growing economies in the world, six are in Africa. These African countries are becoming lucrative areas for investments since most of them are rich in natural resources such as gold, copper, diamond, cocoa and oil. Also, sixty percent of the world's uncultivated arable land is in Africa (Forbes Magazine, 2014). This lays a strong foundation for studies targeting developing economies, especially in Africa.

There is therefore the need for a cross-country assessment of the portfolio selection decisions that target developing economies (emerging and frontier markets), especially in Africa. There is also the need to consider how robust the optimal decision is amidst to the tempestuous economic climate. This study fills these gaps by providing an empirical assessment of the optimal portfolio choice across countries whiles applying a bootstrapping technique to capture various economic conditions that can affect stock returns resulting in variations in stock returns.

1.3 Research Objectives

Primarily, this study assesses a cross-country optimal portfolio selection decision across some emerging and frontier markets in Africa through a static model and bootstrap algorithm. This is addressed using a sample of 11 African countries- Tunisia, Ghana, South Africa, Botswana, Nigeria, Namibia, Morocco, Cote D'Ivoire, Zambia, Kenya and Mauritius, over the 15 year period from 2000 to 2014. Specifically, this objective is achieved by addressing the following specific objectives:

a. To construct a composite optimal risky portfolio across eleven African countries (stock markets).
b. To assess how robust the optimal risky portfolio is to variations in economic conditions through a bootstrap algorithm.
c. To assess the optimal portfolio choices for risk lover and risk averse investors.

1.4 Research Questions

The following research questions are addressed in order to achieve the research objectives of this study:

a. What is the composition of the optimal risky portfolio across eleven African countries?

b. How robust is the optimal risky portfolio to variations in economic conditions?

c. What are the optimal portfolio choices for risk lover and risk averse investors?

1.5 Significance of Study

The study is of value to future research, practice and policy making.

Research: The current study goes beyond the literature on cross-border asset allocation and portfolio diversification by including bootstrapping as a simulation technique to cater for any variations that can occur in stock returns. This study, therefore, serves as a guideline for further research on cross-border asset allocation and portfolio diversification.

Practice: The findings from this study informs traders and investors on the Ghana Stock Exchange and the other ten African stock exchanges on how to allocate their wealth across borders to optimize utility based on their risk behaviours and also benefit from international diversification. This study also gives some level of certainty to investors, since the bootstrapping performed caters for most of the possible variations that can occur in stock returns.

Policy making: The findings of this study create a picture of how variations in the macro-environment can alter the returns of the various portfolios. Hence policy makers in these developing economies are informed on how macroeconomic factors impact investors' wealth and eventually the development of capital markets.

1.6 Research Limitation and Scope

The researcher is unable to consider all countries in Africa. This is as a result of data unavailability which made it impossible to consider all African countries and other capital markets such as the bonds and the commodities market since most African countries possess only stock markets. This research assumes a no short selling constraint. This is because short selling is hardly practised in this part of the world as a result of the illiquid nature of most African stock markets. The study also employs a composite index, thus the market stock price index of each of the eleven countries and does not consider the individual stocks in each country to construct the individual country's portfolios due to data unavailability for the individual stocks across the countries.

1.7 Chapter Outline

This study is divided into five chapters as follows. Chapter one gives a brief introduction to the study. This constitutes the research background, research problem, objectives, significance of study, research limitation and project outline. Chapter two reviews relevant literature on asset allocation, portfolio optimisation and diversification. This chapter also gives an overview of what has been done until now on asset allocation and optimal portfolio selection. Chapter three deals with the methodology. It includes the population, sample, mode of data selection & collection, methods and models. Chapter four consists of the analyses and interpretation of the results. Chapter five entails the conclusion, a summary of the research and its implications for future research.

1.8 Chapter Summary

This chapter summarises what this research sets out to do. That is, to construct a well diversified portfolio across eleven stock markets in Africa. A bootstrapping algorithm is used as both a robust and a form of simulation technique, to determine how appropriate this study's optimal choices are. The research background, problem statement, research objectives & questions, significance and the limitation of this study are also included in this chapter. It then concludes with the order this study follows.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter discusses various concepts such as, portfolio theory, static & dynamic models, portfolio diversification, separation theorem and simulation techniques. Empirical reviews of these concepts would also be explored subsequently.

2.2 Theoretical Review

This section gives a brief overview of concepts, theories, principles and models underlying portfolio optimization, asset allocation and simulation.

2.2.1 Portfolio Theory

The pioneer paper by Markowitz postulates that mean-variance criterion (Markowitz Optimization technique) is superior to the formally used mean criterion in asset allocation, portfolio optimization and selection. This gave rise to the portfolio theory in 1952 by Markowitz. Five assumptions underpin the portfolio theory (Kisaka, Mbithi, & Kitur, 2015). First, the risk investors are exposed to is within a myopic time period (single time period). Second, investors have diminishing marginal utility curves and aim at maximizing their utility. Third, investment choices made by investors depend on their expected return and risk preferences. This is as a result of the assumption undelying the mean-variance approach, that stocks' returns are normally distributed. Hence the mean (first moment) and variance (second moment) are sufficient in the construction and selection of a portfolio. Fourth, for given levels of risk, investors tend to maximize their expected returns. Conversely, for given levels of expected returns, investors minimize their risk. That is, the mean-variance criterion aims at selecting efficient portfolios for investors. Fifth, portfolio risks are computed based on variations in expected returns.

Portfolio theory relates to the basic asset allocation problem (the choice between the level of the risk and return) an investor is faced when selecting a portfolio (Gratcheva & Falk, 2003). Specifically, which assets or portfolios to invest in and in what proportions. Hence, the risk preference of an investor is vital in any asset allocation decision making (Kisaka et al., 2015). This theory also assumes that investors are rational, hence they will seek to invest in securities with higher levels of expected returns (Gennaioli, Shleifer, & Vishny, 2015). However, these higher returns come with higher risks, thus, the risks and expected returns are positively related (Gennaioli et al., 2015; Gerlach, Obadyin, & Zurbruegg, 2015; Sharpe, 1964). In view of these principles, the efficient frontier which enables investors to choose among portfolios based on individual risk-return preferences was formulated (Wang & Forsyth, 2010; Celikyurt & Ozekici, 2007).

The Markowitz optimization technique as mentioned earlier was formulated for a single time period (Wu & Chen, 2015; Liu et al., 2012; Calafiore, 2008). Hence, this has led to controversies about how it can be applied to a multiple time period, resulting in the formulation of dynamic models by some researchers (Gulpinar & Rustem, 2007; Mulvey & Shetty, 2004; Li & Ng, 2000; Mossin, 1968).

2.2.2 Static and Dynamic Models

The popularly known models in portfolio optimization are the static, also known as the tactical asset allocation and the dynamic model (Liu & Zhang, 2015; Wu & Chen, 2015; Calafiore, 2008). With the static or tactical asset allocation investors define the mean-variance approach across a single period (Brennan et al., 1997). For a static model, investors make the asset allocation choice at the start of their investment horizon and wait until the period ends to obtain the expected returns on their portfolios with the aim of maximizing their utility at the end of this period (Mossin, 1968). That is, with the assumption of a static time period, investors can neither include more asset classes, adjust their wealth nor change the composition of the portfolio during the period (Zhang, Liu, & Xu, 2012). However, in the real world, portfolio optimization and asset allocation decisions by some investors deviate from the Markowitz static model (Liu & Zhang, 2015; Gulpinar & Rustem, 2007). This has led to the formulation of multi-period or dynamic models (Wu & Chen, 2015; Yao et al., 2014; Calafiore, 2008; Li & Ng, 2000).

A dynamic model assumes that investment decisions are made over multiple time periods. With the dynamic model, investors are capable of making changes prior to the end of the investment period (Liu et al., 2012). Some investors would want to adjust their wealth from time to time (Liu & Zhang, 2015). Whereas, others would also want to make adjustments to the asset mix that forms the portfolio or in some cases include other asset classes that would offer higher expected returns on their portfolios (Zhang et al., 2012; Mossin, 1968). Mossin (1968) formulates the structure of a multi-period model as follows. Investors pre-determine a future time, at which they can consume their wealth. However, these investors still have the intention of maximizing their expected return at this future date. This model therefore assumes that, the time between the present and the investment period can be separated into several time periods, either of equal lengths or not. Hence, at the end of these periods, the expected returns on the portfolios are obtained and based on these, a decision on the asset mix of the portfolios are made for the subsequent periods.

2.2.3 Portfolio Diversification

Diversification is a fundamental principle in finance, which aims at minimizing the risk investors are faced with (Gaudecker, 2015; Amenc & Martellini, 2011; Goetzmann & Kumar, 2008; De Santis & Gerard, 1997; Meric & Meric, 1989; Klein & Bawa, 1976; Lessard, 1973). This is done by investing in different assets, asset classes and markets that have low, negative or possibly no correlations between their returns, thereby reducing the risk investors are subjected to (Flavin, 2004). The Markowitz mean-variance criterion incorporates the benefits of portfolio diversification (Garcia-Herrero & Vazquez, 2013; Driessen & Laeven, 2007). That is, the theory postulates that, assets cannot be chosen based on only attributes specific to a security. Instead, investors are required to take into consideration the correlation between the various assets (Kisaka et al., 2015). In other words, investors should allocate their wealth among securities that are not highly correlated so that events such as changes in business cycles and macroeconomic conditions, they can benefit from the diversification (Gokgoz & Atmaca, 2012).

The risk portfolios are exposed to, can be decomposed into two, namely, the systematic and unsystematic risk (Bali, Brown, & Caglayan, 2012; Jacquier, Titman, & Yalcin, 2010; Ramchad & Sethapakdi, 2000). Portfolio diversification if performed effectively, can eradicate unsystematic risk (Bodie, Kane, & Marcus, 2009). That is, a portfolio's risk reduces as more assets are added to it, but cannot be entirely eliminated (Bodie et al., 2009; Statman, 1987). The risk that persists even after extensive diversification is referred to as the systematic (market or nondiversifiable) risk (Jeng & Liu, 2012; Los, 1999). This type of risk can be attributed to the various risk factors, markets are exposed to (Weitzman, 2013; Bodie, Kane, & Marcus, 2008). Whereas, the risk that is diversifiable is referred to as the unsystematic risk (Statman, 1987). This is the risk usually unique to a firm or company (Calgary, 2014; Bodie et al., 2008). Hence, also referred to as the firm-specific, unique or diversifiable risk (Bodie, Kane, & Marcus, 2011). Despite, the benefits of diversification to minimize risk, extreme diversification could result in the reduction of the expected return of a portfolio (Kisaka et al., 2015). This is so, because the risks and expected returns of a portfolio are usually positively related (Gerlach et al., 2015; Sharpe, 1964). Hence the lesser the risk, the lesser the expected return on the portfolio.

2.2.4 Separation Theorem

The idea behind the separation theorem is that the optimal choice (optimal portfolio) of investors is independent of their expected returns and risk preferences in the presence of a risk-free asset (Elton & Gruber, 1997). The separation theorem has simplified computations in portfolio selection, in that, now the portfolio issue may be seen as determining the portfolio tangent to a ray which passes through a risk free asset in an expected return-standard deviation cosmos (Elton & Gruber, 1997). This theorem also brought about the mutual fund theorem, which stipulates that all types of investors can obtain their preferred portfolio by combining two mutual funds, the first, consisting of the optimal portfolio and the second, the risk-free asset (Lin & Lu, 2012; Elton & Gruber, 1997).

A popularly known separation theorem is the 2 fund separation theorem (Tobin, 1958). This theorem stipulates that any portfolio on the efficient frontier can be represented linearly by two respective portfolios on the same efficient frontier and on the other hand, any two convex combination of efficient portfolios (portfolios on the efficient frontier) result in an efficient portfolio (Yao et al., 2014). The separation theorem also enhanced the development of the Capital Asset Pricing model (CAPM). That is, the modern portfolio theory in combination with the separation theorem contributed tremendously in the establishment of the CAPM in finance (Sharpe, 1964).

2.2.5 Simulation Techniques

Simulation may be referred to as a system that mimics the events or variations that could, or are likely to occur in an actual world (Winston, 2003). This is done by establishing a simulation model, which is represented by a set of assumptions about how the real world operates. Monte Carlo simulations are the popularly used simulation techniques in most portfolio selection and asset allocation studies (Castellano & Cerqueti, 2014; Branger & Hansis, 2012; Lin & Lu, 2012; Elliott, Siu, & Badescu, 2010). Monte Carlo simulations may also be referred to as static simulations. This is because simulations are computed at a given point in time- that is, each point in time is an independent simulation (Winston, 2003). The distribution of stock returns needs to be explicitly known with this simulation technique. This is because for this technique, the simulation is done from the model, hence the distribution, properties and assumption underlying the model is required in order to simulate. This, therefore, lead to the employing of bootstrap as a simulation technique (Jacobs et al., 2014; Kim, 2009; Corradi & Iglesias, 2008; Ledoit & Wolf, 2008).

Efron's (1979) first article on bootstrapping methods was a breakthrough in world of Statistics This resulted in the merging of the bootstrapping methods to earlier resampling techniques and led to the development of a modified system for handling simulation-based statistical analysis. Academicians were, however, skeptical about this new technique, in that, this new computerized simulation technique would be a replacement for more complex, difficult and error-prone traditional approximations to measures of uncertainty such as variances, confidence intervals and standard errors (Davinson & Hinkley, 1997). Over time, these skeptical academicians and empirical evidence found that the bootstrapping technique was an excellent replacement for previous methods employed, in terms of accuracy, performance, independence of distribution and easiness as compared to traditional approximations mentioned earlier on (Jacobs et al., 2014; Lee, 2014; Souza, Marcato, Dias, & Oliveira, 2012; Chernick, 2008; Corradi & Iglesias, 2008; Efron, 1979).

Bootstrapping can be performed in two ways (Efron & Tibshirani, 1993; Efron, 1979). First, we can bootstrap from the actual or original dataset. This is referred to as a naive bootstrap (Tortosa-Austina, Grifell-tatje, Armero, & Conesa, 2008). That is, if the distribution of the data is uncertain, bootstrap can be done on the original sample dataset. Based on the results obtained after undergoing the bootstrapping procedure, the distribution or assumptions that underpin the sampling data can be used to formulate the model and make inferences to the population parameters of interest (Tortosa-Ausina et al., 2008). This is one of the unique properties bootstrapping has over the Monte Carlo simulation technique. In that, Monte Carlo requires the distribution of the original sampling dataset or model to be explicitly known.

Second, we can bootstrap from the model just as the Monte Carlo if the assumptions underlying the model are known. This is also referred to as the smoothed bootstrap (Tortosa-Ausina et al., 2008). Bootstrap can also be used to check robustness in both parametric and non-parametric situations. (Gnabo, Hvozdyk, & Lahaye, 2014; Kim, 2009; Corradi & Iglesias, 2008; Goncalves & White, 2004; Karolyi & Kho, 2004). For instance, if a study assumes a parametric distribution and incorporates this in its analysis, the bootstrap method, specifically the non-parametric bootstrap can still be used to analyse the robustness of the analysis, despite it being parametric

(Davinson & Hinkley, 1997). This is once again possible due to its unique property of independence.

The bootstrapping technique may be referred to as a computer intensive technique (Tortosa-Ausina et al., 2012; Tortosa-Ausina, 2002; Davinson & Hinkley, 1997). This is because it involves the resampling of the actual data set to create numerous datasets. These datasets mimic various datasets which can result from variations in an economy (Gnabo et al., 2014; Assaf et al., 2011; Karolyi & Kho, 2004). The parameters of interest computed in the original dataset or modelled can, then, be compared to those computed in these new datasets, to determine if there are significant variations. This technique, therefore avoids any complicated or error-prone computations, if the traditional methods of uncertainty were used as mentioned earlier. The name bootstrap came from the famous Baron Munchausen, since the ideology behind the method was similar to the trick Munchausen's used to pull himself out of a lake. He employed his bootstraps as a rope. (Davinson & Hinkley, 1997).

2.3 Empirical Review

This section gives an empirical overview of what have been done on the various theories, concepts and principles mentioned above and how this study adds onto these studies.

2.3.1 Portfolio Theory, Static and Dynamic models

Following the portfolio theory, various studies have been conducted. Siegel and Woodgate (2007) found that, the expected returns and variances increase linearly for every addition of an asset using a sample of six developed countries (Japan, Australia, Germany, United Kingdom, United States of America and Hong Kong) by employing the mean-variance approach. This is attributed to the flexibility of statistical errors to distort the optimisation procedure as more assets are included in the investment set. Also, statistical errors tend to be insignificant in large data sets. Ledoit and Wolf (2003) also constructed a mean-variance optimal portfolio across various US stocks by employing a transformation technique, shrinkage. They realised that by employing this transformation technique, investors are exposed to more information about the market they intend to invest in. This, therefore, reduces estimation errors and enhances appropriate and less biased decisions on optimal choices. Studies of Zhang (2007) and Zhang, Wang, Chen and Nie (2007) also employed possibility means, variances and efficient frontiers (possibility distributions) to a mean-variance technique, instead of the usual probability distributions used by most academic researchers. They found that by employing this possibility technique they are able to solve and construct large, efficient portfolio sets and choices for investors. Both studies employed securities from the Shangai stock exchange. Zhang (2007) applied a sample of 5 securities where as Zhang et al. (2007) applied a sample of 10 securities.

However, other studies have suggested including the third (skewness) and fourth (kurtosis) moments unlike the studies mentioned above, since they assume the distribution of most stock returns deviate from that of a normal, which is the main concept on which the Markowitz Optimization technique (mean-variance criterion) is built upon (Yang & Hung, 2010; Farinelli, Ferreira, Rossello, Thoeny, & Tibiletti, 2008; Kraus & Litzenberger, 1976). While, Yang and Hung (2010) examined 5 stocks, Kraus and Litzenberger (1976) considered stocks listed on the New York Stock Exchange constantly for 10years. They found that the inclusion of these higher moments result in changes in the constructing of optimal portfolios, shapes of efficient frontiers, and the magnitude of risk as compared to using Markowitz's mean-variance analysis (Kerstens, Mounir, & Woestyne, 2011; Yang & Hung, 2010; Prakash, Chang, & Pactwa, 2003; Sun & Yan, 2003; Chunhachinda, Dandapani, Hamid, & Prakash, 1997).

Other studies have also employed a dynamic asset allocation in their portfolio selection, since the Markowitz mean-variance analysis is over a single time period, which is not the case of most investments in the real world. Studies such as Yao et al. (2014), Yu and Huang (2013), Munk and Sorensen (2010) and Xie (2009) employed multi-period or continuous-time asset allocations to dynamic situations. However, though several techniques (dynamic programming, stochastic programming, sample paths, decision trees with quadratic programming, an analytical optimal solution) have been proposed for handling the dynamic asset allocation, most of these techniques are usually problematic when employed in a real world numerical data (Gulpinar & Rustem, 2007; Mulvey & Shetty, 2004; Li & Ng, 2000). For example, dynamic programming when applied to a multiple period asset allocation results in the curse of dimensionality and is not practical, in an actual world multi-period asset allocation decision made by investors (Calafiore, 2008). To the best of my knowledge most of the studies that have proposed a dynamic asset allocation did not apply it in a real world situation due to its complexity (Liu et al, 2012; Calafiore, 2008). Also, the application of a dynamic model would also require a huge data set, since the time period would be sub-divided into several time periods to obtain a multi- period situation, which is usually difficult to attain (Mossin, 1968). This would be extremely difficult to attain in terms of Africa due to data unavailability.

In spite of all these, the Markowitz's mean-variance theory continues to be the foundation of modern portfolio theory because, first, it places huge data requirements on investors. Also, there is no empirical evidence to suggest that including higher moments in the framework changes the appeal of the portfolio selected (Elton & Gruber, 1997). Second, this theory continues to stand firm, because, its significance is highly recognized, well established and greatly acknowledged globally (Elton & Gruber, 1997). This study, therefore, applies a Markowitz optimization technique in the selection of a portfolio across eleven African stock markets.

2.3.2 Portfolio Diversification

There have been several studies on the need and benefits of portfolio diversification (Gaudecker, 2015; Levy & Levy, 2015; Zhou & Nicholson, 2015; Brandtner, 2013; Hung, Liu, & Tsai, 2012; Amenc & Martellini, 2011; Goetzmann & Kumar, 2008; De Santis & Gerard, 1997; Meric & Meric, 1989). Zhou & Nicholson (2015) constructed a diversified portfolio across 3 asset classes in the US economy. They found that by modelling a covariance asymmetry as a result of the asymmetric response correlation and volatility has to possible shocks that could occur in returns, US investors tend to obtain significant gains on a diversified portfolio across these asset classes. Brandtner (2013) also examined the optimal choices for a mean-variance technique as compared to a value at risk (VAR) technique, specifically the spectral risk approach. He found that the benefits of diversification are not optimized when employing this risk (spectral risk) measure since it deviates from the typical trade-off between risk and expected return captured in Markowitz's optimization technique.

Hung et al. (2012) used a sample from the Taiwan Economic Journal, Over the Counter and the Taiwan stock markets to illustrate how managers' diversification (invests into other firms to reduce risk) choices made for their own interest, can in the long run result in the reduction of agency cost. Agency cost arises as a result of agency problems. This is the problem that exists between managers and shareholders, thus when the goals of the managers do not align with or are not in the best interest of the shareholders (Jensen & Meckling, 1976). However, the studies above all focused on the benefits of portfolio diversification within a given economy. This is because, in the actual world we live in, most investors prefer to invest internally or within rather than cross borders (Driessen & Laevan, 2007). Studies such as, that of Miralles-Marcelo, Miralles-Quiros, and Miralles-Quiros (2015) as well as Garcia-Herrero and Vasquez (2013) have, however, indicated the possible benefits of cross-border portfolio diversification when the stock returns of the countries are not highly correlated.

Miralles-Marcelo et al. (2015) examined how to increase the benefits and make cross borders diversification more attractive to prospective US investors using stocks from the Japan, UK and US markets. They found that the benefits of cross border portfolio diversifications are more significant and realised when investors invest in the US currency (dollar) instead of their individual countries' currencies. Garcia-Herrero and Vasquez (2013) found that the benefits of diversification are also realized in the banking industry when banks have subsidiaries by employing a sample of 38 banks from 8 developed economies (US, UK, Spain, France, Germany, Canada, Japan and Italy). They also found that, subsidiaries of banks situated outside their respective home countries, specifically developing economies tend to offer better risk-expected return trade-offs than their parent banks. This, therefore further indicates the substantial benefits of cross border diversification. Despite, the substantial studies on the benefits of portfolio diversification within an economy and a few cross borders, most of the samples of these studies focused either only on or were being dominated by the developed economies to the neglect of developing economies.

For example, Chen et al. (2014) modelled the stock and bond returns of 8 developed economies (US, UK, Australia, France, Germany, Canada, Japan and Italy) using a semi-parametric copula technique. This model was then used to estimate two risk measures (expected shortfall and value at risk) in order to construct diversified portfolios across the two asset classes. Whereas, Driessen and Laevan (2007) employed a sample of 52 countries with 18 being developed economies of which only 6 were African countries. Hence the need for more cross border studies on portfolio diversification in developing economies, specifically Africa. This study therefore, fills this gap.

Africa is gradually becoming a place of interest in terms of investment. Driessen and Laevan (2007) found that investing in Africa tends to offer higher expected returns and better investment opportunities. This is because Africa is gradually becoming a lucrative investment destination as a result of its risky nature and the disintegration of the returns of most African markets. They also realised that the diversified portfolios constructed offered the highest expected returns and best risk-expected return tradeoffs, specifically portfolios which included African stocks. This, therefore, lays a strong foundation for studies targeting developing economies, especially in Africa. Hence, the need for this study. This study, therefore, experts to obtain higher returns at the minimized possible risk and better investment opportunities as compared to a developed economy (US).

2.3.3 Simulation Techniques

There have a tremendous number of studies on the application and the need of simulation techniques in portfolio optimization and asset allocation (Castellano & Cerqueti, 2014; Branger & Hansis, 2012; Engsted & Pedersen, 2012; Lin & Lu, 2012; Pettenuzzo & Timmermann, 2011; Elliott et al., 2010; Ledoit & Wolf, 2008). However, most of these studies employed the Monte Carlo Simulation. For instance, Lin and Lu (2012) focused on the 2 basic decisions faced by investors. The first is how an investor can optimise their consumption levels and the second is which assets to include in an optimal portfolio. They found that if the risks associated with investments and investors' wealth are dependent of each other, investors' risk preference are portrayed in their optimal portfolio. They also found that the risk associated with investors' wealth can be eliminated by constructing a portfolio that protects investors against the risk associated with their investments. These findings were obtained by establishing various paths of consumption with the help of Monte Carlo simulations. Elliott et al. (2010) also employed Monte Carlo simulations in order to create possible paths for the returns of various asset classes in a typical dynamic asset allocation situation. They also utilised a stochastic analysis to construct an equation that can aid in the puzzling mean-variance decision making problem faced by most investors.

However, the various Monte Carlo simulations employed in these studies may be seen as static, since most of the simulations are computed at a given point in time (Winston, 2003). Also, the distribution of the returns for the various asset classes that could be included in investment mixes need to be explicitly known with this technique. This is because with this technique, the simulation is done from the model, hence the distribution, properties and assumptions underlying the model is required in order to simulate (Castellano & Cerqueti, 2014). That is, the various simulations (datasets) that are created in order to obtain the estimates of interest to the researcher are entirely different from the actual dataset which is not the case, in bootstrapping (Bucklew, Ney, & Sadowsky, 1990). This has therefore recently lead to researchers employing bootstrapping as a form of simulation technique (Jacobs et al., 2014; Kim, 2009; Corradi & Iglesias, 2008; Ledoit & Wolf, 2008).

Jacobs et al. (2014) examined various portfolio diversification and optimisation techniques by employing a bootstrapping algorithm over four sectors (Emerging markets, North America, Pacific and Europe). The bootstrap was used in this study to determine if there exist significant differences between the Sharpe ratios of heuristic and optimised portfolios. This study employed bootstrapping, mainly because of its independence to the distribution of stock returns. Hence, this made the results and estimates of the study robust to any non-normality biases. Siegel and Woodgate (2007) also employed a bootstrap algorithm to a sample of four developed countries (Japan, United Kingdom, United States of America and Hong Kong) with the similar motive of not explicitly defining the distribution of stock returns. They found that their optimal choices were also hedged against non-normality biases. In both studies the bootstrap replicated various datasets by resampling the original sample dataset with replacement. Hence, unlike the Monte Carlo simulations, the bootstrap is a more effective, realistic and efficient way of obtaining possible datasets that could result from variations in the economy (Tortosa-Ausina et al., 2012). This is because the tendency of mimicking the actual perturbations that occurs in various economies is higher when using bootstrapping (resamples actual sample data) than Monte Carlo (creates imaginary datasets). This study, therefore employs a bootstrap technique.

The bootstrapping technique employed in this study, makes this research of great relevance to future research. This is because the few studies that performed cross border diversification in portfolio selection rarely performed simulation (Chen et al., 2014; Driessen & Laevan, 2007). Hence, most of these studies did not examine how robust their optimal choices are to possible disturbances that could affect and result in variations in stock returns. It is also worth mentioning that, a few studies have employed other alternatives other than the popularly used Monte Carlo simulations and recently used bootstrapping technique (Bae, Kim, & Mulvey, 2014; Engsted &

Pedersen, 2012; Pettenuzzo & Timmermann, 2011; Binswanger, 2010). Engsted and Pedersen (2012) for example, employed a completely different approach to simulating rather than the bootstrap or the Monte Carlo technique. They used an analytical formula as a result of its properties of biasness, easy understandability, implementation and yielding the same finite sample properties. This analytical bias formula employed in their study is developed mainly for Vector Auto Regressive (VAR) models.

2.4 Overview of Stock Markets in Africa.

There is extant literature on the importance of banks to an economy through their intermediation role (Arestis, Demetriades, & Luintel, 2001; Levine, 1998; Levine & Zervos, 1998; King & Levine, 1993). The role of the banks can therefore not be overemphasized. Stock markets also play an important role in an economy, although it might contribute less to a country's GDP (Rousseau & Wachtel, 2000). Allen and Gale (2000) posit that stock markets serve as a substitute source of funds or investment opportunities for investors. The growth of an equity (stock) market in a country, despite the country's level of development reduces the monopolistic power of banking sector and leads to economic growth (Allen & Gale, 2000; Levine & Zervos, 1996). Stock markets have therefore been identified in the literature to complement the banks in contributing to the long run growth of an economy (Naceur, Ghazouani, & Omran, 2007; Garcia & Liu, 1999; Singh, 1997).

One key difference between developed and developing countries is the presence of the developed equity markets in the developed countries (Pradhan, Arvin, & Bahmani, 2015). In Africa most economies are bank-based and efforts are being made to build up the capital markets in African countries. This is because development of stock markets enhances economic growth in the long run (Enisan & Olufisayo, 2009; Beck & Levine, 2004; Beck & Levine, 2002; Rousseau & Wachtel, 2000). These efforts have paid off because most African countries and other developing economies since 1994 have recorded increases in their stock indices as well as market capitalization (Alagidede, 2011). In 1994 the highest gains in monetary value were recorded from stock markets in Africa (Alagidede, 2011). These gains continued to 1995. On average, stock indices of African countries increased by a margin of 40% in 1995 (value of stocks in Nigeria and Cote D'Ivoire doubled in 1995) and ever since have appreciated gradually over the years with a few shocks. The appreciation in the value of stocks and globilisation accompanied by major technological development has made the African stock market a place of interest (Alagidede, 2011; Magnusson & Wydick, 2010; Appiah-Kusi & Menyah, 2003).

The returns of African stock markets are also characterised with different features, as compared to those of the developed countries, thereby enabling them to offer better investment sets and opportunities for both local and foreign investors (Graham, Kiviaho, & Nikkinen, 2012; Harvey, 1995). Bekaert and Harvey (1997) found that the stock returns of most emerging markets (which includes two of the African countries in this study's sample) tend to be highly volatile as compared to developed economies. Hence, a construction of a well diversified portfolio with African stocks, will result in higher returns since risks and expected returns are positively related (Gennaioli et al., 2015; Gerlach et al., 2015; Harvey, 1995). The risky nature of African stock returns is depicted in figures 1 to 6 below.

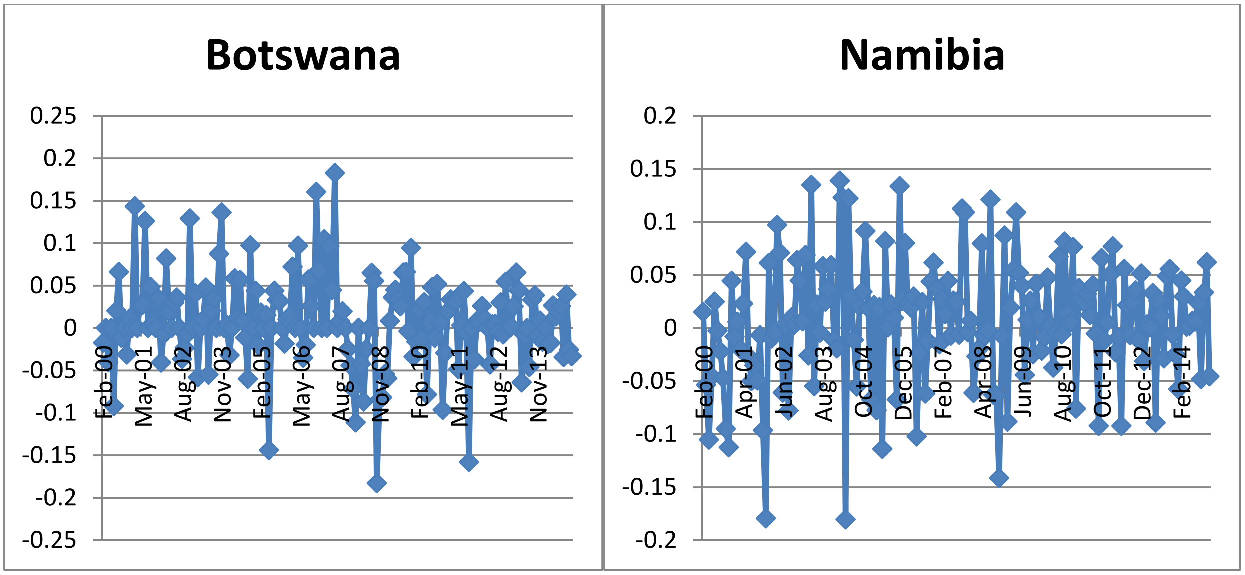


Figure 1: A Representation of Stock Returns for Botswana and Namibia from 2000 to 2014

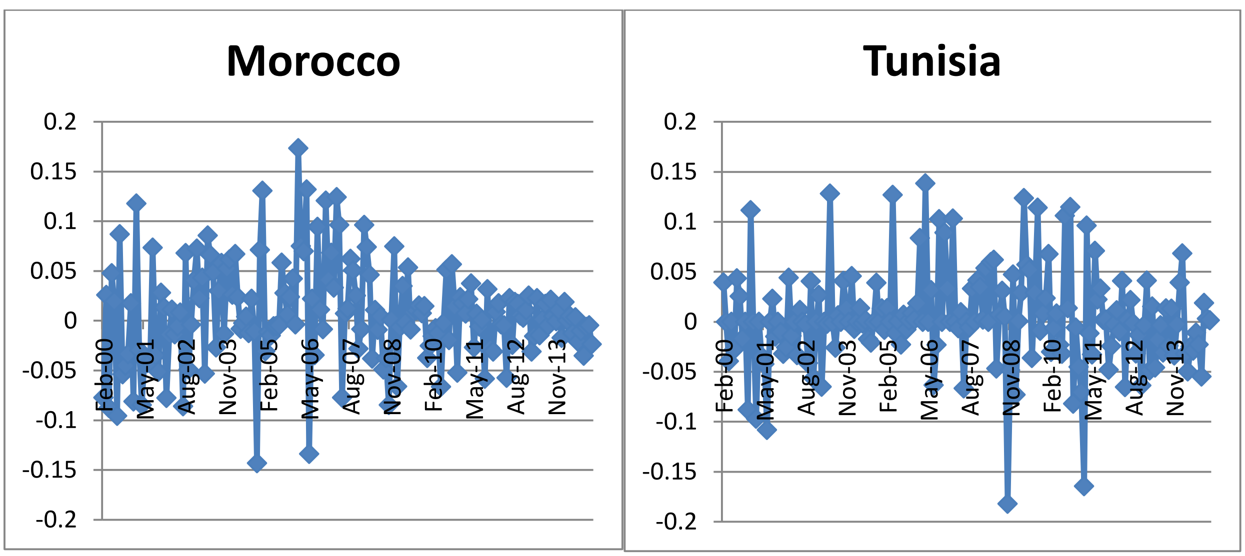


Figure 2: A Representation of Stock Returns for Morocco and Tunisia from 2000 to 2014

Source: Author (2015)

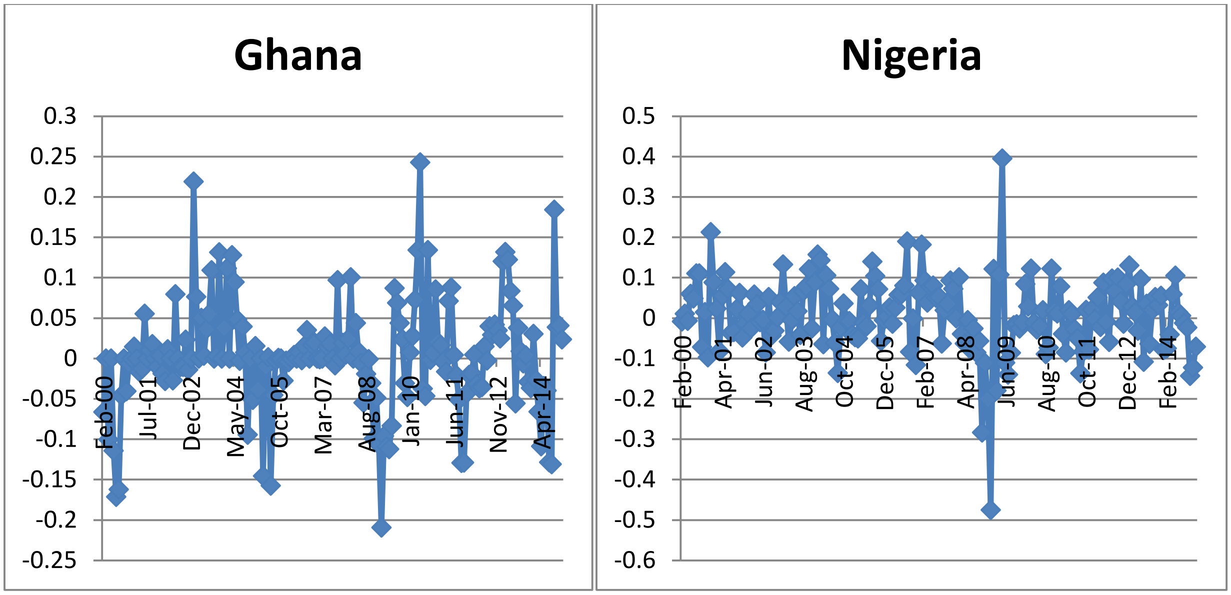


Figure 3: A Representation of Stock Returns for Ghana and Nigeria from 2000 to 2014

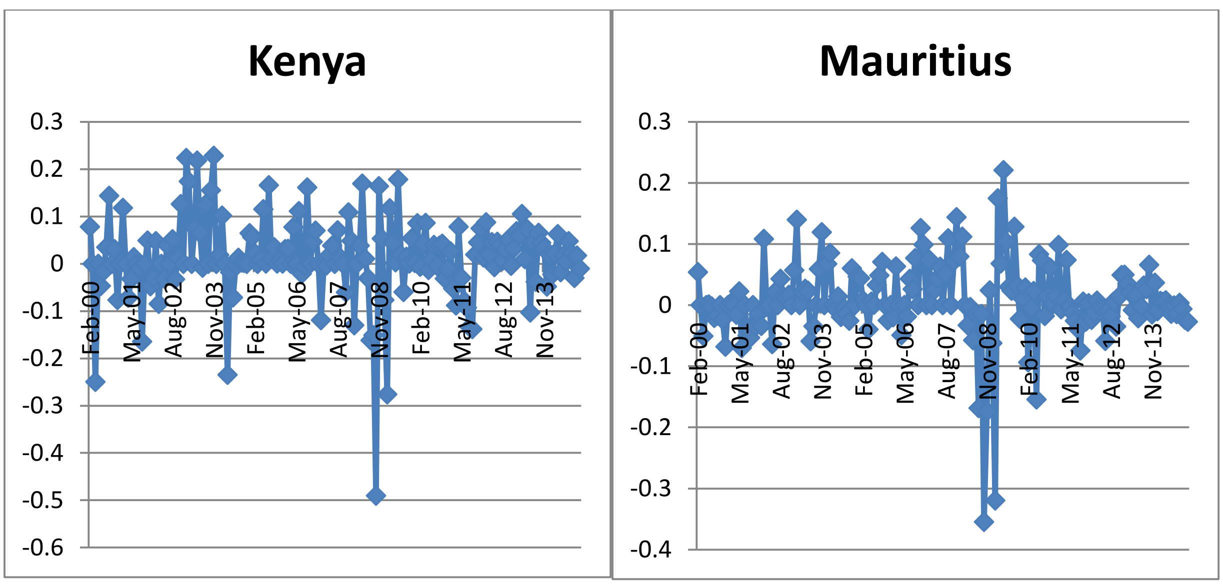


Figure 4: A Representation of Stock Returns for Kenya and Mauritius from 2000 to 2014

Source: Author (2015)

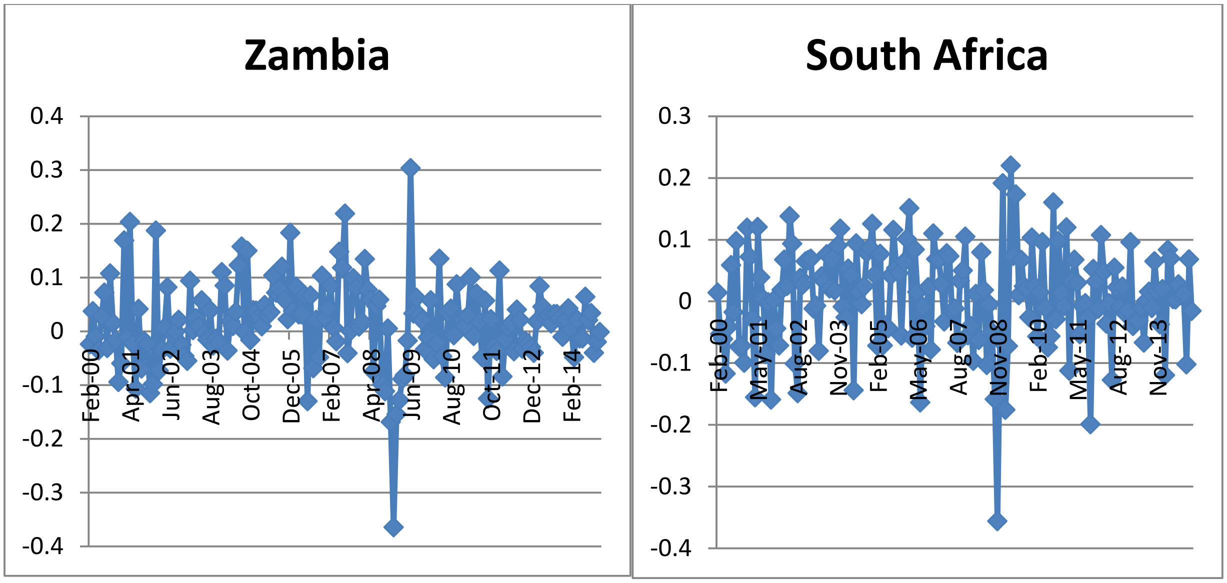


Figure 5: A Representation of Stock Returns for Zambia and South Africa from 2000 to 2014

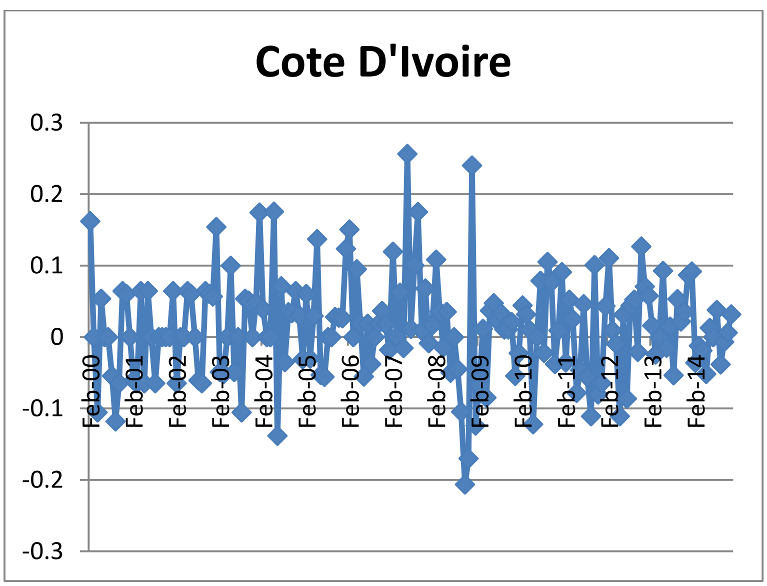


Figure 6: A Representation of Stock Returns for Cote D'Ivoire from 2000 to 2014

Source: Author (2015)

In figures 1 to 6, the stock returns of the eleven African stock markets have not been stable from 2000 to 2014 indicating volatility in the returns.

Another interesting feature of African stock returns is their low or negative correlations with developed and other developing economies which has been found to enhance portfolio diversification (Alagidede & Panagiotidis, 2009). International portfolio investments have, therefore, been of interest in recent years (Rouseau & Wachtel, 2000). Therefore, the low correlations between the stock returns of African and developed markets along with the higher returns they offer as a result of their risky nature (African markets) makes Africa a more attractive pool to invest in (Groot et al., 2012).

2.5 Chapter Summary

This chapter gives a brief overview of relevant theories and concepts in asset allocation and portfolio optimization. Theories and concepts such as, portfolio theory, portfolio diversification, separation theorem, static and dynamic models are discussed. This chapter also gives a general overview of what has been done over the years in terms of asset allocation, portfolio diversification and optimization. Simulation techniques, specifically the bootstrapping technique is also discussed in this chapter and the need for it in any asset allocation study.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

This chapter entails the research design adopted by the researcher in this study and provides information about the population, sample, sampling technique and data analysis plan. The portfolio optimization model as well as the bootstrapping algorithm to be incorporated in this study are briefly explained.

3.2 Research Design

This study employs a quantitative approach to research. According to the literature, quantitative approach to research allows a researcher to test the objective theories by examining the relationship that may exist among variables. With the help of statistical packages and tools these variables are measurable (Creswell, 2009). Also, since the data being used is purely quantitative, the quantitative approach is more appropriate (Baltagi, 2008). The study formulates an optimized portfolio across eleven African stock markets (Cote D'Ivoire, Mauritius, Kenya, Nigeria, Tunisia, South Africa, Morocco, Bostwana, Ghana, Namibia and Zambia) by employing the Makowitz Portfolio Optimization technique and compares it to that of the S&P Dow Jones portfolio. Pairwise correlations are constructed to check for the correlations among the eleven stocks. The correlation matrix is constructed in order to check if there exist low and negative correlations among the markets. Previously, it has been mentioned that in order to benefit from international and portfolio diversification, there should be low and possibly negative correlation between stock returns of the markets (Gerstner, Griebel, Holtz, Goschnick, & Haep, 2008).

A Bootstrap algorithm is used as a simulation technique that will generate various datasets. Simulation is relevant in portfolio optimisation, in that, there are possible variations that could occur in the macroeconomic variables of a country as well as any idiosyncratic errors in measurement (Klassen & Yoogalingam, 2013). Hence the bootstrap process will simultaneously address the stochastic parameters of the sampling process to determine whether there exist significant differences between the simulated results to that of the original (Aslanidis & Casas, 2013; Adachi & Gupta, 2005). Optimization is done on the original dataset as well as the bootstrapped datasets. Each of the bootstrapped datasets captures the possible variations that may occur in an economy such as procyclicality of business trends, economic instability and other forms of idiosyncratic errors. Thus, the bootstrap also serves as a robustness check and corrections would be made if there exists any significant variations in the original and bootstrapped estimates.

3.3 Source of Data

In order to construct an optimal portfolio, there is the need to gather information on dividend-adjusted market returns and treasury bill rates of the countries under consideration. Market indices are sourced from the Thomson Reuters Datastream database but computed by Standard and Poor's. Standard and Poor's has been considered to be one of the largest source of indices concerning capital markets across the globe (S&P Dow Jones, 2015). It gathers the daily, weekly and monthly stock indices globally. This has been the major source of data in papers such as Bae et al. (2014), Munk and Sorensen (2010), Goldfarb and Iyengar (2003) and Elton and Gruber (2000).

Treasury bill rate is used as a proxy for the risk-free asset in the study. These treasury bill rates were derived from the World Development Indicators database. These are computed as the difference between the yearly lending rates and risk premiums on each of the eleven countries. This method is used in obtaining the treasury bill rates since it may be difficult to obtain the treasury bill rates for all eleven countries directly. Hence, for uniformity and prevention of bias this technique is employed.

3.4 Population, Sample, Sampling Technique

The population for the study is all the 24 stock exchanges in Africa (ASEA, 2013). These 24 stock exchanges represent 38 African countries.

The study period is between the years 2000 and 2014. Egypt, South Africa, Morocco, Zimbabwe, Kenya, Nigeria, Tunisia, Mauritius, Bostwana, Ghana, Swaziland, Namibia, Sudan, Zambia, Malawi, Algeria, Tanzania, Cote D'Ivoire, Uganda and Mozambique, therefore qualify for this study, since their stock markets date back to 2000 as shown in Table 1. Countries such as, Cameroon, Cape Verde, Libya, Rwanda, Seychelles and Somalia have been excluded from this study because their stock markets do not date back to 2000. However, out of these 20 countries that qualify to be included in this study, only 11 of them would be used due to data unavailability. This study also focuses on these 11 countries because they may offer better investment opportunities and portfolios with high returns (Groot et al., 2012; Jacobs et al., 2014). The optimal portfolio to be constructed for this study using these countries will maximize an investor's wealth and the S&P Dow Jones (SPDJ) Composite price index is used as the benchmark for the optimal portfolio to be constructed across the eleven African countries. It is assumed that this index has been adjusted for dividend payments.

The data being used is balanced in nature. In computing the stock returns, the monthly stock price indices of each country are used.

Table 1: Stock Exchanges in Africa

No	Economy	Exchange	Location	Founded
1	Egypt	Egyptian Exchange (EGX)	Cairo, Alexandria	1883
2	South Africa	JSE Limited (JSE)	Johannesburg	1887
3	Morocco	Casablanca Stock Exchange (Casa SE)	Casablanca	1929
4	Zimbabwe	Zimbabwe Stock Exchange (ZSE)	Harare	1948
5	Kenya	Nairobi Securities Exchange (NSE)	Nairobi	1954
6	Nigeria	Nigerian Stock Exchange (NSE)	Lagos	1960
7	Tunisia	Bourse de Tunis (BVMT)	Tunis	1969
8	Mauritius	Stock Exchange of Mauritius (SEM)	Port Louis	1988
9	Botswana	Botswana Stock Exchange (BSE)	Gaborone	1989
10	Ghana	Ghana Stock Exchange (GSE)	Accra	1990
11	Swaziland	Swaziland Stock Exchange (SSX)	Mbabane	1990
12	Namibia	Namibia Stock Exchange (NSX)	Windhoek	1992
13	Sudan	Khartoum Stock Exchange (KSE)	Khartoum	1994
14	Zambia	Lusaka Stock Exchange (LuSE)	Lusaka	1994
15	Malawi	Malawi Stock Exchange (MSE)	Blantyre	1995
16	Algeria	Algiers Stock Exchange (ASE)	Algiers	1997
17	Uganda	Uganda Securities Exchange (USE)	Kampala	1997
18	Cote D-Ivoire	Bourse Regionale des Valeurs Mobilieres (BVRM)	Abidjan	1998
19	Nigeria	Abuja Securities and Commodities Exchange (ASCE)	Abuja	1998
20	Tanzania	Dar es Salaam Stock Exchange (DSE)	Dar Es Salaam	1998
21	Mozambique	Bolsa de Valores de Mozambique (BVM)	Maputo	1999
22	Cameroon	Douala Stock Exchange (DSX)	Douala	2001
23	Cape Verde	Bolsa de Valores de Cabo Verde (BVC)	Mindelo	2005
24	Libya	Libyan Stock Market (LSM)	Tripoli	2007
25	Rwanda	Rwanda Stock Exchange (RSE)	Kigali	2008
26	Seychelles	Seychelles Securities Exchange (SSE)	Victoria	2012
27	Somalia	Somalia Stock Exchange (SSTE)	Mogadishu	2012

Source: Author (2015)

3.5 Correlation Matrix

The asset allocation and portfolio optimization literature indicates that in order to take advantage of diversification in portfolio optimization and asset allocation, low and negative correlations need to exist between stock returns of markets or securities (Mensah et al., 2013; Alagidede & Panagiotidis, 2009). This study also performs hypothesis testing to determine if these correlations are significantly different from zero.

3.6 Test of Normality

There are several techniques that can be used in checking normality (Brooks, 2008). Some of these are Shapiro-Wilk, Jarque-Bera, Anderson-Darlington, Pearson's Chi-squared and Kolmogorov-Smirov normality tests. However, the study employs the Jarque-Bera normality test. The reason being that, it is one of the most accurate and popular approaches in testing normality and appropriate when using large sample sizes (Gujarati, 2003). The price indices obtained for the various countries are transformed into returns as stated in the earlier section. These stock returns are then tested to determine if they are normally distributed since it is a requirement when adopting the Markowitz Optimization technique (Guo, Ye, & Yin, 2012).

3.7 Minimum Variance Portfolio

Bodie et al. (2011) define a minimum variance portfolio as one that contains a mixture of risky securities which provides the least variance. That is, this is a portfolio in which changes in the variance with respect to the changes in the weight apportioned to the various securities that constitute the portfolio is zero.

Mathematically, this can be expressed as;

$$\frac{\partial Var(MVP)}{\partial w} = 0$$

Where *MVP* represents the minimum variance portfolio, *w* represents the weight and *Var* represents the variance of the minimum variance portfolio which is derived in model 5 below.

3.8 Tangency Portfolio

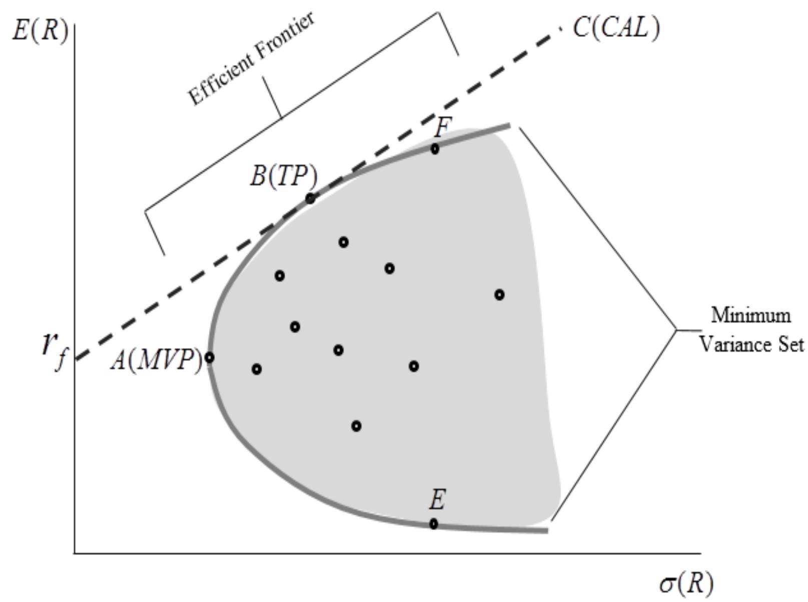
A tangency portfolio may be defined as the best mixture of risky assets which gives the utmost degree of expected return on a portfolio at an optimized risk. This is also referred as an optimal portfolio, thus one that offers the best risk-expected return trade-off (Tepla, 2000). A tangency portfolio is a portfolio on the efficient frontier which is tangent to the capital allocation line (Jorion, 1992). A tangency portfolio is derived using model 6 shown below.

3.9 Efficient Frontier

Efficient frontier is a curve indicating portfolios with the maximum expected levels of returns for given levels of variances (Elton & Gruber, 2011; Xie, 2009). The efficient frontier is obtained by, first, determining the set of possible risk-return combinations that investors may be faced considering the assets available to them (Costa & Araujo, 2008; Topaloglou, Vladimirou, & Zenios, 2002). Thus, given a particular level of expected return, the least variance is obtained. These risk-return combinations are then plotted on a graph as shown in Figure 7 below. This results in an egg-like shape curve known as a hyperbola, which is usually referred to as the minimum variance frontier or set. That is to say, the various investment mixes that plots this minimum variance set (MVS) are considered to be dominating portfolios as compared with those within the set.

Point A indicates a minimum-variance portfolio. This is a portfolio which provides the least risk as seen in Figure 7. Thus, the upper section of the MVS is the efficient frontier, that is, the curve above the minimum-variance portfolio. Hence, efficient portfolios (portfolios on efficient frontiers) provide the most favourable risk and expected return combinations, investors can have (Bodie, Kane, & Marcus, 2005). Based on the risk preference of investors, the capital allocation line is obtained. The capital allocation line (CAL) is a representation of the expected returns and risks investors are subjected when they invest in both risky (portfolio of risky assets) and risk-free assets (Bodie et al., 2005). The point (C) at which CAL is tangent to the efficient frontier is the optimal portfolio as mentioned earlier. It is realised in figure 7 that, for every portfolio, at the lower part of the MVS (Portfolio E), there exists a portfolio (Portfolio F) on the upper part of the MVS at the same given level of risk that offers a higher expected level of return.

Risk averse investors would therefore apportion their wealth or choose portfolios between the optimal portfolio B and the risk-free assets' rate (r_f) (Asness, Frazzini, & Pedersen, 2012). Thus the movement along this line would be based on an individual's level of risk behaviour. However, risk lovers would prefer to select the tangency portfolio, B and beyond it (Frazzini & Pedersen, 2014). These investors would rather prefer to take on more risk by borrowing at this risk-free rate in order to invest beyond a hundred percent in the tangency portfolio, as oppose to risk averse investors who would rather lend (invest) a portion of their wealth at the risk-free rate (Asness et al., 2012; Alexander & Baptista, 2002).



Source: Author (2015)

Figure 7: Graphical Representation of an Efficient Frontier

Where r_f, MVP and TP represent the risk-free, minimum-variance and tangency portfolios respectively.

3.10 Portfolio Optimization Model

This study employs the Markowitz Portfolio Optimization model. This is to determine the optimal risky portfolio that investors can hold by investing in stock markets across the eleven African stock exchanges mentioned above. Given that investors are dealing with price indices of $N = \{N_i | i = 1,2,..N\}$ number markets over $T = \{T_t | t = 1,2,..T\}$ number of months.

The return of market i at a month t is therefore computed by the log returns formula:

$$r_{it} = \ln\left(\frac{P_{it}}{P_{it-1}}\right) \times 100\% \tag{1}$$

where r_{it} is the return of market i at month t, and P_{it} is the price index of a market i at month t. This study employs the natural log $(\ln)$ since it has unique statistical properties which allows returns to be continuously compounded and summed. Dividend payments are not included in the computation of returns since it is assumed in this study that, such effects have been already adjusted for in the computation of the index. The expected return $E(r_i)$ of a market i is computed by:

$$E(r_i) = \frac{1}{T}\sum_{t=1}^{T} r_{it} \tag{2}$$

whereas the market variance σ_i^2, as the measure of risk, is computed by:

$$\sigma_i^2 = E\left[r_{it} - E(r_i)\right]^2 \tag{3}$$

The expected return on the portfolio is therefore;

$$E(r_P) = \sum_{i=1}^{N} w_i E(r_i) \tag{4}$$

where w_i is the weight or the proportion of an investor's budget dedicated to investing in a particular market. Markowitz's model is therefore formulated using the linear programming models (5) and (6) below. Model (5) is a minimization problem that aids in computing the minimum variance portfolio. Conversely, Equation (6) is a maximization problem aimed at maximizing the Sharpe ratio in order to generate the Tangency Portfolio (Mensah et al., 2013).

$$\text{Minimize} \quad \sigma_P^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2\sum_{i,j=1}^{N} w_i w_j \operatorname{cov}(r_i, r_j) \qquad \text{, where } i \neq j$$
$$\text{subject to:}$$
$$\sum_{i,j=1}^{N} w_{ij} = 1 \tag{5}$$
$$\sum_{i=1}^{N} w_i E(r_i) \geq r_p$$
$$w_{i,j} \geq 0 \qquad i, j = 1,2,\ldots, N$$

The objective here is to determine how much to invest in the various markets so as to minimise the variance of this portfolio. However, this is subject to some constraints. The first constraint ensures that the total budget is invested in the markets (Farinelli et al., 2008). This means that the sum of the weights for the various markets should be equal to 1, not less or greater. The second constraint requires the optimal expected return of the portfolio to be either equal to or greater than the minimum portfolio return. The minimum portfolio return is defined as the expected return from investing equally across the markets. It is therefore expected that, the optimal solution is not dominated by this heuristic choice. The final constraint, the non-negativity constraint requires that either nothing or strictly positive weight is to be assigned to a market. It also shows the absence of short-selling. Short selling is not assumed since in developing markets like those in Africa, it is hardly practised due to the illiquid nature of their stock markets (Mensah et al., 2013).

For the tangency portfolio the model is to, maximize the Sharpe ratio subject to the same constraints as in Eqn (5). This is expressed as follows:

$$\text{Maximize} \quad S_p = \left(\sum_{i=1}^{N} w_i E(r_i) - r_f \right) \sigma_p^{-1}, \text{where } i \neq j$$
$$\text{subject to:}$$
$$\sum_{i,j=1}^{N} w_{ij} = 1 \qquad (6)$$
$$\sum_{i=1}^{N} w_i E(r_i) \geq r_p$$
$$w_{i,j} \geq 0 \qquad i, j = 1,2,..., N$$

It must be noted that $\sum_{i=1}^{N} w_i E(r_i)$ is the expected return of the portfolio. r_f is the average risk-free of the markets under study. σ_p^{-1} denotes the inverse of the portfolio's risk which is measured using the standard deviation of the portfolio.

3.11 Bootstrapping as a Simulation Technique

Bootstrapping is related to simulation, except that with simulation data is completely artificially generated for the purpose of making statistical inferences. However, in bootstrapping empirical estimators are generated using data points sampled repeatedly (with replacement) from the sample (Brooks, 2008). The basic assumption underlining the bootstrapping algorithm is that, the sample if well constructed is a good representative of the actual population. Therefore, by resampling randomly from the sampling distribution, we can get an approximation of the exact nature of the population distribution by correcting the bias in the sampling distribution.

Given a sample data of $X = (X_1,..., X_T)$ are available to estimate a desired parameter $\theta(X)$. Since the true population F is unknown, the true parameter $\hat{\theta}(F)$ is also unknown. Therefore, we draw B samples of $X^{(b)} \big| b = 1,..., B$ from the actual data. This provides B number of pseudo samples each of the size T: $X^{(b)} = (X_1^{(b)},..., X_T^{(b)})$. We can therefore compute $\theta^*(X^{(b)})$ for each $i = 1,..., T$. Therefore the relationship between the true and sample estimates can be computed using the relationship between the sample and bootstrapped estimates (De Borger, Kerstens & Staat, 2008)

For example,

$$\left(\hat{\theta}(F) - \theta(X)\right)\Big| X \approx \left(\theta(X) - \theta^*(X^{(b)})\right)\Big| X^{(b)}$$

This means that, the difference between the true (but unknown) estimate $\hat{\theta}(F)$ and the sample estimate $\theta(X)$, is approximately equal to the difference between the sample estimate $\theta(X)$ and the bootstrapped estimate $\theta(X^{(b)})$. The bias in the estimate is therefore computed as:

$$Bias(\theta^*) = \frac{1}{B}\sum_{b=1}^{N} \theta^*\left(X^{(b)}\right) - \theta(X)$$

It should be noted that $\frac{1}{B}\sum_{b=1}^{B} \theta^*\left(X^{(b)}\right)$ represents the mean of the bootstrapped estimate.

The variance for the estimate is also computed by

$$var(\theta^*) = \mathrm{E}[\theta^* - E(\theta^*)]^2$$

Rule of thumb, if $\frac{|Bias(\theta^*)|}{Std(\theta^*)} \leq 0.25$, then deviation of the bootstrapped estimates from the actual sample is insignificant (Efron & Tibshirani, 1993).

where $Std(\theta^*)$ represents the standard deviation of the bootstrapped estimate θ^*.

If the biases found are significant, the bias corrected estimates are computed as the actual estimate from the original sample minus the bias computed. (Efron & Tibshirani, 1993). Mathematically, this is represented as follows;

$$\theta(X_*^*) = \theta(X) - Bias(\theta^*)$$

where $\theta(X_*^*), \theta(X)$ and $Bias(\theta^*)$represents the bias corrected estimate, the actual estimate from the original sample and the bias estimate respectively.

3.12 Independent Sample t-test

Independent (two) sample t-tests are performed in this study to determine if there exists significant differences between the bias corrected expected returns obtained for the minimum-variance and tangency portfolios and the expected returns for each country's individual portfolio. A second independent t-test is also performed between the bias corrected expected returns obtained for the minimum-variance and tangency portfolios and expected return of the Standard and Poor's Dow Jones portfolio. These independent t-tests are performed to determine if the expected returns of two well diversified portfolios (minimum-variance and optimal portfolios) constructed in this study which have undergone the bootstrapping process are statistically different from each of the individual country's portfolios and that of the Standard and Poor's Dow Jones portfolio's expected returns.

3.13 Data Analysis Plan

The price indices are obtained from the Datastream database. The minimum, maximum, average return, skewness and kurtosis are computed so as to get a fair idea of the characteristics and behaviours of the eleven African stock markets. The returns are then computed in an excel spreadsheet and imported onto Matlab software platform for the optimization and bootstrap processes. MatLab is used for all statistical computations.

3.14 Chapter Summary

This chapter entails the methodology the research that is employed in this study and the various assumptions underlying these methods. This consists of the research design, the population, sample, sampling technique and a brief outline of how the analysis is performed. This study strictly follows the Markowitz Optimization technique replicated by Mensah et al. (2013). A bootstrapping algorithm is used as both a robust check and a form of a simulation technique.

CHAPTER FOUR

DATA ANALYSIS

4.1 Introduction

This section entails the sequence of the data analysis performed in this study and the interpretations of these results. This would commence with the summary statistics of the stock returns for each country. Preliminary analyses on each objective of the study would follow subsequently.

4.2 Summary Statistics

This section displays a table of the yearly average returns, standard deviations, minimum & maximum returns and the Sharpe ratios of the eleven African stock markets. The descriptive statistics of the S & P Dow Jones stock returns are also included.

Table 2:Descriptive Statistics of Stock Returns for the Eleven African Countries

Country	Mean	Standard Deviation	Minimum	Maximum	SR
Botswana	0.0940	0.1786	-2.1942	2.1920	0.5261
Namibia	0.0747	0.1965	-2.1643	1.6680	0.3800
Morocco	0.0745	0.1689	-1.7198	2.0814	0.4410
Tunisia	0.0291	0.1648	-2.1858	1.6594	0.1765
Ghana	0.0183	0.2298	-2.5131	2.9120	0.0795
Nigeria	0.0902	0.3098	-5.7045	4.7378	0.2911
Kenya	0.1493	0.2940	-5.8810	2.7395	0.5079
Mauritius	0.0869	0.2223	-4.2539	2.6483	0.3910
Zambia	0.1764	0.2638	-4.3695	3.6432	0.6689
SA	0.0740	0.2758	-4.2672	2.6416	0.2684
CD	0.1499	0.2436	-2.4760	3.0772	0.6152
SPDJ	0.0497	0.1450	-1.8336	1.1783	0.3426

*SA-South Africa, CD-Cote D'Ivoire, SPDJ-S&P Dow Jones

From table 2, Kenya has the least minimum stock return of -5.8810, whereas Morocco has the highest minimum stock return of -1.7198. Tunisia and Nigeria have the lowest and highest maximum stock return of 1.6594 and 4.7378 respectively. The average returns and their associated risks are also indicated. The results revealed that these returns are associated with averagely high risks as expected, since Africa (emerging and frontier markets) is considered to be a very risky environment in terms of investment (Bekaert & Harvey, 2003). The lowest risk is 16.48%, which corresponds to an average return of 2.91%, whereas the highest risk is 30.98%, which is associated with an average return of 9.02%. Zambia's portfolio has the highest average return of 17.64% with a risk of 26.38%, whereas that of Ghana has the least average return of 1.83% with a risk of 22.98%.

The average return and standard deviation of the S&P Dow Jones' portfolio are also reported in this table The S&P Dow Jones' portfolio has an average return of 4.97% with a risk of 14.50%. In comparison to the average returns of the eleven African portfolios, with the exception of the portfolios from Tunisia and Ghana, all the others have a higher average return than that of the S&P Dow Jones. However, the portfolio from S&P Dow Jones tends to have a lower risk than those from the eleven African countries. These results conform to the argument made by Bekaert and Harvey (1997) and Harvey (1995) that, stock returns of developing economies tend to have different characteristics as compared to the developed economies. Hence, given the higher returns African stocks and portfolios seeks to offer, a diversified portfolio across these eleven stock markets would be very beneficial in maximizing the wealth of both foreign investors outside Africa, specifically a US investor and domestic investors within Africa.

The Sharpe ratios (SR) were also estimated using the average return and risk for the individual portfolios of the eleven African countries and that of the S&P Dow Jones. These ratios indicate the reward to volatility, thus the ratio of average return per risk taken, assuming a risk-free rate of zero (Mensah et al., 2013; Bodie et al., 2011). Zambia's portfolio reported the highest reward to volatility of 66.89%, whereas Ghana's reported the least (7.95%). On the average, more than 50% of individual countries (African) portfolios have low Sharpe ratios (less than 50%) indicating that, investors investing in these individual country's portfolio are not being rewarded greatly for the excessive risk taken (Bodie et al., 2011). The same can be said for the S&P Dow Jones, which has a Sharpe ratio of 34.26%.

A graphical representation of the various individual portfolios (eleven African countries' portfolios) with their corresponding average returns and risks are shown below.

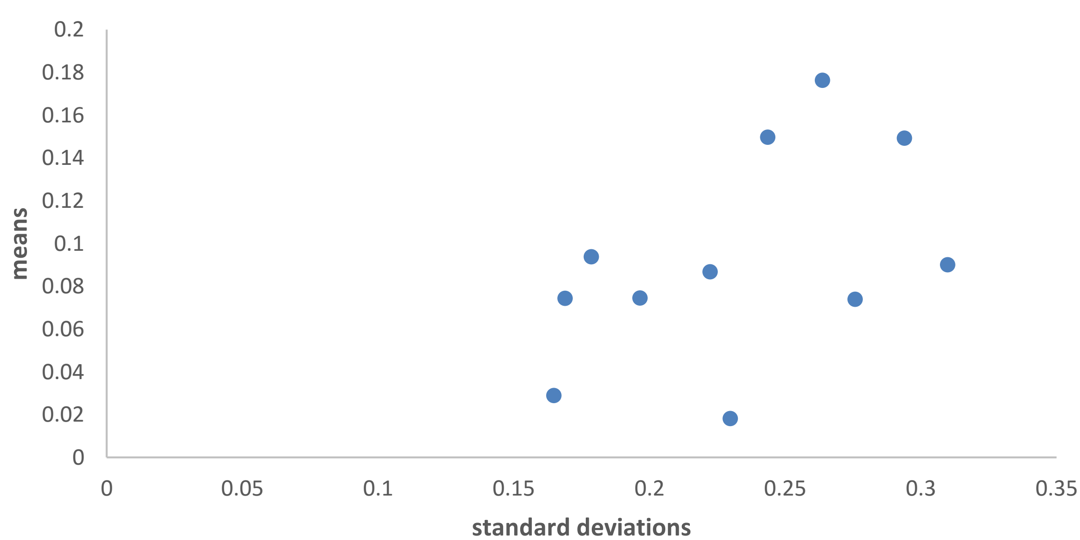


Figure 8 : Graphical Representation of the Average Returns and Standard Deviations

Figure 8 indicates that most of the individual country's portfolios are skewed towards the lower right, indicating very risky portfolios which are also not being rewarded much for excessive risk taken.

4.3 Normality Tests

The stock returns of the various countries were tested for normality. Table 3 reports the skewness, kurtosis, the Jarque-Bera statistic and the p-values.

Table 3:Normality Tests of Stock Returns for the Eleven African Countries

Country	Skewness	Kurtosis	Jarque-Bera	p-value
Botswana	-0.0577	2.3680	38.4985	0.0000
Namibia	-0.3723	0.8013	8.2126	0.0165
Morocco	0.2116	1.1058	9.4089	0.0091
Tunisia	0.0493	2.2849	35.7843	0.0000
Ghana	0.1342	1.9830	27.3027	0.0000
Nigeria	-0.5939	5.9466	256.8091	0.0000
Kenya	-1.3994	7.6920	470.9001	0.0000
Mauritius	-1.4721	9.4959	694.7560	0.0000
Zambia	-0.2245	3.9978	112.2395	0.0000
South Africa	-0.6111	1.9004	35.5086	0.0000
Cote D'Ivoire	0.2682	1.2235	12.0818	0.0024

With the exception of Morocco (0.2116), Tunisia (0.0493), Ghana (0.1342) & Cote D'Ivoire (0.2682), all the other 7 countries' stock returns were negatively skewed. The eleven countries' stock returns have positive kurtosis. However, normal distributions have excess kurtosis and skewness of zero (Anderson, Sweeney, & Williams, 2011). Hence, the stock returns of all eleven African economies deviate from the characteristics of a normal distribution. The Jarque-Bera and its associated p-values were computed to further confirm if, these stock returns deviate from the assumptions of a normal distribution. The results of the test indicated in table 2 confirm that stock returns do violate the properties and assumptions underlying a normal distribution, since their p-values are less than 5%. Hence, these results correspond with the studies of Yang and Hung, (2010), Susmel (2001), Bekaert, Erb, Harvey, and Viskanta, (1998) as well as Chunhachinda et al. (1997) that stock returns do not follow a normal distribution.

However, stock returns need to follow a normal distribution in order to apply the Markowitz Optimization technique. This study is using the bootstrapping algorithm as both a robust and a form of simulation technique. The bootstrap has a unique characteristic of being independent to the distribution stock returns follow, since it does not require the distribution to be explicitly known (Jacobs et al., 2014). Hence the bootstrap would cater for any bias in the estimation of the various portfolios and make the results of this study robust to non-normality, thereby justifying the use of the Markowitz Optimization Technique in this study (Siegel & Woodgate, 2007).

4.4 Correlation Matrix

A correlation matrix for the stock returns of the eleven stock markets and that of the S&P Dow Jones was constructed. The researcher further tested the hypothesis of no correlation, to check if these correlation coefficients are significantly different from zero. Table 4 reports the correlation coefficients of the stock returns between the various African countries and S&P Dow Jones. The least correlation coefficient (0.01) is reported between the stock returns of Ghana and Tunisia, whereas the highest (0.38) is reported between Kenya and Mauritius. It is realized from table 4, that all the correlations are below 50%, indicating that the relationship between the returns of the eleven African stock markets are not strong (Cohen, 1988). These results therefore, do not warrant any of the eleven African countries to be eliminated. Hence portfolio diversification across these countries should result in higher returns or reduce the risk associated with these returns as compared to investing in any of the individual countries (Groot et al., 2012; Li et al., 2003; Harvey, 1995; Obstfeld, 1994).

Table 4: Correlation matrix between the Stocks Returns

	Country	1	2	3	4	5	6	7	8	9	10	11	12
1	**Botswana**	1											
2	**Namibia**	0.23*	1										
3	**Morocco**	0.25*	0.31*	1									
4	**Tunisia**	0.23*	0.18*	0.2*	1								
5	**Ghana**	0.04	-0.04	0.12	-0.01	1							
6	**Nigeria**	0.17*	0.08	0.18*	0.03	0.11	1						
7	**Kenya**	0.13	0.13	0.29*	0.1	0.11	0.09	1					
8	**Mauritius**	0.23*	0.16*	0.31*	0.21*	0.16*	0.27*	0.38*	1				
9	**Zambia**	0.07	0.09	0.02	0.06*	0.09	0.07	-0.02	0.17*	1			
10	**SA**	0.25*	0.06	0.04	0.09	0.1	0.15*	0.18*	0.22*	0.21*	1		
11	**CD**	0.24*	0.25*	0.33*	0.18*	0.13	0.21*	0.17*	0.29*	0.14*	0.07	1	
12	**SPDJ**	0.34	0.32	0.28	0.15	0.01	0.14	0.26	0.14	0.00	0.1	0.21	1

* Correlation is significant at 5% significance level.

It can also be observed that the correlation coefficients reported in table 4 between each of the African countries and S&P Dow Jones are less than 50%. This corresponds and affirms the argument that the stock returns of African (developing) economies and that of the developed, which is represented by S&P Dow Jones are not related to each other (Jacobs et al., 2014; Bekaert & Harvey, 1997). Hence foreign investors may be exposed to higher investment opportunities if they invest and diversify in Africa.

4.5 Average Returns and Covariance Matrix of The Eleven African Countries.

The monthly average returns and covariance matrix for the eleven African stock markets are reported in tables 5 and 6 above. These are used as inputs in the linear programming models stated in chapter 3. The first model aims at minimizing the variance and the second at maximizing the Sharpe ratio in order to generate minimum and tangency portfolios respectively (first objective of the study).

Table 5: Monthly Average Returns of The Eleven African Countries

Country	Average Return
Botswana	0.0078
Namibia	0.0062
Morocco	0.0062
Tunisia	0.0024
Ghana	0.0015
Nigeria	0.0075
Kenya	0.0124
Mauritius	0.0072
Zambia	0.0147
South Africa	0.0062
Cote D'Ivoire	0.0125

Table 6: Covariance matrix between the Stocks Returns of The Eleven African Countries

	Country	1	2	3	4	5	6	7	8	9	10	11
1	**Botswana**	0.0026	0.0007	0.0006	0.0005	0.0002	0.0010	0.0011	0.0011	0.0006	0.0012	0.0010
2	**Namibia**	0.0007	0.0032	0.0008	0.0006	-0.0001	0.0007	0.0012	0.0009	0.0004	0.0003	0.0010
3	**Morocco**	0.0006	0.0008	0.0024	0.0005	0.0003	0.0007	0.0012	0.0009	-0.0001	0.0002	0.0010
4	**Tunisia**	0.0005	0.0006	0.0005	0.0023	-0.0002	0.0007	0.0008	0.0009	0.0001	0.0005	0.0009
5	**Ghana**	0.0002	-0.0001	0.0003	-0.0002	0.0044	0.0004	0.0009	0.0006	0.0006	0.0006	0.0006
6	**Nigeria**	0.0010	0.0007	0.0007	0.0007	0.0004	0.0080	0.0011	0.0017	0.0010	0.0007	0.0015
7	**Kenya**	0.0011	0.0012	0.0012	0.0008	0.0009	0.0011	0.0072	0.0028	0.0004	0.0013	0.0014
8	**Mauritius**	0.0011	0.0009	0.0009	0.0009	0.0006	0.0017	0.0028	0.0041	0.0010	0.0016	0.0017
9	**Zambia**	0.0006	0.0004	-0.0001	0.0001	0.0006	0.0010	0.0004	0.0010	0.0058	0.0014	0.0006
10	**SA**	0.0012	0.0003	0.0002	0.0005	0.0006	0.0007	0.0013	0.0016	0.0014	0.0063	0.0005
11	**CD**	0.0010	0.0010	0.0010	0.0009	0.0006	0.0015	0.0014	0.0017	0.0006	0.0005	0.0049

The diagonal in the covariance matrix (table 6) reports the various monthly variances of the stock returns for the eleven countries. The other values within the matrix represents the monthly co-variances among the stock returns of the various countries. The co-variances between Nambia & Ghana, Zambia & Morocco and Ghana &Tunisia are the only pairs that reported negative co-variances of -0.0001,-0.0001 and -0.0002 respectively. These negative co-variances among the stock returns of these countries would tend to reduce the variances of the equally-weighted, tangency and minimum-variance portfolios to be constructed (Bodie et al., 2011). This is because, in the computation of the variance of a portfolio the covariance between each pair of asset returns are required. Hence a negative covariance would on average reduce the variance of a portfolio.

4.6 Portfolio Optimization

This section entails the equally-weighted, minimum variance and tangency optimal portfolios constructed.

Table 7: The Expected Returns, Risks, Sharpe Ratios and Weights Allocations

	EQWP	**MVP1**	**MVP2**	**TP**
Expected Return	0.0925	0.0652	0.0925	0.1338
Standard Deviation	0.1184	0.0998	0.1066	0.1367
Sharpe Ratio	0.7810	0.6536	0.8671	0.9788
Weights Allocation to each Country				
Botswana	0.0909	0.1264	0.1526	0.1649
Namibia	0.0909	0.1143	0.0932	0.0338
Morocco	0.0909	0.1941	0.2014	0.1638
Tunisia	0.0909	0.2586	0.1446	0.0000
Ghana	0.0909	0.1634	0.0838	0.0000
Nigeria	0.0909	0.0173	0.0145	0.0003
Kenya	0.0909	0.0000	0.0351	0.1262
Mauritius	0.0909	0.0000	0.0000	0.0000
Zambia	0.0909	0.0876	0.1733	0.3054
SA	0.0909	0.0383	0.0252	0.0000
Cote D'Ivoire	0.0909	0.0000	0.0766	0.2057

*EQWP-Equally-weighted, TP-Tangency and MVP-Minimum-Variance Portfolios

Equally-weighted, minimum-variance and tangency (optimal) portfolios are constructed across the eleven African stock markets. The various expected returns, standard deviations and Sharpe ratios associated with these portfolios are reported in table 7. The corresponding weight allocations required to construct the respective portfolios are also reported. The equally-weighted portfolio across the eleven stock markets reported an expected return of 9.25% with a risk of 11.84%. Corresponding to its name (equally-weighted portfolio), since there are eleven countries being considered in this study, investors would have to invest 0.0909 of their entire wealth in each of these countries to obtain the risk and expected return associated with this portfolio.

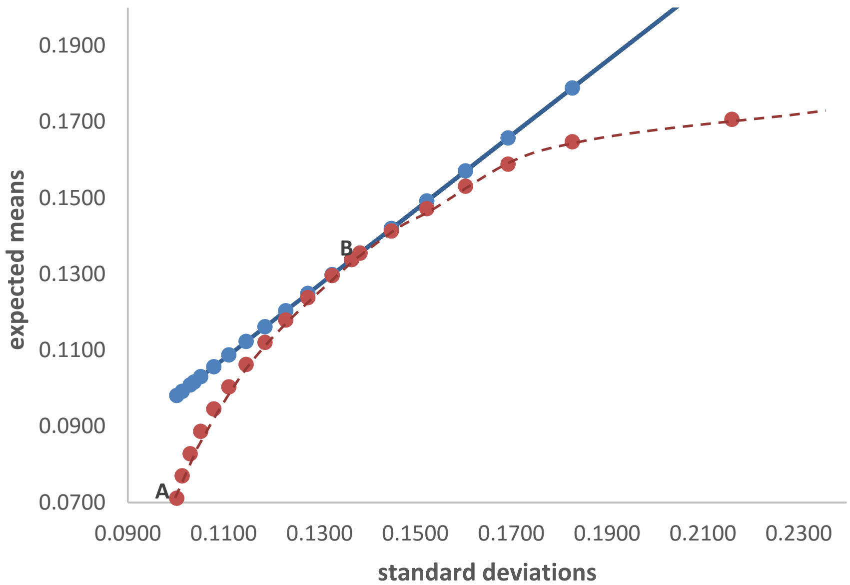
Two minimum-variance portfolios were constructed. The second minimum-variance portfolio was subjected to an additional constraint, which restricts the expected return of this minimum-variance portfolio to be either equal to or greater than that of an equally-weighted portfolio. The first minimum-variance portfolio reported an expected return of 6.52% with a risk of 9.99%, whereas the second reported an expected return of 9.25% with a risk of 10.66%. The additional constraint placed on the second minimum-variance portfolio increased the risk and expected return in comparison to that of the first by 2.73%. Hence, this portfolio dominates the heuristic choice of investing equally among the eleven countries which was the purpose of including this additional constraint. It is therefore realised from the results that with the extra constraint placed on the second minimum-variance portfolio, though it has the same expected return as that of the equally-weighted (9.25%), the risk associated with it is lower than that of equally-weighted portfolio by 1.18%.

Highly risk-averse investors would opt for the first minimum-variance portfolio, since it offered the least risk. Such investors would have to allocate 12.64%, 11.43%, 19.41%, 25.86%, 16.34%, 1.73%, 8.76% & 3.83% to portfolios of Botswana, Namibia, Morocco, Tunisia, Ghana, Nigeria, Zambia & South Africa respectively, and nothing to Kenya, Mauritius and Cote D'Ivoire rspectively so as to ascertain the expected return and risk associated with the first minimum-variance portfolio. Less risk-averse investors would opt for the second minimum-variance portfolio. These investors would, however, have to invest 15.26%, 9.32%, 20.14%, 14.46%, 8.38%, 1.45%, 3.51%, 17.33%, 2.52% & 7.66% to the portfolios of Botswana, Namibia, Morocco, Tunisia, Ghana, Nigeria, Kenya, Zambia, South Africa & Cote D'Ivoire and nothing to Mauritius respectively. It is noticed from table 7 that, such investors have higher Sharpe ratios than the former. This is so, since less risk-averse investors are being rewarded by a margin of 21.35% for the extra risk taken. This also illustrates the positive association between expected return and risk, hence, the higher the risks, the higher the expected returns on the portfolios. (Gennaioli et al., 2015; Gerlach et al., 2015; Sharpe, 1964).

The tangency portfolio reported an expected return of 13.38% with a risk of 13.67%. It is observed that the tangency portfolio provides an optimum solution to the asset allocation problem most investors are faced with, on how much to allocate to each asset or market in order to maximize their wealth at the possible minimum risk (Zhu & Zhou, 2009; Gratcheva & Falk, 2003). This is because this portfolio offers the best blend of risky securities that gives the best risk-expected return trade-off on a portfolio (Xie, 2009; Jorion 1992). The optimal portfolio also had the highest Sharpe ratio of 97.88%, indicating that investors, which invest in the optimal portfolio would attain a compensation of 97.88% for taking a risk of 13.67% to obtain a higher expected return of 13.33% as compared to the equally-weighted and minimum-variance portfolios.

4.7 Efficient Frontier 1

In assessing the optimal choices for a risk lover and risk averse investor, the efficient frontier is established (third objective of the study). This demonstrates the various portfolios investors can choose from based on their risk preferences.



Source: Author (2015)

Figure 9 : Graphical Representation of Efficient Frontier 1

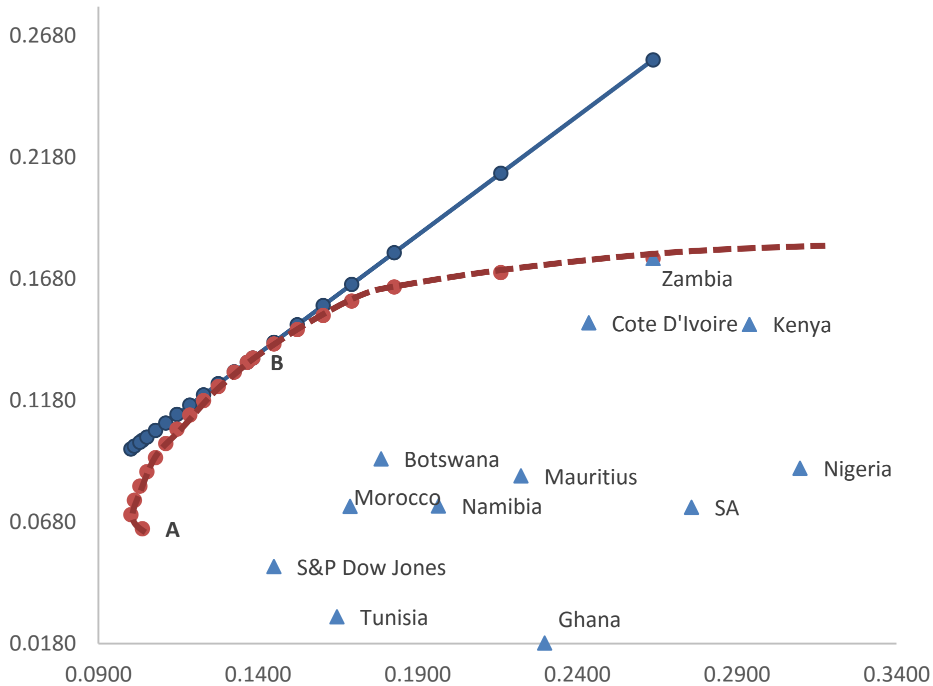
Figure 9 indicates the risks and expected returns for various portfolios on the efficient frontier. Point A represents a minimum-variance portfolio, which can be seen from figure 3, to possess the least risk. The tangency (optimal) portfolio is indicated by portfolio B. This is the point at which the capital allocation line touches the efficient frontier, as depicted in figure 9. With the assumption of investors being rational, they would opt for a portfolio on this efficient frontier. However, the particular portfolio an investor would choose, will depend on his or her risk behaviour. Hence, risk averse investors would choose a portfolio between point A and B, whereas risk lovers would choose a portfolio from point B and beyond. Such investors would prefer to accept more risk by borrowing at the riskless rate to invest more than 100% in the optimal portfolio, as oppose to the former, who would invest less than a 100% of their wealth in the tangency portfolio (Asness et al., 2012; Alexander & Baptista, 2002). However, investors that choose a portfolio beyond the tangency portfolio would not be rewarded for the excessive risk taken. This is because the tangency portfolio is the optimum choice, in that, this portfolio gives the best risk-return trade-off investors can obtain (Tepla, 2000). It is released from table 8 that, beyond the tangency portfolio (second row highlighted in red), the Sharpe ratio begins to fall.

Table 8: The Expected Returns, Risks, Sharpe ratios and Weights Allocated to a given Portfolio on the Efficient Frontier

ER	0.0652	0.0711	0.0769	0.0828	0.1062	0.1121	0.1296	0.1338	0.1355	0.1413	0.1530	0.1647
SD	0.0998	0.1002	0.1013	0.1030	0.1147	0.1186	0.1326	0.1367	0.1384	0.1450	0.1605	0.1827
SR	0.6536	0.7092	0.7592	0.8036	0.9261	0.9445	0.9773	0.9788	0.9786	0.9745	0.9535	0.9014
Weights Allocation to each Country												
Botswana	0.1264	0.1354	0.1411	0.1454	0.1626	0.1669	0.1738	0.1649	0.1607	0.1457	0.0950	0.0000
Namibia	0.1143	0.1117	0.1075	0.1022	0.0812	0.0760	0.0511	0.0338	0.0265	0.0009	0.0000	0.0000
Morocco	0.1941	0.2000	0.2020	0.2018	0.2012	0.2011	0.1845	0.1638	0.1548	0.1230	0.0254	0.0000
Tunisia	0.2586	0.2307	0.2056	0.1825	0.0901	0.0669	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Ghana	0.1634	0.1445	0.1274	0.1109	0.0451	0.0287	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Nigeria	0.0173	0.0175	0.0168	0.0159	0.0124	0.0115	0.0056	0.0003	0.0000	0.0000	0.0000	0.0000
Kenya	0.0000	0.0000	0.0046	0.0161	0.0620	0.0735	0.1126	0.1262	0.1317	0.1510	0.1901	0.1752
Mauritius	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Zambia	0.0876	0.1100	0.1284	0.1453	0.2128	0.2297	0.2863	0.3054	0.3130	0.3394	0.3946	0.5628
SA	0.0383	0.0359	0.0336	0.0304	0.0180	0.0149	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
CD	0.0000	0.0142	0.0331	0.0494	0.1146	0.1309	0.1861	0.2057	0.2133	0.2400	0.2948	0.2620

*ER-Expected return, SD-Standard deviation and SR-Sharpe ratio

4.8 Efficient Frontier 2



Source: Author (2015)

Figure 10: Graphical Representation of Efficient Frontier 2

The various expected returns and standard deviations for each individual country's and the S&P Dow Jones portfolios were plotted on the efficient frontier to determine if either of them lies on the efficient frontier. With the exception of Zambia, all the other individual country's portfolios and that of the S&P Dow Jones lie beneath the efficient frontier. This indicates that, with the exception of the portfolio constructed in Zambia, all the other countries' portfolios are being dominated by the portfolios (minimum-variance and tangency portfolios inclusive) on this efficient frontier. Though, Zambia's portfolio is on the efficient frontier and offers a higher expected return than that of the optimal portfolio, it is exposed to a higher risk. The extra risk, a risk lover (investor who would want to invest in Zambia's portfolio) would take on, so as to obtain this higher expected return would however not be compensated for. This is because

Sharpe ratios start to fall after the optimal portfolio. Hence, investors who invest in a portfolio beyond the optimal would not be compensated for the extra volatility taken. This graph also depicts the benefits of diversification in emerging and frontier markets as indicated in studies of Bekaert and Harvey (2003), DeRoon et al., (2001) and Bekaert and Urias, (1996, 1999).

4.9 Bootstrapped Estimates

In order to determine if the optimization technique employed in the selection of the optimal portfolio can withstand any variations in macroeconomic variables, the bootstrapping algorithm is used as a simulation technique (second objective of the study). The sample of stock prices for the eleven stock markets was bootstrapped 2000 times to obtain 2000 replicates of stock returns. The means, standard deviations, bias estimates and the significances of the biases are shown in tables 9,10 and 11.

Table 9: The bootstrapped Estimates of the Equally-Weighted Portfolio

Estimate	Mean	SD	Bias	Significance
Expected return	0.0922	0.0307	-0.0003	0.0098
Standard deviation	0.1170	0.0129	-0.0014	0.1085

Table 10:The Bootstrapped Estimates of the Minimum-Variance Portfolio

Estimate	Mean	SD	Bias	Significance
Expected return	0.0682	0.0269	0.0030	0.1115
Standard deviation	0.0958	0.0078	-0.0040	0.5128

Table 11: The Bootstrapped Estimates of the Tangency Portfolio

Estimate	Mean	SD	Bias	Significance
Expected return	0.1592	0.0358	0.0254	0.2094
Standard deviation	0.1330	0.0190	-0.0037	0.1947

The means and standard deviations for bootstrapped estimates are reported in tables 9,10 and 11. These are used as inputs in computing the biases. The biases reported in tables 9, 10 and 11 indicates the deviations of the various bootstrapped estimates from that of the actual estimates computed using the original sample. These biases were then tested to determine if they were significant. With the exception, of the minimum-variance's risk, all the biases were statistically insignificant. This is because the significance which was computed as, $\frac{|Bias(\theta^*)|}{Std(\theta^*)}$ (Efron & Tibshirani, 1993) for all the estimates were less than 0.25, except that of the minimum-variance portfolio, which has a significance of 0.5128. Hence the bias corrected estimate was computed for the risk of this portfolio. This was estimated as the difference between the actual estimate and the bias of its corresponding bootstrapped estimate. This is shown in table 12. The various expected returns, standard deviations and Sharpe ratios for a sample of 450 bootstrapped replicates are shown in appendix A.

Table 12: Bias Corrected Estimates

	EQWP	MVP	TP
Expected return	0.0925	0.0652	0.1338
Standard deviation	0.1184	0.1038	0.1367
Sharpe ratio	0.7810	0.6283	0.9788

The bias corrected estimates for the expected returns, risks and Sharpe ratios are stated in table 11. With the exception of the Sharpe ratio and standard deviation of the minimum-variance portfolio, all the estimates' bias corrected values are equivalent to the actual estimates reported in table 7.

4.10 Test of Differences in Mean

The independent t-tests were undertaken to determine if the bias corrected expected returns of the two portfolios (optimal and minimum-variance) were significantly different from the individual country's portfolios and that of the S&P Dow Jones. In order to perform this test, Levene's homogeneity of variance test is performed to determine if the variances that exist between the samples are significantly different from each other.

4.10.1 Levene's Homogeneity of Variances Test

The results of the homogeneity of variance tests are reported in table 13.

Table 13: Homogeneity Tests of Variances

Minimum-Variance Portfolio		
Country	**F-ratio**	**p-value**
Botswana	518.3518	0.0000
Namibia	627.6443	0.0000
Morocco	463.5073	0.0000
Tunisia	441.3906	0.0000
Ghana	858.0549	0.0000
Nigeria	1560.2522	0.0000
Kenya	1404.8011	0.0000
Mauritius	803.5015	0.0000
Zambia	1130.7455	0.0000
South Africa	1236.2505	0.0000
Cote D'Ivoire	964.5028	0.0000
S&P Dow Jones	341.6129	0.0000
Tangency Portfolio		
Country	**F-ratio**	**p-value**
Botswana	88.2306	0.0000
Namibia	106.8337	0.0000
Morocco	78.8953	0.0000
Tunisia	75.1307	0.0000
Ghana	146.0527	0.0000
Nigeria	265.5763	0.0000
Kenya	239.1164	0.0000
Mauritius	136.7670	0.0000
Zambia	192.4683	0.0000
South Africa	210.4267	0.0000
Cote D'Ivoire	164.1716	0.0000
S&P Dow Jones	58.1472	0.0000

The F-ratio statistic and the p-values are reported in table 13. P-values of each pair of variances are less than 5%, indicating that the variances that exist between the samples are significantly different from each other. Hence the independent t-test with unequal variances is used to perform the test of differences in mean.

4.10.2 Independent (Two-Sample) t-tests with Unequal Variances

The two-sample t-test statistic and their p-values are reported in table 13. P-values of each pair of expected returns are less than 5%. This indicates that the bias corrected expected returns for the two portfolios (minimum-variance and tangency) constructed are significantly different from the individual country's portfolios and that of the S&P Dow Jones statistically.

Table 14: Two Sample t-test

Minimum-Variance Portfolio		
Country	**t-Statistics**	**p-value**
Botswana	-6.9164	0.0000
Namibia	-2.1131	0.0360
Morocco	-2.4084	0.0170
Tunisia	-11.0680	0.0000
Ghana	-10.2988	0.0000
Nigeria	-3.4653	0.0007
Kenya	-12.6170	0.0000
Mauritius	-4.1973	0.0000
Zambia	-17.9532	0.0000
South Africa	-2.3131	0.0189
Cote D'Ivoire	-14.8185	0.0000
S&P Dow Jones	-5.4479	0.0000
Tangency Portfolio		
Country	**t-Statistics**	**p-value**
Botswana	-11.7382	0.0000
Namibia	-15.5978	0.0000
Morocco	-18.2091	0.0000
Tunisia	-32.4889	0.0000
Ghana	-25.6639	0.0000
Nigeria	-7.3818	0.0000
Kenya	-2.5731	0.0054
Mauritius	-11.0302	0.0000
Zambia	-7.2325	0.0000
South Africa	-11.2412	0.0000
Cote D'Ivoire	-3.2107	0.0008
S&P Dow Jones	-31.3942	0.0000

The independent t-tests performed indicate that, if investors should invest in the minimum-variance portfolio constructed in the study, they would obtain a significant expected return of 6.52% at a risk of 10.38%. In the case of the tangency portfolio, investors would obtain a significant expected return of 13.38% at a risk of 13.67%. With the exception of Kenya, Cote D'Ivoire and Zambia as shown in table 1, all the other individual country's portfolios have expected returns lesser than that of the optimal portfolio and are even subjected to higher risks at these lesser rates. Though the portfolios of Kenya (14.93%), Cote D'Ivoire (14.99%) and Zambia (17.64%) have higher expected returns than the optimal portfolio, they are exposed to higher risks (29.40% 24.36% and 26.38% respectively). Hence, rational investors who would always want to maximize their wealth at an optimized risk would go in for the optimal portfolio, as it gives the best risk-expected return compromise.

Also, the percentage of investors' wealth, they will apportion to this portfolio will depend on their risk behaviour. However, whatever being the case, rational investors would choose an efficient portfolio, which is represented in figure 3 and their movement along the curve would once again depend on their risk-expected return preferences. The results obtained in table 14 also display the benefits of diversifying across the eleven stock markets (international diversification) as opposed to investing or diversifying within each country (Bekaert & Harvey, 2003; De Roon et al., 2001; Bekaert and Urias, 1999; De Santis & Gerard, 1997). The diversified portfolios across the eleven stock markets reduced the risk significantly as compared to the risks associated with the individual country's portfolios.

The minimum-variance and optimal portfolio constructed in this study offer a better alternative as opposed to the S&P Dow Jones portfolio. This is because the S&P Dow Jones offered an expected return of 4.97%, which is lesser than the expected return of 6.52% and 13.38% of the minimum-variance and tangency portfolios respectively constructed across the eleven African stock markets. The S&P Dow Jones also had a higher risk (14.50%) than that of the two portfolios (10.38% and 13.67% respectively). This, therefore, warrants our claim that, the African stock markets (emerging and frontier markets), though are riskier than the developed markets, tend to offer higher and better investment opportunities than that of the developed (Groot et al., 2012; Bekaert & Harvey, 2003; Harvey, 1995). Hence investors from developed economies should consider investing in African markets since they offer more attractive investment opportunities.

The monthly standard deviations and expected returns of various portfolios constructed in this study are annualised by multiplying the former by the square root of 12 and the latter by 12 (Jorion, 1992).

4.11 Chapter Summary

This chapter entailed the results of the analysis. This study indicated that a well-diversified portfolio across the eleven African stock markets (Cote D'Ivoire, Mauritius, Kenya, Nigeria, Egypt, South Africa, Morocco, Botswana, Ghana, Namibia and Zambia) offers a better investment opportunity than the individual countries' portfolios. The optimal portfolio constructed after having been bootstrapped, had an expected return of 13.38% with a risk of 13.67%. This portfolio in comparison to the individual countries' portfolios and that of the S&P Dow Jones, offered the best expected return-risk trade-off. The outcome of the risks and expected returns of the various diversified portfolios constructed in the study also illustrated the benefits of portfolio diversification in frontier & emerging markets from both a US investor's perspective and that of a domestic investor (investors from any of the eleven African countries). This study also exhibited the risky nature of African markets.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

This chapter gives a summary of what this study is about. It also indicates the key findings and conclusions. Based on these conclusions, appropriate recommendations and suggestions are made.

5.2 Summary

This study sets out to construct an optimal portfolio across eleven African stock markets. This is because Africa has become a very lucrative place to invest in and may offer better investment opportunities. However, studies have found that investors prefer internal diversification as opposed to external and global diversification. Also, foreign investors have the perception that Africa is a highly risky environment. Hence this deters them from investing or diversifying in Africa. The few studies that performed cross-border diversification and asset allocation focused more on developed economies. The few developing economies that were even included in these studies, only a handful came from Africa. Hence, the need for this study.

The study also employed the bootstrapping technique. This is because the few studies on cross-border diversification and asset allocation neglected to check how robust their optimal choices were to the possible variations that could occur in macroeconomic variables and any other idiosyncratic errors. Hence, this study constructed an optimal portfolio across the eleven African stock markets using the Markowitz Optimization technique. It further went on to perform bootstrapping as both a robust check and a form of simulation technique. The study period was from the year 2000 to 2014. In order to construct the various portfolios, the study required market indices of the eleven African countries and that of S & P Dow Jones (SPDJ). The SPDJ composite index was used in constructing the portfolio for a developed economy, since it covers about a 90% of the US economy. This data was sourced from the Thomson Reuters Datastream database but computed by Standard and Poor's. The Market indices computed by the Standard and Poor's have been used by many studies due to its reliability and accuracy.

This study found that a diversified portfolio across the eleven African stock markets offers a better investment opportunity than the individual countries' portfolios. The minimum-variance and the optimal (tangency) portfolios constructed across these eleven stock markets dominated most of the individual countries' portfolios. This is because, with the exception of the portfolio constructed in the economy of Zambia, all the others were beneath the efficient frontier. This is not surprising, since with the exception of Zambia which had an average return of 17.64% with a risk of 26.38%, all the others had a lesser average return but were highly risky. The Sharpe ratios of the individual countries' portfolios confirmed this, with Zambia having the highest Sharpe ratio of 66.89% as compared to that of the minimum-variance (65.36% & 87.61%) and optimal portfolios (97.88%) constructed. Two minimum-variance portfolios were constructed. The first had an expected return of 6.52% with a risk of 9.98%, whereas the second (additional constraint placed on it) had an expected return of 9.25%with a risk of 10.88%. The optimal portfolio offered an expected return of 13.38% with a risk of 13.67%. These portfolios (minimum-variance and optimal portfolios) constructed also outperformed that of the S&P Dow Jones. The S&P Dow Jones portfolio had an average return of 4.57% with a risk of 14.50%.

5.3 Conclusion

Investors can invest and diversify across assets in their individual countries. However, they should sometimes move out of their comfort zone and invest across borders, since it offers higher and better investment opportunities as shown by this study. Though Africa is risky, it offers a better investment opportunity than that of the US economy which is less risky. The portfolios constructed indicated that, though the individual African countries' portfolios are highly risky, a well-diversified portfolio can offer a better risk-return trade-off by reducing the risk and increasing the return. The outcomes of the study also indicated that possible variations that can affect macroeconomic variables resulting in differences in returns can have a significant effect on optimal choices. Hence this study also gives some level of certainty to investors, since the bootstrapping performed caters for most of the possible variations that can occur in stock returns.

5.4 Recommendations

This study goes beyond the literature on cross-border asset allocation and portfolio diversification by including bootstrapping as a simulation technique to cater for any variations that can occur in stock returns. The study, therefore, serves as a guideline for further research on cross-border portfolio optimization, diversification and asset allocation. However, this study did not consider all African countries. Further research should consider a larger sample size and also other capital markets such as the bonds and commodities markets. This study assumed a no short selling constraint. Hence, further research can assume short selling, if it applies in their context of study, to see how different the optimal choices would be.

In terms of practice, the findings from this study can inform companies and investors on the eleven stock exchanges on how to diversify across borders so as to optimize their utility of wealth based on their risk preferences (risk-averse and risk-lover). This study also gives some level of assurance to both domestic and foreign investors, since the bootstrapping performed caters for most of the possible variations that can occur in stock returns. Hence, this study can motivate foreign investors to invest in African markets. This is because they will be rewarded for the extra risk they would take on, investing in Africa, as compared to their less risky environment.

This study portrays how variations in the macro-environment can alter the returns of the various portfolios. Hence this would inform policy makers on how macroeconomic factors can impact an investor's wealth and eventually the development of capital markets. The results of this research also suggest that well diversified portfolios across the eleven stock markets offers very attractive investment opportunities as compared to that of the developed economy. Hence this can attract foreign investors to this part of the world. This, therefore, in the long run can aid in the development of our capital markets and economies.

REFERENCES

Assaf, A. G., Barros, C. P., & Matousek, R. (2011). Productivity and efficiency analysis of Shinkin banks: Evidence from bootstrap and Bayesian approaches. *Journal of Banking & Finance*, *35*(2), 331–342.

Adachi, J., & Gupta, A. (2005). Simulation-based parametric optimization for long-term asset allocation using behavioral utilities. *Applied Mathematical Modelling*, *29*(4), 309–320.

Adjasi, C., Harvey, S. K., and Agyapong, D. and (2008). Effect of exchange rate volatility on the ghana stock exchange. *African Journal of Accounting, Economics, Finance and Banking Research*, *3*(3), 28–47.

Adjasi, C. K. D. (2009). Macroeconomic uncertainty and conditional stock-price volatility in frontier African markets: Evidence from Ghana. *The Journal of Risk Finance*, *10*(4), 333–349.

African Securities Exchanges Association (2013). *About ASEA.* Retrieved on 20th April, 2015 from http://www.african-exchanges.org/

Alagidede, P. (2011). Return behaviour in Africa's emerging equity markets. *Quarterly Review of Economics and Finance*, *51*(2), 133–140.

Alagidede, P., & Panagiotidis, T. (2009). Modelling stock returns in Africa's emerging equity markets. *International Review of Financial Analysis*, *18*(1-2), 1–11.

Alexander, G. J., & Baptista, A. M. (2002). Economic implications of using a mean-VaR model for portfolio selection : A comparison with mean-variance analysis. *Journal of Economic Dynamics and Control*, *26*(7-8), 1159–1193.

Allen, F., & Gale, D. (2000). *Comparing Financial Systems.* Cambridge, MA: MIT press.

Amenc, N., & Martellini, L. (2011). In Diversification We Trust ? *The Journal of Portfolio Management*, *37*(2), 1–2.

Anderson, D. R., Sweeney, D. J., & Williams, T. A. (2011). *Statistics for Business and Economics* (11th ed.). Boulevard, California: South-Western, Cengage Learning.

Appiah-kusi, J., & Menyah, K. (2003). Return predictability in African stock markets. *Review of Financial Economics*, *12*(3), 247–270.

Arestis, P., Demetriades, P. O., & Luintel, K. B. (2001). Financial Development and Economic Growth: The Role of Stock Markets. *Journal of Money, Credit and Banking*, *33*(1), 16–41.

Aslanidis, N., & Casas, I. (2013). Nonparametric correlation models for portfolio allocation. *Journal of Banking & Finance*, *37*(7), 2268–2283.

Asness, C. S., Frazzini, A., & Pedersen, L.H. (2012). "Leverage Aversion and Risk Parity." *Financial Analysts Journal*, *68*(1), 1-23.

Bae, G. I., Kim, W. C., & Mulvey, J. M. (2014). Dynamic asset allocation for varied financial markets under regime switching framework. *European Journal of Operational Research*, *234*(2), 450–458.

Baltagi, B. H. (2008). *Essentials of Econometrics* (4th ed.). Berlin: John, Wiley & Sons.

Bali, T. G., Brown, S. J., & Caglayan, M. O. (2012). Systematic risk and the cross section of hedge fund returns. *Journal of Financial Economics, 106*(1), 114–131.

Beck, T., & Levine, R. (2002). Industry growth and capital allocation : does having a market- or bank-based system. *Journal of Financial Economics, 64*(2), 147–180.

Beck, T., & Levine, R. (2004). Stock markets, banks, and growth : Panel evidence. *Journal of Banking & Finance, 28*(3), 423–442.

Bekaert, G., Erb, C.B., Harvey, C. R., & Viskanta, T. E. (1998). Distributional Characteristics of Emerging Market Returns and Asset Allocation. *The Journal of Portfolio Management, 24*(2), 102-116.

Bekaert, G., & Harvey, C. R. (1997). Emerging equity market volatility. *Journal of Financial Economics, 43*(1), 29-77.

Bekaert, G., & Harvey, C. R. (2003). Emerging markets finance. *Journal of Empirical Finance, 10*(1-2), 3-55.

Bekaert, G., & Urias, M. S. (1996). Diversification, integration and emerging market closed-end funds. *Journal of Finance, 51(3),* 835– 869.

Bekaert, G., & Urias, M. S. (1999). Is there a free lunch in emerging market equities? *The Journal of Portfolio Management, 25*(3), 83– 95.

Berger, D., Pukthuanthong, K., & Yang, J. J. (2011). International diversification with frontier markets. *Journal of Financial Economics*, *101*(1), 227–242.

Binswanger, J. (2010). Understanding the heterogeneity of savings and asset allocation: A behavioral-economics perspective. *Journal of Economic Behavior & Organization*, *76*(2), 296–317.

Black, F., & Litterman, R. (1992). Global Portfolio Optimization. *Financial Analysts Journal*, *48*(5), 28–43.

Brandtner, M. (2013). Conditional Value-at-Risk, spectral risk measures and (non-) diversification in portfolio selection problems – A comparison with mean-variance analysis. *Journal of Banking and Finance*, *37*(12), 5526–5537.

Branger, N., & Hansis, A. (2012). Asset allocation: How much does model choice matter? *Journal of Banking & Finance*, *36*(7), 1865–1882.

Brennan, M. J., Schwartz, E. S., & Lagnado, R. (1997). Strategic asset allocation. *Journal of Economic Dynamic and Control*, *21*(8-9), 1377–1403.

Bodie, Z., Kane, A., & Marcus, A. J. (2005). *Essentials of Investments* (6th ed.). New York: McGraw Hill.

Bodie, Z., Kane, A., & Marcus, A. J. (2008). *Essentials of Investments* (7th ed.). New York: McGraw Hill.

Bodie, Z., Kane, A., & Marcus, A. J. (2009). *Essentials of Investments* (8th ed.). New York: McGraw Hill.

Bodie, Z., Kane, A., & Marcus, A. J. (2011). *Investments* (9th ed.). New York: McGraw Hill.

Brooks, C. (2008). *Introductory Econometrics for Finance* (2nd ed.). New York: Cambridge University Press.

Bucklew, J. A., Ney, P., & Sadowsky, J. S. (1990). Monte Carlo Simulation and Large Deviations Theory for Uniformly Recurrent Markov chains. *Journal of Applied Probability*, *27*(1), 44-59.

Calafiore, G. C. (2008). Multi-period portfolio optimization with linear control policies. *Automatica*, *44*(10), 2463–2473.

Campbell, R., Huisman, R., & Koedijk, K. (2001). Optimal portfolio selection in a Value-at-Risk framework. *Journal of Banking & Finance*, *25*(9), 1789–1804.

Castellano, R., & Cerqueti, R. (2014). Mean–Variance portfolio selection in presence of infrequently traded stocks. *European Journal of Operational Research*, *234*(2), 442–449.

Celikyurt, U., & Ozekici, S. (2007). Multiperiod portfolio optimization models in stochastic markets using the mean – variance approach. *European Journal of Operational Research*, *179*(1), 186–202.

Cesari, R., & Cremonini, D. (2003). Benchmarking, portfolio insurance and technical analysis: a Monte Carlo comparison of dynamic strategies of asset allocation. *Journal of Economic Dynamics and Control*, *27*(6), 987–1011.

Cohen, J. W. (1988). *Statistical power analysis for the behavioural sciences* (2nd ed.). Hillsdale, New Jersey: Lawrence Erlbaum Asssociates.

Corradi, V., & Iglesias, E. M. (2008). Bootstrap refinements for QML estimators of the GARCH(1,1) parameters. *Journal of Econometrics*, *144*(2), 500–510.

Costa, O. L. V., & Araujo, M. V. (2008). A generalized multi-period mean – variance portfolio optimization with Markov. *Automatica*, *44*(10), 2487–2497.

Coval, J. D., & Moskowitz, T. J. (1999). Home Bias at Home : Local Equity Preference in Domestic Portfolios. *The Journal of Finance*, *54*(6), 2045–2073.

Chen, X. B., Silvapulle, P., & Silvapulle, M. (2014). A semiparametric approach to value-at-risk, expected shortfall and optimum asset allocation in stock–bond portfolios. *Economic Modelling*, *42*, 230–242.

Chernick, M.R. (2008). *Bootstrap Methods: A Guide for Practitioners and Reseachers* (2nd ed.) New Jersey: John Wiley & Sons.

Chu, B. (2011). Recovering copulas from limited information and an application to asset allocation. *Journal of Banking & Finance*, *35*(7), 1824–1842.

Chunhachinda, P., Dandapani, K., Hamid, S., & Prakash, J. (1997). Portfolio selection and skewness: Evidence from intemational stock markets. *Journal of Banking and Finance*, *21*(2), 143–167.

Creswell, J. W. (2009). *Research Design: Qualitative, Quantitative and Mixed Methods Approaches* (3rd ed.). California: SAGE Publications Inc.

Davinson, A. C., & Hinkley, D. V. (1997). *Bootstrap methods and their applications*. New York: Cambridge University Press.

De Borger, B., Kerstens, K., & Staat, M. (2008). Transit costs and cost efficiency: Bootstrapping non-parametric frontiers. *Research in Transportation Economics, 23*(1), 53-64.

De Roon, F.A., & Nijman, T.E. (2001) Testing for mean–variance spanning with short sales constraints and transaction costs: The case of emerging markets. *Journal of Empirical Finance, 56*(2), 721–742.

De Santis, G., & Gerard, B. (1997). International asset pricing and portfolio diversification with time-varying risk. *The Journal of Finance*, *52*(5), 1881– 1912.

Detemple, J., Garcia, R., & Rindisbacher, M. (2005). Intertemporal asset allocation: A comparison of methods. *Journal of Banking & Finance*, *29*(11), 2821–2848.

Driessen, J., & Laeven, L. (2007). International portfolio diversification benefits: Cross-country evidence from a local perspective. *Journal of Banking & Finance*, *31*(6), 1693–1712.

Efron, B. (1979). Bootstrap methods: Another look at the jackknife. *The Annals of Statistics, 7*(1), 1-26.

Efron, B., & Tibshirani, R. J. (1993). *An Introduction to the Bootstrap*. London: Chapman and Hall.

Elliott, R. J., Siu, T. K., & Badescu, A. (2010). On mean-variance portfolio selection under a hidden Markovian regime-switching model. *Economic Modelling*, *27*(3), 678–686.

Elton, E. J., & Gruber, M. J. (1978). Taxes and portfolio composition. *Journal of Financial Economics, 6*(4), 399-410.

Elton, E. J., & Gruber, M. J. (1997). Modern Portfolio Theory, 1950 to date. *Journal of Banking and Finance, 21*(11-12), 1743-1759.

Elton, E. J., & Gruber, M. J. (2000). The rationality of asset allocation recommendations. *The Journal of Financial and Quantitative Analysis, 35*(1), 25-41.

Elton, E. J., & Gruber, M. J. (2011). *Investments and Portfolio Performance.* New Jersey: World Scientific Publishing Co. Pte. Ltd.

Engsted, T., & Pedersen, T. Q. (2012). Return predictability and intertemporal asset allocation: Evidence from a bias-adjusted VAR model. *Journal of Empirical Finance*, *19*(2), 241–253.

Enisan, A. A., & Olufisayo, A. O. (2009). Stock market development and economic growth : Evidence from seven sub-Sahara African countries, *Journal of Economics and Business*, *61*(2), 162–171.

Forbes Magazine. (2014). *Top Quotes About Africa At The 2014 World Economic Forum in Davos.* Retrieved on 23rd November, 2014, from http://www.forbes.com/sites/faraigundan/2014/01/28/top-quotes-about-africa-at-the-2014-world-economic-forum-in-davos/

Evans, J. L., & Archer, S. H., (1968). Diversification and the reductiom of dispersion :An Empirical Analysis. *The Journal of Finance*, *23*(5),761-767.

Farinelli, S., Ferreira, M., Rossello, D., Thoeny, M., & Tibiletti, L. (2008). Beyond Sharpe ratio: Optimal asset allocation using different performance ratios. *Journal of Banking & Finance*, *32*(10), 2057–2063.

Fortin, I., & Hlouskova, J. (2011). Optimal asset allocation under linear loss aversion. *Journal of Banking & Finance*, *35*(11), 2974–2990.

Frazzini, A., & Pedersen, L.H. (2014). "Betting against Beta." *Journal of Financial Economics, 111*(1), 1-25.

García-Herrero, A., & Vázquez, F. (2013). International diversification gains and home bias in banking. *Journal of Banking & Finance*, *37*(7), 2560–2571.

Garcia, V. F., & Liu, L. (1999). Macroeconomic determinants of stock market development. *Journal of Applied Economics*, *2*(1), 29–59.

Gaudecker H.-M. V. (2015). How does household portfolio diversification vary with financial literacy and financial advice? *The Journal of Finance*, *70*(2), 489-507.

Gennaioli, N., Shleifer, A., & Vishny, R. (2015). Money doctors. *The Journal of Finance,* 70(1), 91-114.

Gerlach, R., Obaydin, I., & Zurbruegg, R. (2015). The impact of leverage on the idiosyncratic risk and return relationship of REITs around the financial crisis. *International Review of Economics & Finance*, *38*, 207–219.

Gerstner, T., Griebel, M., Holtz, M., Goschnick, R., & Haep, M. (2008). A general asset-liability management model for the efficient simulation of portfolios of life insurance policies. *Insurance: Mathematics and Economics*, *42*(2), 704–716.

Gnabo, J.-Y., Hvozdyk, L., & Lahaye, J. (2014). System-wide tail comovements: A bootstrap test for cojump identification on the S&P 500, US bonds and currencies. *Journal of International Money and Finance*, *48*, 147–174.

Goetzmann, W. N., & Kumar, A. (2008). Equity Portfolio Diversification. *Review of Finance*, *12*(3), 433–463.

Gökgöz, F., & Atmaca, M. E. (2012). Financial optimization in the Turkish electricity market: Markowitz's mean-variance approach. *Renewable and Sustainable Energy Reviews*, *16*(1), 357–368.

Goldfarb, D., & Iyengar , G. (2003). Robust portfolio selection problems. *Mathematics of Operations Research*, *28*(1), 1–38.

Gonçalves, S., & White, H. (2004). Maximum likelihood and the bootstrap for nonlinear dynamic models. *Journal of Econometrics*, *119*(1), 199–219.

Graham, M., Kiviaho, J., & Nikkinen, J. (2012). Integration of 22 emerging stock markets : A three-dimensional analysis. *Global Finance Journal*, *23*(1), 34–47.

Gratcheva, E. M., & Falk, J. E. (2003). Optimal deviations from an asset allocation. *Computers & Operations Research*, *30*(11), 1643–1659.

Groot, W. De., Pang, J., & Swinkels, L. (2012). The cross-section of stock returns in frontier emerging markets. *Journal of Empirical Finance, 19*(5), 796–818.

Guidolin, M., & Hyde, S. (2012). Can VAR models capture regime shifts in asset returns? A long-horizon strategic asset allocation perspective. *Journal of Banking & Finance*, *36*(3), 695–716.

Gujarati, D. N. (2003). *Basic Econometrics* (4th ed.). New York: McGraw-Hill.

Gulpinar, N., & Rustem, B. (2007). Worst case robust decisions for multi-period mean-variance portfolio optimization. *European Journal of Operational Research*, *183*(3), 981-1000.

Guo, X., Ye, L., & Yin, G. (2012). A mean–variance optimization problem for discounted Markov decision processes. *European Journal of Operational Research*, *220*(2), 423–429.

Harlow, W.V. (1991). Asset Allocation in a Downside-Risk Framework. *Financial Analysts Journal*, *47*(5), 28–40.

Harvey, C. R. (1995). Predictable Risk and Returns in Emerging Markets. *The Review of Financial Studies*, *8*(3), 773–816.

Hibiki, N. (2006). Multi-period stochastic optimization models for dynamic asset allocation. *Journal of Banking & Finance*, *30*(2), 365–390.

Horneff, W. J., Maurer, R. H., Mitchell, O. S., & Stamos, M. Z. (2009). Asset allocation and location over the life cycle with investment-linked survival-contingent payouts. *Journal of Banking & Finance*, *33*(9), 1688–1699.

Huang, H.-C., & Lee, Y.-T. (2010). Optimal asset allocation for a general portfolio of life insurance policies. *Insurance: Mathematics and Economics*, *46*(2), 271–280.

Huang, H.-H., Lin, S.-H., Wang, C.-P., & Chiu, C.-Y. (2014). Adjusting MV-efficient portfolio frontier bias for skewed and non-mesokurtic returns. *The North American Journal of Economics and Finance*, *29*, 59–83.

Huang, X. (2008). Portfolio selection with a new definition of risk. *European Journal of Operational Research*, *186*(1), 351–357.

Huberman, G., & Kandel, S., 1987. Mean–variance spanning. *Journal of Finance 42*(4), 873–888.

Hung, M., Liu, Y., & Tsai, C. (2012). Managerial personal diversification and portfolio equity incentives. *Journal of Corporate Finance*, *18*(1), 38–64.

Jacobs, H., Müller, S., & Weber, M. (2014). How should individual investors diversify? An empirical evaluation of alternative asset allocation policies. *Journal of Financial Markets*, *19*, 62–85.

Jacquier, E., Titman, S., & Yalçin, A. (2010). Predicting systematic risk : Implications from growth options, *Journal of Empirical Finance, 17*(5), 991–1005.

Jeng, J.-L., & Liu, Q. W. (2012). Do Idiosyncratic Risks in Multi-Factor Asset Pricing Models Really Contain a Hidden Non-Diversifiable Factor ? A Diagnostic Testing Approach. *Journal of Mathematical Finance*, *2*, 251–263.

Jensen, M., & Meckling, W. (1976). Theory of the firm: Managerial behaviour, agency costs and ownership structure. *Journal of Financial Economics, 3*(4), 305–360.

Jorion, P. (1992). Portfolio Optimization in Practice. *Financial Analysts Journal*, *48*(1), 68–74.

Karolyi, G. A., & Kho, B.-C. (2004). Momentum strategies: some bootstrap tests. *Journal of Empirical Finance*, *11*(4), 509–536.

Kerstens, K., Mounir, A., & Van de Woestyne, I. (2011). Geometric representation of the mean–variance–skewness portfolio frontier based upon the shortage function. *European Journal of Operational Research*, *210*(1), 81–94.

Kim, H. (2009). On the usefulness of the contrarian strategy across national stock markets: A grid bootstrap analysis. *Journal of Empirical Finance*, *16*(5), 734–744.

King, R. G., & Levine, R. (1993). Finance and Growth : Schumpeter Might be Right. *The Quarterly Journal of Economics, 108*(3), 717–737.

Kisaka, S. E., Mbithi, J. A., & Kitur, H. (2015). Determining the Optimal Portfolio Size on the Nairobi Securities Exchange. *Research Journal of Finance and Accounting*, *6*(6), 215–230.

Klassen, K. J., & Yoogalingam, R. (2013). Appointment system design with interruptions and physician lateness. *International Journal of Operations & Production Management*, *33*(4), 394–414.

Klein, R. W., & Bawa, V. S. (1976). The effect of estimating risk on an optimal portfolio choice. *Journal of Financial Economics, 3*(3), 215-231.

Kraus, A., & Litzenberger, R. H. (1976). Skewness preference and the valuation of risk assets. *Journal of Finance*, *31*(4), 1085–1100.

Ledoit, O., & Wolf, M. (2003). Improved estimation of the covariance matrix of stock returns with an application to portfolio selection. *Journal of Empirical Finance*, *10*(5), 603–621.

Ledoit, O., & Wolf, M. (2008). Robust performance hypothesis testing with the Sharpe ratio. *Journal of Empirical Finance*, *15*(5), 850–859.

Lee, S. (2014). Asymptotic refinements of a misspecification-robust bootstrap for generalized method of moments estimators. *Journal of Econometrics*, *178*(3), 398–413.

Lessard, D. R. (1973). International Portfolio Diversification: A multivariate analysis for a group of Latin American countries. *The Journal of Finance, 28*(3), 619-633.

Levine, R. (1998). The Legal Environment , Banks , and Long-Run Economic Growth. *Journal of Money, Credit & Banking*, *30*(3), 596–613.

Levine, R., & Zervos, S. (1996). Stock Market Development and Long-Run Growth. *The World Bank Economic Review*, *10*(2), 323–339.

Levine, R., & Zervos, S. (1998). Stock Markets, Banks and Economic Growth. *The American Economic Review*, *88*(3), 537-558.

Levy, M., & Levy, H. (2015). Keeping up with the Joneses and optimal diversification. *Journal of Banking and Finance*, *58*, 29–38.

Li, D., & Ng, W. L. (2000). Optimal dynamic portfolio selection:multiperiod mean-variance formulation. *Mathematical Finance*, *10*(3), 387-406.

Li, K., Sarkar, A., & Wang, Z. (2003). Diversification benefits of emerging markets subject to portfolio constraints. *Journal of Empirical Finance*, *10*(1-2), 57–80.

Lin, W., & Lu, J. (2012). Insurance : Mathematics and Economics Risky asset allocation and consumption rule in the presence of background risk and insurance markets. *Insurance: Mathematics and Economics*, *50*(1), 150–158.

Liu, Y., & Zhang, W. (2015). A multi-period fuzzy portfolio optimization model with minimum transaction lots. *European Journal of Operational Research*, *242*(3), 933–941.

Liu, Y., Zhang, W., & Xu, W. (2012). Automatica Fuzzy multi-period portfolio selection optimization models using. *Automatica*, *48*(12), 3042–3053.

Los, C. A. (1999). Galton's Error and the under-representation of systematic risk. *Journal of Banking & Finance*, *23*(12), 1793–1829.

Magnusson, M., & Wydick, B. (2010). How Efficient are Africa's Emerging Stock Markets? *Journal of Development Studies*, *38*(4), 141–156.

Marekwica, M. (2012). Optimal tax-timing and asset allocation when tax rebates on capital losses are limited. *Journal of Banking & Finance*, *36*(7), 2048–2063.

Markowitz, H. (1952). Portofolio Selection. *The Journal of Finance*, *7*(1), 77-91.

Markowitz, H. (1999). The early history of portfolio theory: 1600-1960. *Financial Analyst Journal*, *7*(1),77-91.

Mensah, Lord, Avuglah, R. K., & Dedu, V. (2013). Allocation of Assets on the Ghana Stock Exchange (GSE). *International Journal of Financial Research*, *4*(2), 108–114.

Meric, I., & Meric, G. (1989). Potential gains from international portfolio diversification and inter-temporal stability and seasonality in international stock market relationships. *Journal of Banking and Finance*, *13*(4-5), 627-640.

Merton, R. C. (1969). Portfolio selection under uncertainty: The continuous-time case. The *Review of Economics and Statistics*, *51*(3), 247-257.

Miralles-Marcelo, J. L., Miralles-Quirós, M. M., & Miralles-Quirós, J. L. (2015). North American Journal of Economics and Finance Improving international diversification benefits for US investors. *North American Journal of Economics and Finance*, *32*, 64–76.

Mossin, J. (1968). optimal multiperiod portfolio policies. *The Journal of Business*, *41*(2), 215–229.

Mulvey, J. M., & Shetty, B. (2004). Financial planning via multi-stage stochastic optimization. *Computers & Operations Research*, *31*(1), 1–20.

Munk, C., & Sørensen, C. (2010). Dynamic asset allocation with stochastic income and interest rates. *Journal of Financial Economics*, *96*(3), 433–462.

Naceur, S. B., Ghazouani, S., & Omran, M. (2007). The determinants of stock market development in the Middle-Eastern and North African region. *Managerial Finance*, *33*(7), 477–489.

Obstfeld, M., 1994. Risk-taking, global diversification, and growth. *American Economic Review*, *84*, 1310–1329.

Pettenuzzo, D., & Timmermann, A. (2011). Predictability of stock returns and asset allocation under structural breaks. *Journal of Econometrics*, *164*(1), 60–78.

Pradhan, R. P., Arvin, M. B., & Bahmani, S. (2015). Causal nexus between economic growth, inflation and stock market development: The case of OECD countries. *Global Finance Journal*. doi:10.1016/j.gfj.2015.04.006.

Prakash, A. J., Chang, C.-H., & Pactwa, T. E. (2003). Selecting a portfolio with skewness: Recent evidence from US, European, and Latin American equity markets. *Journal of Banking & Finance*, *27*(7), 1375–1390.

Ramchand, L., & Sethapakdi, P. (2000). Changes in systematic risk following global equity issuance. *Journal of Banking & Finance*, *24*(9), 1491–1514.

Rjoub, H., Türsoy, T., & Günsel, N. (2009). The effects of macroeconomic factors on stock returns: Istanbul Stock Market. *Studies in Economics and Finance*, *26*(1), 36–45.

Rousseau, P.L.,& Wachtel, P. (2000). Equity markets and growth: Cross-country evidence on timing and outcomes, 1980–1995. *Journal of Banking & Finance*, *24*(12), 1933–1957.

Sharpe, W. F. (1963). A simplified model for portfolio analysis. *Management Science, 9*(2), 277–293.

Sharpe, W. F. (1964). Capital Asset Prices: A theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance, 19*(3), 425-442.

Siegel, A. F., & Woodgate, A. (2007). Performance of Portfolios Optimized with Estimation Error. *Management Science*, *53*(6), 1005–1015.

Simar, L., & Wilson, P.W. (1998). Sensitivity Analysis of Efficiency Scores: How to Bootstrap in Nonparametric Frontier Models. *Management Science, 44*(1), 49-61.

Simar, L., & Wilson, P.W. (2000). A general methodology for bootstrapping in non-parametric frontier models. *Journal of Applied Statistics, 27*(6), 779–802.

Simar, L., & Wilson, P.W. (2007). Estimation and inference in two-stage, semi-parametric models of production processes. *Journal of Econometrics, 136*(1), 31-64.

Singh, A. (1997). Financial liberalization, stock markets and economic development. *The Economic Journal*, *107*, 771–782.

Souza, R. C., Marcato, A. L. M., Dias, B. H., & Oliveira, F. L. C. (2012). Optimal operation of hydrothermal systems with Hydrological Scenario Generation through Bootstrap and Periodic Autoregressive Models. *European Journal of Operational Research*, *222*(3), 606–615.

Standard & Poor Dow Jones (2015). *About Us.* Retrieved on 20th April, 2015 from http://us.spindices.com/about-sp-indices/

Statman, M. (1987). How many stocks make a diversified portfolio? *The Journal of Financial and Ouantitative Analysis*, *22*(3), 353–363.

Sun, Q., & Yan, Y. (2003). Skewness persistence with optimal portfolio selection. *Journal of Banking & Finance*, *27*(6), 1111–1121.

Susmel, R. (2001). Extreme observations and diversification in Latin American emerging equity markets. *Journal of International Money and Finance*, *20*(7), 971–986.

Tepla, L. (2000). Optimal portfolio policies with borrowing and shortsale constraints. *Journal of Economic Dynamic and Control*, *24*(11-12), 1623–1639.

Tobin, R. (1958). Liquidity Preference as Behavior Towards Risk. *The Review of Economic Studies*, *25*(2), 65–86.

Topaloglou, N., Vladimirou, H., & Zenios, S. a. (2002). CVaR models with selective hedging for international asset allocation. *Journal of Banking & Finance*, *26*(7), 1535–1561.

Tortosa-Ausina, E. (2002). Bank cost efficiency and output specification. *Journal of Productivity Analysis*, *18*(3), 199-222.

Tortosa-Ausina, E., Armero, C., Conesa, D., & Grifell-Tatjé, E. (2012). Bootstrapping profit change: An application to Spanish banks. *Computers & Operations Research*, *39*(8), 1857-1871.

Tortosa-Ausina, E., Grifell-Tatjé, E., Armero, C., & Conesa, D. (2008). Sensitivity analysis of efficiency and Malmquist productivity indices: An application to Spanish savings banks. *European Journal of Operational Research*, *184*(3), 1062–1084.

Xie, S. (2009). Insurance : Mathematics and Economics Continuous-time mean – variance portfolio selection with liability and regime switching. *Insurance: Mathematics and Economics*, *45*(1), 148–155.

Yang, C. W., & Hung, K. (2010). A Generalized Markowitz Portfolio Selection Model with Higher Moments, *International Review of Accounting, banking and Finance, 2*(1), 1–8.

Yao, H., Li, Z., & Chen, S. (2014). Continuous-time mean–variance portfolio selection with only risky assets. *Economic Modelling*, *36*, 244–251.

Wang, J., & Forsyth, P. A. (2010). Numerical solution of the Hamilton–Jacobi–Bellman formulation for continuous time mean variance asset allocation. *Journal of Economic Dynamics and Control*, *34*(2), 207–230.

Wang, J., & Forsyth, P. A. (2011). Continuous time mean variance asset allocation: A time-consistent strategy. *European Journal of Operational Research*, *209*(2), 184–201.

Weitzman, M. L. (2013). Tail-Hedge Discounting and the Social Cost of Carbon. *Journal of Economic Literature*, *51*(3), 873–882.

Winston, L.W. (2003). *Operations Research: Applications and Algorithms* (4th ed.). Cengage Learning.

Wu, H., & Chen, H. (2015). Nash equilibrium strategy for a multi-period mean – variance portfolio selection problem with regime switching. *Economic Modelling*, *46*, 79–90.

Yao, H., Li, Z., & Chen, S. (2014). Continuous-time mean – variance portfolio selection with only risky assets. *Economic Modelling*, *36*, 244–251.

Yu, T.-Y., & Huang, H.-T. (2013). Coupling a memetic algorithm to simulation models for promising multi-period asset allocations. *Computers & Operations Research*, *40*(10), 2585–2597.

Zhang, W.-G. (2007). Possibilistic mean-standard deviation models to portfolio selection for bounded assets. *Applied Mathematics and Computation*, *189*, 1614–1623.

Zhang, W.-G., Wang, Y.-L., Chen, Z.-P., & Nie, Z.-K. (2007). Possibilistic mean-variance models and efficient frontiers for portfolio selection problem. *Information Sciences*, *177*(13), 2787–2801.

Zhang, W., Liu, Y., & Xu, W. (2012). A possibilistic mean-semivariance-entropy model for multi-period portfolio selection with transaction costs. *European Journal of Operational Research*, *222*(2), 341–349.

Zhou, J., & Nicholson, J. R. (2015). Economic value of modeling covariance asymmetry for mixed-asset portfolio diversifications. *Economic Modelling*, *45*, 14–21.

Zhu, Y., & Zhou, G. (2009). Technical analysis: An asset allocation perspective on the use of moving averages. *Journal of Financial Economics*, *92*(3), 519–544.

APPENDICES

APPENDIX A

Table 8: A sample of 450 out of the 2000 bootstrapped estimates of expected returns, risks & sharpe ratios for the three portfolios.

EWP MEAN	EWP SD	EWP SR	MVP MEAN	MVP SD	MVP SR	TP MEAN	TP SD	TP SR
0.1007	0.1221	0.8246	0.0932	0.0992	0.9390	0.1256	0.1141	1.1009
0.0504	0.1157	0.4361	0.0291	0.0919	0.3162	0.0983	0.1319	0.7449
0.1091	0.1254	0.8699	0.0919	0.0970	0.9472	0.1474	0.1148	1.2841
0.1295	0.1021	1.2685	0.0780	0.0912	0.8557	0.2158	0.1262	1.7091
0.0620	0.1016	0.6100	0.0388	0.0865	0.4480	0.1602	0.1319	1.2140
0.0925	0.1179	0.7846	0.0769	0.1007	0.7639	0.1340	0.1188	1.1280
0.1241	0.0989	1.2548	0.0878	0.0883	0.9937	0.1725	0.1111	1.5528
0.1358	0.1130	1.2015	0.1332	0.0896	1.4877	0.1626	0.0965	1.6840
0.0643	0.0992	0.6483	0.0459	0.0828	0.5541	0.1322	0.1144	1.1555
0.1121	0.1254	0.8945	0.0907	0.1041	0.8712	0.1819	0.1399	1.3001
0.1081	0.1137	0.9507	0.0823	0.0952	0.8643	0.1272	0.1163	1.0941
0.0761	0.1064	0.7152	0.0641	0.0920	0.6966	0.1259	0.1211	1.0401
0.1093	0.0984	1.1105	0.0867	0.0895	0.9687	0.1490	0.1097	1.3580
0.0362	0.0986	0.3676	0.0488	0.0816	0.5984	0.1237	0.1085	1.1394
0.0439	0.0919	0.4774	0.0336	0.0829	0.4047	0.1411	0.1436	0.9829
0.0663	0.1267	0.5230	0.0147	0.0969	0.1516	0.1692	0.1727	0.9795
0.0589	0.1128	0.5219	0.0285	0.0935	0.3045	0.1196	0.1383	0.8651
0.1059	0.1135	0.9326	0.0904	0.0942	0.9602	0.1782	0.1142	1.5606
0.0531	0.1358	0.3910	0.0460	0.1086	0.4239	0.1164	0.1520	0.7658
0.0906	0.1252	0.7237	0.0730	0.1027	0.7109	0.1237	0.1210	1.0220
0.1090	0.1349	0.8082	0.0629	0.1007	0.6242	0.2062	0.1695	1.2166
0.1526	0.1040	1.4675	0.1210	0.0886	1.3648	0.2245	0.1141	1.9684
0.0859	0.1023	0.8392	0.0536	0.0895	0.5986	0.1616	0.1338	1.2082
0.0547	0.1174	0.4658	0.0307	0.0947	0.3243	0.1336	0.1456	0.9171
0.0922	0.1249	0.7379	0.0555	0.0994	0.5582	0.1384	0.1326	1.0433
0.1046	0.1100	0.9511	0.0796	0.0958	0.8311	0.1600	0.1295	1.2356
0.1123	0.1281	0.8763	0.0812	0.0989	0.8206	0.1474	0.1179	1.2502
0.1053	0.0969	1.0867	0.0868	0.0803	1.0808	0.1666	0.1105	1.5070
0.1274	0.1311	0.9716	0.0750	0.1091	0.6869	0.1829	0.1536	1.1910
0.0435	0.1001	0.4348	0.0337	0.0900	0.3748	0.1011	0.1262	0.8013
0.0656	0.1194	0.5494	0.0460	0.1041	0.4423	0.2179	0.1629	1.3379
0.0877	0.1304	0.6724	0.0791	0.1060	0.7456	0.1895	0.1485	1.2765
0.0849	0.0962	0.8821	0.0693	0.0833	0.8325	0.1739	0.1298	1.3401
0.0940	0.1261	0.7456	0.0567	0.1021	0.5556	0.1857	0.1580	1.1755
0.0513	0.1114	0.4607	0.0395	0.0914	0.4323	0.1470	0.1449	1.0145
0.0629	0.0908	0.6926	0.0418	0.0839	0.4981	0.1913	0.1307	1.4635

0.0408	0.1078	0.3788	0.0219	0.0909	0.2403	0.1578	0.1728	0.9129
0.0598	0.1130	0.5295	0.0525	0.0972	0.5402	0.1020	0.1182	0.8634
0.0985	0.1363	0.7226	0.0761	0.1039	0.7319	0.1317	0.1271	1.0359
0.1073	0.1251	0.8579	0.0801	0.0984	0.8136	0.1889	0.1442	1.3101
0.1286	0.1180	1.0894	0.1075	0.0908	1.1847	0.1619	0.1086	1.4905
0.1156	0.1050	1.1006	0.1246	0.0897	1.3889	0.1827	0.1081	1.6907
0.1029	0.1219	0.8448	0.0896	0.1038	0.8633	0.1628	0.1368	1.1894
0.1181	0.1236	0.9551	0.0916	0.1009	0.9079	0.2507	0.1549	1.6188
0.0973	0.1245	0.7820	0.0765	0.1060	0.7218	0.1453	0.1396	1.0408
0.1034	0.1419	0.7291	0.0843	0.1045	0.8071	0.1629	0.1296	1.2569
0.0862	0.1068	0.8068	0.0602	0.0943	0.6390	0.1937	0.1375	1.4094
0.0743	0.1220	0.6088	0.0767	0.0987	0.7773	0.1336	0.1227	1.0888
0.1414	0.1266	1.1166	0.0963	0.1027	0.9377	0.1948	0.1348	1.4456
0.0563	0.1019	0.5529	0.0526	0.0916	0.5736	0.1162	0.1203	0.9657
0.0564	0.0974	0.5791	0.0346	0.0812	0.4259	0.1316	0.1257	1.0468
0.0759	0.1253	0.6060	0.0794	0.0990	0.8019	0.1226	0.1170	1.0477
0.0976	0.1080	0.9033	0.0775	0.0935	0.8285	0.1614	0.1288	1.2531
0.0667	0.1043	0.6396	0.0483	0.0888	0.5439	0.1166	0.1146	1.0175
0.0777	0.1138	0.6829	0.0547	0.0935	0.5849	0.1364	0.1340	1.0176
0.0541	0.1010	0.5357	0.0393	0.0889	0.4420	0.1041	0.1217	0.8552
0.0851	0.0907	0.9375	0.0646	0.0804	0.8039	0.1200	0.0964	1.2453
0.0764	0.1336	0.5720	0.0501	0.1032	0.4854	0.1364	0.1492	0.9145
0.1000	0.1443	0.6931	0.0819	0.1114	0.7348	0.1296	0.1326	0.9771
0.0581	0.1292	0.4500	0.0502	0.0997	0.5032	0.1266	0.1458	0.8684
0.1038	0.1125	0.9224	0.0773	0.0990	0.7808	0.1703	0.1393	1.2224
-0.0035	0.1165	-0.0296	-0.0167	0.0990	-0.1684	0.0946	0.1707	0.5540
0.0672	0.0929	0.7233	0.0685	0.0817	0.8385	0.1430	0.1110	1.2880
0.1150	0.1286	0.8944	0.0670	0.1041	0.6440	0.1927	0.1581	1.2188
0.0454	0.1184	0.3837	0.0534	0.1027	0.5197	0.0965	0.1192	0.8102
0.0524	0.1254	0.4182	0.0136	0.1003	0.1352	0.1325	0.1745	0.7595
0.0369	0.1223	0.3020	0.0416	0.1085	0.3833	0.1457	0.1366	1.0669
0.1302	0.1389	0.9369	0.0899	0.1071	0.8393	0.1931	0.1444	1.3378
0.1279	0.1264	1.0125	0.0950	0.1024	0.9283	0.1753	0.1381	1.2696
0.0942	0.1025	0.9186	0.0666	0.0860	0.7743	0.1615	0.1204	1.3418
0.1096	0.1181	0.9280	0.0708	0.1021	0.6939	0.1779	0.1429	1.2448
0.0816	0.1269	0.6431	0.0513	0.0950	0.5402	0.1357	0.1524	0.8908
0.0493	0.1141	0.4317	0.0425	0.0904	0.4706	0.1067	0.1203	0.8870
0.1200	0.1316	0.9118	0.0675	0.1077	0.6264	0.2234	0.1608	1.3889
0.1060	0.1065	0.9959	0.0802	0.0938	0.8556	0.1394	0.1192	1.1689
0.0975	0.1100	0.8859	0.0617	0.0961	0.6426	0.1701	0.1406	1.2094
0.1088	0.1040	1.0461	0.0874	0.0863	1.0125	0.1910	0.1164	1.6405

0.0720	0.1327	0.5429	0.0371	0.0999	0.3716	0.1270	0.1412	0.8994
0.0543	0.1013	0.5358	0.0236	0.0875	0.2695	0.1768	0.1533	1.1534
0.0380	0.1270	0.2995	0.0411	0.1078	0.3813	0.1220	0.1524	0.8005
0.0872	0.1102	0.7910	0.0683	0.0908	0.7523	0.1478	0.1254	1.1781
0.0928	0.1225	0.7579	0.0414	0.0985	0.4204	0.1652	0.1475	1.1196
0.0620	0.1189	0.5219	0.0514	0.0994	0.5171	0.1215	0.1441	0.8427
0.0878	0.1112	0.7895	0.0764	0.0936	0.8164	0.1326	0.1112	1.1926
0.0680	0.1078	0.6308	0.0310	0.0909	0.3411	0.1319	0.1339	0.9850
0.1232	0.1155	1.0672	0.0968	0.0965	1.0025	0.1888	0.1219	1.5488
0.1052	0.1052	1.0001	0.0972	0.0864	1.1244	0.1520	0.1014	1.4983
0.1416	0.1214	1.1657	0.0818	0.0954	0.8573	0.2175	0.1335	1.6297
0.1303	0.1313	0.9923	0.1207	0.1025	1.1780	0.2144	0.1230	1.7422
0.0816	0.1336	0.6106	0.0721	0.1038	0.6940	0.1306	0.1287	1.0145
0.0506	0.1322	0.3827	0.0468	0.1011	0.4635	0.1173	0.1341	0.8746
0.0673	0.1062	0.6334	0.0391	0.0895	0.4374	0.1143	0.1244	0.9191
0.0735	0.1278	0.5748	0.0378	0.0981	0.3856	0.1702	0.1685	1.0104
0.0948	0.1250	0.7585	0.0491	0.1060	0.4634	0.2752	0.2084	1.3205
0.1337	0.1239	1.0785	0.0955	0.0978	0.9767	0.1793	0.1317	1.3615
0.1059	0.1184	0.8943	0.0818	0.0981	0.8342	0.1265	0.1189	1.0643
0.0959	0.1223	0.7835	0.0583	0.0991	0.5888	0.1263	0.1389	0.9091
0.1173	0.1044	1.1232	0.0864	0.0874	0.9887	0.1565	0.1076	1.4546
0.0691	0.1071	0.6451	0.0611	0.0927	0.6586	0.1428	0.1227	1.1640
0.0919	0.0937	0.9807	0.0638	0.0864	0.7386	0.1835	0.1234	1.4867
0.0695	0.1331	0.5219	0.0684	0.1069	0.6396	0.1800	0.1359	1.3245
0.0913	0.1117	0.8174	0.0549	0.0942	0.5831	0.1835	0.1410	1.3015
0.0463	0.1174	0.3940	0.0460	0.0903	0.5089	0.1195	0.1417	0.8438
0.0645	0.1204	0.5362	0.0343	0.0946	0.3627	0.1657	0.1444	1.1480
0.1517	0.1312	1.1564	0.1476	0.1021	1.4459	0.2372	0.1231	1.9273
0.0729	0.1098	0.6633	0.0517	0.0940	0.5498	0.1228	0.1244	0.9872
0.0879	0.1085	0.8099	0.0694	0.1002	0.6931	0.1323	0.1295	1.0216
0.0692	0.1336	0.5184	0.0434	0.1126	0.3858	0.1298	0.1604	0.8089
0.0917	0.1067	0.8598	0.0552	0.0873	0.6325	0.1892	0.1353	1.3983
0.1251	0.0974	1.2842	0.1055	0.0756	1.3960	0.1750	0.0972	1.8010
0.1323	0.1234	1.0722	0.0982	0.1027	0.9559	0.1645	0.1331	1.2357
0.1554	0.1075	1.4453	0.0994	0.0860	1.1565	0.2149	0.1242	1.7296
0.0909	0.1125	0.8084	0.0402	0.0973	0.4134	0.1722	0.1515	1.1363
0.0788	0.1196	0.6589	0.0489	0.0999	0.4898	0.1252	0.1553	0.8064
0.1250	0.1041	1.2006	0.0924	0.0836	1.1053	0.1364	0.1004	1.3584
0.0295	0.1174	0.2513	0.0026	0.0928	0.0284	0.1354	0.1872	0.7234
0.0732	0.1180	0.6205	0.0570	0.0967	0.5897	0.0986	0.1193	0.8263
0.0988	0.1081	0.9142	0.0531	0.0981	0.5412	0.1690	0.1336	1.2650

0.0965	0.1015	0.9507	0.0657	0.0846	0.7766	0.1853	0.1224	1.5141
0.0977	0.1087	0.8990	0.0769	0.0921	0.8348	0.1465	0.1157	1.2663
0.1183	0.1214	0.9742	0.0739	0.0914	0.8084	0.1521	0.1297	1.1726
0.0603	0.1385	0.4354	0.0512	0.1038	0.4931	0.1563	0.1503	1.0397
0.0872	0.1413	0.6171	0.0591	0.1086	0.5444	0.1607	0.1757	0.9148
0.1230	0.1024	1.2008	0.0918	0.0857	1.0708	0.1802	0.1163	1.5493
0.0830	0.1029	0.8058	0.0523	0.0903	0.5791	0.1765	0.1399	1.2609
0.0869	0.0947	0.9183	0.0721	0.0827	0.8717	0.1505	0.1065	1.4126
0.1473	0.1032	1.4266	0.1270	0.0903	1.4071	0.2010	0.1116	1.8005
0.0709	0.1086	0.6530	0.0478	0.0887	0.5384	0.1131	0.1205	0.9382
0.0522	0.1012	0.5161	0.0499	0.0907	0.5505	0.0819	0.1099	0.7447
0.0918	0.1022	0.8981	0.0818	0.0947	0.8634	0.1626	0.1265	1.2852
0.0762	0.1218	0.6257	0.0603	0.1012	0.5954	0.1734	0.1474	1.1767
0.0689	0.1027	0.6710	0.0591	0.0877	0.6738	0.1071	0.1086	0.9859
0.1143	0.1042	1.0964	0.0759	0.0919	0.8264	0.2188	0.1397	1.5656
0.0608	0.1226	0.4963	0.0459	0.1012	0.4533	0.1201	0.1321	0.9085
0.1414	0.0977	1.4476	0.1145	0.0801	1.4294	0.2055	0.1070	1.9212
0.1039	0.1219	0.8517	0.0741	0.0971	0.7635	0.1582	0.1363	1.1606
0.0986	0.1416	0.6965	0.0187	0.1110	0.1681	0.1951	0.1592	1.2254
0.0346	0.0966	0.3584	0.0171	0.0823	0.2072	0.1005	0.1177	0.8541
0.0737	0.1050	0.7016	0.0649	0.0878	0.7390	0.1430	0.1175	1.2165
0.0748	0.1042	0.7180	0.0588	0.0837	0.7017	0.1289	0.1091	1.1821
0.1075	0.1278	0.8410	0.1000	0.1061	0.9424	0.1486	0.1268	1.1718
0.0870	0.1068	0.8145	0.0659	0.0858	0.7681	0.1161	0.1083	1.0718
0.0709	0.1245	0.5695	0.0505	0.1044	0.4839	0.1139	0.1381	0.8244
0.1149	0.1203	0.9555	0.0950	0.0992	0.9573	0.1531	0.1182	1.2950
0.1240	0.1204	1.0298	0.1051	0.0925	1.1356	0.1475	0.1071	1.3774
0.1111	0.1395	0.7962	0.0546	0.1037	0.5259	0.1850	0.1605	1.1529
0.0837	0.1025	0.8166	0.0714	0.0954	0.7486	0.1281	0.1175	1.0900
0.0600	0.0876	0.6848	0.0431	0.0768	0.5614	0.1417	0.1150	1.2323
0.1484	0.1077	1.3782	0.1157	0.0932	1.2405	0.1832	0.1181	1.5505
0.0728	0.1132	0.6425	0.0628	0.0999	0.6287	0.1300	0.1272	1.0214
0.0934	0.1089	0.8570	0.0277	0.0894	0.3095	0.1711	0.1515	1.1294
0.0333	0.1108	0.3003	0.0071	0.0954	0.0747	0.1174	0.1623	0.7232
0.1049	0.1046	1.0022	0.0825	0.0885	0.9321	0.1696	0.1208	1.4039
0.1167	0.1203	0.9698	0.1206	0.1036	1.1644	0.1831	0.1217	1.5052
0.1013	0.1066	0.9504	0.0893	0.0909	0.9827	0.1592	0.1150	1.3846
0.1084	0.1098	0.9874	0.0826	0.0916	0.9020	0.1507	0.1202	1.2538
0.0917	0.1070	0.8571	0.0614	0.0951	0.6458	0.1829	0.1363	1.3416
0.1058	0.1259	0.8406	0.0910	0.0955	0.9530	0.1387	0.1194	1.1617
0.0816	0.1329	0.6137	0.0748	0.1020	0.7335	0.1300	0.1342	0.9690

0.1242	0.1274	0.9746	0.1034	0.1001	1.0324	0.1542	0.1205	1.2796
0.1670	0.1098	1.5211	0.1091	0.0916	1.1915	0.2218	0.1263	1.7559
0.0776	0.1349	0.5757	0.0566	0.1041	0.5441	0.1276	0.1328	0.9612
0.0789	0.0907	0.8701	0.0531	0.0844	0.6283	0.1449	0.1131	1.2814
0.0805	0.1100	0.7319	0.0603	0.0929	0.6489	0.1072	0.1172	0.9149
0.0653	0.1024	0.6376	0.0561	0.0863	0.6499	0.1147	0.1153	0.9942
0.1012	0.1172	0.8641	0.0826	0.1002	0.8248	0.1858	0.1384	1.3421
0.0511	0.1082	0.4718	0.0331	0.0879	0.3768	0.1369	0.1290	1.0612
0.0755	0.1074	0.7026	0.0481	0.0969	0.4959	0.1707	0.1554	1.0988
0.1354	0.1243	1.0894	0.1214	0.0981	1.2380	0.1750	0.1160	1.5092
0.0427	0.1299	0.3283	0.0132	0.1020	0.1299	0.1704	0.1759	0.9687
0.0544	0.1088	0.5001	0.0285	0.0904	0.3152	0.1431	0.1660	0.8618
0.0656	0.1223	0.5363	0.0630	0.0998	0.6315	0.1399	0.1225	1.1423
0.0539	0.1090	0.4946	0.0327	0.0939	0.3478	0.1385	0.1531	0.9046
0.0842	0.1081	0.7787	0.0703	0.0882	0.7964	0.1176	0.1077	1.0913
0.0824	0.1286	0.6413	0.0641	0.1035	0.6193	0.1445	0.1479	0.9766
0.0847	0.1222	0.6931	0.0749	0.1032	0.7264	0.1341	0.1274	1.0532
0.1369	0.1226	1.1168	0.1087	0.0997	1.0897	0.1692	0.1175	1.4402
0.0978	0.1224	0.7992	0.0809	0.0982	0.8239	0.1308	0.1246	1.0498
0.1108	0.1128	0.9825	0.0805	0.1002	0.8038	0.2387	0.1417	1.6849
0.0577	0.1109	0.5206	0.0664	0.0891	0.7445	0.2162	0.1490	1.4504
0.1151	0.1371	0.8396	0.0855	0.1084	0.7886	0.1884	0.1395	1.3501
0.0852	0.1393	0.6114	0.0931	0.0985	0.9455	0.1165	0.1078	1.0806
0.0798	0.1443	0.5528	0.0694	0.1041	0.6664	0.1194	0.1322	0.9029
0.0834	0.1233	0.6768	0.0733	0.0993	0.7381	0.1429	0.1284	1.1134
0.1353	0.0883	1.5331	0.1253	0.0772	1.6226	0.1568	0.0862	1.8203
0.0708	0.1231	0.5753	0.0624	0.1088	0.5734	0.1342	0.1453	0.9238
0.1359	0.1019	1.3335	0.0941	0.0870	1.0813	0.1882	0.1178	1.5978
0.0682	0.1061	0.6423	0.0418	0.0900	0.4643	0.1217	0.1325	0.9186
0.0736	0.1237	0.5945	0.0316	0.1002	0.3155	0.1403	0.1493	0.9394
0.1486	0.1215	1.2231	0.0947	0.0977	0.9688	0.1765	0.1276	1.3831
0.1269	0.0956	1.3281	0.0938	0.0804	1.1671	0.2116	0.1181	1.7914
0.1166	0.1226	0.9507	0.0955	0.0994	0.9615	0.1874	0.1310	1.4311
0.0855	0.1020	0.8380	0.0507	0.0872	0.5813	0.1789	0.1358	1.3174
0.0722	0.1119	0.6452	0.0618	0.0925	0.6683	0.1015	0.1065	0.9529
0.0871	0.1336	0.6516	0.0564	0.1043	0.5405	0.1627	0.1595	1.0202
0.1459	0.1244	1.1724	0.1103	0.0993	1.1111	0.1780	0.1220	1.4585
0.0590	0.1099	0.5372	0.0579	0.0915	0.6326	0.1547	0.1432	1.0806
0.1243	0.1351	0.9206	0.0794	0.1066	0.7447	0.1666	0.1408	1.1830
0.0628	0.1105	0.5679	0.0424	0.0885	0.4792	0.1341	0.1417	0.9464
0.0744	0.1145	0.6494	0.0942	0.0912	1.0328	0.1417	0.1071	1.3225

0.0759	0.1006	0.7547	0.0512	0.0884	0.5789	0.1422	0.1172	1.2137
0.0729	0.1230	0.5931	0.0579	0.0965	0.6004	0.1536	0.1386	1.1080
0.1298	0.1245	1.0427	0.0933	0.0960	0.9720	0.1709	0.1232	1.3874
0.0664	0.1195	0.5557	0.0534	0.0985	0.5416	0.1277	0.1154	1.1068
0.0963	0.1254	0.7676	0.0651	0.1034	0.6300	0.1351	0.1389	0.9727
0.0697	0.1018	0.6842	0.0510	0.0907	0.5616	0.1374	0.1236	1.1113
0.1261	0.1187	1.0616	0.0821	0.1048	0.7836	0.2074	0.1533	1.3531
0.1477	0.1203	1.2278	0.1115	0.0992	1.1243	0.1712	0.1231	1.3911
0.0667	0.0956	0.6978	0.0257	0.0836	0.3070	0.1310	0.1243	1.0541
0.1212	0.1151	1.0525	0.0962	0.0942	1.0222	0.1916	0.1193	1.6067
0.1026	0.1331	0.7712	0.0364	0.1039	0.3507	0.2325	0.1876	1.2394
0.0652	0.1022	0.6378	0.0431	0.0888	0.4856	0.1229	0.1317	0.9333
0.1237	0.1202	1.0289	0.0771	0.0952	0.8095	0.2475	0.1632	1.5165
0.0868	0.1156	0.7514	0.0502	0.0974	0.5150	0.1417	0.1478	0.9588
0.0872	0.1259	0.6930	0.0756	0.1067	0.7086	0.1502	0.1305	1.1510
0.0752	0.0979	0.7680	0.0763	0.0899	0.8480	0.1424	0.1120	1.2716
0.0969	0.1135	0.8532	0.0872	0.0916	0.9520	0.1433	0.1186	1.2086
0.0836	0.1140	0.7335	0.0432	0.0931	0.4645	0.1268	0.1300	0.9753
0.1296	0.1066	1.2153	0.0864	0.0926	0.9331	0.1884	0.1263	1.4926
0.0998	0.1178	0.8474	0.0700	0.1001	0.6999	0.1847	0.1499	1.2324
0.1368	0.1108	1.2346	0.1010	0.0961	1.0518	0.1851	0.1305	1.4189
0.0741	0.1300	0.5705	0.0585	0.1040	0.5625	0.2319	0.1715	1.3520
0.1019	0.1288	0.7911	0.0812	0.1017	0.7983	0.1315	0.1317	0.9982
0.0729	0.1082	0.6743	0.0542	0.0883	0.6136	0.1152	0.1211	0.9514
0.1280	0.1185	1.0797	0.0712	0.1031	0.6910	0.1800	0.1361	1.3230
0.1395	0.1066	1.3084	0.1038	0.0915	1.1341	0.1916	0.1146	1.6727
0.1516	0.1155	1.3125	0.1172	0.0897	1.3064	0.1804	0.1121	1.6093
0.0742	0.1102	0.6730	0.0425	0.0925	0.4589	0.1245	0.1245	1.0001
0.1069	0.1019	1.0491	0.0951	0.0902	1.0554	0.1870	0.1194	1.5660
0.0845	0.1147	0.7367	0.0934	0.0910	1.0261	0.1433	0.1108	1.2930
0.1387	0.1303	1.0641	0.1023	0.0980	1.0435	0.1888	0.1233	1.5306
0.1039	0.1115	0.9316	0.0870	0.0952	0.9140	0.1428	0.1180	1.2101
0.0855	0.1177	0.7261	0.0564	0.0922	0.6123	0.2062	0.1536	1.3427
0.0658	0.1302	0.5053	0.0302	0.1068	0.2830	0.1269	0.1448	0.8763
0.1151	0.1135	1.0136	0.0540	0.0912	0.5924	0.1812	0.1463	1.2381
0.0778	0.1173	0.6629	0.0529	0.0895	0.5906	0.1666	0.1316	1.2655
0.0908	0.1170	0.7762	0.0923	0.1036	0.8903	0.1606	0.1276	1.2581
0.1091	0.1270	0.8589	0.0784	0.1054	0.7437	0.1963	0.1490	1.3170
0.1324	0.1142	1.1592	0.0943	0.0966	0.9754	0.1614	0.1232	1.3100
0.1308	0.1155	1.1320	0.1086	0.0949	1.1433	0.1691	0.1167	1.4495
0.1161	0.1020	1.1382	0.0964	0.0857	1.1249	0.1606	0.1021	1.5725

0.0822	0.1357	0.6054	0.0659	0.1010	0.6526	0.1270	0.1315	0.9657
0.1094	0.1403	0.7802	0.0649	0.1129	0.5744	0.2070	0.1496	1.3834
0.0904	0.1372	0.6591	0.0614	0.1024	0.6002	0.1482	0.1356	1.0933
0.1753	0.1039	1.6872	0.1482	0.0880	1.6845	0.2404	0.1093	2.1993
0.0962	0.1090	0.8819	0.0664	0.0878	0.7560	0.1597	0.1322	1.2082
0.0742	0.0896	0.8278	0.0618	0.0798	0.7750	0.1142	0.1060	1.0773
0.0578	0.1096	0.5276	0.0161	0.0992	0.1625	0.1850	0.1680	1.1015
0.1021	0.1094	0.9333	0.0746	0.0926	0.8065	0.1775	0.1347	1.3176
0.0861	0.1324	0.6498	0.0863	0.1041	0.8289	0.1941	0.1482	1.3097
0.0846	0.1140	0.7419	0.0830	0.0968	0.8569	0.1345	0.1163	1.1564
0.1176	0.1211	0.9708	0.1056	0.0996	1.0607	0.1548	0.1095	1.4142
0.0774	0.1006	0.7693	0.0675	0.0906	0.7446	0.1217	0.1085	1.1210
0.0897	0.0900	0.9976	0.0786	0.0842	0.9336	0.1659	0.1166	1.4230
0.0580	0.1272	0.4561	0.0457	0.1001	0.4566	0.1418	0.1452	0.9765
0.1434	0.1108	1.2948	0.1062	0.0955	1.1124	0.1716	0.1147	1.4959
0.1041	0.1132	0.9192	0.0602	0.0908	0.6626	0.1664	0.1317	1.2633
0.1013	0.1098	0.9224	0.0609	0.0916	0.6653	0.1943	0.1463	1.3282
0.0591	0.1142	0.5176	0.0438	0.0987	0.4439	0.1872	0.1435	1.3043
0.0918	0.1256	0.7310	0.0757	0.0972	0.7792	0.1288	0.1177	1.0944
0.1349	0.1319	1.0225	0.1005	0.1052	0.9554	0.2114	0.1513	1.3971
0.0627	0.0932	0.6723	0.0400	0.0764	0.5238	0.1105	0.1045	1.0574
0.1219	0.1116	1.0923	0.0653	0.0924	0.7070	0.2504	0.1574	1.5911
0.1635	0.1081	1.5134	0.1206	0.0941	1.2819	0.2101	0.1199	1.7520
0.0566	0.1291	0.4383	0.0384	0.0983	0.3911	0.1453	0.1425	1.0194
0.1017	0.0989	1.0279	0.0870	0.0879	0.9896	0.1392	0.1093	1.2745
0.0996	0.1280	0.7784	0.0854	0.1010	0.8458	0.1522	0.1247	1.2201
0.0762	0.1082	0.7046	0.0611	0.0925	0.6603	0.1555	0.1213	1.2824
0.1202	0.1151	1.0445	0.1063	0.1001	1.0620	0.1600	0.1179	1.3569
0.1147	0.1025	1.1190	0.1009	0.0927	1.0891	0.1511	0.1082	1.3973
0.0843	0.0985	0.8566	0.0567	0.0872	0.6495	0.1979	0.1290	1.5338
0.1161	0.0921	1.2606	0.0776	0.0764	1.0159	0.1672	0.1055	1.5840
0.1333	0.1009	1.3211	0.0861	0.0888	0.9689	0.2439	0.1415	1.7237
0.1105	0.1085	1.0186	0.0669	0.0874	0.7658	0.1580	0.1211	1.3040
0.0781	0.1132	0.6898	0.0404	0.0887	0.4551	0.1628	0.1446	1.1261
0.0331	0.1248	0.2649	0.0174	0.1038	0.1674	0.0891	0.1555	0.5734
0.0895	0.1302	0.6879	0.0672	0.1014	0.6629	0.1399	0.1359	1.0293
0.0840	0.1366	0.6151	0.0748	0.1062	0.7049	0.1385	0.1382	1.0021
0.1098	0.1258	0.8726	0.0796	0.1015	0.7843	0.2360	0.1513	1.5596
0.1129	0.1001	1.1270	0.1029	0.0897	1.1469	0.1509	0.1077	1.4005
0.1021	0.1060	0.9631	0.0777	0.0876	0.8876	0.1701	0.1266	1.3436
0.1066	0.1365	0.7808	0.1014	0.1021	0.9937	0.1448	0.1193	1.2139

0.1051	0.1075	0.9784	0.0542	0.0822	0.6595	0.1535	0.1312	1.1705
0.0598	0.1277	0.4685	0.0375	0.0944	0.3969	0.0868	0.1292	0.6724
0.0986	0.1231	0.8009	0.0696	0.0969	0.7179	0.1591	0.1442	1.1028
0.0931	0.1126	0.8271	0.0716	0.0959	0.7469	0.1517	0.1274	1.1910
0.1075	0.1257	0.8547	0.1100	0.0891	1.2344	0.1562	0.1054	1.4819
0.1126	0.1121	1.0047	0.0794	0.0930	0.8538	0.1614	0.1288	1.2530
0.1251	0.1061	1.1789	0.0883	0.0875	1.0093	0.1586	0.1055	1.5034
0.1099	0.1220	0.9005	0.1084	0.0964	1.1245	0.1442	0.1082	1.3328
0.1196	0.1299	0.9202	0.0929	0.1058	0.8778	0.1667	0.1356	1.2292
0.1216	0.1017	1.1956	0.0860	0.0888	0.9685	0.1919	0.1299	1.4777
0.0610	0.1381	0.4418	0.0490	0.1022	0.4800	0.1330	0.1452	0.9160
0.0849	0.1040	0.8161	0.0926	0.0853	1.0854	0.1300	0.0985	1.3197
0.1380	0.0936	1.4747	0.1030	0.0824	1.2502	0.1803	0.1044	1.7273
0.0623	0.1198	0.5199	0.0400	0.0969	0.4131	0.1186	0.1306	0.9078
0.1059	0.1085	0.9761	0.0525	0.0878	0.5977	0.2267	0.1320	1.7169
0.0560	0.1085	0.5163	0.0246	0.0909	0.2709	0.1290	0.1618	0.7974
0.0842	0.1164	0.7231	0.0579	0.1035	0.5591	0.1386	0.1374	1.0082
0.1011	0.1279	0.7902	0.0986	0.0978	1.0084	0.1318	0.1115	1.1820
0.0984	0.1184	0.8312	0.0716	0.1054	0.6795	0.1905	0.1601	1.1903
0.0542	0.1132	0.4785	0.0124	0.0903	0.1375	0.1440	0.1490	0.9661
0.1154	0.0999	1.1555	0.1091	0.0881	1.2388	0.1725	0.1054	1.6358
0.1162	0.1384	0.8400	0.0892	0.1119	0.7971	0.1346	0.1345	1.0006
0.1186	0.1091	1.0878	0.0737	0.0912	0.8077	0.1797	0.1290	1.3931
0.0522	0.1080	0.4832	0.0358	0.0879	0.4076	0.1290	0.1301	0.9916
0.1153	0.0911	1.2646	0.0936	0.0839	1.1148	0.1665	0.1025	1.6238
0.1125	0.1126	0.9984	0.0985	0.0949	1.0374	0.2064	0.1301	1.5864
0.0852	0.1223	0.6970	0.0362	0.1097	0.3300	0.1580	0.1480	1.0678
0.0399	0.1173	0.3403	0.0230	0.0942	0.2438	0.1484	0.1555	0.9542
0.1194	0.1147	1.0405	0.0972	0.1001	0.9705	0.1528	0.1205	1.2679
0.0749	0.0985	0.7605	0.0579	0.0897	0.6448	0.1758	0.1320	1.3319
0.0841	0.1151	0.7312	0.0679	0.0964	0.7039	0.1888	0.1452	1.3004
0.1019	0.1231	0.8276	0.0697	0.1030	0.6769	0.1732	0.1401	1.2358
0.0398	0.1292	0.3080	0.0162	0.0998	0.1626	0.1610	0.1797	0.8958
0.0962	0.1149	0.8368	0.0715	0.1031	0.6930	0.1578	0.1403	1.1251
0.0179	0.1483	0.1207	0.0082	0.1100	0.0742	0.1235	0.1726	0.7155
0.0687	0.1254	0.5480	0.0738	0.0957	0.7710	0.1289	0.1256	1.0260
0.0444	0.1316	0.3374	0.0709	0.1044	0.6794	0.1246	0.1218	1.0229
0.0689	0.0996	0.6918	0.0659	0.0808	0.8156	0.1403	0.1117	1.2560
0.1131	0.1116	1.0141	0.0761	0.0976	0.7797	0.2017	0.1337	1.5081
0.0664	0.1108	0.5991	0.0549	0.0915	0.5995	0.1156	0.1204	0.9603
0.1164	0.1090	1.0679	0.0791	0.0954	0.8288	0.1773	0.1290	1.3743

0.1257	0.0959	1.3103	0.1028	0.0797	1.2909	0.2342	0.1108	2.1136
0.1334	0.1082	1.2328	0.0946	0.0890	1.0632	0.1665	0.1100	1.5133
0.1109	0.1157	0.9589	0.0684	0.0923	0.7412	0.2074	0.1284	1.6152
0.1277	0.1425	0.8965	0.0960	0.1051	0.9133	0.1413	0.1262	1.1192
0.0754	0.1217	0.6196	0.0585	0.0959	0.6097	0.1764	0.1389	1.2701
0.0255	0.1340	0.1904	0.0318	0.0980	0.3242	0.1048	0.1294	0.8101
0.0447	0.1154	0.3875	0.0444	0.0943	0.4712	0.1275	0.1292	0.9869
0.1116	0.1220	0.9142	0.0488	0.0972	0.5019	0.2151	0.1663	1.2934
0.1037	0.1415	0.7329	0.0759	0.1088	0.6976	0.1504	0.1337	1.1248
0.0940	0.1108	0.8484	0.0663	0.0948	0.6998	0.1916	0.1482	1.2929
0.1176	0.1263	0.9307	0.0832	0.1089	0.7643	0.1820	0.1576	1.1550
0.0242	0.1118	0.2163	-0.0029	0.0921	-0.0314	0.0998	0.1541	0.6478
0.0609	0.0994	0.6120	0.0526	0.0840	0.6268	0.1060	0.1083	0.9785
0.0542	0.1123	0.4823	0.0455	0.0915	0.4975	0.0967	0.1243	0.7779
0.1058	0.0962	1.0998	0.0771	0.0865	0.8904	0.1602	0.1086	1.4754
0.1082	0.1518	0.7128	0.0494	0.1086	0.4550	0.2299	0.1971	1.1664
0.0887	0.1078	0.8233	0.0635	0.0911	0.6974	0.1374	0.1252	1.0978
0.1140	0.1218	0.9361	0.0711	0.1038	0.6855	0.1754	0.1495	1.1728
0.1218	0.1275	0.9558	0.0891	0.1008	0.8837	0.1610	0.1277	1.2603
0.1113	0.1159	0.9599	0.0724	0.0957	0.7561	0.1622	0.1285	1.2623
0.0677	0.1230	0.5502	0.0399	0.0978	0.4078	0.1132	0.1375	0.8232
0.0337	0.1334	0.2526	0.0189	0.1055	0.1795	0.1341	0.1692	0.7925
0.1406	0.1183	1.1892	0.1064	0.0964	1.1031	0.1954	0.1280	1.5261
0.0981	0.1228	0.7991	0.0747	0.0949	0.7873	0.1635	0.1290	1.2679
0.0898	0.1244	0.7217	0.0855	0.0944	0.9062	0.1491	0.1193	1.2502
0.0708	0.1170	0.6054	0.0567	0.0973	0.5822	0.1337	0.1266	1.0565
0.0912	0.1176	0.7759	0.0713	0.0944	0.7545	0.1612	0.1303	1.2368
0.0930	0.1028	0.9040	0.0945	0.0855	1.1041	0.1415	0.0990	1.4283
0.0932	0.1244	0.7491	0.0842	0.1001	0.8419	0.1269	0.1225	1.0358
0.0452	0.1282	0.3525	0.0111	0.0985	0.1123	0.1201	0.1341	0.8951
0.1363	0.1107	1.2315	0.0995	0.0911	1.0922	0.2199	0.1232	1.7842
0.0422	0.1327	0.3181	0.0114	0.1067	0.1071	0.1159	0.1753	0.6613
0.0666	0.1106	0.6022	0.0491	0.0942	0.5210	0.1512	0.1401	1.0795
0.1452	0.1277	1.1369	0.1176	0.1027	1.1451	0.1592	0.1204	1.3218
0.1041	0.1233	0.8443	0.0581	0.0990	0.5872	0.1606	0.1452	1.1063
0.0900	0.1175	0.7663	0.0772	0.0981	0.7869	0.2155	0.1368	1.5755
0.0985	0.1068	0.9220	0.0621	0.0837	0.7424	0.1535	0.1218	1.2604
0.0729	0.1292	0.5642	0.0361	0.1022	0.3527	0.1574	0.1600	0.9838
0.1196	0.1043	1.1466	0.0876	0.0934	0.9385	0.1802	0.1169	1.5414
0.0676	0.1084	0.6235	0.0603	0.0879	0.6858	0.1341	0.1262	1.0623
0.1075	0.0984	1.0930	0.0811	0.0843	0.9623	0.1416	0.1073	1.3191

0.0904	0.1244	0.7266	0.0577	0.1000	0.5768	0.2026	0.1681	1.2054
0.1349	0.1271	1.0614	0.0736	0.0980	0.7517	0.2327	0.1546	1.5057
0.0870	0.0875	0.9937	0.0650	0.0772	0.8427	0.1669	0.1138	1.4676
0.0599	0.1194	0.5017	0.0305	0.0921	0.3314	0.1789	0.1481	1.2073
0.0602	0.1136	0.5298	0.0393	0.0980	0.4006	0.1356	0.1423	0.9532
0.0533	0.0988	0.5392	0.0459	0.0820	0.5600	0.1310	0.1254	1.0444
0.0760	0.1083	0.7022	0.0561	0.0908	0.6176	0.1198	0.1221	0.9814
0.0415	0.0988	0.4205	0.0403	0.0899	0.4480	0.1210	0.1302	0.9290
0.0848	0.1120	0.7575	0.0706	0.0963	0.7333	0.1621	0.1370	1.1835
0.1072	0.1323	0.8099	0.0636	0.1082	0.5880	0.2018	0.1750	1.1527
0.0458	0.1235	0.3708	0.0386	0.0904	0.4272	0.1185	0.1645	0.7206
0.0712	0.1400	0.5083	0.0500	0.1100	0.4547	0.1250	0.1445	0.8651
0.0980	0.1000	0.9804	0.0639	0.0881	0.7261	0.2096	0.1411	1.4859
0.0927	0.1391	0.6666	0.0281	0.1077	0.2613	0.1868	0.1720	1.0859
0.0986	0.1161	0.8494	0.0776	0.0979	0.7928	0.1720	0.1301	1.3215
0.1092	0.1040	1.0499	0.0837	0.0850	0.9856	0.1684	0.1125	1.4967
0.0877	0.1057	0.8293	0.0757	0.0925	0.8184	0.1390	0.1158	1.2008
0.1125	0.1267	0.8879	0.1155	0.1053	1.0966	0.2373	0.1428	1.6621
0.0734	0.1281	0.5730	0.0383	0.1015	0.3768	0.1795	0.1628	1.1028
0.1400	0.1062	1.3177	0.0975	0.0920	1.0598	0.1821	0.1166	1.5623
0.0859	0.1240	0.6925	0.0422	0.0982	0.4303	0.1320	0.1379	0.9572
0.0840	0.1097	0.7658	0.0749	0.0946	0.7921	0.1502	0.1227	1.2241
0.1095	0.1131	0.9683	0.0465	0.0885	0.5259	0.1823	0.1334	1.3664
0.1089	0.1042	1.0457	0.0791	0.0902	0.8773	0.1804	0.1228	1.4686
0.0623	0.1281	0.4864	0.0476	0.0988	0.4812	0.1707	0.1574	1.0850
0.0648	0.1101	0.5887	0.0687	0.0936	0.7338	0.1329	0.1097	1.2117
0.1175	0.1045	1.1238	0.1021	0.0899	1.1353	0.1550	0.1072	1.4464
0.0868	0.1152	0.7534	0.0290	0.0966	0.3003	0.1599	0.1559	1.0257
0.0941	0.1246	0.7553	0.0656	0.0992	0.6618	0.1587	0.1397	1.1358
0.1201	0.1164	1.0312	0.0796	0.0935	0.8515	0.1803	0.1310	1.3769
0.1382	0.1108	1.2468	0.1185	0.0898	1.3194	0.1826	0.1086	1.6809
0.0867	0.1302	0.6663	0.0629	0.1045	0.6022	0.2563	0.1703	1.5051
0.1018	0.1266	0.8043	0.0701	0.0964	0.7278	0.1727	0.1467	1.1772
0.0941	0.1178	0.7988	0.0439	0.0913	0.4806	0.1900	0.1467	1.2946
0.0688	0.1150	0.5983	0.0570	0.0972	0.5867	0.1148	0.1288	0.8916
0.0787	0.1080	0.7291	0.0538	0.0910	0.5918	0.2053	0.1522	1.3489
0.0589	0.1273	0.4631	0.0260	0.1052	0.2468	0.1426	0.1357	1.0502
0.0751	0.1186	0.6329	0.0572	0.0996	0.5739	0.1918	0.1530	1.2533
0.0804	0.1186	0.6781	0.0686	0.1018	0.6735	0.1567	0.1443	1.0855
0.0945	0.1049	0.9007	0.0437	0.0862	0.5074	0.1894	0.1402	1.3513
0.0998	0.1217	0.8208	0.0693	0.0920	0.7540	0.1860	0.1405	1.3242

0.1248	0.1285	0.9708	0.0943	0.1006	0.9368	0.1755	0.1335	1.3151
0.1239	0.1028	1.2055	0.1000	0.0889	1.1242	0.1711	0.1124	1.5229
0.1023	0.1079	0.9478	0.1008	0.0908	1.1102	0.1257	0.1000	1.2566
0.0957	0.1133	0.8443	0.0644	0.0973	0.6619	0.1757	0.1515	1.1597
0.0984	0.1139	0.8642	0.0772	0.0945	0.8173	0.1401	0.1139	1.2300
0.0711	0.1015	0.7012	0.0568	0.0895	0.6348	0.1603	0.1249	1.2838
0.1049	0.1300	0.8072	0.0579	0.0971	0.5960	0.1570	0.1456	1.0782
0.0459	0.1176	0.3907	0.0296	0.1005	0.2950	0.1260	0.1509	0.8349
0.0592	0.1079	0.5490	0.0378	0.0947	0.3993	0.1147	0.1235	0.9284
0.0613	0.1294	0.4739	0.0553	0.0978	0.5653	0.1745	0.1445	1.2073
0.1257	0.1412	0.8899	0.0937	0.1089	0.8604	0.1715	0.1404	1.2217
0.0789	0.1216	0.6486	0.0551	0.0911	0.6051	0.1286	0.1276	1.0084
0.0894	0.1216	0.7346	0.0623	0.1020	0.6108	0.1412	0.1302	1.0849
0.0572	0.1153	0.4964	0.0526	0.0997	0.5280	0.1224	0.1314	0.9318
0.0595	0.1340	0.4440	0.0414	0.1122	0.3693	0.1275	0.1413	0.9023
0.0643	0.1140	0.5644	0.0726	0.0976	0.7439	0.1353	0.1230	1.0993
0.0558	0.1247	0.4472	0.0249	0.1002	0.2486	0.1746	0.1783	0.9795
0.0205	0.1070	0.1919	-0.0009	0.0936	-0.0102	0.1207	0.1815	0.6648
0.1460	0.1098	1.3294	0.0916	0.0896	1.0225	0.2588	0.1502	1.7237
0.1116	0.1255	0.8895	0.0626	0.0986	0.6353	0.2007	0.1438	1.3959
0.0690	0.1128	0.6118	0.0736	0.0863	0.8534	0.1511	0.1080	1.3988
0.0593	0.1290	0.4594	0.0706	0.1015	0.6956	0.1137	0.1238	0.9180
0.0904	0.1212	0.7462	0.0683	0.0938	0.7281	0.1357	0.1191	1.1392
0.1491	0.1130	1.3203	0.1075	0.0885	1.2150	0.2268	0.1275	1.7781
0.0926	0.1196	0.7738	0.0494	0.0941	0.5256	0.1616	0.1281	1.2618
0.0862	0.1075	0.8019	0.0858	0.0932	0.9204	0.2030	0.1335	1.5207
0.0795	0.1536	0.5177	0.0353	0.1219	0.2893	0.1420	0.1842	0.7707
0.0667	0.0988	0.6755	0.0711	0.0881	0.8068	0.1416	0.1170	1.2108
0.1218	0.1105	1.1029	0.0895	0.0882	1.0145	0.1712	0.1223	1.3999