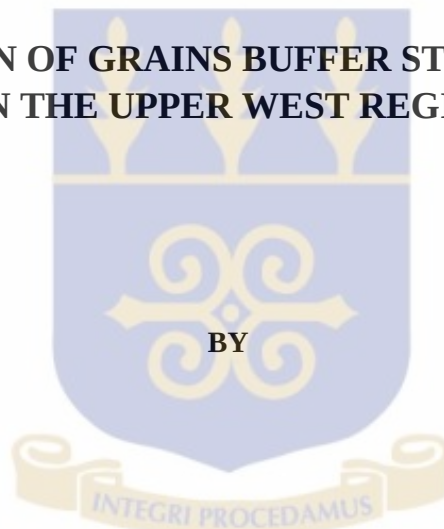


UNIVERSITY OF GHANA



OPTIMISATION OF GRAINS BUFFER STOCK FACILITY LOCATIONS IN THE UPPER WEST REGION OF GHANA



**McCLEAN CONSTANT KOKU AGBALENYO
(10395931)**

**THIS THESIS IS SUBMITTED TO THE UNIVERSITY OF GHANA, LEGON
IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE AWARD OF
MPHIL IN OPERATIONS MANAGEMENT DEGREE**

OCTOBER, 2014

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DECLARATION

I do hereby declare that this work is the result of my own research and has not been presented by anyone for any academic award in this or any other university. All references used in the work have been fully acknowledged.

I bear sole responsibility for any shortcomings.

.....
McClellan Constant Koku Agbalenyo

.....
Date



CERTIFICATION

I hereby certify that this thesis was supervised in accordance with procedures laid down by this University.

Dr Francis Yaw Banuro
SUPERVISOR

Date



Dr Anthony Afful-Dadzie
CO-SUPERVISOR

Date

DEDICATION

Dedicated to the five women who helped to locate me at an optimal level in academia: Aku (my mum), Tornyeli, Beatrice, Mawutor and Celestina; not forgetting Gabriel and all the Boys.



ACKNOWLEDGEMENTS

As is common with projects such as this, there are so many people who contributed directly or indirectly to the success of this thesis as well as to my studies at the University of Ghana, Legon. First and foremost, I glorify God almighty because “unless the Lord builds the house, its builders labour in vain...” (Psalm 127:1). I also owe so much to that wonderful family of mine who have been an ever flowing source of solidarity and encouragement. Daavi Aku, Tornyeli, Beatrice, Mawutor, Gabby, Eric, Martin, Nick, and Divine; I salute you all for being there for me. Miss Celestina Akosua Anyomih too deserves a special mention for her long suffering.

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ABSTRACT

This thesis sets out to optimally locate grains buffer stock warehouses amongst the eleven districts of the Upper West region of Ghana. The research conceptualises the logistical gap between food production and its efficient delivery to households as a location problem. Consequently, it discusses the subject of location analysis and its history, examines commercial versus public sector modeling approaches, catalogues location model typologies, and eventually diagnoses the said problem as a network p -median problem. Subsequently, some exact and heuristic algorithms for the p -median model are demonstrated.

The research methodology employs an exact solution approach executed with *Premium Solver 2014-R2*, a spreadsheet software that runs on the Microsoft Excel platform. Instead of using raw distance measures as proxy for travel cost, the research evens out inequalities in road quality by introducing multipliers that act as incentives or penalties on particular road stretches depending on their nature. Another novelty in here is the unconventional use of a spreadsheet package—rather than traditional combinatorial software—as vehicle for executing location problems.

The results are in the form of different solution sets that recommend optimal locations for between one to ten facilities, out of which decision options may be selected depending on available budget. For instance, if a decision is made to open only one buffer stock facility, the solution recommends Nadowli town as the optimal location. Similarly, if it is decided to build two facilities, the optimal sites are Wa and Lambussie; whereas Wa, Jirapa and Tumu should be opted for in case of three warehouses. Bearing in mind that each solution set is unique, and that the cost of building facilities and that of constructing networks are inversely related, it is recommended that long term plans be factored into the choice of any location-network configuration. Anything to the contrary is tantamount to making decisions based on myopic heuristics, which the data analysis proves as being suboptimal. Also, this work raises some pertinent issues about the classification of location models and proposes a new categorisation concept called the *location-network hierarchy* framework.

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ABBREVIATIONS

FLND	-	Facility location-network design
GFDC	-	Ghana Food Distribution Corporation
GGC	-	Ghana Grains Council
LBC	-	Licensed Buying Company
MoFA	-	Ministry of Food and Agriculture
NAFCO	-	National Food Buffer Stock Company
ND	-	Network design
UN	-	United Nations
UWR	-	Upper West Region

CHAPTER 1: INTRODUCTION

1.1 Background to the Study

The right to food is one of the most inalienable human rights. However, United Nations (UN) estimates indicate there are 842 million people around the globe who are malnourished (FAO, IFAD, & WFP, 2013), a phenomenon that prompts Clover (2003) to view the right to food as the one human right that has been violated with the most impunity. As a UN member country, Ghana does not only share the burden of hunger with the rest of the world, but also bears a piece of the responsibility for minimising food insecurity.

Ghana has attempted solving the hunger problem by largely encouraging abundant food production. This is evidenced by the following three policy initiatives the Ministry of Food and Agriculture (MoFA) has been implementing since 2009 (IFPRI, 2013):

- Increased agricultural mechanisation through the provision of subsidised tractors.
- Fertiliser subsidy programme that absorbs about 36% of the retail price of four widely used fertiliser types.
- Block farm initiative that brings together several farmers into a large production area and provides them with extension services and credit in the form of mechanisation services, certified seed, subsidised fertiliser, and pesticides.

It is sad to note, though, that postharvest losses are probably consuming a good chunk of whatever is being produced. The UN says one-third of all food produced for human consumption (1.3 billion tonnes) is wasted, a statistic the organisation rightly characterises as representing a missed opportunity to improve global food security in a world where one-in-eight is hungry (FAO, 2013). Unfortunately, the situation in Ghana appears to be in tandem with this rather bleak global assessment, as MoFA estimates that over 35% of maize

produced in Ghana, for example, is lost along the postharvest value chain through storage and distribution among other factors (MoFA, 2010).

As a case in point, Ghana's total domestic production of maize in 2010 amounted to 1,871,700 metric tonnes, whereas local demand was 1,561,100 metric tonnes; resulting in a surplus of 310,600 metric tonnes (UNDP-Ghana & NDPC/GOG, 2012). A bigger picture of the production and surplus/deficit levels of some of Ghana's foremost staple foods are shown below:

Staple Food Crop	Domestic Production (‘000 metric tons)			Surplus/Deficit (‘000 metric tons)		
	2008	2009	2010	2008	2009	2010
Maize	1,470.1	1,619.6	1,871.7	65.6	145.6	310.6
Millet	193.8	245.6	219.0	145.2	189.6	165.8
Sorghum	331.0	350.5	324.2	276.3	292.9	269.9
Rice (milled)	301.9	391.4	491.6	-403.8	-372.2	-335.5

Table 1.1: Domestic Production of Staple Food Grain Cereals from 2008 to 2010
(Source: UNDP-Ghana & NDPC/GOG, 2012, p. 22)

An analytical look at Table 1.1 makes things appear on face value as though Ghana is self-sufficient in the production of all the listed grain cereals, except rice. In the case of maize for example, the 2010 figures indicate an improvement on the situation about a decade earlier when Ghana was said to be 99% self-sufficient in maize production (Nyateng & Asuming-Bempong, 2003).

Clearly, Ghana's problem with food insufficiency when it comes to grain cereals is not so much a question of inadequate production, but largely of what happens to the produce, postharvest. Put differently, even though Ghana produces more maize, millet and sorghum than it needs (see Table 1.1), if there are no mechanisms in place to store surplus stocks so as

to release them into the market at appropriate times, the produce may largely go waste or get exported only for the population to become exposed to hunger later. Indeed, Armah & Asante (2006) points out that given the extent of Ghana's self-sufficiency in food production, any difficulties with food supply may stem from inadequate storage. Furthermore, the above paper is of the opinion that, in the absence of a nationally coordinated food storage programme that is necessary to help bridge the gap between food production and its supply, the task falls to private sector actors such as farmers and traders.

However, data collected by Armah & Asante (2006), again, indicate that traders did not see the storage of maize (and probably grain cereals in general) from times of glut in order to sell in the lean season a worthwhile business because it ties down capital. Worse still, farmers too cannot be relied upon to be the ones to store excess grain cereals from the abundant season in order to supply the nation's needs in the lean season. The evidence lies in the stark reality illustrated by Quaye (2008) to the effect that staple foods produced by subsistence farmer households in northern Ghana, for example, lasts for seven months on the average, beyond which families have to come up with some sort of coping mechanism. Specifically, Quaye (2008, p.334) observes that "although farmers in these regions [i.e. northern Ghana] cultivate purposely for household consumption and sell the surplus, food was not available throughout the year in the farmer households interviewed." Most of these farmers are said to be unenthusiastic about storing grain cereals across seasons due to future price unpredictability (Armah & Asante, 2006).

In spite of these dire realities, Ghana is reported to be making good progress towards the attainment of UN Millennium Development Goal No. 1, which seeks to halve the number of hungry people by the year 2015 (UNDP-Ghana & NDPC/GOG, 2012). The positive trend is attributed to a continual improvement in the country's food supply, which in turn is partially

credited to the creation of the National Food Buffer Stock Company (NAFCO) to hold food security buffer stocks and intervene in the market to shore up supplies, forestall shortages, and protect farmer incomes by ensuring price stability at all times (UNDP-Ghana & NDPC/GOG, 2012).

Given the perennial food shortage situation described by Quaye (2008) in northern Ghana, one would have thought that residents of these three regions would have been some of the first to benefit from NAFCO's activities. As a matter of fact, this research was in the first place inspired by a radio news report that carried the outcry of maize farmers in the Upper West region (UWR) to the effect that NAFCO, which they have known as the buyer of last resort, turned away most of their produce in February 2013 because of a lack of further storage facilities. Having zeroed in on the UWR, it is interesting to note that NAFCO does currently buy produce from the region but does not have even a single warehouse there. Any shortage of grains in the region, therefore, would have to be supplied by private traders, or from NAFCO warehouses in the Brong Ahafo region, either of which is a costlier option.

That the nation really needs the buffer stock arrangement is not at all in doubt. The operations manager of NAFCO once remarked: "in the past [i.e. prior to the formation of NAFCO] it was observed that during periods of glut, neighbouring countries like Burkina Faso, Cote d'Ivoire and Niger purchased these foodstuffs from Ghana, and resold them back to Ghanaians when there was a shortage. The creation of NAFCO by the Ministry of Food and Agriculture was therefore meant to reverse the situation" (Djaisi, 2012). To underscore the need for more grains buffer stock facilities, he added that NAFCO needed to "secure more facilities to be able to execute its mandate fully." Thus, whether the issues are evaluated from the perspective of farmers or that of the consumer public, more grain buffer stock warehouses are needed. Much as the deficit in fulfilling the said mandate of NAFCO is not

limited to the Upper West region, this research intends to focus on the region for reasons explained in the previous paragraph.

1.2 Problem Statement

Much as the Ministry of Food and Agriculture of the Republic of Ghana has identified food storage and distribution as bottlenecks in the nation's efforts at achieving food sufficiency, and has gone ahead to create commodity buffer stocks to help correct the situation, the initiative has yet to yield the desired dividends for residents of the Upper West region.

The specific challenge is for NAFCO to find the best locations for its buffer stock warehouses across the region, such that a minimal investment would yield the maximum possible access for all residents of the region in need of the company's services. The decision issues involved is succinctly echoed by Owen & Daskin (1998, p. 424) thus:

The development and acquisition of a new facility is typically a costly, time-sensitive project. Before a facility can be purchased or constructed, good locations must be identified, appropriate facility capacity specifications must be determined, and large amounts of capital must be allocated. While the objectives driving a facility location decision depend on the firm or government agency, the high costs associated with this process make almost any location project a long-term investment. Thus, facilities which are located today are expected to remain in operation for an extended time. Environmental changes during the facility's lifetime can drastically alter the appeal of a particular site, turning today's optimal location into tomorrow's investment blunder. Determining the best locations for new facilities is thus an important strategic challenge.

The issue at stake has to do with accurately anticipating production and consumption levels within the Upper West region, building appropriately-sized storage facilities at the right

locations to hold enough inventories for the region, and then implicitly determining what levels of stock the region should relinquish or ship off to other localities.

While attempting to find optimal locations for grains buffer stock facilities, an eye ought to be kept on the underlying distribution network, as transportation constitutes one of the key elements in the physical distribution function (Jayaraman, 1998). Fact is that the quality of the various roads will affect which locations a customer might be more willing to visit in order to receive service (Cocking, Flessa, & Reinelt, 2012). Thus, the nature of roads linking the various communities in the region will be observed and factored into the process of determining if a NAFCO warehouse should be placed at one location or another; though the formulation will not go as far as to model road network design as an endogenous part of the mathematical problem to be solved.

In a nutshell, this research seeks to analyse the supply system of maize and other grain cereals in the Upper West Region of Ghana and to employ the use of a mathematical model that recommends locations for buffer stock facilities relative to an existing transportation network. Additionally, investment trade-offs between opening facilities and that of improving upon the road network will be examined.

1.3 Specific Objectives of the Study

In a bid to ultimately find optimal locations for grain cereals in the Upper West Region, the under-listed specific objectives will be achieved:

1. To assess the mechanisms by which Ghana runs its grains buffer stock programme, especially in the Upper West region of the country.

2. To *review* different location models and solution approaches in order to adopt the most suitable for effectively appraising, modelling and solving the problem of finding optimal locations for cereal buffer stock warehouse facilities within the study area.
3. To draw conclusions from the resulting location-network configurations, *formulate* recommendations on logistics issues in mitigating food insecurity, and advance the frontiers of knowledge in the computational and practical aspects of location analysis.

1.4 **Significance of the Study**

There appears not to be in existence as yet any comprehensive plans for tackling postharvest losses in Ghana through optimisation of storage facilities. The current research seeks to fill this gap, and to subsequently analyse possible impacts the resulting location configuration(s) might have on the road network and travel route options.

It is also significant that though location science is not a new subject, not much of it appears to have been applied within the context of Ghana apart from its use to model health facility sites in the Eastern region of the country (Murawski & Church, 2009; Oppong, 1996). Even more noteworthy is the fact that no evidence has been discovered by this study of anyone ever using optimisation to model the locations of cereal grains buffer stock facilities anywhere in the world. Hopefully, the findings of this research should become a cardinal reference document in this direction for policy makers and future researchers, academic or otherwise. Also, the approach for the current work could become a template for solving real-life location problems in Ghana and beyond.

Should NAFCO or MoFA no longer be interested in establishing the buffer stock warehouses in the region, private businesses which are into trading cereals, or indeed any other

universally demanded products in the region, could make use of the procedures or recommendations advanced herein to locate their facilities or trade hubs.

1.5 Methodology and Data Analysis

Secondary data will be used to estimate demand and travel distances. Once these data are extracted and customised, an exact solution approach based on Microsoft Excel will be used to execute a solution, after which several scenario analysis will be performed to produce a band of solutions that will enable planners and policy makers evaluate the problem from various angles depending on available budget.

Though the research appears to focus on only the demand side of the supply-storage-demand relationship, it is assumed that the supply side is implicitly catered for due to the fact that supply sources and demand destinations are the same. As a matter of fact, some of the people who sell produce to the buffer stock centres could return to make purchases when their own stocks run out (Quaye, 2008).

1.6 How the Research is Organised

As explained earlier, a combination of literature review and empirical analysis will be employed in the study. The research flow proceeds as follows:

- **Chapter 1 (Introduction):** This chapter seeks to set the tone for the research, demonstrate its motivation and relevance, and briefly explains how the work is organised.
- **Chapter 2 (Context of the Study):** Chapter 2 will discuss the peculiar circumstances of the study area, shed light on the grains buffer stock scheme in Ghana, and also

explain the workings of NAFCO; all in a bid to give a broader background to the project.

- **Chapter 3 (Literature Review):** Relevant existing literature will be presented in Chapter 3. The approach is to first take a general view on location typology to help place the specific research problem into context. Next, the dichotomy between private versus public sector location modelling will be drawn, and a specification of the problem will be diagnosed. Lastly, solution approaches for the specific problem type will be discussed.
- **Chapter 4 (Research Methodology):** The research methodology segment will outline the steps for utilising how the secondary data to be collected will be handled in order to accurately quantify problem parameters. Also the specific solution method will be described.
- **Chapter 5 (Data Presentation and Analyses):** In this chapter, a step-by-step procedure of solving the problem will be outlined, while analysing any data generated along the way. Optimal locations for the NAFCO warehouses will be generated alongside an analysis of how a set of optimal locations might affect the transportation network.
- **Chapter 6 (Discussion and Conclusion):** When all is said and done, inferences and conclusions will be drawn and stated, together with recommendations that would aim to contribute knowledge to the subject under discussion, and propose areas for further research as well.

CHAPTER 2: CONTEXT OF THE STUDY

This chapter contextualises the role of food buffer stock schemes in fighting hunger and achieving food security; and by extension takes a close look at the structure and workings of the National Food Buffer Stock Company (NAFCO). The discourse first profiles the UWR and touches on peculiar food security issues faced by the region, which is why a research to locate buffer stock warehouses in such a place should be considered an exigency. The chapter ends with a brief look at some alternative measures of attacking food insecurity besides the buffer stock warehousing approach offered by NAFCO.

2.1 The Upper West Region in Context

Being one of ten administrative regions of Ghana, the Upper West Region (UWR) is located in the North-Western corner of the country, and is itself subdivided into eleven districts. The region is bounded by the Upper East Region to the East, Northern Region to the South while its Northern and Western limits touches Burkina Faso.

UWR holds the unenviable accolade of being the poorest and the least developed of all the ten administrative regions of Ghana (Garba, 2013). The Ghana Statistical Service estimates that 88% of the region's residents fall below the poverty line (IFAD, 2012); and its annual per capita income of GH¢130.00 compares so unfavourably with the national average of GH¢397.00 (GSS, 2008).



Figure 2.1: Map of Upper West Region (Source: Verizon.net; GSS)

The 2010 Population and Housing Census reports the UWR as the country's least populated region with a total of 702,110 inhabitants, 84% of whom live in rural areas. The region happens to be the seventh largest, contributes 2.8 percent of Ghana's population, and occupies an area of 18,476 square kilometres which translates to 7.7% of the country's total land area. The population density stands at 31.2 persons per square kilometre (GSS, 2013).

Observing that the essence of literacy is to help improve socio-economic development, the Ghana Statistical Service finds that 59.5% of Upper Westerners are non-literate, as compared to a national figure of 25.9%.

KPMG's *Doing Business in Ghana* report for 2012 says the region's economy is predominantly agrarian, with more than 80% the population being engaged in the subsistence production of such staple foods as maize, millet, rice, yams, beans, groundnuts, and cowpea.

The main cash crop cultivated here is cotton, of which 40% of the country's total output originates from this region. The region is endowed with shea nut which grows in the wild but rakes in considerable cash for residents. Local industries dominant in the Upper West include pottery, shea butter processing, groundnut oil extraction, soap making, and weaving (KPMG, 2012).

One positive story of socio-economic development emerging from the region in recent years is the improvement of access to electricity from three percent (Sackey, 2005) to 30.9 (GSS, 2013). Also, the illiteracy rate has dropped from 75.5% (Agyei-Mensah, 2006; Luginaah, 2008) to 59.5% (GSS, 2013).

This positive outlook notwithstanding, the reality is that Upper Westerners are confronted with socio-economic challenges that are generally worse than national averages. It is against the background of such dire socio-economic realities that we are of the conviction that the region's food security situation ought to be tackled very urgently. The fact that the region (alongside the rest of Northern Ghana) has a declining rainfall pattern that ensures only one farming season in a year (as compared to two in the South) whereas over 80% of the population derive their livelihood from rain-fed agriculture makes the issue of food storage all the more expedient.

2.2 The Buffer Stock Concept and its History in Ghana

As expressed in the words of Commandeur (2013, p.20), the aim of "cereal and grain banks, is to both improve national or local food security and to mitigate price fluctuations." Wiggins & Keats (2009) expands the scope a bit, postulating three reasons for which cereal stocks may be held publically. These are:

- to ensure continued supply on the market,
- to induce price stability, and
- to create a ready backup stock that can quickly be deployed in case of emergency food aid needs.

To further emphasise on the phenomenon of nations seeking to use buffer stock schemes to achieve economic goals, von Braun (2009, p.6) remarks that “many governments have also used public buffer stocks, price regulation, and trade policy instruments to prevent market volatility and to stabilize prices.” Indeed, as far back as in ancient Egypt the Bible records that Joseph, on behalf of the King of the empire, bought excess produce from farmers during seven years of bumper harvest which he then stored and later re-sold to the populace when food shortage occurred and persisted for the next seven years.

Ghana’s first national effort at food storage and distribution as a way of ensuring food security came in 1971 when the Food Marketing Corporation and the Food Research Institute were merged into the Ghana Food Distribution Corporation (GFDC). The new state-owned enterprise was tasked to buy agricultural produce from farmers and sell them to local and international markets, though it was unable to achieve the latter objective due to lack of capacity (Dzisi, Mensah, & Oduro, 1998; Uddin & Tsamenyi, 2005). Under the Acheampong military regime’s “Operation Feed Yourself” initiative, the GFDC’s scope of operation was expanded to include the monitoring of prices and the prevention of commodity price spikes for that matter. In response to the 1983 drought the corporation’s mandate was further widened for it to adopt a more holistic approach to food security. Uddin & Tsamenyi (2005) found that as the enterprise grew, so did political interference in its affairs increase. Eventually, well-intended administrative control mechanisms waxed into bureaucratic red tape, turning an otherwise profit-making venture into an unwieldy parastatal that run at a loss.

As the losses mounted and the drain on national resources became unbearable, the government eventually officially closed down the GFDC in 2003.

2.3 NAFCO and its Mandate

The mandate of the National Food Buffer Stock Organisation (NAFCO), basically, is to buy grain cereals from farmers during times of bumper harvest and store them for sale in the lean season; thereby guaranteeing higher incomes for producers and lower prices for consumers (IFPRI, 2013).

A buffer stock scheme seeks to stabilise prices, either in an entire economy, or in an individual commodity market. The workings of a typical commodity buffer stock scheme are such that the intervening entity (usually a government parastatal) first sets *floor* and *ceiling* prices for the given commodity. Upon closely monitoring price movements and realising that the market price is about to fall below the floor price (usually around harvest seasons) the scheme operator begins to buy up stock so prices will not fall further. Similarly, when the market price approaches a previously determined price ceiling the entity in charge begins to release its stock into the market to help tame prices (IFPRI, 2013).

2.3.1 Detailed Mandate

The above ideas are well reflected in the official mandate upon which NAFCO was incorporated. The full range of the company's mission is (NAFCO, 2013; IFPRI, 2013):

- To buy up excess produce from farmers and store in order to reduce postharvest losses resulting from spoilage due to poor storage, thereby protecting farm incomes.

- To guarantee an assured income to farmers by providing a minimum guaranteed price and a ready market.
- To purchase, sell, preserve and distribute food stuffs.
- To employ a buffer stock mechanism to ensure stability in demand and supply.
- To expand the demand for food grown in Ghana by selling to state institutions such as the military, schools, hospitals, prisons etc.
- To manage government's emergency food security
- To facilitate the export of excess stock
- To carry out such other activities that are incidental to the attainment of the above objectives or such other duties as may from time to time be assigned by the Minister of Food and Agriculture.

The importance of NAFCO would be appreciated even the more in light of an observation by its operations manager to the effect that, prior to the company's formation, traders from neighbouring countries sometimes purchased excess foodstuffs from Ghana during the harvest seasons only to re-sell them to Ghanaians in times of shortage (Djaisi, 2012).

2.3.2 Structure of NAFCO

NAFCO was incorporated on March 11, 2010 with registration number CA-72,140 as a business wholly owned by the Government of Ghana in accordance with the Companies Code of Ghana 1963, Act 179. The company is managed by a chief executive officer (CEO) supervised by an eight-member board. At the onset, NAFCO was endowed with *GHC*15,000,000 seed money from the Government of Ghana and its donor partners; plus a handover of those assets of the defunct Ghana Food Distribution Corporation that had not been divested (IFPRI, 2013).

In addition, regional directors of the Ministry of Food and Agriculture double up as NAFCO managers in their respective territories on behalf of the company's CEO and national secretariat which is based in Accra. At present NAFCO, warehouses exist in only three of Ghana's ten regions; and this does not include our study area, the Upper West region. According to the Upper West regional representative of NAFCO, the organisation does buy produce in the region but then transports all of it to the Brong Ahafo region since that is where the nearest warehouses are located. This fact makes the current research even more imperative.

2.3.3 NAFCO's Operational Model

At the moment NAFCO interfaces with mainly small-holder farmers in rural areas through its 75 Licensed Buying Companies (LBCs). The LBCs travel to farm gates especially around the harvest season when the farmers need their services most in order to ensure timely access to market and to prevent spoilage. These stocks are then sold to NAFCO which stores them for sale usually in the off-harvest season (NAFCO, 2013). Figure 2.2 illustrates the transactional interaction at NAFCO.



Figure 2.2: NAFCO's Operational Model (Source: Adapted from NAFCO, 2013)

Although the organisation is mandated to trade in food crops in general, it has now restricted itself to cereal grains such as maize, paddy rice and soya beans, with a view to widening the scope in future to cover other food crops (NAFCO, 2013). The main reason for this temporary self-imposed restriction is that these three crops are the predominant staple foods

in Ghana and should naturally be tackled as they are produced in comparatively larger volumes. Additionally, IFPRI (2013) observed that out of lessons learned from the demise of the erstwhile GFDC, NAFCO deliberately chose to avoid trading in produce that have short shelf life; and hence the organisation's concentration on grain cereals. This way, the mechanism of carrying savings from the bumper season to lean periods could then be achieved.

In an exercise to review the impact of NAFCO, IFPRI (2013) breaks down the activities of the organisation into three, namely *inputs*, *outputs* and *outcomes*. A simplified adaptation of their proposed flow model now follows:

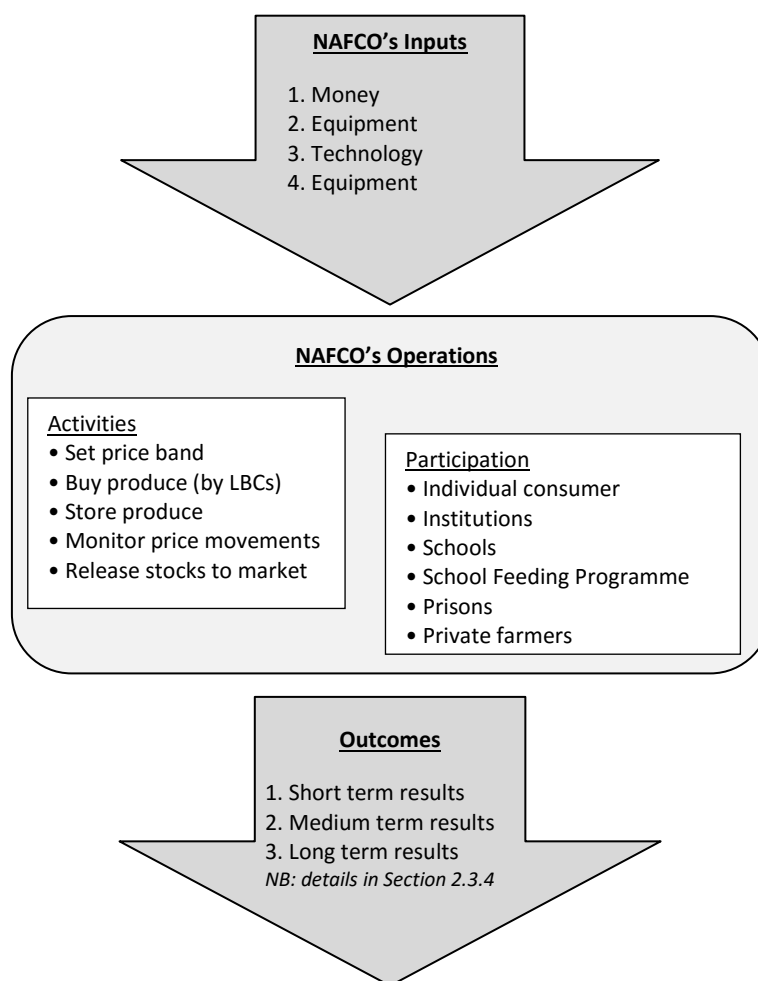


Figure 2.3: NAFCO Impact Pathway (Source: adapted from IFPRI, 2013)

2.3.4 Outcome Specifics of NAFCO Operations

The outcome part of Figure 2.3 is further detailed by IFPRI (2013), as explained below.

While the lists are adopted from IFPRI (2013), explanatory notes are ours.

Short term results of NAFCO's interventions include:

- Stabilise prices of food produce
- Stabilise food grain supplies
- Maintain minimum price of produce
- Avoid price hikes
- Increase in acreage

In a way, the above could be described as the primary aims of the NAFCO project. Already, there is a strong belief that these goals are being attained to a large extent (Commandeur, 2013; Duffuor, 2011; UNDP-Ghana & NDPC/GOG, 2012) though some think more research is needed for confirmation (IFPRI, 2013).

Medium term aims of NAFCO are to:

- Create employment
- Provide incentive to farmers
- Supply raw materials
- Technology adoption

Though NAFCO itself has small staff strength and mostly depend on employees of the Ministry of Food and Agriculture, the organisation's employment creation agenda might have been on track given that the 75 Licensed Buying Companies each offers some employment. Up to 800 people are said to be employed by 52 out of the 75 LBCs (data was available for only the 52) as at 2013 (IFPRI, 2013). On the issue of technology adoption, one simple technology tool that would be useful for NAFCO's agents is an instrument for testing the

moisture content of produce they are buying. Concerning the supply of inputs, it must be noted that one of the reasons why the GFDC could not survive was the inclusion of supply of machinery and other input to farmers in its mandate (Dzisi et al., 1998).

Beyond the above, there are also **long term** results that are intended in the design and set-up of NAFCO. These include:

- Earn foreign exchange
- Improve emergency food reserves
- Stable supply of raw material for agro processing industries
- Job creation
- Expansion of NAFCO (increase volume of purchases)

2.4 **Some Alternatives to the NAFCO Approach**

The approach adopted by both NAFCO and its predecessor organisation, the Ghana Food Distribution Corporation, is to buy up produce which they then store and later sell off in times of scarcity. An alternative approach is for farmer groups to operate common warehouses. An example of such a scheme is the Ghana Inventory Credit Project which was initiated by the government in 1989 (Wiggins & Keats, 2009). It involved 20 to 50 small-scale farmer groups forming cooperatives that receive soft loans from the government for onward distribution amongst themselves to be invested into their individual grain farms. Once the cereal grains are harvested, group members pool their produce into a common warehouse for which the group has joint responsibility for its preservation treatment, safety, and subsequent sale. Net income from the sales is then distributed in proportion with each member's input. Wiggins & Keats (2009) reports that so much success was achieved by this scheme that the Agricultural Development Bank sought to replicate it across the country.

Today the Ghana Grains Council (GGC) runs a similar arrangement whereby actors in the cereal grains value chain (i.e. farmers, aggregators, millers, processors, etc) hand in their cereal grains to certified warehouse operators, where the produce are tested for quality before being stored. When prices become more favourable, the produce are either sold on behalf of the owners, or they themselves would retrieve their share of the stocks and put them on sale (Ghana Grains Council, 2013). The GGC says its scheme smoothens variability in seasonal prices for grains, introduces a trustworthy quality and weighing system, gives depositors a receipt/certificate with which they could acquire credit, and supplies up-to-date market information to its members and the general public.

Other options to the physical storage mechanism adopted by NAFCO, GGC and others have been proposed. These include trade liberalisation so countries experiencing shortage will simply import; finding alternatives for animal feed and industrial uses so more food could be made available for human consumption; and an idea that if some structured information could be shared by international partners, shortages could be foreseen and forestalled through international collaborations (Wiggins & Keats, 2009). Proposals such as these are usually recommended by researchers who feel that buffer stock schemes are too expensive to run (Rashid, Minot, & Msu, 2010).

2.5 **Summary**

The discussion in this chapter points out that the Upper West region of Ghana does need the buffer stock facilities in order to improve the grains supply situation, and that NAFCO is in a position to contribute immensely towards this end if it resourced to achieve its stated mandate. Whereas NAFCO has so far stayed away from most of the temptations that brought

about the demise of its predecessor organisation, the Ghana Food Distribution Corporation, some of the organisations's medium and long term goals listed earlier are likely to push it down the same path. Even though some have proposed other storage and non-storage approaches to ensuring food security, the buffer stock scheme is what Ghana has now and needs to make the most of it.

CHAPTER 3: LITERATURE REVIEW

As already explained, the purpose of this work is to help determine optimal locations for buffer stock warehouse facilities in the Upper West Region of Ghana as a means of making food more readily available, thereby helping to stave off hunger and malnutrition. Chapter 1 outlined three specific objectives to be attained with this research, the second of which states:

To *explore* different location models and solution approaches in order to adopt the most suitable for effectively appraising, modelling and solving the problem of finding optimal locations for cereal buffer stock warehouse facilities in the Upper West region.

In line with the above objective, this chapter explores different location model options that could be applied in solving the dual problem of facility location and its underlying transportation network. This will be achieved by defining and contextualising location science, and then reviewing location modelling typologies with a view to exploring what features could be adopted from the various model formulations to help solve the problem at hand. The fact of having several models reviewed should also help to highlight the dilemma faced by location scientists in deciding which model to choose for a particular problem, as well as to implicitly explain the factors that go into resolving such dilemma.

The chapter first defines location analysis, traces the history of this field of inquiry, and touches on the objectives and criteria a typical location model would seek to achieve among other issues. Furthermore, the special issues that make public sector facility locations especially problematic to model as compared to those in the private sector are discussed. Subsequently, quite a comprehensive catalogue of facility location problems is presented, grouping the various problems appropriately under *static* and *dynamic* models. More attention is paid to static models since they form the foundations for their dynamic

counterparts (De Lotto & Ferrara, 2002). Under static models, continuous, discrete, network, and analytic models are discussed, each (with the exception of analytic models) being illustrated with copious problem examples. The approach adopted here is to catalogue a variety of models so that a selection will be made within the context of what alternative models do or do not offer. After outlining the problem typologies, a summary of the review is presented in the form of a conceptual relationship summing up the material presented.

The literature review rounds up with a justification of why the p -median is the best-fit model for the current problem and goes on to explain some of its solution methods.

3.1 Location Analysis

Facility location problems do arise under a range of circumstances. Applications may include a private sector manufacturer seeking to find a single optimal location for a production plant, a local government authority seeking to find the best place for a landfill site, or a non-governmental organization in search of the right locations for boreholes among a given cluster of communities. Whether a single or multiple facilities are involved, the objective is to find an optimal location or set of locations (Cocking et al., 2012). It is also worth noting that facility location problems need not focus on only the establishment of new facilities, but also the modification of existing ones (Arabani & Farahani, 2012).

Location analysis (ReVelle & Eiselt, 2005) or location science (Farahani, SteadieSeifi, & Asgari, 2010), as understood from the Operations Research perspective, is a decision making tool that involves the modelling, formulation, and solution of problems seeking to help site or locate facilities in some given space (ReVelle & Eiselt, 2005). This definition takes the view that any location analysis can only be conducted in the context of a predefined geographical

space, which in the current study is the Upper West region of the Republic of Ghana. Arabani & Farahani (2012) concurred with the geographical space contextualisation, and even expanded it by defining facility location problems in terms of **space** and **time**. They explain ‘space’ as the planning area where facilities are located; while referring to ‘time’ as the time of location within the framework of a given planning horizon envisaged (i.e. either immediately or at some future time) by the decision maker.

As an illustration, the geographical space inhabited by a facility location problem could be thought of as a network in certain instances, especially given that the current problem encompasses the determination of locations for facilities in the context of a certain transportation network configuration. Tansel, Francis, & Lowe, (1983, p.482) might have had this additional notion in mind when they proffered the following definition for network location modelling: “network location problems occur when ... facilities are to be located on a network. The network of interest may be a road network, an air transport network, a river network, or a network of shipping lanes.”

While networks restrict access to facilities via predetermined links—whether existing or proposed—non-network models assume connection by Euclidian or metropolitan paths among others. Really, network location models constitute just one of four broad categorisations in location analysis. Based jointly on the decision maker’s objective, and the space inhabited by the model as the criterion, ReVelle, Eiselt, & Daskin (2008) lists the four categories as *continuous*, *network*, *discrete*, and *analytic* models. A similar categorization exercise by Arabani & Farahani (2012) puts location models into three groups, omitting analytic models. Using solution space as a basis for their classifications, Nagy & Salhi (2007) and Mladenović, Brimberg, Hansen, & Moreno-Pérez (2007) were similarly silent on analytic models while mentioning the other three. The relatively widespread omission of analytic

models as one of the categories of location problems, is perhaps justified by ReVelle et al.'s (2008) observation that they are rare and largely unexplored.

ReVelle & Eiselt (2005) enhance the location analysis discussion by stating the four main elements that characterise location problems as:

- *Customers* (presumed to be already located at points or on routes)
- *Facilities* to be located
- *Space* in which customers and facilities are located
- *Metric* that indicates cost (distances or times between customers and facilities).

At this point we dare suggest that ‘*time*’ should have been added to the above list as a fifth element, as the planning horizon envisaged for a problem instance may play a key role in how its mathematical model is formulated (Arabani & Farahani, 2012).

Summing up in the words of Chhajed, Francis, & Lowe (1992), location analysis makes use of mathematical models that represent the relationships between the key elements of the location decision factors (i.e. customers/demand, facilities, space, time horizon, and metric for defining objectives), with the aim of aiding decision in choosing new facility locations.

3.2 **The History of Location Analysis**

The human race must have been faced with facility location dilemmas since the beginning of time, but it was not until 1909 AD that Alfred Weber instituted a formal scientific investigation into the field (Owen & Daskin, 1998). It is believed in certain quarters that the science of facility location originated independently from Pierre de Fermat, Battista Cavallieri, and Evangelistica Torricelli (a student of Galileo) in the seventeenth century

(Farahani et al., 2010), but it was Weber's work that really set the ball rolling in what would become known as location analysis, location science, or facility location modelling among other labels.

The mere embryonic status location science had attained at the time is perhaps reflected in the fact that the seminal work by Weber considered how to position only a single warehouse, with a view to minimising the total distance between it and several demand locations. Five decades or so after Weber's research, not much advancement appears to have been made in rolling back the frontiers of knowledge as far as location analysis is concerned. In 1964, however, Hakimi (1964) inspired renewed interest in the study of location modelling by making a publication that sought to locate switching centres in a communications system; and followed this up with another article that sought to locate police stations within a highway network (Hakimi, 1965 as cited in Shankar, Basavarajappa, & Kadadevaramath, 2012). Indeed, Hakimi's work could be described as a watershed moment in facility location modelling as evidenced by one catalogue depicting the history of network location publications up to 1982 (Tansel et al., 1983). The catalogue has it that just two publications predates that of Hakimi's, with 103 occurring from 1964 to 1982. Domschke & Drexl (1985) lists more than 1500 research pieces carried out on location and layout problems. Much as these catalogues may have been disconnected pieces of effort, and therefore not so exhaustive, they do tell a story about how the study of location theory has flourished over the past few decades (Owen & Daskin, 1998). As location science matured, accompanying mathematical formulations too advanced from single-period, deterministic models to dynamic ones that increasingly capture the "complex time and uncertainty characteristics of most real-world problem instances" (Owen & Daskin, 1998, p. 432).

A new chapter in the ever deepening subject of location science is the emergence of text books such as *Discrete Location Theory* edited by Mirchandani & Francis (1990); *Facility Location: A Survey of Applications and Methods* edited by Drezner (1995); *Facility Location: Applications and Theory* edited by Drezner & Hamacher (2002), *Location Theory: A Unified Approach* by Nickel & Puerto (2005); *Facility Location: Concepts, Models, Algorithms and Case Studies* edited by Farahani & Hekmatfar (2009); and *Foundations of Location Analysis* edited by Eiselt & Marianov (2011).

Yet another evidence of location science's advancement is the fact that the American Mathematical Society (AMS) recognises it enough to have created specific codes for location problems. For example, the AMS code 90B80 represents discrete location and assignment problems, while code 90B85 is for continuous location problems (Melo, Nickel, & Saldanha-da-gama, 2009).

Another exciting development has been the discovery that rather than concentrate on just locating facilities, it is sometimes more cost-effective to construct links (as in road network) so that customers from different locations are able to access a common facility, a situation that would have been unthinkable if link construction was ignored (Melkote & Daskin, 2001a). This innovation in location analysis belong to a problem class called *integrated facility location and network design*, and constitutes a huge inspiration for this very research.

In terms of the country-specific context of Ghana, not much location analysis research has been found in the literature apart from the work by Oppong (1996) which turns out to be one of a few classical location analysis researches set in Sub-Saharan Africa. Using the location of healthcare facilities in the Suhum district of Ghana's Eastern region as a case, the research argues that ignoring the impact of the rainy season in deciding model parameters could render an otherwise optimal solution useless once the rains set in and certain roads and paths

become impassable. Over a decade later, Murawski & Church (2009) re-evaluated the Suhum problem with a special focus on road network design rather than facility location. Beyond these, Ghana does not appear to have featured much in location analysis research and applications. It is hoped that the current research will contribute to what Ghana has to offer to facility location science as a field of inquiry.

3.3 Objectives in Location Modelling

A catalogue of factors that shape the typology of a location model would include not only the customers, facility type, geographical space, or distances, but also the objective the decision maker seeks to achieve (ReVelle & Eiselt, 2005). As happens in mathematical programming generally, location analysis requires that some sort of objective function is minimised or maximised. For example, a problem may be modelled to minimise either a sum of transportation costs between customers and facilities or to minimise the total number of new facilities built (Tansel et al., 1983).

Given the specific problem of finding optimal locations for buffer stock warehouses, objective alternatives may include trying to situate facilities as close as possible to their intended customers, to minimise the facility set-up costs, or both. The philosophies that go into deciding which objective or set of objectives to pursue in a location problem are discussed next.

3.3.1 Broad Categorization of Location Objectives

In general terms, there are ‘pull’ and ‘push’ objectives to be considered in facility location analysis, depending on whether the facility in question is *desirable* or *undesirable* (ReVelle

& Eiselt, 2005). The foregoing authors explain desirable facilities as those having ‘pull’ objectives, meaning that the closer these facilities are to customers, the better. In locating a borehole, for instance, the idea will be to have it as close as possible (‘pull’) to its intended clients. For noxious or obnoxious facilities such as landfill sites or nuclear power plants, where nearness is undesirable to the customer, the objective would be to ‘push’ the facility away from human habitation—and towards infinity—as much as possible (Erkut & Neuman, 1989). The caution here though, is that, since the cost of accessing the so-called obnoxious facility (for example, cost of transporting waste to a landfill, or of transmitting power from a nuclear power plant) is directly or indirectly borne by the customer, some kind of trade-off ought to be struck between ‘pull’ and ‘push’ objectives in order to achieve a balance. The way to achieve this in a mathematical model is to introduce upper bounds on distances between desirable facilities and their customers, and some combination of upper and lower bounds on unpleasant facilities (Owen & Daskin, 1998; ReVelle & Eiselt, 2005).

An extension of the idea of balancing facility location objectives leads to what is termed ‘equity objectives’; meaning these objectives attempt to equalise the treatment of all customers (ReVelle & Eiselt, 2005). Indeed, Church & ReVelle (1974) considers the *average* distance between customers and the facility that is closest to them as one of the cardinal objectives in location modelling. According to ReVelle & Eiselt (2005) models that seek to achieve equalisation are constructed such that client-to-facility distances may be equivalent to each other as much as practicable. As average travel cost (i.e. travel distance or travel time) increases for a given location, the relevant facility becomes less accessible, less attractive, and less useful for that matter (Owen & Daskin, 1998). Erkut (1995), Marsh & Schilling (1994), and Eiselt & Larpote (1995) provide further deliberations on equity objectives; and Mandell (1991) demonstrates how to implement these.

It has been observed by Owen & Daskin (1998) that facility location researchers are fond of assuming that customers would naturally opt for the closest facility, an assumption that could be defeated by the fact that customers could look beyond travel cost and consider facility attributes such as size, cleanliness and level of congestion (Ghosh & Craig; Brandeau & Chiu as cited in Owen & Daskin, 1998). Owen & Daskin (1998) actually did generate a model that minimises the total cost comprising of travel cost and the customer's perceived cost of waiting to be served due to queue formation at the facilities. What this means for the case of the buffer stock storage facilities being studied in this research is that since the various warehouses belong to the same state-owned agency and are not expected to be engaged in a competitive relationship, a uniform standard for facility attributes such as efficiency of service has to be ensured in order to discourage customers from opting for facilities other than the ones closest to them.

Farahani et al. (2010) takes the discussion on location objectives or criteria to a higher level by listing energy cost, land use and construction cost, congestion, noise, quality of life, pollution, fossil fuel crisis and tourism as some of the issues that cluster into what they call *environmental* and *social* objectives.

3.3.2 Criterion Specificity

In an attempt to crystalise the categories of objectives or criteria related to location theory, Farahani et al. (2010) itemised the most frequently used objectives or criteria for modelling as follows:

- Minimising the total setup cost.
- Minimising the longest distance from the existing facilities.

- Minimising fixed cost.
- Minimising total annual operating cost.
- Maximising service.
- Minimising average time/ distance travelled.
- Minimising maximum time/ distance travelled.
- Minimising the number of located facilities.
- Maximising responsiveness.

While listing out all the above objectives as the most common in location objective types, Farahani et al. (2010) surveyed 66 published papers in order to find out which criterion is more prevalent in practice. The exercise yielded the following broad groupings:

- Minimisation of cost, time, distance or risk
- Maximisation of profit or availability of service
- Multi-criteria/multi-objective

Of the 66 articles surveyed, there were 48, 7 and 11 that fall in to the above three groupings respectively. Both Farahani et al. (2010) and Tansel et al. (1983) are optimistic that interest in multi-criteria/multi-objective location problems is growing.

3.4 Public Sector versus Private Sector Location Modelling

One way of characterising location models is to categorise them into *private* and *public* sector problems (Revelle et al., 1970). Whereas the decision process of locating private sector facilities seeks to optimise monetary functions such as minimising costs or maximising profits, the issue in public sector facility locations is to optimise access or some other measure of utility for members of the beneficiary population (ReVelle & Eiselt, 2005).

Shariff et al (2012, p. 1000), while speaking specifically about the location of health facilities, made the point that public facility location decisions ought to be made with a view to “minimising social cost (i.e. non-economic, non-monetary cost) or equivalently maximising the benefits of the people.” Furthermore, Revelle et al. (1970, p. 692) summarises the discussion on the difference between the two: “Private sector and public sector location problems are alike in that they share the objective of maximising some measure of utility to the owners while at the same time satisfying constraints on demands and other conditions... They differ in the way that these objectives and constraints are formulated.” Similarly, one other researcher is of the opinion that public sector facilities are the ones whose physical positioning is chosen with no competition in mind; and that the way in which objective functions are formulated constitute the main difference between competitive facility location models, and those of the public sector (Datta, 2012).

Thus, the difference between the two goes beyond the superficial issue of ownership; it launches into the realm of mathematical modelling. For example, while both private and public sector models alike may seek to minimise the average or total customer-facility distance or travel time (minisum objective), only public sector models would usually seek to minimise the longest distance between a facility and its customers (minimax objective). While Harsanyi (1975) and Rawls (1971) recommend minisum and minimax objectives respectively as the most appropriate proxies for accessibility as far as public sector location modelling is concerned, ReVelle & Eiselt (2005) believe the choice between the two is a matter of mere opinion. Before opting for either of these, it is worth considering a further clarification provided by Current, Daskin, & Schilling (2001), to the effect that minisum and minimax objectives correspond with the *efficiency* and *equity* seeking location criteria respectively.

Beyond minisum and minimax objectives, Revelle et al. (1970) mentions a third proxy which has to do with the creation of demand. In this approach, instead of regarding demand as fixed, the focus is on the location, size and number of facilities; so that the more attractive customers find a given facility, the more efficient it is deemed to be. Finer details of exactly how this model works were not clarified by the authors, but we suggest that issues of congestion and queuing might influence the formulation of such a model.

In the midst of this seeming absence of consensus, the expediency of multi-objective modelling comes in handy whenever possible though this comes with some complexities (ReVelle & Eiselt, 2005). Owing to this, (ReVelle & Eiselt, 2005) insist that defining objectives in public sector models is much more difficult than in the private sector. Revelle et al. (1970) takes the opinion that the difficulties in modeling public sector location problems arise from the dilemma in having to justify and quantify whatever surrogate metrics one chooses to use to account for unquantifiable problem parameters. Daskin & Murray (2012) also attribute the said modelling difficulty to the fact that public sector facilities are characterised by the following four complexity issues:

- **Multiple constituents:** The issue of multiple constituents has to do with competing interests of stakeholders to a public sector facility. Brill, Jr (1979, p. 416) explains that “different members of society may not be able to agree on public goals and that if a common goal is accepted there is often disagreement over how to achieve the goal, or even on the criteria for evaluating different mechanisms...” A decision on the best locations for buffer stock warehouses, for instance, would likely be influenced by lobbying on the part of farmer groups and consumers who may both want the warehouses to be closer in order to reduce their access costs, whereas traders who may see the whole project through the lens of competition might campaign for it to be

taken as far away as possible. Still, some communities may insist on hosting the facility because they see it as just another prestigious social amenity.

- **Multiple competing objectives:** It is generally accepted that, having originated from the family of mathematical programming or optimisation, location models naturally seek to achieve efficient solutions (Datta, 2012). However, the nature of public sector location problems are such that equity issues—and lately, environmental sustainability issues also—have been ranked as being equally important; meaning that *efficiency* would have to be traded off to an extent for these other objectives.
- **Highly political decision making atmosphere:** The sheer weight of political pressure is enough to make a decision maker set aside optimal locations in order to forestall ethnic or other sectionalist discontent. Unsurprisingly, when Cocking et al. (2012, p. 168) modelled health facility locations in Burkina Faso, they shied away from attempting to measure such factors as “tribal rivalries that sometimes influence which villages a [patient] is willing to visit” to access healthcare services. Operations Research practitioners seem to have long recognised that an optimal solution ought not be seen as an undisputed “answer,” but rather as an aid to decision-making (Cocking et al., 2012). Brill, Jr, (1979) cautioned that an optimal solution, after all, is based on the narrow views of economic efficiency and social optimality, and should not be the reason for which decision makers should refuse resolving political issues that may arise when public sector facilities are to be located.
- **The incidence of limited data and constrained planning resources:** Some of the most vivid demonstrations of constrained data in developing world contexts are

offered by the works of Cocking et al. (2012, p. 168) and Oppong (1996). Working in the rural Nouna district of Burkina Faso, the former lamented: “As is common when working in developing nations, obtaining reliable data on Nouna was not easy. The most recent detailed road map of the area was from 1971, and the records kept by the officials in the district regarding village locations and populations are often handwritten.”

Regarding peculiar difficulties associated with modelling the locations of public sector facilities, Brill, Jr, (1979) says there are no known model formulation approaches that can account for all factors. Over three decades later a solution appears not to have been found as to how to account for the relevant equity and environmental objectives. The evidence lies in an admission that “in order to model the [Nouna] situation mathematically, we have to make some simplifying assumptions, and ignore less-pertinent or more-difficult-to-model factors” (Cocking et al., 2012, p. 168).

3.5 Categorisation of Location Problems: Static versus Dynamic Models

In an attempt to find the most appropriate model to employ in solving the problem this research seeks to address, it is expedient at this point to review classification of location models. This is to help situate the eventual ‘ideal’ model within the context of competing alternatives. Hamacher & Nickel (1998) extol the usefulness of systematic classification as helping the modeller to structure problem statement concisely and to avoid ambiguity.

As earlier noted, Arabani & Farahani (2012) describe location problems in terms of *space* and *time*. The authors went ahead to explain that static models deal with the space issue while dynamic models answer to the question of time. In effect, “static models try to optimize system performance for one representative period [whereas] dynamic models reflect data

(cost, demand, capacities, etc.) varying over time within a given planning horizon” (Klose & Drexel, 2005, p. 5).

In other words, the use of a static model implies the modeller has a single planning period in mind while a dynamic model accounts for multiple planning periods (Nagy & Salhi, 2007). One important point made by De Lotto & Ferrara (2010) is that the static model forms the basis upon which its dynamic counterpart is built; in the sense that the transient moment within which a given activity takes place in a dynamical system could be captured and described as a static scenario, albeit just for an instance.

On the strength of such argument, the current research takes the view that instead of linking *space* and *time* to static and dynamic models respectively (Arabani & Farahani, 2012), dynamic models should have been characterised as taking care of both ‘space and time’—not only ‘time’—issues.

To be more explicit, this section presents a categorisation of location model typology based on the ***static*** versus ***dynamic*** classifications. The classification approach employed here is largely inspired by the work of Arabani & Farahani (2012), Klose & Drexel (2005), Owen & Daskin (1998), and ReVelle & Eiselt (2005).

3.6 **Static Models**

As mentioned above, the inputs to static models are not time-dependent. Rather, a single representative set of inputs is used to solve a problem for a single representative period (Daskin, 1995).

Before delving further into model classification and a discussion of specific models thereof, we do agree with Arabani & Farahani (2012) that static facility locations are not limited to the list presented herein (see Sections 3.6.1 to 3.6.4 below), nor is the approach used here the only classification method there is. The four model categories—namely *continuous*, *discrete*, *network*, and *analytic* models—are discussed as follows:

3.6.1 Continuous Models

A facility location problem is said to be *continuous* if facilities can be located anywhere within the planning area under consideration, as opposed to discrete location problems which places facilities at only specific possible points (Arabani & Farahani, 2012; Dantrakul, Likasiri, & Pongvuthithum, 2014; ReVelle et al., 2008). Put differently, in continuous location models, it is feasible to locate facilities at any and every point in the plane (Klose & Drexler, 2005). A good example of a continuous location problem would be the siting of rain gauges in a given forest. In such an instance, any location in the forest may be good enough.

Arabani & Farahani (2012) mentions the manner in which distance is measured as a key characteristic of continuous location models. Metrics for measuring distance may be the Manhattan (right-angle distance metric) or the Euclidean (straight-line distance metric), among others. Also, Dantrakul et al. (2014) briefly touch on situations where a problem may be of continuous nature alright but have its results being discretised. Specific continuous problems considered here are *single-facility location problems*, *multiple-facility location problems*, and *facility location–allocation problems*.

Single-facility location Problem: The famous generalised Weber problem which forms the foundation for most of location science (Wesolowsky, 1973) is a good example of single

facility location problems. It entails locating a new facility such that the weighted Euclidean (or other distance metric) between it and other facilities are minimised (Arabani & Farahani, 2012; Revelle et al., 1970). The mathematical model, which minimises the total incurred cost is given as (Arabani & Farahani, 2012; Wesolowsky, 1973):

$$\text{Minimise } Z = \sum_{i \in F} w_i d(X, P_i) \quad (3.1)$$

Where w_i = weight transforming distances into costs for existing facility i ;

X = position for a new facility;

P_i = position for existing facility i ;

$d(X, P_i)$ = distance between X and P_i ; and

F = set of existing facilities.

Being the simplest location model and computationally the least cumbersome (Church, 2002), the earliest location scientists were initially fixated on the single-facility location problem for a long time before developing the confidence to venture into more complex model variants (Thanh, Bostel, & Péton, 2008).

Multiple-facility location Problem: The relationship between the multiple-facility location problem and the single-facility version (described above) is that with minor adjustments in problem parameters of the latter, the new model is transformed to allow for several new facilities finding their optimal locations within the given geographic space (Arabani & Farahani, 2012):

$$\text{Minimise } Z = \sum_{i \in F} \sum_{j \in D} w_{ij} d(X_j, P_i) \quad (3.2)$$

Where w_{ij} = weight between existing facility i and new facility j ;

X_j = position for a new facility j ;

P_i = position for existing facility i ;

$d(X_j, P_i)$ = distance between X_j and P_i ;

F = set of existing facilities; and

D = set of new facilities.

It has to be noted that the above model does not consider weights between new facilities. Daneshzand & Sholeh (2009) addresses such variations.

Facility Location-Allocation Problem: The location-allocation model determines the optimal number of facilities, as well as their locations and the source and level of demand allocated to each (Hamadani, Abouei Ardakan, Rezvan, & Honarmandian, 2013; Min, Patterson, & Oh, 1998). This means the location-allocation model optimally *locate* facilities and also optimally *allocate* (or assign) these facilities to specific customer groups depending on demand levels (Arabani & Farahani, 2012).

Shariff, Moin, & Omar (2012, p. 1000) point out that the location-allocation model “provides a framework for investigating accessibility problems, comparing the quality (in terms of efficiency) of previous location decisions, and providing alternative solutions to change and improve the existing system,” and that the manner in which demand is allocated “has a direct impact on the whole system’s efficiency.” Arabani & Farahani (2012) give the model as follows:

$$\text{Minimise } Z = \sum_{i \in D} \sum_{j \in F} C(i, j) y_{ij} \quad (3.3)$$

Subject to:

$$\sum_{j \in F} x_j = p \quad (3.4)$$

$$\sum_{j \in F} y_{ij} = 1 \quad \forall i \in D \quad (3.5)$$

$$x_j \geq y_{ij} \quad \forall i \in D, j \in F \quad (3.6)$$

$$x_j = \begin{cases} 1 & \text{if candidate node } j \text{ includes a facility, } \forall j \in F \\ 0 & \text{otherwise} \end{cases} \quad (3.7)$$

$$y_{ij} = \begin{cases} 1 & \text{if candidate node } j \text{ gives service to demand node } i, \forall i \in D, \forall j \in F \\ 0 & \text{otherwise} \end{cases} \quad (3.8)$$

Where $C(i,j)$ = cost of assigning customer i to facility j ;

p = number of facilities opened;

F = set of candidate locations where facilities may be sited; and

D = set of demand nodes.

Furthermore, constraint (3.5) ensures that each demand node is assigned; (3.6) shows that only facilities that are opened can be assigned to demand nodes; and (3.7) and (3.8) represent binary conditions. One outstanding example of the use of the facility location-allocation model is a case where it was applied in allocating parking facilities to the various departments of a large firm (Hamadani et al., 2013).

3.6.2 Discrete Facility Location Problems

In discrete facility location problems, demand is deemed to occur at specific geographic points (Arabani & Farahani, 2012). Unlike continuous location problems where a new facility may be sited anywhere within the solution space, a discrete facility location model

ensures that the possible sites for new facilities is restricted to a finite set of available candidate locations (Melo et al., 2009). Whereas *candidate locations* are discrete for both continuous and discrete facility location problems, it is only the latter which is deemed to have discrete *demand*. This means that whether the case is continuous or discrete, potential facility locations are thought of as occurring at discrete points, though demand in the latter case is regarded to be spread over a geographical area.

One group of researchers underscored the importance of discrete location models by contending that they form the basics of almost all facility location applications in healthcare (Beliën, De Boeck, Colpaert, Devesse, & Van den Bossche, 2013). Two specific model types considered for further exploration are the *quadratic assignment problem* and *plant location problem*.

Quadratic Assignment Problem (QAP): First proposed by Koopmans and Beckmann (1957 as cited in Arabani & Farahani, 2012), this model assigns each customer group to some facilities. Essentially, the model minimises the total cost incurred by the assignment of facilities to customers. Both sets of authors mentioned above concur on the following model:

$$\text{Minimise } Z = \sum_{i=1}^D \sum_{j=1}^D \sum_{k=1}^D \sum_{l=1}^D C(i, j) d(k, l) x_{ik} x_{jl} \quad (3.9)$$

Subject to:

$$\sum_{i=1}^m x_{ij} = 1 \quad j = 1, \dots, D \quad (3.10)$$

$$\sum_{j=1}^m x_{ij} = 1 \quad i = 1, \dots, D \quad (3.11)$$

$$x_{ij} = \begin{cases} 1 & \text{if facility } i \text{ is assigned to customer } j, \\ 0 & \text{otherwise} \end{cases} \quad i, j = 1, \dots, D \quad (3.12)$$

Where $C(i,j)$ = cost of assigning facility i to customer j ;

$d(k,l)$ = distance between facility k and customer l ; and

D = number of facilities and number of customers (their number should equal).

Constraints (3.10) and (3.11) maintain the core condition in assignment problems, which says that each facility is assigned to one customer and vice versa. Classically, the main model constraints in QAP ensure that each facility can serve just one client, and every client can receive service from just one facility (Arabani & Farahani, 2012). The notion of one customer to one facility, classical as it may be in assignment modelling generally, is only a symbolic assumption because model parameters can always be tweaked to assign more than one customer to a facility. The opposite, though, will not hold.

Zanjirani et al. (2012) researched on the scenario whereby beyond the customer-facility relationship, the quadratic assignment model depicts a relationship between the facilities being located.

Plant Location Problem: The plant location problem, otherwise known as the *simple plant location problem*, *uncapacitated facility location problem*, or *warehouse location problem* (Marianov & Serra, 2004; Melkote & Daskin, 2001b), aims to site an undetermined number of facilities at designated candidate locations. It is the mathematical formulation and its solution that determines the number and location of the facilities, each of whose capacity is

boundless (i.e. uncapacitated), so that each customer is served by a specific facility (Hamadani et al., 2013; ReVelle & Eiselt, 2005). By contrast, in capacitated facility location problems, facilities to be located come with some lower or upper bounds on their capacities, and the modelling ensures that each customer location is able to receive its demand from several facilities (Hamadani et al., 2013). Simple facility location problems are modelled such that, the sum of setup costs and the cost of serving customers are minimised. ReVelle et al. (2008) presents this formulation:

$$\text{Minimise } Z = \sum_{j \in F} C_j x_j + \alpha \sum_{j \in F} \sum_{i \in D} w_j d(i, j) y_{ij} \quad (3.13)$$

Where C_j = cost incurred by candidate facility j per time unit;

w_j = weight transforming distances into costs for existing facility j ;

α = factor that converts demand-weighted distances to cost units;

$d(i, j)$ = distance from demand node i to candidate facility j ; and

D = set of demand nodes.

Besides objective function (3.13), constraints (3.5) to (3.8) also apply here.

3.6.3 Network Facility Location Problems

In probably one of the oldest comprehensive research on network location problems, ReVelle et al. (1970) explained that the solution space of this class of models reside within a pre-specified network, meaning both demand and facility locations may occur only at the nodes or links of the given network. Klose & Drexl (2005) and Contreras, Fernández, & Reinelt (2012) further emphasised the inherence of network-imposed restrictions in this model.

Reproducing and extending a definition of network location problems offered by Tansel, Francis, & Lowe (1983, p. 482), as partially mentioned elsewhere in this literature review (see Section 3.1), should provide a compact summary of the problem-type being discussed:

Network location problems occur when ... facilities are to be located on a network. The network of interest may be a road network, an air transport network, a river network, or a network of shipping lanes. For a given network location problem, the new facilities are often idealised as points, and may be located anywhere on the network; constraints may be imposed upon the problem so that new facilities are not too far from existing facilities. Usually some objective function is to be minimised. For single objective function problems, typically the objective is to minimise either a sum of transport costs proportional to network travel distances between existing facilities and closest new facilities, or a maximum of "losses" proportional to such travel distances, or the total number of new facilities to be located.

Network facility location models include *median problems*, *covering problems*, *centre problems*, *hub location problems*, and *hierarchical location problems* (Arabani & Farahani, 2012).

Median Problem: While Arabani & Farahani (2012) discussed the median problem under network location problems, Daskin (2008) tackled it within the context of discrete location problems. Keen followers of location science won't be surprised about this seeming discordance because they know all too well the level of fluidity that exists in approaches of subdividing the "broad spectrum of location models" (Daskin, 2008, p. 283).

Concisely, median problems—also known as minisum problems—aim to minimise the sum [or average thereof] of the travel costs (Cocking, 2008). Speaking in terms of a public sector facility in a rural setting, Oppong (1996) explained that minimising aggregate travel distance,

implies cheaper travel cost for the majority while the minority, typically those on the outskirts of the geographic space under review, incur higher travel costs.

A median problem may be 1-median ($p = 1$) or p -median ($p > 1$). Just as the classical Weber problem aims to find a median position for a single facility in a continuous space but could be remodelled to locate multiple facilities (Klose & Drexel, 2005), the 1-median problem could be modified slightly to locate p number of facilities, hence the name *p-median* (Arabani & Farahani, 2012). The p -median model, which seeks to minimise the demand-weighted total distance (equivalent to minimising the demand-weighted average distance once demands are known), is formulated by ReVelle et al. (2008) as follows:

$$\text{Minimise } Z = \sum_{i \in D} \sum_{j \in F} w_i d_{ij} y_{ij} \quad (3.14)$$

Variables are defined as before, and constraints (3.4) to (3.8) also apply.

Covering Problem: In seeking to minimise average distance, median models (described above) make facilities more accessible to the majority while making it prohibitive for a minority of customers located on the outskirts of the demand area to access the facilities (Oppong, 1996). The inherent inequity imposed on minority customers (Rahman & Smith, 2000) under the median model is eliminated when the coverage model is used.

With the covering problem, each customer can access any facility provided the distance (or time) between them is within a pre-specified coverage distance or radius (Arabani & Farahani, 2012; Farahani et al., 2012; ReVelle & Eiselt, 2005). A good example of a covering problem would be to locate a minimum number of police posts in a crime-prone locality such

that no domestic or commercial building would be more than five minutes travel time from at least one police post. In this instance, any dwelling or public building that is satisfied in accordance with the stated criteria (i.e. is within five minutes of a police post) is said to have been *covered*.

Many variants exist for the covering problem (see Farahani et al., 2012) but the most popular of these are the *set covering* and *maximum covering* problems. Whereas the objective of a set covering model is to minimise the number of facilities required to achieve full coverage, that of a maximum covering model seeks to maximise the covered population with a given number of facilities or servers (Marianov & Serra, 2004). Toregas, et al. (1974 as cited in Arabani & Farahani, 2012; ReVelle et al., 2008) formulated the *set covering* problem as below:

$$\text{Minimise } Z = \sum_{j \in F} x_j \quad (3.15)$$

Subject to:

$$\sum_{j \in F_c} x_j \geq 1 \quad \forall i \in D \quad (3.16)$$

The objective here is to minimise the number of facilities, x_j , required to cover the total demand. Constraint (3.16) stipulates the need for at least one candidate facility location from the set F_c .

For the *maximum covering* variation, the model is (Arabani & Farahani, 2012; Church & ReVelle, 1974):

$$\text{Maximise } Z = \sum_{i \in D} w_i z_i \quad (3.17)$$

Subject to:

$$z_i - \sum_{j \in F_C} x_j \leq 0 \quad \forall i \in D \quad (3.18)$$

$$z_i = \begin{cases} 1 & \text{if demand node } i \text{ is covered,} \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in D \quad (3.19)$$

Aiming to maximise the number of covered demands (3.17) with a predetermined number of facilities (x_j), w_i denotes a weight transforming distances travelled by customer i . Constraint (3.18) says that for demand node i to be covered there must necessarily be a facility at one of the candidate locations serving the node. Additionally, constraints (3.4) to (3.7) complete this model.

Marianov & Serra (2004) makes a case for the covering problem's importance—alongside with median problems—in location science by stating that almost all location models could be classified as falling under one or the other. And the authors demonstrate this conviction by actually grouping all examples in the particular write-up under these two broad model classifications.

We are of the view that such classification is rooted in the basic principles of location analysis. Why this position? Median and covering problems are widely regarded as representing the most fundamental instances of minisum and minimax location objectives/criteria (Farahani et al., 2010); the pair in turn form the fundamental basis for all location modelling (ReVelle & Eiselt, 2005). Thus, to label median and covering models as together constituting the benchmark for all location models and model extensions is not a matter of stating a mere opinion; it should be considered as a profound observation that captures the very essence of location problem classifications.

Centre Problems: The centre problem is similar to the covering problem in the sense that it too seeks to ‘cover’ all demand. However, instead of feeding a pre-specified coverage distance into the model, the given number of facilities to be sited (probably subject to resource availability) is decided beforehand and the model determines minimal coverage distance and hence locations for the facilities (Arabani & Farahani, 2012; Owen & Daskin, 1998). Also known as the *minimax* problem (Owen & Daskin, 1998) or *p-centre problem* (in reference to its goal of trying to locate p number of facilities), Owen & Daskin (1998) explain that the centre problem may come in the form of a *vertex centre problem* (facilities restricted at the nodes of the network) or an *absolute centre problem* (facilities locating anywhere on the network). The location of lorry stations within population centres, as against the siting of emergency ambulances either in towns or anywhere on a road network provide good illustrations for the *vertex* and *absolute* centre problems respectively. With ‘Z’ representing the total distance, ReVelle et al. (2008) presents the following model for the vertex centre problem:

$$\text{Minimise } Z \quad (3.20)$$

Subject to:

$$Z - \sum_{j \in F} d(i, j) y_{ij} \geq 0 \quad \forall j \in F \quad (3.21)$$

Together, objective function (3.20) and constraint (3.21) minimises the maximal facility customer distance. As explained previously, $d(i, j)$ denotes the distance from demand node i to candidate facility j . To complete the model, we include constraints (3.4) to (3.8).

Hub Location Problems: One definition that well clarifies the concept of hub location problems is the one by Alumur & Kara (2008, p. 1). Writing on the topic “Network hub location problems: the state of the art,” they said:

Hubs are special facilities that serve as switching, transshipment and sorting points in many-to-many distribution systems. Instead of serving each origin–destination pair directly, hub facilities concentrate flows in order to take advantage of economies of scale. Flows from the same origin with different destinations are consolidated on their route to the hub and are combined with flows that have different origins but the same destination. The consolidation is on the route from the origin to the hub and from the hub to the destination as well as between hubs. The hub location problem is concerned with locating hub facilities and allocating demand nodes to hubs in order to route the traffic between origin–destination pairs.

The myriad of possible specific hub location models may be grouped under *single allocation* and *multiple allocation* models. In the *single allocation* model, flow to or from a terminal node is linked to exactly one hub node; and for the *multiple allocation* model demand can flow to or from more than one hub (Alumur & Kara, 2008; Klose & Drexl, 2005). In certain instances the links connecting terminal nodes to a hub are called ‘spokes’ (much like the hub and spokes in a bicycle wheel), hence the occasional use of the name *hub-and-spoke network* (Rahmaniani & Shafia, 2013).

The airline industry provides a good illustration of the hub-and-spoke location concept, with some industry players even striking strategic alliances with competitors so each would move passengers from various non-hub airports into hub airports where they then exchange them for onward transmission to other locations. ReVelle & Eiselt (2005, pp. 13 - 14) alluded to this idea thus: “hub networks are assembled that require customers to travel from their origin to a central hub, from there either to their destination or to another hub, and if to a second hub, then to continue on to their final destination.”

It is noteworthy to mention that combined facility location and network design solution approaches may sometimes be employed in solving hub-and-spoke network problems (Contreras et al., 2012). One mathematical formulation for a p -hub location problem is presented here (Arabani & Farahani, 2012; O’Kelly, 1987):

Minimise $Z =$

$$\sum_{i \in D} \sum_{k \in D} C(i, k) y_{ik} \left(\sum_{j \in F} f(i, j) \right) + \sum_{k \in D} \sum_{i \in D} C(k, i) y_{ik} \left(\sum_{j \in F} f(i, j) \right) + B \sum_{i \in D} \sum_{j \in F} \sum_{k \in D} \sum_{l \in D} f(i, j) C(k, l) y_{ik} y_{jl} \dots (3.22)$$

Where $C(i, j)$ = cost of moving from i to j

$f(i, j)$ = flow between nodes i and j ;

B = discount factor

The objective function minimises the total cost. Constraints (3.4) to (3.8) complete the model.

Hierarchical Location Problem: One illustrative example of hierarchical facility location in the public sector occurs in the health system, where local clinics occupy the lowest echelon of the hierarchy, followed by district and regional hospitals respectively, with referrals-only hospitals making up the topmost layer of the hierarchy.

In hierarchical facility location generally, both demand and services are organised in a series of levels or echelons and are mapped to each other depending on the complexity of function or service involved; and all these issues are taken care of in the mathematical modelling (Marianov & Serra, 2004).

Melo et al. (2009) laments about the relative dearth in hierarchical facility location models as only one-third of papers they had reviewed addressed this area though in real life this class of location models are quite ubiquitous. Şahin & Süral (2007) came out with a formulation that

minimises the total demand weighted distance for a kind of hierarchical location relationship referred to by Arabani & Farahani (2012) as ‘single-flow two-level system:’

$$\text{Minimise } Z = \sum_j \sum_k f_1(j, k)C(j, k) + \sum_i \sum_j f_2(i, j)C(i, j) \quad (3.23)$$

Subject to:

$$\sum_{j \in F_1} f_1(j, k) = d_k \quad \forall k \in D \quad (3.24)$$

$$\sum_{j \in F_2} f_2(i, j) = \sum_{k \in D} f_1(j, k) \quad \forall j \in F_1 \quad (3.25)$$

$$\sum_{k \in D} f_1(j, k) \leq M_j y_j \quad \forall j \in F_1 \quad (3.26)$$

$$\sum_{j \in F_1} f_2(i, j) = M_i x_i \quad \forall i \in F_2 \quad (3.27)$$

$$\sum_{j \in F_1} y_j = p_1 \quad (3.28)$$

$$\sum_{i \in F_2} x_i = p_2 \quad (3.29)$$

$$x_i = \begin{cases} 1 & \text{if level 2 facility is located at node } i, \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in F_2 \quad (3.30)$$

$$y_j = \begin{cases} 1 & \text{if level 1 facility is located at node } j, \\ 0 & \text{otherwise} \end{cases} \quad \forall j \in F_1 \quad (3.31)$$

Where $C(j, k)$ = cost per unit flow across levels;

$C(i, j)$ = cost per unit flow between level 1 facilities and demand nodes;

$f_1(j,k)$ = flow from facility node j and demand node k (in level 1);

$f_2(i,j)$ = flow from facility node i (at level 2) and facility node j (at level 1);

F_1 = set of candidate locations where facilities may be sited for level 1;

F_2 = set of candidate locations where facilities may be sited for level 2;

M_j = capacities of facilities at level 1;

M_i = capacities of facilities at level 2;

p_1 = number of facilities expected to be located/opened at level 1;

p_2 = number of facilities expected to be located/opened at level 2; and

d_k = the demand placed by demand node k

Constraint (3.24) ensures each demand node is satisfied while (3.25) equates the total demand a level 1 facility receives to the one it transfers outwards to a facility at another level. While (3.26) and (3.27) are related to facility capacities at the two levels, (3.28) and (3.29) indicate the number of facilities to be located for number of facilities at each level.

Competitive Location Problem: Built on the assumption that customers patronise the nearest facility, competitive location problems seek to find competitively advantageous locations with a view to maximising market share at the expense of rival facilities (Drezner, Drezner, & Salhi, 2002; Klose & Drexl, 2005). In other words, in competitive facility location models, facilities compete with one another to attract as many customers as possible. An example is when a retail oil company attempts to find the best fuel station locations in a neighbourhood where its competitors already have a presence.

ReVelle & Eiselt (2005) *extend two cautions about* competitive location problems. The first is that any such model should look beyond merely locating facilities, and take pricing into

account (meaning the resulting model will seek the positions and prices which maximise market share), since price wars can make nonsense of whatever optimal location configuration that has been arrived at. Secondly, an inherent weakness in this type of model is that, a seemingly minor change in problem parameters will alter the solution.

3.6.4 Analytic Models

Analytic models are said to be the simplest to formulate and solve (Daskin, 2008). Just like the continuous versions, analytic models locate facilities anywhere within a solution space, but have their solution methods usually based on calculus or other simple techniques instead of on linear or non-linear optimisation (Daskin, 2008; ReVelle et al., 2008). Analytic models also come with simplifying assumptions such as demands being uniformly distributed.

It is quite interesting that almost all of the literature we have sighted so far that speaks of analytic models are written by Mark Daskin (Daskin, 2008; ReVelle et al., 2008), either solely or jointly with others; or the sources are quoting a work he is involved in.

3.7 Dynamic Models

The choice to site a facility at one location or another is a strategic one, meaning that a certain time or planning horizon has to be factored into the decision making process (Daskin, Hesse, & ReVelle, 1998). The *raison d'être* of dynamic location models is to account for the factor of time period, as opposed to the single instance timing assumed in static scenarios (Ambrosino, Scutell, & Grazia Scutellà, 2005).

Current, Ratick, & Revelle (1997) explain that opting for a dynamic model implies coming out with a definite planning period and working out (by means of the dynamic model

developed or adopted) which facilities to locate initially, and at what points in time to locate additional ones, and/or to shut or relocate any of them. The idea is to build a model that is robust enough to proactively provide for likely future fluctuations in demand and other parameters over a planning period; resulting in a schedule for opening and/or closing facilities at specific times and locations. Thanh, Bostel, & Péton, (2008, p. 679) succinctly summarises the basic meaning of dynamic location problems as follows:

In this class of problems, the decisions are spread out over a long-term planning horizon, and the decision variables are time-dependent. The purpose is to minimise or maximise an objective function over this horizon. For example, how should a growing company develop its logistic system in the five forthcoming years so that the demand and the logistic constraints remain satisfied at a minimum cost?

Furthermore, Current et al. (1997) speak of ‘explicitly dynamic,’ and ‘implicitly dynamic’ models. In implicitly dynamic models, the initial set of locations is determined taking into account parameter variations over the planning horizon. An example is the location of fire stations in an area where the frequency of fire occurrence from sub-area to sub-area oscillates seasonally. An implicitly dynamic approach would locate the fire stations such that, depending on the time of year, certain stations will be busier than others, though all of them will be kept operational throughout. Conversely, an explicitly dynamic model in the same situation would first and foremost show where to locate the initial fire stations, when to close or relocate any of these, and also recommend when and where to locate additional ones.

Like most of location science, dynamic location modelling has progressed significantly enough to be applied in multiple facility location problems, as opposed to the single facility the first ever published paper in the field (Ballou, 1968) sought to locate.

Instead of outlining a complete typology of dynamic location problems as has been done in the static case, we believe it suffices to mention just a few ‘popular’ ones. The reason for brevity here is that this work does not anticipate dynamic modelling. This means that once optimal (or near optimal) facility locations are found, what most probably needs to be done would be to expand capacity on an ad hoc basis as and when the need arises.

Much as this approach may seem myopic, it surely is a legitimate option according to Schilling (1980). The only issue here is that such reactive planning may produce sub-optimal solutions in the long term. However, Schilling (1980) also warns that much as it is okay to go explicitly dynamic, and in so doing produce a currently optimal solution together with a schedule for future expansion or relocation, any ‘mis-prediction’ of future parameter uncertainties could make nonsense of the supposed optimality of the resulting solution. Thus, we contend it is not scientifically out of place to ignore the effect of time horizon in location modelling. We now proceed with a brief discussion of just five of the most utilised dynamic location models:

Dynamic deterministic facility location problems simply transforms a static problem by considering more than one time period, and then modifying the input parameters in order to re-solve the problem for each time-period scenario (Arabani & Farahani, 2012; Wesolowsky, 1973).

In discussing **Facility Location-Relocation Problems**, Arabani & Farahani (2012) stress the fact that the essence of dynamic models lies in the issue of location and relocation timelines, and that these can be divided into discrete and continuous time. With discrete time, definite points in time at which relocation will take effect are predetermined; but when it comes to continuous time, the model endogenously spells out the relocation times over the chosen time horizon.

A close variant of the above is the **Multi-period (discrete time) Location Problems** which adds some new decision variables to an essentially static model in order to make for future transportation costs and time-staged opening/closing of new/existing facilities (Wesolowsky & Truscott, 1975 as cited in Arabani & Farahani, 2012).

In situations where demand varies throughout the planning horizon under consideration, as in the case of commodities with daily or seasonal demand fluctuations, **Time-Dependent Facility Location Models** are the obvious option. The main difference between this one and the rest of dynamic location modelling is that the time-staged relocations for each season is easily predictable (Drezner & Wesolowsky, 1991 as cited in Arabani & Farahani, 2012), meaning the incidence of uncertainty is less. According to the last authors, the main issue of concern to address in this class of models is to determine the time of changing a location and the new place for siting in each season. Finally, **Stochastic/probabilistic Location Problems** are said to occur when parameters for future times take on probabilistic values (Arabani & Farahani, 2012).

3.8 Facility Location versus Network Design

With part of this research's aim being to observe the impact of eventual facility locations on road network, it has been considered essential to briefly touch on combined facility location and network design problems. Melkote & Daskin (2001a) observed that the configuration of a given problem's underlying network may have a profound impact on the nature of its solution. This observation was based on an earlier work which concluded that in certain situations, it may be more economical to reconfigure the underlying network than to build new facilities (Daskin, Hurter, & Van Buer, 1993).

By definition, facility location-network design (FLND) problems simultaneously find optimal facility locations together with the ideal transportation network that makes those locations optimal (Rahmaniani & Shafia, 2013). Such a situation arise when trade-offs have to be considered between facility costs, network design costs, and operating costs (Melkote & Daskin, 2001b). As an illustration, suppose a certain community or district in this research's study area (i.e. Upper West region of Ghana) does not have a particularly large population to warrant the siting of a buffer stock warehouse, but one has to be placed there just because the nearest warehouse is too far away. The trade-off dilemma would be to either build the warehouse, or to construct a road or bridge that links the community to an existing or proposed warehouse via a shorter route. In doing so, the issue is about comparing the set-up cost (i.e. cost of establishing facility), network design cost (i.e. cost of constructing a new road or improving an existing one), and the cost of operation (i.e. the cost of customers accessing the facility). With public sector facilities, for example, whether the cost relates to the building of roads or warehouses, or of customers travelling, it would be incurred directly or indirectly by the same taxpayer public. This is the rationale behind minimising the three different costs together via an FLND formulation. Prior to the emergence of FLND models, facility location problems (as illustrated in all the models presented previously) and network design problems (as illustrated by Drezner & Wesolowsky, 2003) were solved as separate problems.

The mathematical formulation of an FLND problem is essentially an extension of one of the network problems previously reviewed. So, one could have an FLND based on the p -median (Cocking et al., 2012; Melkote & Daskin, 2001a, 2001b) or the maximum covering (Contreras et al., 2012) models, for example. Some tweaking of the network part of the problem could also be performed. Examples include assuming a complete absence of networks (Rahmaniani & Ghaderi, 2013), or setting different qualities of networks so that the

solution may recommend a new road or the improvement of an existing one (Cocking et al., 2012). In a similar vein, Rahmaniani & Ghaderi (2013) undertook the novelty of imposing upper limits on the capacities particular links (roads) can handle. There is also uncapacitated FLNDs, in which no upper bounds are placed on the capacities of the facilities being located (Melkote & Daskin, 2001a), as against capacitated FLNDs (Melkote & Daskin, 2001b). Other exciting developments include the modelling of budget constrained FLNDs, which involves imposing the maximum budget within which the three cost components of FLND problems must fit (Cocking et al., 2012; Ghaderi & Jabalameli, 2013). Another novelty is the introduction of dynamic FLND models by Ghaderi & Jabalameli (2013).

As further explorations and model extensions are being sought to FLNDs, Rahmaniani & Ghaderi (2013) reports that the most complicated FLND problem solved to date (as at the year 2013) comprised 100 nodes and 500 links.

3.9 Contextualisation

A pattern has emerged from the literature reviewed so far on the primary location typologies. Analytic models aside, it has been observed that the three other categories of static location models (i.e. continuous, discrete and network models) are conceptually the same, with the deepening layers of restriction making all the difference; and that FLNDs and pure network design problems appear to be just another level of the hierarchical relationship between these models. The relationships, which we propose to be called the **location-network hierarchy**, are summarised below, and also illustrated in Figure 3.1. Note that the listing below corresponds with rung numberings in Figure 3.1.

1. *Continuous* facility location problems locate facilities anywhere within the solution space (Arabani & Farahani, 2012; ReVelle et al., 2008). This class of problems would be located on the first rung of the location-network hierarchy.
2. If the given problem is formulated such that potential facility locations are restricted to a finite set of possible sites (instead of being allowed to locate anywhere), we have what is called a *discrete* location problem (Arabani & Farahani, 2012).

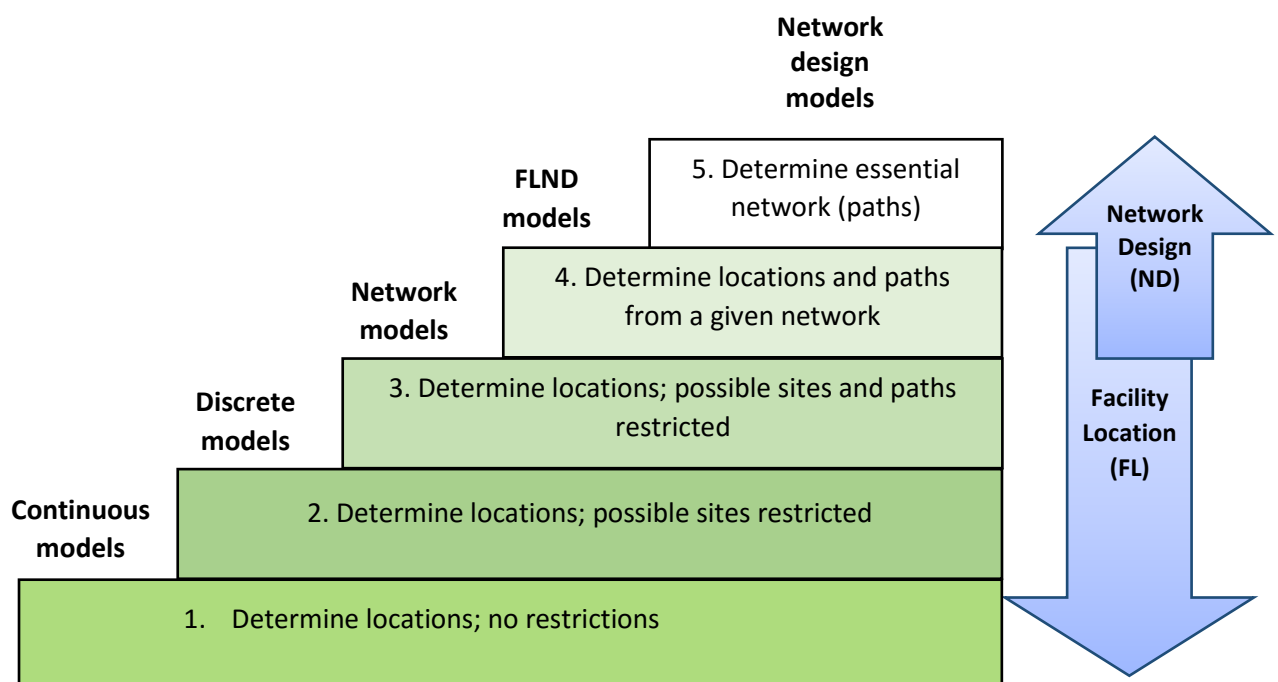


Figure 3.1: The Location-Network Hierarchy (Source: Research data)

3. A location problem that is discrete in nature but has the additional condition of access from one node to another (and between customers and facilities for that matter) being restricted via a set of specific routes is known as a *network* location problem (Arabani & Farahani, 2012; Tansel et al., 1983).
4. If a network location problem has an additional feature of the network configuration (such as road or rail network) itself being determined as part of the

solution, the problem becomes a *combined facility location-network design* (FLND) problem (Melkote & Daskin, 2001a, 2001b).

5. Finally, pure *network design* problems (not included in this literature review) are shown on the highest rung of the *location-network hierarchy*. These typically begin with a given network of potential links, from which a sub-network is to be found that connects all the original vertices (nodes) subject to certain constraints. The objective is to minimise the total construction and transportation costs (Drezner & Wesolowsky, 2003; Johnson, Lenstra, & Kan, 1978).

The two arrows located on the extreme right of the *location-network hierarchy* show models from the fourth rung downwards are concerned with facility location, while from that same fourth rung upwards are concerned with network design. The obvious result is that FLND models address both issues, hence an overlap of the two arrows.

While the above observation may not be claimed as a discovery made by this research, it is quite profound that to date, no author appears to have directly stated this simple but perceptive idea. The essence of the foregoing observation is that, what has always been presented as location problem typology (Arabani & Farahani, 2012; Daskin, 2008; ReVelle et al., 2008; ReVelle & Eiselt, 2005) could also been seen as a progressive deepening of restrictions on essentially the same problem type, and that the philosophy could be extended to rope in FLNDs and pure network design problems. If for nothing at all, it would help students of location science gain a more instructive understanding of the philosophies behind model classifications.

3.10 Problem Categorisation

To specify the parameters of the location problem at hand, it is necessary to first clarify which broad problem category it belongs to, and then to narrow down to a specific model type. Firstly, one has to consider whether problem circumstances demand a *static* or a *dynamic* approach. While the former approach determines facility locations for one representative period, its dynamic counterpart works out solutions that are time-phased over a defined planning horizon (Klose & Drexl, 2005; Nagy & Salhi, 2007).

A comparison of Upper West region's census figures from 1960, 1970, 1984, 2000 (National Population Council, 2004) and 2010 (GSS, 2013) shows the region has not experienced any abnormal population growth as has happened in, say, the Greater Accra and Ashanti regions. Such an easily predictable growth in demand diminishes the need for dynamic modeling given its complications and shortcomings (Schilling, 1980). In other words, since demand is predictable any need for future capacity upgrades could be determined beforehand and be factored into the initial construction without having to forfeit the advantages of the simplicity offered by *static* models. Under such a strategy, the focus is on finding currently optimal locations with room for future capacity expansion. Another reason which makes dynamic modelling unnecessary is the fact that NAFCO is most unlikely to invest into a warehouse only to close it down and move to another location.

Having chosen to adopt a static model, the next decision is to sort out which of *continuous*, *discrete*, *network* and *analytic* location models to settle on. Analytic models come across as the first to eliminate from further consideration because they are deemed to be less useful in practical circumstances (ReVelle et al., 2008).

Furthermore, we perceive our problem would be located on rung 4 of the *location-network hierarchy*; hence a **network location problem**, since candidate locations are restricted to specific discrete nodes, and access between nodes is via an existing road network.

3.10.1 Specific Problem Definition: a p -Median Model

Understanding the core mandate of NAFCO is central to deciding which specific network location model to employ. The organisation may either be seen as one rendering not-for-profit public services or as a commercial entity. While both private (commercial) and public sector models alike may seek to minimise the average customer-facility travel cost, only public sector models would usually seek to minimise the longest distance between a facility and its customers. The two approaches are said to be aiming at minisum and minimax objectives respectively (ReVelle & Eiselt, 2005).

It is just natural to view NAFCO as a public sector organisation, but that does not make it a not-for-profit entity. As a matter of fact, in spite of the food security mandate it carries, NAFCO is reportedly oriented to be purely commercial and self-sustaining (NAFCO, 2013). Being desirable and commercial facilities, a minisum strategy has to be adopted in modelling the locations of grains buffer stock warehouses (as compared to maxisum approach for undesirable facilities). Farahani et al. (2010) argues that *median* problem formulations provide the most eloquent mathematical expression of *minisum* objectives (just as *covering* models apply to *minimax* objectives), hence the choice of a median model as the vehicle of solving the problem at hand.

According to Lee & Yang (2009), the p -median problem represent the most classic case of those location problems that focus on minimising the average or total distance between customers and facilities. In more express terms, the p -median problem seeks to find the

optimal location(s) of a predetermined p number of facilities so as to minimise the demand-weighted average distance of a system (Marianov & Serra, 2004); in effect, seeking to locate facilities in closer proximity to their users (Dzator & Dzator, 2013). Yet another definition describes the p -median problem as “an abstract for real world problems like warehouse location” (Maroszek & Rettig, 2008), a notion that vindicates the adoption of the p -median as the model of choice in the current studies to locate buffer stock warehouses.

The p -median problem was first formulated by Hakimi (1964) to locate switching centres on a power transmission network; and Gould & Leinback (1966) provides one of the earliest implementations of the p -median model in the location of hospitals in Guatemala and the assignment of each to their nearest community. Similarly, Oppong (1996) and Mehretu, Wittick, & Pigozzi (1983) too solved examples of the p -median problem for the location of health facilities in Ghana and Burkina Faso respectively. The model has also been applied in locating emergency services (Dzator & Dzator, 2013), political districting, design of sales force territories, cluster analysis, quantitative psychology, vehicle routing, and topological design of computer communication networks among others (Goldengorin, Krushinsky, & Pardolas, 2013). The mathematical formulation of the p -median problem is embodied in objective (3.14) and constraints (3.4) to (3.8), but will be further explained in Chapter 5.

3.11 Solution Methods for the p -Median Model

After being able to correctly formulate a location problem, an even more daunting challenge is to be able to solve the problem to optimality (Current et al., 2004). In the parlance of network complexity theory, p -median problems are known to be NP-hard, implying they are computationally very difficult to solve due to the large number of variables usually involved, especially when p is not fixed (Current et al., 2004). Whereas the ideal situation is to work

towards optimal solutions through the use of exact solution methods, the p -median's inherently complicated nature means that *near optimal* solutions are instead achieved sometimes using heuristic or meta-heuristic approaches (Dzator & Dzator, 2013).

Arabani & Farahani (2012) posits that an exact solution approach finds a feasible solution within 5% of optimality while heuristics and meta-heuristics provide solutions that fall outside this margin. Unlike what happens with exact solution methods, heuristics do not test every possible solution but rather quickly and intelligently generate a solution very close to optimality (Khoban & Ghadimi, 2009). The only problem is that, one has no way of knowing how close a heuristic solution may be from optimality except another means is found to generate the optimal solution to enable a comparison (Maroszek & Rettig, 2008). Metaheuristics provide a framework within which elements from different simple heuristic methods are combined to generate procedures that yield higher quality solutions than is possible from the use of individual heuristics (Mladenović et al., 2007), some of which so easily gets trapped in local optima (Khoban & Ghadimi, 2009). Reese (2005) enumerates metaheuristics including variable neighbourhood search, genetic algorithms, GRASP, scatter search, tabu search, simulated annealing, and neural networks. Details of most of these will not be discussed here.

Any algorithm—such as the greedy or myopic type—which works out a solution from scratch, is classified amongst *construction algorithms*. *Improvement* algorithms, on the other hand, use feasible solutions from other heuristics as their starting solution (Daskin, 1995). There is also the *Lagrangian relaxation* approach which belongs to a categorisation of its own.

3.11.1 Greedy Heuristics

Greedy or *myopic* heuristics operate by locating facilities one after the other, as opposed to generating all the locations in a single solution. While evaluating options in order to site each facility, the greedy heuristic behaves as though that single facility is the last one to be sited, only to restart the procedure and add on the remaining facilities, one at a time, until the required number of facilities, p , is achieved. The procedure is such that in siting the n^{th} facility, any previous ones located i.e. $(n-1)^{\text{th}}$, $(n-2)^{\text{th}}$, etc, stays in the solution, a situation that may result in grossly suboptimal results (Dzator & Dzator, 2013). To illustrate, take the following 5-node network with demands and distances as shown in Figure 3.2:

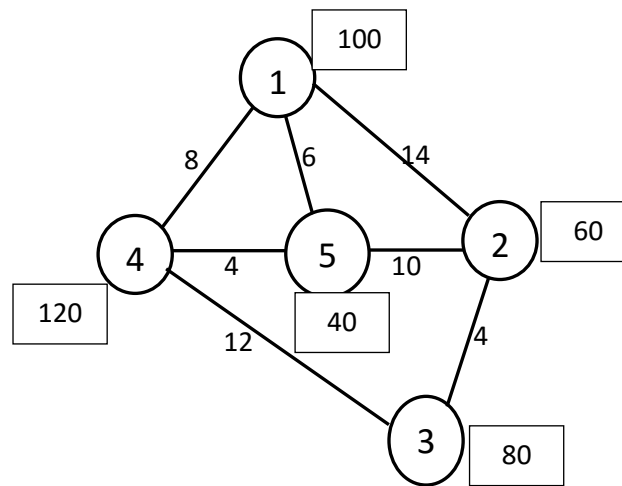


Figure 3.2: Network for example problem (Source: Research data)

Finding the shortest path between non-adjacent pairs of nodes, the distance matrix, d_{ij} , and the demand-weighted distance, $w_i d_{ij}$, matrix are presented below:

$$d_{ij} = \begin{bmatrix} 0 & 14 & 18 & 8 & 6 \\ 14 & 0 & 4 & 14 & 10 \\ 18 & 4 & 0 & 12 & 14 \\ 8 & 14 & 12 & 0 & 4 \\ 6 & 10 & 14 & 4 & 0 \end{bmatrix}, \quad w_i d_{ij} = \begin{bmatrix} 0 & 1400 & 1800 & 800 & 600 \\ 840 & 0 & 240 & 840 & 600 \\ 1440 & 320 & 0 & 960 & 1120 \\ 960 & 1680 & 1440 & 0 & 480 \\ 240 & 400 & 560 & 160 & 0 \end{bmatrix}$$

To locate $p = 2$ facilities using the Greedy-Add heuristic, a 1-median problem is first solved by first generating column totals from the w_id_{ij} matrix, and the node corresponding to the least of these is selected as the location for the first facility. From the above data, $Z_1^1 = 3480$, $Z_2^1 = 3800$, $Z_3^1 = 4040$, $Z_4^1 = \underline{2760}$, and $Z_5^1 = 2800$. A facility would therefore be cited at Node 4. To locate the second facility, demand-weighted distances, w_id_{ij} , for Node 4 (row 4) are deleted and the objective value re-computed:

$$w_id_{ij} = \begin{bmatrix} 0 & 1400 & 1800 & 800 & 600 \\ 840 & 0 & 240 & 840 & 600 \\ 1440 & 320 & 0 & 960 & 1120 \\ 0 & 0 & 0 & 0 & 0 \\ 240 & 400 & 560 & 160 & 0 \end{bmatrix}$$

The second facility locates at Node 2 since it now has the least column total: $Z_1^2 = 2520$, $Z_2^2 = \underline{2120}$, $Z_3^2 = 2600$, $Z_4^2 = \infty$, and $Z_5^2 = 2320$. The second facility locates at Node 2. Solution for the 2-median is therefore $P = \{2,4\}$. The above procedure is repeated until locations are found for all the required number of facilities. Specifically, the above algorithm is called *Greedy-Add*, as there is also a *Greedy-Drop* which starts by locating facilities at all candidate sites and then dropping locations that have the highest objective value one after the other until p facilities remain (Current et al., 2004).

3.11.2 Improvement Heuristics

As the name suggests, *improvement* or *search* heuristics seek to improve on feasible solutions, perhaps as a cure for the fact that solutions from myopic heuristics, though computationally easier to execute, are mostly not reliable (Current et al., 2004) and usually

needs to be improved upon. Though numerous improvement heuristics does exist (refer to Reese, 2005) only the *neighbourhood search* and *exchange algorithms* will be discussed here.

- Neighbourhood search (NS) heuristic:** The NS heuristic uses a feasible solution that has been previously generated through the myopic heuristic or some other approach as its starting point. Neighbourhoods are created around current facility locations based on the assumption that each demand node is served by the facility closest to it, so that each facility together with the demand nodes it serves forms a *cluster* or *neighbourhood*. For example, after finding the 2-medians in the preceding example using the greedy approach, two neighbourhoods are formed around the two facilities nodes $P = \{2,4\}$, with Node 2 serving Node 3 and Node 4 serving Nodes 1 and 5:

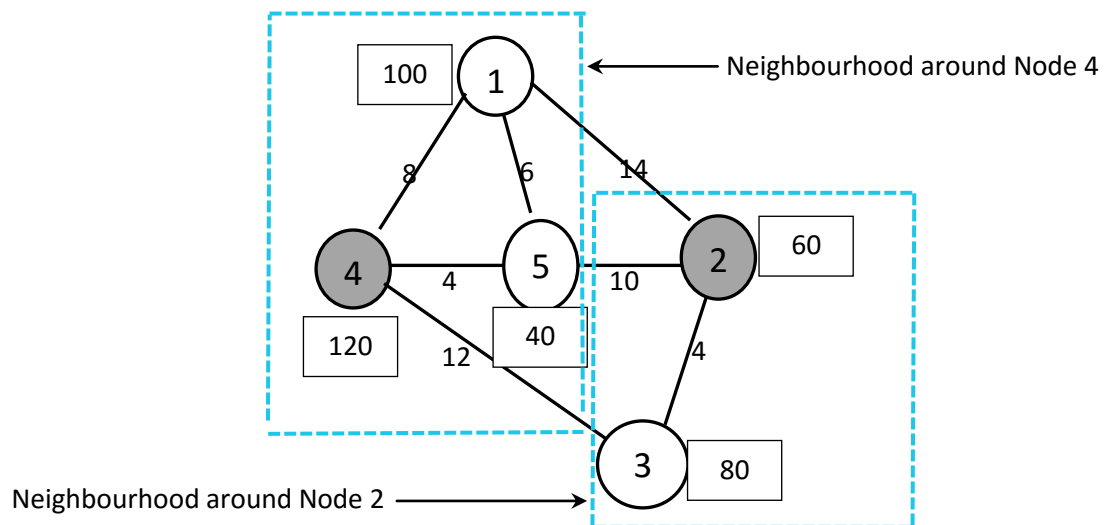


Figure 3.3: Neighbourhoods associated with a greedy 2-median (Source: Research data)

First proposed by Marazana (1964 as cited in Current et al., 2004), the neighbourhood search heuristic works by treating the sub-problem in each neighbourhood as a 1-median problem which can then be solved to optimality. Once a solution relocates a

facility to another node, new neighbourhoods are formed and the procedure is repeated for each neighbourhood generated until the iterations cease to make any changes to locations or neighbourhoods. Daskin (1995) proposes the flowchart in Figure 3.4 to summarise how the neighbourhood search algorithm is implemented.

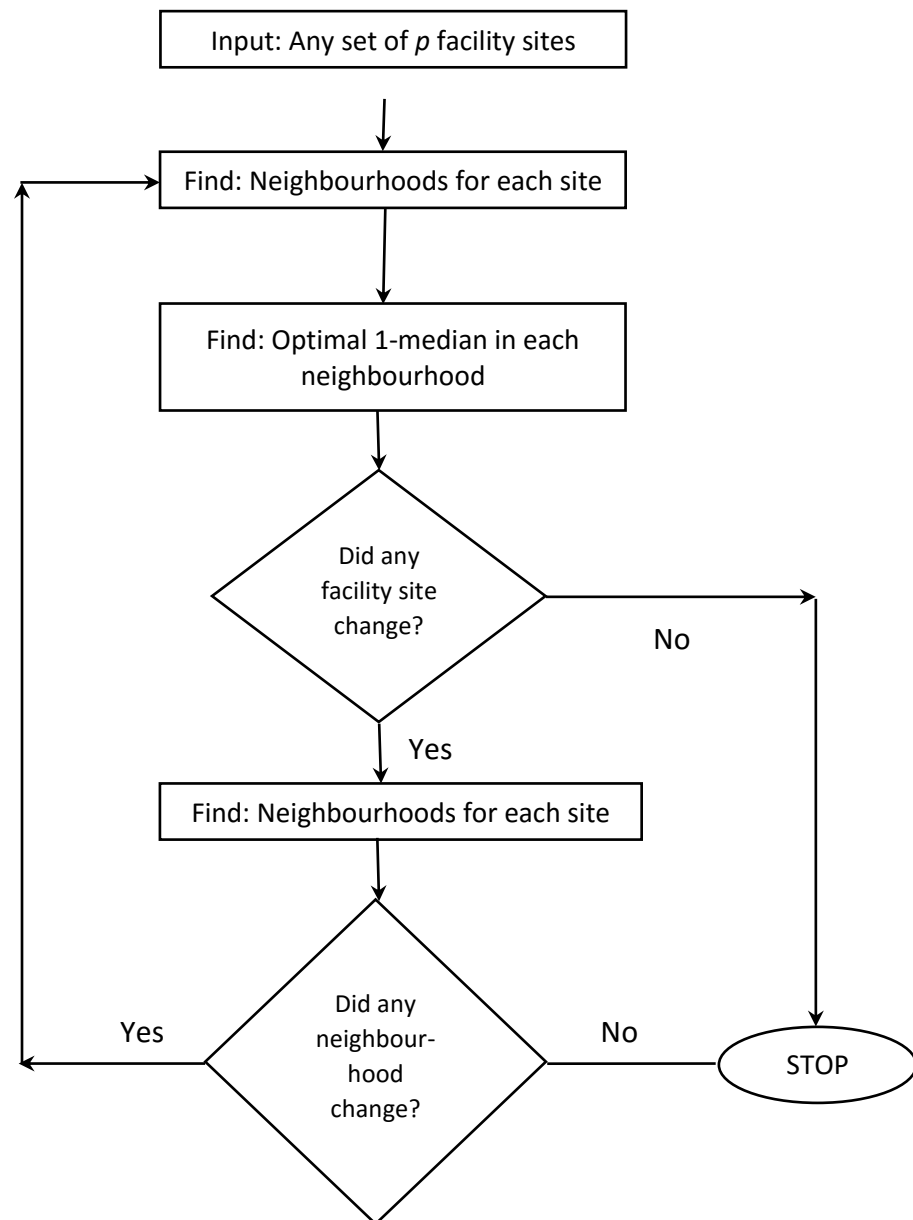


Figure 3.4: Flowchart of neighbourhood search algorithm (Source: Daskin, 1995)

- **Exchange heuristics:** The most successful *exchange*, *interchange* or *substitution* heuristic is that of Teitz and Bart (1968 as cited in Current et al., 2004; Dzator &

Dzator, 2013) though others like ‘fast interchange’ proposed by Whitaker (Current et al., 2004) and *heuristic concentration* approach put forward by Rosing and ReVelle (Current et al., 2004) are also in use to a lesser extent. Focus here is limited to the Teitz and Bart procedure as illustrated in Figure 3.5.

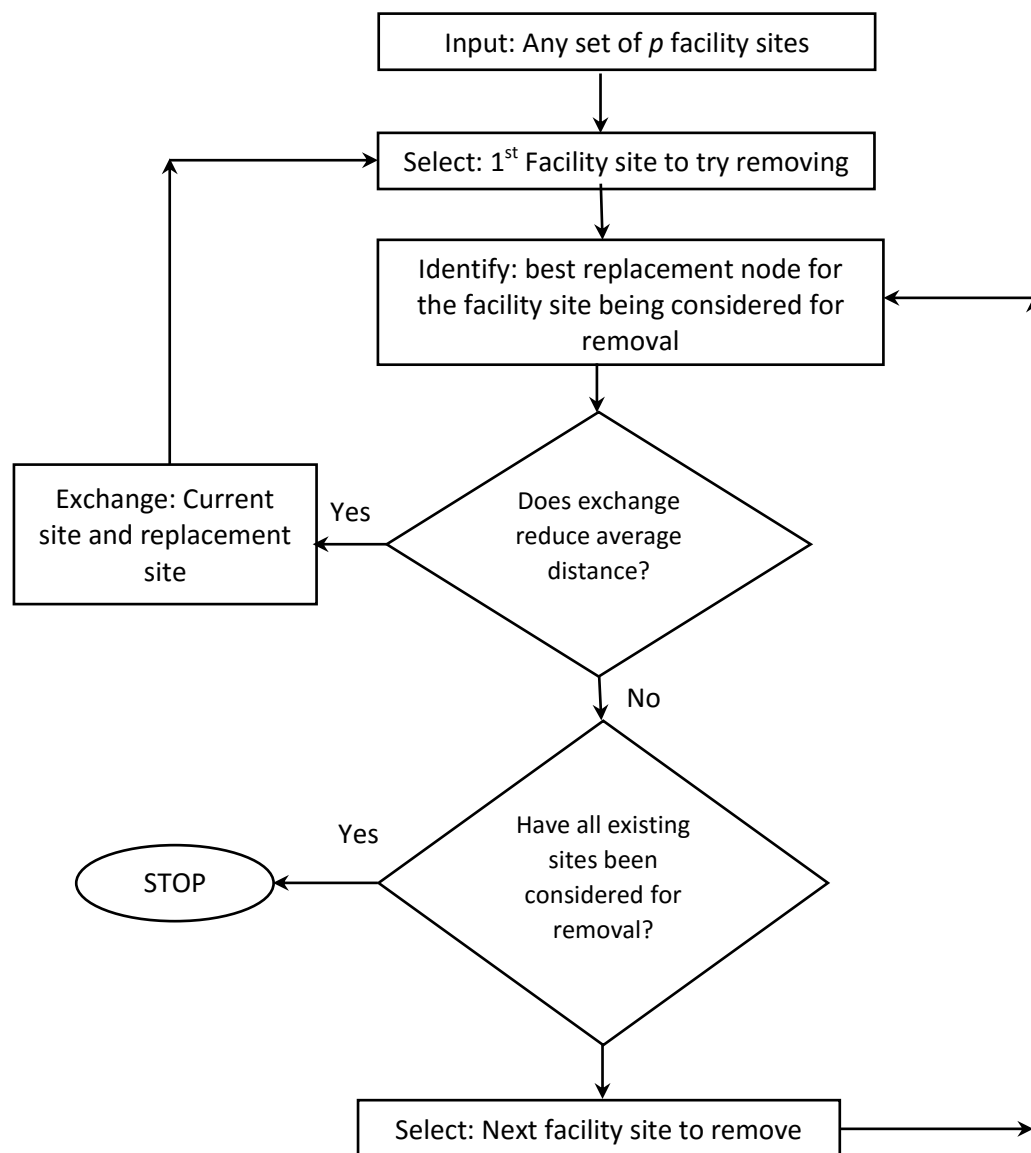


Figure 3.5: Flowchart of the exchange algorithm (Source: Daskin, 1995)

In summary, the exchange heuristic is performed by moving sited facilities to unused nodes and re-computing the objective value in the hope that, should a better value be

obtained the new location will be accepted. The procedure terminates when no further improvements accrue by exchanging.

3.11.3 The Lagrangean Relaxation Approach

The *Lagrangean relaxation*, though another heuristic (Current et al., 2004), is reviewed here as an exact method (Dzator & Dzator, 2013) due to its proclaimed ability to obtain solutions that are within 1% of optimality (Arabani & Farahani, 2012), and the fact that the gap between a given solution and the optimal is verifiable (Current et al., 2004).

The Lagrangean relaxation approach replaces the original problem with a dual Lagrangean problem which relaxes one or more of the constraints and computes upper and lower bounds that gives an indication as to the range within which the optimal solution of the original problem should fall. The solution approach begins by arbitrarily estimating some Lagrange multipliers, and successive iterations seek to estimate more precise multiplier values (Daskin, 1995; Current et al., 2004).

Here we will use the example problem introduced in Section 3.11.1 to illustrate the procedure. Again we want to find a solution for a 2-median. Relaxing constraint (3.5), we reproduce the Lagrangean relaxation (dual problem) from Eiselt & Sandblom (2004) as follows:

$$\begin{aligned} \max_{\lambda} \min_{x,y} L &= \sum_{i \in D} \sum_{j \in F} w_i d_{ij} y_{ij} + \sum_{i \in D} \lambda_i (1 - \sum_{j \in F} y_{ij}) \\ &= \sum_{i \in D} \sum_{j \in F} (w_i d_{ij} - \lambda_i) y_{ij} + \sum_{i \in D} \lambda_i \end{aligned} \quad (3.32)$$

Subject to:

$$\sum_{j \in F} x_j = p \quad (3.4)$$

$$y_{ij} \geq x_j \quad \forall i \in D, j \in F \quad (3.6)$$

$$x_j = \begin{cases} 1 & \text{if candidate node } j \text{ includes a facility, } \forall j \in F \\ 0 & \text{otherwise} \end{cases} \quad (3.7)$$

$$y_{ij} = \begin{cases} 1 & \text{if candidate node } j \text{ gives service to demand node } i, \forall i \in D, \forall j \in F \\ 0 & \text{otherwise} \end{cases} \quad (3.8)$$

Objective function (3.32) is minimised with the original location and allocation variables (x_j and y_{ij} , respectively) but is maximised with respect to the Lagrange multipliers (λ_i). The largest L achieved over all iterations represents the lower bound on the objective function for the original p -median problem. The procedure is as follows:

Step 1: Determine the matrix $G = (g_{ij}) = (w_i d_{ij})$, which represents demand-weighted distance.

We reproduce our $w_i d_{ij}$ matrix from Section 3.11.1:

$$G = \begin{bmatrix} 0 & 1400 & 1800 & 800 & 600 \\ 840 & 0 & 240 & 840 & 600 \\ 1440 & 320 & 0 & 960 & 1120 \\ 960 & 1680 & 1440 & 0 & 480 \\ 240 & 400 & 560 & 160 & 0 \end{bmatrix}$$

Determine another matrix V_j such that $V_j = (U_{ij}) = \min \{0, w_i d_{ij} - \lambda_i\}$

$$U_{ij} = \begin{cases} w_i d_{ij} - \lambda_i, & \text{if } w_i d_{ij} < \lambda_i \\ 0, & \text{otherwise} \end{cases} \quad (3.33)$$

But we first need the Lagrange multipliers λ_i to be able to proceed. With an eye on the figures in the w_id_{ij} matrix, we arbitrarily choose $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 500$.

Now executing (3.33),

$$U_{ij} = \begin{bmatrix} -500 & 0 & 0 & 0 & 0 \\ 0 & -500 & -260 & 0 & 0 \\ 0 & -180 & -500 & 0 & 0 \\ 0 & 0 & 0 & -500 & -20 \\ -260 & -100 & 0 & -340 & -500 \end{bmatrix}$$

The column sums $V_j = [-760, \mathbf{-780}, -760, \mathbf{-840}, -520]$. With columns 2 and 4 constituting the least two sums, our solution is $P = \{2, 4\}$.

Step 2: Now that we have a feasible solution we have to find out whether to terminate the procedure or not. If $LB = UB$ stop; otherwise compute the step size t and use it to update the Lagrange multipliers.

$$\begin{aligned} \text{Lower Bound (LB)} &= \sum_{i \in D} \sum_{j \in F} (w_id_{ij} - \lambda_i)y_{ij} + \sum_{i \in D} \lambda_i & (3.32) \\ &= (-780 - 840) + (500 + 500 + 500 + 500 + 500) \\ &= -1620 + 2500 \\ &= 880 \end{aligned}$$

Note: y_{ij} in objective (3.32) above indicates that only Nodes 2 and 4, where $y_{ij} = 1$ because they host facilities, does matter here. For all other nodes, $y_{ij} = 0$.

Upper bound (UB) = objective value for current solution, i.e. (i.e. $P = \{2, 4\}$)

Given that each demand area will access the closest facility, we compute the demand-weighted distance between each node and the facility sites, $P = \{2,4\}$, and pick the minimum in each case.

$$\begin{aligned}
 \text{Min } \{w_i d_{ij}\} &= \min \{w_1 d_{12}, w_1 d_{14}\} = \{1400, \underline{800}\} = 800 \\
 &= \min \{w_2 d_{22}, w_2 d_{24}\} = \{\underline{0}, 840\} = 0 \\
 &= \min \{w_3 d_{32}, w_3 d_{34}\} = \{\underline{320}, 960\} = 320 \\
 &= \min \{w_4 d_{42}, w_4 d_{44}\} = \{1680, \underline{0}\} = 0 \\
 &= \min \{w_5 d_{52}, w_5 d_{54}\} = \{400, \underline{160}\} = 160 \\
 \therefore \text{UB} &= \text{Objective value of } P \{2,4\} = \underline{1280}.
 \end{aligned}$$

As it turns out, $LB \neq UB$ [as $LB = 880$ and $UB = 1280$] we have to perform another iteration.

$$\text{Now, } t = \frac{\alpha(UB - LB)}{\sum_i \left(\sum_j y_{ij} - 1 \right)^2} \quad (3.34)$$

The expression in the denominator represents the sum of squared violations of the assignment constraint (3.5) which was originally relaxed. To explain, let's compute a matrix for y_{ij} . Nodes 4 and 5 now include facilities, therefore all non-zero entries in these two columns would be set to 1, all other entries = 0. (NB: it is the non-zero columns which contributed towards the V_j values that attracted the facilities to these nodes in the first place).

$$U_{ij} = \begin{bmatrix} -500 & 0 & 0 & 0 & 0 \\ 0 & -500 & -260 & 0 & 0 \\ 0 & -180 & -500 & 0 & 0 \\ 0 & 0 & 0 & -500 & -20 \\ -260 & -100 & 0 & -340 & -500 \end{bmatrix}, \quad y_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

But the allocations depicted from y_{ij} violate the relaxed constraint $\sum y_{ij} = 1$. Instead of each row sum being 1, we instead have 0, 1, 1, 1 and 2 as row totals respectively. For the sum of squared violations,

$$\sum_i \left(\sum_j y_{ij} - 1 \right)^2 \quad (3.35)$$

$$= (0 - 1)^2 + (1 - 1)^2 + (1 - 1)^2 + (1 - 1)^2 + (2 - 1)^2 = 1 + 0 + 0 + 0 + 1 = 2$$

Setting $\alpha = 2$ [according to Daskin (1995) $\alpha = 2$ is typically used but if there is no improvement (decrease) in the LB after some number of iterations, say four iterations, the value of α is halved].

$$t = 2(1280 - 880)/2 = \mathbf{400}$$

Step 3: To update the Lagrange multipliers for the $(n+1)^{\text{th}}$ iteration,

$$\lambda_i^{n+1} = \max \left\{ 0; \lambda_i^n - t^n \left(\sum_j y_{ij}^n - 1 \right) \right\} \quad (3.36)$$

$$\lambda_1^2 = \max \{0; 500 - 400 (-1) = \max \{0; \underline{900}\} = 900$$

$$\lambda_2^2 = \max \{0; 500 - 400 (0) = \max \{0; \underline{500}\} = 500$$

$$\lambda_3^2 = \max \{0; 500 - 400 (0) = \max \{0; \underline{500}\} = 500$$

$$\lambda_4^2 = \max \{0; 500 - 400 (0) = \max \{0; \underline{500}\} = 500$$

$$\lambda_5^2 = \max \{0; 500 - 400 (1)\} = \max \{0; \underline{100}\} = 100$$

The new Lagrange multipliers computed would be used in the next iteration. The algorithm is terminated when any one of the following conditions occur (Daskin, 1995):

- A pre-specified number of iterations are completed.
- The lower bound of the current iteration equals, or is very close to, the best (smallest) upper bound value.
- α^n becomes very small, as the changes in λ_i also become too small to help achieve a solution.

3.12 Summary of Literature Review

This section seeks to put the foregoing literature review into perspective. According to Hamadani et al. (2013) the *theoretical framework* for location analysis was first introduced by Weber. Thus, the Weber Problem has come to form the foundation for most of location science, which in our context means the use of Operations Research methods to find optimal locations for facilities. The famous Weber problem is all about optimally locating a single facility such that the demand-weighted total distance is minimised. With time, other researchers developed model extensions as well as solution algorithms to solve the said problem. Thanks to continuing research, models now exist that—theoretically at least—are able to generate any number of facility locations.

The historical background of location science was also reviewed with key issues being the contribution of pioneers such as Alfred Weber and Hakimi, as well as an exploration of the extent to which location analysis has been applied in Ghana. Also, the objectives that shape location problem typology—depending on whether the facility to be located is desirable (e.g.

buffer stock warehouses) or undesirable (e.g. landfill site)—have been discussed (ReVelle et al., 2008). As the discourse continued into the realm of balancing the various objectives that might compete to influence how a particular location problem is modelled, it naturally led to a comparison of private versus public sector location problems. Predominantly, private sector (commercial) facilities mostly seek to achieve *efficiency* and go by minimax objectives, while their public sector counterparts pursue mostly *equity* (i.e. attempting to treat all customers equally) and are usually minimax in their objectives (ReVelle & Eiselt, 2005).

In a bid to make sense of the multiplicity of approaches to classifying location problems, it emerged that whereas all location problems are defined within the context of a given geographical *space*; some can only be fully explained when the issue of *time* is considered as an added dimension. The point was made in the review that a *static* model solves a problem for one representative period (i.e. considers the location ‘space,’ but not its ‘time’ horizon) while a *dynamic* one accounts for multiple planning periods (considers both location ‘space’, and ‘time’ horizon) (Arabani & Farahani, 2012; Nagy & Salhi, 2007).

A decision was made not to review dynamic models into detail since the problem instance at hand did not necessarily need dynamic modelling. As such, only a few dynamic models were mentioned in passing, unlike the detailed treatment given to the static ones. The various static models reviewed are summarised in Figure 3.6.

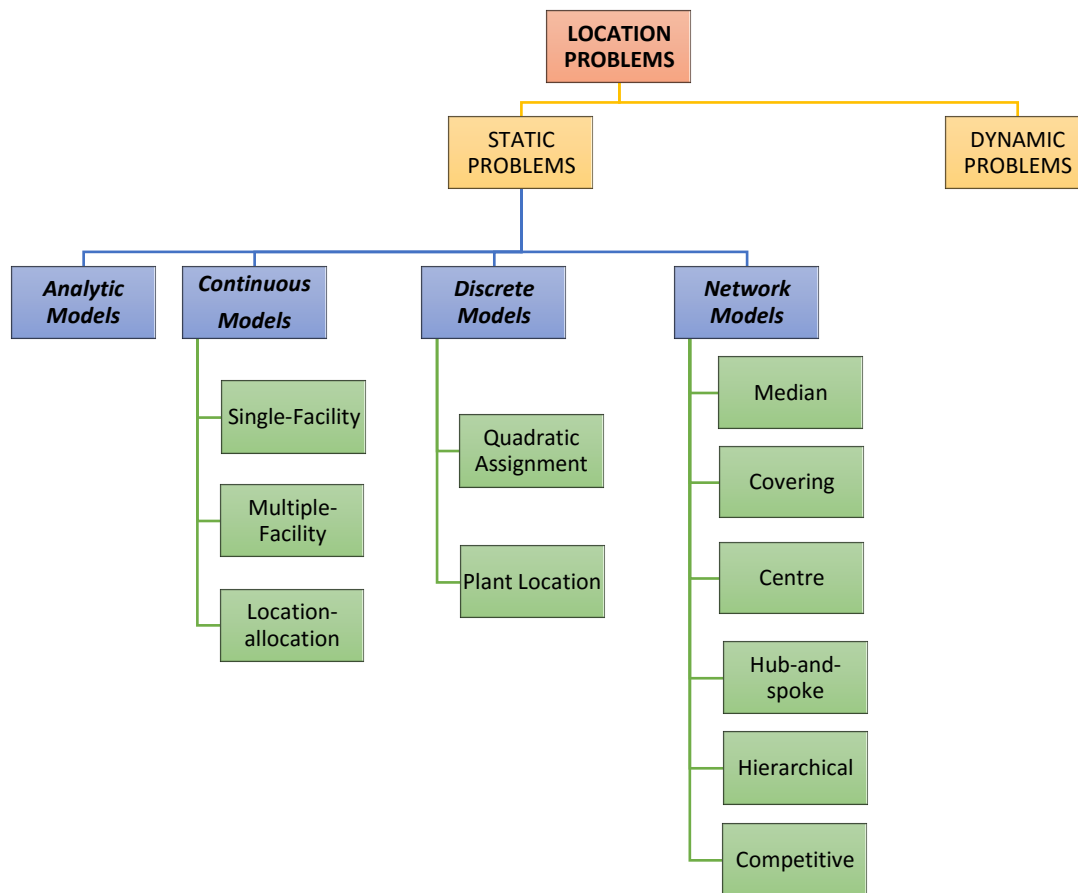


Figure 3.6: A Taxonomy of location models (Source: Research data)

It ought to be mentioned that whereas this research adopted the location modelling taxonomy illustrated in Figure 3.6, alternative approaches do exist (Daskin, 2008; ReVelle & Eiselt, 2005). The review also touched on combined facility location and network design (FLND) models. Subsequently, a point was made that the various static models, together with FLND and network design models, are only distinguished by the extent to which model parameters are restricted. This concept, illustrated in Figure 3.6, has led to our proposal of the *location-network hierarchy*, a new classification framework.

Finally, a justification was made as to why the p -median (or ‘median’) model best suits the problem at hand. The p -median approach was discussed, and some of its solution procedures such as the *greedy algorithm*, *improvement heuristics* and the *Lagrangian relaxation* approaches were demonstrated.

CHAPTER 4: RESEARCH METHODOLOGY

The three previous chapters explained the need for finding optimal locations for grains buffer stock warehouses in the Upper West region of Ghana, presented a catalogue of modeling approaches to choose from, and finally zeroed in on the p -median model as being the most appropriate for the situation. This chapter systematically describes the approach adopted in implementing a solution. It begins with a summary of the assumptions upon which our particular problem is built, outlines how data was collected and processed, and also demonstrates how the model is solved unorthodoxly with Microsoft Excel.

4.1 Model Assumptions

As a reminder, the problem is being modelled as a discrete network location problem (the family of location problems to which the p -median belongs) because the demand points (also potential facility sites) are restricted to specific nodes (i.e. towns), and travel from one town to another is assumed to be possible only via an existing road network. To help place into context the way in which data was extracted and organised, specific model assumptions are outlined below:

1. Each node represents a demand point, as well as a potential facility site. Specifically, this problem has eleven nodes, each representing a capital of the eleven districts in the Upper West region. As explained in the next section, the demand indicated against each district capital represents an aggregation of the demand from the entire respective district.
2. Each node (district capital) hosts at most one facility.

3. It is a customer-to-server system, where demands (i.e. customers) themselves travel to the facilities.
4. Users will opt to travel to the buffer stock warehouse that is closest to them (i.e. cheapest to access).
5. There is a given existing road (links) network that restricts travel route options; and link construction is not part of the problem being solved, because such decisions do not lie in NAFCO's domain. However, the impact a given location configuration may have on the road network (i.e. travel route options) will be examined.
6. Links are uncapacitated as the UWR is a sparsely populated area, and it is impractical to envisage a situation where the inter-district road network is utilised to the point of congestion (Rahmaniani & Ghaderi, 2013).
7. Facilities are uncapacitated, hence they could theoretically serve an unlimited amount of demand.

4.2 Data Collection and Transformation

Two sets of secondary data were collected, namely population data and distance data. The whole of the Upper West region represents the solution space. It follows that the entire regional population of 702,110 is under consideration. The other data set needed involves collecting driving distances on the major roads linking the various districts. This represents the cost of travelling from one district capital to another. Note that the terms 'distance' and 'travel cost' are used interchangeably.

4.2.1 Population Data

As mentioned above, this research presents population data as a surrogate for demand; thus, the higher the population of a locality, the bigger its demand is deemed to be. Population statistics for the region, disaggregated by districts, was extracted from the 2010 Population and Housing Census reports (GSS, 2013). Since buffer stock warehouses are commercial facilities, they would most likely be located in townships. For this reason, all eleven (11) district capitals in the region are designated as demand points, and also as potential facility locations.

The next challenge has to do with whether to use only population statistics for the towns (district capitals) themselves or to rope in a whole district's population and assign it all to the capital. These choices had to be thought through especially due to the fact that the total population of the major towns or district capitals accounts for less than 19% of the region's population, with the remainder strewn across a vast number of small settlements. Going by the Ghana Statistical Service's definition of an urban community being one which has a population of at least 5,000 (GSS, 2013), only Wa, Tumu, Jirapa, Nandom, Lawra and Hamile (the first four are district capitals) make the mark. This means that if the major towns' populations only are used, more than 81% of demand would have been discounted from the data and subsequent solution.

With district capitals being the focal points for socio-economic development in Ghana, for which reason they receive a lot of state and donor resources on behalf of their entire district-wide populations (Owusu, 2004), it was thought wise to aggregate the entire population of each district and assign it all to their respective capitals. Population-wise, all the district capitals, except two, represent the biggest communities in their various districts, anyway. The exceptions exist in Hamile being more populous than its capital, Lambussie (of Lambussie-

Karni district), and Daffiama surpassing Issa, the capital of the Daffiama-Bussie-Issa district. This notwithstanding, the research opts for making the capitals represent the total demand for their respective districts for reasons of consistency.

Given the way demand for each district is being lumped, questions may naturally emerge as to why demand across a town or village, let alone a whole district, be considered as a mathematical point. ReVelle & Eiselt (2005) stresses that one factor which is common to almost all location models is the assumption that demand occurs only at specified points in the plane or nodes in a network; meaning several customers are aggregated and modeled as a single demand point for the sake of modeling convenience. It ought to be known that the issue of demand aggregation is all about trading off less model accuracy for more model tractability, or vice versa (NaimiSadigh & Fallah, 2009). The former is opted for because the alternative would involve time and logistics constraints that this research project cannot accommodate. A district-by-district *impact aggregation* implies the cost of accessing a buffer stock facility if it is located within a customer's district is assumed to be zero.

It is acknowledged that not every resident of the region would be interested in patronising the NAFCO warehouses, as the case may be for some farmers. To account for this, different outputs from the 2010 Population and Housing Census as well as from other empirical studies were be used to categorise potential customers and the extent to which each group may need service from NAFCO warehouses. The result from this represents the true demand to be used as input for the problem.

4.2.2 Distance Data

This work uses distance data as proxy for *travel cost* and *accessibility*. The only practical means of moving across districts in the Upper West region is by road, as there is no railway

or other travel network infrastructure. The *shortest routes* between district capitals by road were estimated in kilometres using interactive tools from google.com.gh/maps. In order to maintain accuracy and consistency, all distances were estimated using the same method, and the results triangulated against any available published distance data.

Initially, travel cost between two nodes is measured as the *network distance* (i.e. measured along existing roads) linking the two; with the *Floyd's algorithm* (Taha, 2007a) being executed with the TORA software (Taha, 2007b) to evaluate the shortest distance between each pair of non-adjacent nodes.

It is a known fact that road quality affects ease of travel, thus any deterioration in the quality of a road link serves as a disincentive for customers plying such a route, a fact that should be accounted for in a location model where possible (Cocking et al., 2012). To implement this, raw distance data obtained was multiplied by a factor representing road quality. The higher the quality of a road link, the smaller its multiplying factor, and hence the lower the travel cost. Most of the inter-district roads are untarred but relatively usable except in rainy seasons. For the purposes of this research, roads surfaced with asphalt or bitumen and still in good condition are designated as 'good' quality; followed by 'normal' quality roads, which represents the best untarred, regularly-maintained gravel roads. Finally, we designate pothole-riddled or hardly maintained gravel/dirt roads as 'bad' quality roads. Roads whose qualities are worse than the above are not considered as potential routes as customers are unlikely to use them, anyway. Cocking et al. (2012) modelled three widely disparate road qualities with subjective multipliers of 1.00, 1.50 and 2.00. Based on observation and experience, equally subjective but less disparate multipliers of 0.75, 1.00 and 1.25 were used to evaluate travel costs over 'good,' 'normal,' and 'poor' quality roads respectively. To compute travel cost on a 20km 'normal' road (e.g Jirapa to Lambussie), we multiply 20 by 1.00 to give us a travel cost of 20. The same distance on a 'good' road such as Wa to Nadowli

would be easier to travel, and so would yield a travel cost of 15 (i.e. 20×0.75). In the same way, 20km on a ‘bad’ road such as Issa to Tumu (via Walembele) would have a travel cost of 25 (i.e. 20×1.25) since it is the most difficult to travel. Distance data so transformed provides the primary distance cost matrix for solving the problem.

4.3 Software Employed

Though most location analysts use special purpose software such as LINDO, MATLAB, SAS, GAMS AMPL, and CPLEX to solve location and other combinatorial problems, it has been argued that spreadsheet packages are handier and so should be encouraged (Ipsilandis, 2008). The use of spreadsheet software such as Microsoft Excel Solver Add-In in location analysis is user friendlier because they are widely available, do not require knowledge in algebraic language or custom coding, and they can be structured in novel ways, free from the rigidity imposed by traditional Operations Research software.

It is not all combinatorial problems that can be implemented in the Solver Add-In, but to demonstrate that it is possible, we reproduce in Figures 4.1 to 4.3 below the initial set-up and computational results for the example problem introduced in Section 3.11.1 as per a format proposed by Lee & Yang (2009).

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	
1		Demand-weighted Distances							Location and Allocation								
2		$w_i d_{ij}$	1	2	3	4	5		y_{ij}	1	2	3	4	5	Total		
3	1		0	1400	1800	800	600	1		0	0	0	0	0	0		
4	2		840	0	240	840	600	2		0	0	0	0	0	0		
5	3		1440	320	0	960	1120	3		0	0	0	0	0	0		
6	4		960	1680	1440	0	480	4		0	0	0	0	0	0		
7	5		240	400	560	160	0	5		0	0	0	0	0	0		
8																	
9									Locations								
10									y_{ij}	1	2	3	4	5	Total		
11										0	0	0	0	0	0		
12																	
13																	
14																	

Figure 4.1: Screenshot of initial Solver set-up for example problem (Source: Research data)

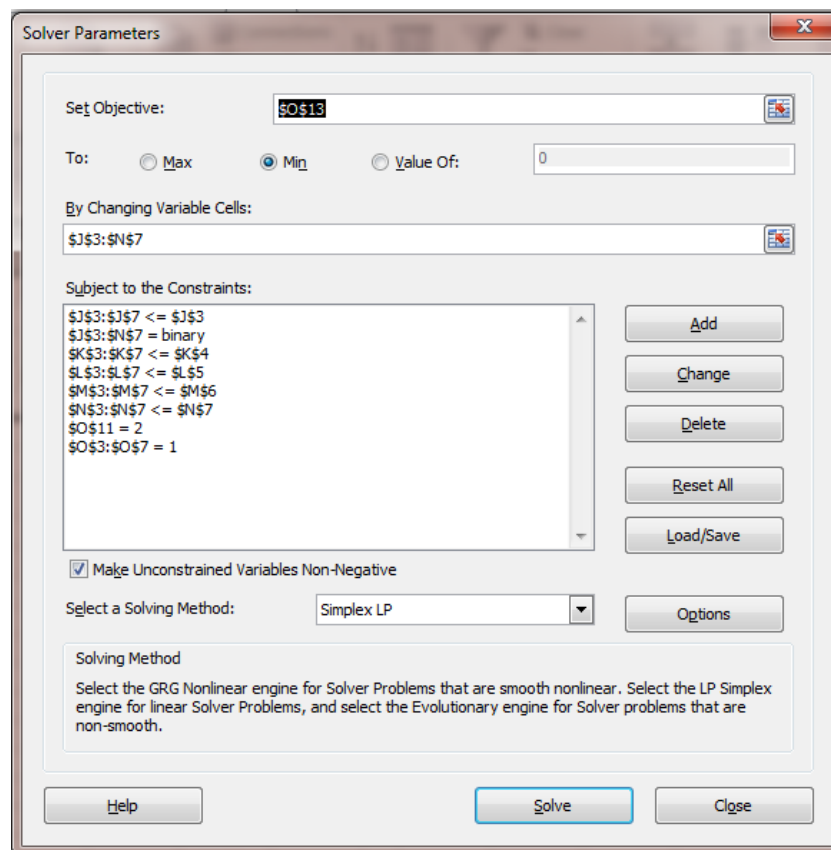


Figure 4.2: Solver dialogue box showing problem parameters (Source: Research data)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1		Demand-weighted Distances							Location and Allocation							
2		$w_i d_{ij}$	1	2	3	4	5		y_{ij}	1	2	3	4	5	Total	
3		1	0	1400	1800	800	600		1	0	0	0	1	0	1	
4		2	840	0	240	840	600		2	0	0	1	0	0	1	
5		3	1440	320	0	960	1120		3	0	0	1	0	0	1	
6		4	960	1680	1440	0	480		4	0	0	0	1	0	1	
7		5	240	400	560	160	0		5	0	0	0	1	0	1	
8																
9									Locations							
10									y_{ij}	1	2	3	4	5	Total	
11										0	0	1	1	0	2	
12																
13															Minimum Costs:	1200
14																

Figure 4.3: Computation results of example problem showing facilities optimally located at Nodes 3 and 4 [see Cells L11 and M11] (Source: Research data)

Limitations of Solver Add-in: The Microsoft Excel Solver Add-In has three different engines, each representing a different algorithm, heuristics or solution method. These are the *Simplex LP*, *GRG Nonlinear*, and *Evolutionary* engines. Trials show that none of the three is able to handle our substantial problem obviously because it is larger than the 200 variables that the Solver Add-In could cope with. Regardless of which engine was chosen, the regular Solver Add-In was able to solve the 5-Node trial problem discussed above. However, for each trial on an 11-Node simulated problem, the *Solver* dialogue box returned the message: “The problem is too large for *Solver* to handle. *Solver* is limited to 200 variable cells and 100 constraints, plus bounds on the variable cells.” Once it becomes clear that the regular Solver Add-In does not possess the power to solve our substantial problem, the *Premium Solver 2014-R2* was opted for. The process of setting up and implementing the solution is the same; the only difference being that the commercially available *Premium Solver 2014-R2* is able to accommodate a larger number of decision variables and constraints, and is also able to solve the problem to optimality.

CHAPTER 5: DATA PRESENTATION AND ANALYSES

This chapter presents the details of how the real-life problem of optimally locating grains buffer stock warehouses in the Upper West region of Ghana was actually synthesised, mathematically modeled and solved. The chapter presents how population numbers and travel distances are adjusted to reflect practical constraints; and in the end, produced a multi-dimensional solution that is robust enough to cater for different budget scenarios that the National Food Buffer Stock Company might approach the problem with.

5.1 Population Figures and Demand Data

According to figures from the 2010 Population and Housing Census, the total population of the Upper West region (UWR) stands at 702,110 (GSS, 2013). Such data is easily obtainable from publications by the Ghana Statistical Service but these were not enough as the census was conducted when there were only nine districts in the Upper West region and published data is organised as such. Now there are two more districts: Daffiama-Busie-Issa, carved from Nadowli, and Nandom from the Lawra district. Additional information was obtained from the Ghana Statistical Service and the Ministry of Local Government and Rural Development to help separate population data for the newly created districts from their parent districts.

Though it has been established in the methodology that population data would be used as proxy for demand, it became apparent that using the raw district-by-district population figures would cause a distortion. For instance, the Wa Municipality has a population of 107,214 from 18,891 households. About 31% (5,841) of these households are into agriculture and so might be supplying some of their own food needs, and would therefore have less need

for buffer stock warehouses. However, 17.1% of the said 5,841 households which are into agriculture do not grow crops. They are into fish farming, livestock rearing, and tree growing. This means they are just like those households who are not into farming at all, and are therefore in need of the sort of services NAFCO warehouses are meant to render.

A spreadsheet that takes all these factors into consideration has been designed to help adjust population data (see Appendix 1), so that the adjusted figures would more accurately reflect the situation on the ground. It includes the following district level information from a Ghana Statistical Service publication entitled *2010 Population & Housing Census: Regional Analytical Report – Upper West Region*:

- Population
- Number of households
- Number of farmer households
- Percentage of farmer households who are into crop farming
- Average size of farmer households

The above information was used to estimate the total number of people in each district who are into crop farming and so are likely to supply their own grain cereals, at least to some extent. Deducting this number from a district's total population gives us an estimate of those who are not into crop cultivation whether they are farmers or not.

Another key metric that helped determine demand data comes from the findings of a research conducted by Quaye (2008) to the effect that farmer households in the Upper West region on the average finish off any grains they might have stored from a season's harvest in seven months, leaving them vulnerable for the year's five (5) remaining months (as against 6 months for Upper East and 5 months for the Northern regions). Non-crop households, on the other hand, would be in need for all twelve (12) months of the year. For this reason, factors of

$\frac{5}{12}$ and $\frac{12}{12}$ are applied to the populations coming from crop-growing and non-crop-growing households, respectively. A summary is presented in Table 5.1, but finer details of the spreadsheet, which shows all formulae and arithmetic computations, may be observed from Appendix 1. Taking the case of Lawra, for example, the district has a population of 55,633 but as per our computations, only about 49.6% of these (27,612) may need to buy grain cereals.

Node	District/Demand Area	Capital	Original Population	Demand (Population in need of Grain Cereals)	Percentage over Total Population
1	Wa West	Wechiau	81,348	37,788	46.5
2	Wa Manucipal	Wa	107,214	85,747	80.0
3	Nadowli-Kaleo	Nadowli	61,804	31,558	51.1
4	Jirapa	Jirapa	88,402	43,842	49.6
5	Lawra	Lawra	55,633	27,612	49.6
6	Lambussie-Karni	Lambussie	51,654	25,797	49.9
7	Nandom	Nandom	45,296	22,481	49.6
8	Sissala-West	Gwollu	49,573	22,675	45.7
9	Sissala-East	Tumu	56,528	27,579	48.8
10	Daffiama-Bussie-Issa	Issa	32,584	16,638	51.1
11	Wa East	Funsi	72,074	33,292	46.2
Total			702,110	375,009	Overall = 53.4

Table 5.1: Demand data (Source: GSS, 2013; Research data)

With total demand now amounting to 375,009, NAFCO should therefore be planning with this number per annum instead of the original population of 702,110. Overall, the change represents 53.4% of the original demand (population). If all districts had seen the same proportion of effect, the above exercise would not have had any significance. To the contrary, there is a low of 45.7% in the Sissala West, and a high of 80.0% for the Wa Municipality. The figures are understandable given that the Sissala West district has a 100% rural population whereas Wa Municipal is only 33.7% rural. The uneven effect in demand figures

proves the point that using the raw demand data would have meant a misrepresentation of the real facts.

5.2 **Distance Data and Travel Cost**

It has already been established that this research is modelling the districts in the UWR as demand areas, the district capitals as potential facility sites (on behalf of their respective districts), and the existing road network as the means of access between districts. This presupposes that the solution space is being modeled into a network with district capitals as nodes and access between them via road constituting the links or arcs of the network.

Ordinarily, distances are presumed as travel cost in network theory. However, in a real-life problem instance such as ours, practical issues such as varying qualities of road links, and ease of access for that matter, has to be accommodated. As explained in the previous chapter, quality of the roads observed in the region has been classified into three levels namely, ‘Good’, ‘Normal’ and ‘Bad’ for the purposes of this study. These represent tarred roads, relatively best quality gravel roads, and poor quality gravel roads, in that order. A network depicting access among the various district capitals is shown in Figure 5.1 below. Note that the figures written against the various nodes are populations of the various districts, and the links are of different qualities in line with the way the roads currently exist, as this illustration seeks to show raw data before any transformation of population into *demand* and distance into *travel cost*. Also, the node numbering follows the order in Table 5.1.

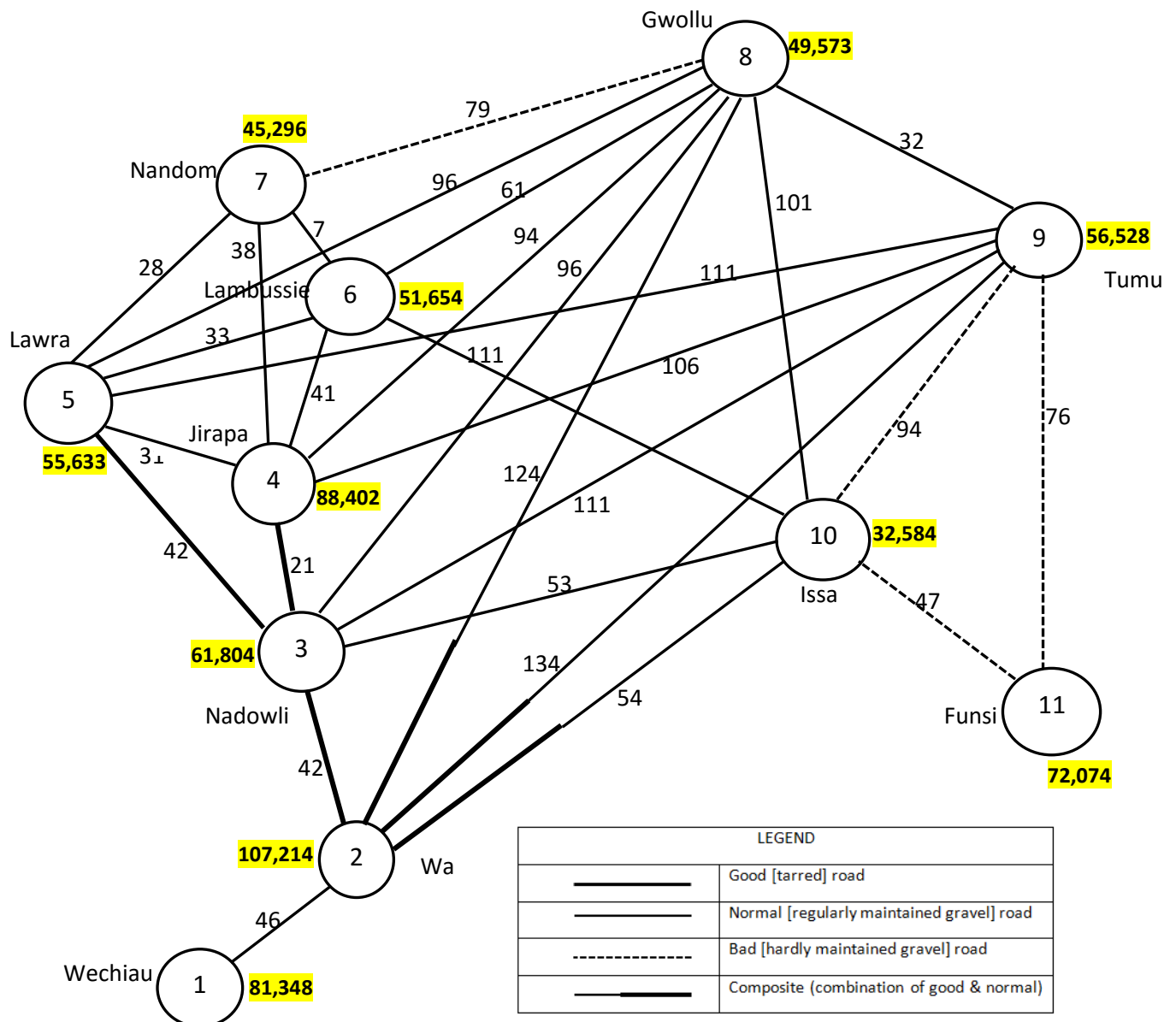


Figure 5.1: Network depicting districts/district capitals in UWR (Source: Research data)

The above network has 28 links of different quality levels. These are transformed into travel cost as shown in Table 5.2.

No (a)	Link (b)	Raw Dist. (km) (c)	Disaggregated Raw Distances			Transformed Distances (Travel cost)			Total Travel Cost (j) [(g)+(h)+(i)]	Change (k)
			Good (d)	Normal (e)	Bad (f)	Good (d) X [0.75] (g)	Normal (e) X [1.00] (h)	Bad (f) X [1.25] (i)		
1	1 - 2	46	-	46	-	-	46	-	46	-
2	2 - 3	42	42	-	-	32	-	-	32	-10
3	2 - 8	124	31	93	-	23	93	-	116	-8
4	2 - 9	134	31	103	-	23	103	-	126	-8
5	2 - 10	54	31	23	-	23	23	-	46	-8
6	3 - 4	21	21	-	-	16	-	-	16	-5
7	3 - 5	42	42	-	-	32	-	-	32	-10
8	3 - 8	96	-	96	-	-	96	-	96	-
9	3 - 9	111	-	111	-	-	111	-	111	-
10	3 - 10	53	-	53	-	-	53	-	53	-
11	4 - 5	31	-	31	-	-	31	-	31	-
12	4 - 6	41	-	41	-	-	41	-	41	-
13	4 - 7	38	-	38	-	-	38	-	38	-
14	4 - 8	94	-	94	-	-	94	-	94	-
15	4 - 9	106	-	106	-	-	106	-	106	-
16	5 - 6	33	-	33	-	-	33	-	33	-
17	5 - 7	28	-	28	-	-	28	-	28	-
18	5 - 8	96	-	96	-	-	96	-	96	-
19	5 - 9	111	-	111	-	-	111	-	111	-
20	6 - 7	10	-	10	-	-	10	-	10	-
21	6 - 8	61	-	61	-	-	61	-	61	-
22	6 - 10	111	-	111	-	-	111	-	111	-
23	7 - 8	79	-	-	79	-	-	99	99	20
24	8 - 9	32	-	32	-	-	32	-	32	-
25	8 - 10	101	-	101	-	-	101	-	101	-
26	9 - 10	94	-	-	94	-	-	118	118	24
27	9 - 11	76	-	-	76	-	-	95	95	19
28	10 - 11	47	-	-	47	-	-	59	59	12

Table 5.2: Distances transformed into Travel Cost (Source: Research data)

From Table 5.2 it will be seen that road stretches designated as ‘Good’ (column ‘d’), ‘Normal’ (column ‘e’), and ‘Bad’ (column ‘f’) are multiplied by 0.75 (column ‘g’), 1.00 (column ‘h’), and 1.25 (column ‘i’) respectively. The road type labelled as ‘Normal’ is the most predominant in the region so it was given a weighting of 1.00 so that even after

applying the multiplier it will remain the same for the purpose of easy comparison with ‘Good’ and ‘Bad’ road types.

Having now transformed population into **demand** and factored road quality into raw distance measurements to obtain **travel cost** (now with all links assumed to be of uniform quality), we unify these in an updated network in Figure 5.2 to reflect the two sets of information.

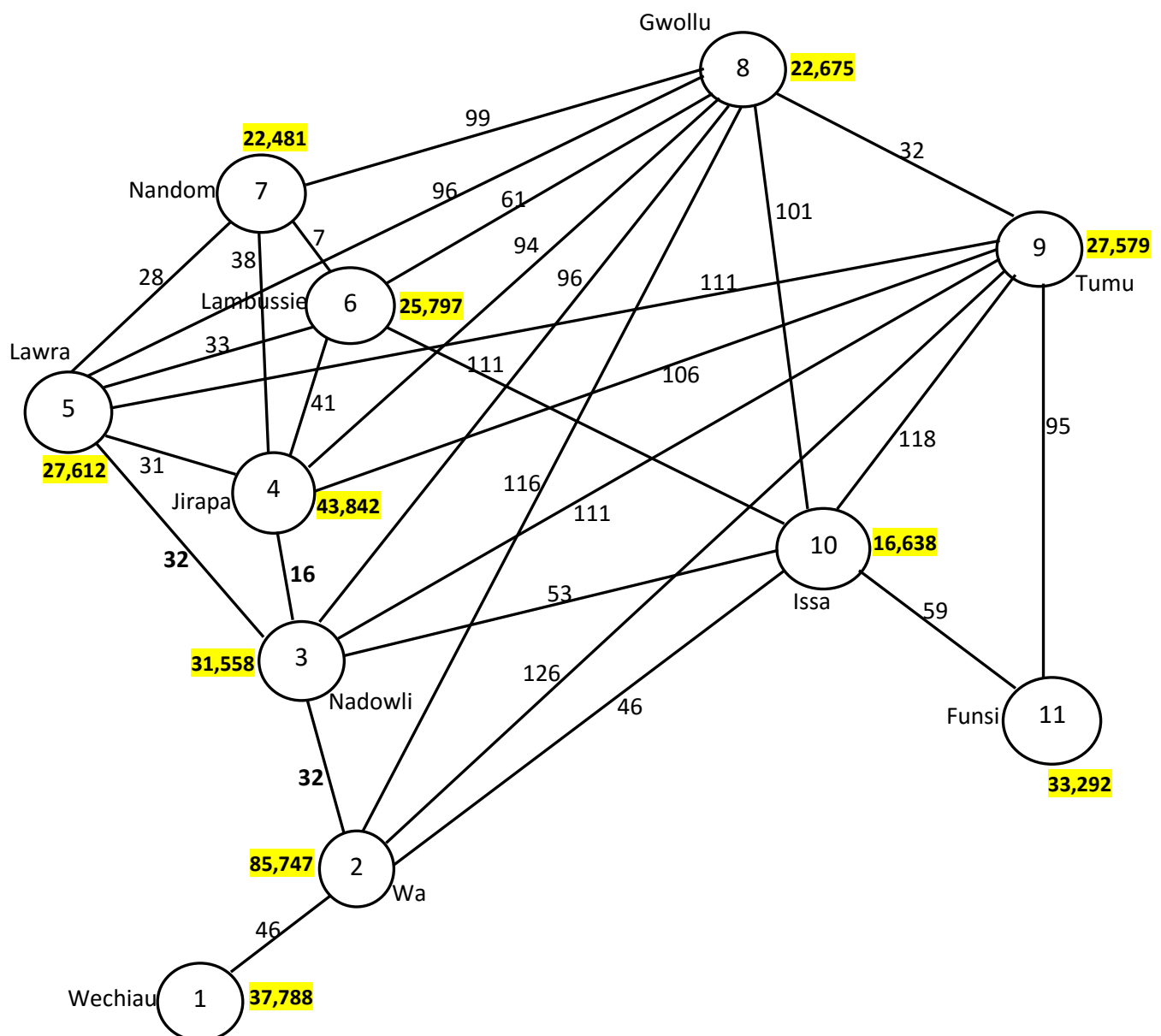


Figure 5.2: Network with modified demand and travel cost (Source: Research data)

The transformed distances or travel costs obtained above results in the cost (distance) matrix in Table 5.3 below. Note that if two locations are not directly connected by the links on the network the distance between them is labelled with the ' ∞ ' (infinity) sign.

Nodes		1	2	3	4	5	6	7	8	9	10	11
		Wech.	Wa	Nadowli	Jirapa	Lawra	Lambuss	Nandom	Gwollu	Tumu	Issa	Funsi
1	Wechiau	0	46	∞	∞	∞	∞	∞	∞	∞	∞	∞
2	Wa	46	0	32	∞	∞	∞	∞	116	126	46	∞
3	Nadowli	∞	32	0	16	32	∞	∞	96	111	53	∞
4	Jirapa	∞	∞	16	0	31	41	38	94	106	∞	∞
5	Lawra	∞	∞	32	31	0	33	28	96	111	∞	∞
6	Lambus.	∞	∞	∞	41	33	0	7	61	∞	111	∞
7	Nandom	∞	∞	∞	38	28	7	0	99	∞	∞	∞
8	Gwollu	∞	116	96	94	96	61	99	0	32	101	∞
9	Tumu	∞	126	111	106	111	∞	∞	32	0	118	95
10	Issa	∞	46	53	∞	∞	111	∞	101	118	0	59
11	Funsi	∞	∞	∞	∞	∞	∞	∞	∞	95	59	0

Table 5.3: Matrix showing Distances transformed into Travel Cost (Source: Research data)

Based on the above matrix, all-pairs shortest routes were computed using Floyd's algorithm executed with the *TORA Optimisation System* software that is included in a pack accompanying Hamdy Taha's *Operations Research: An Introduction – 8th edition*.

Nodes		1	2	3	4	5	6	7	8	9	10	11
		Wech.	Wa	Nadowli	Jirapa	Lawra	Lambuss	Nandom	Gwollu	Tumu	Issa	Funsi
1	Wechiau	0	46	78	94	110	135	132	162	172	92	151
2	Wa	46	0	32	48	64	89	86	116	126	46	105
3	Nadowli	78	32	0	16	32	57	54	96	111	53	112
4	Jirapa	94	48	16	0	31	41	38	94	106	69	128
5	Lawra	110	64	32	31	0	33	28	94	111	85	144
6	Lambus.	135	89	57	41	33	0	7	61	93	110	169
7	Nandom	132	86	54	38	28	7	0	68	100	107	166
8	Gwollu	162	116	96	94	94	61	68	0	32	101	127
9	Tumu	172	126	111	106	111	93	100	32	0	118	95
10	Issa	92	46	53	69	85	110	107	101	118	0	59
11	Funsi	151	105	112	128	144	169	166	127	95	59	0

Table 5.4: Distance/Cost matrix showing all-pairs shortest routes (Source: Research data)

Appendix 2 presents the TORA output showing all the shortest paths and their magnitudes. A part of that information has been used to update the distance/cost matrix as presented in Table 5.4 above.

5.3 Solution Approach

Now that all the data needed has been extracted and transformed, the next step is to implement a solution. The problem is modelled and solved as an uncapacitated p -median problem. The buffer stock warehouses being ‘uncapacitated’ means there is no limit to the amount of demand a facility can serve. In practice, this may mean building or acquiring facilities that have room for expansion, or locating additional facilities at the same node (same town or district) when the need arises.

5.3.1 The p -Median Formulation

The manner in which data was organised in the above sections was influenced by the mathematical structure of the uncapacitated p -median model formulation (Arabani & Farahani, 2012; Current et al., 2004; ReVelle et al., 2008) mentioned in Chapter 3. The model is hereby repeated and better explained:

$$\text{Minimise } Z = \sum_{i \in D} \sum_{j \in F} w_i d_{ij} y_{ij} \quad (5.1)$$

Subject to:

$$\sum_{j \in F} x_j = p \quad (5.2)$$

$$\sum_{j \in F} y_{ij} = 1 \quad \forall i \in D \quad (5.3)$$

$$y_{ij} \geq x_j \quad \forall i \in D, j \in F \quad (5.4)$$

$$x_j = \begin{cases} 1 & \text{if candidate node } j \text{ includes a facility, } \forall j \in F \\ 0 & \text{otherwise} \end{cases} \quad (5.5)$$

$$y_{ij} = \begin{cases} 1 & \text{if candidate node } j \text{ gives service to demand node } i, \forall i \in D, \forall j \in F \\ 0 & \text{otherwise} \end{cases} \quad (5.6)$$

Where

i = index of demand points (sources);

j = index of facility sites (destinations);

p = number of facilities opened.

D = total demand in the space of interest;

F = total number of potential facility locations;

w_i = weight associated with each demand point;

d_{ij} = distance between demand area i and facility j ;

The model consists of objective function (5.1) which seeks to minimise the average or total demand-weighted cost (distance) between customers and the facilities, subject to constraints (5.2) to (5.6). Constraint (5.2) fixes the number of facilities, (5.3) ensures that each demand node is assigned to exactly one facility; (5.4) links location variables x_j and allocation variables y_{ij} , so that demand at node i can only be assigned to a facility at node j ($y_{ij} = 1$) if a facility is located at node j ($x_j = 1$); and (5.5) and (5.6) represent binary conditions.

5.3.2 Implementing the Model

- **Demand-weighted cost (distance):** Since facility sites are not known at the onset, and every node is a potential facility site, we begin by computing d_{ij} for all pairs of nodes. With our 11-Node network, we will have 121 d_{ij} values as presented in Table 5.4. Also, w_i represents demand (or weight or load) associated with each node. For

instance, w_3 for our problem is the demand at Node 3 (Nadowli) which is 31,558. If, for example, a facility should locate at Node 5 (Lawra) to serve demand from Node 3 (Nadowli), the demand-weighted distance, $w_i d_{ij}$, between the customer (Node 3; Nadowli) and the facility (Node 5; Lawra) will be $w_5 d_{35}$, which is 31,558 (i.e. w_5) multiplied by 32 (i.e. d_{35} or distance between Nodes 3 to 5) to give us 1,009,856. As explained earlier, we begin by computing $w_i d_{ij}$ in similar manner for all 121 i to j pairs since we do not yet know where a facility will eventually be located. The third variable that completes the objective function (i.e. y_{ij}) is a binary condition. For instance, if no facility is eventually located at Lawra—or it is located but is not assigned to serve demand from Nadowli—the demand-weighted cost (distance) $w_3 d_{35} y_{35}$, will then amount to zero ($31,558 \times 32 \times 0$) and so will contribute nothing to the resulting *objective value*.

- **Number of Facilities (p):** The Upper West regional coordinator of NAFCO revealed the company is contemplating facilities in the region but does not yet know how many facilities the company might establish, nor at which locations. In the absence of such vital information the model was solved in the first instance as a 1-median to provide an optimal location should NAFCO choose to build only one facility. It was then re-solved as a 2-median, 3-median and so on, all the way up to $p = 10$ in order to cater for all kinds of possibilities. However, explanations on problem set-up and primary analysis is based on the 3-median.

5.3.3 Problem Set-up for Microsoft Excel

Table 5.5 presents a unified table showing both demand (w_i) and travel cost (d_{ij}). When the demand on a given row is multiplied by each travel cost on the row, we have the demand-weighted matrix in Table 5.6. The problem is then solved using these values.

Node	Demand	1	2	3	4	5	6	7	8	9	10	11
1	37,788	0	46	78	94	110	135	132	162	172	92	151
2	85,747	46	0	32	48	64	89	86	116	126	46	105
3	31,558	78	32	0	16	32	57	54	96	111	53	112
4	43,842	94	48	16	0	31	41	38	94	106	69	128
5	27,612	110	64	32	31	0	33	28	94	111	85	144
6	25,797	135	89	57	41	33	0	7	61	93	110	169
7	22,481	132	86	54	38	28	7	0	68	100	107	166
8	22,675	162	116	96	94	94	61	68	0	32	101	127
9	27,579	172	126	111	106	111	93	100	32	0	118	95
10	16,638	92	46	53	69	85	110	107	101	118	0	59
11	33,292	151	105	112	128	144	169	166	127	95	59	0

Table 5.5: Demand (w_i) and Travel cost (d_{ij}) (Source: Research data)

Node	1	2	3	4	5	6	7	8	9	10	11
1	0	1,738,248	2,947,464	3,552,072	4,156,680	5,101,380	4,988,016	6,121,656	6,499,536	3,476,496	5,705,988
2	3,944,362	0	2,743,904	4,115,856	5,487,808	7,631,483	7,374,242	9,946,652	10,804,122	3,944,362	9,003,435
3	2,461,524	1,009,856	0	504,928	1,009,856	1,798,806	1,704,132	3,029,568	3,502,938	1,672,574	3,534,496
4	4,121,148	2,104,416	701,472	0	1,359,102	1,797,522	1,665,996	4,121,148	4,647,252	3,025,098	5,611,776
5	3,037,320	1,767,168	883,584	855,972	0	911,196	773,136	2,595,528	3,064,932	2,347,020	3,976,128
6	3,482,595	2,295,933	1,470,429	1,057,677	851,301	0	180,579	1,573,617	2,399,121	2,837,670	4,359,693
7	2,967,492	1,933,366	1,213,974	854,278	629,468	157,367	0	1,528,708	2,248,100	2,405,467	3,731,846
8	3,673,350	2,630,300	2,176,800	2,131,450	2,131,450	1,383,175	1,541,900	0	725,600	2,290,175	2,879,725
9	4,743,588	3,474,954	3,061,269	2,923,374	3,061,269	2,564,847	2,757,900	882,528	0	3,254,322	2,620,005
10	1,530,696	765,348	881,814	1,148,022	1,414,230	1,830,180	1,780,266	1,680,438	1,963,284	0	981,642
11	5,027,092	3,495,660	3,728,704	4,261,376	4,794,048	5,626,348	5,526,472	4,228,084	3,162,740	1,964,228	0

Table 5.6: Demand-weighted Travel cost ($w_i d_{ij}$) (Source: Research data)

Figures 5.3 (a) to (c) and Table 5.7 respectively show the initial problem set-up and spreadsheet formulae employed.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	DEMAND													
2			Capital/											
3		District	Location	Node	Demand									
4	Wa West	Wechiau	1	37,788										
5	Wa Manucipal	Wa	2	85,747										
6	Nadowli-Kaleo	Nadowli	3	31,558										
7	Jirapa	Jirapa	4	43,842										
8	Lawra	Lawra	5	27,612										
9	Lambussie-Karni	Lambussie	6	25,797										
10	Nandom	Nandom	7	22,481										
11	Sissala-West	Gwollu	8	22,675										
12	Sissala-East	Tumu	9	27,579										
13	Daffiama-Bussie-Is	Issa	10	16,638										
14	Wa East	Funsi	11	33,292										
15			Total Demand	375,009										
16														
17	DISTANCE MATRIX (d_{ij})													
18	Node	1	2	3	4	5	6	7	8	9	10	11		
19	1	0	46	78	94	110	135	132	162	172	92	151		
20	2	46	0	32	48	64	89	86	116	126	46	105		
21	3	78	32	0	16	32	57	54	96	111	53	112		
22	4	94	48	16	0	31	41	38	94	106	69	128		
23	5	110	64	32	31	0	33	28	94	111	85	144		
24	6	135	89	57	41	33	0	7	61	93	110	169		
25	7	132	86	54	38	28	7	0	68	100	107	166		
26	8	162	116	96	94	94	61	68	0	32	101	127		
27	9	172	126	111	106	111	93	100	32	0	118	95		
28	10	92	46	53	69	85	110	107	101	118	0	59		
29	11	151	105	112	128	144	169	166	127	95	59	0		
30														

Figure 5.3 (a): Partial Initial problem set-up in Microsoft Excel: demand and travel cost (w_i and d_{ij}) (Source: Research data)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
30														
31	DEMAND-WEIGHTED DISTANCE MATRIX ($w_i d_{ij}$)													
32	Node	1	2	3	4	5	6	7	8	9	10	11		
33	1	0	1,738,248	2,947,464	3,552,072	4,156,680	5,101,380	4,988,016	6,121,656	6,499,536	3,476,496	5,705,988		
34	2	3,944,362	0	2,743,904	4,115,856	5,487,808	7,631,483	7,374,242	9,946,652	10,804,122	3,944,362	9,003,435		
35	3	2,461,524	1,009,856	0	504,928	1,009,856	1,798,806	1,704,132	3,029,568	3,502,938	1,672,574	3,534,496		
36	4	4,121,148	2,104,416	701,472	0	1,359,102	1,797,522	1,665,996	4,121,148	4,647,252	3,025,098	5,611,776		
37	5	3,037,320	1,767,168	883,584	855,972	0	911,196	773,136	2,595,528	3,064,932	2,347,020	3,976,128		
38	6	3,482,595	2,295,933	1,470,429	1,057,677	851,301	0	180,579	1,573,617	2,399,121	2,837,670	4,359,693		
39	7	2,967,492	1,933,366	1,213,974	854,278	629,468	157,367	0	1,528,708	2,248,100	2,405,467	3,731,846		
40	8	3,673,350	2,630,300	2,176,800	2,131,450	2,131,450	1,383,175	1,541,900	0	725,600	2,290,175	2,879,725		
41	9	4,743,588	3,474,954	3,061,269	2,923,374	3,061,269	2,564,847	2,757,900	882,528	0	3,254,322	2,620,005		
42	10	1,530,696	765,348	881,814	1,148,022	1,414,230	1,830,180	1,780,266	1,680,438	1,963,284	0	981,642		
43	11	5,027,092	3,495,660	3,728,704	4,261,376	4,794,048	5,626,348	5,526,472	4,228,084	3,162,740	1,964,228	0		
44														

Figure 5.3 (b): Partial Initial problem set-up: demand-weighted travel cost ($w_i d_{ij}$) (Source: Research data)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
44														
45														
46	LOCATION AND ALLOCATION (y_{ij}, x_j)													
Node	1	2	3	4	5	6	7	8	9	10	11	Total		
1	0	0	0	0	0	0	0	0	0	0	0	0		
2	0	0	0	0	0	0	0	0	0	0	0	0		
3	0	0	0	0	0	0	0	0	0	0	0	0		
4	0	0	0	0	0	0	0	0	0	0	0	0		
5	0	0	0	0	0	0	0	0	0	0	0	0		
6	0	0	0	0	0	0	0	0	0	0	0	0		
7	0	0	0	0	0	0	0	0	0	0	0	0		
8	0	0	0	0	0	0	0	0	0	0	0	0		
9	0	0	0	0	0	0	0	0	0	0	0	0		
10	0	0	0	0	0	0	0	0	0	0	0	0		
11	0	0	0	0	0	0	0	0	0	0	0	0		
59														
60														
61	SUMMARY OF LOCATIONS ($\sum x_j = p$)													
Node	1	2	3	4	5	6	7	8	9	10	11	Total		
1	0	0	0	0	0	0	0	0	0	0	0	0		
64														
65												Total Cost	0	
66														

Figure 5.3 (c): Partial Initial problem set-up: locations and allocations (x_j and y_{ij})

(Source: Research data)

Cell	Formula	Copied to
M65	=SUMPRODUCT(B33:L43,B48:L58)	
B33	=B19*\$E\$4	C33:L33
B34	=B20*\$E\$5	C34:L34
B35	=B21*\$E\$6	C35:L35
Etc...up to...		...
B43	=B29*\$E\$14	C43:L43
M48	=SUM(B48:L48)	M49:M58
B63	=B48	
C63	=C49	
D63	=D50	
Etc...up to...	...	
L63	=L58	
M63	=SUM(B63:L63)	

Table 5.7: Spreadsheet Formulae (Source: Research data)

In line with cell references in the above Microsoft Excel screenshots and $p = 3$, the *Objective* and *Constraints* are set up as follows:

- Minimise M65 by Changing B48:L58 [Objective 5.1]
- Subject to:
 - B48:L58=binary [Constraints 5.5 & 5.6]
 - M48:M58=1 [Constraint 5.3]
 - B48:B58<=B48
 - C48:C58<=C49
 - D48:D58<=D50
 - E48:E58<=E51
 - F48:F58<=F52
 - G48:G58<=G53
 - H48:H58<=H54
 - I48:I58<=I55
 - J48:J58<=J56
 - K48:K58<=K57
 - L48:L58<=L58
 - M63=3

[Constraint 5.4]

[Constraint 5.2; $p = 3$]

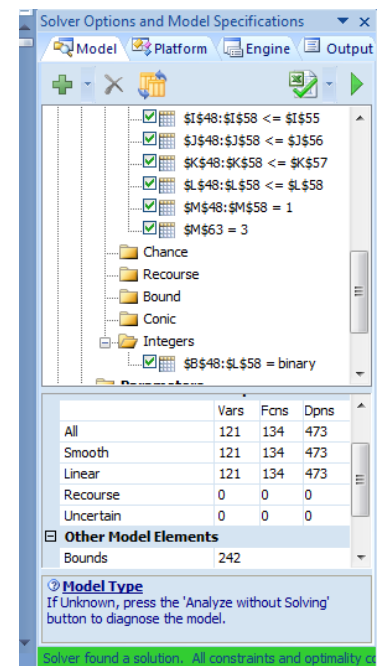
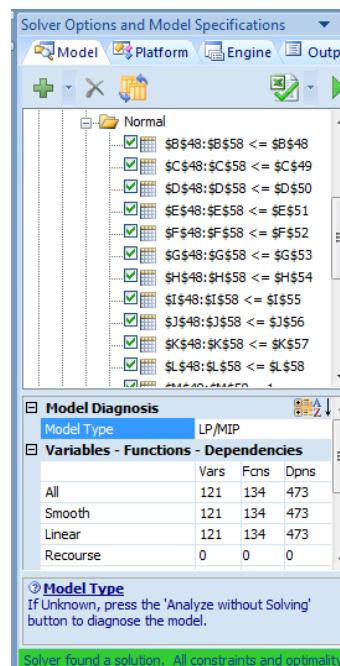
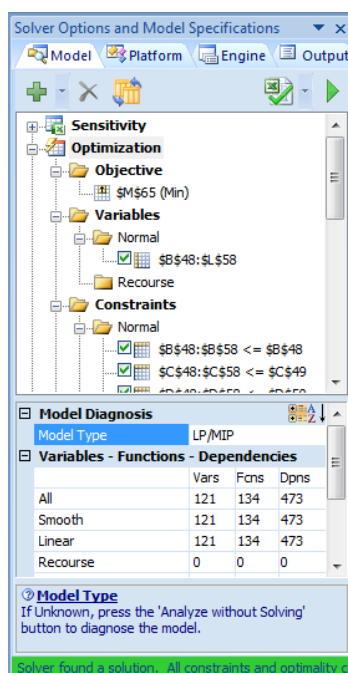


Figure 5.4: Premium Solver Constraints set-up – different views (Source: Research data)

5.3.4 Premium Solver and Computational Issues

The *Premium Solver* platform diagnosed our 11-Node problem as an LP/MIP model with 121 variables (Premium Solver limit = 8,000), 133 normal constraints (limit = 8,000), 242 bound constraints (limit = 16,000), and 121 integer constraints (limit = 2,000), meaning it is beyond the capacity of the Microsoft Excel Solver Add-In. The *Premium Solver 2014-R2* platform is a commercially available upgrade of the Solver Add-In. A 15-day trial version was downloaded from Frontline Solvers (solver.com), developers of the original Excel Solver Add-In, for the purpose of analysing the current problem.

The platform has five engines namely, the *Standard LP/Quadratic*, *Standard LSGRG Nonlinear*, *Standard Evolutionary*, *Standard Interval Global*, and the *Standard SOCP Barrier* engines. Of these, the *Standard LP/Quadratic* and *Standard Evolutionary* engines appear to be the most useful for our problem. Several trials performed on the substantive 11-Node problem but with randomly changing values of p on an Intel(R) Pentium(R) CPU G2030 @ 3.00GHz; 2.00 GB RAM desktop computer with Microsoft Excel 2007 show that the former generates a solution in an average of 2.03 seconds as compared to 38 seconds for its evolutionary counterpart. Furthermore, the *Standard Evolutionary* engine's solutions are slightly worse off numerically, and comes with the message: "Solver cannot improve the current solution. All constraints are satisfied."

An inbuilt help guide interprets this caution to mean that further improvements were not possible though optimality may not have been attained, but that all constraints were satisfied. It has been observed, however, that if Solver is re-run the *Standard Evolutionary* engine produces an optimal solution similar to that of the *Standard LP/Quadratic* engine.

Messages accompanying the *Standard LP/Quadratic* result read: “Solver found a solution. All constraints and optimality conditions are satisfied.” This means that the *Standard LP/Quadratic* engine finds the optimal or “best” solution under the circumstances.

Trials similar to the above were run using the *Standard LSGRG Nonlinear*, *Standard Interval Global*, and the *Standard SOCP Barrier* engines as well. The *Standard LSGRG Nonlinear* and *Standard Interval Global* both generated a message that reads: “Solver could not find a feasible solution” after an average of 1.81 seconds; and the values generated all violated the integer bounds in both cases. The *Standard SOCP Barrier* engine performed 4356 iterations with objective values hovering around zero and negative figures and then terminated by itself after 661 seconds with a message reading: “Solver could not find a feasible solution.”

From the foregoing, it was concluded that the *Standard LP/Quadratic* engine is the most appropriate Microsoft Excel based software tool for tackling a problem of this type and size.

5.4 Solutions

5.4.1 The 3-Median Solution

After solving the 3-median problem, the *location* and *allocation* part of the spreadsheet set-up (Figure 5.3c) changes as in Figure 5.4 showing the three (3) facilities (see Cell M63) optimally located at Wa (Node 2; Cell C49 or C63), Jirapa (Node 4; Cell E51 or E63) and Tumu (Node 9; Cell J56 or J63) with the objective value being 9,664,791. This represents the total demand-weighted cost to be incurred if the above location configuration is implemented.

Note that each of the three facilities forms neighbourhoods around themselves, which are the demand areas they serve. For example, the facility at Node 2 also serves demand from Nodes

1 and 10 (i.e. cells with the value of 1, rather than 0, in the same column). Likewise, Node 9 serves its own demand as well as those of Nodes 8 and 11, and so on.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
44														
45														
46	LOCATION AND ALLOCATION (y_{ij}, x_j)													
47	Node	1	2	3	4	5	6	7	8	9	10	11	Total	
48	1	0	1	0	0	0	0	0	0	0	0	0	1	
49	2	0	1	0	0	0	0	0	0	0	0	0	1	
50	3	0	0	0	1	0	0	0	0	0	0	0	1	
51	4	0	0	0	1	0	0	0	0	0	0	0	1	
52	5	0	0	0	1	0	0	0	0	0	0	0	1	
53	6	0	0	0	1	0	0	0	0	0	0	0	1	
54	7	0	0	0	1	0	0	0	0	0	0	0	1	
55	8	0	0	0	0	0	0	0	0	1	0	0	1	
56	9	0	0	0	0	0	0	0	0	1	0	0	1	
57	10	0	1	0	0	0	0	0	0	0	0	0	1	
58	11	0	0	0	0	0	0	0	0	1	0	0	1	
59														
60														
61	SUMMARY OF LOCATIONS ($\sum x_j = p$)													
62	Node	1	2	3	4	5	6	7	8	9	10	11	Total	
63	1	0	1	0	1	0	0	0	0	1	0	0	3	
64														
65													Total Cost	9664791
66														

Figure 5.5: 3-Median solution (Source: Research data)

To verify the objective value, we sum the demand-weighted travel cost ($w_i d_{ij}$) values for the cells whose corresponding y_{ij} values are now 1 (i.e. Cells C33, C34, C42, E35, E36, E37, E38, E39, J40, J41, J43 matched against C48, C49, C57, E50, E51, E52, E53, E54, J55, J56, J58 respectively).

5.4.2 Impact of 3-Median on Road Network

Though this problem was not modelled to endogenously solve for road network design as is done with FLND problems, the solutions do provide pointers to which roads do have any relevance for the solution set. With the location and allocation configuration recommended by the 3-median solution, we delete all other links from the network except those linking non-facility nodes to their respective nearest facilities (see Figure 5.5).

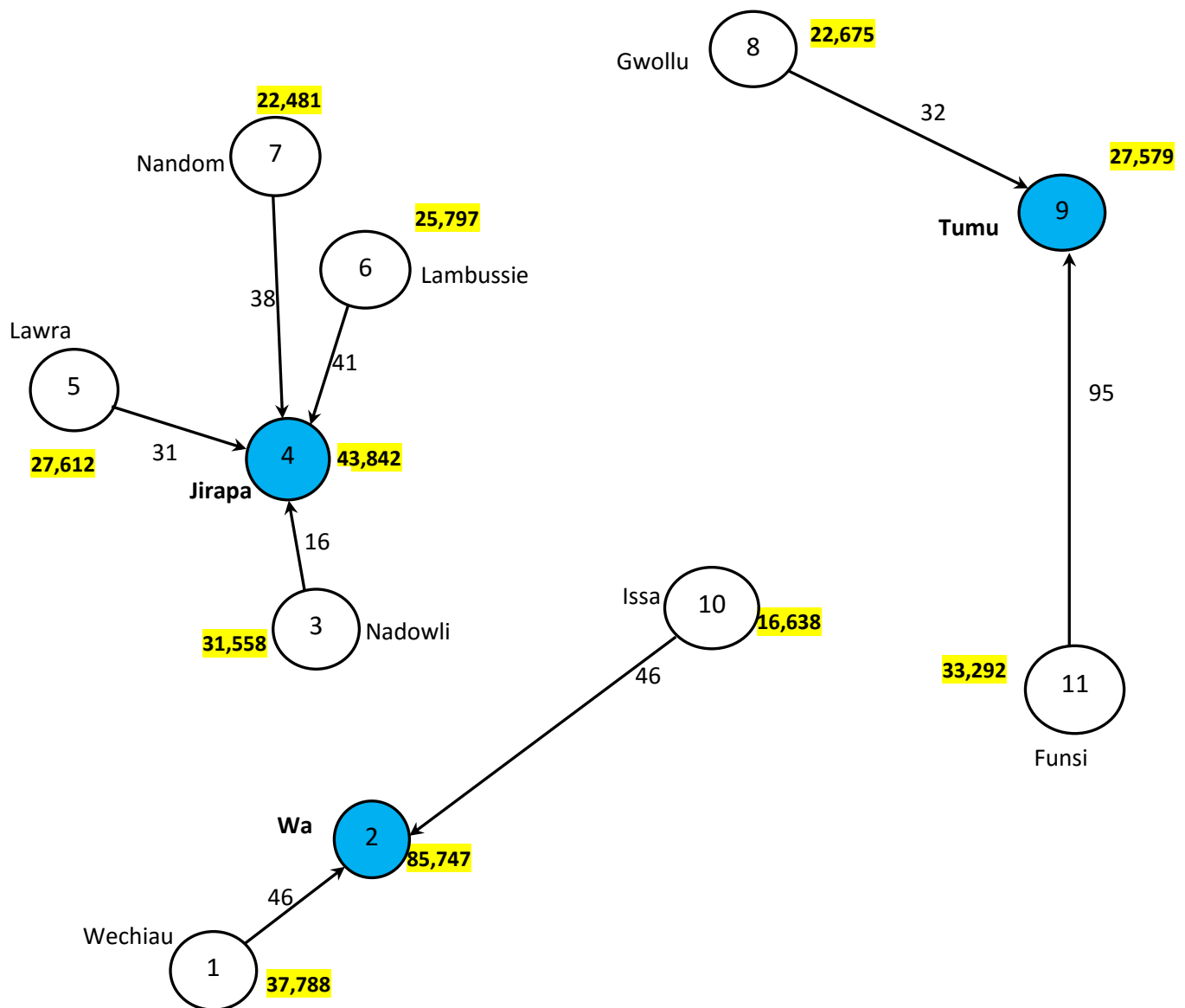


Figure 5.6: Modified network for 3-median solution (Source: Research data)

The results suggest the road links connecting Wechiau-Wa-Issa, Gwollu-Tumu-Funsi (via Walembale), Jirapa-Lawra, and Nadowli-Jirapa-Lambussie-Nandom are the only ones with any relevance to the 3-median solution. It should be noted that Jirapa-Lambussie and Jirapa-Nandom are not different road links as the network might have portrayed. It is the road from Jirapa to Nandom that branches off at a point to Lambussie. After the three facilities are established, an improvement on these roads would reduce travel cost for customers. As the

number of facilities increase, road links that are essential for smooth service reduces, and vice versa.

5.4.3 Changing the Number of Facilities

Having solved the problem severally with the number of facilities being revised between one ($p = 1$) to ten ($p = 10$), the location and allocation configurations generated are summarised in Table 5.8 (a) and (b). Note that *average travel cost* per cluster or neighbourhood is based on the travel cost incurred by customers from **non-facility nodes** traveling to the nearest facility to obtain service. Likewise, *overall average travel cost* is computed by including each and every component **inter-district** travel cost from every neighbourhood. As an example, the 3-median solution yields three per cluster (i.e. Wa, Jirapa and Tumu) average travel costs, plus an overall average travel cost, which are computed as shown below:

$$\begin{aligned}
 \text{Average travel cost} &= (d_{12} + d_{10,2})/2; (d_{34} + d_{54} + d_{64} + d_{74})/4; (d_{89} + d_{11,9})/2 \\
 &= (46 + 46)/2; (16 + 31 + 41 + 38)/4; (32 + 95)/2 \\
 &= 92/2; 126/4; 127/2 \\
 &= 46; 31.5; 63.5
 \end{aligned}$$

$$\begin{aligned}
 \text{Overall average travel cost} &= (d_{12} + d_{10,2} + d_{34} + d_{54} + d_{64} + d_{74} + d_{89} + d_{11,9})/8 \\
 &= (46 + 46 + 16 + 31 + 41 + 38 + 32 + 95)/8 \\
 &= 345/8 \\
 &= 43.1
 \end{aligned}$$

No. of Facilities	Objective Value	Optimal Locations	Customers/Demand Assigned	Average Travel Cost per Cluster	Overall Average Travel Cost
1	19,809,414	Nadowli	All	64.1	64.1
2	13,823,219	Wa	Wa, Wechiau, Nadowli, Issa, Funsi	57.3	51.6
		Lambussie	Lambussie, Jirapa, Lawra, Nandom, Gwollu, Tumu	47.0	
3	9,664,791	Wa	Wa, Wechiau, Issa	46.0	43.1
		Jirapa	Jirapa, Nadowli, Lawra, Lambussie, Nandom	31.5	
		Tumu	Tumu, Gwollu, Funsi	63.5	
4	6,502,051	Wa	Wa, Wechiau, Issa	46.0	35.6
		Jirapa	Jirapa, Nadowli, Lawra, Lambussie, Nandom	31.5	
		Tumu	Tumu, Gwollu	32.0	
		Funsi	Funsi	-	
5	4,687,839	Wa	Wa, Wechiau, Issa	46.0	29.2
		Jirapa	Jirapa, Nadowli	16.0	
		Tumu	Tumu, Gwollu	32.0	
		Funsi	Funsi	-	
		Nandom	Nandom, Lawra, Lambussie,	17.5	
6	2,949,591	Wa	Wa, Issa	46.0	25.8
		Jirapa	Jirapa, Nadowli	16.0	
		Tumu	Tumu, Gwollu	32.0	
		Funsi	Funsi	-	
		Nandom	Nandom, Lawra, Lambussie,	17.5	
		Wechiau	Wechiau	-	
7	2,153,243	Wa	Wa, Issa	46.0	25.3
		Jirapa	Jirapa, Nadowli	16.0	
		Tumu	Tumu, Gwollu	32.0	
		Funsi	Funsi	-	
		Wechiau	Wechiau	-	
		Lawra	Lawra	-	
		Lambussie	Lambussie, Nandom	7.0	
8	1,387,859	Wa	Wa	-	18.3
		Jirapa	Jirapa, Nadowli	16.0	
		Tumu	Tumu, Gwollu	32.0	
		Funsi	Funsi	-	
		Wechiau	Wechiau	-	
		Lawra	Lawra	-	
		Lambussie	Lambussie, Nandom	7.0	
		Issa	Issa	-	

Table 5.8(a): Location & Allocation configurations: 1 to 8 Facilities (Source: research data)

No. of Facilities	Objective Value	Optimal Locations	Customers/Demand Assigned	Average Travel Cost per Cluster	Overall Average Travel Cost
9	662,295	Wa	Wa	-	11.5
		Jirapa	Jirapa, Nadowli	16.0	
		Tumu	Tumu	-	
		Funsi	Funsi	-	
		Wechiau	Wechiau	-	
		Lawra	Lawra	-	
		Lambussie	Lambussie, Nandom	7.0	
		Issa	Issa	-	
		Gwollu	Gwollu	-	
10	157,367	Wa	Wa	-	7.0
		Jirapa	Jirapa	-	
		Tumu	Tumu	-	
		Funsi	Funsi	-	
		Wechiau	Wechiau	-	
		Lawra	Lawra	-	
		Lambussie	Lambussie, Nandom	7.0	
		Issa	Issa	-	
		Gwollu	Gwollu	-	
		Nadowli	Nadowli	-	

Table 5.8(b): Location & Allocation configurations: 9 to 10 Facilities (Source: research data)

It would be observed that, as the investment in facilities increase (as in more facilities being established), average inter-district travel costs decrease, and so does the objective value or demand-weighted travel cost; implying lesser and lesser investment being required in network (road) construction. The reverse would, of course, hold true in conformance with the observation that facility building costs and network construction costs trade-off against each other as they relate inversely (Daskin et al., 1993; Melkote & Daskin, 2001a, 2001b). Further discussions on the above findings are made in the next chapter.

CHAPTER 6: DISCUSSION AND CONCLUSIONS

The overall aim of this work has been to employ an Operations Research approach in optimally locating grains buffer stock warehouses in the Upper West region of Ghana as a means to help mitigate food insecurity. It was also expected that any advancement in location analysis that might be found in the course of the research would be a treasured by-product. The purpose of this chapter is to clarify the solutions sets, and to show how this work enriches practice, policy and future research. It commences with a discussion of the results presented in Chapter 5, goes on to draw general conclusions on the entire research, puts forward recommendations that flow from the conclusions, and eventually suggests angles from which this work could be advanced.

6.1 Discussion of Results

Studying the location-allocation configurations for the various solutions presented in Table 5.8, we observe as follows:

6.1.1 Alternative Solutions

1. If only **one** facility is to be established, the optimal location is **Nadowli**. The average travel cost in this case would be 64.1, the demand-weighted travel cost or objective value—which represents the total cost incurred if all the demand (i.e. prospective customers) from outside Nadowli district were to travel exactly once to Nadowli to access service—stands at 19,809,414.

2. If the number of facilities is to be **two**, Nadowli will no longer be an optimal location. Rather, the two facilities will have to be placed at **Wa** and **Lambussie**. It is interesting that if the number of facilities to be built is incrementally added on in this manner, Nadowli does not become an optimal site again, unless up to 10 facilities are built. With two facilities, customers from Wechiau, Nadowli, Issa and Funsu will access the Wa warehouse at an average travel cost of 57.3, while all other customers go to Lambussie at a lower cost of 47.0 on average. With this configuration, the average travel cost for all inter-district customers is 51.6. Objective value at this point reduces to 13,823,219, meaning a second facility would reduce the aggregate demand-weighted total cost to 69.8% of what would have been incurred if there were only one facility.
3. For **three** facilities, the optimal sites should be **Wa, Jirapa** and **Tumu**. Here again, Lambussie falls out of optimality while two new locations become optimal. Lambussie only becomes optimal again if up to seven facilities are located. On siting three facilities, the burden of traveling reduces considerably to 43.1 overall, while that for the Wa, Jirapa and Tumu neighbourhoods are 46.0, 31.5 and 63.5 respectively. Comparing, it is clear customers from Gwollu and Funsu who are travelling to Tumu would do so at a relatively prohibitive cost. With the Gwollu-Tumu cost being 32, customers from Funsu (cost is 95) will be the ones bearing the brunt; no wonder at the next opportunity (i.e. when facilities are increased to four in number), the model locates one at Funsu. Also, establishing three facilities further lessens the overall burden of travelling down to 6,502,051 which constitutes less than half (48.8%) of what prevails for a single facility.
4. Supposing the facilities are **four**, the optimal locations will be **Wa, Jirapa, Tumu** and **Funsu**. The only difference here from the previous solution is that, Funsu, which was highly penalised under the 3-median solution now gets a facility of its own, and would

have no need to travel elsewhere, reducing the overall average travel cost to 35.6. At this point demand-weighted travel cost whittles down to about one-third (32.8%) of what the residents would incur if only one facility were to be built.

5. If the facilities are to be **five** in number, the locations will be all of the above, plus **Nandom**; overall average cost reducing from 35.6 to 29.2. Again, building five facilities means customers as a whole would save three-quarters of the cost they would have borne if one facility were to be opted for.
6. Again, the **6-median** solution maintains every location in the 5-median and adds on **Wechiau**. Whereas Wechiau ever becomes optimal only if up to six warehouses are planned, it remains in the optimal solution set from this point onwards; and Nandom falls out of solution beyond this point never to return unless each and every location has to get a facility (i.e. 11-median). Overall average cost at this level is 25.8; and objective value is 2,949,591 (i.e. 14.9% of cost if $p = 1$).
7. If NAFCO opts to establish **seven** facilities, all the locations recommended for the 6-median except Nandom stay in solution; **Lambussie** returns and **Lawra** enters solution for the first time. If overall average distance is the only basis of increasing facility numbers, then there is little to choose between the 6-median and 7-median solutions given the minimal reduction in travel cost (from 25.8 to 25.3). In that sense it may not be too economical building seven facilities; it should either be six or eight. Aggregate travel cost between the two, however, is a bit more disparate, reducing from 2,949,591 to 2,153,243; representing just 10.9% of that of the 1-median solution.
8. From the **eight** to **tenth** facilities, all the locations for the 7-median remain optimal and **Issa**, **Gwollu** and **Nadowli** are added in that order. This implies 18.3, 11.5, and 7.0

overall average costs; and objective values of 1,387,859 (7.0% of aggregate cost for 1-median), 662,295 (3.3%) and 157,367 (0.8%) respectively.

9. Needless to say, solving the 10-median problem is the limit for an 11-Node problem. Solving for 11-medians out of 11 locations is a straight forward matter that does not need any modelling with all the trouble that goes with it.

6.1.2 Additional Comments

1. *Effect of Travel Cost Conversion:* Though Wechiau and Funsu stays in solution from the 6-median onwards they serve only their own demands. In the case of Wechiau, this may be because demand from every other area would have to go through Wa in order to get there; and once there is a facility at Wa, the one at Wechiau can only serve its own demand. The situation would have been different if the facilities were capacitated. For Funsu, it is clear the nature of the road is the problem. Ordinarily, if there is a warehouse at Funsu it should serve Issa, instead of the former going to Wa for service. With the unconverted distances, Issa-Funsu is 47km and Issa-Wa is 54km. While Issa-Wa is tarred up to a point, reducing travel cost from 54 to 46, Issa-Funsu has deteriorated enough to increase the travel cost from 47 to 59. Thus, the exercise this research embarked on to account for road quality has brought out a reality that would have been overlooked otherwise.
2. *Solutions are Independent:* The situation that made certain locations remain in the optimal solution as the number of facilities is increased is a mere coincidence, as several trials performed based on different input data proved a more erratic behaviour. This means that each solution set is an optimal solution in its own right, unrelated to what

happens with another solution based on a different value of p . Thus, the above location recommendations are not to be taken as a ranking of which locations are more important.

3. *Long term Planning*: In order for the implementing organisation to take advantage of the optimal solutions sets generated above, it has to decide beforehand how many facilities it is going to establish within the Upper West region in the medium to long term, even if all the resources are not yet in hand. This way, the specific solution set could be chosen out of the above. For example, if NAFCO knows it will be building only one facility in the region within the medium to long term, then the optimal location is Nadowli. Likewise, if the plan is to build two, then Wa and Lambussie would be chosen even if both warehouses cannot be built immediately. If on the other hand, NAFCO opts for any one of the above solutions, only to decide to add on more facilities then optimality as far as the above solution sets are concerned may no longer be achieved. In such a situation the problem has to be remodelled taking the existing facilities as open and fixed.

6.1.3 Comparing Solution from Greedy Heuristics

It has been demonstrated in Chapter 3 how the *greedy* or *myopic* heuristics locate facilities one after the other, instead of doing so in a single solution attempt; and how this approach usually leads to sub-optimal solutions. When the steps outlined earlier for the greedy heuristics were followed, the solution for the 1-median was unsurprisingly the same as what the exact method employed on the Premium Solver platform achieved: Nadowli. However, the 2-median solution added Wa, and the 3-median added on Jirapa. Comparing this to the 3-median solution from Premium Solver's exact optimisation approach, we make the following observations:

- Nadowli serves no other demand aside of itself.
- The Wa facility also serves Wechiau, Issa, and Funsu at an average travel cost of 65.7.
- The facility at Jirapa clustered all other nodes around itself with an average cost of 62.0. From this and the above case, it will be seen that the only occasion when the solution from the Premium Solver came up with such high cost was when only one facility was located.
- The overall average travel cost amounts to 46.1 as against 43.1 for the exact 3-median solution discussed earlier, and at 13,822,007, the objective value exceeds that from the exact solution approach (9,664,791; see Table 5.8a) by more than 30%.

Obviously, solutions from the substantive approach adopted are much more superior.

6.2 **Conclusions**

This section revisits the objectives to assess the extent to which they have been met. To recap, the three specific objectives the research sets out to achieve are:

1. To *assess* the mechanisms by which Ghana runs its grains buffer stock programme, especially in the Upper West region of the country.
2. To *review* different location models and solution approaches in order to adopt the most suitable for effectively appraising, modelling and solving the problem of finding optimal locations for cereal buffer stock warehouse facilities within the study area.

3. To *formulate* conclusions from the resulting location-network configurations, offer recommendations on logistics issues in mitigating food insecurity, and advance the frontiers of knowledge in the computational and practical aspects of location analysis.

The following conclusions are hereby advanced, without attempting to necessarily categorise them into strict conformance with the three objectives:

1. *Operation Mechanisms of NAFCO*: As explored in Chapter 2 (Context of the Study), the workings of the National Food Buffer Stock Company has been found to be such that its agents do reach out to the remotest of communities to purchase produce, but the company does not consider populations from these localities as its primary customers should their own stocks run out. Though it was meant to be an independent commercial trader, the company still relies on the benevolence of the state through the Ministry of Food and Agriculture to manage aspects of its programmes.
2. *Model and Heuristic Typologies*: In the course of reviewing the literature, two very important observations were made that have the potential to significantly change the way location model typologies are organised and studied. If for no other reason, this will help in defining the basic concepts of the subject for the benefit of those new to it. These are:
 - a) In classifying location models into *static* and *dynamic*, most researchers have taken the view that the two are respectively linked to the *space* and *time* dimensions of modelling. This research takes the view that the above thinking, though widespread, is not fully accurate. The argument here is that static models result when we are considering only a solution *space* (such as the geographic space of UWR considered in this research), but that dynamic models consider

both *space* and *time horizon*. Hence the right labelling for dynamic models should have been ‘space and time’—not only ‘time’ as is usually done.

- b) It was also observed that the most dominant system of classifying location models could actually be conceived of as dealing with the same problem type but with a progressive deepening of restrictions. Specifically, when you take a continuous model and restrain it from freely locating anywhere within a solution space, you have a discrete problem, which in turn becomes a network problem if a restriction is imposed on specific access routes between the discrete possible location sites. Again if instead of taking the network as given, it is actually modelled endogenously into the model, we have combined facility location and network design (FLND) problem. To the best of our knowledge no author has yet adopted this simple but profound characterisation directly in explaining the relationships between the model types. Consequently, the study has consolidated this idea into a new proposed *location-network hierarchy* framework (Figure 3.1).

3. *Optimal Locations*: On the substantive problem that constitutes the subject for this research, optimal locations have been found depending on how many facilities NAFCO plans to establish in the region. For example, if there is going to be only one facility the optimal site is Nadowli in the Nadowli-Kaleo district. However, if NAFCO wishes for two facilities, it has to locate them at Wa and Lambussie. The concept of optimality is such that depending on the number of facilities envisaged, each solution set is uniquely optimal in its own right and remains so if and only if exactly the number of facilities the said solution is designed for is what will actually be built.

4. *The Impact of Location on Road Network:* It has also been established that if even a network problem does not endogenously solve for network design, the optimal solution for a network problem can provide useful inferences for planning road or other network improvement projects. For example, when three facilities (at Wa, Jirapa and Tumu) were recommended by our model it showed that the roads that are relevant for lessening travel cost to these facilities are: Gwollu-Tumu-Walembale-Funsi, Wechiau-Wa-Issa, Nadowli-Jirapa-Nandom (including branch road to Lambussie), and Jirapa-Nadowli. Thus, investments made into facilities vary inversely, though not necessarily proportionally, with those spent on road (network) construction.
5. *Computational Approach:* Lastly, it has been realised from this research that location and other combinatorial problem analysts are failing to explore the benefits offered by Microsoft Excel based software tools. For example, out of the hundreds of scholarly works on combinatorial analysis viewed or reviewed for this research, only two (Ipsilandis, 2008; Lee & Yang, 2009) applied this despite the spreadsheet's ubiquity, flexibility, applicability, and computational efficiency among others.

6.3 **Recommendations**

Based on the conclusions drawn, we make the following recommendations:

1. Researchers in location science should take another look at the issue of classification of model typology in order to remove all the vagueness there might be, as a streamlined system is the starting point for deepening knowledge in the field. In this direction, other researchers should critique the proposed *location-network hierarchy* framework for fine tuning if any.

2. Much as the optimal location solutions obtained in this research is recommended to NAFCO, we caution that anyone implementing these or other multiple optimisation recommendations ought to do thorough planning before adopting a solution so as to ensure that assumptions under which a problem was developed is same as what the implementer has in mind.
3. Institutions in charge of constructing and rehabilitating roads should take food security issues into consideration in their activities, and to do so based on the principles of optimisation, when planning road network improvement in the Upper West region and elsewhere.
4. More work has to be done to make the spreadsheet approach more widely accepted and applied in solving combinatorial problems, as they are user friendlier than the conventional software that are based on algebraic language. In this direction, more trials aimed at comparing the computational efficiency of spreadsheet packages with those of competing combinatorial software are needed.

6.4 Further Research

Two directions are suggested for future research:

1. As explained in the Methodology, the entire demand in each district has been lumped together and assigned to their respective district capitals in order to make the problem less tractable. While this may have sufficed for the time being it will be interesting how the solution might alter if the various major towns, at least, in each district are

modelled as individual demand areas. This will help throw more light on the issue of demand aggregation versus the trade-off between model accuracy and tractability.

2. Other location models that may fit the buffer stock warehouses problem should be used to re-solve it in order to determine if any of these might be a more suitable.
3. Elements of uncertainty could be introduced into the current problem to transform it into a dynamic location problem.

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APPENDICES

APPENDIX 1: TRANSFORMATION OF DEMAND DATA

District	Population (a)	Total No. of Households (b)	No. of Farmer Households (c)	Individuals from Households with own Food Crop farms				Demand			
				% of Farmer Households Growing Crop (d)	Est. No. of Crop Growing Households (e) = (c)*(d)	Average size of Farmer Households (f)	Est. Pop. from Crop Growing Households (g) = (e) * (f)	Est. Pop. from non-Crop Households (h) = (a) - (g)	For Crop Household members (i) = (g)* ⁵ / ₁₂	For non-Crop Households members (j) = (h)* ¹² / ₁₂	Total Demand (k) = (i) + (j)
Wa West	81,348	11,486	10,524	97.2	10,229	7.3	74,674.09	6,673.91	31,114	6,674	37,788
Wa Municipal	107,214	18,891	5,841	82.9	4,842	7.6	36,800.64	70,413.36	15,334	70,413	85,747
Nadowli-Kl*	61,804	9,959	8,305	94.6	7,856	6.6	51,850.97	9,953.03	21,605	9,953	31,558
Jirapa	88,402	13,911	11,508	96.2	11,071	6.9	76,387.80	12,014.20	31,828	12,014	43,842
Lawra**	55,633	9,159	7,722	97.2	7,506	6.4	48,036.34	7,596.66	20,015	7,597	27,612
Lambussie-Kn	51,654	7,524	6,325	96	6,072	7.3	44,325.60	7,328.40	18,469	7,328	25,797
Nandom**	45,296	7,458	6,287	97.2	6,111	6.4	39,110.85	6,185.15	16,296	6,185	22,481
Sissala-West	49,573	7,116	6,535	98	6,404	7.2	46,110.96	3,462.04	19,213	3,462	22,675
Sissala-East	56,528	8,652	7,339	96.6	7,089	7	49,626.32	6,901.68	20,678	6,902	27,579
Daffiama-B-Issa*	32,584	5,251	4,378	94.6	4,142	6.6	27,336.61	5,247.39	11,390	5,247	16,638
Wa East	72,074	10,768	10,167	97.6	9,923	6.7	66,484.05	5,589.95	27,702	5,590	33,292

* The Daffiama-Bussie-Issa district was part of Nadowli-Kaleo District when the 2010 Census was conducted. Data for for columns (b) and (c) are pro rata calculations based on their respective populations as against their joint population of 94,388. For columns (d) and (f), data for the old Nadowli district was replicated for Daffiama-Bussie-Issa.

** At the time of 2010 Census, Nandom was part of Lawra with a total joint population of 100,929. Data for columns (b), (c), (d) and (f) are generated as explained above.

APPENDIX 2**SHORTEST ROUTE -- ORIGINAL DATA**

Title: uwr

	N1:Wechia	N2:Wa	N3:Nadowl	N4:Jirapa	N5:Lawra	N6:Lambus
N1:Wechia		46.	infinity	infinity	infinity	infinity
N2:Wa	46.		32.	infinity	infinity	infinity
N3:Nadowl	infinity	32.		16.	32.	infinity
N4:Jirapa	infinity	infinity	16.		31.	41.
N5:Lawra	infinity	infinity	32.	31.		33.
N6:Lambus	infinity	infinity	infinity	41.	33.	
N7:Nandom	infinity	infinity	infinity	38.	28.	7.
N8:Gwollu	infinity	116.	96.	94.	96.	61.
N9:Tumu	infinity	126.	111.	106.	111.	infinity
N10:Issa	infinity	46.	53.	infinity	infinity	111.
N11:Funsi	infinity	infinity	infinity	infinity	infinity	infinity
	N7:Nandom	N8:Gwollu	N9:Tumu	N10:Issa	N11:Funsi	
N1:Wechia	infinity	infinity	infinity	infinity	infinity	
N2:Wa	infinity	116.	126.	46.	infinity	
N3:Nadowl	infinity	96.	111.	53.	infinity	
N4:Jirapa	38.	94.	106.	infinity	infinity	
N5:Lawra	28.	96.	111.	infinity	infinity	
N6:Lambus	7.	61.	infinity	111.	infinity	
N7:Nandom		99.	infinity	infinity	infinity	
N8:Gwollu	99.		32.	101.	infinity	
N9:Tumu	infinity	32.		118.	95.	
N10:Issa	infinity	101.	118.		59.	
N11:Funsi	infinity	infinity	95.	59.		

SHORTEST ROUTES -- FLOYD'S OUTPUT SUMMARY

Title: uwr

From	To	Distance	Route
1-Wechia	2-Wa	46.	1- 2
1-Wechia	3-Nadowl	78.	1- 2- 3
1-Wechia	4-Jirapa	94.	1- 2- 3- 4
1-Wechia	5-Lawra	110.	1- 2- 3- 5
1-Wechia	6-Lambus	135.	1- 2- 3- 4- 6
1-Wechia	7-Nandom	132.	1- 2- 3- 4- 7
1-Wechia	8-Gwollu	162.	1- 2- 8
1-Wechia	9-Tumu	172.	1- 2- 9
1-Wechia	10-Issa	92.	1- 2- 10
1-Wechia	11-Funsi	151.	1- 2- 10- 11
2-Wa	1-Wechia	46.	2- 1
2-Wa	3-Nadowl	32.	2- 3
2-Wa	4-Jirapa	48.	2- 3- 4
2-Wa	5-Lawra	64.	2- 3- 5
2-Wa	6-Lambus	89.	2- 3- 4- 6
2-Wa	7-Nandom	86.	2- 3- 4- 7
2-Wa	8-Gwollu	116.	2- 8
2-Wa	9-Tumu	126.	2- 9
2-Wa	10-Issa	46.	2- 10
2-Wa	11-Funsi	105.	2- 10- 11
3-Nadowl	1-Wechia	78.	3- 2- 1
3-Nadowl	2-Wa	32.	3- 2
3-Nadowl	4-Jirapa	16.	3- 4
3-Nadowl	5-Lawra	32.	3- 5
3-Nadowl	6-Lambus	57.	3- 4- 6
3-Nadowl	7-Nandom	54.	3- 4- 7
3-Nadowl	8-Gwollu	96.	3- 8
3-Nadowl	9-Tumu	111.	3- 9
3-Nadowl	10-Issa	53.	3- 10
3-Nadowl	11-Funsi	112.	3- 10- 11
4-Jirapa	1-Wechia	94.	4- 3- 2- 1
4-Jirapa	2-Wa	48.	4- 3- 2
4-Jirapa	3-Nadowl	16.	4- 3
4-Jirapa	5-Lawra	31.	4- 5
4-Jirapa	6-Lambus	41.	4- 6
4-Jirapa	7-Nandom	38.	4- 7

4-Jirapa	8-Gwollu	94.	4- 8
4-Jirapa	9-Tumu	106.	4- 9
4-Jirapa	10-Issa	69.	4- 3- 10
4-Jirapa	11-Funsi	128.	4- 3- 10- 11
5-Lawra	1-Wechia	110.	5- 3- 2- 1
5-Lawra	2-Wa	64.	5- 3- 2
5-Lawra	3-Nadowl	32.	5- 3
5-Lawra	4-Jirapa	31.	5- 4
5-Lawra	6-Lambus	33.	5- 6
5-Lawra	7-Nandom	28.	5- 7
5-Lawra	8-Gwollu	94.	5- 6- 8
5-Lawra	9-Tumu	111.	5- 9
5-Lawra	10-Issa	85.	5- 3- 10
5-Lawra	11-Funsi	144.	5- 3- 10- 11
6-Lambus	1-Wechia	135.	6- 4- 3- 2- 1
6-Lambus	2-Wa	89.	6- 4- 3- 2
6-Lambus	3-Nadowl	57.	6- 4- 3
6-Lambus	4-Jirapa	41.	6- 4
6-Lambus	5-Lawra	33.	6- 5
6-Lambus	7-Nandom	7.	6- 7
6-Lambus	8-Gwollu	61.	6- 8
6-Lambus	9-Tumu	93.	6- 8- 9
6-Lambus	10-Issa	110.	6- 4- 3- 10
6-Lambus	11-Funsi	169.	6- 4- 3- 10- 11
7-Nandom	1-Wechia	132.	7- 4- 3- 2- 1
7-Nandom	2-Wa	86.	7- 4- 3- 2
7-Nandom	3-Nadowl	54.	7- 4- 3
7-Nandom	4-Jirapa	38.	7- 4
7-Nandom	5-Lawra	28.	7- 5
7-Nandom	6-Lambus	7.	7- 6
7-Nandom	8-Gwollu	68.	7- 6- 8
7-Nandom	9-Tumu	100.	7- 6- 8- 9
7-Nandom	10-Issa	107.	7- 4- 3- 10
7-Nandom	11-Funsi	166.	7- 4- 3- 10- 11
8-Gwollu	1-Wechia	162.	8- 2- 1
8-Gwollu	2-Wa	116.	8- 2
8-Gwollu	3-Nadowl	96.	8- 3
8-Gwollu	4-Jirapa	94.	8- 4
8-Gwollu	5-Lawra	94.	8- 6- 5

8-Gwollu	6-Lambus	61.	8- 6
8-Gwollu	7-Nandom	68.	8- 6- 7
8-Gwollu	9-Tumu	32.	8- 9
8-Gwollu	10-Issa	101.	8- 10
8-Gwollu	11-Funsi	127.	8- 9- 11
9-Tumu	1-Wechia	172.	9- 2- 1
9-Tumu	2-Wa	126.	9- 2
9-Tumu	3-Nadowl	111.	9- 3
9-Tumu	4-Jirapa	106.	9- 4
9-Tumu	5-Lawra	111.	9- 5
9-Tumu	6-Lambus	93.	9- 8- 6
9-Tumu	7-Nandom	100.	9- 8- 6- 7
9-Tumu	8-Gwollu	32.	9- 8
9-Tumu	10-Issa	118.	9- 10
9-Tumu	11-Funsi	95.	9- 11
10-Issa	1-Wechia	92.	10- 2- 1
10-Issa	2-Wa	46.	10- 2
10-Issa	3-Nadowl	53.	10- 3
10-Issa	4-Jirapa	69.	10- 3- 4
10-Issa	5-Lawra	85.	10- 3- 5
10-Issa	6-Lambus	110.	10- 3- 4- 6
10-Issa	7-Nandom	107.	10- 3- 4- 7
10-Issa	8-Gwollu	101.	10- 8
10-Issa	9-Tumu	118.	10- 9
10-Issa	11-Funsi	59.	10- 11
11-Funsi	1-Wechia	151.	11- 10- 2- 1
11-Funsi	2-Wa	105.	11- 10- 2
11-Funsi	3-Nadowl	112.	11- 10- 3
11-Funsi	4-Jirapa	128.	11- 10- 3- 4
11-Funsi	5-Lawra	144.	11- 10- 3- 5
11-Funsi	6-Lambus	169.	11- 10- 3- 4- 6
11-Funsi	7-Nandom	166.	11- 10- 3- 4- 7
11-Funsi	8-Gwollu	127.	11- 9- 8
11-Funsi	9-Tumu	95.	11- 9
11-Funsi	10-Issa	59.	11- 10
