MODELLING RATES OF INFLATION IN GHANA:

AN APPLICATION OF AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTIC (ARCH) TYPE MODELS

BY

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THIS THESIS IS SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES, UNIVERSITY OF GHANA, LEGON IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE AWARD OF THE MPHIL STATISTICS DEGREE

JUNE, 2013

DECLARATION

CANDIDATE’S DECLARATION

This is to certify that this thesis is the result of my own research work and that no part of it has been presented for another degree in this university or elsewhere.

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SUPERVISORS’ DECLARATION

We hereby certify that this thesis was prepared from the candidate’s own work and supervised in accordance with guidelines on supervision of thesis laid down by the University of Ghana.

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ABSTRACT

The research is based on financial time series modelling with special application to modelling inflation data for Ghana. In particular the theory of time series is explored and applied to the inflation data spanning from January 1965 to December 2012 which were obtained from the Ghana Statistical Service. Three Autoregressive Conditional Heteroscedastic (ARCH) family type models (traditional ARCH, Generalized ARCH (GARCH), and the Exponential GARCH (EGARCH)) models were fitted to the data. This was especially so because the data were characterized by changing mean and variance. The Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) were used to assess the performance of each of the fitted models such that the model with the minimum value of AIC and BIC was adjudged the best model. The results revealed that the ARCH – family type models, particularly, the EGARCH (2, 1) was superior in performance in forecasting Ghana's monthly rates of inflation. The results also showed that the monthly rates on inflation were not weakly stationary and although there was the presence of asymmetric effects in the volatility in the monthly rates of inflation, there was an absence of leverage effects as positive shock increased the volatility in the monthly rate of inflation more than a negative shock of equal magnitude. The study recommends that policy makers and all interested in modelling and forecasting monthly rates of inflation in Ghana should consider using the Heteroscedastic models as it is able to properly capture the volatilities in the monthly rates of inflation. Analysis were done using MINITAB 16.0 and EVIEWS 5.0.

DEDICATION

This work is dedicated to the Lord God Almighty for the divine wisdom and strength given me to go through this research successfully.

It is also dedicated to Very Rev. Andrew Mbeah- Baiden and Mrs. Rebecca Mbeah-Baiden, my parents and also to my sweet siblings, Andrew, Christian and Gifty for their love, care and support.

Lastly, this work is dedicated to Miss Deborah Esi Edzie for her gargantuan support and encouragement.

ACKNOWLEDGEMENT

This thesis would not have been possible if it were not for the tireless guidance and support I got from my supervisors Dr. Julius B. Dasah and Dr. Ezekiel N.N. Nortey; I say thank you.

To all other faculty members, I owe you my deepest gratitude for your support and assistance; you stood by me every step of the way, both during the course work and during the period of thesis write up.

I am indebted to my colleagues; your presence provided a source of warmth and gave me a source of hope in challenging and difficult periods through the research. I would like to express my sincere gratitude to Victoria Agyepong for her unwavering support and encouragement.

Also sincere appreciation is extended to the Carnegie Corporation of New York for funding this research under the New Generation of Academics in Africa through Office of Research, Innovation and Development (ORID), University of Ghana.

Lastly, it is my pleasure to thank my cousins Antoinette, Gifty, Georgina and all those not mentioned by names but whose support made this thesis a success.

May the Almighty God richly bless you all.

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LIST OF ABBREVIATIONS

ACF	Auto Correlation Function
AIC	Akaike Information Criterion
APARCH	Augmented Power Auto Regressive Conditional Heteroscedastic
ARCH	Auto Regressive Conditional Heteroscedastic
ARFIMA	Auto Regressive Fractionally Integrated Moving Average
ARSV	Auto Regressive Stochastic Volatility
BIC	Bayesian Information Criterion
BVAR	Bayesian Vector Auto Regressive
COICOP	Classification of Individual Consumption by Purposes
CPI	Consumer Price Index
EGARCH	Exponential Generalized Auto Regressive Conditional Heteroscedastic
ERP	Economic Recovery Programme
GARCH	Generalized Auto Regressive Conditional Heteroscedastic
GARCH-M	Generalized Auto Regressive Conditional Heteroscedastic in Mean
GDP	Gross Domestic Product
GED	Generalized Error Distribution
GNPA	Ghana National Petroleum Authority
GJR	Glosten Jagannathan Runkle
GSS	Ghana Statistical Service
HQ	Hannan-Quinn
IGARCH	Integrated Generalized Auto Regressive Conditional Heteroscedastic
IT	Inflation Targeting
LA-VAR	Lag-Augmented Vector Auto Regressive
MOFEP	Ministry of Finance and Economic Planning
MT	Monetary Targeting

PACF	Partial Auto Correlation Function
PARCH	Power Auto Regressive Conditional Heteroscedastic
QTM	Quantum Theory of Money
RBF	Radial Basis Function
RW	Random Walk
SAP	Structural Adjustment Programme
SAR	Simple Auto Regressive
SARIMA	Seasonal Auto Regressive Integrated Moving Average
SVAR	Structural Vector Auto Regressive
SVM	Support Vector Machine
TAR	Threshold Auto Regressive
TGARCH	Threshold Generalized Auto Regressive Conditional Heteroscedastic
VaR	Value at Risk
VAR	Vector Auto Regressive
VAT	Value Added Tax

CHAPTER ONE

INTRODUCTION

1.0 Background of the Study

Price stability (stable inflation) is one of the main objectives of every government as it is an important economic indicator that the government, politicians, economists and other stakeholders use as their basis of argument when debating on the state of the economy (Suleman and Sarpong, 2012). In recent years, rising inflation has become one of the major economic challenges facing most countries in the world especially developing countries like Ghana. David (2001) described inflation as a major focus of economic policy worldwide. This is rightly so as inflation is the frequently used economic indicator of the performance of a country's economy as it has a direct effect on the state of the economy. In Ghana, the debate of achieving a single digit inflation value has been the major concern for both the government and the opposition parties. While the government boasts of a stable economy with consistent single digit inflation, the opposition parties' doubts these figures and believe that the figures had been cooked up and do not reflect the true situation in the economy. Despite the different opinions on the inflation figures, it is important to point out that, both the government and the opposition parties are concerned about the inflation (general level of prices) in the country as it affects all sectors of the economy.

Webster (2000) defined inflation as the persistent increase in the level of consumer prices or a persistent decline in the purchasing power of money. Hall (1982) also expresses inflation as a situation where the demand for goods and services exceeds their supply in the economy. Inflation and its volatility entail large real costs to the economy (Moreno, 2004). Among the harmful effects of inflation volatility are the higher risk of permia for long term arrangement, unforeseen redistribution of wealth and higher costs for hedging against inflation risks (Rother, 2004). Thus inflation volatility can impede growth even if inflation on the average remains restrained (Awogbemi and Oluwaseyi, 2011) and hence monetary policy makers are more interested in containing and reducing inflation through price stability (Amos, 2010). Policy makers will be content and satisfied if they are able to understand the underlying dynamics of inflation and how it evolves. Ngailo (2011) observes that inflation dynamics and evolution can be studied using a stochastic modelling approach that captures the time dependent structure embedded in the time series inflation data.

Traditional time series models assume a constant conditional variance. However, to a large extent most economic and financial series often exhibit non–constant conditional variance (Heteroscedastic) and hence traditional time series do not perform well when used to forecast such series. The heteroscedasticity affects the accuracy of forecast confidence limits and thus has to be handled by constructing appropriate non – constant variance models (Amos, 2010). According to Maddela and Rao (1996), until some fifteen years ago, the focus of statistical analysis of time series centred on the conditional first moments. The increased role played by risk and uncertainty in models of economic decision making and the finding that common measures of risks and volatility exhibit strong variation over time lead to the developments of new time series techniques for modelling time-variants in the second moments.

Several models such as the Autoregressive Conditionally Heteroscedastic (ARCH) model and its variants like the Generalised ARCH (GARCH) and Exponential GARCH (EGARCH) models have therefore been developed to model the non-constant volatility of such series. The ARCH model was introduced by Engle (1982) and later it was modified by Bollerslev (1986) to a more generalized form known as the GARCH. The GARCH model has been used most widely for the specification of the ARCH. The GARCH model imposed restrictions on the parameters to assure positive variances. Nelson (1991) therefore presented an alternative to the GARCH model by modifying the GARCH to Exponential GARCH (EGARCH) model. Unlike the GARCH, the EGARCH does not need the inequality restrictions on the parameters to assume a positive variance.

A practitioner has the option to use any or a combination of the models based on the performance of such a model in the particular series he or she is estimating. As a result of this, there is the need for performance evaluation to be done so that the practitioner would be able to choose the optimal model among competing class of models.

Several evaluation criteria have been developed to measure the performance of these models. One criterion that is popularly used is by estimating the maximum likelihood of the models and observing which of them has the highest log-likelihood value (Shephard, 1996). In situations when the models do not have the same number of parameters, the principle of parsimony is applied and a suitable model selection criterion such as the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and the Hannan–Quinn (HQ) is used to choose the best model. Another model criterion is to submit estimated models to misspecification tests and observe each model perform under each scenario.

Although there are several empirical approaches and findings on the relative performance of time series models that are based on the non–constant variance, these works are primarily on time series data from the developed countries such as the US and Europe. It is therefore necessary to show or exhibit more international evidence on the relative performance of these models especially in developing countries and that is the essence of this study.

This study investigates and provides empirical evidence of the relative performance of the ARCH, GARCH and EGARCH models using the univariate time series analysis in which the analysis is on the present and past values of single series. That is the study seeks to investigate the time series of the monthly inflation rates in Ghana using the ARCH, GARCH and EGARCH models and determines the best among these models in forecasting the inflation rate values in Ghana.

1.1 Statement of Problem

Inflation is the measure of the persistent and continuous rise in the general price levels in an economy or a country. Inflation is one economic factor that affects all other levels of the economy and every country or government aims to control inflation as a result of this. Due to the fact that inflation levels affect all other sectors of the economy especially business transactions, it is important to be able to forecast or estimate the value of inflation in the future so that such values are incorporated in decisions affecting all these other sectors.

Empirical researches have been carried out in the area of inflation modelling and forecasting in Ghana. Examples include Aidoo (2010), Alnaa and Ahiakpor (2011), Suleman and Sarpong (2012), etc. All these researchers attempted to model inflation in Ghana using models that did not capture the conditional heteroscedasticity of the time series inflation data. Gujarati (2004) asserted that the underlying characteristic of most financial time series is that, in their level form they are random walks, i.e. they are non–stationary.

It is been argued by Campbell, Lo and MacKinlay (1997) that it is both statistically inefficient and logically inconsistent to use models that are based on the assumption of constant variance over some period when the resulting series progress over time. In the case of financial data for example, large and small errors occur in clusters which implies that large returns are followed by more large returns and small returns are also followed by further small returns. When applied to inflation time series data, it is equivalent to saying that periods of high inflation are usually followed by further periods of high inflation while low inflation is likely to be followed by further periods of low inflation (Amos, 2010).

Time series models that capture the conditional heteroscedasticity of time series inflation data had been developed to model and forecast the rates of inflation using time series analysis. These models have been used and empirical evidence on their relative performance has been given for developed economies like the US and Europe. However, limited or no studies have been done in the context of developing countries. This indicates a gap in literature or information on the relative performance of these models in the context of developing countries and poses a challenge as to which of these models is the optimal choice for modelling and forecasting economic and financial data (in particular inflation rates) for developing countries.

In view of this, the study intends to model inflation in Ghana using the ARCH-type models and to choose the most appropriate model suitable for inflation modelling and forecasting in Ghana.

1.2 Objectives of the Study

1.2.1 General Objective

The main objective of this study is to investigate the relative performance of three selected Autoregressive Heteroscedastic time series models (ARCH, GARCH and EGARCH).

1.2.2 Specific Objectives

The specific objectives would be to;

i. Fit each of the three time series models(ARCH,GARCH and EGARCH) using the inflation rates in Ghana;
ii. Identify the optimal model;
iii. Predict a one year out–sample forecast based on the optimal model and;
iv. Identify the presence of asymmetric and leverage effects in the volatility in the monthly rates of inflation.

1.3 Significance of the Study

The empirical results and findings from this study would be significant to industry practitioners and policy makers such as the government, businesses and the general public as well as academics and researchers due to the following reasons:

First of all by identifying the optimal model based on their relative performance, better and robust forecasts of inflation values which will be very useful in the planning activities of the government, businesses and the public in general would be obtained.

Secondly, the results from this study will benefit academia and research by contributing to existing literature by closing or the elimination of the gap in literature or information on the relative performance of these models in the context of developing countries. It will also serve as a basis for further research for both academic researchers and industry practitioners.

1.4 Scope and Methodology

The study was carried out in a developing country specifically Ghana. Secondary data consisting of year-on-year inflation data for each month from January 1965 to December 2012 was used in this study. The total number of data points is therefore 576. The year-on-year inflation is the percentage change in the consumer price index (CPI) over a twelve-month period which is used to measure changes over time in the general price level of goods and services that households acquire for the purpose of consumption. The monthly year-on-year inflation is collected by the Ghana Statistical Service.

The data was analysed and the three selected time series models (i.e. the ARCH, GARCH and the EGARCH) for the non–constant conditional variance series were estimated using the maximum likelihood estimation process. The estimation of the model consists of four stages namely; testing for ARCH effects, identification, estimation of parameters and the diagnostic checking stages.

The ARCH effects were tested using the Ljung-Box statistics Q (m) test (McLeod and Li, 1983) and the Lagrange multiplier test of Engle (1982) as this forms the basis for building ARCH-type models. The partial autocorrelation function (PACF) of the squared residuals was used to determine the order. Next the estimation of the parameters for the tentative models was carried out using the maximum likelihood estimation method. Three likelihood functions are commonly used in ARCH – typed model estimation depending on the assumption of the distribution that the residuals follow. The distributions are the normal distribution, heavy-tailed distribution such as the standardised student-t distribution and the generalised error distribution (GED). In this study, it is assumed that the residuals are normally distributed since it is the most commonly used distribution and it makes the estimation of the parameters relatively easier.

Lastly, the estimated models were checked to verify if it adequately represents the series. Diagnostic checks were performed on the residuals to see the validity of the distribution assumption. In particular, the measure of skewness, kurtosis and Quantile-to Quantile plot (Q-Q plot) of the residuals was used to check for the validity of the distribution assumption. All the analyses were carried out with statistical software MINITAB 16.0 and EVIEWS 5.0.

1.5 Organisation of the study

The study is divided into five chapters. Chapter one introduces the research study, providing the background to the study, statement of the problem, objectives, the significance, scope and brief methodology used in the study. Chapter two focuses on the conceptual framework and review related literature that pertains to the study whilst Chapter three presents the detailed methodology used for the study. Chapter four covers the data analysis, presentation and discussion of the results. Finally, Chapter five encompasses the summary, conclusion, recommendations, and direction for future research.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This chapter reviews the relevant theories and concepts associated with inflation and related works that has been carried out by other researchers on the topic area. The chapter is divided into two main headings namely: The Concept of inflation and Review of related works.

2.1 The Concept of Inflation

2.1.1 Introduction

According Webster (2000), inflation is the persistent and continuous rise in the levels of the consumer prices in an economy. Inflation can also be seen as the persistent decline in the purchasing power of money. That is, inflation means that your money can not buy today as much as what it could have bought yesterday.

There are different theories that have been proposed by economists to explain the occurrence of inflationary situation. These numerous theories can be grouped into two main broad theories; the excess- demand theory and the cost- push theory. The excess-demand theory argued that the excess demand for goods and services over supply in the economy is the main source of inflation as expressed by Hall (1982). On the other hand, the cost-push theory of inflation believes that inflation can be triggered by the increase in the cost of production of firms. The increase in the cost of production will affect the profit margins of these firms and hence they will have to pass on the extra to consumers by increasing the prices of their products.

The effects of inflation include among other things, people losing confidence in the currency as the real value of the currency is severely reduced. Inflation can also lead to the ‘wage-price spiral’. This is the situation in which there are higher wage demands as people try to maintain their real living standards. This leads to businesses to increase prices to maintain profits and higher prices then put further pressure on wage. Furthermore, inflation can lead to a build up of inflation expectations that can worsen the trade-off between unemployment and inflation. Lastly, the uncertainty created by the rising inflation can also disrupt business planning since budgeting becomes difficult. Bailey (1956) observed that inflation has negative effects on the economy through its cost on welfare. Furthermore he stated that the costs associated with unanticipated inflation are the distributive effects from creditors to debtors, increasing uncertainty affecting consumption, savings, borrowing and investment decisions.

2.1.2 Consumer Price Index as a Measure of Inflation

Various indexes have been devised to measure inflation. These indexes include consumer price index, producer price index, cost of living index, commodity price index and the Gross Domestic Product (GDP) deflator. However, the consumer price index is the most common way of measuring inflation.

The consumer price index is a measure for capturing changes over time (monthly, quarterly, yearly) in the general price level of goods and services. This is determined at a beginning period called the base period and according to a fixed pattern of consumption called weight assigned to a representative sampled basket of goods and services. The consumer price index is then used to calculate the inflation rate as shown below.

Let P_t be the current average price level of an economic basket of goods and services and P_{t-1} be the average price level of the same basket a year ago, then the inflation rate I_t at time t is calculated as;

$$I_t = \frac{P_t - P_{t-1}}{P_{t-1}} * 100\%$$

2.1.3 Construction of Consumer Price Index in Ghana

In Ghana, the consumer price index is calculated by the Ghana Statistical Service (GSS) which is a department of the Ministry of Finance and Economic Planning (MOFEP). The Ghana Living Standards Survey generates the basket of goods and services classified into 12 main classes using the classification of individual consumption by purpose (COICOP) system. These baskets of goods and services are then used in the construction of the CPI based on the weight assigned to each item in the basket. In all there are 242 items and the weight assigned to each item depends on the expenditure on that item such that high volume expenditure items carry the most weight and therefore would have the most material impact on the calculated index (GSS Newsletter, 2009).

The CPI covers prices collected from a national sample of 40 markets. The markets are made up of 9 urban and 31 rural markets across the country. Prices are collected every first and third week of the month from 6 traders in the urban markets and 3 traders in the rural markets for all goods excepts those with fixed prices such as stamps. The prices are then used to construct the CPI which is in turn use to calculate the inflation rates. Currently, the construction is based on 2002 base year having been changed from 1963 firstly in 1977 and then in 1997.

Table 2.1.3 shows the 12 main classes of items used in constructing the CPI with their corresponding weights and the number of items in each class.

Table 2.1.3: 12 main classes of items used in constructing the CPI in Ghana

CLASS	WEIGHT	NUMBER OF ITEMS
Food and Non-alcoholic beverages	44.91	76
Alcoholic beverages, tobacco and narcotics	2.23	11
Clothing and Footwear	11.29	59
Housing, Water, Electricity, Gas and Other Fuels	6.98	10
Furnishings, Household Equipment and Routine Maintenance of the house	7.83	43
Health	4.33	9
Transport	6.21	9
Communication	0.31	3
Recreation and Culture	3.04	6
Education	1.60	2
Restaurants and Hotels	8.28	7
Miscellaneous Goods and Services	2.99	7
Total	**100**	**242**

Source: Ghana Statistical Service

2.1.4 The Ghanaian Experience of Inflation

This section attempts to take a look at Ghana’s inflation experience since the attainment of independence. This is relevant as a good appreciation of the need to model and forecast Ghana’s inflation certainly require an understanding of where Ghana has come from, where Ghana is currently in terms of inflation rates. Ghana has experienced high rates of inflation for several decades. However since July 2009, inflation has fallen consistently even to a single digit level being achieved since June 2010.

Ocran (2007) asserts that the inflation in Ghana from independence to 2003 can be characterised as episodic, identifying four distinct episodes: the immediate post independence period which was up to 1966; immediate post Nkrumah period (1966-1972); the deterioration period of 1972-1982 and the most recent period (1982-2003), which he termed the stabilization inflationary experience. This study would adopt Ocran (2007) episodic characterisation of inflation in Ghana and modify it slightly by adding a new episode. Hence the study would review Ghana’s inflation experience under five distinct episodes as follows: the immediate post independence period up to 1966; immediate post-Nkrumah period (1966-1972); the deterioration period (1972-1983); recent period (1984-2000) and the most recent period of 2001-2012 which would be termed the single-digit inflationary experience.

The first five years in the post independence period (1957-1962) saw inflation centring on a single-digit. This stability in prices could be attribute to the trickling effects of Ghana been a member of the West African Currency Board (WACB) which consisted of the four British Colonies of West Africa - Ghana, Nigeria, Sierra Leone and The Gambia. The currency board had no control of the discretionary monetary policy and as a result, market forces determined the money supply in the member countries. Ocran (2007) pointed out that during the currency board years, inflation was in single digits and indeed inflation rates were typically estimated at less than 1%. Between 1960 and 1962, inflation averaged 8% per annum and then increased to 23% per annum between 1964 and 1966, by which time the trickling benefits of Ghana having been a member of the WACB had been eroded.

From 1966 to 1972, inflation rates were in the range of 31.2% (January 1966) to -12.1% (July 1967), with annual averages in the range of -8.3% to 10.2%. During this period, there was a devaluation of the cedi by 30% against the US dollar and massive retrenchment exercise in the public sector (Hutchful, 2002). This led to a deflation of about 8% in 1962. It is worth noting that from December 1966 through to December 1967, the inflation was less than zero (negative).

The period 1972 to 1983 arguably has been the period that inflation rates increased the most in the economic history of Ghana. According to Apaloo (2001), inflation was running at about 100% at the beginning of 1979. In the mid-1979, however, the rate dropped dramatically by about 25% following the coup-de-etat in June 1979. With the exception of the first five months in 1980, the inflation rates ranged between 40% and 88% in 1980. Throughout the period 1981to1983, inflation rates were over 100% for all the months with the rates reaching a peak of 174% at the end of June 1983 before declining to 142 % in December. In sum, the inflationary experience in most of the period 1972-1983 was largely due to expansionary fiscal and loose monetary policies and the attempt to using controls such as fixed exchange rate, import licensing and administered prices for goods and services to hold down inflation (Ocran, 2007). In particular, the high rate of inflation recorded in 1983 could be attributed to the devaluation of the cedi, the drought and famine.

The recent episode of inflation started on the backdrop of the introduction of the Economic Recovery Programme (ERP) and the successive Structural Adjustment Programme (SAP) in 1983. The ERP had two stages of implementation. The first stage, ERP I (1983-1986) had a stabilization package aimed at reducing inflation and fostering external balance. The second stage, ERP II had the structural adjustment which was undertaken with the aim of removing the distortions in the incentive structure and thereby facilitating production as well as restoring broken down social and economic infrastructure (Ocran, 2007). The ERP was able to bring down inflation to an average of about 40% and subsequently lowering it to a single digit levels by the end of 1985. The success of ERP and SAP at bringing down inflation was short lived though as between 1986 and 1990, year-on-year inflation was in the range of 19% to 46%. The average inflation was between 25% and 40% per annum, far exceeding the official targets set within ERP (Apaloo, 2001). The rates of inflation fell continually from the beginning of 1991 till the beginning of 1992(the first two quarters) where single digit inflation rates were recorded. The rate of inflation was relatively kept under control till the end of 1992, largely on the account of the good harvests of the previous year and conscious efforts at monetary control (Apaloo, 2001).

In January 1993, there was over 60% increase in inflation rate as the rates stood at 21.50% compared to 13.3% in December 1992. The average rate of inflation in the 1993 was 24.9% compared to the average of 10% in 1992. This increase in inflation rate was attributed to an increase in petroleum prices. The next two years (1994-1995) that followed was even worse off as there was a sharp increase in inflation rates from 22.80% in January 1994 to 70.80% in November 1995. According to Aidoo (2010), this sharp increase could be attributed to several factors. These factors included a triple year to year increase in petroleum prices in 1993, 1994 and 1995, the depreciation of the local currency (cedi) at the exchange rate level relative to the US dollar the same year, a poor performance of agriculture in 1995 and the introduction of a new tax system known as the Value Added Tax (VAT). The value of the VAT was higher than the previous sales tax and that led to an increase in general prices of commodity.

Inflation fell consistently from 69.1% from the beginning of January 1996 till May 1999 except for a brief increase in March and April 1998. In May 1999, the rate was 9.4%. This drastic drop in inflation rate was due to the improvement in agriculture productivity giving credence to the fact that the food component plays a significant role in the level of inflation rates. This was short lived as the level of prices started rising again in June 1999 from 10.3% to 40.5% at the end of December 2000. This was attributed to the increase in world oil prices and a decrease in world market cocoa prices as well as reduction in agriculture performance in the year 2000 (Aidoo,2010).

The most recent inflation episode started with a new government in office in January 2001. In the first quarter of 2001, inflation was still higher ranging between 40.1% and 41.9%. However, inflation rates started dropping from 39.5% in the second quarter of 2001 to 12.9% in the third quarter of 2002. In the last quarter of 2002, inflation rose again ending the year at 15.2%. Between 2003 and 2006, inflation ranged from 33.6% (August, 2003) and 10.7% (November, 2004) with an average of 29.8%, 18.2%, 15.5% and 11.7% for 2003, 2004, 2005 and 2006 respectively. Ghana adopted a monetary policy called the Inflation Targeting (IT) in 2007. This was after the Monetary Targeting (MT) framework used in the management of inflation was not effective due to the intractability of the underlying causes (Kwakye, 2004). The aim of the Bank of Ghana (BoG) was to target inflation rate and then attempt to direct actual inflation rate towards the target. The target set by the BoG was to bring inflation rate below 10%. The target of an inflation below 10% was not successful until June 2010 as the inflation figures hovered between 10.1% (October, 2007) and 20.7% (June, 2009) with an annual average of 10.7% (2007), 16.5% (2008), 19.3% (2009). Since June 2010, the inflation has since been below 10.0%, meeting the target of the inflation targeting.

In conclusion, it could be seen that Ghana has had its own share of unstable and high inflation rates. However, in the last few years, the country can be seen to be wining the fight against inflation as inflation has been kept at single digits. This notwithstanding, the authorities in charge of price stability in the country should however take note of the potential threat posed by the oil production, boost in the government expenditure through the implementation of the single spine salary structure as these factors could exert both demand and cost pressures on inflation.

2.2 Review of Related Works

In this section, a review of the numerous related works that has been carried out by other researchers using time series techniques and other forecasting techniques is taken into consideration. These include Vector Auto Regressive (VAR), Bayesian Vector Auto Regressive (BVAR), Structural Vector Auto Regressive (SVAR), Seasonal Auto Regressive Integrated Moving Average (SARIMA), Simple Auto Regressive (SAR), random walk (RW) and Auto Regressive Fractionally Integrated Moving Average (ARFIMA). The rest are the ARCH-typed models including the traditional Auto Regressive Conditional Heteroscedastic (ARCH) model with extensions such as Generalized ARCH (GARCH), Exponential ARCH (EGARCH), Integrated Generalized ARCH (IGARCH), Power ARCH (PARCH) and Glosten - Jagannathan Runkle GARCH (GJR - GARCH).

The related works reviewed would be categorised into two: works done in Ghana and other African countries and works done in the rest of the world.

2.2.1 Review of Related Works in Ghana and Other African Countries

Minkah (2007) examined the forecasting ability of three widely used time series volatility models namely, the Historical Variance, the Generalized Autoregressive Conditional Heteroscedastic (GARCH) Model and the Risk Metrics Exponential Weighted Moving Average (EWMA). The characteristics of these volatility models were explored using data on the Standard & Poor's (S&P) 500 Index, Dow Jones Industrial Average (DJIA), OMX Swedish Stock Exchange (OMXS30) index, Dow Jones-AIG Commodity Index (DJ-AIGCI), the 3 Months US Treasury Bill Yield, the Ghanaian Cedi and the US Dollar (CEDI/USD) exchange rates. It was observed that the complex models i.e. GARCH (1, 1) and Risk Metrics EWMA outperformed the simple Historical Variance in the In-Sample volatility forecasts. The Out-of-Sample forecasting accuracy comparisons also revealed that for shorter forecasting horizons, the GARCH (1, 1) performed better whereas at longer horizons the simple Historical Variance outperformed all in most markets. This was due to the fact that complex models have more parameters and thus add to the estimation errors and its forecasts are consistently poor in Out-of-Sample.

Owusu (2010) used the ARIMA models to model inflation and forecast the monthly inflation on short-term basis. The study used different ARIMA models to model the inflation rates from 1990 – 2009. The period under consideration was split into two sub-periods: 1990 – 2000 and 2001 – 2009. The results showed that the best inflation model for the period of 1990 – 2000 was ARIMA (1, 2, 2) whilst that of the period 2001 – 2009 was ARIMA (2, 2, 1). Furthermore, the study concluded that the inflation for the period of January 2001 to December 2009 was less than that of January 1990 to December 2000.

Ocran (2007) in modelling Ghana's inflation experience sought to ascertain the key determinants of inflation in Ghana for the past 40 years. Stylized facts about Ghana's inflation experience indicated that since Ghana's exit from the West African Currency Board soon after independence, inflation management has been ineffective despite two decades of vigorous reforms. He used the Johansen co-integration test and an error correction model and the results identified inflation inertia, changes in money supply and changes in government Treasury bill rates, as well as changes in the exchange rate, as determinants of inflation in the short run. Of these determinants, inflation inertia was the most dominant and therefore the study suggested that to make Treasury bill rates more effective as a nominal anchor, inflationary expectations, ought to be reduced considerably.

In an attempt to analyse and forecast the macroeconomic impact of oil price fluctuations in Ghana using annual data from 2000 - 2011, Abledu and Agbodah (2012) focused on the feasibility forecast using nested conditional mean (ARIMA) and conditional variance (GARCH, EGARCH and GJR) family of models as the market conditions were too volatile. The best model was the ARIMA (1, 1, 0) and it was used to predict the oil prices in Ghana National Petroleum Authority (GNPA) till the end of 2016.

Using the Seasonal Autoregressive Integrated Moving Average (SARIMA) model, Aidoo (2010) examined the inflation rates in Ghana. Monthly inflation data from July 1991 to December 2009 were used. The results revealed that the ARIMA $(1,1,1) \times (0,0,1)_{12}$ can best represent the behaviour of inflation rates in Ghana.

Suleman and Sarpong (2012) applied the Box-Jenkins approach to model monthly inflation data in Ghana. The study applied the SARIMA model to inflation rates from January 1990 to January 2012. The study concluded that the best model was the $\text{ARIMA}(3,1,3) \times (2,1,1)_{12}$.

Alnaa and Ahiakpor (2011) also used the ARIMA approach to predict inflation in Ghana. The monthly data from June 2000 to December 2010 was used and it was found that ARIMA $(6,1,6)$ was the best fitted model for forecasting inflation in Ghana. Inflation was predicted highest for the months of March, April and May to be 8.95%, 10.07% and 10.24% respectively. The researchers recommended that the appropriate measures must be put in place to prevent inflation spiral from setting in motion. Since their model suggests that, inflation has a long memory and that once the inflation spiral is set in motion, it will take at least 12 periods (months) to bring it to a stable state.

Frimpong and Oteng-Abayie (2006) in studying volatility of returns on the Ghana Stock Exchange (GSE) used the random walk (RW), GARCH, EGARCH and TGARCH models. The unique 'three days a week' Databank Stock Index (DSI) was used to study the dynamics of the GSE volatility over a 10-year period. Their results revealed that the DSI exhibited the stylized facts such as volatility clustering, leptokurtosis and asymmetry effects associated with stock market returns on more advanced stock markets. The random walk hypothesis was also rejected and overall, the GARCH $(1, 1)$ model outperformed the other models under the assumption that the innovation follows a normal distribution.

The ARCH-type models were used by Wagala, Nassioma and Islam (2011) to model the volatility of the Nairobi Stock Exchange weekly returns. The models applied in the study included the ARCH (p), standard GARCH (p, q), IGARCH (p, q) and TGARCH (p, q). The results demonstrated that the ARCH (8) was found to be the most adequate for the NSE index, Bamburi and KQ while ARCH (9) provided the best order for the NBK series. Furthermore four different p and q values were tested for the GARCH (p, q), EGARCH (p, q) and TGARCH (p, q). These were $(1, 1)$, $(1, 2)$, $(2, 1)$ and $(2, 2)$. The order $(1, 1)$ was the best choice in all cases and it was consistent with results obtained from most GARCH research works. Comparing the diagnostics and the goodness of fit statistics, the IGARCH $(1, 1)$ outperformed the ARCH, EGARCH and TGARCH models due to its stationarity in the strong sense. However, because the IGARCH model was unable to capture the asymmetry exhibited by the stock data, the EGARCH (1,1) and the TGARCH (1,1) provided the best options to describe the dependence in variance for all the four series since they were able to model asymmetry and parsimoniously represent a higher order ARCH (p).

Amos (2010) examined financial time series with special application to modelling inflation data for South Africa. The data spanned from January 1994 to December 2008. The study considered two families of time series namely the autoregressive integrated moving averages (ARIMA) with extension to the Seasonal ARIMA

(SARIMA) model and the autoregressive conditional Heteroscedastic (ARCH) with extensions to the generalized ARCH (GARCH) model. The study concluded that the SARIMA $(1,1,0) \times (0,1,1)_s$ was the best fitting model from the ARIMA family of models while the GARCH (1, 1) was chosen to be the best fit from the ARCH-GARCH models. Furthermore, a comparison of the two selected models based on the goodness of fit and the forecasting power of the two models was carried out. It was established that the GARCH (1, 1) model was superior to the SARIMA (1, 1, 0) × (0, 1, 1) model according to both criteria as the data was characterized by changing mean and variance.

Awogbemi and Oluwaseyi (2011) described the volatility in the consumer prices of some selected commodities in the Nigerian market. The researchers examined the presence or otherwise of the volatility in their prices using ARCH and GARCH models with monthly Consumer Price Index (CPI) of five selected commodities over a period of 1997 – 2007. The results showed that ARCH and GARCH models are better models because they give lower values of AIC and BIC as compared to the conventional Box and Jenkins ARMA models. The researchers also observed that since volatility seems to persist in all the commodity items, people who expect a rise in the rate of inflation (the 'bullish crowd') will be highly favoured in the market of the said commodity items.

Ngailo (2011) modelled financial time series with special application to modelling inflation data for Tanzania. In particular the theory of univariate non linear time series analysis was explored and applied to the inflation data spanning from January 1997 to December 2010. He fitted the ARCH and GARCH models to the data. Based on the AIC and BIC values, the results revealed that the best fit models tend to be the GARCH (1, 1) and GARCH (1, 2). However after diagnostic and forecast accuracy tests were performed, the GARCH (1, 1) model was adjudged to be the best model for forecasting.

Inflation and Inflation forecasting in Uganda from 1993 to 2009 was examined by Mugume and Kasekende (2009). They employed various inflation forecasting models like Philips curve, P-star model based on Quantum Theory of Money (QTM), and the price equation and ARIMA model. They also employed M3 and the results of both short-run dynamics and long run equilibrium showed that inflation had not been a result of money growth. The long run inflation equation seemed to show that exchange rate depreciation could have had a stronger impact in driving inflation upwards than money supply, although it had no short run impact.

Igogo (2010) employed the ARCH family of models to measure the effect of real exchange volatility on trade flows in Tanzania for the period of 1968 to 2007. He fitted the GARCH (1, 1) and EGARCH (1, 1) models. The results indicated that GARCH (1, 1) model violated the non-negativity conditions and hence to resolve the problem, the EGARCH (1, 1) was used. The adequacy of the EGARCH (1, 1) model to measure the real exchange rate volatility was confirmed by testing for ARCH effect after running the model. Furthermore, the study revealed that it is the real exchange rate rather than its volatility that is found to have a significant effect on trade flows although the effect is larger on exports than imports. He concluded therefore that in the short run, imports are mainly affected by the domestic income while exports are mainly affected by the real exchange rate.

Ezzat (2012) studied volatility of daily stock returns listed on the Egyptian Exchange during the political turmoil of 2011. This was particular because modelling volatility during a financial crisis where massive shocks are generated presents an ideal environment for investigating the dynamics of volatility during periods of extreme fluctuations for comparison with volatility during more tranquil periods. The analysis was based on employing both GARCH and EGARCH models. Daily closing prices of four Egyptian stock market indices, the EGX 30, EGX70, EGX 100, and the EGX 20 capped were used in the analysis. The time frame was from the inception of each index to the 30th of June 2012. The sample period covers the period of pre-and post Egyptian revolution which was shaped by extreme volatile fluctuations in stock returns. The EGARCH model was the method of choice for modelling the volatility in order to investigate the long memory and the leverage effect in the volatilities of the two periods. The findings revealed higher volatility during the revolution period for all indices reflected in higher standard deviations for both daily returns and absolute returns, with the EGX 70 displaying the highest volatility. The leverage effect was more apparent during the revolution period. However, long memory was more apparent during the pre-revolution period.

2.2.2 Review of Related Works in the Rest of the World

Engle (1982) studied the ARCH model and revealed that these models were designed to deal with the assumption of non-stationarity found in real life financial data. The researcher based the ARCH model on the idea that a natural way to update a variance forecast was to average the squared deviation of the rate of return from its mean just like the principle used in standard deviation. The ARCH process allowed the conditional variance to change over time as a function of past errors leaving the unconditional variance constant. Empirical evidence revealed that the ARCH model required a relatively long lag in the conditional variance equation and so to avoid the problems with negative variance parameters, a fixed lag structure was typically imposed.

Bollerslev (1986) proposed a generalized ARCH (GARCH) to overcome the limitations of the traditional ARCH model of Engle (1982). The GARCH model allowed for both a longer memory and a more flexible lag structure. In the ACRH process, the conditional variance is specified as a linear function of past sample variance only whereas the GARCH process allows lagged conditional variances to enter in the model as well.

Both the ARCH and GARCH models of Engle (1982) and Bollerslev (1986) could not tell how the variance of return was influenced differently by positive and negative news. Hence Nelson (1991) extended the ARCH framework in order to better describe the behaviour of return volatilities. His study broke the rigidity of the ARCH and GARCH model specification. He proposed the Exponential GARCH (EGARCH) model to test the hypothesis that variance of return was influenced differently by positive and negative excess returns. The results revealed that the hypothesis was true and also the excess returns were negatively related to stock market variance.

Asri and Mohammad (2011) proposed an alternative model for modelling the volatility of the conditional variances: A (Radial Basis Function) RBF-EGARCH Neural Networks Model. Their proposed forecasting model combines a RBF neural network for the conditional mean and a parametric EGARCH model for the conditional volatility. They used the regression approach to estimate the weight and the parameters of the EGARCH model. They carried out a simulation based on a sample of Bank Rakyat Indonesia TBK stock returns and the results indicated that their proposed model is able to accurately predict 63% upward and downward movements of future predictions. They concluded that the simulation results obtained in the forecasting performances motivates further work, which will involve comparing a different method of parameters model estimation.

Kunst (1997) studied the augmented ARCH models which encompasses most linear ARCH-type models. He considered the two basic ARCH variants for auto-correlated series; conditional variance lagged by errors (Engle, 1982) or conditional variance lagged by observations (Weiss, 1984). He evaluated whether the restrictions evolving from these two ARCH variants are valid in practice. Time series of stock market indexes for some major stock exchanges (Standard and Poor 500 index, Stock market index for German, French, British and Japanese) were considered. For the important US Standard & Poor 500 Index and for Japanese and German stock index, the evidence indicated more or less convincingly that fourth-moments structures in financial series may be more complicated than the traditional ARCH models. A non - parametric comparison of sample moments also supported this result. The statistical evidence presented was stronger than the weak evidence on more general structures found by Tsay (1987) in an exchange rate series. For two other countries, France and the United Kingdom, the statistical description achieved by the standard ARCH model appears to be sufficient.

Su (2010) employed both GARCH and EGARCH models in studying the financial volatility in China. He applied the daily stock returns data from January 2000 to April 2010 and split the time series into two parts: before the crisis and during the crisis period. The empirical results suggested that EGARCH model fits the sample data better than GARCH model in modelling the volatility of Chinese stock returns. The result also showed that long term volatility was more volatile during the crisis period whilst Bad news produced stronger effect than good news for the Chinese stock market during the crisis.

Malmsten (2004) used a unified framework for testing the adequacy of an estimated EGARCH model. The tests were Lagrange multiplier type tests and included testing an EGARCH model against a higher-order one and testing parameter constancy. Furthermore, various existing ways of testing the EGARCH model against GARCH models were also investigated as another check of model adequacy. This was done by size and power simulations. Simulations revealed that the simulated LR test is more powerful than the encompassing test and that the size of the test may be a problem in applying the pseudo-score test. Finally, the simulation results indicated that in practice, the robust versions of their tests should be preferred to non robust ones and they can be recommended as standard tools when it comes to testing the adequacy of an estimated EGARCH (p, q) model.

The stylized facts of financial time series using three popular models were studied by Malmsten and Terasvirta (2004). The models used were the GARCH, EGARCH and Autoregressive Stochastic Volatility (ARSV) models and they focused on how well these models are able to reproduce characteristic features (stylized facts) of financial series. Their study used stock returns as a case study of the financial series. The results showed that the GARCH model and EGARCH models were at their best when characterizing models based on time series with relatively low kurtosis and high first-order autocorrelation of squares, assuming normality of errors. However the ARSV (1) model is a better option for time series displaying a combination of high kurtosis and high autocorrelations.

Blake and Kapetanios (2005) investigated the extent of the effect of neglected nonlinearity on the properties of ARCH testing procedures. They proposed and used a new ARCH testing procedures based on neural networks which are robust to the presence of neglected nonlinearity. The neural networks were used to purge the residuals of the effects of nonlinearity before applying an ARCH test. Thus they correctly sized the ARCH test while retaining good power for the ARCH test. Results based on Monte Carlo simulations showed that the new method alleviated the problem posed by the presence of neglected nonlinearity to a very large extent. Empirical evidence or results based on the application of the new test procedures to exchange rate data indicated substantial evidence of spurious rejection of the null hypothesis of no ARCH effects. There was also further evidence that exchange rates exhibited complicated, dynamic behaviour, with important nonlinearity and volatility effects.

Karanasos and Kim (2003) considered the moment structure of the general ARMA (r, s) -EGARCH (p, q) model and compared it with the standard GARCH model and APARCH model. In particular, they derived the autocorrelation function of any positive integer power of the squared errors and also obtained the autocorrelations of the squares of the observed process and cross correlations between the levels and the squares of the observed process assuming that the error terms are drawn from either a normal, double exponential or generalised error distributions. Daily data on four East Asia stock indices – Korean Stock price index (KOSPI), Japanese Nikkei index (Nikkei) and the Taiwanese SE Weighted index (SE) for the period 1980:01 – 1997:04 and the Singaporean Straits Times price index (ST) for the period 1985:01 – 1997:04. They concluded that there were differences in the moment structure between the ARMA (r, s) – EGARCH (p, q) model and the standard GARCH model. The study also concluded that, to help with model identification, results of the autocorrelations of the squared deviations can be applied to the observed data and its properties compared with the theoretical properties of the models. Based on that, it was observed that the EGARCH model can more accurately reproduce the nature of the sample autocorrelations of squared returns than the GARCH models.

Lee and Brorsen (1996) also studied the relative performance of the GARCH model and the EGARCH model by using a Cox-type non-nested test that used the Monte Carlo hypothesis tests. The approach used by Lee and Brorsen (1996) was similar to the approach used by Pesaran and Pesaran (1993). Whilst the approach of the Pesaran and Pesaran (1993) assumed asymptotic normality, Lee and Brorsen (1996) approach did not assume asymptotic normality. They estimated that the GARCH and EGARCH models of the daily spot prices of Deutsche Mark in terms of the United States dollars using the maximum likelihood procedure. The GARCH model was rejected whilst the EGARCH model was not rejected. The study therefore concluded that the EGARCH models were preferable to the GARCH models in modelling Deutsche mark/dollar exchange rate.

The effects of good and bad news on volatility in the Indian stock markets using asymmetric ARCH models during the global financial crises of 2008-2009 was investigated by Goudarzi and Ramanaraynan (2011). The asymmetric volatility models considered were the EGARCH and TGARCH models and the BSE 500 stock index was used as a proxy to the Indian stock market. The study found out that the BSE 500 return series reacted to good news and bad news asymmetrically. That is, the BSE 500 return series reacted differently to good news and to bad news. The EGARCH (1,1) and TGARCH (1,1) models were estimated for the BSE 500 stock returns series using the robust method of Bollerslev-Wooldridge’s quasi-maximum likelihood estimation (QMLE) assuming the Gaussian standard normal distribution.

The results indicated that the conditional means are significant in both estimated models. Hence the SBIC information criterion was applied to select the fittest model to the data. The TGARCH (1,1) model was selected and the study therefore concluded that the TGARCH (1,1) model can be possible representative of the asymmetric conditional volatility process for daily return series of BSE 500 as compared to the EGARCH (1,1).

Jean-Philippe (2001) examined the forecasting performance of four GARCH-typed models. The comparison focused on two different aspects; the difference between symmetric GARCH model (traditional GARCH model) and asymmetric models (EGARCH, GJR and APARCH) and the difference between normal tailed symmetric, fat-tailed symmetric and fat tailed asymmetric distributions (i.e. normal distributions against student-t and skewed student-t distributions). The study concluded that noticeable improvements were made when using an asymmetric GARCH in the conditional variance and that the APARCH and GJR outperformed the EGARCH. Furthermore, non-normal distributions provided better in-sample results than Gaussian distributions.

Alberg, Shalit and Yosef (2008) carried a comprehensive empirical analysis of the mean return and conditional variance of Tel Aviv Stock Exchange (TASE) indices using various GARCH models. The prediction performance of these conditional changing variance models were compared to newer asymmetric GJR and APARCH models. The results indicated that the asymmetric GARCH model with fat tailed densities improved overall estimation for measuring conditional variance. The EGARCH model using a skewed student-t distribution is the most successful for forecasting TASE indices as compared to the asymmetric GARCH, GJR and APARCH models.

Angelidis, Benos and Degiannakis (2003) evaluated the performance of an extensive family of ARCH models (GARCH, TARCH and EGARCH) in modelling daily Value-at-Risk (VaR) of perfectly diversified portfolios in five stock indices using a number of distributional assumptions and sample sizes. The five perfectly diversified portfolios were the S&P 500, Nikkei 225, FTSE 100, CAC 40 and DAX 30. The different distributions were normal, student-t and generalised error distribution whilst the sample sizes were 500, 1000, 1500 and 2000. Their results show that under the evaluation framework based on the proposed quartile loss function, there was strong evidence that the combination of the student-t distribution with the simplest EGARCH models produce the most adequate VaR forecasts for the majority of the markets. Furthermore, the size of the rolling sample used in estimation turned out to be rather important since in simpler models and low confidence levels, a sample size smaller than 2000 improves probability values. In more complex models where leptokurtic distributions are used and when the confidence level is high, a small sample size led to lack of convergence in the estimation algorithms. Finally, there was no consistent relation between the sample sizes and the optimal models as there were significant differences in the VaR forecasts for the same model under the four sample sizes.

Yuksel and Bayram (2005) investigated the stock market volatility in Turkish, Greek and Russian stock markets using the total return indexes based on the domestic currencies of the corresponding countries. The data set covers a period from 1994 - 2004. The study concluded that the GARCH-M (1, 1) was the best model for modelling the volatility in the stock markets in Turkey. In the case of the stock markets of Greece, the TARCH (1, 2) was the best model whilst the TARCH (1,1) was the best model for the Russian stock markets.

Irfan, Irfan and Awais (2010) modelled the volatility of short term interest rates in Pakistan and India using the ARCH family models. The study used the Karachi Inter Bank Offering Rate (KIBOR) and Mumbai Inter Bank Offering Rate (MIBOR) in Pakistan and India respectively and the various time series models examined included GARCH, EGARCH,TGARCH and PARCH. The results from all the ARCH family models indicated that high volatility is present in KIBOR returns while volatility shock is moderately present in MIBOR returns. Also all the ARCH family models were compared using the within sample forecasting performance on basis of root mean squared error (RMSE) and Mean Absolute Error (MAE) and the comparison suggested that MIBOR forecasted better than KIBOR as it had minimum errors. Lastly, the TGARCH was adjudged the best model in both returns because they had all the parameters being significant whilst the PARCH (1, 1) model is selected the second best model based on the criteria of the students t-distribution.

Anna (2011) examined the relationship between inflation, inflation uncertainty and output growth with evidence from the G-20 countries using several GARCH and GARCH-M models in order to generate a measure of inflation uncertainty. The study adopted two approaches to test for the impact of inflation uncertainty on inflation and vice versa. The first approach was based on the GARCH-M model that allows for simultaneous feedback between the conditional mean and variance of inflation while the second approach was based on a two-step procedure where Granger methods were employed using the conditional variance of a simple GARCH model. The results of the study suggested significant positive relationship between inflation uncertainty and inflation in most countries. These results go to support the Cukierman-Matter and Friedman-Ball hypothesis. Also the results of the study provided evidence for the Holland theory; that uncertainty lead to lower and in the case of the effect of inflation uncertainty in output growth, there was little evidence that inflation uncertainty has negative real effects.

Chatfield (2000) asserted that the idea behind a GARCH model was similar to that behind the ARMA model with respect to the fact that a higher order AR or MA model may often be approximated by a mixed ARMA model with fewer parameters using a rational polynomial approximation. He described the GARCH model as an approximation to a higher-order ARCH model. He noted that the GARCH (1, 1) model has become the standard model for describing non constant variance due to its relative simplicity. Empirical evidence has revealed that often $(\alpha + \beta) < 1$ so that the stationarity condition may be met. However if the $(\alpha + \beta) = 1$, the process ceases to have a finite variance although it can be shown that the squared observations are stationary after taking first differences. In such a situation a better model Integrated GARCH (IGARCH) developed by Engle and Bollerslev (1986) is recommended.

Rafique and Ur-Rehman (2011) compared the volatility behaviour and variance structure of high (daily) and low (weekly, monthly) frequencies of stock returns in Pakistan. The study used data from 1991 to 2008 of the KSE-100 index. By employing the EGARCH model, they found that there are significant asymmetric shocks (leverage effect) to volatility in the three series but the intensity of the shock were not equal for all the series. Furthermore, it was concluded that the variance structure of high frequencies (daily) data is dissimilar from the low frequencies (weekly, monthly) data.

Karanasos, Karanassou and Fountas (2004) also examined the relationship between inflation and inflation uncertainty in the US using a GARCH model that allows for simultaneous feedback between the conditional mean and variance of inflation. The results showed that there was a strong positive bi - directional relationship between inflation and inflation uncertainty. The results are also in agreement with the predictions of economic theory expressed by the Friedman-Ball and Cukierman-Meltzer hypothesis; however, it was in conflict with existing empirical evidence. The study also compared the properties of the observed time series with the theoretical properties of GARCH models to illustrate how theoretical results on correlation structure can facilitate model identification. The results showed that the AR-GARCH-M-L model can approximate reality well.

Ling and Li (1997) considered fractionally integrated autoregressive moving average time series models with conditional heteroscedasticity, which combined the popular generalised autoregressive conditional Heteroscedastic (GARCH) and fractional ARIMA models.

Drost and Klassen (1997) constructed adoptive and hence efficient estimators in a general GARCH –M in mean type context including integrated GARCH models.

A time lag between a change in money supply and the inflation rate response was examined by Jehovanes (2007). He employed a modified GARCH model to monthly inflation data for the period 1994 to 2006. The maximum likelihood estimation technique was used to estimate the parameters of the model and to determine significance of the lagged values. Results showed that the GARCH model was a better fit and indicated that a change in supply of money would affect inflation rate considerably in seven months ahead.

Brooks (2008) studied the stochastic volatility models and found that most time series models such as GARCH will have forecasts that tend towards the unconditional variance of the series as the prediction horizon increases. This implies that if they are at a low level relative to their historic average they will have a tendency to rise back towards the average and this feature is accounted for in GARCH volatility forecasting models.

Mushtaq, Shah and Ur-Rehman (2011) examining the relationship between stock exchange market volatility and macroeconomic variables volatility with respect to Pakistan. To measure this time series relationship for Pakistan, exponential generalized autoregressive conditional heteroscedasticity (EGARCH) and lag-augmented vector auto regression (LA-VAR) models were used. It was found that there is a positive relationship of consumer price index (CPI) and foreign direct investment (FDI) with stock market; however, exchange rate (ER) and T-bill rate (TBR) are inversely related to stock market volatility. On the other hand, they found strong evidence that there is a bilateral relationship of FDI and ER with stock prices, while a unidirectional relationship was found between TBR and stock market prices, with the direction from stock prices to treasury bills interest rates. However, a significant causal relationship was not found between CPI and stock prices. The analysis of this study reveals that the stock market of Pakistan is relatively less efficient as compared to US and other developed economies of the world.

Nakajima (2008) proposed the EGARCH model with jumps and heavy-tailed errors, and studied the empirical performance of different models including the stochastic volatility models with leverage, jumps and heavy-tailed errors for daily stock returns. In the framework of a Bayesian inference, the Markov Chain Monte Carlo estimation methods for these models were illustrated using a simulation study. The model comparison based on the marginal likelihood estimation was carried out with data on the U.S. stock index. Based on the estimates of the marginal likelihood, the study found that the jumps and heavy-tails raise the marginal likelihood of the EGARCH model. The EGARCH model with jumps and heavy-tails and the SV model with heavy-tails and leverage fit to the data better than other competing models for their dataset.

Ou and Wang (2010) used a probabilistic method called the Relevance Vector Machine (RVM) to predict GARCH, EGARCH and GJR based volatilities of the Hang Seng Index (HSI) for two stage out-of-sample forecasts. The RVM is a powerful tool for prediction problems as it uses a Bayesian approach whose functional form is identical to a well-known Support Vector Machine (SVM). Their goal was to compare the model with an SVM approach and classical GARCH, EGARCH and GJR models. The experimental results suggested that the proposed models can capture two different asymmetric effects of news impacts, and hence outperforms the other models; particularly, the RVM based GJR generated a best ability for first stage forecast and the RVM based EGARCH was superior for the second stage forecast of HSI volatility, in terms of the evaluation metrics: RMSE, MSE, MAD, NMSE, and linear regression R squared.

Duan, Gauthier, Simonato and Sasseville (2006) extended the analytical approach to pricing European options in the GARCH framework developed earlier in Duan, Gauthier and Simonato (1999). They extended the approximation to two other popular GARCH specifications namely the GJR-GARCH and EGARCH using the cumulative asset return as their data set. The study provided the corresponding formula and also examined their numerical performance. In each case, the resulting formula was the Black-Scholes formulae plus adjustment terms accounting for skewness and kurtosis. Also their results suggested that the approximations were adequate, particularly for shorter-maturity options. The results also revealed that their analytical approximation formula can be useful for a large-scale GARCH option pricing model where computation time can be a serious concern.

Ramasamy and Munisamy (2012) compared three simulated exchange rates of Malaysian Ringgit with actual exchange rates using GARCH, GJR and EGARCH models. For testing the forecasting effectiveness of GARCH, GJR and EGARCH the daily exchange rates of four currencies - Australian Dollar, Singapore Dollar, Thailand Bhat and Philippine Peso - were used. The forecasted rates, using Gaussian random numbers, were compared with the actual exchange rates of year 2011 to estimate errors. Both the forecasted and actual rates were then plotted to observe the synchronisation and validation. The results showed more volatile exchange rates are predicted well by the GARCH models efficiently than the hard currency exchange rates which are less volatile. Among the three models the effective model was indeterminable as these models forecast the exchange rates in different number of iterations for different currencies. The leverage effect incorporated in GJR and EGARCH models did not improve the results much.

Shamiri and Hassan (2005) examined and estimated the three GARCH(1,1) models (GARCH, EGARCH and GJR-GARCH) using the daily price data of two Asian stock indices, Strait Times Index in Singapore (STI) and Kuala Luampur Composite Index in Malaysia (KLCI) over a 14- years period. The competing models GARCH, EGARCH and GJR-GARCH were developed based on three different distributions, Gaussian normal, Student-t, Generalized Error Distribution. The estimation results showed that the forecasting performance of asymmetric GARCH Models (GJR-GARCH and EGARCH), especially when fat-tailed asymmetric densities are taken into account in the conditional volatility, was better than symmetric GARCH. Moreover, it was found that the AR (1)-GJR model provided the best out-of-sample forecast for the Malaysian stock market, while AR(1)-EGARCH provided a better estimation for the Singaporean stock market.

Jiang (2011) examined the relationship between inflation and inflation uncertainty in China. He believed that it was worthy to investigate the inflation and inflation uncertainty relationship in China as it is commonly believed that one possible channel that inflation imposes significant economic costs is through its effect on inflation uncertainty. Jiang (2011) addressed the relationship of inflation and its uncertainty in China's urban and rural areas separately given the huge urban-rural gaps. The GARCH(1,1) and E-GARCH(1,1) models were used to generate the measure of inflation uncertainty and then Granger causality tests were performed to test for the causality between inflation and inflation uncertainty. GARCH (1, 1)-M models were also employed to further investigate the inflation-uncertainty nexus. The results provided strong statistical supportive evidence that higher inflation raises inflation uncertainty. On the other hand, the evidence on the effect of inflation uncertainty on inflation was mixed and depended on the sample period and areas examined.

Hassan, Moud and Ekonomi (2006) explored the varying volatility dynamic of inflation rates in Malaysia for the period from August 1980 to December 2004. The GARCH and EGARCH models were used to capture the stochastic variation and asymmetries in the economic instruments. Also, an in-sample evaluation of the sub-periods volatility was done using both models. The results indicated that, the EGARCH model gave better estimates of sub-periods volatility as compared to the GARCH model.

Berument, Kivilcim and Neyapti (2001) used the EGARCH to model inflation uncertainty in Turkey. Their study used the monthly CPI inflation covering the period from 1986 to 2000. Their study gave further contribution to literature due to the inclusion of seasonal terms in the conditional variance equation. The results of the study provided evidence to show that in Turkey, the effect on inflation uncertainty of positive shocks to inflation are greater than that of negative shocks to inflation. Also, when monthly dummies were used in modelling both inflation and inflation uncertainty, the effect of lagged inflation on inflation uncertainty disappeared. They concluded that there is no significant lagged effect of inflation on inflation uncertainty. Lastly, there was evidence of significant seasonal effects of inflation on conditional variability.

Alam and Rahman (2012) explored the application of GARCH type models such as GARCH; EGARCH; TARCH; and PARCH; to modelling the BDT/USD exchange rate using the daily foreign exchange rate series fixed up by Bangladesh Bank. The BDT/USD time series from July 03, 2006 to April 30, 2012 were used for the study purpose out of which in-sample and out-of-sample date set covered from July 03, 2006 to May 13, 2010 and May 14, 2010 to April 30, 2012 respectively. They benchmarked their results with AR and ARMA models. They found that all GARCH type models demonstrated that past volatility of exchange rate significantly influenced current volatility. Both the AR and ARMA models were found as the best model as per in-sample statistical performance results, whereas according to out-of-sample, GARCH model was the best model with transaction costs and the TARCH model was nominated as the best model without transaction costs. The EGARCH and TARCH models outperform all the other models as per to in-sample and out-of-sample trading performance outcomes respectively including transaction costs.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

This chapter deals with the methodology for the study and it has been sub divided into five sections aside the introductory section. Section one looks briefly at time series and its basic concepts like stationarity; Sections two to four will give a detailed description and explanation of the theory and concept of the ARCH-type models (i.e. ARCH, GARCH and EGARCH models) that would be used in the chapter four to analyse the data. The final section would be the conclusion.

3.1 Time Series and its Basic Concepts

Chatfield (2000) defines time series as a series or sequence $\{x_t\}$ of data points measured typically at successive times. The data points are commonly spaced equal in time. Time series analysis comprises methods that attempt to understand the underlying generation process of the data points and constructs a mathematical model to represent the process. The constructed model is then used to forecast future events based on known past events. Time series often makes use of the natural one-way ordering of time so that values in a series for a given time will be expressed as being derived from past values rather than future values. A time series model usually reflects the fact that observations close together in time domain are more correlated as compared to observations further apart. That is, there is ‘volatility clusters’- small (large) shocks are again followed by small (large) shocks.

Original time series data are made up of various patterns that are derived on casual factors which are identified by time series analysis methods. The four patterns that characterize economic and business series are the long-run development known as the trend, cyclical or periodic component, seasonal component and the error or residual component. The trend component deals with the general and overall pattern of the time series; the cyclical component refers to the variation in the series which arise out of the phenomenon of business cycles. It usually spans within periods of more than one year. The seasonal variations refers to the periodic and repetitive ups and downs in the series that occur within a year and lastly the error term is the component that contains all moments which neither belong to the trend nor to the cycle nor to the seasonal component.

The models for time series data can have many forms and represents different stochastic processes which could be linear or non-linear. Among the linear models include autoregressive (AR) model of order (p), moving average (MA) of order (q) and autoregressive moving average (ARMA) model of order(p, q). A combination of the above models produce the autoregressive integrated moving average (ARIMA) model with a generalized model known as the autoregressive fractionally integrated moving average (ARFIMA) model.

The non-linear time series model represent or reflect the changes of variance along with time known as heteroscedasticity. With these models, changes in variability are related to and/or predicted by recent past values of the observed series. The wide variety of non-linear models include the symmetric models such as Autoregressive Conditional Heteroscedastic (ARCH) model with order (p) and Generalized ARCH (GARCH) model with order(p, q). Other asymmetric models are the Power ARCH

(PARCH), Threshold GARCH (TGARCH), Exponential GARCH (EGACRH), Integrated GARCH (IGARCH), etc. All these asymmetric models have order (p, q). The above mentioned non-linear models form part of a large family of the ARCH-type models. In this study, three of such models – ARCH, GARCH and EGARCH- would be fitted to the data set. The theory and concepts of these models are explained in detail in later sections of this chapter. Other forms of non-linear models include the bilinear model, threshold autoregressive (TAR), state-dependent model, markov switching models, etc.

3.1.1 Stationary and Non Stationary Processes

The foundation of time series analysis is stationarity. That is, before time series analysis is carried out, one needs to verify whether the series is stationary or otherwise. However, an assumption of stationarity is usually made. In this section, we define and describe stationarity (non-stationarity).

A series is said to be stationary if the mean and auto covariances of the series do not depend on time. There are two forms of stationarity - strict stationarity and weak stationarity. Under strict stationarity, the common distribution function of the stochastic process does not change by a shift in time. That is, a time series $\{x_t\}$ is said to be strictly stationary if the joint distribution of $(x_1, \cdots, x_k)$ is identical to that of $(x_{1+t}, \cdots, x_{k+t})$ for all t, where k is an arbitrary positive integer and $(1, \cdots, k)$ is a collection of k positive integers. The shifting of the time origin by t has no effect on the joint distribution which depends only on the intervals between the two sets of points given by t which is called a lag.

The concept of strict stationarity is difficult to apply in practice and hence weak stationarity or stationarity in the second moment is often assumed. A time series $\{x_t\}$ is weakly stationary if both the mean of x_t and the covariance between x_t and x_s are time-invariant. More specifically, $\{x_t\}$ is weakly stationary if:

(a) $\mathbb{E}\ (x_t)$ = μ, which is a constant, and

(b) $cov\ (x_t, x_s)$ = γ which is only a function of the time distance between the two random variables and does not depend on the actual points in time t.

3.2 ARCH (m) MODEL

An ARCH process is a mechanism that includes past variance in the explanation of future variances (Engle, 2004). The ARCH model was developed by Engle (1982) and it provides a systematic framework for volatility modelling. ARCH models specifically take the dependence of the conditional second moments in consideration when modelling.

Let $\{x_t\}$ be the mean-corrected return, ε_t be the Gaussian white noise with zero mean and unit variance and I_t be the information set at time t given by $I_t = \{x_1, x_2, \dots, x_{t-1}\}$. Then the ARCH ($m$) model is specified as:

$$x_t = \sigma_t \varepsilon_t \tag{3.1a}$$

$$\sigma^2{}_t = \alpha_0 + \alpha_1 x^2{}_{t-1} + \dots . + \alpha_m x^2{}_{t-m} \tag{3.1b}$$

where $\alpha_0 > 0$ and $\alpha_i \geq 0$, $i = 1, \dots, m$

and

$$\mathbb{E}(x_t|I_t) = \mathbb{E}[\mathbb{E}(x_t|I_t)] = \mathbb{E}[\sigma_t \mathbb{E}(\varepsilon_t)] = 0 \quad (3.2a)$$

$$V(x_t|I_t) = \mathbb{E}({x^2}_t) = {\sigma^2}_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i {x^2}_{t-i} \quad (3.2b)$$

and the error term ε_t is such that

$$\mathbb{E}(\varepsilon_t|I_t) = 0 \quad (3.3a)$$

and

$$V(\varepsilon_t|I_t) = 1 \quad (3.3b)$$

From equations (3.3a) and (3.3b), it can be seen that the error term ε_t is a conditional standardised martingale difference. A stochastic series $\{x_t\}$ is said to be a martingale difference if its expectation with respect to past values of another stochastic series $\{y_i\}$ is zero (Amos, 2010).

That is

$$\mathbb{E}\left(x_{t+i}|y_{i,}y_{i-1,} \dots\right) = 0 \text{ for } i = 1,2, \dots \quad (3.4)$$

From the structure of the model, it can be seen that the dependence of the present volatility $\{x_t\}$ is a simple quadratic function of its lagged values. The coefficients $\alpha_i, i = 0, \dots, m$ can consistently be estimated by regressing $\{{x^2}_t\}$ on ${x^2}_{t-1}, {x^2}_{t-2}$, … , ${x^2}_{t-m}$. To ensure that the conditional variance ${\sigma^2}_t$ is always positive for all t, it is required that $\alpha_0 > 0$ and $\alpha_i \geq 0$, $i = 1, \dots, m$. From equations (3.1a) and (3.1b) it follows that large past squared values $\{{x^2}_{t-i}\}, i = 1, \dots, m$ imply a large conditional variance ${\sigma^2}_t$ for the present volatility$\{x_t\}$. Consequently, $\{x_t\}$ tends to assume a large value in absolute value. Hence under the ARCH framework, large shocks tend to be followed by another large shock. We would take a particular case of the ARCH (m) model where $m = 1$, ARCH(1) to help understand the ARCH(m) better.

3.2.1 ARCH (1) Model

The ARCH (1) model is a special case of the general ARCH (m) model. Let $\{x_t\}$ be the mean-corrected return, ε_t be the Gaussian white noise with zero mean and unit variance. If I_t is the information set available at time t given by $I_t = \{x_1, x_2, \dots, x_{t-1}\}$, then the process $\{x_t\}$ is ARCH (1) where $m = 1$, if

$$x_t = \sigma_t \varepsilon_t \tag{3.5a}$$

$$\sigma^2{}_t = \alpha_0 + \alpha_1 x^2{}_{t-1} \tag{3.5b}$$

where α_0 and α_1 are unknown parameters. The process $\{x_t\}$ can be stated conditionally in terms of I_t similar to the variance $\sigma^2{}_t$ under the normality assumption of the error term ε_t. Again to ensure that the conditional variance is always positive, the constraints $\alpha_0 > 0$ and $\alpha_i \geq 0$, $i = 1, \dots, m$ is required. Since the ARCH (1) is a special case of ARCH (m), whatever applies to the ARCH (m) model also applies for the ARCH (1). Hence it can be concluded from equations (3.5a) and (3.5b) that a large past squared mean-corrected return $\{x^2{}_{t-i}\}$, $i = 1, \dots, m$ implies a large conditional variance$(\sigma^2{}_t)$, resulting in x_t being large in absolute value. For the ARCH (m) models to be valid, the presence of ARCH effects should be statistically significant and hence the presence of the ARCH effects should be tested for.

3.2.2 Testing for ARCH Effects

The presence of conditional heteroscedasticity is referred to as the ARCH effects. To determine the presence of the ARCH effect, a formal statistical test is required. Two tests are available. These are Ljung-Box Statistics $Q\ (m)$ test and Lagrange Multiplier (LM) test, which would be discussed in detail in the next two subsections.

Let $x_t = r_t - u_t$ be the mean corrected return, where r_t is the return of an asset, u_t is the conditional mean of r_t. The squared series $\{{r^2}_t\}$ is then used to check for the presence of ARCH effects.

3.2.2.1 Ljung – Box Test

The null hypothesis (H_0) for this test is that the first m lags of the autocorrelation function of the series $\{{r^2}_t\}$ is zero against the alternative hypothesis (H_1) that not all the first m lags of the autocorrelation function of the series is zero.

The test statistic is given as;

$$Q = T \sum_{i=1}^{m} \widehat{p(\iota)}^2 \tag{3.6a}$$

where $\widehat{p(\iota)}$ is the consistent estimator of the autocorrelation function and T is the sample size. Under the null hypothesis, Q is asymptotically distributed as chi-square with m degrees of freedom. For small samples, the test statistic is given as;

$$Q^* = T(T + 2) \sum_{i=1}^{m} \frac{\widehat{p(\iota)}^2}{T-i} \tag{3.6b}$$

Q^* is also asymptotically distributed as chi-square with m degrees of freedom under the null hypothesis. The decision rule is to reject the null hypothesis of non-autocorrelation of the residuals if Q or Q^* are too large than the corresponding critical value of the distribution with m degrees of freedom for a specified significance level (α) or if the p value of Q or Q^* is less than the significance level (α).

3.2.2.2 Lagrange Multiple (LM) Test

This test is equivalent to the usual F statistic for testing $\alpha_i = 0$, $(i = 1, \dots, m)$ in the linear regression

$$x^2{}_t = \alpha_0 + \alpha_1 x^2{}_{t-1} + \dots + \alpha_m x^2{}_{t-m} + \epsilon_t, \quad t = m+1, \dots, T \tag{3.7}$$

where ϵ_t denotes the error term, m is the pre-specified positive integer and T is the sample size. Specifically the null hypothesis is

$H_0 = \alpha_1 = \dots = \alpha_m = 0$

with the test statistic given by

$$F = \frac{(SSR_0 - SSR_1)/m}{SSR_1/(T-2m-1)}$$

where $SSR_0 = \sum_{t=m+1}^{T} (x^2{}_t - \overline{w})^2$, $\overline{w} = \frac{\sum_{t=1}^{T} x^2{}_t}{T}$ is the sample mean of $x^2{}_t$ and $SSR_1 = \sum_{t=m+1}^{T} \widehat{\epsilon^2}_t$, $\widehat{\epsilon_t}$ is the least square residual of the prior linear regression. Under the null hypothesis, F is asymptotically distributed as chi-squared distribution with m degrees of freedom. The decision rule is to reject the null hypothesis if the F is greater than the corresponding critical value of the chi-square distribution with m degrees of freedom for a specified significance level (α) or if the p-value of F is less than the significance level (α).

3.2.3 Determination of the order of ARCH (m) Model

If the presence of the ARCH effect is significantly established, the ARCH model is valid and can be used to model the series. However to model the ARCH (m), the order m should be determined. The partial autocorrelation function (PACF) of the ${x^2}_t$ is used to determine the order m, of the ARCH (m) model.

Given that $x_t = \sigma_t \varepsilon_t$ and ${\sigma^2}_t = \alpha_0 + \alpha_1 {x^2}_{t-1} + ... + \alpha_m {x^2}_{t-m}$ as shown by equations (3.1a) and (3.1b), for a given sample, ${x^2}_t$ is an unbiased estimate of ${\sigma^2}_t$ and hence ${x^2}_t$ is expected to be linearly related to ${x^2}_{t-1}$, ..., ${x^2}_{t-m}$ similar to that of an autoregressive model of order m. It should be noted that a single ${x^2}_t$ is generally not an efficient estimate of ${\sigma^2}_t$. However, it serves as an approximate value that could be informative in specifying the order m.

3.2.4 Estimation of the ARCH (m) and ARCH (1) Models

3.2.4.1 Estimation of the ARCH (m) model

There are three likelihood functions that are commonly used in ARCH (m) estimation depending on the distributional assumption made on the error term ε_t. The three common distributions are the normal distribution, standardized student-t distribution which is a heavy tailed distribution and the generalised error distribution (GED). This study assumes that the error term ε_t is normally distributed.

Based on the assumption of normality made on the error term ε_t, the likelihood function of an ARCH (m) model is given as:

$$f(x_1, \cdots, x_t|\theta) = f(x_t|x_{t-1})\, f(x_{t-1}|x_{t-2}) \cdots f(x_{m+1}|x_m)\, f(x_1, \cdots, x_m|\theta)$$

$$= \prod_{t=m+1}^{T} \frac{1}{\sqrt{2\pi\sigma^2{}_t}} \exp\left(\frac{-x^2{}_t}{2\sigma^2{}_t}\right) f(x_1, \cdots, x_m|\theta) \tag{3.8}$$

where $\theta = (\alpha_0, \alpha_1, \cdots, \alpha_m)'$ and $f(x_1, \cdots, x_m|\theta)$ is the joint probability density function of $x_1, \cdots, x_m$. Since the exact form of $f(x_1, \cdots, x_m|\theta)$ is complicated and difficult to obtain, it is commonly dropped from the prior likelihood function, especially when the sample size is sufficiently large. Rather it is practically easier to condition on the first $x_1, \cdots, x_m$ since they are usually known and equal to its observed values. This results in the conditional likelihood function being:

$$f(x_1, \cdots, x_t|\theta; x_1, \cdots, x_m) = \prod_{t=m+1}^{T} \frac{1}{\sqrt{2\pi\sigma^2{}_t}} \exp\left(\frac{-x^2{}_t}{2\sigma^2{}_t}\right) \tag{3.9}$$

where $\sigma^2{}_t$ can be evaluated recursively. Under the normality assumption, the estimates $\widehat{\alpha_0}$, $\widehat{\alpha_1}$, $\cdots$, $\widehat{\alpha_m}$, are obtained by maximising the prior likelihood function called the conditional maximum likelihood estimates (MLE) (Tsay, 2002).

Maximising the conditional likelihood function can be difficult to handle. An equivalent way which is easier to handle is to maximise the logarithm of the conditional likelihood function. Accordingly, the conditional log likelihood function is given as

$$\ell(x_{m+1},\cdots,x_t|\theta;x_1,\cdots,x_m) = \sum_{t=m+1}^{T}\left(-\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln \sigma^2{}_t - \frac{x^2{}_t}{2\sigma^2{}_t}\right)$$

$$= -\sum_{t=m+1}^{T}\left(\frac{1}{2}\ln \sigma^2{}_t + \frac{x^2{}_t}{2\sigma^2{}_t}\right) + K \qquad (3.10)$$

where $K = \frac{-(T-m)}{2}\ln(2\pi)$

since the first term $\frac{1}{2}\ln 2\pi$ does not involve any parameter and hence its exclusion has no effect on the estimation process. Again $\sigma^2{}_t = \alpha_0 + \alpha_1 x^2{}_{t-1} + ... + \alpha_m x^2{}_{t-m}$ can be evaluated recursively.

3.2.4.2 Estimation of the ARCH (1) Model

Given that the order (m) is $m = 1$, the ARCH (1) can be estimated. Based on the assumption of normality made on the error term, ε_t, the maximum likelihood estimation is used. Let $\{x_t\}$ be a realization from an ARCH (1) process. Then the likelihood of the data is written as a product of the conditionals as

$$f(x_1,\cdots,x_t|\theta) = f(x_t|x_{t-1})\, f(x_{t-1}|x_{t-2}) \cdots f(x_2|x_1)\, f(x_1|\theta)$$

$$= \prod_{t=2}^{T} \frac{1}{\sqrt{2\pi\sigma^2{}_t}} \exp\left(\frac{-x^2{}_t}{2\sigma^2{}_t}\right) f(x_1|\theta) \qquad (3.11)$$

Where $\theta = (\alpha_0, \alpha_1)'$. Since it is complicated and difficult to obtain the exact form of $f(x_1|\theta)$, it is more practical and easier to condition on the first x_1 since x_1 is usually known and equal to its observed value. This result in the conditional likelihood function being:

$$f(x_1,\cdots,x_t|\theta;x_1) = f(x_t|x_{t-1})\, f(x_{t-1}|x_{t-2}) \cdots f(x_2|x_1)\, f(x_1|\theta;x_1)$$

$$= \prod_{t=2}^{T} \frac{1}{\sqrt{2\pi\sigma^2{}_t}} \exp\left(\frac{-x^2{}_t}{2\sigma^2{}_t}\right) \qquad (3.12)$$

Since $x_t|I_t \sim N\ (0, \sigma^2{}_t)$ with a probability density function of

$$f(x_t|I_t) = \frac{1}{\sqrt{2\pi\sigma^2{}_t}} \exp\left(\frac{-x^2{}_t}{2\sigma^2{}_t}\right)$$

where $\sigma^2{}_t = \alpha_0 + \alpha_1 x^2{}_{t-1}$.

The conditional log likelihood function is expressed as

$$\ell(x_2, \cdots, x_t|\theta; x_1) = \sum_{t=2}^{T} \left(-\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln\sigma^2{}_t - \frac{x^2{}_t}{2\sigma^2{}_t}\right)$$

$$= -\sum_{t=2}^{T} \left(\frac{1}{2}\ln\sigma^2{}_t + \frac{x^2{}_t}{2\sigma^2{}_t}\right) \qquad (3.13)$$

since the first term $\frac{1}{2}\ln 2\pi$ does not involve any parameter and hence its exclusion has no effect on the estimation process and $\sigma^2{}_t = \alpha_0 + \alpha_1 x^2{}_{t-1}$ can be evaluated recursively. The maximum likelihood estimates are obtained by maximising this function with respect to α_0 and α_1 . Since the function is non-linear in these parameters, its maximisation must be done using appropriate non-linear optimization routine.

Let $\{x_t\}$, $t = 1, \ldots, T$ be a series generated by an ARCH (1) process, where T is the sample size. By conditioning on the initial observation (x_1), the joint probability density function is written as

$$f(x) = \prod_{t=2}^{T} f(x_t|I_t) = \prod_{t=2}^{T} \frac{1}{\sqrt{2\pi\sigma^2{}_t}} \exp\left(\frac{-x^2{}_t}{2\sigma^2{}_t}\right) \text{ as in equation (3.12)}$$

with the conditional log likelihood function expressed as

$$\ell(x_2, \cdots, x_t|\theta; x_1) = \sum_{t=2}^{T} \left(-\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln\sigma^2{}_t - \frac{x^2{}_t}{2\sigma^2{}_t}\right)$$

$$= -\sum_{t=2}^{T}\left(\frac{1}{2}\ln{\sigma^2}_t + \frac{{x^2}_t}{2{\sigma^2}_t}\right) \text{ as in equation (3.13)}$$

The conditional maximum likelihood estimates of α_0 and α_1 are obtained by taking the derivatives of the conditional log likelihood with respect α_0 and α_1 to as given below:

$$\frac{\partial \ell}{\partial \alpha_0} = \frac{1}{2{\sigma^2}_t}\left(\frac{{x^2}_t}{{\sigma^2}_t} - 1\right)\frac{\partial \ell}{\partial \alpha_0}$$

$$= \frac{1}{2{\sigma^2}_t}\left(\frac{{x^2}_t}{{\sigma^2}_t} - 1\right)\frac{\partial \ell}{\partial {\sigma^2}_t} \times \frac{\partial {\sigma^2}_t}{\partial \alpha_0} \qquad (3.14a)$$

and

$$\frac{\partial \ell}{\partial \alpha_1} = \frac{1}{2{\sigma^2}_t}\left(\frac{{x^2}_t}{{\sigma^2}_t} - 1\right)\frac{\partial \ell}{\partial \alpha_1}$$

$$= \frac{1}{2{\sigma^2}_t}\left(\frac{{x^2}_t}{{\sigma^2}_t} - 1\right)\frac{\partial \ell}{\partial {\sigma^2}_t} \times \frac{\partial {\sigma^2}_t}{\partial \alpha_1} \qquad (3.14b)$$

Generally, the partial derivative of ℓ is

$$\frac{\partial \ell}{\partial \theta} = \sum_{t=2}^{T}\left(\frac{\partial \ell}{\partial {\sigma^2}_t} \times \frac{\partial {\sigma^2}_t}{\partial \theta}\right)$$

$$= -\frac{1}{2}\sum_{t=2}^{T}\left(\frac{1}{{\sigma^2}_t} - \frac{{x^2}_t}{{\sigma^4}_t}\right)\begin{pmatrix}1\\ {x^2}_{t-1}\end{pmatrix}$$

$$= \frac{1}{2}\sum_{t=2}^{T}\left(\frac{{x^2}_t}{{\sigma^2}_t} - 1\right)\frac{1}{{\sigma^2}_t}\begin{pmatrix}1\\ {x^2}_{t-1}\end{pmatrix} \qquad (3.15)$$

recalling that ${\sigma^2}_t = \alpha_0 + \alpha_1 {x^2}_{t-1}$.

The Hessian ($\mathbb{H}$) is then given by

$$\frac{\partial^2 \ell}{\partial \theta \partial \theta'} = \sum_{t=2}^{T}\left(\frac{\partial^2 \ell}{\partial {\sigma^4}_t} \times \frac{\partial {\sigma^2}_t}{\partial \theta} \times \frac{\partial {\sigma^2}_t}{\partial \theta'}\right)$$

$$= -\frac{1}{2}\sum_{t=2}^{T}\left(\frac{x^2{}_t}{(\sigma^2{}_t)^3} + \left(\frac{x^2{}_t}{\sigma^2{}_t} - 1\right)\frac{1}{\sigma^4{}_t}\right)\begin{pmatrix}1 & x^2{}_t\\ x^2{}_t & x^4{}_t\end{pmatrix} \tag{3.16}$$

Since $\frac{\partial^2\sigma^2{}_t}{\partial\theta\partial\theta'} = 0$

The Fisher information matrix defined as the negative expected value of the Hessian and denoted usually by g is given as

$$g = -\mathbb{E}\left(\frac{\partial^2\ell}{\partial\theta\partial\theta'}\right)$$

Now since $\mathbb{E}_{x_t|I_t}\left\{\left(\frac{x^2{}_t}{\sigma^2{}_t} - 1\right)\frac{1}{\sigma^4{}_t}\begin{pmatrix}1 & x^2{}_t\\ x^2{}_t & x^4{}_t\end{pmatrix}\right\} = 0$

and $\mathbb{E}_{x_t|I_t}\left\{\frac{x^2{}_t}{(\sigma^2{}_t)^3}\right\} = \left\{\frac{\mathbb{E}_{x_t|I_t}(x^2{}_t)}{(\sigma^2{}_t)^3}\right\} = \frac{1}{\sigma^4{}_t}$,

It follows then that

$$g = \frac{1}{2}\sum_{t=2}^{T}\left(\frac{1}{\sigma^4{}_t}\right)\begin{pmatrix}1 & x^2{}_t\\ x^2{}_t & x^4{}_t\end{pmatrix} \text{ as in Engle (1982).} \tag{3.17}$$

Non-linear optimization routines are iterative, thus if θ^i denotes the parameter estimates after the i^{th} iterations, then θ^{i+1} has the form

$$\theta^{i+1} = \theta^i + \lambda M^{-1}\left\{\frac{\partial\ell}{\partial\theta}\right\} \tag{3.18}$$

Where λ is a step-length chosen to maximise the likelihood function in the direction of $\frac{\partial\ell}{\partial\theta}$. For the Newton Raphson based routines $\lambda = 1$ and $= \frac{\partial^2\ell}{\partial\theta\partial\theta'}$, and for the Fisher scoring method $\lambda = 1$ and $M = g$ (Mills, 1994 and Engle 1982).

3.2.5 Forecasting with the ARCH Model

One important aim of developing a time series model is to estimate future values before they are realized. The ARCH model is no exception. The ARCH model provides good estimates of the series before it is realized. The theory of forecasting with the ARCH models is presented and discussed in detail in this section**.**

Let $x_1, x_2, \dots, x_t$ be an observed time series, then the $\zeta -$ step ahead forecast for ζ= 1, 2,⋯ at the origin, denoted as $x_t\ (\zeta)$ is taken to be the minimum mean squared error predictor. That is the value of ζ that minimizes the function

$$\mathbb{E}\big(x_{t+\,\zeta} - f(x)\big)^{\mathbf{2}} \qquad (3.19)$$

where $f(x)$ is a function of the observation.

Then

$$x_t\ (\zeta) = \mathbb{E}\big(x_{t+\,\zeta} \big| x_1, x_2, \dots, x_t\big) \text{ Tsay (2002).} \qquad (3.20)$$

For the ARCH (1) model $x_t\ (\zeta) = \mathbb{E}\big(x_{t+\,\zeta} \big| x_1, x_2, \dots, x_t\big) = 0$ according to Shepard (1996).

It is important to note that the forecast for the x_t series provide no much helpful information and therefore it is imperative to look at the squared returns ${x^2}_t$ given as

$${x^2}_t\ (\zeta) = \mathbb{E}\big({x^2}_{t\,+\,\zeta} \big| {x^2}_1, {x^2}_2, \dots, {x^2}_t\big). \qquad (3.21)$$

For the ARCH (m) model, the 1- step forecast for ${x^2}_t$ at the origin t is given by

$${x^2}_t\ (1) = \widehat{\alpha_0} + \widehat{\alpha_1}{x^2}_t + \cdots + \widehat{\alpha_m}{x^2}_{t+1-\,m}$$

$$= {\sigma^2}_t\ (1) \qquad (3.22)$$

where $\widehat{\alpha_\iota}$, $i = 0, 1, \dots, m$ are the conditional maximum likelihood estimates of α_i ,$i = 1, \dots, m$.

The 2-step ahead forecast for $x^2{}_t$ is given as

$$x^2{}_t\,(1) = \widehat{\alpha_0} + \widehat{\alpha_1} x^2{}_t(1) + \widehat{\alpha_2} x^2{}_t \;\cdots + \widehat{\alpha_m} x^2{}_{t+2-\,m}$$

$$= \widehat{\alpha_0} + \widehat{\alpha_1} \sigma^2{}_t\,(1) + \widehat{\alpha_2} x^2{}_t \;\cdots + \widehat{\alpha_m} x^2{}_{t+2-\,m} \qquad (3.23)$$

and in general, the ζ- step ahead forecast for $x^2{}_t$ is given as

$$x^2{}_t\,(\zeta) = \sigma^2{}_t\,(\zeta) = \widehat{\alpha_0} + \sum_{i=1}^{m} \widehat{\alpha_\iota} \sigma^2{}_t\,(\zeta - i)$$

where $\sigma^2{}_t\,(\zeta - i) = x^2{}_{t+\,\zeta-i}$ if $\zeta - i \le 0$, Tsay (2002). (3.24)

In the special case of the ARCH (1) model the 1- step forecast for $x^2{}_t$ at the origin t is given by

$$x^2{}_t\,(1) = \mathbb{E}(x^2{}_{t+1}\,|x_t)$$

$$= \widehat{\alpha_0} + \widehat{\alpha_1} x^2{}_t$$

$$= \sigma^2{}_t\,(1) \qquad (3.25)$$

Since $\sigma^2{}_t\,(1) = \mathbb{E}(\sigma^2{}_{t+1}\,|x_t) = \widehat{\alpha_0} + \widehat{\alpha_1} x^2{}_t$

where $\widehat{\alpha_0}$ and $\widehat{\alpha_1}$ are the conditional maximum likelihood estimates of α_0 and α_1.

Similarly a 2-step ahead forecast for $x^2{}_t$ is given as

$$x^2{}_t\,(2) = \mathbb{E}(x^2{}_{t+2}\,|x_t)$$

$$= \mathbb{E}(\sigma^2{}_{t+2}\,|x_t)$$

$$= \widehat{\alpha_0} + \widehat{\alpha_1}\mathbb{E}({x^2}_{t+1} \,|x_t)$$

$$= \widehat{\alpha_0} + \widehat{\alpha_1}(\widehat{\alpha_0} + \widehat{\alpha_1}{x^2}_t)$$

$$= \widehat{\alpha_0}\,(1 + \widehat{\alpha_1}) + \widehat{\alpha_1}^2\,{x^2}_t = {\sigma^2}_t\,(2) \tag{3.26}$$

and the ζ- step ahead forecast for ${x^2}_t$ is given as

$${x^2}_t\,(\zeta) = \mathbb{E}\left({x^2}_{t+\zeta} \,|x_t\right)$$

$$= \mathbb{E}\left({\sigma^2}_{t+\zeta} \,|x_t\right)$$

$$= \widehat{\alpha_0}\,(1 + \widehat{\alpha_1} + \widehat{\alpha_1}^2 + \cdots + \widehat{\alpha_1}^{\zeta-1}) + \widehat{\alpha_1}^{\zeta}\,{x^2}_t = {\sigma^2}_t\,(\zeta) \tag{3.27}$$

Despite the advantages of the ARCH models, there are problems in using the ARCH models. First of all, the ARCH models assume that positive and negative shocks have the same effects on volatility because it depends on the square of the previous shocks, which is not the case in practice. Also the ARCH formulation can lead to complexity if the order of the model is higher. This necessitated the introduction of the GARCH model as an extension of the ARCH models (Tsay, 2002).

3.3 The GARCH (m, s) Model

The Generalized ARCH (GARCH) model was developed by Bollerslev (1986) as an extension of the ARCH model in the same way the ARMA process is an extension of the AR process. The principle of parsimony may be violated when a model has a large number of parameters resulting in difficulties in using the model to adequately describe the data. In particular, although the ARCH model is simple, it may require many parameters as there might be a need for a large value of lag q and hence the principle of parsimony would be violated in such a case. An ARMA model may have fewer parameters compared to the AR model and similarly, a GARCH model may contain fewer parameters when compared to an ARCH model. Thus a GARCH model may be preferred to an ARCH model using the principle of parsimony.

Let $x_t = r_t - u_t$ be the mean corrected return, where r_t is the return of an asset, u_t is the conditional mean of x_t. Then x_t follows a GARCH (m, s) model if

$$x_t = \sigma_t \varepsilon_t \tag{3.28a}$$

$$\sigma^2{}_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i \, x^2{}_{t-i} + \sum_{j=1}^{s} \beta_j \, \sigma^2{}_{t-j} \tag{3.29b}$$

where $\{\varepsilon_t\}$ is a sequence of independent, identically distributed random variables with mean zero and unit variance and the parameters of the model are α_i , $i = o, \cdots, m$ and β_j ,$j = 1, \cdots, s$ such that $\alpha_i \geq 0$ and $\beta_j \geq 0$; $\sum_{i=1}^{v}(\alpha_i + \beta_i) < 1$, where $v = \max(m, s)$ and $\alpha_i = 0$ for $i > m$ and $\beta_j = 0$ for $j > s$. The constraints on $\alpha_i + \beta_i$ implies that the unconditional variance of x_t is finite, whereas its conditional variance $\sigma^2{}_t$ evolves over time. From the equations (3.28a) and (3.28b), it is seen that the GARCH (m, s) model employs the same equation (3.1a) for the mean corrected return x_t as in the ARCH (m) but the equation for the volatility includes s new terms. Therefore equations (3.28a) and (3.28b) reduces to a pure ARCH (m) model if $s = 0$. Thus the GARCH model generalizes the ARCH model by introducing values of $\sigma^2{}_{t-1}, \sigma^2{}_{t-2}, \cdots$. The parameters α_i and β_j are respectively referred to as the ARCH and GARCH parameters.

The GARCH (m, s) model can be stated differently. Let $\eta_t = x^2{}_t - \sigma^2{}_t$ so that

$\sigma^2{}_t = x^2{}_t - \eta_t$. By substituting $\sigma^2{}_{t-i} = x^2{}_{t-i} - \eta_{t-i}$, $(i = o, \cdots, m)$ into equation (3.28b), the GARCH (m, s) can be written as

$$x^2{}_t = \alpha_0 + \sum_{i=1}^{v} (\alpha_i + \beta_i)\, x^2{}_{t-i} + \eta_t - \sum_{j=1}^{s} \beta_j\, \eta_{t-j} \tag{3.29}$$

where $v = \max(m, s)$, $\alpha_i = 0$ for $i > m$ and $\beta_j = 0$ for $j > s$.

Thus the equation of $\sigma^2{}_t$ has an ARMA (m, s) representation and it can be seen that $\{\eta_t\}$ is a martingale difference series (i.e.$\mathbb{E}(\eta_t) = 0$ and $cov\ (\eta_t, \eta_{t-j})$ for $j \geq 1$). However, the $\{\eta_t\}$ is not an independent, identically distributed random sequence. In order to find the GARCH (m, s) process, we solve for α_0 in the equation (3.29) by letting the variance of x_t be $\sigma^2{}_t$. This yields

$$\alpha_0 = \sigma^2{}_t \left(1 - \sum_{i=1}^{m} \alpha_i - \sum_{j=1}^{s} \beta_j\right) \tag{3.30}$$

And substituting the value of α_0 as given by equation (3.29) into equation (3.30) gives

$$\begin{aligned} x^2{}_t &= \sigma^2{}_t\left[1 - \sum_{i,j=1}^{v}(\alpha_i + \beta_j)\right] + \left[\sum_{i,j=1}^{v}(\alpha_i + \beta_j)\right] x^2{}_{t-i} - \sum_{j=1}^{s} \beta_j\, \eta_{t-j} + \eta_t \\ &= \sigma^2{}_t + \sum_{i,j=1}^{v}(\alpha_i + \beta_j)\,(x^2{}_{t-i} - \sigma^2{}_t) - \sum_{j=1}^{s} \beta_j\, \eta_{t-j} + \eta_t \end{aligned} \tag{3.31}$$

Therefore

$$x^2{}_t - \sigma^2{}_t = \sum_{i,j=1}^{v}(\alpha_i + \beta_j)\,(x^2{}_{t-i} - \sigma^2{}_t) - \sum_{j=1}^{s} \beta_j\, \eta_{t-j} + \eta_t \tag{3.32}$$

Multiplying both sides of equation (3.32) by $(x^2{}_{t-k} - \sigma^2{}_t)$ results in

$$\begin{aligned} (x^2{}_{t-k} - \sigma^2{}_t)(x^2{}_t - \sigma^2{}_t) = {} & \sum_{i,j=1}^{v}(\alpha_i + \beta_j)\,(x^2{}_{t-i} - \sigma^2{}_t)(x^2{}_{t-k} - \sigma^2{}_t) - \\ & \sum_{j=1}^{s} \beta_j\, \eta_{t-j}(x^2{}_{t-k} - \sigma^2{}_t) + \eta_t(x^2{}_{t-k} - \sigma^2{}_t) \end{aligned} \tag{3.33}$$

And taking expectations of equation (3.33), we have

$$\mathbb{E}[({x^2}_{t-k} - {\sigma^2}_t)({x^2}_t - {\sigma^2}_t)] = \mathbb{E}\left[\sum_{i,j=1}^{v}(\alpha_i + \beta_j)({x^2}_{t-i} - {\sigma^2}_t)({x^2}_{t-k} - {\sigma^2}_t)\right] - \mathbb{E}\left[\sum_{j=1}^{s}\beta_j \eta_{t-j}({x^2}_{t-k} - {\sigma^2}_t)\right] + \mathbb{E}\,[\eta_t({x^2}_{t-k} - {\sigma^2}_t)] \qquad (3.34)$$

But $\mathbb{E}\,[\eta_t({x^2}_{t-k} - {\sigma^2}_t)] = \mathbb{E}\,[({x^2}_{t-k} - {\sigma^2}_t)\mathbb{E}(\eta_t|x_t)] = 0$ since η_t is a martingale difference and also

$$\mathbb{E}[\beta_j\eta_{t-j}({x^2}_{t-k} - {\sigma^2}_t)] = \mathbb{E}\,[({x^2}_{t-k} - {\sigma^2}_t)\mathbb{E}(\eta_{t-j}|x_{t-j})] = 0 \text{ for } k < j.$$

Thus the autocovariance of the squared returns for the GARCH (m, s) model is given by

$$cov\,({x^2}_t, {x^2}_{t-k}) = \mathbb{E}\left[\sum_{i,j=1}^{v}(\alpha_i + \beta_j)({x^2}_{t-i} - {\sigma^2}_t)({x^2}_{t-k} - {\sigma^2}_t)\right]$$

$$= \sum_{i,j=1}^{v}(\alpha_i + \beta_j)\; cov\,({x^2}_t, {x^2}_{t-k+i}) \qquad (3.35)$$

Dividing both sides of equation (3.35) by ${\sigma^2}_t$ gives the autocorrelation function at lag k as

$$\rho_k = \sum_{i,j=1}^{v}(\alpha_i + \beta_j)\,\rho_{k-i}\,, \text{ for } k \geq (m+1) \qquad (3.36)$$

This result is analogous to the Yule-Walker equations for an AR process. Hence the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the squared returns in a GARCH process has the same pattern as those of an ARMA process. The ACF and PACF are useful in determining the orders m and s of the GARCH (m, s) process. Also the ACF is used in checking model accuracy; in which case, the ACF's of the residuals indicates the presence of a white noise if the model is adequate.

The parameters $\alpha_0, \alpha_1, \cdots, \alpha_m; \beta_1, \beta_2, \cdots, \beta_s$ affect the autocorrelation but given the ρ_k , $\cdots$, ρ_{m+1-v}, the autocorrelation at higher lags are determined uniquely by the expression in equation (3.33) (Bollerslev,1986) as cited in Ngailo (2011). Denoting the v^{th} partial autocorrelation for ${x^2}_t$ by $\emptyset_{vv}$ then

$$\rho_k = \sum_{i,j=1}^{v} \emptyset_{vv}\, \rho_{k-i}\ , k = 1, \cdots, v = \max(m, s) \tag{3.37}$$

It can be seen from equation (3.33) that, there are cut offs after lag m for an ARCH (m) process such that $\emptyset_{vv} \neq 0$ for $k \leq m$ and $\emptyset_{vv} = 0$ for $k > m$ and it is similar to the AR (m) process and decays exponentially (Bollerslev, 1986). To understand the theory and concepts of the GARCH model, we would focus on the special case of the GARCH (1, 1) model.

3.3.1 GARCH (1, 1) Model

The GARCH (1,1) model is a particular case of the GARCH (1,1) model where the orders m and s are both equal to one (i.e. $m = s = 1$). Let $\{x_t\}$ be the mean corrected return, ε_t be a Gaussian white noise with mean zero and unit variance. If I_t is the information set available at time t given by $I_t = \{x_1, x_2, \cdots, x_{t-1}\ ;\ {\sigma^2}_1, {\sigma^2}_2, \cdots, {\sigma^2}_{t-1}\}$, then the process $\{x_t\}$ follows a GARCH (1, 1) model if

$$x_t = \sigma_t \varepsilon_t \tag{3.38a}$$

$${\sigma^2}_t = \alpha_0 + \alpha_1 {x^2}_{t-1} + \beta_1 {\sigma^2}_{t-1} \tag{3.38b}$$

where α_0, α_1 and β_1 are the parameters of the model such that $\alpha_0 \geq 0, \alpha_1 \geq 0\ , \beta_1 \geq 0$ and $(\alpha_i + \beta_i) < 1$. The constraints on the parameters are to ensure that the conditional variance ${\sigma^2}_t$ is positive. Clearly from (3.38a) and (3.38b), it is evidenced that large past mean corrected return ${x^2}_{t-1}$ or past conditional variance ${\sigma^2}_{t-1}$ give rise to large values of ${\sigma^2}_t$ (Tsay,2002). It can be seen that $\{x_t\}$ is martingale difference as the conditional mean is zero (i.e. $\mathbb{E}\,(x_t|I_t) = 0$).

Taking $\eta_t = {x^2}_t - {\sigma^2}_t$ so that ${\sigma^2}_t = {x^2}_t - \eta_t$, the GARCH (1,1) can be represented differently. By substituting ${\sigma^2}_{t-1} = {x^2}_{t-1} - \eta_{t-1}$, into equation (3.38b), the GARCH (1,1) can be written as

$$
\begin{aligned}
{x^2}_t &= \alpha_0 + (\alpha_1 + \beta_1){x^2}_{t-1} + \eta_t - \beta_1\eta_{t-1} \\
&= \alpha_0 + \alpha_1{x^2}_{t-1} + \beta_1({x^2}_{t-1} - \eta_{t-1}) + \eta_t \qquad (3.39)
\end{aligned}
$$

Again it can be seen that $\{\eta_t\}$ is a martingale difference series as $\mathbb{E}\,(\eta_t\,|I_t) = 0$ (i.e. $\mathbb{E}(\eta_t) = 0$ and $cov\,(\eta_t, \eta_{t-j})$ for $j\ \geq 1$) and $\{\eta_t\}$ is an uncorrelated sequence. This implies from equation (3.39) that

$$\mathbb{E}({x^2}_t) = {\sigma^2}_t = \alpha_0 + (\alpha_1 + \beta_1)\,\mathbb{E}({x^2}_{t-1})$$

$$\Rightarrow {\sigma^2}_t = \alpha_0 + (\alpha_1 + \beta_1)\,\mathbb{E}({\sigma^2}_t\,{\varepsilon^2}_t)$$

$$\Rightarrow {\sigma^2}_t = \alpha_0 + (\alpha_1 + \beta_1)\,{\sigma^2}_t\mathbb{E}({\varepsilon^2}_t)$$

$$\Rightarrow {\sigma^2}_t = \alpha_0 + (\alpha_1 + \beta_1)\,{\sigma^2}_t \text{ , since } \mathbb{E}({\varepsilon^2}_t) = Var\,({\varepsilon^2}_t) = 1$$

$$\Rightarrow \alpha_0 = (1 - \alpha_1 - \beta_1){\sigma^2}_t$$

$$\Rightarrow {\sigma^2}_t = \frac{\alpha_0}{[1 - (\alpha_1 + \beta_1)]} \text{ , provided } |\alpha_1 + \beta_1| < 1 \qquad (3.40)$$

3.3.2 Estimation of GARCH (m, s) model

Once the orders m and s have been identified, the parameters $\alpha_0, \alpha_1, \cdots, \alpha_m; \beta_1, \beta_2, \cdots, \beta_s$ of the GARCH (m, s) model can then be estimated. The maximum likelihood estimation is used to estimate the parameters of the model. The initial values of both the squared returns and past conditional variances are needed in estimating the parameters of the model. Bollerslev (1986) and Tsay (2002) suggest that the unconditional variance given in equation (3.28b) or the past sample variance of the returns may be used as initial values. Therefore assuming $x_1, x_2, \cdots, x_m$; $\sigma^2{}_1, \sigma^2{}_2, \cdots, \sigma^2{}_s$ are known, the conditional log-likelihood is given by

$$\ell\left(x_{m+1}, \cdots, x_{t;\ \sigma^2{}_{s+1},\ \cdots, \sigma^2{}_t} \middle| \theta; x_1, x_2, \cdots, x_m\ ;\ \sigma^2{}_1, \sigma^2{}_2, \cdots, \sigma^2{}_s\right) =$$

$$\sum_{t=v+1}^{T}\left(-\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln \sigma^2{}_t - \frac{x^2{}_t}{2\sigma^2{}_t}\right) \tag{3.41}$$

where $\theta = (\alpha_0, \alpha_1, \cdots, \alpha_m; \beta_1, \beta_2, \cdots, \beta_s)$ and $v = \max(m, s)$

It follows that the conditional maximum likelihood estimates are obtained by maximizing the conditional log-likelihood function given by equation (3.41)

3.3.3 Estimation of the GARCH (1, 1)

The estimation of the GARCH (1, 1) model is done in the same way as in the ARCH (1) model. The initial value of the past conditional variance $(\sigma^2{}_1)$ is needed since the conditional variance of the GARCH (1, 1) model depends also on the past conditional variance. The unconditional variance of x_t can be taken as an initial value for this variance. That is, it can be taken as $\frac{\alpha_0}{[1-(\alpha_1+\beta_1)]}$. In some cases, the sample variance of the return series can be taken to be the initial value of the past conditional variance $(\sigma^2{}_1)$. Let $x_1, x_2, \cdots, x_n$ be a sample of log-returns. The distribution of x_t conditional on α_0,α_1, $\beta_1$and x_{t-1} is normal with zero mean and variance $\sigma^2{}_t$. Thus $x_t|\alpha_0,\alpha_1, \beta_1$, x_{t-1} is normal such that

$$\mathbb{E}(x_t|\alpha_0,\alpha_1, \beta_1, x_{t-1}) = 0 \tag{3.42a}$$

and

$$Var\,(x_t|\alpha_0,\alpha_1, \beta_1, x_{t-1}) = \sigma^2{}_t = \alpha_0 + \alpha_1 x^2{}_{t-1} + \beta_1\sigma^2{}_{t-1} \tag{3.42b}$$

The likelihood function is given by

$$f(x_1, \cdots, x_n|\alpha_0, \alpha_1, \beta_1) = \prod_{t=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2{}_t}} \exp(\frac{-x^2{}_t}{2\sigma^2{}_t}) \tag{3.43}$$

Such that $\sigma^2{}_t = \alpha_0 + \alpha_1 x^2{}_{t-1} + \beta_1\sigma^2{}_{t-1}$.

The log-likelihood function of α_0,α_1, β_1 is given as

$$\ell(x_1, \cdots, x_n|\alpha_0, \alpha_1, \beta_1) = \sum_{t=1}^{n}\left(-\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln \sigma^2{}_t - \frac{x^2{}_t}{2\sigma^2{}_t}\right) \tag{3.44}$$

To obtain the estimates $\widehat{\alpha_0}, \widehat{\alpha_1}$ and $\widehat{\beta_1}$ of α_0, α_1 and β_1, maximize the log likelihood function by taking the partial derivatives of $\ell(x_1, \cdots, x_n|\alpha_0, \alpha_1, \beta_1)$ with respect to α_0, α_1 and β_1. Respectively the partial derivatives of α_0, α_1 and β_1 are

$$\frac{\partial \ell(x_1, \cdots, x_n|\alpha_0, \alpha_1, \beta_1)}{\partial \alpha_0} = -\frac{1}{2}\sum_{t=1}^{n}\frac{1}{\sigma^2{}_t} + \frac{1}{2}\sum_{t=1}^{n}\frac{x^2{}_t}{\sigma^4{}_t}$$

(3.45a)

$$\frac{\partial \ell(x_1, \cdots, x_n | \alpha_0, \alpha_1, \beta_1)}{\partial \alpha_1} = -\frac{1}{2}\sum_{t=1}^{n} \frac{x^2_{t-1}}{\sigma^2_t} + \frac{1}{2}\sum_{t=1}^{n} \frac{x^2_t \, x^2_{t-1}}{\sigma^4_t}$$

(3.45b)

$$\frac{\partial \ell(x_1, \cdots, x_n | \alpha_0, \alpha_1, \beta_1)}{\partial \beta_1} = -\frac{1}{2}\sum_{t=1}^{n} \frac{\sigma^2_{t-1}}{\sigma^2_t} + \frac{1}{2}\sum_{t=1}^{n} \frac{x^2_t \, \sigma^2_{t-1}}{\sigma^4_t}$$

(3.45c)

Recalling that $\sigma^2_t = \alpha_0 + \alpha_1 x^2_{t-1} + \beta_1 \sigma^2_{t-1}$ and equating equations (3.45a), (3.45b) and (3.45c) to zero, three (3) systems of equations with three (3) unknowns are obtained as below:

$$\sum_{t=1}^{n} \left(\frac{1}{\widehat{\alpha_0} + \widehat{\alpha_1} x^2_{t-1} + \widehat{\beta_1} \sigma^2_{t-1}} \right) = \sum_{t=1}^{n} \left(\frac{x^2_t}{\left(\widehat{\alpha_0} + \widehat{\alpha_1} x^2_{t-1} + \widehat{\beta_1} \sigma^2_{t-1}\right)^2} \right) \qquad (3.46a)$$

$$\sum_{t=1}^{n} \left(\frac{x^2_{t-1}}{\widehat{\alpha_0} + \widehat{\alpha_1} x^2_{t-1} + \widehat{\beta_1} \sigma^2_{t-1}} \right) = \sum_{t=1}^{n} \left(\frac{x^2_t \, x^2_{t-1}}{\left(\widehat{\alpha_0} + \widehat{\alpha_1} x^2_{t-1} + \widehat{\beta_1} \sigma^2_{t-1}\right)^2} \right) \qquad (3.46b)$$

$$\sum_{t=1}^{n} \left(\frac{\widehat{\alpha_0} + \widehat{\alpha_1} x^2_{t-2} + \widehat{\beta_1} \sigma^2_{t-2}}{\widehat{\alpha_0} + \widehat{\alpha_1} x^2_{t-1} + \widehat{\beta_1} \sigma^2_{t-1}} \right) = \sum_{t=1}^{n} \left(\frac{x^2_t\left(\widehat{\alpha_0} + \widehat{\alpha_1} x^2_{t-2} + \widehat{\beta_1} \sigma^2_{t-2}\right)}{\left(\widehat{\alpha_0} + \widehat{\alpha_1} x^2_{t-1} + \widehat{\beta_1} \sigma^2_{t-1}\right)^2} \right) \qquad (3.46c)$$

where σ^2_{t-1} and σ^2_{t-2} can be expressed in terms of the log returns only, given that some initial values of σ^2_0 and σ^2_1 are known. The maximum likelihood estimator of σ^2_t is given as $\widehat{\sigma^2}_t = \frac{\sum_{i=1}^{n} x^2_i}{n}$. Alternatively, the above sums can start from $t = 2$ and $t = 3$. Numerical methods are used in solving for the estimates $\widehat{\alpha_0}, \widehat{\alpha_1}$ and $\widehat{\beta_1}$ and it can be verified that the Hessian matrix evaluated at $\alpha_0 = \widehat{\alpha_0}$, $\alpha_1 = \widehat{\alpha_1}$ and $\beta_1 = \widehat{\beta_1}$ defined by

$$\mathbb{H} = \begin{bmatrix} \frac{\partial^2 \ell}{\partial {\alpha_0}^2} & \frac{\partial \ell}{\partial \alpha_0 \alpha_1} & \frac{\partial \ell}{\partial \alpha_0 \beta_1} \\ \frac{\partial \ell}{\partial \alpha_1 \alpha_0} & \frac{\partial \ell}{\partial {\alpha_1}^2} & \frac{\partial \ell}{\partial \alpha_1 \beta_1} \\ \frac{\partial \ell}{\partial \beta_1 \alpha_0} & \frac{\partial \ell}{\partial \beta_1 \alpha_1} & \frac{\partial \ell}{\partial {\beta_1}^2} \end{bmatrix} \tag{3.47}$$

is a negative matrix and so $\widehat{\alpha_0}, \widehat{\alpha_1}$ and $\widehat{\beta_1}$ are the maximum likelihood estimates of α_0, α_1 and β_1. Stated earlier, since the conditional variance of the GARCH (1, 1) model depends on the past conditional variance, an initial value of the past conditional variance $({\sigma^2}_1)$ is needed. Usually, the unconditional variance of x_t is used as an initial value for the past conditional variance. That is ${\sigma^2}_1$ is taken to be $\frac{\alpha_0}{[1-(\alpha_1+\beta_1)]}$. However, the sample variance of the return series can be taken to be the initial value.

3.3.4 Forecasting with GARCH (m, s) model

Forecasting of a GARCH model can be obtained using methods similar to those of an ARMA model. Thus the conditional variance of $\{x_t\}$ is obtained by taking the conditional expectation of the squared mean corrected returns. Consider the GARCH (m, s) model as stated in equations (3.28a) and (3.28b). Assuming a forecasting origin of t, then the ζ-step ahead volatility forecast is given by

$${x^2}_t(\zeta) = \mathbb{E}\left({x^2}_{t+\zeta} \middle| x_t\right)$$

$$= \alpha_0 + \sum_{i=1}^{m} (\alpha_i + \beta_i)\, \mathbb{E}({x^2}_{t+\zeta-i} | x_t) + \eta_t - \sum_{j=1}^{s} \beta_j\, \mathbb{E}(\eta_{t+\zeta-j} | x_t)$$

$$= {\sigma^2}_t(\zeta); \tag{3.48}$$

where ${x^2}_t, \cdots, {x^2}_{t+1-m}$; ${\sigma^2}_t, \cdots, {\sigma^2}_{t+1-s}$ are assumed known at time t and the true parameters α_i; $(i = 1, \cdots, m)$ and β_j ; $(j = 1, \cdots, s)$ values are replaced by their estimates. Furthermore, $\mathbb{E}\left({x^2}_{t+\zeta}\middle|x_t\right)$ for $i < \zeta$ can be obtained recursively. For $j \geq \zeta$ $\mathbb{E}\left(\eta_{t+\zeta-j}\middle|x_t\right) = 0$ and for $< \zeta$, $\mathbb{E}\left(\eta_{t+\zeta-j}\middle|x_t\right) = \eta_{t+\zeta-j}$.

Considering the special case of GARCH (1, 1) model in equations (3.38a) and (3.38b) and assuming that the forecast origin of t, the 1-step ahead volatility forecast is given by

$$ {\sigma^2}_t\,(1) = {x^2}_t\,(1) = \alpha_0 + \alpha_1 {x^2}_t + \beta_1 {\sigma^2}_t \tag{3.49}$$

where x_t and ${\sigma^2}_t$ are known at the time index t.

For a multi - step ahead forecast, we use ${x^2}_t = {\sigma^2}_t\ {\varepsilon^2}_t$ and rewrite the volatility equation in (3.38b) as

$$ {\sigma^2}_{t+1} = \alpha_0 + (\alpha_1 + \beta_1)\,{\sigma^2}_t + \alpha_1\,{\sigma^2}_t\,({\varepsilon^2}_t - 1) \tag{3.50}$$

when $t = h + 1$, then equation (3.50) becomes

$$ {\sigma^2}_{h+2} = \alpha_0 + (\alpha_1 + \beta_1)\,{\sigma^2}_{h+1} + \alpha_1\,{\sigma^2}_{h+1}\,({\varepsilon^2}_{h+1} - 1) \tag{3.51}$$

A 2-step ahead volatility forecast at the forecast origin t is given as

$$ {\sigma^2}_t\,(2) = {x^2}_t\,(2) = \alpha_0 + (\alpha_1 + \beta_1){\sigma^2}_t\,(1) \tag{3.52}$$

and in general, the ζ- step ahead forecast is given as

$$ {\sigma^2}_t\,(\zeta) = {x^2}_t\,(\zeta) = \alpha_0 + (\alpha_1 + \beta_1){\sigma^2}_t\,(\zeta - 1),\ \zeta > 1 \tag{3.53}$$

This result is exactly the same as that of an ARMA (1, 1) model with AR polynomial $1 - (\alpha_1 + \beta_1)\mathcal{B}$. By repeated substitutions in equation (3.49), the ζ- step ahead volatility forecast can be written as

$$ {\sigma^2}_t\,(\zeta) = \frac{\alpha_0[1-(\alpha_1+\beta_1)^{\zeta-1}]}{[1-(\alpha_1+\beta_1)]} + (\alpha_1 + \beta_1)^{\zeta-1}{\sigma^2}_t\,(1) \tag{3.54}$$

Therefore

$\sigma^2{}_t\,(\zeta) \rightarrow \frac{\alpha_0}{[1-(\alpha_1+\beta_1)]}$ as $\zeta \rightarrow \infty$ provided that $(\alpha_1 + \beta_1) < 1.$

Consequently, the multi-step ahead volatility forecast of a GARCH (1, 1) model converges to the unconditional variance of x_t , as the forecast horizon increase to infinity provided that the variance of x_t $(\sigma^2{}_t)$ exists (Tsay,2002).

Despite the added advantage that the GARCH model brought to the ARCH – type models, the GARCH model had the same weakness as the ARCH model. It also assumes that the return volatilities (conditional variance) respond equally to positive and negative shocks. That is the GARCH model is a symmetric model and does not capture the asymmetry effect that is inherent in most real life financial data (Frimpong and Oteng - Abayie, 2006). To circumvent this problem of asymmetric effects on the conditional variance, Nelson (1991) extended the ARCH framework by proposing the Exponential GARCH (EGARCH) model.

3.4 EGARCH (m, s) Model

The Exponential GARCH model was proposed by Nelson (1991) to overcome some weakness of the GARCH model in dealing with financial time series. In particular, the EGARCH model is used to allow for asymmetric effects between positive and negative asset returns. Nelson (1991) considered the weighted innovation (error term)

$$g(\varepsilon_t) = \theta\varepsilon_t + \gamma[|\varepsilon_t| - \mathbb{E}(|\varepsilon_t|)] \qquad (3.55)$$

where θ and γ are real coefficients such that $\mathbb{E}[g(\varepsilon_t)\ ] = 0$ since both $|\varepsilon_t|$ and $\mathbb{E}(|\varepsilon_t|)$ are identical and independently distributed sequence with continuous distributions with zero mean. The asymmetry of $g(\varepsilon_t)$ can be easily seen by rewriting $g(\varepsilon_t)$ as

$$g(\varepsilon_t) \ = \begin{cases} (\theta + \gamma\ )\varepsilon_t \ - \ \gamma\mathbb{E}(|\varepsilon_t|)\ ,\ \varepsilon_t \geq 0 \\ (\theta - \gamma\ )\varepsilon_t \ - \ \gamma\mathbb{E}(|\varepsilon_t|),\ \varepsilon_t < 0 \end{cases} \tag{3.56}$$

As stated earlier in section 3.2.4.1, the error term (innovation) ε_t is assumed to be a standard normal (Gaussian), standardised student-t distribution or generalized error distribution (GED). For a standard normal random variable ε_t, $\mathbb{E}(|\varepsilon_t|) = \sqrt{2/\pi}$ and for the standardised student –t distribution, we have $\mathbb{E}(|\varepsilon_t|) = \frac{2\sqrt{\nu-2}\,\Gamma\,[(\nu+1)/2]}{(\nu-1)\Gamma(\nu/2)\sqrt{\pi}}$ where ν is the degrees of freedom.

Let $x_t = r_t$ - u_t be the mean corrected return, where r_t is the return of an asset, u_t is the conditional mean of x_t. Then x_t follows an EGARCH (m, s) model if

$$x_t = \sigma_t\varepsilon_t \quad \text{and} \tag{3.57a}$$

$$\ln({\sigma^2}_t) = \alpha_0 + \frac{1 + \beta_1\mathcal{B} + \cdots + \beta_{s-1}\mathcal{B}^{s-1}}{1 - \alpha_1\mathcal{B} + \cdots + \alpha_m\mathcal{B}^m} \cdot g(\varepsilon_{t-1}) \tag{3.57b}$$

where α_0 is a constant, $\mathcal{B}$ is the lag (back-shift) operator such that $\mathcal{B}g(\varepsilon_t) = g(\varepsilon_{t-1})$, $1 + \beta_1\mathcal{B} + \cdots + \beta_{s-1}\mathcal{B}^{s-1}$ and $1 - \alpha_1\mathcal{B} + \cdots + \alpha_m\mathcal{B}^m$ are polynomials with zeros outside the unit circle and have no common factors. By zeros outside the circle we mean the absolute values of the zeros are greater than 1.

From equations (3.57a) and (3.57b), it is seen that the EGARCH (m,s) model uses the usual ARMA parameterization to describe the evolution of the conditional variance of x_t and hence some of the properties of the EGARCH model can be obtained in a similar manner as those of the GARCH model. An example of such properties is that the unconditional mean of $\ln({\sigma^2}_t) = \alpha_0$. However, unlike the GARCH (m,s) model where there are positivity constraints made on the model constraints to ensure that the conditional variance $({\sigma^2}_t)$ is positive, the EGARCH (m,s) model relaxes the positivity constraints on the model parameters. This is due to the fact that the EGARCH (m,s) models the logarithm of the conditional variance $\ln({\sigma^2}_t)$ instead of the conditional variance $({\sigma^2}_t)$ itself. Also, the use of the $\ln({\sigma^2}_t)$ enables the model to respond asymmetrically to positive and negative lagged values of x_t.

The EGARCH (m,s) model can be stated alternatively as

$$x_t = \sigma_t \varepsilon_t \tag{3.58a}$$

$$\ln({\sigma^2}_t) = \alpha_0 + \frac{\sum_{i=1}^{s} \alpha_i |x_{t-i}| + \gamma_i x_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{m} \beta_j \ln\left({\sigma^2}_{t-j}\right) \tag{3.58b}$$

A positive x_{t-i} contributes $\alpha_i(1 + \gamma_i)|\varepsilon_{t-i}|$to the log volatility, whereas a negative x_{t-i} contributes $\alpha_i(1 - \gamma_i)|\varepsilon_{t-i}|$, where $\varepsilon_{t-i} = x_{t-i}/\sigma_{t-i}$. The parameter γ signifies the leverage effect and is expected to be negative. A simple EGARCH model of order (1,1) is considered to help better understand the theory and concept of the EGACRCH model.

3.4.1 EGARCH (1, 1) Model

The EGARCH (1, 1) model is a particular case of the EGARCH (m, s) model with order $(1, 1)$ (i.e. $m = s = 1$). Let $\{x_t\}$ be the mean corrected return, ε_t be an identical and independently distributed standard normal white noise. Then the process follow an EGARCH (1, 1) model if

$$x_t = \sigma_t \varepsilon_t \tag{3.59a}$$

$$(1 - \alpha\beta)\ln({\sigma^2}_t) = (1 - \alpha)\alpha_0 + g(\varepsilon_{t-1}) \tag{3.59b}$$

Given that ε_t be an identical and independently distributed standard normal with mean $\mathbb{E}(|\varepsilon_t|) = \sqrt{2/\pi}$, the model for $\ln({\sigma^2}_t)$ becomes

$$(1 - \alpha\beta)\ln({\sigma^2}_t) = \begin{cases} \alpha_* + (\gamma + \theta)\varepsilon_{t-1}, & \varepsilon_{t-1} \geq 0, \\ \alpha_* + (\gamma - \theta)(-\varepsilon_{t-1}), & \varepsilon_{t-1} < 0. \end{cases} \tag{3.60}$$

where $\alpha_* = (1 - \alpha)\alpha_0 - \gamma\sqrt{2/\pi}$

Equation (3.60) is a nonlinear function and hence for this simple EGARCH model, the conditional variance $({\sigma^2}_t)$ evolves in a nonlinear manner depending on the sign of x_{t-1}.

Specifically, we have

$${\sigma^2}_t = {\sigma^{2\alpha}}_{t-1}.\exp(\alpha_*)\begin{cases} \left[(\gamma + \theta)\frac{x_{t-1}}{\sigma_{t-1}}\right], & if\ x_{t-1} \geq 0, \\ \left[(\gamma - \theta)\frac{|x_{t-1}|}{\sigma_{t-1}}\right], & if\ \ x_{t-1} \geq 0. \end{cases} \tag{3.61}$$

The coefficients $(\gamma + \theta)$and $(\gamma - \theta)$ show the asymmetric response to positive and negative lagged returns (x_{t-1}). The model is therefore nonlinear if $\theta \neq 0$. θ is expected to be negative since negative shocks tend to have larger impacts and for higher order EGARCH models, the nonlinearity becomes much more complicated.

Using the alternative form of the EGARCH (m, s) model, the EGARCH $(1, 1)$ can be written as

$$x_t = \sigma_t \varepsilon_t \tag{3.62a}$$

$$\ln({\sigma^2}_t) = \alpha_0 + \frac{\alpha_1|x_{t-1}| + \gamma_1 x_{t-1}}{\sigma_{t-1}} + \beta_1\ln({\sigma^2}_{t-1}) \tag{3.62b}$$

3.4.2 Forecasting with EGARCH Model

The forecasting using an EGARCH model is done using methods similar to those of an ARMA model since the EGARCH model uses the usual ARMA parameterization to describe the evolution of the conditional variance of x_t . The simple EGARCH (1, 1) model would be used to illustrate the multi - step ahead forecasts of the EGACRH models.

Assuming that the model parameters are known and the distributional assumption made on the error term is the standard Gaussian, the EGACRH (m, s) model is given as

$$\ln({\sigma^2}_t) = (1 - \alpha_1)\alpha_0 + \alpha_1 \ln({\sigma^2}_{t-1}) + g(\varepsilon_{t-1}) \tag{3.63}$$

where $g(\varepsilon_{t-1}) = \theta\varepsilon_{t-1} + \gamma[|\varepsilon_{t-1}| - \mathbb{E}(|\varepsilon_{t-1}|)]$

$$= \theta\varepsilon_{t-1} + \gamma\left[|\varepsilon_{t-1}| - \sqrt{2/\pi}\right].$$

Assuming a forecasting origin of t, then the 1- step ahead volatility forecast is given by

$${\sigma^2}_t\,(1) = {\sigma^2}_{t+1} = {\sigma^{2\alpha_1}}_t\,.\,exp[(1 - \alpha_1)\alpha_0]exp[g(\varepsilon_t)] \tag{3.64}$$

where all of the quantities on the right-hand side are known.

For a 2 - step ahead volatility forecast, at the forecast origin t, equation (3.36) gives

$${\sigma^2}_t\,(2) = {\sigma^2}_{t+2} = {\sigma^{2\alpha_1}}_{t+1}\,.\,exp[(1 - \alpha_1)\alpha_0]exp[g(\varepsilon_{t+1})] \tag{3.65}$$

Taking conditional expectation on equation (3.65) at time t, we have

$${\sigma^2}_t\,(2) = {\sigma^2}_{t+2} = {\sigma^{2\alpha_1}}_{t+1}\,.\,exp[(1 - \alpha_1)\alpha_0]\mathbb{E}_t\{exp[g(\varepsilon_{t+1})]\}$$

where $\mathbb{E}_t$ denotes a conditional expectation taken at the time origin t. The prior expectation can be obtained as follows

$$\mathbb{E}\{exp[g(\varepsilon)]\} = \int_{-\infty}^{\infty} exp\left[\theta\varepsilon + \gamma\left(|\varepsilon| - \sqrt{2/\pi}\right)\right] f(\varepsilon)d\varepsilon$$

$$= exp\left(-\gamma\sqrt{2/\pi}\right)\left[\int_{0}^{\infty} e^{(\theta+\gamma)\varepsilon} . \frac{1}{\sqrt{2\pi}} . e^{-\frac{\varepsilon^2}{2}} d\varepsilon + \int_{-\infty}^{0} e^{(\theta-\gamma)\varepsilon} . \frac{1}{\sqrt{2\pi}} . e^{-\frac{\varepsilon^2}{2}} d\varepsilon\right]$$

$$= exp\left(-\gamma\sqrt{2/\pi}\right)\left[e^{\frac{(\theta+\gamma)^2}{2}}\Phi(\theta+\gamma) + e^{\frac{(\theta-\gamma)^2}{2}}\Phi(\gamma-\theta)\right] \qquad (3.66)$$

where $f(\varepsilon)$ and $\Phi(x)$ are the probability density function and cumulative density function of the standard normal distribution respectively. Consequently, the 2-step ahead volatility forecast is

$$\sigma^2{}_t\,(2) = \sigma^2{}_{t+2} = \sigma^{2\alpha_1}{}_t \,.\, exp\left[(1-\alpha_1)\alpha_0 - \sqrt{2/\pi}\,\right]$$

$$\times\left\{exp\left[\frac{(\theta+\gamma)^2}{2}\right]\Phi(\theta+\gamma) + exp\left[\frac{(\theta-\gamma)^2}{2}\right]\Phi(\gamma-\theta)\right\} \qquad (3.67)$$

Repeating the previous procedure, a recursive formula for the ζ- step ahead forecast is obtained as

$$\sigma^2{}_t\,(\zeta) = \sigma^{2\alpha_1}{}_t\,(\zeta-1)\exp(\omega) \times \left\{exp\left[\frac{(\theta+\gamma)^2}{2}\right]\Phi(\theta+\gamma) + exp\left[\frac{(\theta-\gamma)^2}{2}\right]\Phi(\gamma-\theta)\right\} \qquad (3.68)$$

where $\omega = (1-\alpha_1)\alpha_0 - \sqrt{2/\pi}$.

The values of $\Phi(\theta + \gamma\ )$ and $\Phi(\theta - \gamma\ )$ can be obtained from the following approximation. The cumulative density function of $\Phi(x)$ where x is a standard normal random variable can be approximated by

$$\Phi(x) = \begin{cases} 1 - f(x)[c_1 k + c_2 k^2 + c_3 k^3 + c_4 k^4 + c_5 k^5] & if\ x \geq 0, \\ 1 - \Phi(-x) & if\ x < 0. \end{cases} \tag{3.69}$$

where $(x) = \frac{\exp(-x^2/2)}{\sqrt{2\pi}}$; $k = \frac{1}{(1+0.2316419x)}$, $c_1 = 0.319381530$, $c_2 = -0.356563782$, $c_3 = 1.781477937$, $c_1 = -1.821255978$ and $c_5 = 1.330274429$

Further, with the advancement of technology, it possible for the value of $\Phi(\theta + \gamma\ )$ and $\Phi(\theta - \gamma\ )$ to be obtained from most statistical packages (Tsay, 2002).

3.5 Model Selection Criteria

The ACF and PACF assist in determining the order of the model but this is just a suggestion of where the model can be built from and it is imperative to build the model around the suggested model order (Aidoo, 2010). Several models with different orders can be considered and the ultimate (most suitable) model be selected from the family of candidate models that characterize the ordering data. The information criteria have been widely used in time series analysis to determine the appropriate order of a model. The idea behind the information criteria is to provide a measure of information in terms of the order of the model, which strikes a balance between the measure of goodness of fit and parsimonious specification of the model. The information criteria make use of the Kullback-Leibler effect in determining the suitable model. The Kullback-Leibler quantity of information contained in a model is the distance from the ‘true’ model and is measured by the log likelihood function (Aidoo, 2010).

Several selection criteria have been proposed to aid in selecting the most appropriate model. Among others, we have the Akaike Information criteria (AIC) by Akaike (1974), Bayesian Information criterion (BIC) by Schwartz (1978), Hannan-Quinn (HQ) by Hannan and Quinn (1979), the coefficient of determination (R^2), etc.

The several competing models are ranked according to their AIC, BIC or HQ values with the model having the lowest information criterion value being adjudged the best. If two or more competing models have the same or similar AIC, BIC or HQ values, then the principle of parsimony is applied to select the most appropriate model. The principle of parsimony states that a model with fewer parameters is usually better than a complex model. Alternatively to the use of the principle of parsimony, forecast accuracy tests between the competing models can be used (Aidoo, 2010).

In general, the model selected as the most appropriate model by two different criteria may differ and thus it should be noted that the selection of an ARCH-type model depends on the selection criteria used (Talke, 2003).

3.5.1 Akaike Information Criterion (AIC)

The Akaike Information Criterion (AIC) was introduced by Hirotogu Akaike in 1973. It was the first model selection criterion to gain widespread acceptance. The AIC was an extension to the maximum principle and consequently the maximum likelihood principle is applied to estimate the parameters of the model once the structure of the model has been specified. The AIC is defined as

$$\text{AIC} = 2(N) - 2(loglikelihood) \tag{3.70}$$

where N denotes the number of parameters in the model.

Given a family of competing models of various structures, the maximum likelihood estimation is used to fit the model and the AIC is computed based on each model fit. The selection of the most appropriate model is then made by considering the model with the minimum AIC. Akaike's idea was to combine estimation and structural determination into a single procedure. The first term of the AIC in equation (3.70) measures the goodness of fit of the model whereas the second term is called the penalty function of the criterion since it penalizes a candidate model by the number of parameters used.

The main advantage of the AIC is that it is useful for both in-sample and out-of-sample forecasting performance of a model. In-sample forecasting indicates how the chosen model fits the data in a given sample while out-of-sample forecasting is concerned with determining how a fitted model forecast future values of the regressed given the values of the regressors. Secondly, the AIC is useful for both nested and non-nested models.

Despite the advantages of the AIC such as mentioned above, the AIC has been criticised because of its inconsistency and tendency to over-fit a model. This inconsistency was shown by Shibata (1976) for autoregressive models (AR) and Hannan (1982) for ARMA models as cited in Shittu and Asemota (2009). To overcome this problem especially that of inconsistency, Schwartz (1978) proposed the Bayesian Information Criterion.

3.5.2 Bayesian Information Criterion

The Bayesian Information criterion (BIC) is related to the Bayes factor and is useful for selecting the most appropriate model out of a candidate of families of models. The BIC is obtained by replacing the non-negative factor $2(N)$ in equation (3.70) by $k\ln(n)$. Hence, the BIC is defined as

$$\text{BIC} = k\ln(n) - 2(loglikelihood) \tag{3.71}$$

Where k denotes the number of parameters in the model, n is the length of the time series or the sample size. Again, the maximum likelihood estimation is used to fit the model and the BIC is computed for each of the models in a family of competing models and the fitted model with the minimum BIC is considered to be most appropriate model. Comparing equations (3.70) and (3.71), it is can be seen that the BIC imposes a harsher penalty than the AIC especially for models with many parameters (i.e. complex models).

The advantages of the Bayesian information criterion is that for a wide range of statistical problems, it is order consistent (i.e. when the sample size goes to infinity, the probability of choosing the right model converges to unity) leading to more parsimonious model. Also, like the AIC, the BIC can be used to compare in-sample or out-of-sample forecasting performance of a model.

3.6 Model diagnostic checks and adequacy

The model diagnostic checks are performed to determine the adequacy or goodness of fit of a chosen model. The model diagnostic checks are performed on residuals and more specifically on the standardized residuals (Talke, 2003). The residuals are assumed to be independently and identically distributed following a normal distribution (Tsay, 2002). Plots of the residuals such as the histogram, the normal probability plot and the time plot of residuals can be used. If the model fits the data well the histogram of residuals should be approximately symmetric. The normal probability plot should be a straight line while the time plot should exhibit random variation (Bowerman and O'Connell, 1997). The ACF and the PACF of the standardized residuals are used for checking the adequacy of the conditional variance model. The Lagrange multiplier and the Ljung Box Q-test (given in section 3.2) are used to check the validity of the ARCH effects as well as test for autocorrelation in the data. To test the presence of ARCH effects, the null hypothesis of no ARCH effects is rejected if the significance probability value (p-value) is less than specified level of significance. In case of testing for the presence of autocorrelation, the null hypothesis of no autocorrelation is rejected if the Ljung –Box (Q) statistics (as defined under section 3.2.2.1 by equations 3.6a and 3.6b) of some of the lags are significant. Thus if the probability value of Ljung –Box (Q) statistics of some of the lags are less than the specified level of significance, then the null hypothesis of no autocorrelation is rejected. Once the estimated model satisfies all these model assumptions, it can be seen as an appropriate representation of the data. Having established that the model fits the data well, the model can then be used to compute forecasts of the series under consideration.

3.7 Model Validation

The data set was divided into two parts; an initialization or training set and a verification or test set. The training set was used to estimate the model parameters whilst the test set was used to validate the model. This validation process is necessary to evaluate the model for how accurate it is in forecasting. If a chosen model is able to describe the testing set well, then the model is considered valid and adequate and hence it can be used in forecasting the series under consideration.

3.8 Assessment of Predictiveness or Forecast Accuracy of a Model

As pointed earlier in section 3.5, forecast accuracy test can be used as criteria for selecting the best model. Several measures for assessing the forecast accuracy of ARCH-type models have been proposed. Some of these measures are the mean square error (MSE), mean absolute error (MAE) and Theil's U – statistic.

The MSE is defined as the average of the squared difference between the actual variance and the volatility forecast (${\sigma^2}_t$). In the absence of the observed true variance, the squared time series observation ${x^2}_t$is used. The MSE is given by

$$\text{MSE} = \frac{\sum_{t=1}^{T}\left({x^2}_t - \widehat{\sigma^2}_t\right)^2}{T} \qquad (3.72)$$

where $\widehat{\sigma^2}_t,\ t = 1, \cdots, T$ is the estimated conditional variance obtained from fitting the ARCH-type model. The MSE is criticised because ${x^2}_t$ is noisy and unstable although the ${x^2}_t$ is a consistent estimator of ${\sigma^2}_t$ (Tsay, 2002). Alternatively, other measures have been proposed. The mean absolute error (MAE) was proposed by Lopez (1999) and defined as

$$\text{MAE} = \frac{\sum_{t=1}^{T}\left|x^2{}_t - \widehat{\sigma^2}{}_t\right|^2}{T} \qquad (3.73)$$

The last but not the least criterion is the Theil's U-statistic which is used to test the accuracy of the future predictions. The Theil's U-statistic is defined as

$$U = \sqrt{\frac{\sum_{t=1}^{T-1}(FPE_{t+1} - APE_{t+1})^2}{\sum_{t=1}^{T-1}(APE_{t+1})^2}} \qquad (3.74)$$

where $FPE_{t+1} = \frac{(\widehat{x_{t+1}} - x_t)}{x_t}$ is the forecasted relative change, and APE_{t+1} $\frac{(x_{t+1} - x_t)}{x_t}$ is the actual relative change. If the forecasts are good then U should be close to zero. A U-statistic of one implies that the model under consideration and the benchmark model are equal. The models could equal in terms of accuracy or inaccuracy. A U-statistic of less than one implies that the model is superior to the benchmark while a U-statistic of greater than one implies the model is inferior to the benchmark model.

3.9 Conclusion

The chapter has provided an overview of time series and its basic concepts as well as a detailed description – order determination, estimation and forecasting – of the three main autoregressive Heteroscedastic models that were used in the study. Furthermore, various selection criteria that help to select the best fit model among a class of competing models were discussed. Finally, there was an exposition on model validation as well as assessing the predictiveness or forecast accuracy of a model. The empirical findings based on the methods discussed under this chapter follow in the next chapter.

CHAPTER FOUR

DATA ANALYSIS AND DISCUSSION OF RESULTS

4.0 Introduction

This chapter presents the analysis and the discussion of the results obtained from the study. The chapter is further organised into five sub sections excluding the introductory section. A description of the data with respect to basic statistics is done under section 4.1. Section 4.2 deals with the preliminary analysis of the data. Model estimation and fitting as well as model evaluation and diagnostics are presented under sections 4.3 and 4.4 respectively. The last section, section 4.5 focused on forecasting of monthly inflation rates based on the model selected as the most appropriate model under section 4.4. The analysis was carried out using both MINITAB 16 and EVIEWS 5.0 statistical software. The MINITAB 16 was used to obtain the various graphs due to its pictorial clarity whilst the EVIEWS 5.0 was used for the rest of the analysis such as descriptive statistics, estimation, etc.

4.1 Summary Statistics and Data Description

The sample data consists of Five Hundred and Seventy Six (576) observations of the monthly rates of inflation in Ghana. It covers a forty - eight (48) year period spanning from January 1965 to December 2012. The data was divided into two parts; a training set consisting of the monthly inflation rates from January 1965 to December 2011 which was used to estimate the model parameters and a test set consisting of the monthly rates on inflation from January 2012 to December 2012 which was used to validate the chosen model. The data were obtained from the Ghana Statistical Service (GSS) as published on their official website www.gss.gov.org. The GSS is the official government institution mandated to provide the rate of inflation in Ghana on periodic bases such as monthly rates of inflation. Table 4.1.1 shows the descriptive statistics of the monthly rates of inflation.

Table 4.1.1: Descriptive Statistics of Monthly Rates of Inflation in Ghana (1965 - 2012)

Statistic	Value	Statistic	Value
Mean	29.82	Range	186.20
S.E Mean	1.28	Skewness	2.14
Std. deviation	30.73	Kurtosis	7.58
Median	20.20	Jarque-Bera	931.27
Maximum	174.10	Probability	0.0000
Minimum	-12.10	Sample	576

Source: Researcher’s Calculation based on sampled data

From Table 4.1.1, the results show that the mean of the monthly inflation rate is 29.82 with a standard error of 1.28 and a standard deviation of 30.73. The maximum rate of inflation was 174.10 whilst the minimum rate of inflation was -12.10. Thus the range of the rate of inflation over the 28 years period under consideration was 186.20. Also the rates of inflation were centred on a median of 20.20. Further, the data had a positive skewness of 2.14 implying that the distribution of the data has a long right tail and a kurtosis of 7.58 (i.e. a high excess kurtosis of 4.58) indicating that the distribution of the monthly rate of inflation was leptokurtic. The Jarque-Bera statistic of 931.27 is statistically significant at 1% level of significance. Thus it can be concluded that the monthly rates of inflation has a non-normal distribution. These confirm the non-normality and positive skewness of the monthly rates of inflation as revealed by Figure 4.1.1. The histogram of the residuals has a long right tail indicating positive skewness whilst the normal probability (QQ) plot of the residuals shows the data in a curvilinear form implying a deviation from normality.

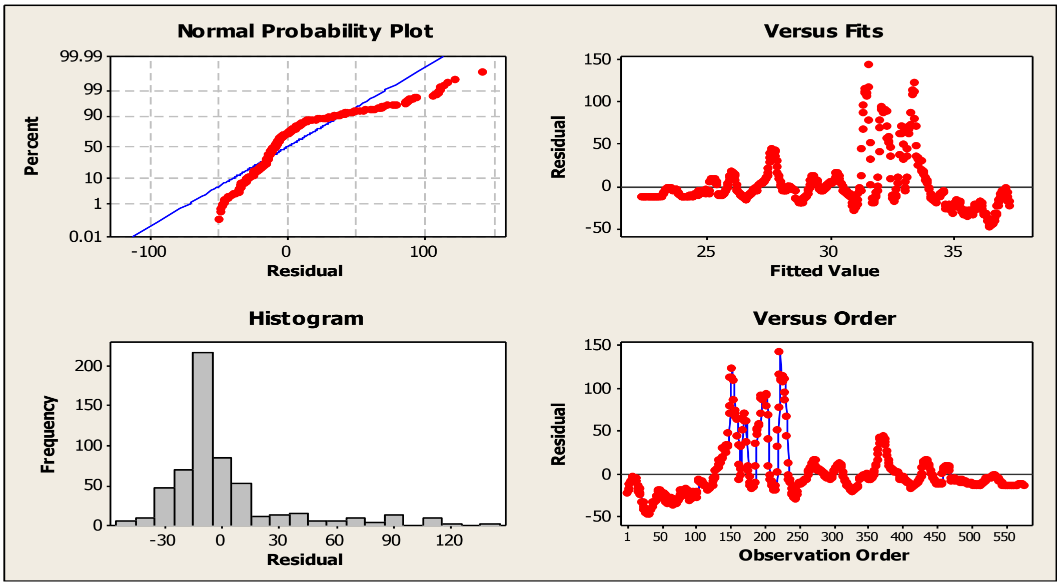

Figure 4.1.1: Residual Plots of the Monthly Rates of inflation in Ghana (1965 - 2012)

4.2 Preliminary Analysis

The plot of the monthly rates of inflation for the period January 1965 to December 2012 is given by Figure 4.2.1. From Figure 4.2.1, it is evident that both the mean and variance are changing over time. That is, the monthly rate of inflation series is characterised by a non-constant mean and an unstable variance. The changing mean and variance over time is an indication of the non-stationarity of the monthly rates of inflation. Moreover, the trend analysis as shown in Figure 4.2.2 reveals a decreasing trend.

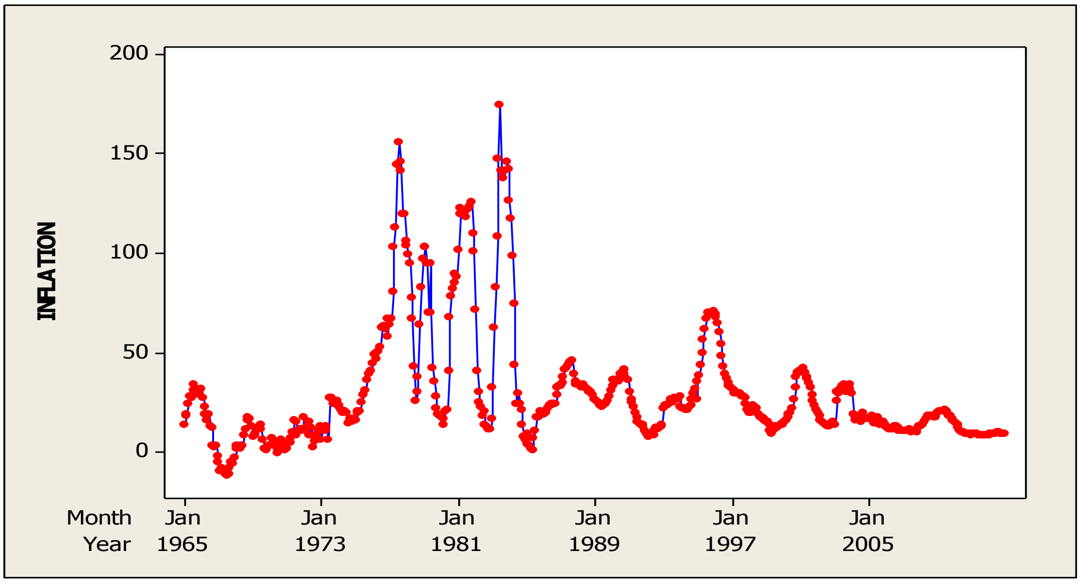

Figure 4.2.1: **Time Series plot of the Monthly rates of Inflation in Ghana (1965 - 2012)**

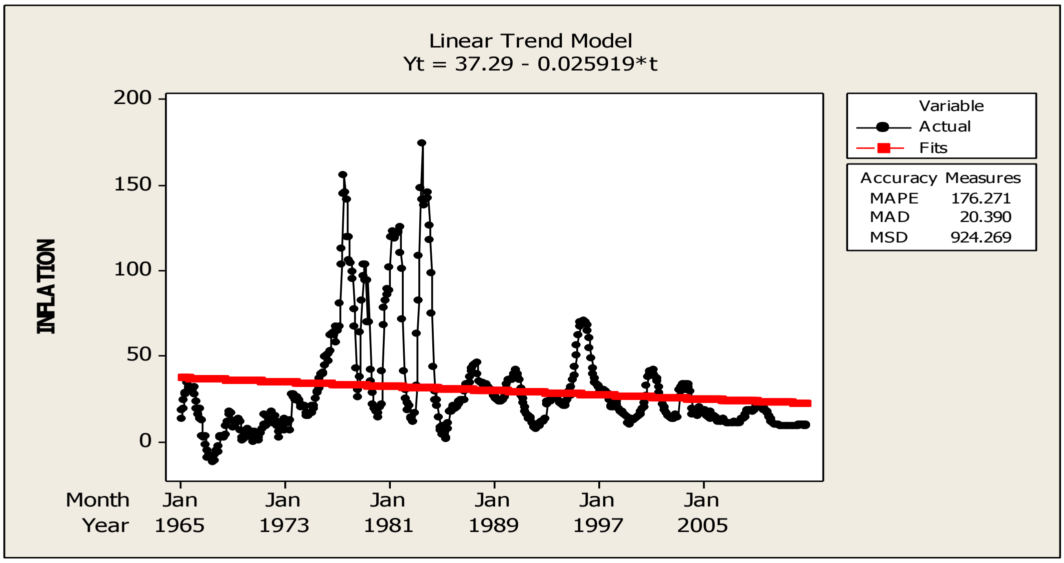

Figure 4.2.2: **Trend Analysis Plot for Monthly rates of inflation in Ghana (1965 - 2012)**

To confirm the presence of stationarity, the Augmented Dickey-Fuller (ADF) test was performed. The test fails to reject the null hypothesis of unit root at 5% level of significance and thus it can be concluded that the rate of inflation is not stationary over the period January 1965 to December 2012. The result of the Augmented Dickey-Fuller (ADF) test is shown on Table 4.2.1.

Table 4.2.1: Augmented Dickey-Fuller (ADF) Unit Root Test for the monthly Rates of Inflation in Ghana (1965 - 2012)

Model Type	Test Statistic	Critical Value	P-value
Constant	-2.65	-2.87	0.08
Constant + Trend	-2.74	-3.42	0.22
None	-1.77	-1.94	0.07

Source: Researcher's Calculation based on sampled data

A transformation was carried on the data to bring it to stationarity, which is a desirable characteristic feature in most time series models. There are several transformations that are usually used to achieve the desired characteristics in a time series data. Examples of such transformations include ordinary differencing, seasonal differencing, taking of natural logarithms, taking square roots, etc. The ordinary differencing was preferred because of the following reasons. First of all, some of the monthly rate of inflation had negative values and as such taking the natural logarithm or square roots of the data would have resulted in missing values. Secondly, the data does not reveal any form of seasonality (see Figure 2B and Figure 3B in Appendix B) and hence seasonal differencing was not necessary. Thus the choice of the ordinary difference was used. Figure 4.2.3 gives the plot of the first ordinary difference of the monthly rates of inflation.

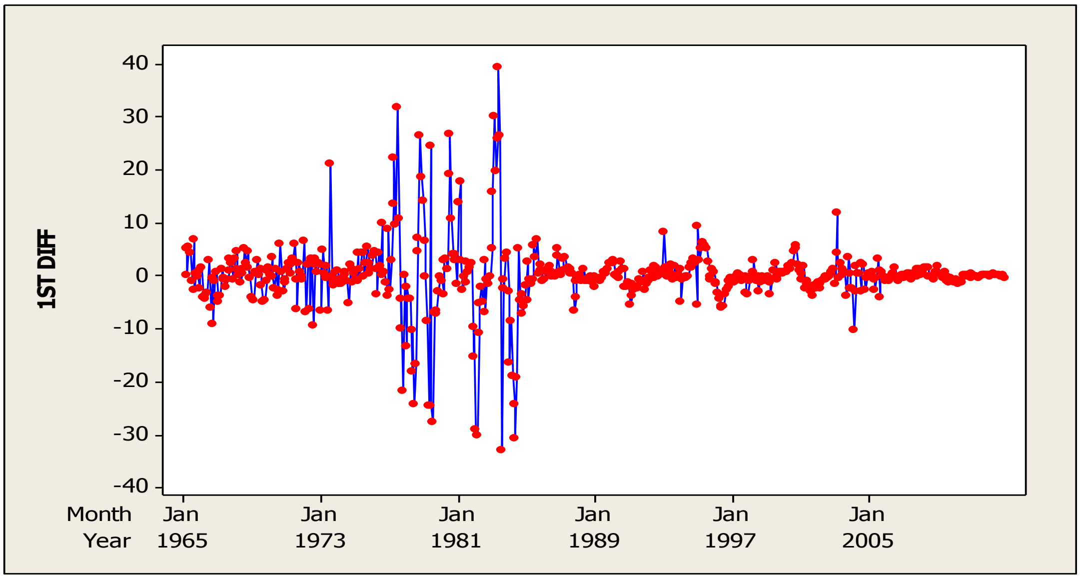

Figure 4.2.3: Time series plot of the first difference of the monthly rates of inflation in Ghana (1965 - 2012)

The plot in Figure 4.2.3 reveals that the first ordinary difference monthly rate of inflation series appears to be stable in both the mean and variance over time implying there is stationarity in the first ordinary difference monthly rate of inflation series. This is also confirmed by the residual plot and trend analysis of the first ordinary difference monthly rate of inflation series (see Figure 4B and Figure 5B respectively in Appendix B).

From Figure 4.2.1, the monthly rate of inflation series exhibits heteroscedasticity (changing variance over time). A formal test for heteroscedasticity was however carried out to confirm the presence of heteroscedasticity (ARCH effects). The Ljung-Box (Q) test was performed and the results for some selected lags are shown in Table 4.2.2 (the results of all lags are given in Table 1B in Appendix B). From the results, the p-values are less than 5% level of significance indicating that, the Ljung-Box test statistic is significant at all lags giving an evidence of the presence of heteroscedasticity (ARCH effects) in the monthly rate of inflation series.

Table 4.2.2: Test for Heteroscedasticity (ARCH effects) in monthly rates of inflation in Ghana (1965 - 2012)

Lag	Q – statistic	P-value
1	551.52	0.0000
6	2425.30	0.0000
12	3073.10	0.0000
18	3263.20	0.0000
24	3608.50	0.0000
30	4120.20	0.0000
36	4499.30	0.0000

Source: Researcher's Calculation based on sampled data

The test for heteroscedasticity (ARCH effects) was also performed for the first difference monthly rate of inflation series and the results as shown by Table 4.2.3 revealed that there was significant evidence of heteroscedasticity (ARCH effects) although it has been reduced as compared to the case of the original monthly rate of inflation series.

Table 4.2.3: Test for Heteroscedasticity (ARCH effects) in the first difference monthly rates of inflation in Ghana (1965 -2012)

Lag	Q - statistic	P-value
1	130.74	0.0000
6	225.81	0.0000
12	387.17	0.0000
18	533.88	0.0000
24	537.62	0.0000
30	570.25	0.0000
36	584.07	0.0000

Source: Researcher's Calculation based on sampled data

Furthermore, the test for serial correlation (autocorrelation) in the monthly rate of inflation series was also performed. This was done by obtaining the Autocorrelation function (ACF) and Partial Autocorrelation function (ACF) plots of the monthly rate of inflation series as given by Figures 4.2.4 and 4.2.5 respectively. Significant spikes at lags 1 and 12 of the PACF given by Figure 4.2.5 may be an indication of seasonal variation. However, from the seasonal analysis done (see Figures 2B and 3B in Appendix B), the data does not reveal any form of seasonality and hence the spikes could be attributed to random effects. Furthermore, from the plots of the ACF and PACF shown by Figure 4.2.4 and Figure 4.2.5 respectively, there was an indication of correlation in the monthly rates of inflation. Engle (1982) asserts that any autocorrelation in a time series has to be removed before any ARCH-family models is constructed. To eliminate the autocorrelation, the first difference transformation of the monthly rates of inflation was obtained. Figures 4.2.6 and Figure 4.2.7 gives the ACF and PACF of the first difference of the monthly rates of inflation.

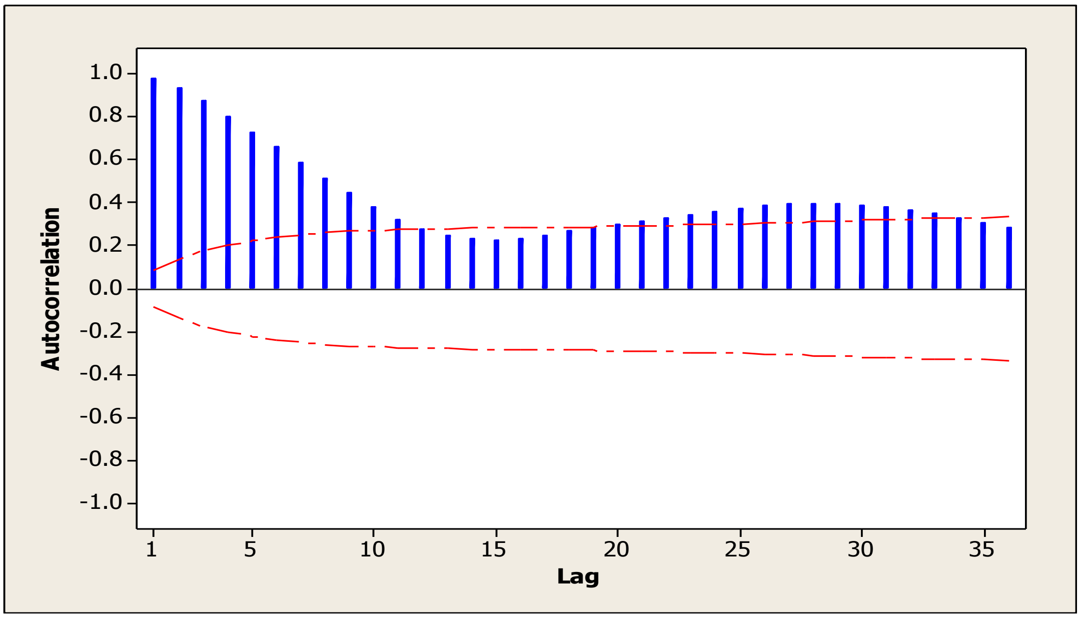

Figure 4.2.4: Autocorrelation function (ACF) plots of the monthly rates of inflation (1965 - 2012)

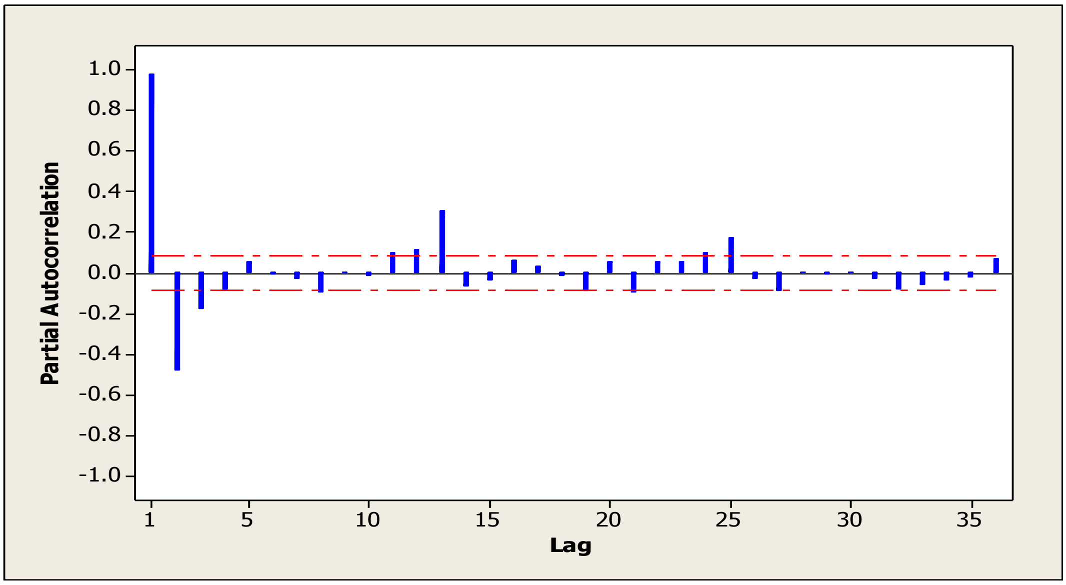

Figure 4.2.5: Partial Autocorrelation Function (ACF) plots of the monthly rates of inflation (1965 -2012)

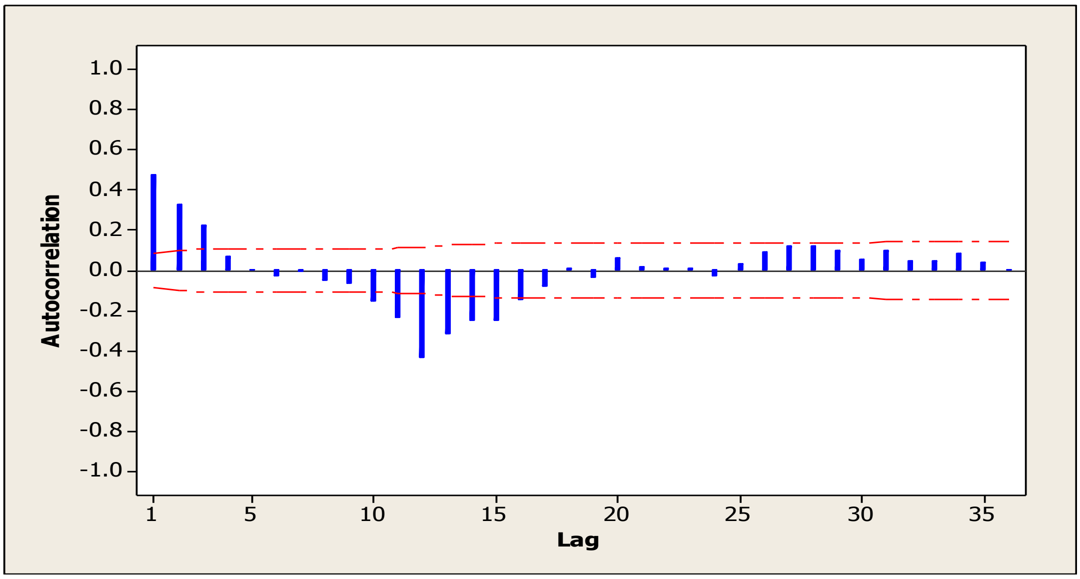

Figure 4.2.6: Autocorrelation Function (ACF) plots for the first difference of Monthly Rates of Inflation (1965 -2012)

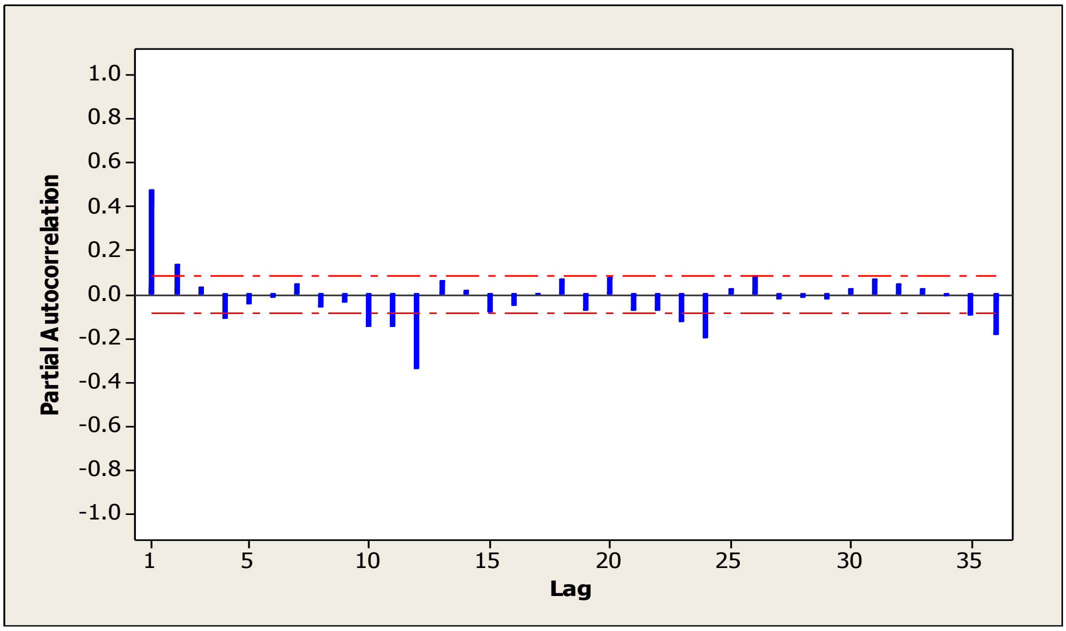

Figure 4.2.7: Partial Autocorrelation Function (PACF) plots for the first difference of Monthly Rates of Inflation (1965 - 2012)

The ACF dies in a sine wave form whilst the PACF shows significant number of spikes also dying down in a sine wave fashion. This indicates that there is no significant correlation in the first difference monthly rates of inflation.

From the fore going analysis, it can be concluded that the first difference monthly rates of inflation satisfy all the data assumption or characteristics for a volatility model such as the ARCH-family models. Hence in subsequent analysis, the first differenced monthly rates of inflation was used or considered.

The next step in time series model building procedure after all the assumptions or properties of the series has been satisfied is the determination of the order of the model. The Autocorrelation Function (ACF) and Partial Autocorrelation Function (ACF) plots of the series were used to determine the order. From the ACF and PACF plots of the first difference in Figure 4.2.6 and Figure 4.2.7 respectively, the ACF tails of at lag 2 whilst the PACF spike at lag 1. This suggests that the order $m = 2$ and $s = 1$ and hence therefore an AR (2) and MA (1) models were suspected combining to give an ARMA (2, 1) model.

In conclusion, it is has been observed that the monthly rates of inflation had a unit root or was non-stationary and the first difference transformation brought stationarity. It was also observed that the first difference monthly rates of inflation have both significant heteroscedasticity (ARCH effects) and autocorrelation present in the series. Lastly from the ACF and PACF plots of the first difference of the monthly rates of inflation, the AR (2) and MA (1) were suspected giving an indication of an ARMA (2, 1) model around which to build the ARCH-family models.

4.3 Model Fitting and Estimation

After the determination of the order of the model and consequently the model identification has been done, the parameters of the model can now be estimated. The method used to estimate the parameters is the maximum likelihood method. The maximum likelihood function that was used in the estimation was based on the distributional assumption of normality made on the error term or residuals as discussed under section 3.2.4.1. This distributional assumption of normality made on the error term or residuals was confirmed by histogram of the residuals of the first difference of the monthly rates of inflation as shown by Figure 6B in Appendix B.

Since the order determined is usually a suggestion of the order around which the most appropriate model is found, several models of different order that lies close to the suggested model of ARMA (2, 1) was fitted and the most appropriate model was selected based on the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), Log Likelihood, R-square and the significance tests. The criteria is that the smaller the AIC, BIC and larger the R-square the better. Also the idea is to have a parsimonious model that captures as much variation in the data as possible. The EVIEWS 5.0 software was used to perform the trial and error modelling to determine the best fitting model. Table 4.3.1 gives the various suggested models for the ARCH model with their respective fit statistics.

Table 4.3.1: Comparison of suggested ARCH (m) models with fit statistics

Model	AIC	BIC
ARCH(1)	5.36	5.41
ARCH(2)	5.31	5.36
ARCH(3)	5.31	5.37

Source: Researcher's Calculation based on the sampled data

From Table 4.3.1, both ARCH (2) and ARCH (3) have the same AIC and almost the same BIC value and hence any of these models might be appropriate. Therefore the most adequate model is specified by

$$x_t = \sigma_t \varepsilon_t \tag{4.1a}$$

for $\sigma^2{}_t = \alpha_0 + \alpha_1 x^2{}_{t-1} + \alpha_2 x^2{}_{t-2}$ (4.1b)

in the case of ARCH (2) or

$$x_t = \sigma_t \varepsilon_t \tag{4.1c}$$

for $\sigma^2{}_t = \alpha_0 + \alpha_1 x^2{}_{t-1} + \alpha_2 x^2{}_{t-2} + \alpha_3 x^2{}_{t-3}$ (4.1d)

in the case of ARCH (3).

Assuming that the residuals are normally distributed, the Table 4.3.2 and Table 4.3.3 present the model outputs respectively for ARCH (2) and ARCH (3).

Table 4.3.2: Model output for ARCH (2)

	Coefficient	Std. Error	z-Statistic	Prob.
Mean Equation				
C	-0.154409	0.100296	-1.539531	0.1237
AR(1)	0.038025	0.080133	0.474520	0.6351
AR(2)	0.247293	0.038061	6.497345	0.0000
MA(1)	0.236626	0.091648	2.581898	0.0098
Variance Equation				
α_0	2.581578	0.169496	15.23090	0.0000
α_1	1.071298	0.093814	11.41942	0.0000
α_2	0.598491	0.061488	9.733461	0.0000

Source: Researcher's Calculation based on sampled data

Table 4.3.3: Model output for ARCH (3)

	Coefficient	Std. Error	z-Statistic	Prob.
Mean Equation				
C	-0.212855	0.113236	-1.879746	0.0601
AR(1)	0.892542	0.093720	9.523500	0.0000
AR(2)	-0.110620	0.045480	-2.432313	0.0150
MA(1)	-0.627033	0.080582	-7.781275	0.0000
Variance Equation				
α_0	2.533706	0.169868	14.91570	0.0000
α_1	1.111763	0.104060	10.68385	0.0000
α_2	0.644038	0.101071	6.372114	0.0000
α_3	-0.001810	0.015143	-0.119515	0.9049

Source: Researcher's Calculation based on sampled data

From Table 4.3.2, the estimates of the coefficients of ARCH (2) as seen in the variance equation are all significant whilst although the estimates of the coefficients in Table 4.3.3 as seen in the variance equation meet the general requirement of an ARCH (3) model, the estimate of α_3 is not statistically significant at the 5% level of significance. Thus the ARCH (3) parameter adds little explanatory power to model. Therefore, the model can be simplified by dropping the non significant parameter.

Hence it can be concluded that the ARCH (2) model is the most appropriate among the ARCH (m) models and hence from Table 4.3.2 equations (4.1a) and (4.1b) can now be written as

$$x_t = \sigma_t \varepsilon_t$$

$$\sigma^2{}_t = 2.5816 + 1.0713x^2{}_{t-1} + 0.5985x^2{}_{t-2}.$$

The next member of the ARCH-family models that were considered was the GARCH (m, s) models. Table 4.3.4 gives the various suggested models for the GARCH (m, s) model with their respective fit statistics.

Table 4.3.4: Comparison of suggested GARCH (m, s) models with fit statistics

Model	AIC	BIC
GARCH(1,1)	5.17	5.22
GARCH(1,2)	5.16	5.22
GARCH(2,1)	5.12	5.18
GARCH(2,2)	5.17	5.24

Source: Researcher's Calculation based on sampled data

From Table 4.3.4, the GARCH (2, 1) have the smallest AIC and BIC values of 5.12 and 5.18 respectively and hence GARCH (2, 1) model is the most appropriate among the GARCH (m, s) models. Therefore the most adequate model is specified by

$$x_t = \sigma_t \varepsilon_t \qquad (4.2a)$$

$$\sigma^2{}_t = \alpha_0 + \alpha_1 x^2{}_{t-1} + \alpha_2 x^2{}_{t-2} + \beta_1 \sigma^2{}_{t-1} \qquad (4.2b)$$

Assuming that the residuals are normally distributed, the model output obtained for GARCH (2, 1) is given by Table 4.3.5.

Table 4.3.5: Model output for GARCH (2, 1)

	Coefficient	Std. Error	z-Statistic	Prob.
		Mean Equation		
C	-0.050480	0.061193	-0.824919	0.4094
AR(1)	0.354733	0.206899	1.714528	0.0864
AR(2)	0.168479	0.084851	1.985587	0.0471
MA(1)	-0.053134	0.218579	-0.243089	0.8079
		Variance Equation		
α_0	-0.009901	0.004244	-2.332809	0.0197
α_1	0.711921	0.048894	14.56061	0.0000
α_2	-0.503197	0.046471	-10.82820	0.0000
β_1	0.864969	0.008080	107.0518	0.0000

Source: Researcher's Calculation based on sampled data

From Table 4.3.5, the estimates of the coefficients of GARCH (2, 1) are all significant at the 5% level of significance. Hence from Table 4.3.5 equations (4.2a) and (4.2b) can now be written as

$$x_t = \sigma_t \varepsilon_t$$

$$\sigma^2{}_t = -0.0099 + 0.7119x^2{}_{t-1} - 0.5032x^2{}_{t-2} + 0.8650\sigma^2{}_{t-1}.$$

Lastly the EGARCH (m, s) models were considered. Table 4.3.6 gives the various suggested models for the EGARCH (m, s) model with their respective fit statistics.

Table 4.3.6: Comparison of suggested EGARCH (m, s) models with fit statistics

Model	AIC	BIC
EGARCH(1,1)	5.16	5.23
EGARCH(1,2)	5.15	5.22
EGARCH(2,1)	5.09	5.16
EGARCH(2,2)	5.16	5.24

Source: Researcher's Calculation based on sampled data

From Table 4.3.6, the EGARCH (2, 1) have the smallest AIC and BIC values of 5.09 and 5.16 respectively and hence EGARCH (2, 1) model is the most appropriate among the EGARCH (m, s) models. Therefore the most adequate model using the alternative model form is specified by

$$x_t = \sigma_t \varepsilon_t \tag{4.3a}$$

$$\ln(\sigma^2{}_t) = \alpha_0 + \frac{\alpha_1 |x_{t-1}|}{\sigma_{t-1}} + \frac{\alpha_2 |x_{t-2}|}{\sigma_{t-2}} + \gamma \frac{x_{t-1}}{\sigma_{t-1}} + \beta_1 \ln(\sigma^2{}_{t-1}) \tag{4.3b}$$

Assuming that the residuals are normally distributed, the model output obtained for EGARCH (2, 1) is given by Table 4.3.7

Table 4.3.7: Model output for EGARCH (2, 1)

	Coefficient	Std. Error	z-Statistic	Prob.
		Mean Equation		
C	0.106006	0.126240	0.839721	0.4011
AR(1)	0.697105	0.203489	3.425760	0.0006
AR(2)	0.007368	0.085391	0.086289	0.9312
MA(1)	-0.458052	0.195324	-2.345089	0.0190
		Variance Equation		
α_0	-0.168646	0.017756	-9.497944	0.0000
α_1	0.866121	0.069383	12.48327	0.0000
α_2	-0.597416	0.069788	-8.560454	0.0000
γ	0.124346	0.024108	5.157891	0.0000
β_1	0.991669	0.002699	367.4403	0.0000

Source: Researcher's Calculation based on sampled data

From Table 4.3.7, the estimates of EGARCH (2, 1) as seen by the variance equation are all significant at the 5% level of significance. From Table 4.3.5 equations (4.2a) and (4.2b) can now be written as

$x_t = \sigma_t \varepsilon_t$ and

$$\ln(\sigma^2{}_t) = -0.1686 + \frac{0.8661|x_{t-1}|}{\sigma_{t-1}} - \frac{0.5974|x_{t-2}|}{\sigma_{t-2}} + 0.1243\left(\frac{x_{t-1}}{\sigma_{t-1}}\right) + 0.9917 \ln(\sigma^2{}_{t-1})$$

Having constructed the most adequate model for each ARCH-family type model, the next task was to assess how well these models fit the data (model diagnostics or adequacy). The model diagnostic check was based on the residuals of the constructed model.

4.4 Diagnostic Checks and Adequacy for estimated Models

The model diagnostic checks are performed to determine the adequacy of a chosen model. These checks were done through the analysis of the residuals from the fitted model. If the model fits the data well, the residuals are expected to be random, independent and identically distributed following the normal distribution. Plots of the residuals such as the histogram, the normal probability plot and the time plot of residuals were used. The histogram of residuals as well as the normal probability plot was used to check for normality approximately symmetric. The ACF and the PACF of the standardized residuals were used for checking the adequacy of the conditional variance model whilst the Lagrange multiplier and the Ljung Box Q-test were used to check the validity of the ARCH effects in the data.

4.4.1 Diagnostic Checks and Adequacy for the ARCH (2) Model

The time plot of the residuals was used to check whether the residuals were random and the result is given by Figure 4.4.1.1. From the plot, the residuals exhibit random variation about their mean and hence it can be concluded that the residuals appear to be random. Also from the normal probability plot of the standardized residuals in

Figure 4.4.1.2 the plot of the standardized residuals were almost linear. The linearity of the plot implied the distribution of the standardised residuals is normal. This was confirmed by the histogram of the standardised residuals shown by Figure 4.4.1.3. From Figure 4.4.1.3, the histogram is symmetric implying that the standardised residuals have a normal distribution. Furthermore, the results of Lagrange Multiplier test for ARCH effects as shown by Table 4.4.1.1 leads to the rejection of the null hypothesis of no ARCH effects since the test statistic was 2.5549 with a probability value of 0.0000, which is less than 5% significance level.

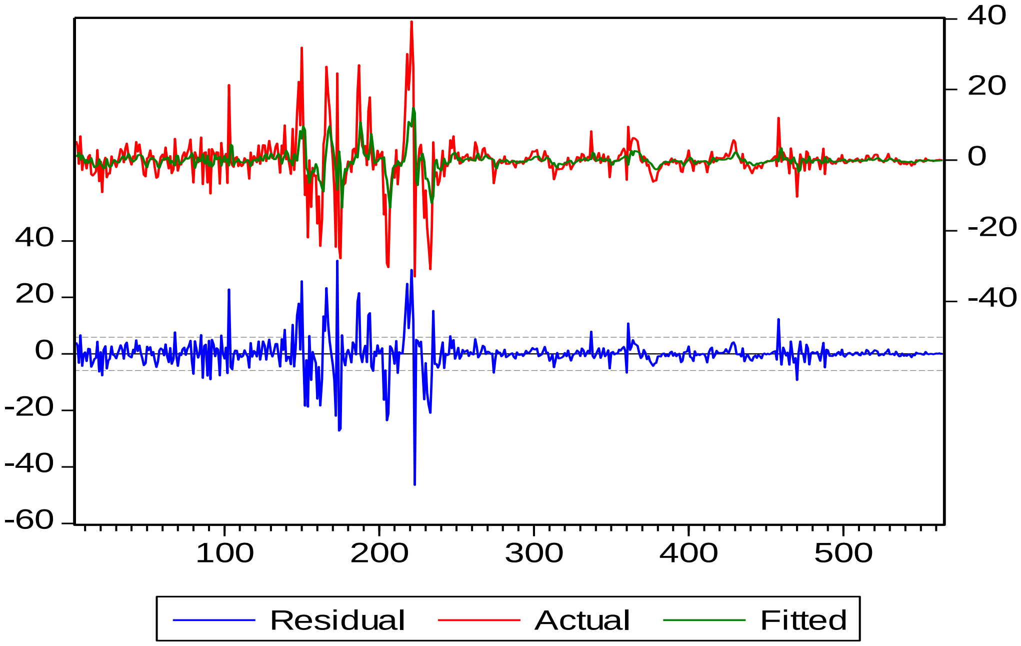

Figure 4.4.1.1: Time plot of the residuals from ARCH (2) model

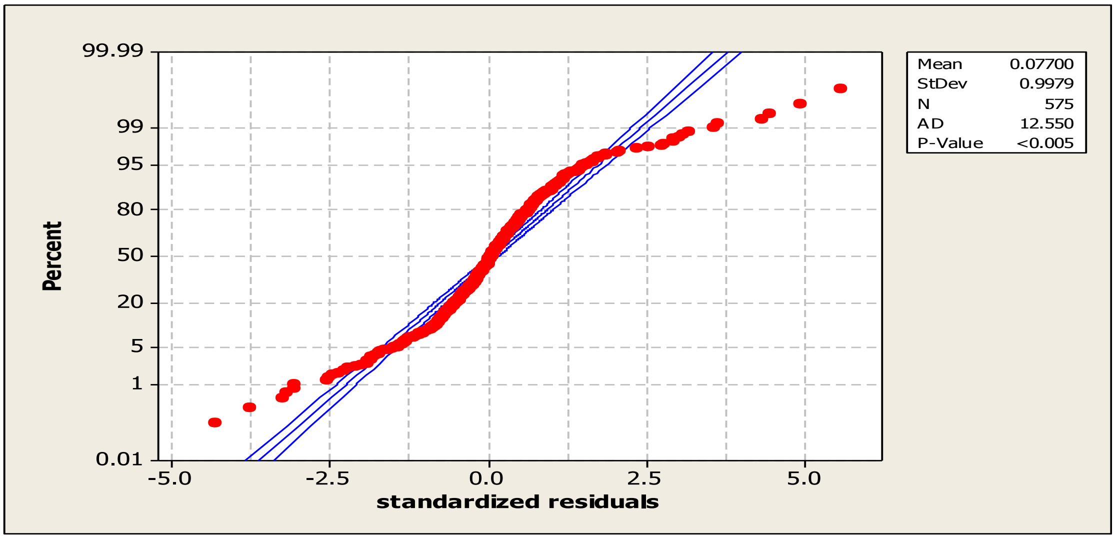

Figure 4.4.1.2: Normal Probability Plot of the Standardised Residuals from ARCH (2)

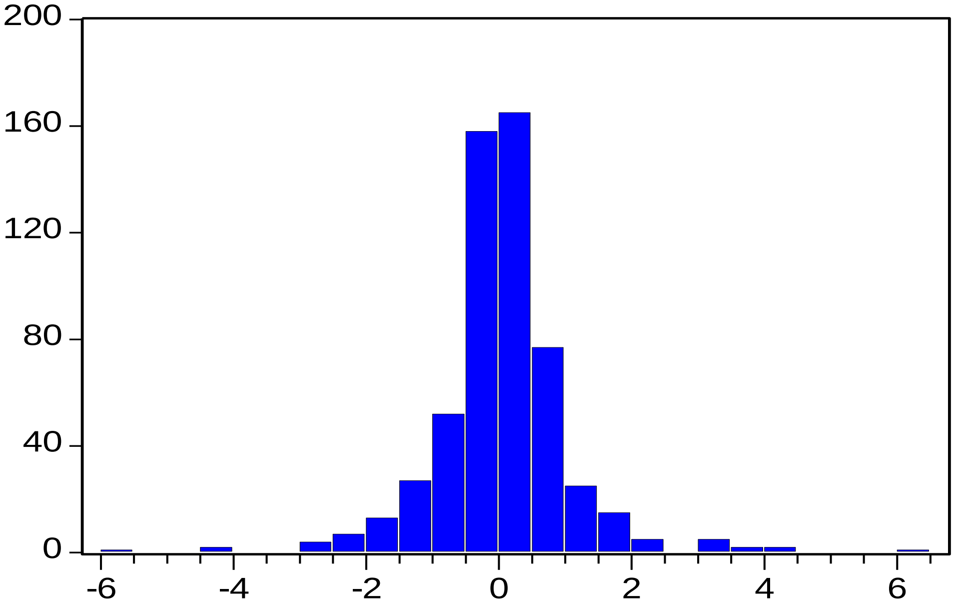

Figure 4.4.1.3: A histogram of the Standardized Residuals from ARCH (2)

Table 4.4.1.1: Lagrange Multiplier ARCH Test for ARCH (2) model

F-statistic	2.554944	Probability	0.000004
Obs*R-squared	83.25909	Probability	0.000013

Source: Researcher’s Calculation based on sampled data

The Ljung-Box (Q) statistic had a probability value of less than 5% level of significance for most of the lags as shown by Table 13C in the Appendix C. Hence the null hypothesis of no autocorrelation was rejected.

Thus it can be concluded that the ARCH (2) model do not provide an adequate representation of the data since most of the model adequacy conditions were not satisfied.

4.4.2 Diagnostic Checks and Adequacy for the GARCH (2, 1) Model

The time plot of the residuals was used to check whether the standardized residuals were random and the result is given by Figure 4.4.2.1. From the plot, the standardized residuals exhibit random variation about their mean and hence it can be concluded that the standardized residuals appear to be random.

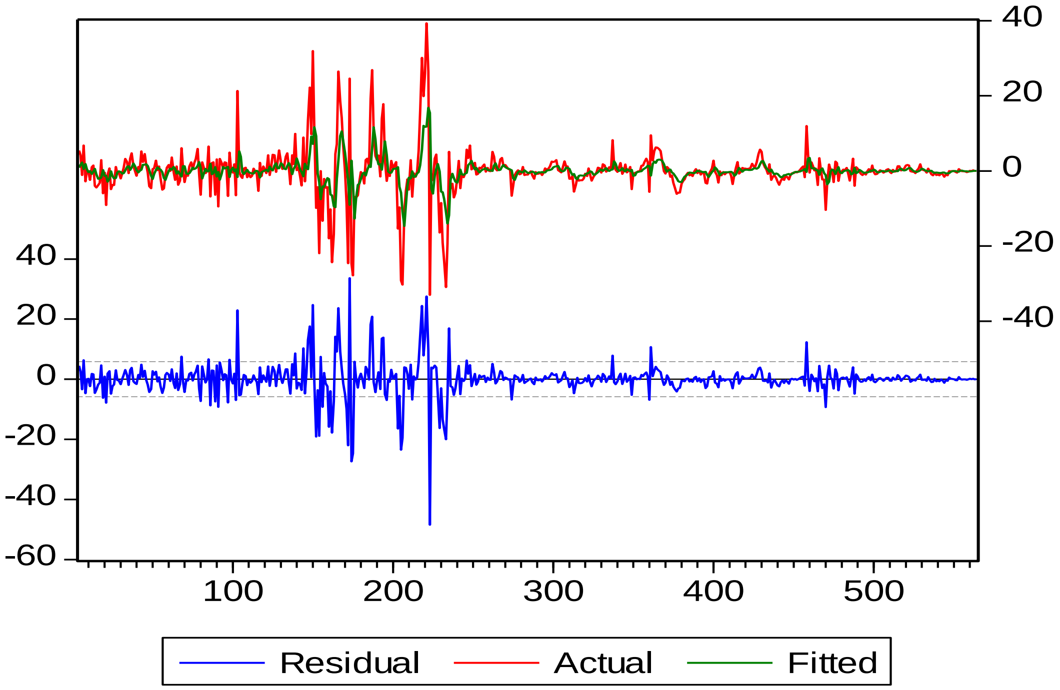

Figure 4.4.2.1: A Time plot of the standardized residuals from the GARCH (2, 1) model

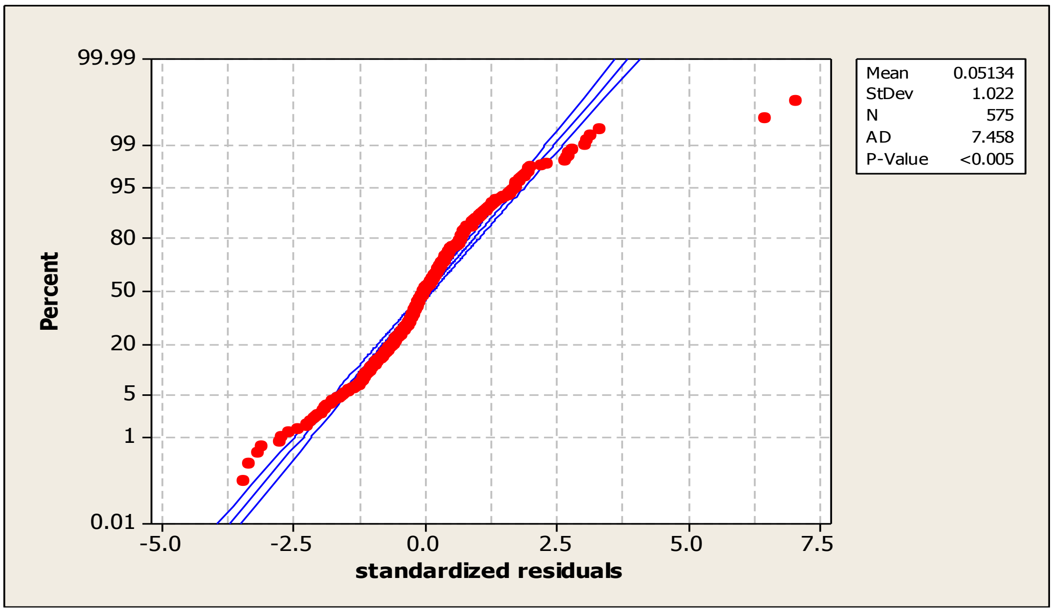

Figure 4.4.2.2: Normal Probability plot of the standardized residuals from the GARCH (2, 1) model

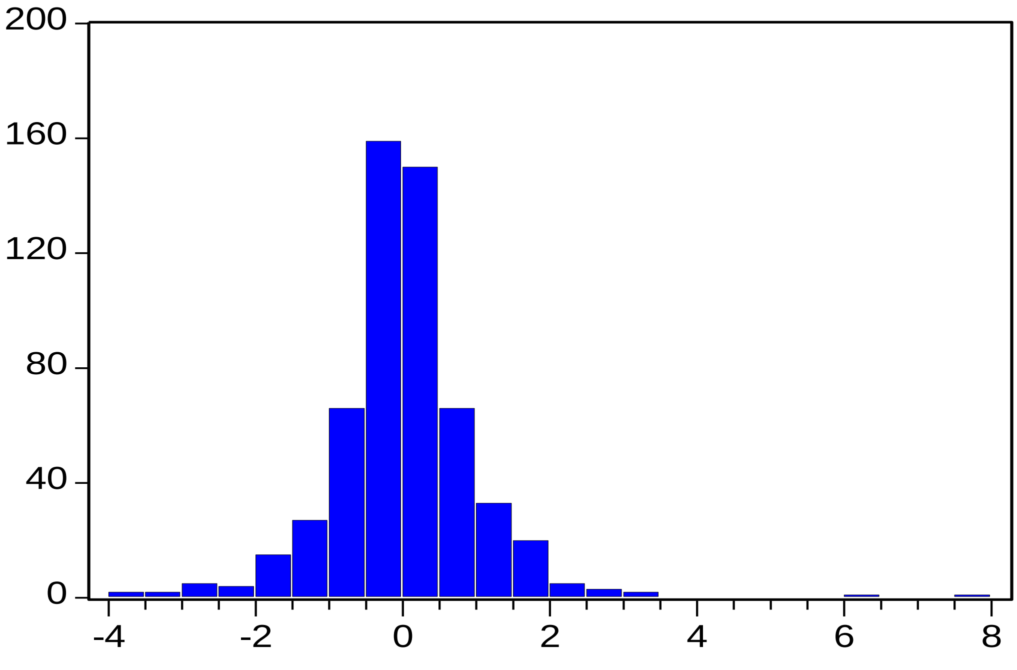

Figure 4.4.2.3: A histogram of the standardized residuals from the GARCH (2, 1) model

Table 4.4.2.1: Lagrange Multiplier ARCH Test for GARCH (2, 1) model

F-statistic	0.649902	Probability	0.943659

Obs*R-squared	24.01883	Probability	0.936672

Source: Researcher’s Calculation based on sampled data

Also from the normal probability plot of the standardized residuals in Figure 4.4.2.2 the plot of the standardized residuals was almost linear. The linearity of the plot implied the distribution of the standardised residuals is normal. This was confirmed by the histogram of the standardised residuals shown by Figure 4.4.2.3. From Figure 4.4.2.3, the histogram is symmetric implying that the standardised residuals have a normal distribution. Furthermore, the results of the Lagrange Multiplier test for ARCH effects as shown by Table 4.4.2.1 leads to failing to reject the null hypothesis of no ARCH effects since the test statistic was 0.6499 with a probability value of 0.9437, which is greater than 5% significance level.

The Ljung-Box (Q) statistic had a probability value of less than 5% level of significance for almost all the lags as shown by Table 15C in the Appendix C. Hence the null hypothesis of no autocorrelation was rejected. Thus it can be concluded that the GARCH (2, 1) model provides an adequate representation of the data since most of the model adequacy conditions were satisfied.

4.4.3 Diagnostic Checks and Adequacy for the EGARCH (2, 1) Model

The time plot of the standardized residuals was used to check whether the standardized residuals were random and the result is given by Figure 4.4.3.1. From the plot, the standardized residuals exhibit random variation about their mean and hence it can be concluded that the standardized residuals appear to be random. Also from the normal probability plot of the standardized residuals in Figure 4.4.3.2 the plot of the standardized residuals were almost linear. The linearity of the plot implied the distribution of the standardised residuals is normal. This was confirmed by the histogram of the standardised residuals shown by Figure 4.4.3.3.

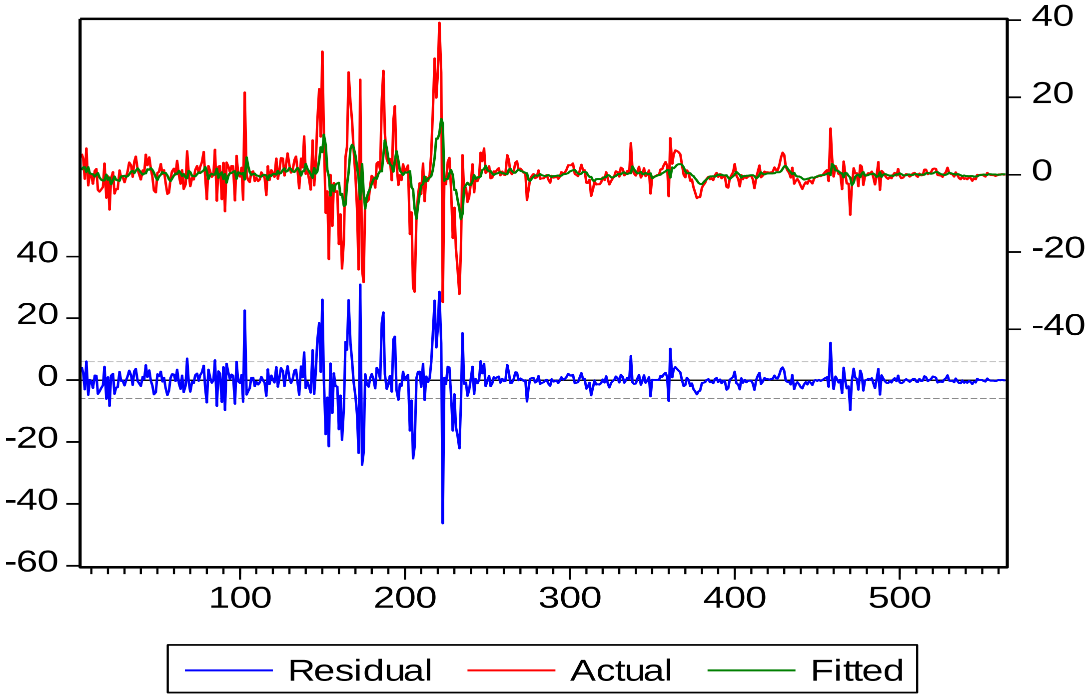

Figure 4.4.3.1: Time plot of the standardized residuals from the EGARCH (2, 1) model

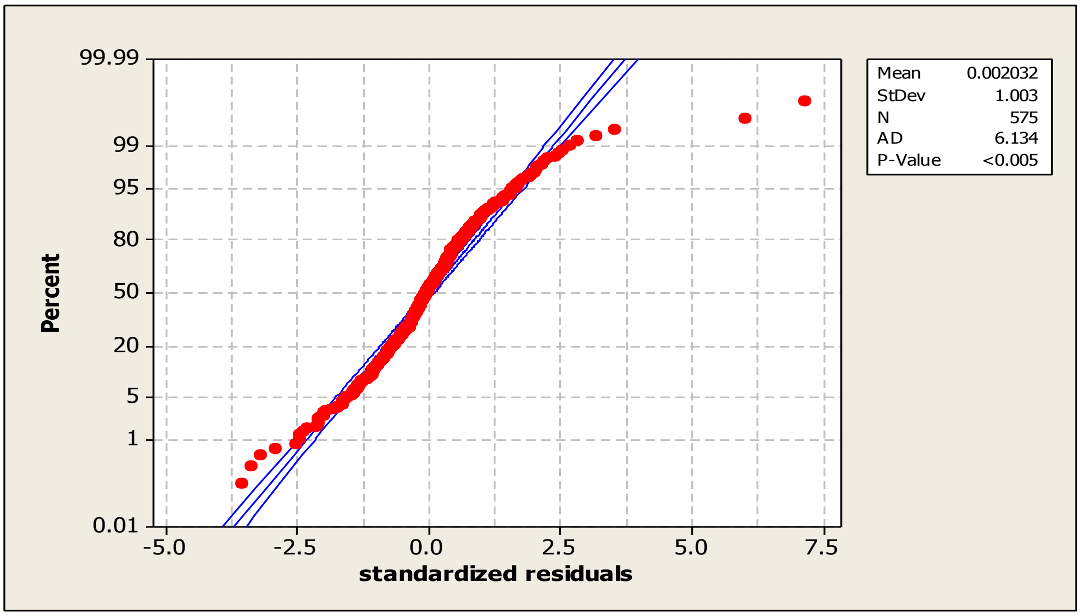

Figure 4.4.3.2: Normal Probability plot of the standardized residuals from EGARCH (2, 1)

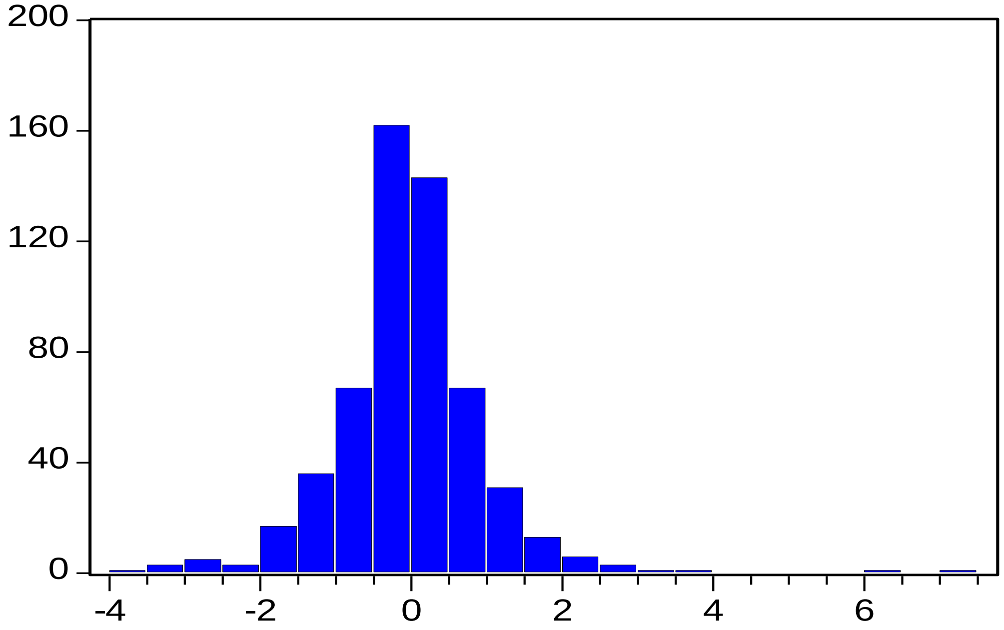

Figure 4.4.3.3: A histogram of the standardised residuals from EGARCH (2, 1) model

Table 4.4.3.1: Lagrange Multiplier ARCH Test for the EGARCH (2, 1)

F-statistic	0.509935	Probability	0.992536
Obs*R-squared	19.03354	Probability	0.990923

Source: Researcher's Calculation based on sampled data

From Figure 4.4.3.3, the histogram is symmetric implying that the standardised residuals have a normal distribution. Furthermore, the results of the Lagrange Multiplier test for ARCH effects as shown by Table 4.4.3.1 leads to the failure to the reject the null hypothesis of no ARCH effects since the test statistic was 0.5099 with a probability value of 0.9925, which is greater than 5% significance level.

The Ljung-Box (Q) statistic had a probability value of less than 5% level of significance for almost all the lags as shown by Table 17C in the Appendix C. Hence the null hypothesis of no autocorrelation was rejected. Thus it can be concluded that the EGARCH (2, 1) model provides an adequate representation of the data since most of the model adequacy conditions were satisfied.

4.5 Most Appropriate Model Selection

Based on the diagnostics checks above, it is seen that among the three selected models (i.e. ARCH (2), GARCH (2, 1) and EGARCH (2, 1) models), the ARCH (2) model did not provide an adequate representation of the data. However, both the GARCH (2, 1) and EGARCH (2,1) provided an adequate representation of the data. Hence to choose the most appropriate model out of these two models, various selection criteria as mentioned earlier under section 3.5 were used. Table 4.5.1 gives the values of AIC and BIC of the GARCH (2, 1) and EGARCH (2, 1) models.

Table 4.5.1: Selection Criteria Values for GARCH (2, 1) and EGARCH (2, 1) models

Model	AIC	BIC
GARCH(2,1)	5.12	5.18
EGARCH(2,1)	5.09	5.16

Source: Researcher's Calculation based on sampled data

From Table 4.5.1, it can be seen that the EGARCH (2, 1) had smaller values in both the AIC and BIC as compared to the GARCH (2, 1) model. Hence it can be concluded that the EGARCH (2, 1) model is the most appropriate representation of the data and therefore it would be used to forecast the future values of the monthly rates of inflation series.

4.6 Forecasting Evaluation and Accuracy Criteria

The models were also evaluated in terms of their forecasting ability of future monthly rates of inflation. This was necessitated as most previous research studies had found that the selected model was not necessary the model that provides best forecasting. Common measures of forecast evaluation such as the RMSE, MAE, MAPE and TIC were used. The model that exhibits the lowest values of the error measurements is considered to be the best. In Table 4.6.1, the results of the forecast performance are shown. Table 4.6.1 revealed that the EGARCH (2, 1) had the least value of all but one of the performance measurements. It had a RMSE of 5.79, MAE of 2.88, and MAPE of 15.70% and a TIC of 0.07. Thus the EGARCH (2, 1) model outperformed all the other models and was adjudged the best performing model. This results confirmed the earlier conclusion obtained based on the selection criteria such as the AIC, BIC, etc. The ARCH (2) model performed the least in forecasting the conditional volatility of the monthly rates of inflation.

Table 4.6.1 Forecast Performance of Estimated Models

Measure	ARCH(2)	GARCH(2,1)	EGARCH(2,1)
Root Mean Squared Error (RMSE)	5.75	**5.70**	5.79
Mean Absolute Error (MAE)	2.89	2.89	**2.88**
Mean Abs. Percent Error (MAPE)	15.88%	15.91%	**15.70%**
Theil’s Inequality Coefficient (TIC)	0.07	0.07	0.07
Overall Model Performance rank	3	2	**1**

Source: Researcher’s Calculation based on sampled data

4.7 Comparison of the EGARCH (2, 1) and ARIMA (2, 1, 1) Models

To provide a base for comparison of the EGARCH (2,1) model to models that assume constant mean and variance, the ARIMA (2,1,1) model was estimated (details of the output are shown by Table 18C under appendix C). Table 4.7.1 shows the AIC, BIC, RMSE, MAE, MAPE and TIC values of the ARIMA (2, 1, 1) and EGARCH (2, 1) models.

Table 4.7.1 Comparison of Performance Results of ARIMA (2, 1, 1) and EGARCH (2, 1)

Statistic	ARIMA (2,1,1)	EGARCH (2,1)
AIC	6.4	**5.1**
BIC	6.4	**5.2**
RMSE	**5.7**	5.8
MAE	2.9	2.9
MAPE	16.1%	**15.7%**
TIC	0.07	0.07

Source: Researcher’s Calculation based on sampled data

From Table 4.7.1, it was revealed that the EGARCH (2, 1) had the least AIC, BIC and MAPE whereas the ARIMA (2, 1) had the least RMSE. However, the MAE and TIC values were the same. Thus the EGARCH (2, 1) model outperformed the ARIMA (2, 1) model in three of the selection criteria used. Hence it was concluded that the EGARCH (2, 1) model was the best performing model on the basis of these results. Hence a one year out-sample forecast of the monthly rates of inflation based on the EGARCH (2, 1) model was obtained. Table 4.7.2 shows the actual monthly rates of inflation for January, 2012 to December, 2012 and the corresponding forecast values for the same period based on the EGARCH (2, 1) model as well as the 95% confidence interval of the forecasted values. Comparing the forecasted values to the actual values, it can be seen that the forecasted values are close to the actual values. Also, all the actual values fall within the confidence interval. Hence it is concluded that the EGARCH (2, 1), model is adequate to be used to forecast the monthly rates of inflation in Ghana.

Table 4.7.2: One Year out – sample forecast of the monthly inflation rate from the EGARCH (2, 1) Model

Month/Year	Actual	Forecast	Forecast Error	95% Confidence Interval	
				Lower Limit	Upper Limit
Jan-2012	8.70	8.70	0.00	8.0	9.3
Feb-2012	8.60	8.80	0.20	8.2	9.4
Mar-2012	8.80	8.70	-0.10	8.0	9.3
Apr-2012	9.10	8.90	0.20	8.3	9.5
May-2012	9.30	9.30	0.00	8.5	10.0
Jun-2012	9.40	9.50	0.10	8.9	10.0
Jul-2012	9.50	9.50	0.00	9.0	10.1
Aug-2012	9.50	9.60	0.10	9.1	10.1
Sep-2012	9.40	9.60	0.20	9.0	10.1
Oct-2012	9.20	9.40	0.20	8.9	10.0
Nov-2012	9.30	9.20	0.10	8.6	9.8
Dec-2012	8.80	9.40	0.60	8.9	9.9

Source: Researcher’s calculation based on sampled data

4.8 Discussion of Results

The implications of the results obtained are discussed under this section. However since the GARCH (2,1) and EGARCH (2,1) proved to be an adequate representation of the data whilst there was evidence that the ARCH (2) model do not provide adequate representation of the data, the discussion would neglect the results from the ARCH (2,1) model and focus on the results obtained from the GARCH (2,1) and the EGARCH (2,1) models.

The results from the GARCH (2, 1) revealed that the volatility in the current month's rate of inflation is explained by approximately 86% of the volatility in the previous month's rate of inflation. Also there was no evidence of weakly stationarity in the volatility in the monthly rates of inflation as the sum of the ARCH parameters and the GARCH parameter exceeds one (i.e.$0.7119 - 0.5032 + 0.8650 = 1.0737$). This implies that there is volatility persistence in the monthly rates of inflation and that the volatility in the monthly rates of inflation is explosive and dies slowly. The persistence in the volatility in the monthly rate of inflation means that the impact of new shocks or information on the monthly rate of inflation will last for a longer period.

The EGARCH (2, 1) model also provided evidence to the effect that the volatility in the current month's rate of inflation is perfectly explained by the volatility in the previous month's rate of inflation. Also the volatility persistence in the monthly rates of inflation observed in the GARCH (2, 1) model is also confirmed by the EGARCH (2, 1) model and that the volatility persistence was explosive as the sum of the ARCH parameters (0.8661,-0.5974) and the GARCH parameter (0.9917) is greater than one (i.e. $0.8661 - 0.5974 + 0.9917 = 1.2604$). Furthermore, there was the presence of asymmetric effects on the volatility of the monthly rates of inflation. Thus positive shocks (news) and negative shocks (news) would have different impacts on the volatility of the monthly rates of inflation. However, there was no evidence of leverage effects as the asymmetric term positive (0.1243). The absence of leverage effects indicates that the impact of a positive shock on the volatility of the monthly rates of inflation exceeds that of a negative shock of equal magnitude. From the results, a positive shock would change the volatility in the monthly rates of inflation by 0.9738 while a negative shock of the same magnitude would change the volatility in the monthly rates of inflation by 0.7584.

The monthly rates of inflation over the period were mostly low and centred around 20%. However, there is a high amount of variation in the monthly rates of inflation over the period and this might pose great challenges to other economic variables such as exchange rates, interest rates, stock returns, insurance premium, etc. Also the autoregressive Heteroscedastic models were superior in forecasting the monthly rates of inflation. This is consistent with Amos (2010), Awogbemi and Oluwaseyi (2011), Ezzat (2012), Igogo (2010), Su (2010), Lee and Brorsen (1996), Karanasos and Kim (2003), Alberg et al (2008); and Angelidis et al (2003). The superiority in performance of the autoregressive models is attributed to their ability to capture the stochastic nature of the monthly rates of inflation as evident in the pattern of the forecast errors. Lastly, looking at the upward trend of the out – sample forecasts, it can be predicted that Ghana would experience double digit inflation for the year 2013. This would impact on several aspects of the economy and could erode the economic gains made in the year 2012. The policy maker (Bank of Ghana) needs to put in place coherent monetary and fiscal policies that would put the anticipated increase in inflation under control.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMEMNDTIONS

5.0 Introduction

The chapter presents a summary of the findings from the study as well as the conclusions, recommendations and the areas for future research. Thus the main results and findings on the performance of the three selected Heteroscedastic models are presented.

5.1 Summary

Most empirical researches carried out to investigate the volatilities of financial series have been carried out using the Autoregressive integrated moving averages (ARIMA) models of Box and Jenkins (1976) until the introduction of the Heteroscedastic models. In Ghana, several of such empirical researches have been carried out in the area of inflation modelling and forecasting. However, these Box and Jenkins models are based on the assumption of constant variance, which is uncharacteristic of most financial series.

This study was designed to investigate the relative performance of three selected autoregressive Heteroscedastic models – Autoregressive Heteroscedastic Conditional (ARCH), Generalised ARCH (GARCH) and the Exponential GARCH (EGARH) models – in modelling the monthly rates of inflation in Ghana from January 1965 to December 2012. Also the study sought to compare the performance of the best fit model among the three selected autoregressive Heteroscedastic models with models from the Box and Jenkins family type models such as the ARIMA and finally to check for the presence of asymmetric and leverage effects in the volatility in the monthly rates of inflation.

Series of tentative models were developed for each of the selected models based on the different values of the order (m, n) of the model. For the ARCH (m) models, the order 1 2, and 3 were developed whilst in the case of the GARCH (m, n) and EGARCH (m, n) models, four different combinations of the order (m, n) were developed. These order combinations were order (1, 1), (1, 2), (2, 1) and (2, 2). After considering the AIC and BIC values of the tentative models, the model with the minimum AIC and BIC was adjudged the best fit model among each of the ARCH - family type models.

Based on the data analyses, the key findings are summarized as follows:

- The average monthly rate of inflation for the period under study was 29.82 with a standard error of 1.28 and a standard deviation of 30.73. The high value of the standard deviation implied that there is a greater amount of variability among the monthly rates of inflation in Ghana over the period under study. The distribution of the monthly rates of inflation was leptokurtic with a long – tail to the right.
- The distribution of the monthly rate of inflation was characterised by a non constant mean and an unstable variance implying a non-stationary series. Also a decreasing trend was observed for the series. The series also exhibited the presence of heteroscedasticity (ARCH effects) and autocorrelation.

- The ARCH (1), ARCH (2) and ARCH (3) models were developed among the ARCH (m) models. The AIC values for ARCH (1), ARCH (2) and ARCH (3) were 5.36, 5.31 and 5.31 respectively whilst the BIC values were also 5.41, 5.36 and 5.37 respectively. Hence the ARCH (2) had the minimum value for both AIC and BIC among the ARCH (m) model developed.
- The GARCH (1, 1), GARCH (1, 2), GARCH (2, 1) and GARCH (2, 2) had the AIC values of 5.17, 5.16, 5.12 and 5.17 respectively. For the BIC values, GARCH (1, 1) had 5.22, GARCH (1, 2) had 5.22, and GARCH (2, 1) had 5.18 whilst GARCH (2, 2) had 5.24. The GARCH (2, 1) had the minimum value for both AIC and BIC values among the GARCH (m, s) models developed.
- The EGARCH (1, 1), EGARCH (1, 2), EGARCH (2, 1) and EGARCH (2, 2) had AIC values of 5.16, 5.15, 5.09 and 5.16 respectively whilst their corresponding BIC values were 5.23, 5.22, 5.16 and 5.24. The EGARCH (2, 1), therefore, had the minimum value for both AIC and BIC values.
- The EGARCH (2, 1) model was the overall most appropriate model as it had the minimum value for both AIC and BIC values among the ARCH –type models fitted.
- Though there was asymmetric effects in the volatility in the monthly rates of inflation, there was however an absence of leverage effects.
- The ARIMA (2, 1, 1) model was fitted for the Box and Jenkins family models. The AIC and BIC values were 6.35 and 6.38 respectively for the ARIMA (2, 1 1) model.

5.2 Conclusion

The ARCH (2) model was selected as the best fit model for predicting the monthly rate of inflation amongst the ARCH (m) models. In the case of the GARCH (m, n) and EGARCH (m, n) models, the order (1, 2) was the best choice amongst the four different order combinations. Thus the GARCH (1,2) and EGARCH (1,2) models were selected as the best fit models amongst the GARCH (m, n) and EGARCH (m, n) models respectively. With respect to the Box and Jenkins models, the ARIMA (2, 1, 1) model was adjudged the best fit model for modelling monthly rates of inflation in Ghana

Subsequently, the three selected autoregressive Heteroscedastic best fit models – AR (2), GARCH (2, 1) and EGARCH (2, 1) - were compared based on their forecast performance. The goodness of fit models that were used included the root mean squared error, mean absolute error, mean absolute percent error and the Theil's Inequality coefficient. The EGARCH (2, 1) was adjudged the most appropriate model amongst the three best fit models in modelling the monthly rates of inflation in Ghana as it had the minimum value for all the goodness of fit statistics.

An asymmetric effect was also evident in the volatility in the monthly rates of inflation. However, there was an absence of leverage effects as positive shock changed the volatility in the monthly rate of inflation more than a negative shock of equal magnitude. Finally, when the EGACRH (2, 1) model was compared to the ARIMA (2, 1, 1) model, the EGACRH (2, 1) was found to be superior in modelling the rate of inflation in Ghana.

5.3 Recommendations

Based on the findings and conclusions made from the study, the following recommendations are made both in the area of policy formulation and further research. First of all, policy makers, industry players and all those interested in modelling and forecasting future rates of inflation in Ghana should consider using the Heteroscedastic models instead of the traditional Box and Jenkins models since the Heteroscedastic models are able to capture the volatilities in the monthly rates of inflation. Also by using the Heteroscedastic models, policy makers and industry players would be able to properly capture the volatility persistence in the monthly rates of inflation and leverage effects and hence forecasts and estimates would be more accurate.

Secondly in the area of researchers, similar studies could be carried out using other Heteroscedastic models such as the Glosten-Jagannathan and Runkle (GJR) model, Threshold ARCH (TARCH) model, Power ARCH (PARCH) model, Integrated PARCH (IPARCH) and others. This would help identify the best model among the family of Heteroscedastic models that fits the monthly rate of inflation better for Ghana.

Finally, it is recommended that multivariate time series models, where other economic variables that could influence the volatilities in the monthly rate of inflation such as exchange rates, amount of money supply, interest rates and others will be modelled along the rates of inflation. The inclusion of these other variables could help identify which of them contribute more to the variability in the monthly rates of inflation in Ghana.

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APPENDICES

APPENDIX A

Table 1A: Monthly Consumer Price Index, Rates of Inflation and the first difference of the Rates of Inflation in Ghana from January 1965 to December 2012.

OBS	YEAR/MONTH	CPI	INFLATION	FIRST DIFFERENCE
1	1965M01	128.10	13.30	NA
2	1965M02	135.30	18.40	5.10
3	1965M03	137.80	18.50	0.10
4	1965M04	145.20	23.90	5.40
5	1965M05	152.00	28.10	4.20
6	1965M06	155.60	27.00	-1.10
7	1965M07	162.80	33.80	6.80
8	1965M08	157.50	31.00	-2.80
9	1965M09	159.40	31.40	0.40
10	1965M10	158.50	31.20	-0.20
11	1965M11	158.50	28.80	-2.40
12	1965M12	165.00	29.80	1.00
13	1966M01	168.20	31.30	1.50
14	1966M02	172.30	27.30	-4.00
15	1966M03	169.30	22.90	-4.40
16	1966M04	173.00	19.10	-3.80
17	1966M05	176.20	15.90	-3.20
18	1966M06	184.90	18.80	2.90
19	1966M07	183.70	12.80	-6.00
20	1966M08	177.00	12.40	-0.40
21	1966M09	164.70	3.30	-9.10
22	1966M10	162.10	2.30	-1.00
23	1966M11	163.20	3.00	0.70
24	1966M12	161.90	-1.90	-4.90
25	1967M01	158.80	-5.60	-3.70
26	1967M02	156.10	-9.40	-3.80
27	1967M03	155.30	-8.30	1.10
28	1967M04	157.50	-9.00	-0.70
29	1967M05	159.40	-9.50	-0.50
30	1967M06	163.70	-11.50	-2.00
31	1967M07	161.50	-12.10	-0.60
32	1967M08	157.10	-11.20	0.90
33	1967M09	151.50	-8.00	3.20
34	1967M10	153.10	-5.60	2.40
35	1967M11	153.10	-6.20	-0.60
36	1967M12	157.20	-2.90	3.30
37	1968M01	161.60	1.80	4.70
38	1968M02	161.00	3.10	1.30
39	1968M03	160.10	3.10	0.00
40	1968M04	160.30	1.80	-1.30
41	1968M05	164.00	2.90	1.10
42	1968M06	169.20	3.40	0.50

43	1968M07	175.40	8.60	5.20
44	1968M08	174.40	11.00	2.40
45	1968M09	175.00	15.50	4.50
46	1968M10	179.30	17.10	1.60
47	1968M11	178.50	16.60	-0.50
48	1968M12	176.80	12.50	-4.10
49	1969M01	174.30	7.90	-4.60
50	1969M02	174.60	8.40	0.50
51	1969M03	174.60	9.10	0.70
52	1969M04	179.40	11.90	2.80
53	1969M05	183.80	12.10	0.20
54	1969M06	191.70	13.30	1.20
55	1969M07	195.40	11.40	-1.90
56	1969M08	185.50	6.40	-5.00
57	1969M09	178.00	1.70	-4.70
58	1969M10	180.50	0.70	-1.00
59	1969M11	181.30	1.60	0.90
60	1969M12	182.50	3.20	1.60
61	1970M01	182.50	3.20	0.00
62	1970M02	186.40	6.80	3.60
63	1970M03	186.20	6.60	-0.20
64	1970M04	186.90	4.20	-2.40
65	1970M05	193.70	5.40	1.20
66	1970M06	194.90	1.70	-3.70
67	1970M07	193.60	-0.90	-2.60
68	1970M08	195.20	5.20	6.10
69	1970M09	188.40	5.80	0.60
70	1970M10	185.60	2.80	-3.00
71	1970M11	185.00	2.00	-0.80
72	1970M12	183.80	0.70	-1.30
73	1971M01	185.90	1.90	1.20
74	1971M02	194.20	4.20	2.30
75	1971M03	194.70	4.60	0.40
76	1971M04	199.50	6.70	2.10
77	1971M05	212.80	9.90	3.20
78	1971M06	225.60	15.80	5.90
79	1971M07	222.70	15.00	-0.80
80	1971M08	211.90	8.60	-6.40
81	1971M09	208.30	10.90	2.30
82	1971M10	207.10	11.60	0.70
83	1971M11	205.30	11.00	-0.60
84	1971M12	203.80	10.90	-0.10
85	1972M01	218.30	17.40	6.50
86	1972M02	214.70	10.60	-6.80
87	1972M03	219.40	12.70	2.10
88	1972M04	229.50	15.00	2.30
89	1972M05	231.10	8.60	-6.40
90	1972M06	252.10	11.70	3.10
91	1972M07	227.50	2.20	-9.50
92	1972M08	223.40	5.40	3.20
93	1972M09	224.00	7.50	2.10

94	1972M10	224.10	8.20	0.70
95	1972M11	226.90	10.50	2.30
96	1972M12	229.60	12.70	2.20
97	1973M01	231.40	6.00	-6.70
98	1973M02	238.10	10.90	4.90
99	1973M03	243.90	11.20	0.30
100	1973M04	254.10	10.70	-0.50
101	1973M05	259.90	12.50	1.80
102	1973M06	267.10	6.00	-6.50
103	1973M07	289.60	27.30	21.30
104	1973M08	284.00	27.10	-0.20
105	1973M09	281.60	25.70	-1.40
106	1973M10	277.40	23.80	-1.90
107	1973M11	282.30	24.40	0.60
108	1973M12	287.80	25.30	0.90
109	1974M01	285.90	23.60	-1.70
110	1974M02	292.60	22.90	-0.70
111	1974M03	295.70	21.20	-1.70
112	1974M04	304.70	19.90	-1.30
113	1974M05	313.20	20.50	0.60
114	1974M06	319.50	20.50	0.00
115	1974M07	319.50	19.60	-0.90
116	1974M08	331.00	14.30	-5.30
117	1974M09	331.00	16.50	2.20
118	1974M10	324.20	15.10	-1.40
119	1974M11	322.60	16.30	1.20
120	1974M12	329.40	16.70	0.40
121	1975M01	334.10	16.10	-0.60
122	1975M02	343.80	20.30	4.20
123	1975M03	348.80	19.20	-1.10
124	1975M04	380.10	20.30	1.10
125	1975M05	390.10	24.60	4.30
126	1975M06	411.50	28.80	4.20
127	1975M07	425.50	28.50	-0.30
128	1975M08	433.40	30.90	2.40
129	1975M09	442.20	36.40	5.50
130	1975M10	447.70	38.80	2.40
131	1975M11	459.00	39.30	0.50
132	1975M12	469.00	40.40	1.10
133	1976M01	495.80	44.20	3.80
134	1976M02	519.40	48.90	4.70
135	1976M03	533.80	50.00	1.10
136	1976M04	556.40	46.40	-3.60
137	1976M05	587.60	50.60	4.20
138	1976M06	627.30	52.40	1.80
139	1976M07	690.70	62.30	9.90
140	1976M08	704.40	62.50	0.20
141	1976M09	721.90	63.30	0.80
142	1976M10	725.10	62.00	-1.30
143	1976M11	725.80	58.10	-3.90
144	1976M12	783.30	67.00	8.90

145	1977M01	814.10	64.20	-2.80
146	1977M02	868.20	67.20	3.00
147	1977M03	964.50	80.70	13.50
148	1977M04	1128.70	102.90	22.20
149	1977M05	1249.80	112.70	9.80
150	1977M06	1534.60	144.60	31.90
151	1977M07	1764.20	155.40	10.80
152	1977M08	1729.20	145.50	-9.90
153	1977M09	1741.40	141.20	-4.30
154	1977M10	1589.90	119.30	-21.90
155	1977M11	1591.60	119.30	0.00
156	1977M12	1613.60	106.00	-13.30
157	1978M01	127.20	103.90	-2.10
158	1978M02	132.70	99.40	-4.50
159	1978M03	144.20	95.10	-4.30
160	1978M04	153.20	77.10	-18.00
161	1978M05	159.90	66.90	-10.20
162	1978M06	167.70	42.60	-24.30
163	1978M07	170.10	25.80	-16.80
164	1978M08	172.70	30.30	4.50
165	1978M09	183.50	37.50	7.20
166	1978M10	199.90	64.00	26.50
167	1978M11	222.70	82.60	18.60
168	1978M12	243.30	96.70	14.10
169	1979M01	258.60	103.30	6.60
170	1979M02	269.50	103.10	-0.20
171	1979M03	310.90	94.40	-8.70
172	1979M04	284.80	69.80	-24.60
173	1979M05	310.90	94.40	24.60
174	1979M06	284.80	69.80	-24.60
175	1979M07	241.60	42.00	-27.80
176	1979M08	233.40	35.10	-6.90
177	1979M09	234.90	28.00	-7.10
178	1979M10	242.90	21.50	-6.50
179	1979M11	264.20	18.60	-2.90
180	1979M12	287.80	18.30	-0.30
181	1980M01	303.30	17.30	-1.00
182	1980M02	307.00	13.90	-3.40
183	1980M03	329.90	16.80	2.90
184	1980M04	356.60	20.10	3.30
185	1980M05	377.30	21.40	1.30
186	1980M06	400.30	40.60	19.20
187	1980M07	404.70	67.50	26.90
188	1980M08	416.20	78.30	10.80
189	1980M09	428.30	82.30	4.00
190	1980M10	449.80	85.20	2.90
191	1980M11	500.10	89.30	4.10
192	1980M12	540.60	87.80	-1.50
193	1981M01	611.80	101.70	13.90
194	1981M02	673.80	119.50	17.80
195	1981M03	733.60	122.40	2.90

196	1981M04	783.50	119.70	-2.70
197	1981M05	828.40	119.60	-0.10
198	1981M06	873.50	118.20	-1.14
199	1981M07	893.80	120.90	2.70
200	1981M08	922.00	121.50	0.60
201	1981M09	955.70	123.10	1.60
202	1981M10	1014.40	125.50	2.40
203	1981M11	1050.80	110.10	-15.40
204	1981M12	1083.30	100.40	-9.70
205	1982M01	1047.40	71.20	-29.20
206	1982M02	949.10	40.90	-30.30
207	1982M03	955.40	30.20	-10.70
208	1982M04	978.60	24.90	-5.30
209	1982M05	1016.50	22.70	-2.20
210	1982M06	1028.00	17.70	-5.00
211	1982M07	1078.30	20.60	2.90
212	1982M08	1048.60	13.70	-6.90
213	1982M09	1080.90	13.10	-0.60
214	1982M10	1131.50	11.50	-1.60
215	1982M11	1171.10	11.40	-0.10
216	1982M12	1264.10	16.70	5.30
217	1983M01	1388.00	32.50	15.80
218	1983M02	1543.60	62.60	30.10
219	1983M03	1743.80	82.50	19.90
220	1983M04	2039.60	108.40	25.90
221	1983M05	2517.80	147.70	39.30
222	1983M06	2818.20	174.10	26.40
223	1983M07	2599.30	141.10	-33.00
224	1983M08	2520.30	140.30	-0.80
225	1983M09	2570.00	137.80	-2.50
226	1983M10	2728.60	141.10	3.30
227	1983M11	2875.10	145.50	4.40
228	1983M12	3064.40	142.40	-3.10
229	1984M01	3136.80	126.00	-16.40
230	1984M02	3357.40	117.50	-8.50
231	1984M03	3459.90	98.40	-19.10
232	1984M04	3552.30	74.20	-24.20
233	1984M05	3606.80	43.30	-30.90
234	1984M06	3491.20	23.90	-19.40
235	1984M07	3352.10	29.00	5.10
236	1984M08	3132.00	24.30	-4.70
237	1984M09	3106.50	20.90	-3.40
238	1984M10	3105.10	13.80	-7.10
239	1984M11	3102.10	7.90	-5.90
240	1984M12	3247.90	6.00	-1.90
241	1985M01	3408.30	8.70	2.70
242	1985M02	3495.40	4.10	-4.60
243	1985M03	3575.90	3.40	-0.70
244	1985M04	3613.50	1.70	-1.70
245	1985M05	3647.90	1.10	-0.60
246	1985M06	3728.00	6.80	5.70

247	1985M07	3693.40	10.20	3.40
248	1985M08	3665.10	17.00	6.80
249	1985M09	3648.20	17.40	0.40
250	1985M10	3672.10	18.30	0.90
251	1985M11	3737.00	20.50	2.20
252	1985M12	3881.20	19.50	-1.00
253	1986M01	4047.20	18.70	-0.80
254	1986M02	4179.00	19.60	0.90
255	1986M03	4301.50	20.30	0.70
256	1986M04	4394.10	21.60	1.30
257	1986M05	4499.20	23.30	1.70
258	1986M06	4590.90	23.10	-0.20
259	1986M07	4575.20	23.90	0.80
260	1986M08	4542.30	23.90	0.00
261	1986M09	4517.80	23.80	-0.10
262	1986M10	4734.90	28.90	5.10
263	1986M11	4959.80	32.70	3.80
264	1986M12	5175.00	33.30	0.60
265	1987M01	5400.40	33.40	0.10
266	1987M02	5616.30	34.40	1.00
267	1987M03	5914.20	37.50	3.10
268	1987M04	6195.60	41.00	3.50
269	1987M05	6404.30	42.30	1.30
270	1987M06	6605.00	43.90	1.60
271	1987M07	6634.00	45.00	1.10
272	1987M08	6610.90	45.50	0.50
273	1987M09	6591.80	45.90	0.40
274	1987M10	6596.00	39.30	-6.60
275	1987M11	6710.50	35.30	-4.00
276	1987M12	6943.80	34.20	-1.10
277	1988M01	7232.00	33.90	-0.30
278	1988M02	7533.60	34.10	0.20
279	1988M03	7875.00	33.20	-0.90
280	1988M04	8208.00	32.50	-0.70
281	1988M05	8553.50	33.60	1.10
282	1988M06	8808.00	32.50	-1.10
283	1988M07	8722.00	31.50	-1.00
284	1988M08	8625.60	30.50	-1.00
285	1988M09	8571.50	30.00	-0.50
286	1988M10	8569.80	29.90	-0.10
287	1988M11	8639.30	28.70	-1.20
288	1988M12	8787.80	26.60	-2.10
289	1989M01	9132.70	26.30	-0.30
290	1989M02	9456.90	25.50	-0.80
291	1989M03	9829.00	24.80	-0.70
292	1989M04	10209.10	24.40	-0.40
293	1989M05	10544.20	23.30	-1.10
294	1989M06	10813.10	22.80	-0.50
295	1989M07	10775.00	23.50	0.70
296	1989M08	10677.40	23.80	0.30
297	1989M09	10663.20	24.40	0.60

298	1989M10	10764.10	25.60	1.20
299	1989M11	11062.70	28.10	2.50
300	1989M12	11464.40	30.50	2.40
301	1990M01	12150.40	33.00	2.50
302	1990M02	12856.30	35.90	2.90
303	1990M03	13374.20	36.10	0.20
304	1990M04	13885.10	36.00	-0.10
305	1990M05	14298.60	35.60	-0.40
306	1990M06	14751.00	36.40	0.80
307	1990M07	14978.00	39.00	2.60
308	1990M08	14971.00	40.20	1.20
309	1990M09	15073.40	41.40	1.20
310	1990M10	14996.80	39.30	-2.10
311	1990M11	15182.80	37.20	-2.10
312	1990M12	15580.30	35.90	-1.30
313	1991M01	15841.70	30.40	-5.50
314	1991M02	16277.90	26.60	-3.80
315	1991M03	16701.90	24.90	-1.70
316	1991M04	16980.70	22.30	-2.60
317	1991M05	17125.70	19.80	-2.50
318	1991M06	17302.80	17.30	-2.50
319	1991M07	17267.20	15.30	-2.00
320	1991M08	17157.80	14.60	-0.70
321	1991M09	17066.40	13.20	-1.40
322	1991M10	17088.50	13.90	0.70
323	1991M11	17140.00	12.90	-1.00
324	1991M12	17178.70	10.30	-2.60
325	1992M01	17217.70	8.70	-1.60
326	1992M02	17535.30	7.70	-1.00
327	1992M03	17925.40	7.30	-0.40
328	1992M04	18374.70	8.20	0.90
329	1992M05	18645.10	8.90	0.70
330	1992M06	18751.30	8.40	-0.50
331	1992M07	19034.40	10.20	1.80
332	1992M08	19164.10	11.70	1.50
333	1992M09	19025.60	11.50	-0.20
334	1992M10	19090.90	11.70	0.20
335	1992M11	19324.20	12.70	1.00
336	1992M12	19469.00	13.30	0.60
337	1993M01	20912.40	21.50	8.20
338	1993M02	21562.70	23.00	1.50
339	1993M03	22096.70	23.30	0.30
340	1993M04	22609.30	23.00	-0.30
341	1993M05	23106.70	23.90	0.90
342	1993M06	23634.90	26.00	2.10
343	1994M07	23826.70	25.20	-0.80
344	1993M08	23997.20	25.20	0.00
345	1993M09	24141.20	26.90	1.70
346	1993M10	24148.90	26.50	-0.40
347	1993M11	24466.50	26.60	0.10
348	1993M12	24853.20	27.70	1.10

349	1994M01	25682.00	22.80	-4.90
350	1994M02	26298.60	22.00	-0.80
351	1994M03	26855.00	21.50	-0.50
352	1994M04	27368.80	21.10	-0.40
353	1994M05	27958.40	21.00	-0.10
354	1994M06	28572.50	20.90	-0.10
355	1994M07	29145.40	22.30	1.40
356	1994M08	29680.90	23.70	1.40
357	1994M09	30441.70	26.10	2.40
358	1994M10	31258.90	29.40	3.30
359	1994M11	32223.20	31.70	2.30
360	1994M12	33347.70	26.10	-5.60
361	1995M01	34819.50	35.60	9.50
362	1995M02	36394.40	38.40	2.80
363	1995M03	38561.40	43.60	5.20
364	1995M04	41034.40	49.90	6.30
365	1995M05	43647.60	56.10	6.20
366	1995M06	46246.00	61.90	5.80
367	1995M07	48731.20	67.20	5.30
368	1995M08	50438.60	69.90	2.70
369	1995M09	51691.00	69.80	-0.10
370	1995M10	52871.40	69.10	-0.70
371	1995M11	54856.00	70.20	1.10
372	1995M12	56964.20	70.80	0.60
373	1996M01	58914.00	69.20	-1.60
374	1996M02	61154.40	68.00	-1.20
375	1996M03	63543.00	64.80	-3.20
376	1996M04	65763.00	60.30	-4.50
377	1996M05	67323.30	54.20	-6.10
378	1996M06	68639.20	48.40	-5.80
379	1996M07	69511.80	42.60	-5.80
380	1996M08	70218.00	39.20	-3.40
381	1996M09	70564.70	36.50	-2.70
382	1996M10	71001.40	34.30	-2.20
383	1996M11	73050.70	33.20	-1.10
384	1996M12	75569.70	32.70	-0.50
385	1997M01	77477.10	31.50	-1.20
386	1997M02	79841.00	30.60	-0.90
387	1997M03	82108.00	29.20	-1.40
388	1997M04	84894.30	29.10	-0.10
389	1997M05	87232.50	29.60	0.50
390	1997M06	88576.50	29.00	-0.60
391	1997M07	89788.80	29.20	0.20
392	1997M08	90033.10	28.20	-1.00
393	1997M09	90133.40	27.70	-0.50
394	1997M10	90467.00	27.40	-0.30
395	1997M11	90723.50	24.20	-3.20
396	1997M12	91311.80	20.80	-3.40
397	1998M01	103.00	19.80	-1.00
398	1998M02	106.00	19.60	-0.20
399	1998M03	109.60	20.30	0.70

400	1998M04	115.90	23.10	2.80
401	1998M05	119.00	22.90	-0.20
402	1998M06	119.70	21.80	-1.10
403	1998M07	118.20	18.70	-3.10
404	1998M08	118.40	18.60	-0.10
405	1998M09	117.40	17.40	-1.20
406	1998M10	115.80	17.10	-0.30
407	1998M11	115.60	16.20	-0.90
408	1998M12	116.90	15.70	-0.50
409	1999M01	118.70	15.30	-0.40
410	1999M02	121.90	15.00	-0.30
411	1999M03	124.60	13.70	-1.30
412	1999M04	127.80	10.20	-3.50
413	1999M05	130.20	9.40	-0.80
414	1999M06	132.00	10.30	0.90
415	1999M07	133.20	12.70	2.40
416	1999M08	132.60	12.00	-0.70
417	1999M09	131.20	11.80	-0.20
418	1999M10	130.40	12.60	0.80
419	1999M11	130.90	13.20	0.60
420	1999M12	133.00	13.80	0.60
421	2000M01	135.70	14.30	0.50
422	2000M02	140.10	14.90	0.60
423	2000M03	144.00	15.60	0.70
424	2000M04	150.10	17.50	1.90
425	2000M05	154.50	18.70	1.20
426	2000M06	158.20	19.80	1.10
427	2000M07	162.60	22.10	2.30
428	2000M08	167.90	26.60	4.50
429	2000M09	173.60	32.30	5.70
430	2000M10	179.20	37.40	5.10
431	2000M11	182.70	39.50	2.10
432	2000M12	187.00	40.50	1.00
433	2001M01	191.20	40.90	0.40
434	2001M02	196.40	40.10	-0.80
435	2001M03	204.40	41.90	1.80
436	2001M04	209.40	39.50	-2.40
437	2001M05	213.10	37.90	-1.60
438	2001M06	216.50	36.80	-1.10
439	2001M07	219.40	34.90	-1.90
440	2001M08	221.70	32.00	-2.90
441	2001M09	222.70	28.30	-3.70
442	2001M10	225.00	25.60	-2.70
443	2001M11	226.00	23.70	-1.90
444	2001M12	226.80	21.30	-2.40
445	2002M01	229.20	19.90	-1.40
446	2002M02	232.30	18.30	-1.60
447	2002M03	237.10	16.00	-2.30
448	2002M04	240.60	14.90	-1.10
449	2002M05	243.70	14.30	-0.60
450	2002M06	246.10	13.70	-0.60

451	2002M07	249.00	13.50	-0.20
452	2002M08	250.80	13.10	-0.40
453	2002M09	251.40	12.90	-0.20
454	2002M10	254.70	13.20	0.30
455	2002M11	257.60	14.00	0.80
456	2002M12	261.20	15.20	1.20
457	2003M01	109.40	13.50	-1.70
458	2003M02	121.30	25.50	12.00
459	2003M03	126.50	29.80	4.30
460	2003M04	127.10	29.30	-0.50
461	2003M05	130.70	31.60	2.30
462	2003M06	132.60	32.90	1.30
463	2003M07	132.60	33.00	0.10
464	2003M08	134.70	33.60	0.60
465	2003M09	132.20	29.80	-3.40
466	2003M10	135.40	33.20	0.40
467	2003M11	137.00	33.60	-2.30
468	2003M12	138.50	31.30	-2.30
469	2004M01	141.10	29.00	-10.40
470	2004M02	143.90	18.60	-3.00
471	2004M03	146.30	15.60	1.70
472	2004M04	149.00	17.30	0.30
473	2004M05	153.70	17.60	0.40
474	2004M06	156.50	18.00	0.40
475	2004M07	152.50	15.00	-3.00
476	2004M08	158.40	17.50	2.50
477	2004M09	158.20	19.60	2.10
478	2004M10	158.30	16.90	-2.70
479	2004M11	159.60	16.50	-0.40
480	2004M12	161.30	16.40	-0.10
481	2005M01	164.80	16.80	0.40
482	2005M02	168.30	17.00	0.20
483	2005M03	172.30	17.80	0.80
484	2005M04	174.60	17.10	-0.70
485	2005M05	176.00	14.50	-2.60
486	2005M06	178.50	14.00	-0.50
487	2005M07	179.00	17.30	3.30
488	2005M08	179.50	13.30	-4.00
489	2005M09	180.80	14.30	1.00
490	2005M10	181.90	14.90	0.60
491	2005M11	183.00	14.70	-0.20
492	2005M12	183.70	13.90	-0.80
493	2006M01	185.80	12.80	-1.10
494	2006M02	188.90	12.30	-0.50
495	2006M03	191.70	11.30	-1.00
496	2006M04	194.20	11.20	-0.10
497	2006M05	196.70	11.70	0.50
498	2006M06	198.80	11.40	-0.30
499	2006M07	202.10	12.90	1.50
500	2006M08	202.00	12.60	-0.30
501	2006M09	201.90	11.70	-0.90

502	2006M10	201.70	10.90	-0.80
503	2006M11	202.60	10.70	-0.20
504	2006M12	203.80	10.90	0.20
505	2007M01	206.10	10.90	0.00
506	2007M02	208.60	10.40	-0.50
507	2007M03	211.30	10.20	-0.20
508	2007M04	214.50	10.50	0.30
509	2007M05	218.40	11.00	0.50
510	2007M06	220.00	10.70	-0.30
511	2007M07	222.60	10.10	-0.60
512	2007M08	223.00	10.40	0.30
513	2007M09	222.50	10.20	-0.20
514	2007M10	222.10	10.10	-0.10
515	2007M11	225.70	11.40	1.30
516	2007M12	229.80	12.70	1.30
517	2008M01	232.50	12.80	0.10
518	2008M02	236.20	13.20	0.40
519	2008M03	240.40	13.80	0.60
520	2008M04	247.40	15.30	1.50
521	2008M05	255.30	16.90	1.60
522	2008M06	260.50	18.40	1.50
523	2008M07	263.40	18.30	-0.10
524	2008M08	263.40	18.10	-0.20
525	2008M09	262.30	17.90	-0.20
526	2008M10	260.60	17.30	-0.60
527	2008M11	265.10	17.40	0.10
528	2008M12	271.50	18.10	0.70
529	2009M01	278.60	19.90	1.80
530	2009M02	284.20	20.30	0.40
531	2009M03	289.80	20.50	0.20
532	2009M04	298.20	20.60	0.10
533	2009M05	306.50	20.10	-0.50
534	2009M06	314.60	20.70	0.60
535	2009M07	317.30	20.50	-0.20
536	2009M08	315.10	19.60	-0.90
537	2009M09	310.50	18.40	-1.20
538	2009M10	307.60	18.00	-0.40
539	2009M11	309.90	16.90	-1.10
540	2009M12	314.80	16.00	-0.90
541	2010M01	319.80	14.80	-1.20
542	2010M02	324.70	14.20	-0.60
543	2010M03	328.40	13.30	-0.90
544	2010M04	333.00	11.70	-1.60
545	2010M05	339.20	10.70	-1.00
546	2010M06	344.50	9.50	-1.20
547	2010M07	347.30	9.50	0.00
548	2010M08	344.90	9.40	-0.10
549	2010M09	339.70	9.40	0.00
550	2010M10	336.40	9.40	0.00
551	2010M11	338.00	9.10	-0.30
552	2010M12	341.80	8.60	-0.50

553	2011M01	348.90	9.10	0.50
554	2011M02	354.40	9.20	0.10
555	2011M03	358.34	9.13	-0.70
556	2011M04	363.02	9.02	-0.11
557	2011M05	369.41	8.90	-0.12
558	2011M06	374.13	8.59	-0.31
559	2011M07	376.50	8.39	-0.20
560	2011M08	373.88	8.41	0.02
561	2011M09	368.18	8.40	-0.01
562	2011M10	365.22	8.56	0.16
563	2011M11	366.90	8.55	-0.01
564	2011M12	371.16	8.58	0.03
565	2012M01	379.30	8.70	0.12
566	2012M02	385.00	8.60	-0.10
567	2012M03	389.80	8.80	0.20
568	2012M04	396.10	9.10	0.30
569	2012M05	403.90	9.30	0.20
570	2012M06	409.50	9.40	0.10
571	2012M07	412.40	9.50	0.10
572	2012M08	409.20	9.50	0.00
573	2012M09	402.90	9.40	-0.10
574	2012M10	399.00	9.20	-0.20
575	2012M11	401.10	9.30	-0.10
576	2012M12	404.00	8.80	-0.50

APPENDIX B

Figure 1B: Histogram of Monthly Rates of Inflation in Ghana from January 1965 to December 2012

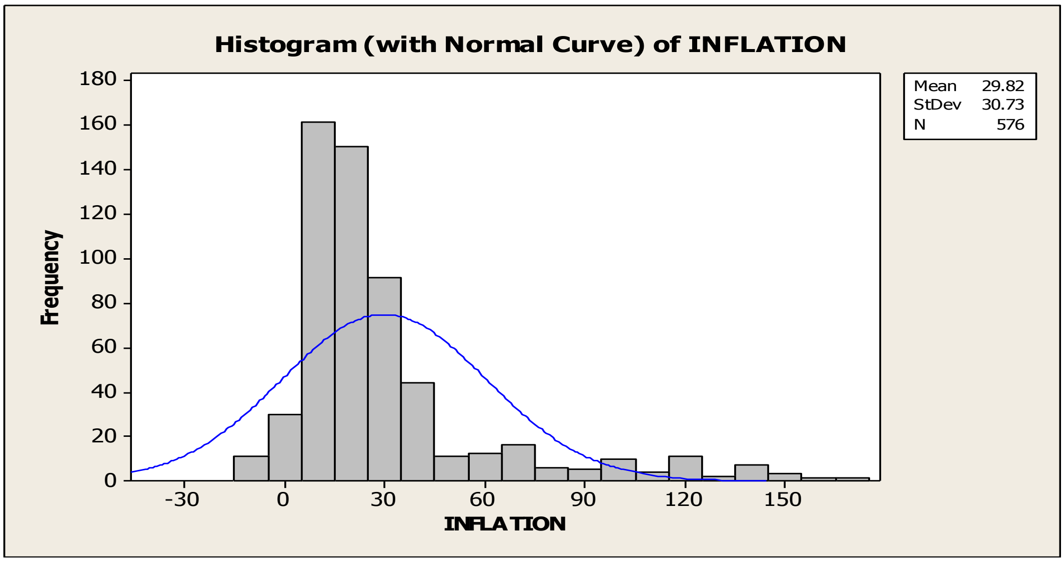


Figure 2B: Seasonal Component Analysis of Monthly Rates of Inflation in Ghana from January 1965 to December 2012

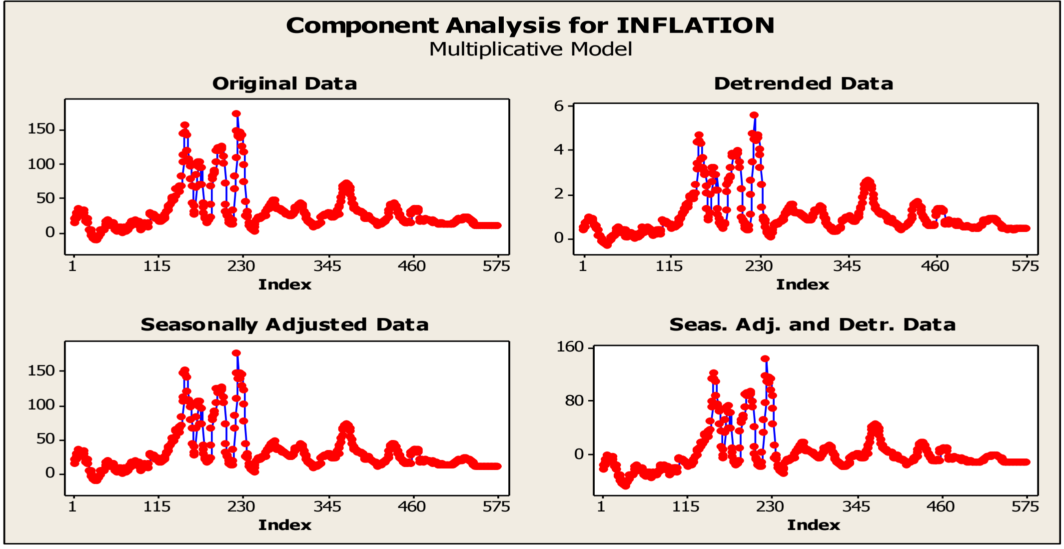


Figure 3B: Seasonal Component Analysis of Monthly Rates of Inflation in Ghana from January 1965 to December 2012

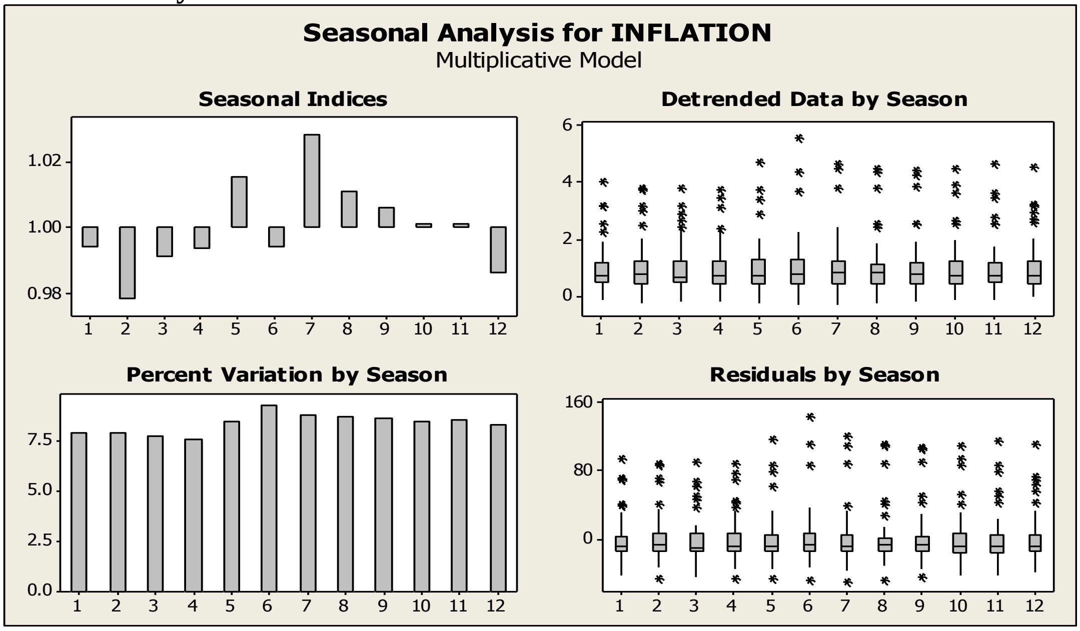


Figure 4B: Residual Plots for the first difference of Monthly Rates of Inflation in Ghana from January 1965 to December 2012

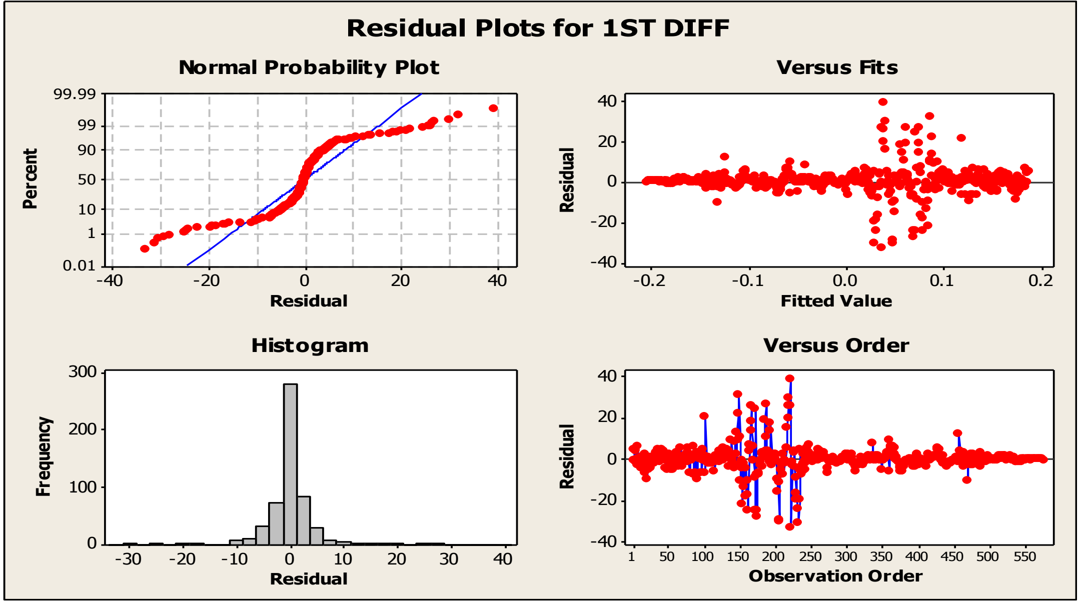
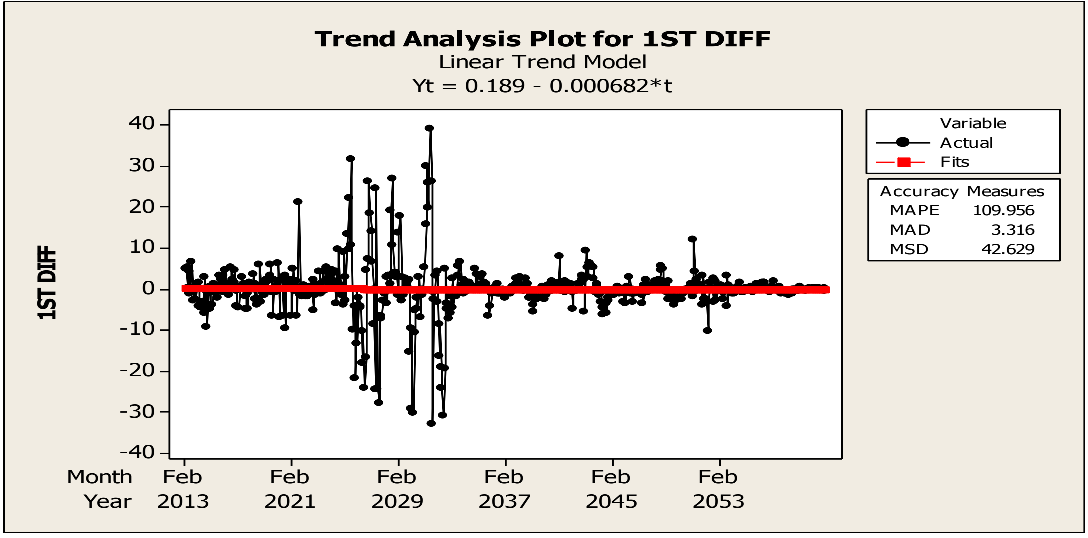
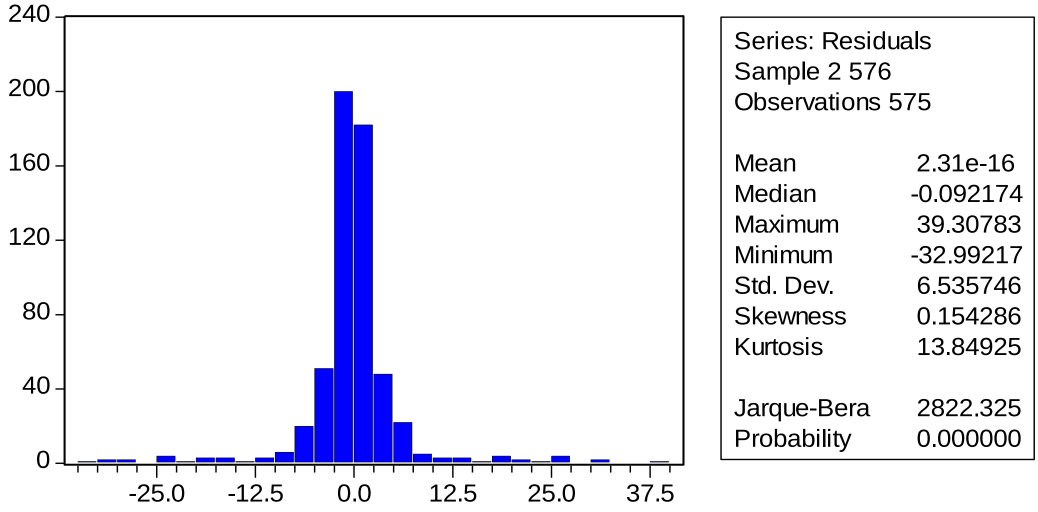


Figure 5B: Trend Analysis for the first difference of Monthly Rates of Inflation in Ghana from January 1965 to December 2012



Accuracy Measures	
MAPE	109.956
MAD	3.316
MSD	42.629

Figure 6B: Histogram of the first difference of the monthly rates of inflation series.



Series: Residuals
Sample 2 576
Observations 575

Mean	2.31e-16
Median	-0.092174
Maximum	39.30783
Minimum	-32.99217
Std. Dev.	6.535746
Skewness	0.154286
Kurtosis	13.84925
Jarque-Bera	2822.325
Probability	0.000000

Table 1B: Test for Heteroscedasticity (ARCH Effects) in Monthly rates of Inflation series in its level form

Lag	Q-Statistic	P-value
1	551.52	0.0000
2	1053.90	0.0000
3	1494.20	0.0000
4	1877.00	0.0000
5	2175.40	0.0000
6	2425.30	0.0000
7	2623.60	0.0000
8	2776.40	0.0000
9	2890.70	0.0000
10	2973.20	0.0000
11	3031.60	0.0000
12	3073.10	0.0000
13	3106.00	0.0000
14	3134.80	0.0000
15	3162.40	0.0000
16	3191.90	0.0000
17	3225.00	0.0000
18	3263.20	0.0000
19	3306.60	0.0000
20	3356.10	0.0000
21	3410.90	0.0000
22	3471.20	0.0000
23	3537.00	0.0000
24	3608.50	0.0000
25	3686.20	0.0000
26	3769.70	0.0000
27	3857.50	0.0000

28	3946.80	0.0000
29	4035.10	0.0000
30	4120.20	0.0000
31	4200.80	0.0000
32	4275.20	0.0000
33	4342.80	0.0000
34	4403.20	0.0000
35	4455.30	0.0000
36	4499.30	0.0000

APPENDIX C

Table 1C: Model Output for ARCH (1)

Dependent Variable: D(INFLATIO)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/26/13 Time: 17:18
Sample (adjusted): 4 564
Included observations: 561 after adjustments
Convergence achieved after 31 iterations
MA backcast: 3, Variance backcast: ON
GARCH = C(5) + C(6)*RESID(-1)^2

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.498281	0.073911	-6.741625	0.0000
AR(1)	0.844158	0.032263	26.16456	0.0000
AR(2)	-0.026945	0.017772	-1.516176	0.1295
MA(1)	-0.704173	0.024205	-29.09191	0.0000
		Variance Equation		
C	2.847795	0.192694	14.77883	0.0000
RESID(-1)^2	2.373269	0.144997	16.36766	0.0000

R-squared	0.140449	Mean dependent var	-0.017683
Adjusted R-squared	0.132705	S.D. dependent var	6.613337
S.E. of regression	6.158914	Akaike info criterion	5.364787
Sum squared resid	21052.38	Schwarz criterion	5.411094
Log likelihood	-1498.823	F-statistic	18.13714
Durbin-Watson stat	1.370483	Prob(F-statistic)	0.000000

Inverted AR Roots	.81	.03
Inverted MA Roots	.70	

Table 2C: Model Output for ARCH (2)

Dependent Variable: D(INFLATIO)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/26/13 Time: 17:30
Sample (adjusted): 4 564
Included observations: 561 after adjustments
Convergence achieved after 22 iterations
MA backcast: 3, Variance backcast: ON
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*RESID(-2)^2

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.154409	0.100296	-1.539531	0.1237
AR(1)	0.038025	0.080133	0.474520	0.6351
AR(2)	0.247293	0.038061	6.497345	0.0000
MA(1)	0.236626	0.091648	2.581898	0.0098
	Variance Equation			
C	2.581578	0.169496	15.23090	0.0000
RESID(-1)^2	1.071298	0.093814	11.41942	0.0000
RESID(-2)^2	0.598491	0.061488	9.733461	0.0000

R-squared	0.219029	Mean dependent var	-0.017683
Adjusted R-squared	0.210571	S.D. dependent var	6.613337
S.E. of regression	5.875939	Akaike info criterion	5.305401
Sum squared resid	19127.77	Schwarz criterion	5.359426
Log likelihood	-1481.165	F-statistic	25.89555
Durbin-Watson stat	1.741881	Prob(F-statistic)	0.000000

Inverted AR Roots	.52	-.48
Inverted MA Roots	-.24	

Table 3C: Model Output for ARCH (3)

```
Dependent Variable: D(INFLATIO)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/26/13   Time: 17:30
Sample (adjusted): 4 564
Included observations: 561 after adjustments
Convergence achieved after 103 iterations
MA backcast: 3, Variance backcast: ON
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*RESID(-2)^2 + C(8)*RESID(
        -3)^2
==============================================================================
                          Coefficient     Std. Error     z-Statistic     Prob.
==============================================================================
           C               -0.212855       0.113236       -1.879746     0.0601
         AR(1)              0.892542       0.093720        9.523500     0.0000
         AR(2)             -0.110620       0.045480       -2.432313     0.0150
         MA(1)             -0.627033       0.080582       -7.781275     0.0000
==============================================================================
                              Variance Equation
==============================================================================
           C                2.533706       0.169868        14.91570     0.0000
      RESID(-1)^2           1.111763       0.104060        10.68385     0.0000
      RESID(-2)^2           0.644038       0.101071        6.372114     0.0000
      RESID(-3)^2          -0.001810       0.015143       -0.119515     0.9049
==============================================================================
R-squared                   0.202374   Mean dependent var            -0.017683
Adjusted R-squared          0.192278   S.D. dependent var             6.613337
S.E. of regression          5.943630   Akaike info criterion          5.305566
Sum squared resid           19535.68   Schwarz criterion              5.367309
Log likelihood             -1480.211   F-statistic                    20.04393
Durbin-Watson stat          1.651470   Prob(F-statistic)              0.000000
==============================================================================
Inverted AR Roots               .74             .15
Inverted MA Roots               .63
==============================================================================
```

Table 4C: Model Output for GARCH (1, 1)

```
Dependent Variable: D(INFLATIO)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/26/13   Time: 17:35
Sample (adjusted): 4 564
Included observations: 561 after adjustments
Convergence achieved after 112 iterations
MA backcast: 3, Variance backcast: ON
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*GARCH(-1)
==========================================================================
                      Coefficient    Std. Error    z-Statistic      Prob.
==========================================================================
         C            -0.194598     0.202762     -0.959734        0.3372
       AR(1)           0.442329     0.234397      1.887088        0.0591
       AR(2)           0.177790     0.121285      1.465887        0.1427
       MA(1)          -0.054806     0.232681     -0.235542        0.8138
==========================================================================
                            Variance Equation
==========================================================================
         C             0.176700     0.031041      5.692498        0.0000
    RESID(-1)^2        0.349895     0.026864      13.02445        0.0000
     GARCH(-1)         0.736577     0.011384      64.70119        0.0000
==========================================================================
R-squared              0.236821   Mean dependent var           -0.017683
Adjusted R-squared     0.228555   S.D. dependent var            6.613337
S.E. of regression     5.808621   Akaike info criterion         5.168316
Sum squared resid      18692.01   Schwarz criterion             5.222341
Log likelihood        -1442.713   F-statistic                   28.65179
Durbin-Watson stat     1.955985   Prob(F-statistic)             0.000000
==========================================================================
Inverted AR Roots         .70          -.25
Inverted MA Roots         .05
==========================================================================
```

Table 5C: Model Output for GARCH (1, 2)

```
Dependent Variable: D(INFLATIO)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/26/13   Time: 17:37
Sample (adjusted): 4 564
Included observations: 561 after adjustments
Convergence achieved after 172 iterations
MA backcast: 3, Variance backcast: ON
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*GARCH(-1) + C(8)
        *GARCH(-2)
==========================================================================
                        Coefficient   Std. Error   z-Statistic       Prob.
==========================================================================
          C              -0.183748     0.182732    -1.005559        0.3146
        AR(1)             0.412817     0.274547     1.503629        0.1327
        AR(2)             0.181130     0.133279     1.359031        0.1741
        MA(1)            -0.031922     0.277152    -0.115179        0.9083
==========================================================================
                           Variance Equation
==========================================================================
          C               0.166547     0.037240     4.472260        0.0000
     RESID(-1)^2          0.430331     0.037721     11.40839        0.0000
      GARCH(-1)           0.420231     0.110561     3.800907        0.0001
      GARCH(-2)           0.261598     0.089936     2.908721        0.0036
==========================================================================
R-squared                 0.237631   Mean dependent var          -0.017683
Adjusted R-squared        0.227980   S.D. dependent var           6.613337
S.E. of regression        5.810785   Akaike info criterion        5.160210
Sum squared resid         18672.17   Schwarz criterion            5.221953
Log likelihood           -1439.439   F-statistic                  24.62432
Durbin-Watson stat        1.945846   Prob(F-statistic)            0.000000
==========================================================================
Inverted AR Roots        .68            -.27
Inverted MA Roots        .03
==========================================================================
```

Table 6C: Model Output for GARCH (2, 1)

```
Dependent Variable: D(INFLATIO)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/26/13   Time: 17:38
Sample (adjusted): 4 564
Included observations: 561 after adjustments
Convergence achieved after 93 iterations
MA backcast: 3, Variance backcast: ON
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*RESID(-2)^2 + C(8)
        *GARCH(-1)
==========================================================================
                           Coefficient    Std. Error   z-Statistic     Prob.
==========================================================================

         C                  -0.050480     0.061193     -0.824919      0.4094
       AR(1)                 0.354733     0.206899      1.714528      0.0864
       AR(2)                 0.168479     0.084851      1.985587      0.0471
       MA(1)                -0.053134     0.218579     -0.243089      0.8079
==========================================================================
                              Variance Equation
==========================================================================

         C                  -0.009901     0.004244     -2.332809      0.0197
     RESID(-1)^2             0.711921     0.048894      14.56061      0.0000
     RESID(-2)^2            -0.503197     0.046471     -10.82820      0.0000
      GARCH(-1)              0.864969     0.008080      107.0518      0.0000
==========================================================================

R-squared                    0.231523   Mean dependent var          -0.017683
Adjusted R-squared           0.221795   S.D. dependent var           6.613337
S.E. of regression           5.834015   Akaike info criterion        5.121990
Sum squared resid            18821.76   Schwarz criterion            5.183732
Log likelihood              -1428.718   F-statistic                  23.80073
Durbin-Watson stat           1.790040   Prob(F-statistic)            0.000000
==========================================================================

Inverted AR Roots          .62            -.27
Inverted MA Roots          .05
==========================================================================
```

Table 7C: Model Output for GARCH (2, 2)

Dependent Variable: D(INFLATIO)

Method: ML - ARCH (Marquardt) - Normal distribution

Date: 07/26/13 Time: 17:39

Sample (adjusted): 4 564

Included observations: 561 after adjustments

Convergence achieved after 71 iterations

MA backcast: 3, Variance backcast: ON

GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*RESID(-2)^2 + C(8)
*GARCH(-1) + C(9)*GARCH(-2)

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.240779	0.197155	-1.221265	0.2220
AR(1)	0.400804	0.225635	1.776334	0.0757
AR(2)	0.193982	0.113446	1.709898	0.0873
MA(1)	-0.002159	0.235977	-0.009148	0.9927
Variance Equation				
C	0.734895	0.098358	7.471611	0.0000
RESID(-1)^2	0.579129	0.048323	11.98445	0.0000
RESID(-2)^2	0.543006	0.046946	11.56649	0.0000
GARCH(-1)	-0.349990	0.028228	-12.39858	0.0000
GARCH(-2)	0.564453	0.018221	30.97825	0.0000

R-squared	0.237200	Mean dependent var	-0.017683
Adjusted R-squared	0.226145	S.D. dependent var	6.613337
S.E. of regression	5.817689	Akaike info criterion	5.169987
Sum squared resid	18682.72	Schwarz criterion	5.239447
Log likelihood	-1441.181	F-statistic	21.45621
Durbin-Watson stat	1.980253	Prob(F-statistic)	0.000000

Inverted AR Roots	.68	-.28
Inverted MA Roots	.00	

Table 8C: Model Output for EGARCH (1, 1)

Dependent Variable: D(INFLATIO)

Method: ML - ARCH (Marquardt) - Normal distribution

Date: 07/26/13 Time: 17:41

Sample (adjusted): 4 564

Included observations: 561 after adjustments

Convergence achieved after 70 iterations

MA backcast: 3, Variance backcast: ON

LOG(GARCH) = C(5) + C(6)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +

C(7)*RESID(-1)/@SQRT(GARCH(-1)) + C(8)*LOG(GARCH(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.348038	0.252564	-1.378021	0.1682
AR(1)	1.001650	0.108285	9.250135	0.0000
AR(2)	-0.116967	0.077527	-1.508734	0.1314
MA(1)	-0.683044	0.084169	-8.115105	0.0000
Variance Equation				
C(5)	-0.220183	0.021570	-10.20764	0.0000
C(6)	0.388365	0.030889	12.57297	0.0000
C(7)	0.060083	0.026349	2.280243	0.0226
C(8)	0.979004	0.005184	188.8448	0.0000

R-squared	0.205014	Mean dependent var	-0.017683
Adjusted R-squared	0.194950	S.D. dependent	6.613337
S.E. of regression	5.933788	Akaike info criterion	5.164555
Sum squared resid	19471.04	Schwarz criterion	5.226298
Log likelihood	-1440.658	F-statistic	20.37276
Durbin-Watson stat	1.745592	Prob(F-statistic)	0.000000

Inverted AR Roots	.87	.13
Inverted MA Roots	.68	

Table 9C: Model Output for EGARCH (1, 2)

Dependent Variable: D(INFLATIO)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/26/13 Time: 17:42
Sample (adjusted): 4 564
Included observations: 561 after adjustments
Convergence achieved after 57 iterations
MA backcast: 3, Variance backcast: ON
LOG(GARCH) = C(5) + C(6)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(7)*RESID(-1)/@SQRT(GARCH(-1)) + C(8)*LOG(GARCH(-1)) +
C(9)*LOG(GARCH(-2))

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.185278	0.171929	-1.077641	0.2812
AR(1)	0.304909	0.283926	1.073905	0.2829
AR(2)	0.195520	0.135199	1.446161	0.1481
MA(1)	0.094811	0.291694	0.325035	0.7452
		Variance Equation		
C(5)	-0.369560	0.034676	-10.65742	0.0000
C(6)	0.639181	0.048335	13.22406	0.0000
C(7)	-0.011265	0.033812	-0.333174	0.7390
C(8)	0.588073	0.087998	6.682786	0.0000
C(9)	0.384565	0.087501	4.394973	0.0000

R-squared	0.239104	Mean dependent var	-0.017683
Adjusted R-squared	0.228077	S.D. dependent var	6.613337
S.E. of regression	5.810422	Akaike info criterion	5.152872
Sum squared resid	18636.07	Schwarz criterion	5.222333
Log likelihood	-1436.381	F-statistic	21.68262
Durbin-Watson stat	1.985774	Prob(F-statistic)	0.000000

Inverted AR Roots	.62	-.32
Inverted MA Roots	-.09	

Table 10C: Model Output for EGARCH (2, 1)

```
Dependent Variable: D(INFLATIO)
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 07/26/13   Time: 17:43
Sample (adjusted): 4 564
Included observations: 561 after adjustments
Convergence achieved after 36 iterations
MA backcast: 3, Variance backcast: ON
LOG(GARCH) = C(5) + C(6)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
        C(7)*ABS(RESID(-2)/@SQRT(GARCH(-2))) + C(8)*RESID(-1)
        /@SQRT(GARCH(-1)) + C(9)*LOG(GARCH(-1))
==========================================================================
                          Coefficient    Std. Error    z-Statistic    Prob.
==========================================================================
          C                 0.106006      0.126240      0.839721     0.4011
        AR(1)               0.697105      0.203489      3.425760     0.0006
        AR(2)               0.007368      0.085391      0.086289     0.9312
        MA(1)              -0.458052      0.195324     -2.345089     0.0190
==========================================================================
                            Variance Equation
==========================================================================
         C(5)              -0.168646      0.017756     -9.497944     0.0000
         C(6)               0.866121      0.069383      12.48327     0.0000
         C(7)              -0.597416      0.069788     -8.560454     0.0000
         C(8)               0.124346      0.024108      5.157891     0.0000
         C(9)               0.991669      0.002699      367.4403     0.0000
==========================================================================
R-squared                   0.208413   Mean dependent var          -0.017683
Adjusted R-squared          0.196941   S.D. dependent var           6.613337
S.E. of regression          5.926447   Akaike info criterion        5.089933
Sum squared resid           19387.77   Schwarz criterion            5.159394
Log likelihood             -1418.726   F-statistic                  18.16669
Durbin-Watson stat          1.624597   Prob(F-statistic)            0.000000
==========================================================================
Inverted AR Roots               .71           -.01
Inverted MA Roots               .46
==========================================================================
```

Table 11C: Model Output for EGARCH (2, 2)

Dependent Variable: D(INFLATIO)

Method: ML - ARCH (Marquardt) - Normal distribution

Date: 07/26/13 Time: 17:44

Sample (adjusted): 4 564

Included observations: 561 after adjustments

Convergence achieved after 75 iterations

MA backcast: 3, Variance backcast: ON

LOG(GARCH) = C(5) + C(6)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(7)*ABS(RESID(-2)/@SQRT(GARCH(-2))) + C(8)*RESID(-1)
/@SQRT(GARCH(-1)) + C(9)*LOG(GARCH(-1)) + C(10)
*LOG(GARCH(-2))

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.270043	0.156617	-1.724228	0.0847
AR(1)	0.237689	0.219705	1.081852	0.2793
AR(2)	0.269255	0.111432	2.416312	0.0157
MA(1)	0.170693	0.226417	0.753889	0.4509

Variance Equation

C(5)	-0.616292	0.045604	-13.51399	0.0000
C(6)	0.517074	0.037174	13.90945	0.0000
C(7)	0.552703	0.042327	13.05794	0.0000
C(8)	-0.137644	0.032864	-4.188309	0.0000
C(9)	0.026609	0.025045	1.062449	0.2880
C(10)	0.929400	0.025143	36.96508	0.0000

R-squared	0.234738	Mean dependent var	-0.017683
Adjusted R-squared	0.222239	S.D. dependent var	6.613337
S.E. of regression	5.832353	Akaike info criterion	5.159967
Sum squared resid	18743.01	Schwarz criterion	5.237145
Log likelihood	-1437.371	F-statistic	18.77947
Durbin-Watson stat	2.009781	Prob(F-statistic)	0.000000

Inverted AR Roots	.65	-.41
Inverted MA Roots	-.17	

Table 12C ARCH LM Test for ARCH (2)

ARCH Test:

F-statistic	2.554944	Probability	0.000004
Obs*R-squared	83.25909	Probability	0.000013

Test Equation:
Dependent Variable: STD_RESID^2
Method: Least Squares
Date: 07/27/13 Time: 03:42
Sample (adjusted): 40 564
Included observations: 525 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.381014	0.237746	1.602610	0.1097
STD_RESID^2(-1)	-0.031904	0.045266	-0.704799	0.4813
STD_RESID^2(-2)	-0.050002	0.045278	-1.104330	0.2700
STD_RESID^2(-3)	-0.049072	0.045106	-1.087943	0.2772
STD_RESID^2(-4)	-0.048959	0.045156	-1.084230	0.2788
STD_RESID^2(-5)	-0.061629	0.045204	-1.363355	0.1734
STD_RESID^2(-6)	-0.012361	0.045249	-0.273180	0.7848
STD_RESID^2(-7)	0.062110	0.045179	1.374755	0.1698
STD_RESID^2(-8)	0.004966	0.045186	0.109892	0.9125
STD_RESID^2(-9)	-0.076530	0.045153	-1.694889	0.0907
STD_RESID^2(-10)	0.017390	0.045287	0.383998	0.7011
STD_RESID^2(-11)	0.000463	0.044818	0.010342	0.9918
STD_RESID^2(-12)	0.118461	0.044805	2.643923	0.0085
STD_RESID^2(-13)	0.102222	0.045118	2.265655	0.0239
STD_RESID^2(-14)	0.024005	0.045340	0.529442	0.5967
STD_RESID^2(-15)	0.031070	0.045339	0.685292	0.4935
STD_RESID^2(-16)	0.041521	0.045337	0.915820	0.3602
STD_RESID^2(-17)	0.324544	0.045368	7.153521	0.0000
STD_RESID^2(-18)	0.016219	0.047687	0.340115	0.7339
STD_RESID^2(-19)	0.007628	0.047674	0.160000	0.8729
STD_RESID^2(-20)	0.021533	0.045397	0.474330	0.6355
STD_RESID^2(-21)	0.031233	0.045372	0.688370	0.4915
STD_RESID^2(-22)	0.024573	0.045351	0.541844	0.5882
STD_RESID^2(-23)	0.024267	0.045356	0.535022	0.5929
STD_RESID^2(-24)	-0.017210	0.045151	-0.381174	0.7032
STD_RESID^2(-25)	-0.014523	0.044858	-0.323764	0.7463
STD_RESID^2(-26)	0.144309	0.044845	3.217915	0.0014
STD_RESID^2(-27)	-0.004719	0.045316	-0.104139	0.9171
STD_RESID^2(-28)	0.027821	0.045171	0.615908	0.5382
STD_RESID^2(-29)	-0.059968	0.045182	-1.327248	0.1850
STD_RESID^2(-30)	0.056355	0.045184	1.247227	0.2129
STD_RESID^2(-31)	0.041584	0.045250	0.918990	0.3586
STD_RESID^2(-32)	0.008435	0.045190	0.186655	0.8520
STD_RESID^2(-33)	-0.015535	0.045130	-0.344234	0.7308
STD_RESID^2(-34)	-0.100775	0.045077	-2.235594	0.0258

STD_RESID^2(-35)	0.015850	0.045240	0.350363	0.7262
STD_RESID^2(-36)	0.007196	0.045213	0.159162	0.8736

R-squared	0.158589	Mean dependent var	1.017323
Adjusted R-squared	0.096517	S.D. dependent var	3.184223
S.E. of regression	3.026658	Akaike info criterion	5.120665
Sum squared resid	4470.402	Schwarz criterion	5.421133
Log likelihood	-1307.174	F-statistic	2.554944
Durbin-Watson stat	1.999704	Prob(F-statistic)	0.000004

Table 13C: Ljung Box Q-Statistics for ARCH (2)

Date: 07/27/13 Time: 03:37
Sample: 4 564
Included observations: 561
Q-statistic probabilities adjusted for 3 ARMA term(s)

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>.|. |</td><td>.|. |</td><td>1</td><td>-0.002</td><td>-0.002</td><td>0.0031</td><td></td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>2</td><td>-0.031</td><td>-0.031</td><td>0.5622</td><td></td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>3</td><td>-0.029</td><td>-0.029</td><td>1.0382</td><td></td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>4</td><td>-0.003</td><td>-0.004</td><td>1.0420</td><td>0.307</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>5</td><td>-0.006</td><td>-0.008</td><td>1.0624</td><td>0.588</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>6</td><td>0.015</td><td>0.014</td><td>1.1954</td><td>0.754</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>7</td><td>0.081</td><td>0.081</td><td>4.9535</td><td>0.292</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>8</td><td>-0.018</td><td>-0.017</td><td>5.1300</td><td>0.400</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>9</td><td>-0.043</td><td>-0.038</td><td>6.1880</td><td>0.402</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>10</td><td>0.035</td><td>0.039</td><td>6.8871</td><td>0.441</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>11</td><td>0.006</td><td>0.003</td><td>6.9076</td><td>0.547</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>12</td><td>0.098</td><td>0.100</td><td>12.482</td><td>0.187</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>13</td><td>0.128</td><td>0.131</td><td>21.931</td><td>0.015</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>14</td><td>0.030</td><td>0.034</td><td>22.458</td><td>0.021</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>15</td><td>0.003</td><td>0.023</td><td>22.463</td><td>0.033</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>16</td><td>0.012</td><td>0.029</td><td>22.547</td><td>0.047</td></tr>
<tr><td>.|** |</td><td>.|** |</td><td>17</td><td>0.276</td><td>0.285</td><td>66.648</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>18</td><td>0.006</td><td>0.024</td><td>66.666</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>19</td><td>0.004</td><td>0.017</td><td>66.673</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>20</td><td>0.009</td><td>0.018</td><td>66.718</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>21</td><td>-0.011</td><td>0.009</td><td>66.786</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>22</td><td>-0.017</td><td>0.000</td><td>66.959</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>23</td><td>0.028</td><td>0.020</td><td>67.428</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>24</td><td>0.030</td><td>-0.025</td><td>67.943</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>25</td><td>0.006</td><td>-0.016</td><td>67.963</td><td>0.000</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>26</td><td>0.127</td><td>0.138</td><td>77.443</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>27</td><td>0.004</td><td>-0.020</td><td>77.453</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>28</td><td>0.021</td><td>0.022</td><td>77.712</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>*|. |</td><td>29</td><td>-0.006</td><td>-0.063</td><td>77.732</td><td>0.000</td></tr>
</table>

```
       .|*    |            .|.    |        30    0.121    0.055    86.399    0.000
       .|.    |            .|.    |        31    0.049    0.041    87.858    0.000
       .|.    |            .|.    |        32    0.011    0.008    87.924    0.000
       .|.    |            .|.    |        33    0.008   -0.019    87.959    0.000
       .|.    |            *|.    |        34   -0.023   -0.106    88.280    0.000
       .|.    |            .|.    |        35    0.002    0.010    88.283    0.000
       .|.    |            .|.    |        36    0.015    0.002    88.422    0.000
```

Table 14C: ARCH –LM Test for the GARCH (2,1) Model

ARCH Test:

F-statistic	0.649902	Probability	0.943659
Obs*R-squared	24.01883	Probability	0.936672

Test Equation:
Dependent Variable: STD_RESID^2
Method: Least Squares
Date: 07/27/13 Time: 06:50
Sample (adjusted): 40 564
Included observations: 525 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.132305	0.337992	3.350098	0.0009
STD_RESID^2(-1)	-0.018270	0.045262	-0.403648	0.6866
STD_RESID^2(-2)	0.007529	0.045242	0.166416	0.8679
STD_RESID^2(-3)	-0.007648	0.045243	-0.169035	0.8658
STD_RESID^2(-4)	-0.028048	0.045245	-0.619908	0.5356
STD_RESID^2(-5)	-0.017295	0.045252	-0.382183	0.7025
STD_RESID^2(-6)	-0.006831	0.045259	-0.150941	0.8801
STD_RESID^2(-7)	-0.027006	0.045233	-0.597043	0.5508
STD_RESID^2(-8)	-0.004652	0.045251	-0.102805	0.9182
STD_RESID^2(-9)	-0.030257	0.045245	-0.668743	0.5040
STD_RESID^2(-10)	-0.032541	0.045242	-0.719256	0.4723
STD_RESID^2(-11)	-0.009626	0.045231	-0.212808	0.8316
STD_RESID^2(-12)	0.098382	0.045223	2.175472	0.0301
STD_RESID^2(-13)	-0.001776	0.045285	-0.039213	0.9687
STD_RESID^2(-14)	-0.001904	0.044967	-0.042344	0.9662
STD_RESID^2(-15)	-0.025881	0.044957	-0.575687	0.5651
STD_RESID^2(-16)	-0.024633	0.044971	-0.547753	0.5841
STD_RESID^2(-17)	-0.009271	0.044982	-0.206103	0.8368
STD_RESID^2(-18)	-0.001222	0.044973	-0.027176	0.9783
STD_RESID^2(-19)	-0.020014	0.044921	-0.445547	0.6561
STD_RESID^2(-20)	-0.016203	0.044927	-0.360656	0.7185
STD_RESID^2(-21)	-0.014759	0.044941	-0.328418	0.7427
STD_RESID^2(-22)	-0.023622	0.044923	-0.525825	0.5992
STD_RESID^2(-23)	0.117809	0.044938	2.621620	0.0090

STD_RESID^2(-24)	0.084390	0.045252	1.864890	0.0628
STD_RESID^2(-25)	-0.022234	0.045195	-0.491949	0.6230
STD_RESID^2(-26)	-0.038882	0.045199	-0.860234	0.3901
STD_RESID^2(-27)	0.029993	0.045208	0.663454	0.5074
STD_RESID^2(-28)	-0.018394	0.045203	-0.406921	0.6842
STD_RESID^2(-29)	0.002990	0.045216	0.066131	0.9473
STD_RESID^2(-30)	0.034765	0.045199	0.769162	0.4422
STD_RESID^2(-31)	-0.010246	0.045218	-0.226590	0.8208
STD_RESID^2(-32)	-0.022175	0.045212	-0.490465	0.6240
STD_RESID^2(-33)	-0.006922	0.045203	-0.153130	0.8784
STD_RESID^2(-34)	-0.010711	0.045206	-0.236939	0.8128
STD_RESID^2(-35)	-0.034403	0.045211	-0.760955	0.4471
STD_RESID^2(-36)	-0.013422	0.045246	-0.296640	0.7669

R-squared	0.045750	Mean dependent var	1.008184
Adjusted R-squared	-0.024645	S.D. dependent var	3.558203
S.E. of regression	3.601783	Akaike info criterion	5.468605
Sum squared resid	6330.747	Schwarz criterion	5.769073
Log likelihood	-1398.509	F-statistic	0.649902
Durbin-Watson stat	2.000645	Prob(F-statistic)	0.943659

Table 15C:Ljung Q- Statistics for the GARCH (2,1)

Date: 07/27/13 Time: 03:52
Sample: 4 564
Included observations: 561
Q-statistic
probabilities adjusted
for 3 ARMA term(s)

```
  Autocorrelation      Partial Correlation        AC     PAC    Q-Stat   Prob

     .|.    |             .|.    |           1   0.047   0.047  1.2543
     .|.    |             .|.    |           2   0.002  -0.000  1.2570
     .|.    |             .|.    |           3   0.058   0.058  3.1427
     .|.    |             .|.    |           4  -0.047  -0.053  4.3964  0.036
     .|.    |             .|.    |           5   0.057   0.063  6.2615  0.044
     .|.    |             .|.    |           6   0.007  -0.003  6.2873  0.098
     .|.    |             .|.    |           7  -0.009  -0.003  6.3350  0.175
     .|*    |             .|.    |           8   0.069   0.061  9.0885  0.106
     .|.    |             .|.    |           9   0.060   0.060  11.152  0.084
     .|.    |             .|.    |          10   0.035   0.028  11.860  0.105
     .|.    |             .|.    |          11  -0.007  -0.019  11.890  0.156
  ***|.     |          ***|.     |          12  -0.322  -0.327  71.565  0.000
     .|.    |             .|.    |          13  -0.036  -0.013  72.292  0.000
     .|.    |             .|.    |          14  -0.042  -0.050  73.331  0.000
     *|.    |             .|.    |          15  -0.080  -0.043  77.072  0.000
     .|.    |             .|.    |          16   0.000  -0.025  77.073  0.000
```

```
     .|.    |            .|.    |          17  -0.053  -0.022   78.700   0.000
     .|.    |            .|.    |          18  -0.033  -0.034   79.352   0.000
     .|.    |            .|.    |          19  -0.022  -0.024   79.626   0.000
     .|.    |            .|.    |          20   0.006   0.062   79.647   0.000
     *|.    |            .|.    |          21  -0.060  -0.014   81.755   0.000
     .|.    |            .|.    |          22  -0.017   0.024   81.918   0.000
     *|.    |            *|.    |          23  -0.101  -0.106   87.891   0.000
     *|.    |           **|.    |          24  -0.099  -0.201   93.628   0.000
     .|.    |            .|.    |          25   0.039   0.038   94.547   0.000
     .|.    |            .|.    |          26   0.032   0.029   95.151   0.000
     .|.    |            .|.    |          27   0.000  -0.028   95.151   0.000
     .|.    |            .|.    |          28   0.006  -0.009   95.173   0.000
     .|.    |            .|.    |          29   0.018  -0.001   95.359   0.000
     .|*    |            .|.    |          30   0.069   0.053   98.207   0.000
     .|.    |            .|.    |          31   0.055   0.053   99.977   0.000
     .|.    |            .|.    |          32  -0.027   0.021   100.41   0.000
     .|.    |            .|.    |          33  -0.005  -0.022   100.42   0.000
     .|.    |            .|.    |          34   0.012   0.007   100.51   0.000
     .|*    |            .|.    |          35   0.068  -0.015   103.26   0.000
     .|*    |            .|.    |          36   0.114   0.002   111.03   0.000
```

Table 16C: ARCH – LM Test for the EGARCH (2,1) Model

ARCH Test:

F-statistic	0.614495	Probability	0.963137
Obs*R-squared	22.74981	Probability	0.957970

Test Equation:
Dependent Variable: STD_RESID^2
Method: Least Squares
Date: 02/06/13 Time: 00:13
Sample (adjusted): 38 576
Included observations: 539 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.135220	0.329098	3.449491	0.0006
STD_RESID^2(-1)	-0.023639	0.044676	-0.529127	0.5970
STD_RESID^2(-2)	-0.000374	0.044678	-0.008375	0.9933
STD_RESID^2(-3)	-0.011996	0.044674	-0.268513	0.7884
STD_RESID^2(-4)	-0.028650	0.044668	-0.641402	0.5216
STD_RESID^2(-5)	-0.034691	0.044676	-0.776508	0.4378
STD_RESID^2(-6)	-0.005466	0.044700	-0.122271	0.9027
STD_RESID^2(-7)	-0.028152	0.044699	-0.629805	0.5291
STD_RESID^2(-8)	-0.019171	0.044703	-0.428858	0.6682
STD_RESID^2(-9)	-0.042396	0.044709	-0.948269	0.3434
STD_RESID^2(-10)	-0.032947	0.044747	-0.736280	0.4619

STD_RESID^2(-11)	-0.006376	0.044768	-0.142433	0.8868
STD_RESID^2(-12)	0.162948	0.044763	3.640253	0.0003
STD_RESID^2(-13)	0.012889	0.045351	0.284199	0.7764
STD_RESID^2(-14)	-0.002968	0.045345	-0.065443	0.9478
STD_RESID^2(-15)	-0.012159	0.045343	-0.268152	0.7887
STD_RESID^2(-16)	-0.012457	0.045348	-0.274689	0.7837
STD_RESID^2(-17)	0.022313	0.044983	0.496032	0.6201
STD_RESID^2(-18)	0.007660	0.044990	0.170254	0.8649
STD_RESID^2(-19)	0.016136	0.044990	0.358644	0.7200
STD_RESID^2(-20)	-0.012533	0.044988	-0.278592	0.7807
STD_RESID^2(-21)	-0.006908	0.044996	-0.153528	0.8780
STD_RESID^2(-22)	-0.011661	0.045000	-0.259121	0.7956
STD_RESID^2(-23)	0.016000	0.045008	0.355500	0.7224
STD_RESID^2(-24)	0.024597	0.045013	0.546446	0.5850
STD_RESID^2(-25)	-0.023031	0.044472	-0.517868	0.6048
STD_RESID^2(-26)	-0.013946	0.044479	-0.313554	0.7540
STD_RESID^2(-27)	-0.000198	0.044456	-0.004456	0.9964
STD_RESID^2(-28)	0.005024	0.044408	0.113133	0.9100
STD_RESID^2(-29)	0.025699	0.044368	0.579228	0.5627
STD_RESID^2(-30)	0.011546	0.044363	0.260256	0.7948
STD_RESID^2(-31)	-0.012071	0.044356	-0.272145	0.7856
STD_RESID^2(-32)	-0.023938	0.044324	-0.540063	0.5894
STD_RESID^2(-33)	-0.021284	0.044324	-0.480204	0.6313
STD_RESID^2(-34)	-0.008021	0.044322	-0.180974	0.8565
STD_RESID^2(-35)	-0.022807	0.044319	-0.514610	0.6071
STD_RESID^2(-36)	-0.016467	0.044312	-0.371619	0.7103

R-squared	0.042207	Mean dependent var	1.003799
Adjusted R-squared	-0.026479	S.D. dependent var	3.100266
S.E. of regression	3.141043	Akaike info criterion	5.193163
Sum squared resid	4952.809	Schwarz criterion	5.487633
Log likelihood	-1362.557	F-statistic	0.614495
Durbin-Watson stat	1.997652	Prob(F-statistic)	0.963137

Table 17C: Ljung Q-Statistics for the EGARCH (2,1)

```
Date: 07/27/13   Time: 16:02
Sample: 4 564
Included observations: 561
        Q-statistic
       probabilities
       adjusted for 3
      ARMA term(s)
==========================================================================
                          Partial
   Autocorrelation      Correlation        AC      PAC     Q-Stat     Prob
==========================================================================
      .|*    |          .|*    |       1  0.077   0.077   3.3316
      .|*    |          .|.    |       2  0.070   0.065   6.1268
      .|*    |          .|.    |       3  0.070   0.060   8.8797
      .|.    |          .|.    |       4 -0.036  -0.050   9.6068      0.002
      .|*    |          .|*    |       5  0.074   0.073   12.757      0.002
      .|.    |          .|.    |       6  0.001  -0.009   12.757      0.005
      .|.    |          .|.    |       7  0.001  -0.002   12.758      0.013
      .|*    |          .|.    |       8  0.072   0.062   15.715      0.008
      .|.    |          .|.    |       9  0.060   0.059   17.798      0.007
      .|.    |          .|.    |      10  0.019  -0.004   17.998      0.012
      .|.    |          .|.    |      11 -0.030  -0.048   18.524      0.018
    ***|.    |        ***|.    |      12 -0.345  -0.352   86.874      0.000
      *|.    |          .|.    |      13 -0.066  -0.027   89.408      0.000
      *|.    |          .|.    |      14 -0.067  -0.023   91.962      0.000
      *|.    |          .|.    |      15 -0.090  -0.038   96.623      0.000
      .|.    |          .|.    |      16 -0.009  -0.017   96.666      0.000
      *|.    |          .|.    |      17 -0.083  -0.036   100.65      0.000
      .|.    |          .|.    |      18 -0.016  -0.010   100.79      0.000
      .|.    |          .|.    |      19 -0.044  -0.035   101.94      0.000
      .|.    |          .|*    |      20 -0.003   0.072   101.94      0.000
      *|.    |          .|.    |      21 -0.067  -0.014   104.61      0.000
      .|.    |          .|.    |      22 -0.027   0.016   105.03      0.000
      *|.    |          *|.    |      23 -0.083  -0.102   109.04      0.000
      *|.    |         **|.    |      24 -0.085  -0.206   113.29      0.000
      .|.    |          .|.    |      25  0.050   0.049   114.75      0.000
      .|.    |          .|.    |      26  0.036   0.053   115.53      0.000
      .|.    |          .|.    |      27  0.029  -0.003   116.03      0.000
      .|.    |          .|.    |      28  0.028   0.007   116.49      0.000
      .|.    |          .|.    |      29  0.042   0.006   117.54      0.000
      .|.    |          .|.    |      30  0.057   0.039   119.48      0.000
      .|.    |          .|.    |      31  0.055   0.037   121.28      0.000
      .|.    |          .|.    |      32 -0.018   0.017   121.47      0.000
      .|.    |          .|.    |      33  0.016  -0.005   121.63      0.000
      .|.    |          .|.    |      34  0.019  -0.008   121.84      0.000
      .|.    |          .|.    |      35  0.064  -0.026   124.33      0.000
      .|*    |          .|.    |      36  0.107  -0.023   131.27      0.000
==========================================================================
```

Table 18C: Model Output for ARIMA (2, 1, 1)

Dependent Variable: D(INFLATIO)
Method: Least Squares
Date: 07/27/13 Time: 12:26
Sample (adjusted): 4 564
Included observations: 561 after adjustments
Convergence achieved after 15 iterations
Backcast: 3

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.021918	0.544724	-0.040236	0.9679
AR(1)	0.492862	0.309025	1.594893	0.1113
AR(2)	0.096179	0.154143	0.623961	0.5329
MA(1)	-0.082511	0.310235	-0.265961	0.7904

R-squared	0.240747	Mean dependent var	-0.017683
Adjusted R-squared	0.236658	S.D. dependent var	6.613337
S.E. of regression	5.778036	Akaike info criterion	6.353109
Sum squared resid	18595.83	Schwarz criterion	6.383981
Log likelihood	-1778.047	F-statistic	58.87209
Durbin-Watson stat	1.999697	Prob(F-statistic)	0.000000

Inverted AR Roots	.64	-.15
Inverted MA Roots	.08	

Figure 1C: Residual Plot of the ARIMA (2,1.1) Model

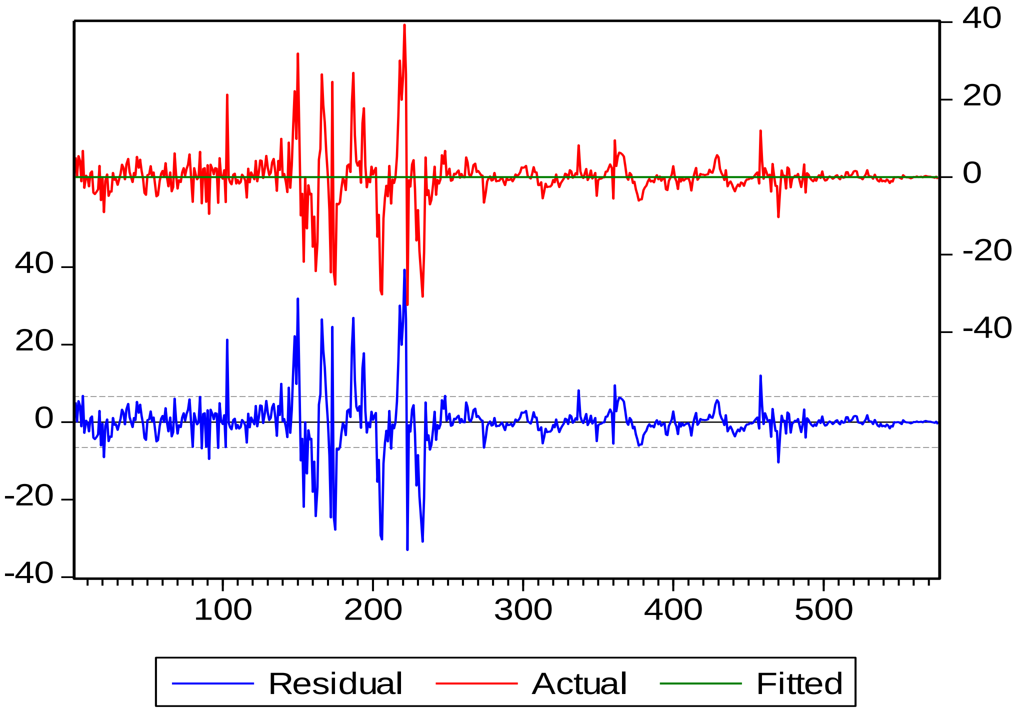
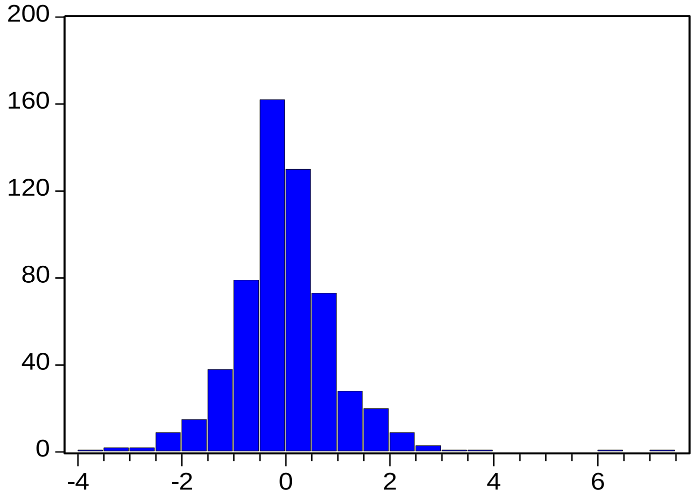


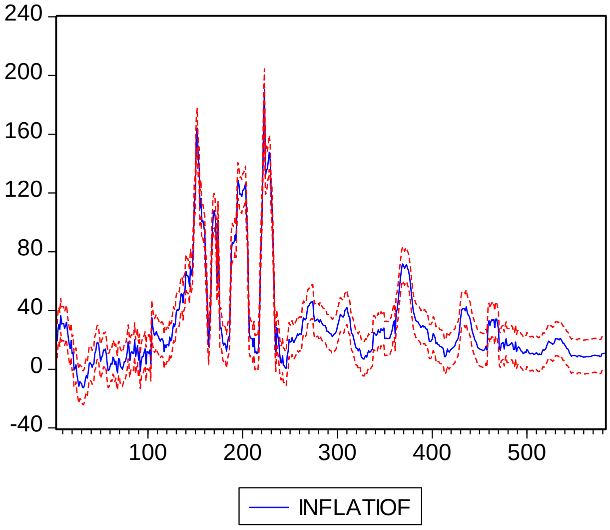
Figure 2C: Histogram of the Standardised Residuals of the ARIMA (2,1,1)



Series: Standardized Residuals
Sample 2 576
Observations 575

Mean	0.002032
Median	-0.039047
Maximum	7.175665
Minimum	-3.559597
Std. Dev.	1.002809
Skewness	0.962002
Kurtosis	10.11439
Jarque-Bera	1301.328
Probability	0.000000

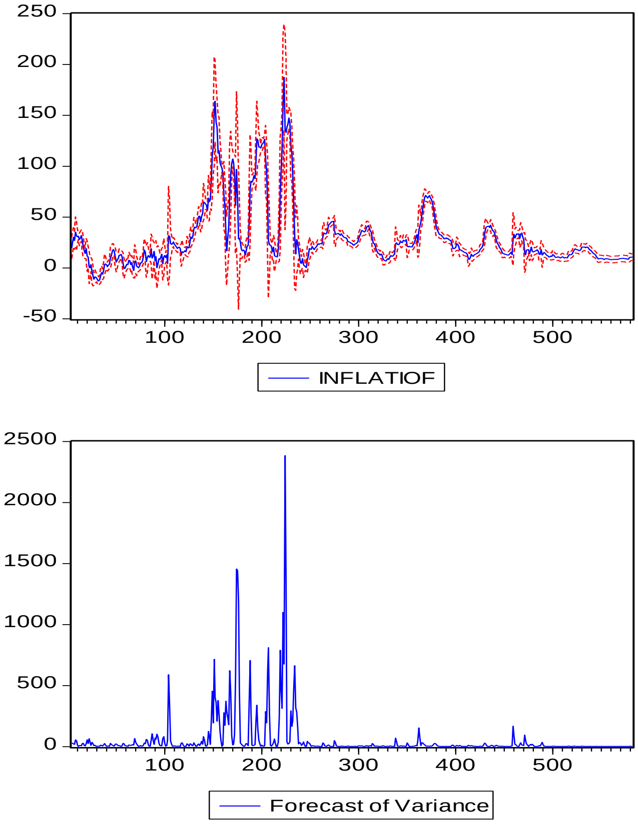
Figure 3C: Static Forecast Graph and Performance of ARIMA (2, 1, 1).



Forecast: INFLATIOF
Actual: INFLATIO
Forecast sample: 1 582
Adjusted sample: 4 582
Included observations: 579

Root Mean Squared Error	5.667538
Mean Absolute Error	2.852212
Mean Abs. Percent Error	16.12051
Theil Inequality Coefficient	0.066034
Bias Proportion	0.000000
Variance Proportion	0.012588
Covariance Proportion	0.987412

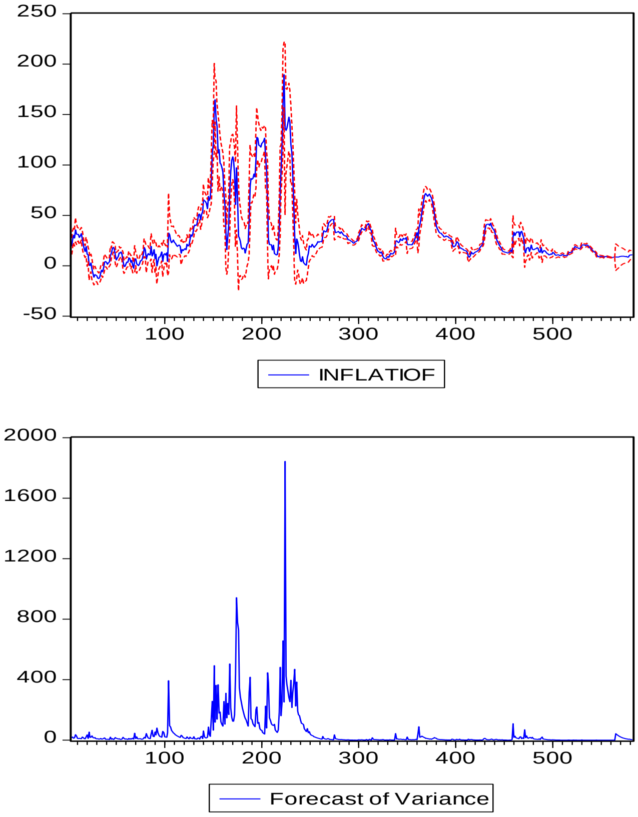
Figure 4C: Static Forecast Graph and Performance for ARCH (2)



```
Forecast: INFLATIOF
Actual: INFLATIO
Forecast sample: 1 582
Adjusted sample: 4 582
Included observations: 579

Root Mean Squared Error          5.748092
Mean Absolute Error              2.892042
Mean Abs. Percent Error          15.88456
Theil Inequality Coefficient     0.067110
    Bias Proportion              0.000195
    Variance Proportion          0.006589
    Covariance Proportion        0.993216
```

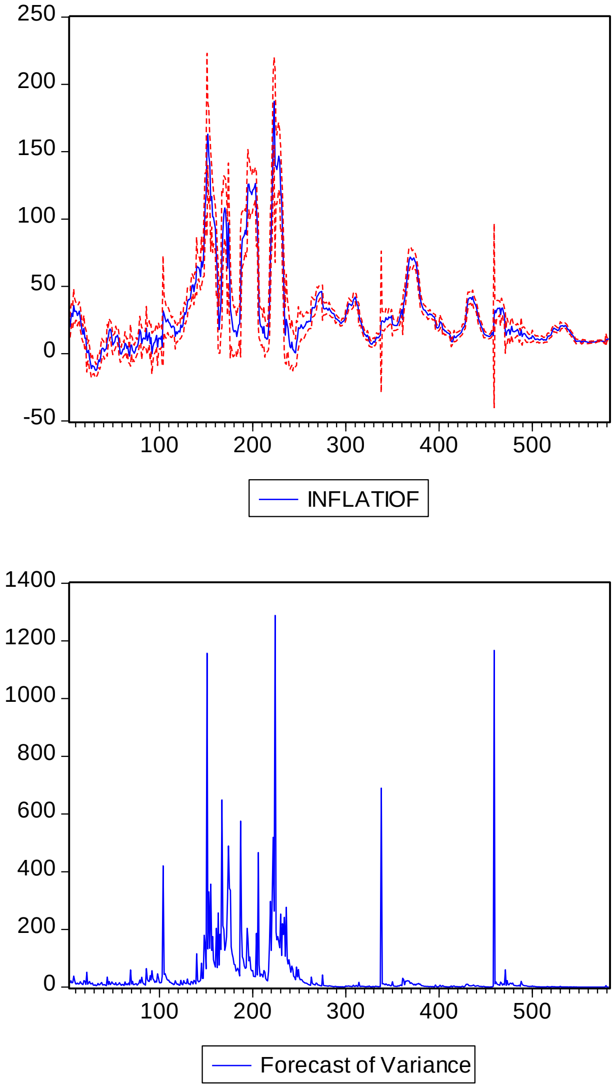
Figure 5C: Static Forecast Graph and Performance for GARCH (2, 1)



```
Forecast: INFLATIOF
Actual: INFLATIO
Forecast sample: 1 582
Adjusted sample: 4 582
Included observations: 579

Root Mean Squared Error          5.701873
Mean Absolute Error              2.866906
Mean Abs. Percent Error          15.90991
Theil Inequality Coefficient     0.066466
    Bias Proportion              0.000010
    Variance Proportion          0.010760
    Covariance Proportion        0.989230
```

Figure 6C: Static Forecast Graph and Performance for EGARCH (2, 1).



```
Forecast: INFLATIOF
Actual: INFLATIO
Forecast sample: 1 582
Adjusted sample: 4 582
Included observations: 579

Root Mean Squared Error       5.786916
Mean Absolute Error           2.880496
Mean Abs. Percent Error       15.70252
Theil Inequality Coefficient  0.067412
     Bias Proportion          0.000137
     Variance Proportion      0.010423
     Covariance Proportion    0.989440
```