ARBITRAGE OPPORTUNITY IN THE GHANAIAN STOCK MARKET AN ARFIMA APPROACH

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DECLARATION

This is to certify that, no part of this work has been submitted to any academic institution elsewhere for the award of a degree and this thesis is a result of my own research with the guidance of my able supervisors.

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ABSTRACT

Most of the methodologies employed in analyzing stock time series data are based on the assumption of Efficient Market Hypothesis which does not assume long range memory or dependence in the data generating process. However empirical evidence from stock data fails to support the lack of dependence especially in developing countries. This study investigated the long range memory in some selected equities on the Ghanaian stock market using non-parametric and parametric methods. Using the fact that, markets that are described by fractional Brownian motion possesses an arbitrage opportunity, an ARFIMA model which is a discretized version of fractional Brownian motion was fitted to the selected equities to investigate the presence of memory. The study found long memory in most of the stock returns of the equities selected. The study also explored the presence of long memory in the absolute return and squared returns. The study found memory in the absolute and squared return which was in general, lager than the return series. The long range memory in the absolute return and the square return can be used by herd fund managers in forecasting of future returns.

DEDICATION

I dedicate this work to my mother Ama Mesrɛnyame who also served as my father in my life.

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ACRONYMS

ARFIMA: Autoregressive Fractionally Integrated Moving Average.

ARIMA: Autoregressive Integrated Moving Average.

MLE: Maximum Likelihood Estimation.

QMLE: Quasi Maximum Likelihood Estimation.

I(d): Integrated process of order d.

EMH: Efficient Market Hypothesis.

MAPE: Mean Absolute Percentage Error.

EQUITIES

BOPP: Benso Oil Palm Plantation.

CALbank: CAL Bank Limited.

ETI: Ecobank Transnational Incorporation.

FML: Fan Milk Limited.

GCB: Ghana Commercial Bank Limited.

GGBL: Guinness Ghana Breweries Ltd.

GOIL: Ghana Oil Company Limited.

HFC: HFC Bank Ltd.

MLC: Mechanical Lloyd Company Ltd.

PBC: Produce Buying Company Ltd.

SOGEGH: Societe Generale Ghana Limited.

SIC: SIC Insurance Company Limited.

TOTAL: Total Petroleum Ghana Ltd.

UNIL: Unilever Ghana Limited. UTB:

UT Bank Limited

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CHAPTER ONE

1.1 INTRODUCTION

In this chapter, a general introduction of the study in terms of the background, problem statement, objectives and also a summarized version of the methods that was used in the study is given. This chapter also presents the scope of the study.

1.2 BACKGROUND

As it has always been, the main interest of investors and traders is to maximize profit that is, returns obtained from an investment in a field where almost everything is time varying. In financial time series, fluctuations are ubiquitous and financial researchers and investors are keen in understanding these fluctuations. The fluctuations are caused by infinite number of factors including past issues, present and even future anticipated issues. Given all these factors, it is incumbent on the analyst to make assumption about the cause of the fluctuations and try to model it. There are a lots of assumptions considered by traders when considering investment. Mathematical statisticians have tried to model the uncertainties in financial time series since mankind started generating financial data based on assumption of the data generating process. The popular random walk theory was applied to stock data in the early 20th century by Bachelier (1900), and since then most researchers have applied this assumption to stock data. Over the last fifty years, the studies of stock prices and other economic variables have been based on the assumption of Efficient Market Hypothesis (EMH) which agrees with the random walk and was first presented by (Fama, 1965). This is a hypothesis based on the assumption that, all information there is to know about the market is available to every player in the industry and the market is self-open with high competition. This hypothesis means a trader cannot make any much more profit but the average of the return simply because, all the information needed to make any future prediction is contained in the current price of a stock which is available to all traders. There has been a lots of critique of the EMH since its introduction even though it still remains the choice of most researchers in finance. The EMH assumption is contrary to the assumption made by the technical analyst who studies the past stock prices to predict future observations and the fundamental analyst who believe that the study of information can help an investor to improve gains in stock returns. The argument against the EMH can be seen in two folds. The first group of critiques uses psychological and sentimental behaviour of the market as an argument against the EMH. Whereas the other argument against the EMH has been empirical evidence using stock data that is, if the market is indeed efficient, then the short term correlation of indexes will be zero. However, this independence in stock prices failed to stand the test when empirical data was tested indicating some level of dependence in financial data. The question sometimes posed by some researchers and financial analyst is that, is there any way to make much money in a market that has unlimited competitors? This question has inspired researchers to look for many possible ways to beat the market and make more profit. Long memory in security data if it exist can be used as a form of arbitrage opportunity by arbitragers where they will sell securities whose prices will fall and buy those whose prices increase as they use the long memory property as a tool of prediction. Long term correlation or dependence and its application in time series analysis was introduced by Hurst in the 1950's when he studied the dependence of the flow of the River Nile using fractal dimension. (Peters, 1991, 1994), introduced the theoretical framework for fractal market hypothesis. His empirical studies was consistent with the claim that Gaussian statistics could not be applied to stock return because their distribution showed fatter tail than the normal distribution, he concluded that the data for the returns from the equities on the stock market does not follow the random walk process but rather a believe that the return has some degree of correlation for which investors and traders can use as a gateway for potential prediction. Investors have always been interested in understanding the movement of stock prices so as to beat the market. For this reason, investors and traders are the largest group of people who are interested in understanding the correlation structure of stock returns in quest of making more profit.

Many researchers have recently studied the correlation structure of markets to receive an insight into their behaviour in both developed and emerging markets. (Peters, 1994);(Ding & Granger, 1996) (Baillie, 1996); (Baillie & Chung, 2002) studied different markets and found out that the markets possess long range memory property with different degrees. In fact, (Man Lui & Chong, 2013) aside their empirical evidence in support of structural correlation, provided empirical evidence in support of skilled traders outperforming beginner traders suggesting that there exist methods of beating the market. This in fact gives the support that the markets behaviour is not a random walk process but there exist some structural and systematic pattern that can be utilized by experienced traders. Even though the EMH produced mixed results, the use of models that accounts for long range memory are not the automatic alternative to the random walk models when the assumption of EMH is violated. This is because, the applicability of the long range memory models are dependent on the existence of a long range memory in the data. For example, according to (Ding, Granger, & Engle, 1993), absolute returns of the daily S&P 500 index have in most cases, positive autocorrelation lasting over ten years. (Cheung & Lai, 1995) explored for evidence on long memory in some international markets using the Morgan Stanley Capital International stock index data for eighteen countries and was not able to find a significant evidence of long memory. Similar results was obtained by (Crato, 1994) when he analysed data from G-7 countries based on the Maximum likelihood estimation method. These and many more literature on long range memory property of stock data for developed countries have made interesting revelations about the fractal dimension of stock market of developed market. However, the findings of these studies are likely to be different for emerging markets. The works of (Costa & Vasconcelos, 2003) studied the Ibovespa index of the São Paulo Stock Exchange and found the market to be displaying a memory effect which was found to last for up to six months. The Greek stock market has also been studied by Barkoulas, Baum, and Travlos (2000) and (Panas, 2001). Using the spectra regression, Barkoulas et al. (2000) found a robust estimate of long memory parameter in the market whereas Panas (2001) used the Levy's index to establish the presence of long memory in the Greek Market. (Rege & Martín, 2011) discovered pronounced long memory effects in the stock market of Portugal using the Hurst exponent. The evidence of long memory is not difference in the developing markets of Asia as was found by (Sadique & Silvapulle, 2001). They found that, there was evidence of persistent long memory in emerging markets of Korea, Malaysia, Singapore and New Zealand but they were not able to find long memory persistence in developed markets such as USA, UK and Japan. The results from these findings suggest that there are differences in terms of long memory in emerging markets and developed markets.

Financial reforms in developing countries including Ghana are aimed at moving the market from one that is based on banking to securities which has led to rapid growth in the interest of investors in stock because stock trade gives more growth to an economy.

Since the late 1980's when the Ghana Stock Exchange was established and its subsequent operational activities from 1991 up until recent date and the large momentum gained since then, has motivated considerable academic inquisitiveness and interest about the relevance of its operations to Ghana's developmental process (Osei, 2005). With the boom in the stock trade and the rapid change in the Ghanaian market, most investors are keen to knowing the behaviour of the Ghanaian stock market. This require clear understanding of the process that generates the Ghanaian financial time series. In the quest to understand the dynamics of the Ghanaian stock market, Kallah-Dagadu (2013) described the Ghanaian data as skewed and heavy tailed as with most stock data. (Nortey, Asare, & Mettle, 2015) modelled all-index on the Ghana stock exchange using Extreme value theorem. There is almost no literature on the fractal demission of the equities that has been listed on the Ghanaian stock market. This research intends to investigate the Ghanaian stock market whether it has a long memory property and fit a long range memory model to some selected equities on the Ghanaian stock market to investigate the existence of a long range memory in the market.

1.3 PROBLEM STATEMENT

Uncertainties are always present in economic variables for which the Ghanaian equity market is no different. More often, Investors and traders are always interested in understanding the uncertainties around the economic variable and use it as a guide in their usual activities. In the early 20th century, (Bachelier, 1900), modeled stock returns as a random walk where the return was assumed to follow a Brownian motion. Due to the unrealistic nature of his assumption (that is allowing negative price), (Samuelson, 1965) corrected the model proposed by Louis Bachelier by allowing the log of the return to follow a Brownian motion. This type of model is referred to as Geometric Brownian Motion. Since then, financial analysts have used this assumption to study stock data. However, the Geometric Brownian Motion method of modelling stock return is based on the assumption that return time series data are independent of each other which is the Efficient Market Hypothesis. This means that in the long run, one can only have the expected return and cannot use old information or prices to gain any advantage of predicting the future return. The violation of the assumption governing the Geometric Brownian Motion has led to studies of other models for analyzing stock returns that can be robust to these violations. Even though the Ghanaian stock market is quite young, there has been a number of research to understand the behaviour of the stock prices and their return. Most of the research studied the behaviour of the stock return based on the assumption of the Efficient Market Hypothesis and the Geometric Brownian Motion assumption. This research aims at investigating the fractal dimension on some selected equities and their use as a long range memory property in analyzing stock return.

1.4 STUDY OBJECTIVES

The main study objective is to investigate how stock market data in the Ghanaian market can be modelled using a long range dependence model.

Specifically, this study seeks;

1) To assess the presence of long memory in the selected equities.
2) To model the stock price of some equities listed on the Stock Exchange if objective (1) above is applicable
3) To compare the long range memory model to a short range model.

1.5 SIGNIFICANCE STUDY

There has been some literature on modeling of financial time series data using models that rely on the assumption of random walk in the Ghanaian market while some researchers have studied the performance of stock return. Little is known about the long range property and its use as an arbitrage in the Ghanaian stock market. This research seek to broaden the spectrum of assumption guiding the data generating process governing the stock returns of the equities on the Ghana stock market. The results from this study will allow financial analyst to use the long range dependence property if they exist in the market to improve on their profit.

1.6 METHODOLOGY

In this thesis, the ADF and the KPSS test of stationarity were used to obtain a general idea about the data generation process. Graphs of spectra density function were also used to explore the data for the presence of long range memory in the selected equites. The study employed the Modified rescaled range statistic (M-R/S), which is a non-parametric method of investigating long range property to classify the selected equities as possessing long memory or not. Then, a parametric Autoregressive fractional integrated Moving Average (ARFIMA) was used to simultaneously model both the long range and short range memory of the selected equities. The study used the Quasi Maximum likelihood (an approximation of the likelihood function) to estimate both the short range and long range dependence parameters.

1.7 SCOPE OF THE STUDY

In this study, the focus was on investigating the presence of long memory dependence in the Ghanaian stock market. The study analyzed weekly return data from some selected equities listed on the Ghana stock market from Jan 2010 to May 2017. Daily closing prices was downloaded from the Ghana Stock exchange website and weekly prices were selected as the price of every Fridays.

1.8 ORGANIZATION OF STUDY

The study has been organized in five chapters. The introduction of the study is given in the first chapter and the second and third chapter gives related literature and methodology respectively. In chapter four, the analysis is given. The summary and recommendation of the study is given in chapter five.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

This part of the study dwells on review of literature that are related to the study of long range memory time series data and its application to stock markets. This chapter will also review the history of the Ghanaian stock market. Then a review of statistical models for modeling stock data will be given including works that have been done on the Ghanaian stock market. A detailed literature on the use of long range memory models (ARFIMA) will be given in terms of its background, testing and parameter estimation and its application to stock data both in developed and emerging markets including Africa.

2.2 THE GHANAIAN STOCK MARKET AND RESEARCH

The gain of independence in 1957 of Ghana from its colonial masters the Great Britain, saw the new birthed sub-Saharan country move to a middle-upper class nation with a $400 dollars per capita income. However, the economic mismanagement coupled with several coup d'état few years after independence plunged the economy into chaos requiring the intervention of the World Bank. The government implemented the World Bank's Structural Adjustment Program (SAP) that was aimed at liberalizing the economy allowing the state to promote the market approach to development. The SAP in 1983 saw the birth of the stock market and trade in Ghana when the Ghana Stock Exchange (GSE) was incorporated in 1989. The emergence of the GSE brought a lots of economic boom into Ghana which was acknowledge by the World Bank when it described Ghana's situation as a success and a model case for the other countries of Africa. With the great success of the Ghana stock market, the curiosity of researchers and traders to understand the market has also increased greatly. Kuwornu and Owusu-Nantwi (2011) studied the relationship between some microeconomic variables and stock returns. Using maximum likelihood estimation method, the study found a significant relationship between consumer price index, exchange rate and treasury bills whereas price of crude oil was found not to affect stock returns. Adu (2012) argued that, in addition to the microeconomic factors found by (Kuwornu & Owusu-Nantwi, 2011) that affect stock returns, prices of crude oil was also found to be affecting the stock market returns. This result was obtained when he used arbitrage pricing method to explain the variation in the Ghana stock market and used Ordinary least square regression combined with co-integration to study both the short term and long term relation of the variables.

Kallah-Dagadu (2013) had studied the empirical distribution of weekly returns of indexes on the Ghanaian stock market using different parameter estimation method including maximum likelihood approach of estimating the parameters of the α-stable distribution. The study concluded that the weekly return were heavily tailed and asymmetric based on the MLE estimate which produced efficient estimate. Nortey et al. (2015) fitted a model to the tails of daily stock return of all share indexes on the Ghanaian stock market using Extreme Values Theorem. They found the daily stock data to be fat tailed and asymmetric. The study concluded that, Peak over Threshold approach of modeling extreme Generalized Pareto Distribution is very efficient in modeling extreme event on the Ghanaian stock market. The volatility of some selected equities on the Ghanaian stock market has been studied by (Omari-Sasu, Frempong, Boateng, & Boadi, 2015) where they used the GARCH model. The study concluded that even though there was volatility in the returns there was no persistence in the volatility of the analyzed returns.

2.3 STATISTICAL MODELS FOR STOCK MARKETS

In recent decades, there has been some imperative changes in the activities of stock markets behaviour as technology, communication and even life style of people are changing.

Statistical models for stock markets have also metamorphosed from using basic assumption to higher sophisticated assumption so as to keep up with the ever changing behaviour of stock markets.

Academicians and researchers have tried to understand the predictability of stock indices and prices as it throws more light on the dynamics of the market. The first application of statistical model for stock market was proposed in the early 20th century by (Bachelier, 1900) when he described the random characteristic of the behaviour of stock prices using mathematical equations and since then, stock markets have been studied with different statistical models. Generally, the models applied to stock data can be categorized into, Time Series Forecasting, Technical Analysis, Fundamental Analysis and Machine Learning Methods. Basically technical analysis is a method where the prediction of stock prices and volumes are based on studying the past market data using patterns from chats to describe the historic data. Technical analysis is believed to have been first used by Joseph de la Vega in his account of analysis of the Dutch stock exchange in the 17th century ("The Origins of Technical Analysis on the Foreign Exchange Market," 2017). In Technical analysis, instead of looking at other external drivers of the stock prices such as, fundamental and news events, technicians believe that the market's price is a reflection of all information that is relevant to stock price determination and hence the history of the security's trading pattern is used to inform the price of the stock. Because the behaviour of investors and traders tend to follow a repeated pattern causing the prices to repeat, technicians focus on identifiable trends and conditions. In recent times, Murphy and Kalayjian (2015) describes new invention of technical trading where a computer-implemented method is used to provide pre-programmed component that receives alert strategy and represent a diagram of the strategy on the screen and validating the strategy using real-time charts and diagram. When given trading instrument, the program executes the trading strategy.

Fundamental analysts study stock prices using the intrinsic value of a stock where they invest in a security if the estimated current value is lower compared to its intrinsic value. The theoretical framework had been studied by (Bernard, 1994) and in recent years has been applied by (Wafi, Hassan, & Mabrouk, 2015). Figurska and Wisniewski (2016) also studied the prospect of applying fundamental analysis to real estate markets based on the principle of the existing capital market.

The Machine learning methods uses samples and traces patterns in the data so that the underlying function that generates the data can be approximated. The patterns may be linear or non-linear.

Pérez-Rodríguez, Torra, and Andrada-Felix (2005) studied non-linear models for stock prediction by investigating Smoothed Transition Autoregressive models(STAR), Nearest Neighbour (NN) and Artificial Neural Networks (ANN) in terms of their ability to do out of sample forecasting over a one year horizon. The paper concluded that the non-linear models outperformed the linear autoregressive and the random walk models. Jasemi, Kimiagari, and Memariani (2011)studied a modern neural network model for stock market mining and for the technical analysis of Japanese candlestick. Their method suggests a regression model such that the independent variables are vital indications and factors which are used as technical analysis patterns and uses the trend of the market in the near future as it's dependent variable.

In Traditional Time Series analysis, researchers and traders try to obtain linear predictive models that are able to trace patterns in historic data. The classification of these models is based on the number of stock data under investigation as either univariate or multivariate. The Box-Jenkins ARIMA models have been applied to stock data by Ariyo, Adewumi, and Ayo (2014) using published data from Nigeria stock Exchange (NSE) and New York Stock

Exchange (NYSE). Mondal, Shit, and Goswami (2014) also studied the effectiveness of ARIMA model in stock prediction using fifty six indexes from the Indian stock market.

The use of box Jenkins models only allow for modelling time series data that are generated by short memory process that is an exponential decay of the autocorrelation function. The remaining sections of this chapter are dedicated to long memory time series model.

2.4 ARFIMA

The phenomenon of long range memory was known long before the development and application of stochastic models to time series as scientist and statisticians in diverse study areas observed that, some empirical autocorrelation functions decay slower than would be expected under the ARMA assumption. The first statistical study of Long memory process dates as far back in 1951 when Hurts proved long range memory in the Nile River (Hurst, 1951). However, the idea of fractional integration was formally introduced by Granger and Joyeux (1980). They introduced a term of infinite filters that corresponds to $(1-L)^d$ such that when applied to a white noise, a time series of an interesting characteristics especially in the low frequency was obtained and this was a good characteristics for predicting long memory process. Hosking (1981) also defined fractionally integrated process as an infinite binomial expansion in the powers of the Backward-shift operator and showed that fractionally integrated process possesses long range memory. However the use of fractionally integrated models as long range memory processes became popular in financial data in the 1980's when (Geweke & Porter-Hudak, 1983) published their work on fractionally integrated models. In their work, they generalized the definition of fractionally Gaussian process. They also proposed an estimator for the long range memory parameter where they performed a regression on the log periodogram based on a deterministic regressor. This estimator was obtained as the ordinary least square estimate formed using just the lower frequency ordinate in the periodogram. Parke (1999) showed a simple construction of an error duration model that could generate fractional integration and long memory process. The work showed that, an error-duration model exist for the usual ARMA models which could be used as alternative models to explain autoregression in terms of the dynamics of persistence of a time series data.

2.4.1 ESTIMATION AND TESTING

As usual with statistical models, parameter estimation and testing is an important stage in the context of model development and application. The development of long memory models with its application in time series analysis came with its own challenges in terms of which parameter best determines the long range parameter and their asymptotic properties. After the work of Hurst to demonstrate the existence of long range memory in time series of the hydrological data from the River Nile, the amount of research work to estimate the degree of fractional integration increased greatly ranging from parametric to non-parametric.

The R/S statistic was proposed by Hurst as a measure of long range parameter when he studied the flow of river in the Nile but was latter modified by the work of (Mandelbrot, 1972) which was used to analyze financial time series by (Booth, Kaen, & Koveos, 1982) and concluded that there exist long range memory in the analyzed time series data. However Lo (1989) also modified the R/S statistics proposed by Hurst where he added correlations between lags to the standard deviation. The use of non-parametric method for testing of fractionally integrated process has received quite a lots of attention from academicians and researchers in terms of long range memory process. Lobato and Robinson (1998) proposed a non-parametric approach of establishing the presence of long range memory in time series data. Their method was proposed to test the null hypothesis of an I(0) process against an I(d) process where the d is allowed to be real using a non-parametric approach that made no assumption on the spectra density of the time series data. Crato and Ray (2000) also studied another version of the Hurst R/S statistics when they studied the memory of returns and volatility of future contracts. They used a biased corrected version of the R/S statistic, a non-parametric test and spectra regression estimate of the long range memory. In their study, even though they found no evidence of long range memory in future returns, there was overwhelming evidence of long range memory in the vitality of future returns. Giraitis, Kokoszka, Leipus, and Teyssière (2003) have also in recent times proposed a new method of testing the presence of long memory by using the rescaled variance statistic and since has been a good estimator of long range memory in time series analysis. This estimate has pretty simple asymptotic distribution and has a good balance of size and power compared to the modified R/S statist proposed by (Lo, 1989).

In the semi-parametric paradigm, there are several of the fractional integration estimators which has been proposed by different researchers both in time and space domain. The semi-parametric estimator proposed by (Geweke & Porter-Hudak, 1983) obtained regression of the log periodogram at zero frequency. (Hassler, 1993) supplemented the proof of the log periodogram suggested by Geweke and Porter-Hudak by assuming a Gaussian distribution. They found that, using the smoothed periodogram method produces estimate of the difference parameter with variance that vanishes faster compared to the pure periodogram regression method. Even though the work could not establish the asymptotic properties of the difference estimator based on this method, a computer experiment showed that the smoothed periodogram regression may be superior to pure periodogram regression.

(Robinson, 1994) also published his work titled “semi-parametric analysis of long range memory time series” where he studied semi-parametric inference of long range memory time series that are covariance stationary with spectrum which vary regularly at the origin. The statistics used in the work was the discretized average periodogram which was based on the degenerating band around the frequency as opposed to a narrow band used by (Geweke & Porter-Hudak, 1983). The consistency of the proposed fractional difference parameter was established under mild condition. Moulines and Soulier (1999) also improved on the work of Robinson by building a model of the spectra density of the process for all the range of the frequency. They estimated the memory property of the spectra density by using truncated Fourier series estimate of the spectra density which they called broadband estimator. With little or mild assumptions, the asymptotic of the broadband estimator was shown to be normal. In their work, they showed that, the broadband estimator attained an asymptotic mean square error of $\odot\left(log\left(n\right)/n\right)$ which outperformed all then semi-parametric estimators of long range memory parameter.

The use of wavelet analysis in the study of long range memory time series data has also received a considerable attention in literature. As an alternative to the periodogram approach by (Geweke & Porter-Hudak, 1983), Jensen (1999) proposed a new paradigm of long range parameter estimation by using wavelet analysis. He transformed the long range memory process by using a wavelet transform. Wavelet is also a functional transform that works in the same spirit as the Fourier transforms but with interesting properties that allow the efficient identification of the long range memory or the short range memory. He established a log linear relationship between the wavelet coefficient variance and the scaling parameter. When he replaced the population wavelet coefficient with the sample wavelets coefficients, the ordinary least square estimator of the long range memory parameter was found to be consistent compared to the Geweke and Porter-Hudak estimate.

Whitcher and Jensen (2000) studied the case of long range memory where the parameter is time varying based on the time scale using the wavelet analysis. Again they used the log linear relationship between the wavelet coefficient and the scaling parameter to measure the long range memory in the time series. The wavelets are able to capture the time varying property of the locally stationary long memory process due to the fact that wavelets are localized in the time domain and due to the fact that the wavelets are localized in the scale allows the capture of the self-similar property exhibited by the time series. Whitcher and Jensen used these two characteristics of the wavelet transform to obtain an approximate relationship which was log linear in nature between the time varying variance of the wavelet coefficient and the wavelet scale which was proportional to the local long range memory parameter.

2.4.1.1 PARAMETRIC METHODS

The fractional difference parameter has been studied and estimated using parametric approaches. Most of the parametric methods of estimating the fractional difference parameter uses the maximum likelihood estimation (MLE) method. Hipel and McLeod (1978) discussed the computational requirement to be considered in the maximum likelihood estimation of the long range memory parameter opening the window of MLE estimation in long range memory process. The parametric estimation methods of the long range memory process are usually referred to as frequency domain methods which was first studied by Fox and Taqqu (1986) when they defined a strongly Gaussian time series in terms of its spectra density and defined and obtained the parameters that maximizes the spectra density. Sowell (1992) obtained the unconditional exact likelihood function of a time series that is fractionally integrated and stationary which could allow simultaneous estimation of the parameters using the exact likelihood function. Sowell used cholesky's method of decomposing the covariance matrix obtained where he expressed the covariance in terms of the parameters. However, the cholesky's algorithm may fail in some situation especially for large samples. In that case, the Levinson–Durbin algorithm proposed in the work of (Levinson, 1947) and Durbin (1960) has been used to compute the autocovariannce matrix of time series data by many researchers including (Pham & Le Breton, 1991); (Palma, 2007). The Levinson–Durbin algorithm is basically a numerical procedure that was designed to exploit Toeplitz structure of the variance–covariance matrix of a second-order stationary process. In recent time Baillie and Chung (2002) have studied the split method of estimating the autocovariance matrix. In the split method of autocovariance approximation, there need not be a restriction on the roots of the autocorrelation function.

The Exact maximum likelihood method is computationally expensive when trying to compute the inverse and the determinant of the variance-covariance matrix and this have inspired research in other approximation of the likelihood function. Estimate obtained from approximating the likelihood function are denoted as Quasi Maximum likelihood estimators. The Autoregressive (AR) approximation has been employed by (Beran, 1994);(Stoffer & Shumway, 2000) and (Bhansali & Kokoszka, 2003). The AR approximation has also been adopted by Haslett and Raftery (1989) to estimate the parameters of the Autoregressive Fractionally Integrated Moving Average process and it is this method that is used for parameter estimation in this thesis.

2.4.2 ASYMPTOTIC

As parameters are proposed in statistical models, investigation of their asymptotic properties are necessary for inferential statistics. This subsection is dedicated to literature that have studied the asymptotic properties of long range memory parameters.

De Jong and Davidson (2000) introduced the first functional central limit theorem for processes that are fractionally integrated using general assumptions where he used the usual statistics that is, the deviations of the statistic from the true parameter. This statistic used, involves the sample covariance of the stationary process normally referred to as the regressors and the stationary series called the disturbances which when weakly related, has their normalized limit as stochastic martingales but when the disturbance is a long memory process, the partial sum could not converge to a martingale. Davidson (2004) derived weak limit theorem for fractionally integrated process.

2.4.3 APPLICATION OF LONG RANGE MEMORY

Long memory time series processes have been applied in hydrology, climatology, physics and other fields of study.

The use of ARFIMA models to study long range memory process in stock dates back to the work of Granger and Joyeux (1980). Literature from works that study stock market using long memory process have mixed results on long memory in the analyzed stock markets with some finding evidence of memory whiles others do not. There has been quite a number of research works that studied long memory in stock data for most developed markets. McKenzie (2001) studied the Australian stock market where he used the rescaled Range (R/S) analysis to investigate whether there was presence of long memory in the market. Cheung and Lai (1995) had also given evidence from a viewpoint of long range memory models using Morgan Stanley Capital International stock index data for eighteen countries. When the R/S statistics was used, there was no evidence of long memory in the analyzed stocks however when the GHP estimator was used, the study found evidence of long range memory for the Australian, Belgian, Italian, Spanish and Japanese stock market. Henry (2002) also studied nine international stock market for the presence of long range memory using a combination of parametric and semi-parametric methods. The ARFIMA model found four of the international stock markets to possess memory whereas there was no evidence of memory in the rest of the market. Kasman, Kasman, and Torun (2009) concluded that, there are more evidence of long memory in developing markets than there is in developed market.

Kiliĉ (2004) studied the stock market of Istanbul by exploring the existence of long range memory of the return, square and also the absolute return using semi-parametric and parametric methods. The GHP estimator of the long range memory parameter was used to investigate the presence of memory in the stock Index. The results from the study showed that the return data had no long memory. However when he applied the FIGARCH method to the conditional variance, there was evidence of long range memory. Kasman et al. (2009) also studied the dual memory in stock data of some selected eastern European countries. They used a combination of long range models including ARFIMA, HYGARCH, FIGARCH and the GPH estimator. The use of the ARFIMA and GHP showed that five of the countries exhibited long range memory in their stock market. They concluded that even though the data can be modeled by both ARFIMA-HIGARCH and ARFIAM-FIGARCH, the latter gives superior out of sample prediction. They also concluded that there was evidence of long memory in both the conditional mean and conditional variance of the stock data.

Kumar (2004) studied the turn over series of the Indian stock market using the maximum likelihood method of parameter estimation by applying the ARFIMA model. The paper used some other method of estimating the long range memory parameter and found the stock market to possess long memory as the memory was robust by all the different test. Due to the heteroscedasticity in the data, he used ARFIMA-GARCH model to fit the turn over data. Aye et al. (2014) investigated the presence of long range memory in the absolute returns of the Russian, Brazilian, South African, Chinese and Indian stock market. The stock data was modeled using the ARFIMA model and they compared the out of sample prediction of the ARFIMA model to the non-ARFIMA model. The paper concluded that there is an evidence of long memory in the stock data from the five countries over different horizons.

Africa being an emerging market has few literature on long range memory in stock return and volatility associated with it. McMillan and Thupayagale (2008) studied the efficiency of the South African stock market using the parametric ARFIMA model. In the paper, they found evidence of long range memory in the volatility series whereas there was no evidence of long memory in the stock returns. Ngene, Tah, and Darrat (2017) studied seven major African markets for the presence of long range memory using the ARFIMA-FIGARCH method. They also tested for structural break using the semi-parametric Robinson test in the stock data and found evidence of structural breaks. However, when the structural breaks was introduced in the testing model and accounted for, the data was shown to be a short range memory process in all the markets.

CHAPTER THREE

METHODLOGY

3.1 INTRODUCTION

This chapter is dedicated to the methodology employed in this study. Definition of technical terms that are relevant to the scope of the study are given and the methods employed to obtain the study objectives are reviewed.

3.2 DATA COLLECTION AND SOURCE

The data used in this study is weekly returns of some randomly selected equities listed on the Ghana stock exchange. A sample of 16 of the listed companies on the Ghana stock market was randomly selected for analysis. The data was obtained from the Ghana Stock Exchange (GSE) website consisting of daily prices of the stock equities that are listed on GSE. Weekly data was taken as the closing prices of the equities on each Friday. For Fridays that happened to be a holidays, the Thursdays closing price were used.

3.3 DEFINITION OF TERMINOLOGIES

3.3.1 ARBITRAGE

A market is said to possess arbitrage if a trader or an investor can make money at no risk. It is done by exploiting the fact that some assets are mispriced where arbitragers make a portfolio that take a short position in the overpriced assets and a long position in the under-priced ones.

Theorem 1

Let S_t be a stochastic process modeling stock price which takes on values in R, if S_t is not a semi martingale, then there exist an approximative arbitrage in the class A^{si} . Where A^{si} is a set of simple and self-financing portfolio.

If S_t^H is a fractional Brownian motion with Hurst exponent $H > 0.5$, then S_t^H is not a semi-martingale which implies that there is an approximative arbitrage in set of simple and self-financing portfolio. The aim of the research is to use the discrete version of the fractional Brownian motion thus ARFIMA to look for evidence of long memory in some selected equities listed on the Ghana stock exchange.

Definition 3.1 ***TIME SERIES***

A time series is a collection of data points that are taken with an ordered index. The order of collection may or may not be regular and the data may or may not be continuous. A time series stochastic process $\{\varepsilon_t\}_{t=1}^{\infty}$ is said to be a purely random process if each ε_t is independent from all the other observations that is for ε_s *where* $t \neq s$. In most time series analysis, researchers are interested in accounting for all systematic and deterministic characteristics of the data generating process such that the residuals will have a purely random process that is constant variance and mean zero usually referred to as white noise.

Definition 3.2 ***STATIONARY PROCESS***

A time series data is said to be stationary if the statistical property of the time series does not change over time. That is for a given times or lags $k_1, k_2, \ . \ . \ . \ , k_\iota$ the joint statistical distribution of $\{Y_{k_1}, Y_{k_2}, \ . \ . \ , Y_{k_\iota}\}$ is the same as the joint statistical distribution of $\{Y_{k_{1+\Gamma}}, Y_{k_{2+\Gamma}}, \ . \ . \ . \ , Y_{k_{\iota+\Gamma}}\}$ $\forall \, \Gamma \, and \, \iota$. This definition implies that for any two observation, the joint statistical distribution does not depend on time of observation but rather the distance between the two observations.

Definition 3.3 ***WEAK STATIONARITY***

The definition 3.2 of stationarity for time series is strict and unrealistic for most application in real time data. So, the assumption is relaxed in some cases where the mean and the variance of the data do not change over time such that the covariance between two observations depends only on the time lags between the two observations. Such a time series is called weakly stationary. The thesis will adopt this definition of stationarity.

Most often than not, economic variables exhibit non-stationarity in their mean. Non-stationarity can be in the mean or the variance as time changes. It is very important to determine the most appropriate form of the non-stationarity in the data and account for it in the analysis. Time series data are investigated for the presence of unit root or non-stationarity to determine if there is the need to detrend by differencing or regressing on a deterministic function of time (lags) to render the series stationary. If the time series is not stationary, classical statistical methods and hypothesis testing based on the time series will not produce relevant result. Non stationarity in time series can be seen from the plot of the time series. Standard statistical test exist that can be used to check for a unit root in time series data. In this thesis, the Augmented Dickey-Fuller Test (ADF) proposed by (Dickey & Fuller, 1981) and the KPSS test will be used to assess the stationarity of the series.

3.4 TEST OF STATIONARITY

3.4.1 AURGUMENTED DICKEY-FULLER TEST

Dickey-Fuller test proposed by (Dickey & Fuller, 1979), is among the most known test for stationarity in time series and it is widely used by researchers to investigate the unit root property of a time series. The derivation of the Dickey-Fuller test is based on the model of the first-order autoregressive process of Box and Jenkins (1976) which is given as

$$y_t = \phi_1 y_1 + \varepsilon_t \quad for \quad t = 1, 2, \ldots, T \qquad (1.1)$$

where ϕ_1 is the autoregression parameter and the innovation term ε_t is the non-systematic component of the model with a constant variance and a zero mean. The Hypothesis test was designed to distinguish between a null hypotheses

$H_o : |\phi_1| = 1$ **(**The process is an $I(1)$ and contains a unit root and therefore it is non-stationary),

and an alternative hypothesis

$H_1 : |\phi_1| < 1$ (The process does not contain a unit root and is stationary $I(0)$)

The Dickey Fuller Test which is based on linear regression is problematic in the case when the non-systematic component in DF models is auto correlated, in this case the Augmented Dickey-Fuller test is constructed to account for the autocorrelation (Dickey & Fuller, 1981). $Eqn(1.1)$ is then transformed and the appropriate test statistic is obtained with the limiting distribution of the ADF test statistic being identical to the distribution of DF test. The ADF test statistic and its critical values are tabulated in (Dickey & Fuller, 1979) and (MacKinnon, 1991) for $T \rightarrow \infty$. However, with the advancement in computational powers of computers, the test statistics of the ADF and its p-values are easily obtained.

The original idea of ADF test was to distinguish between an $I(1)$ null hypothesis and an $I(0)$ alternative however, (Diebold & Rudebusch, 1991) and (Hassler & Wolters, 1994) showed that the ADF test is not powerful in determining an $I(d)$ process when $d < 1$ thus the ADF test concludes stationarity only if the process does not have a unit root.

3.4.2 KPSS TEST

While the ADF tests mentioned above is used in testing the null hypothesis that the data to be investigated is integrated of order one, that is $I\ (1)$. The KPSS was developed to test the exact opposite i.e. testing the null hypothesis that the time series is an $I(0)$, verses an alternative of an $I(1)$ process.(Kwiatkowski, Phillips, Schmidt, & Shin, 1992).

The KPSS test of stationarity is formulated as

$$y_t = \rho t + \xi_t + \varepsilon_t \tag{3.2}$$

Where ε_t are iid stationary process and ξ_t is a random walk which is represented mathematically

$$\xi_t = \xi_{t-1} + \mu_t$$

where $\mu_t \sim iid\ (0, \sigma_u^2)$. The null hypothesis of stationarity is given as

$$H_0: \sigma_u^2 = 0 \text{ Or } \xi_t \ is\ constant.$$

Nabeya and Tanaka (1988) developed a statistic to test the KPSS test hypothesis. The statistic is given by

$$LM = \frac{\sum_{t=1}^{T} S_t^2}{\sigma_e^2}$$

where $\widehat{\sigma_e^2}$ is the residuals variance from the regression and S_t is the partial sum of e_t given

by $S_t = \sum_{t=1}^{T} e_t^{\ 2}\ for\ t = 1, 2, 3, \ldots T$

and e_t are the residuals from a regression y_t on a constant and time trend. The asymptotic distribution of the LM statistics is given in (Nabeya & Tanaka, 1988).

In fractionally integrated process, researchers have to be careful not to use only the ADF to adjudge stationarity of a time series as they have less power in detecting an $I(1)$ and an $I(d)$ for $d \neq 0$

Baillie, Chung, and Tieslau (1996) combined the use of ADF, PP and KPSS test in adjudging stationarity of a time series that requires fractional integration and outlined the possible conclusions that can be made from investigating the stationarity of time series using these tests.

The outline is given below as;

1) If both ADF and PP rejects the null hypothesis and KPSS fails to reject, then the data generating process is considered stationary $I(0)$ process;
2) If both ADF and PP fail to reject the null hypothesis but the KPSS rejects then there is an indication of a unit root $I(1)$ process;
3) Failure to reject by all ADF, PP and KPSS is considered that the data being investigated is insufficiently informative to ascertain the long-run characteristics of the process
4) Rejection by all ADF, PP and KPSS indicates that the process is described by neither $I(0)$ nor $I(1)$ processes and therefore it is probably better described by the fractional integrated alternative.

In this thesis, the combination of the ADF and the KPSS were the stationarity tests used to ascertain which of the equities are fractionally integrated process but the interpretation still remains the same as outline above.

Definition 3.4 ***Integrated process*.**

Let y_t be a non-stationary stochastic process such that it is differenced d times to become invertible, stationary and can have an ARMA representation, then y_t is said to be an integrated series with order d represented mathematically as $y_t \sim I(d)$.

3.5 DETERMINING THE DEGREE OF FRACTIONAL INTEGRATION

In time series analysis of financial data, it is very important to check if the time series has a long range memory property as with the main aim of the research. A measure of duration of long range dependence of the stochastic process plays a very important role in the study processes. There are many methods of measuring the long range memory property of a time series. However, the best method is still a debate among researchers. This is partly because most of the available methods do not perform very well through finite sample experiment. Even though in this study, the Modified R/S statistic estimate for the Hurst exponent will be used for assessing the long range property of the time series in the preliminary analysis, a review of some other methods of measuring long term memory property is given. The method of estimating the long range memory property can be grouped as non-parametric, semi-parametric and parametric approach. The non-parametric methods include the R/S method, the M-R/S method while the Geweke-Porter-Hudak estimator and the wavelet method are classified as semi-parametric methods and ARFIMA method is classified as parametric method.

3.5.1 THE HURST EXPONENT

Estimation of the Hurst exponent for experimental data is used to test for the independence of time series and to inform on the presence of long memory or long range correlations in time series.

Harold Ewin Hurst in his quest to understand the regularities in the level of water in the Nile River introduced the Hurst Exponent as a measure of long range memory property in time series. The Hurst exponent is one of the oldest measure of long range memory in time series which has gained grounds in many field including financial time series analysis. The R/S method which uses the rescaled range statistic (R/S statistic) or the Modified R/S method will be used as the non-parametric method of estimating the long range memory in time series during the preliminary analysis.

The R/S statistic is simply defined as the range computed from the partial sums of deviations of a time-series from its mean, and then rescale it by the standard deviation. Mandelbrot (1972) outlined steps in estimating the Hurst exponent (H) as follows

1) Divide the time series into n subseries $Z_{i:m}$ of length m.
2) Find the mean E_m of each of the subseries.
3) Normalize the subseries by subtracting the mean. $X_{i,m} = Z_{i,m} - E_m$
4) Create the cumulative series $Y_{i,m} = \sum_{j=1}^{i} X_{i,m},\ for\ i = 1,2,3,,,,,,n$
5) Find the range $R_m = \max\{Y_{1,m}, Y_{2,m}, Y_{3,m}, \ldots, Y_{n,m}\} - \min\{Y_{1,m}, Y_{2,m}, Y_{3,m}, \ldots, Y_{n,m}\}$
6) Rescale the range $R = R_m / S_m$
7) Find the mean value $R = (R/S)_n$
8) The R/S statistics asymptotically follows $(R/S)_n \to cn^H$

The Hurst exponent is obtained as the ordinary least square (OLS) estimate of H from regressing the log of the R/S statistics on the log of the number of observation

$$\log(R/S)_n = \log(c) + H\log(n) \tag{3.3}$$

where $$S_m = \frac{1}{m}\left(\sum_j \left(Y_j - \overline{Y_m}\right)^2\right)^{\frac{1}{2}} \tag{3.4}$$

The Hurst exponent $0 < H < 1$ as a measure of long range property of time series has a correlation function $\rho(k)$ decaying at $\rho(k) \propto k^{2H-2}$ *with lag* $k \to \infty$. The values of the Hurst exponent indicates the level at which the time series data is fractionally integrated. When the estimated $H = 0.5$, this means there is no long memory in the time series. This has two implication that is, either the time series observations are independent in which case the autocovariance is insignificant for all non-zero lags or the process is characterized by short memory in which autocovariances are significant at low lags, insignificant at high lags and decay exponentially. If $H > 0.5$, then the time series is said to have a long range memory with positive correlation at higher lags and these time series generating processes are termed as persistent. If $H < 0.5$, the process is termed as anti-persistent and the time series will have negative correlation at higher lags.

The R/S estimate of H is known to be biased and sensitive to short range memory and heteroscedasticity (Lo & MacKinlay, 1989). Due to the sensitiveness of the R/S statistics to short term memory, if the predicted values differs from the theoretical values, it may not necessarily be as a result of long memory but rather a mistaking of short range memory as a long range memory property. This inspired (Lo & MacKinlay, 1989) to propose a method that deals with the short coming of the R/S by adding a correlation term to the computation of the standard deviation to account for the effect of short range memory in the time series. The modified standard deviation is given as

$$S_m = \sqrt{S_m - 2\sum_{k=1}^{p} \omega_k(q)\gamma_k} \tag{3.5}$$

where $\omega_k(q) = 1 - \frac{k}{q+1}$ and γ_k is the sample auto covariance.

The parameter q represents the lags in the weighted autocovariances. This method of estimating the Hurst exponent is called the modified Rescaled Range method (M-R/S)

3.5.2 SEMI-PARAMETRIC METHODS

In the semi parametric paradigm, Geweke and Porter-Hudak (1983) suggested a semi-parametric method that can be used to obtain an estimate of the fractional differencing parameter d using the slope of the spectral density function around the angular frequency ($x{=}0$) . The fractional differencing parameter is estimated as the coefficient when the logarithm of the periodogram is regressed against the function of the Fourier frequencies of the sample. The asymptotic properties of $d \in (0, 0.5)$ was proved by Geweke and Porter-Hudak (1983). In this research, the spectra density which is a Fourier transform of the autocorrelation at time lags will be used as a graphical method of looking for evidence of long range memory.

3.5.3 PARAMETRIC APPROACH

The non-parametric methods of estimating the long range memory property of a time series are used to establish the presence of a long range memory and the level of persistence in the data. However, the aim of investors and financial analyst goes beyond the knowledge of presence of long range memory in a financial time series. Rather, they are interested in using these properties to enhance their forecasting abilities.

The parametric approach employed in estimating the long range parameter in this thesis is the Autoregressive Fractional Integrated Moving Average (ARFIMA) method. The section that follows is an introduction of the ARFIMA model and then the parameter estimation stage will follow.

Before the ARFIMA model is introduced, a definition of some phenomena that are relevant to the understanding of long memory process are given.

3.6 DEFINITION OF LONG MEMORY TERMS

Definition 3.5 ***Fractional Brownian motion*.**

H is called the Hurst constant and it belongs to the interval $(0, 1)$. A fractional Brownian motion, which is denoted by B_t^H is a continuous and centred Gaussian process with covariance function

$$E\left(B_t^H B_s^H\right) = \frac{1}{2}\left(t^{2H} + s^{2H} - |t-s|^{2H}\right) \tag{3.6}$$

And the time increment of $B_{t-h}^H - B_t^H$ and $B_{s-h}^H - B_s^H$ with $s + h < s \, and \, t - s = nh$ have a covariance function $\rho H(k) = \frac{1}{2} h^{2H}\left[(n+1)^{2H} + (n-1)^{2H} - 2n^{2H}\right]$.

It can be seen that, when $H = 0.5$, the covariance function $\rho H(k)$ becomes 0 in which case B_t^H is the standard Brownian motion with independent increments.

Definition 3.6 ***Long-range dependence***

A stationary sequence $X(n)_{n \in N}$ exhibits long-range dependence if the auto covariance function $\rho(n) = cov(X_k, X_{k+n})$ such that

$$lim_{n \to \infty} \frac{\rho(n)}{cn^{-\alpha}} = 1 \tag{3.7}$$

$for \; some \; constant \; c \; and \; \alpha \in (0,1)$

This implies that the dependence between $X_k \; and \; X_{k+n}$ decays slowly as $n \to \infty$ and

$$\sum_{n=1}^{\infty} \rho(n) = \infty$$

This definition is equivalent to stating that the spectral density is unbounded at lower frequency for data that has long memory.

Definition 3.7 ***Self-similarity***

A nontrivial stochastic process which is an $\Re^d$ –valued random process $X(t)_{t>0}$ is said to be a self-similar process or satisfies the property of self-similarity if for every $a > 0$ there exist $b > 0$ such that

$$law(X_{at>o}, t \geq 0) = law(X_{bt>o}, t \geq 0)$$

Self-similarity simply means that the time series process is composed of sub-sub-units that is contained in sub-units on multiple levels that statistically resemble the structure of the whole object that is at lower scales. In financial time series analysis, this property can be viewed in the sense that, the pattern of the observation in the short term is a repetition of a lager pattern but on a small time scale. If $b = a^{-H}$ then $X = X(t)_{t>0}$ is defined as a self-similar process with Hurst index H. For a given α and H, there is direct relationship which is given by $H = \frac{1}{\alpha}$ where α is defined in $eqn(3.7)$. And this means there are correlation at higher order lags but the degree of the correlation cannot be determined from the definition.

The ARFIMA model is a generalization of the ARIMA model where the Integrating parameter is allowed to take on real values instead of integers in the ARIMA models. This section is dedicated to the review of the Box-Jenkins ARIMA models and then extended to the model which allows for fractional integration.

3.7 BOX-JENKINS ARIMA

3.7.1 AUTOCORRELATION AND PARTIAL AUTOCORRELATION

The application of ARIMA models to time series data requires the determination of the Autoregressive (AR) and Moving Average (MA) lags to be used in the model and the degree of differencing or integration d to be used. Any of such model is described by Box-Jenkin using the formal notation ARIMA *(p, d, q).*

The sample autocorrelation coefficient of a time series data is defined mathematically as

$$\rho(k) = \frac{\sum_{i=1}^{n-k}(y_i - \bar{y})((y_{i+k} - \bar{y})}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{3.8}$$

This autocorrelation coefficient gives a measure of linear relationship between observations separated by k lags in the time series. A plot of the autocorrelation coefficient against time lags is used to determine the appropriate order *p* of the model that is, the number of previous observations that should be used to estimate the current value of the process. For example, for an MA (p) process, autocorrelation is zero at lag $p + 1$ and greater.

The partial autocorrelation coefficient for k lagged observation, denoted by $\gamma(k)$, is a measure of the correlation between y_t and y_{t-k} after adjusting for the presence of $y_{t-1}, y_{t-2}, y_{t-3}, \ldots, y_{t-(k-1)}$. This adjustment is done to eliminate the partial effect of $y_{t-1}, y_{t-2}, y_{t-3}, \ldots, y_{t-(k-1)}$ on the correlation between y_t and y_{t-k} which gives the idea of the number of MA terms to be included in the analysis.

A visual inspection of the PACF and ACF plot gives an idea of whether to choose an MA or AR or an ARMA model. The model selection is based on the number of lags in the ACF and PACF that are significant and the rate at which both functions decay to zero.

3.7.2 AR PROCESS

For an ACF that tails off while the PACF shows spikes, an autoregressive (AR) model with order p which is equal to the number of significant PACF spikes is considered to be the "best" model. In this case the present value of the time series is said to be affected by the past p observations. This is mathematically written as

$$Y_t = \propto + \propto_1 Y_{t-1} +, \ . \ . \ . \ , + \propto_p Y_{t-p} + \varepsilon_t \tag{3.9}$$

Where p is the order of the AR process and $\propto_1, \propto_2, \ . \ . \ . \ , \propto_p$ are the parameters of the model and ε_t is the error terms in the model which is usually a white noise with zero means a constant variance σ^2.

3.7.3 MA PROCESS

In the case where the PACF tails off whiles the ACF shows spikes, the best model chosen is a moving average (MA) model with order q equal to the number of significant ACF. This is explained as a model where the current observation is affected by the q past shocks in the model. This is mathematically written as

$$Y_t = \propto_1 e_{t-1} + \propto_2 e_{t-2} +, \ . \ . \ . \ , + \propto_q e_{t-q} + \varepsilon_t \tag{3.10}$$

If the data generating process can be described by both AR and MA terms then this is called an ARMA model where the present values of the process is expressed in terms of the AR and MA lags.

In this case, both the ACF and the PACF trail off. An ARMA of with p and q for AR and MA respectively is mathematically represented as

$$Y_t = +\pi_1 e_{t-1} + \pi_2 e_{t-2} + . \ . \ . + \pi_p e_{t-q} + \propto_1 e_{t-1} + \propto_2 e_{t-2} . \ . \ . + \propto_p e_{t-q} + \varepsilon_t \tag{3.11}$$

with the innovations ε_t being white noise and $\pi_1, \pi_1, \ . \ . \ , \pi_p$ and $\propto_1, \propto_2 \ , \ . \ . \ . \ , \propto_q$ are the coefficients of the model.

3.7.4 ARIMA PROCESSES

The above description is for trend stationary process. But in most financial time series analysis, the data is not stationary. Hence fitting the Box-Jenkins model requires that, the analyst transforms the time series to a stationary one which can be achieved by differencing or the use of other appropriate methods and integrate the difference in the analysis. Differencing is a data-processing technique used to remove trends or seasonal components. In this step of the analysis, one simply considers the difference between pairs of observations with appropriate time separations, such as, the first difference, which is denoted as:

$$\nabla X_t = (1-L) X_t = X_t - X_{t-1} \tag{3.12}$$

For higher order differencing, an order of difference d is given by

$$\nabla^d X_t = (1-L)^d X_t \tag{3.13}$$

Single difference is used to remove linear trend whereas difference of order d greater than one is used to remove higher order polynomial trend in a time series. The differencing parameter d is the number of times the time series is differenced to obtain a stationary series. In the Box-Jenkins method, the differenced data is now fitted to the $ARMA$ model described above. Such models are termed as Autoregressive Integrated Moving Average denoted as $ARIMA(p,d,q)$ where d is the order of integration required to obtain a stationary series and p the number of autoregressive lags and q is the number of moving average lags.

So far, the models described above considered data-generating processes that are either stationary or integrated of an integer order higher than zero. However, cases where the order of integration is not an integer requires another technique to handle as the $ARIMA$ only allows for integer order integration. Also the ACF of the $ARIMA$ models are characterized by an exponential decay so in situation where the higher lags have an ACF significantly different from zero, the ARMA model cannot account for the slowly decaying ACF. In these cases, the *ARFIMA* model can be applied to allow for the slow decay of the ACF.

3.8 ARFIMA PROCESS

Let v_t be an invertible and covariance stationary process such that it can be described by an ARMA (p, q) process which is bounded away from zero at all frequency such that

$$(1-L)^d y_t = v_t \tag{3.14}$$

where $(1-L)^d$ is the fractional deference operator defined in terms of its Maclaurins series expansion. Then y_t is said to be fractionally integrated process of order d . This implies that, the application of the fractional differencing to y_t produces an ARMA process with the regular representation. When v_t is a white noise, then y_t is said to be a fractionally integrated white noise process. By applying the usual representation of ARMA to the of the fractionally integrated process v_t , the ARFIMA process can be represented mathematically as

$$\Theta(L)(1-L)^d y_t = \phi(L)\varepsilon_t \quad \text{t=1, 2,3,...} \tag{3.15}$$

where

$$\phi(L) = \left(1-\phi_1 L - . \ . \ . - \phi_p L^p\right)$$

is the moving average polynomial and

$$\Theta(L) = \left(1-\theta_1 L - . \ . \ . - \theta_q L^q\right)$$

is the Autoregressive polynomial in the lag operator L such that the roots of the MA and AR polynomials lie outside the unit circle and $\Theta(L)=0$ and $\phi(L)=0$ do not have a common root. In the ARFIMA model, p and q are integers whereas d is a real number and $(1-L)^d$ is the fractional difference operator of the *ARFIMA* process.

The interesting thing about $ARFIMA$ models is that, the d parameter is allowed to take on real values and this makes the ACF decay hyperbolically unlike the exponential decay in the $ARIMA$. For $d \in (0, 0.5)$ the process is stationary and has a finite variance whereas for $d >$ 0.5 the variance of the long memory is infinite. The method of estimation in this thesis will based on cases where $d \in (0, 0.5)$.

3.8.1 ACF AND PACF OF ARFIMA PROCESS

Consider a long memory process as defined in definition (3.14) then an integrated process of order d can be written as,

$$(1-L)^d y_t = \psi(L)\varepsilon_t \quad for\, t = 1, 2, \ldots \tag{3.17}$$

where the absolute or square summability $\psi(L)$ is given and is finite that is,

$$\sum_{j=1}^{\infty} |\psi(L)| < \infty \, and \sum_{j=1}^{\infty} \psi(L)^2 < \infty$$

Eqn (3.17) can be rearranged to obtain $y_t = (1-L)^{-d} \psi(L)\varepsilon_t$ and define a function for the scalar z.

$$f(z) = (1-z)^{-d}$$

The derivatives of $f(z)$ are

$$\frac{df}{dz} = d(1-z)^{-d-1}$$

$$\frac{d^2 f}{dz^2} = d(1+d)(1-z)^{-d-2}$$

$$\downarrow$$

$$\downarrow$$

$$\frac{d^j f}{dz^j} = (d-j-1)(d-j-2),,, \ d(1+d)(1-z)^{-d-j}$$

the fractional difference operator $(1-L)^d$ is defined as the binomial expression

$$(1-L)^d = \sum_{j=1}^{\infty} \binom{d}{j} (-L)^j \tag{3.18}$$

by making use of a power series expansion around $z = 0$ and the binomial theorem. Now, the coefficient of the sequence $\binom{d}{j}(-L)^j$ are square summable and can be expressed in terms of the gamma function.

$$\binom{d}{j}(-L)^j = \frac{\Gamma(j-d)}{\Gamma(-d)\Gamma(j+1)} \tag{3.19}$$

where Γ is the usual gamma function.

$$(1-L)^d = 1 - dL - \frac{1}{2}d(1-d)L^2 - \frac{1}{3!}d(1-d)(2-d)L^3 \ldots$$

(3.20)

The autocovariance of the process y_t is given by

$$\gamma_k = E(y_t, y_{t-k}) = \frac{(-1)^k(-2d)!}{(k-d)!(-k-d)!} \tag{3.21}$$

And the autocorrelation function is given as ρ

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{(-d)!(k+d-1)!}{(d-1)!(k-d)!} \quad k = 1,2,3,\ldots \tag{3.22}$$

$$\rho_k \sim \frac{(-d)!}{(d-1)!} k^{2d-1}$$

3.8.2 PARAMETER ESTIMATION

The Hurst exponent which is estimated by the methods described in section 3.5.1 are used to estimate the fractional difference parameter d using the relationship $H = d + 0.5$. In studies of long range memory time series, beyond the knowledge of presence of a long range memory in time series, one would want to do forecasting. This requires estimation of the long range memory parameter and the short range memory parameters to allow for data description and forecasting. There are basically two ways of accounting for both the long range memory and the short range memory: the one-stage and the two-stage method.

3.8.2.1 TWO-STAGE METHOD

The two-stage method uses any of the non-parametric estimation method to obtain the fractional difference parameter d. Then the data is fractionally differenced to obtain the ARMA process from which the autoregressive and the moving average lags can be estimated using the usual Box-Jenkins method.

The two-stage method is outlined bellow

From the definition of an $ARFIMA(p,d,q)$ process y_t, the $ARIMA(p,q)$ part can be obtained as $u_t = (1-L)^d y_t$ and also $v_t = \frac{\Phi(L)}{\Theta(L)} y_t$ is an $ARFIMA(0,d,0)$. (Hosking, 1981) gave the outline as follows:

1) Estimate d from the original time series in the $ARFIMA(p,d,q)$ and denote it d

2) Calculate $u_t = (1-L)^d y_t$

3) Use Box-Jenkins method to obtain Φ and Θ

4) Compute $v_t = \frac{\Phi(L)}{\Theta(L)} y_t$

5) Estimate d^* in the model $(1-L)^{d^*} v_t = \varepsilon_t$

The new value d^* is the values of the long range parameter in the time series

6) Step 2 through to 5 is repeated until convergence is attained for d, Φ and Θ. The convergence is usually attained after the first iteration.

The one-stage method is a parametric approach that estimates all the parameters simultaneously. In this thesis the Quasi Maximum Likelihood will be used in the one-stage parameter estimation method.

3.8.2.2 SIMULTANEOUS PARAMETER ESTIMATION (MLE)

In this section, a review of the simultaneous method of estimating the long range parameter d and the short memory part of the ARFIMA model is given.

There are many ways of estimating parameters of statistical models ranging from Moment Estimation, Least Square Estimation, Maximum likelihood estimation which are most often referred to as frequent approach to the Bayesian estimation Method.

A review of the Maximum likelihood estimator of the ARFIMA models using the proposition of (Haslett & Raftery, 1989) is given in this section. Since the seminal work by Granger and Joyeux (1980) and Hosking (1981), there has been increase in literature about estimating parameter of long range parameters. Generally, parameter estimation method can be categorized into two main domains as far as parameter estimation for long range memory processes are concerned. These categories are time domain and spectra domain. The methods in the time domain include the Exact Maximum likelihood Estimation and the quasi Maximum likelihood estimation whereas the spectra domain includes methods such as the one proposed by Whittle and some other semi-parametric estimators.

3.8.2.2.1 MAXIMUM LIKELIHOOD ESTIMATION METHOD (MLE)

The principle of Maximum Likelihood Estimation is not hard to understand as can be inferred from its name. The intuition about Maximum Likelihood Estimation (MLE) is simply to maximize the likelihood function for a given data over the range space of the parameter. For example,

Let $Y = (y_1, y_2, \ldots, y_t)'$ be a sample data obtained from a population such that we can represent the likelihood of obtaining the sample as a function of the parameter is given as;

$$L(\lambda) = f(Y) \tag{3.23}$$

Then, the maximum like estimate denoted by $\hat{\lambda}$ is the values of λ that maximizes $L(\lambda)$. More often than not, maximizing the likelihood function is hard to achieve. The log likelihood function which is simply the log of the likelihood function is monotonic and easy to maximize over the range space of λ. The maximization is done by differentiating the log likelihood function with respect to the parameter and setting the score function to zero so to solve for the parameter. In some cases, solving for the parameters in the score function is not straight forward and requires some iteration method such as the Newton Raphson method. The section that follows gives an introductory note to the Exact method of estimating the parameter with approximating the likelihood function.

3.8.2.2.1 EXACT MLE

Consider a process y_t which is Gaussian and has zero mean, then the log-likelihood function may be written as

$$l(\lambda) = -\frac{1}{2}\log\left(\det\left(\Sigma_\lambda\right)\right) - \frac{1}{2}\left(Y'\Sigma_\lambda^{-1}Y\right) \tag{3.24}$$

where $Y = (y_1, y_2, y_3, \ . \ . \ , y_t)'$ and Σ_λ is the variance of Y and λ is the parameter vector to be estimated. Then the MLE of λ is obtained by maximizing $l(\lambda)$ with respect to λ . However, obtaining the MLE is not straight forward as the MLE requires computation of the determinant and invers of Σ_λ and this is sometimes computationally expensive. There are many existing methods employed in computations such as the well-known Cholesky decomposition algorithm, the Levinson–Durbin algorithm and the Kalman filters. The Exact Maximum likelihood estimation is computationally expensive making researchers result to other method of Maximum likelihood parameter estimation approach. The method employed in this thesis is the Quasi Maximum likelihood Estimation method where the approximation of the log-likelihood function given in Eqn (3.23) is maximized. There are approximation methods for computing the likelihood function and the choice of which one to use depends on the researcher. The study reviews the Auto regressive approximation method as was applied by (Haslett & Raftery, 1989). There are other methods such as the Moving average approximation of the likelihood but will not be considered in this thesis.

3.8.2.2.2 QUASI MAXIMUM LIKLELIHOOD ESTIMATION (QMLE)

Consider a long range memory process y_t characterized by an $AR(\infty)$ expansion according to Wold decomposition of time series

$$y_t - \pi_1(\lambda) y_{t-1} - \pi_2(\lambda) y_{t-2} ... = \varepsilon_t . \ t=1,2,... \tag{3.25}$$

where $\pi_j(\lambda)$ are the coefficient of $\Theta(L)^{-1} \phi(L)(1-L)^d$. There are infinite terms in (3.25) which is in contrast to what is observed in reality where only finite $\{y_1, y_2, \ . \ . \ , y_n\}$ are observed from the process so (3.25) can be truncated with the first m terms which is given as

$$y_t - \pi_1(\lambda) y_{t-1} - \ldots - \pi_m(\lambda) y_{t-m} = \varepsilon_t \tag{3.26}$$

where $m < t < n$ the MLE estimator $\hat{\lambda}$ is the vector λ that maximizes

$$l(\lambda) = \sum_{t=m+1}^{n} \left[y_t - \pi_1(\lambda) y_{t-1} - \ldots - \pi_m(\lambda) y_{t-m} \right]^2 \tag{3.27}$$

Consider (3.17) such that we have an approximate one step predictor of y_t is given as

$$\hat{y}_t = \Theta(L)^{-1} \phi(L) \sum_{j=1}^{t-1} \varnothing_{tj} y_{t-j} \tag{3.28}$$

with prediction error defined as

$$v_t = var(y_t - \hat{y}_t) = \sigma_y^2 k \prod_{j=1}^{t-1} \left(1 - \varnothing_{jj}^2\right) \tag{3.29}$$

σ_y^2 is $var(y_t)$ and k is the ratio of the innovation variance to the variance of the ARMA(p,q) process and

$$\varnothing_{tj} = -\binom{t}{j} \frac{\Gamma(j-d)\Gamma(t-d-j+1)}{\Gamma(-d)\Gamma(t-d+1)} \; for \; j = 1, 2 \;\; , \;\; , t$$

$\varnothing_{tj}$ is said to be a partial linear regression coefficient of an $ARFIMA(0, d, 0)$.

There is a further approximation of the last term in (3.28) to avoid computation of large coefficient of $\varnothing_{tj}$

$$\sum_{j=1}^{t-1} \varnothing_{tj} y_{t-j} \cong \sum_{j=1}^{m} \varnothing_{tj} y_{t-j} - \sum_{j=m+1}^{t-1} \pi_j(\lambda) y_{t-j} \tag{3.30}$$

since $\varnothing_{tj} \sim -\pi_j(\lambda)$ for lags larger than m and $a_i \sim b_i$ means $\frac{a_i}{b_i} = 1$ as $i \to \infty$

The MLE $\hat{\lambda}$ is obtained by maximizing

$$l(\lambda) = constant - \frac{1}{2} nlog\left[\sigma_y^2(\lambda)\right] \tag{3.31}$$

Where

$$\sigma_y^2(\lambda) = \sum_{t=1}^{n} \frac{(y_t - \hat{y})^2}{y_t} \tag{3.32}$$

When $m = n$ then, this method is the same as the exact maximum likelihood estimator and

$$\lambda = \{d, \Theta, \Phi\}$$

3.9 MODELL SELECTION CRITERIA

Model parsimony is a phenomenon that simply selects any model with the least parameters at a constrain of a measure of good fit. Thus, it tells us that when there are alternative explanation of an event, the least complex one is likely to be correct. In this thesis, the Akaike Information Criterion (AIC) will be used to select the number of AR and MA lags to be included in both the ARFIMA and the ARIMA models. The AIC selects a model with the minimum Kullback-Leibler distance between the model and the truth from a pool of models. The AIC is mathematically given by

$$AIC = -2(\ln(likelihood)) + 2k \tag{3.33}$$

Where k is the number of parameters and the likelihood is defined as the probability of obtaining the data given the model. Due to sample size discrepancies, the sample size can be introduced to penalize for a more complex model with a relatively small sample size. The AIC is then given as

$$AIC = -2(\ln(likelihood)) + 2k\left(\frac{n}{n-k-1}\right) \tag{3.34}$$

From (12), as the sample size N increases $\frac{N}{N-K-1} \rightarrow 1$ so that AICc converges to AIC. For different models with the same likelihood, the smaller the AICc, the better the model.

3.10 PERFORMANCE EVALUATION METHOD

The Mean Absolute Percentage Error MAPE is a measure of prediction accuracy that is used to compare out of sample prediction of different models. As can be suggested by the name, the MAPE is computed by averaging the absolute percentage error for the predicted values. It is mathematically given as

$$MAPE = \frac{1}{M}\sum_{m=1}^{M}\left|\frac{y_t - y_t}{y_t}\right| \times 100 \qquad (3.35)$$

Where M is the number of out of sample prediction. The MAPE is a scale independent measure of accuracy

CHAPTER FOUR

ANALYSIS AND DISCUSSION

This chapter presents a step by step implementation of the methodology to analyse the data to obtain the objectives as set out by the thesis. Table 1 gives a brief description of the equities that were selected and the time range for which data was obtained for the analysis. Except for Societe General Ghana limited, for which only data for 192 weeks could be obtained, data was obtained from 04-01-2010 to 18-May-2017which spans 340 weeks for all the other selected equites.

Table 1: Selected List of companies from Ghana Stock Exchange

Symbol	From	To	No of observation
BOPP	04-Jan-2010	18-May-2017	340
CAL	04-Jan-2010	18-May-2017	340
ETI	04-Jan-2010	18-May-2017	340
FML	04-Jan-2010	18-May-2017	340
GCB	04-Jan-2010	18-May-2017	340
GOIL	04-Jan-2010	18-May-2017	340
GGBL	04-Jan-2010	18-May-2017	340
HFC	04-Jan-2010	18-May-2017	340
MLC	04-Jan-2010	18-May-2017	340
PBC	04-Jan-2010	18-May-2017	340
SIC	04-Jan-2010	18-May-2017	340
SOGEGH	04-Jan-2010	10-Dec-2013	192
TOTAL	04-Jan-2010	18-May-2017	340
UNIL	04-Jan-2010	18-May-2017	340
UTB	04-Jan-2010	18-May-2017	340

To begin with the statistical analysis, a preliminary study of the empirical distribution of the time series by investigating the mean, skewness and the kurtosis of the data is an important stage. The complete Table 2 is shown in the appendix. From Table 2, all the selected equities have positive mean except for MLC, SIC, PBC and UT Bank Limited which had negative mean. This means investors who put their monies in these stock equities will lose their investment in the long run. Goil had the highest expected return of 0.00677 whereas PBC had a mean return of -0.00198 being the equity with the highest negative return. As usual with stock returns, all the selected equities had positive kurtosis often referred to as leptokurtic with TOTAL and FML having kurtosis of 130 and 126 respectively exhibiting very heavy tails. ETI and HFC had kurtosis of 2.7 and4.6 respectively being very close to the normal kurtosis of 3. In terms of the skewness of the time series data, there is a mix of skewness with TOTAL being highly negative in terms of skewness followed by FML whereas BOPP showed the highest positive skewness. However, few of the equities showed symmetry as their skewness was not far from zero.

As was described in the methodology, the ADF and the KPSS test were used for establishing the existence of stationarity in the time series of the stock return for the selected equities. In Table 2, the test statistics and the corresponding p-values for the Augmented Dickey-Fuller test and the KPSS test are given. The ADF test shows that the stock returns for the selected equities are all stationary at zero differencing which means that, there is no need for integer differencing to obtain stationarity and that the stock returns were not $I(1)$ process. The KPSS test for stationarity was also performed on the time series data and the null hypothesis of $I(0)$ process was investigated. On the contrary, the KPSS test rejected the null hypothesis of an $I(0)$ process for four of the equites whereas there was no statistical evidence against an I (0) process for the returns of the remaining equities. The conclusion from these two tests of stationarity that is the ADF and the KPSS test suggest that, the stock returns time series data from four equites can best be described by an *I(d)* where $d \neq 0$ *and* $d \neq 1$. Thus they are neither an $I(1)$ process or an $I(0)$ process suggesting that fractional integration should be incorporate in the time series analysis.

Another important procedure in preliminary statistical analysis is to understand the distribution from which the data is obtained. In this thesis, the Shapiro-Wilks test is used to access normality of the time series data. The test statistics and the p-values are given in Table 2. The Shapiro Wilks test of normality rejected the claim that the returns follows normal distribution for all the selected equities and the same conclusion is arrived when the Kolmogorov-Simonov test was applied.

The main aim of the study is to look for evidence of long range memory that can be used in arbitrage strategy for the stock return of the selected equities. The non-parametric Hurst exponent computed from the Modified Rescaled Range M-R/S statistics estimation method was used to determine the presence of long range memory in the time series data. The value of the fractional dimension parameter d can then be obtained by subtracting 0.5 from the Hurst exponent. The result of the Hurst exponent is given in Table 2. The larger the value of d, the more memory that can be found in the time series data. Most of the estimated values of the Hurst exponent are all greater than 0.5 suggesting that, the fractional difference parameter d is not equal to zero except for SOGEGH and SIC which had an estimated Hurst exponent of 0.476113 and 0.491793 respectively. The implication of Hurst exponent less 0.5 is that, these series do not exhibit long term memory property and that the series are antipersistent and therefore will be termed as a short memory process. Equities that had Hurst exponent less than 0.5 will not be considered in the parametric estimation of long range memory model.

Table 2: Table of summary statistics

Equity	ADF Statistic	ADF P-value	KPSS statistic	KPSS P-value	Hurst Exponent	Shapiro statistics	Shapiro p-value
BOPP	-6.2271	0.01	0.230244	0.100000	0.577056	0.662301	3.39E-26
Calbak	-6.99747	0.01	0.247490	0.100000	0.564464	0.877525	2.32E-16
ETI	-6.77433	0.01	0.170680	0.100000	0.548948	0.903869	2.23E-14
FML	-6.59263	0.01	0.080659	0.100000	0.535840	0.396552	6.08E-33
GCB	-5.41845	0.01	0.642920	0.018735	0.575105	0.776779	2.90E-20
GGBL	-6.09313	0.01	0.608714	0.021844	0.667402	0.632426	3.94E-27
GOIL	-6.96358	0.01	0.190577	0.100000	0.603667	0.831971	3.67E-19
HFC	-6.09593	0.01	0.314582	0.100000	0.623326	0.786796	1.98E-21
MLC	-6.50481	0.01	0.292384	0.100000	0.612497	0.703342	8.41E-25
PBC	-7.64471	0.01	0.147957	0.100000	0.534871	0.738890	1.82E-23
SCB	-7.68182	0.01	0.065988	0.100000	0.519428	0.410615	1.19E-32
SIC	-8.20963	0.01	0.171094	0.100000	0.491793	0.736970	1.53E-23
SOGEGH	-5.71905	0.01	0.033841	0.100000	0.476113	0.761137	2.27E-16
TOTAL	-6.99508	0.01	0.512374	0.038880	0.592573	0.419735	1.84E-32
UNIL	-4.09606	0.01	0.894954	0.010000	0.650019	0.607302	7.16E-28
UTB	-6.35105	0.01	0.171721	0.100000	0.557984	0.818303	2.89E-19

Also a plot of the spectra density function was used as a graphical tool to assess the presence of long range memory in the time series data considered in this research. Four of the plots are presented in this chapter whereas the remaining plots are in the appendix. Figure 1 shows the spectra density plot of the SOGEGH and SIC equities. These two equities had no long range memory according to the estimated Hurst exponent. From the graph, it can be observed that, the power of the spectra density at zero frequency is flat showing that higher order cycle observations do not contribute much in the variation in the time series data. However, it can be observed that, there are poles around 0.05 and 0.15 frequency for SOGEG representing a cycle of $1/0.05 = 20$ weeks and $1/0.15 = 6.7$ weeks respectively being important in the variation of the observed time series. The spectra density of the SIC data shows that, the power is much concentrated around frequency 0.5 indicating that, cycles of 2 weeks are important in the variation of the time series data.

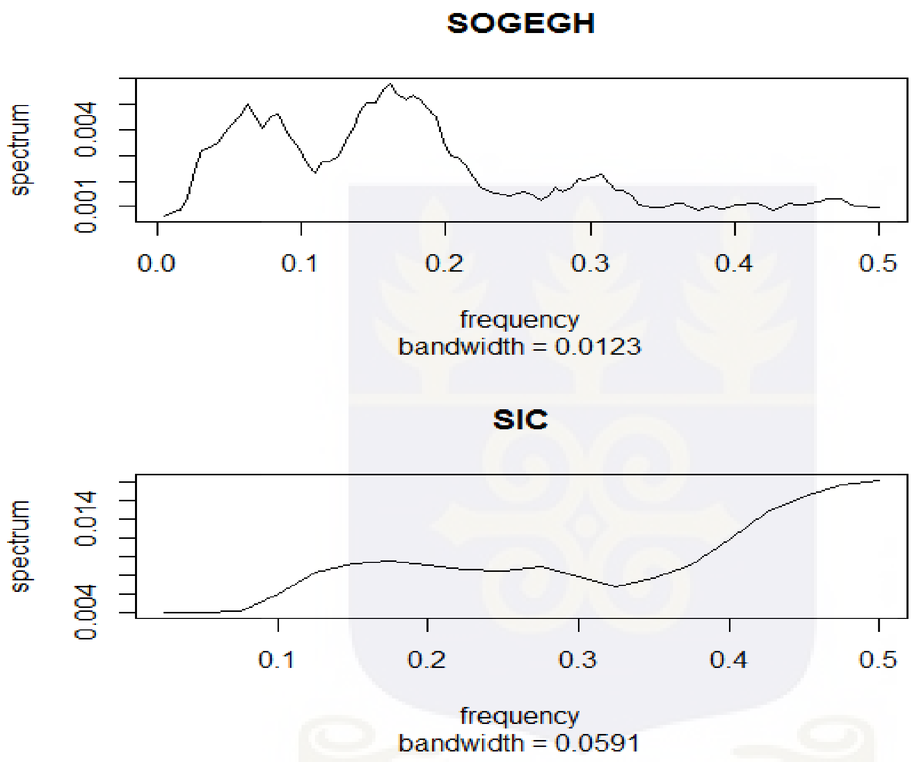

Figure 1: Spectra Density of SOGEGH and SIC

Figure 2 is a graph for two of the time series data that exhibited long range memory by the Hurst exponent namely GCB and UNIL. From the spectra density graph, there is a pole at the zero frequency for the two data sets. This is an indication that, the correlation between two observations that are separated by higher lags are significant and relevant in the variability of the time series. The plots for the two time series looks pretty much the same except that the spectra density of GCB decays in an exponential form whiles that of UNIL data has a spectra density whose power drops from the zero frequency and remains on almost the same level for the rest of the frequencies.

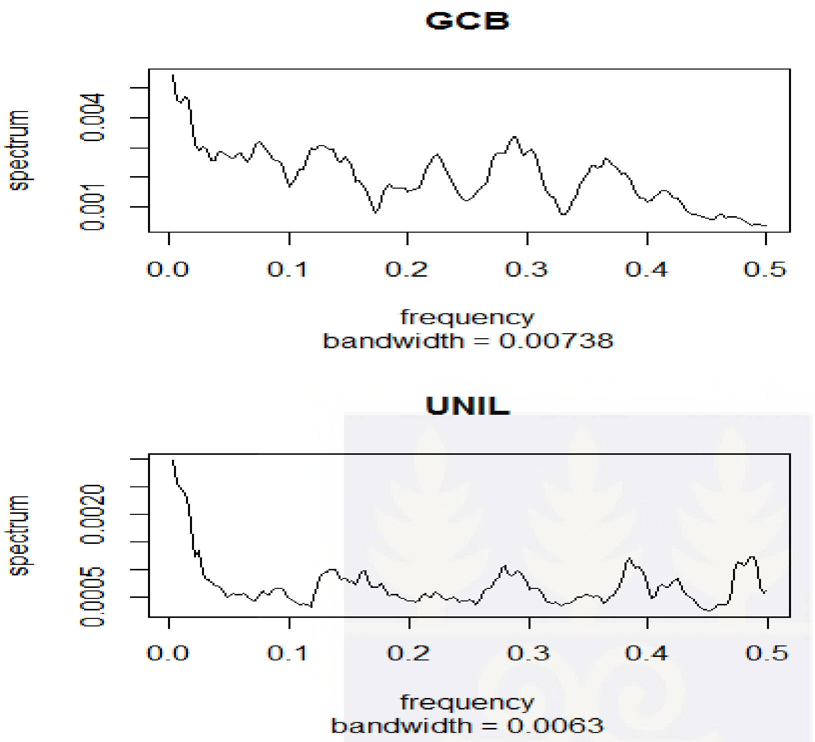

Figure 2: A spectra density plot of GCB and UNIL

The spectra density graphs in the appendix showed that 11 of the equities had presence of long memory in their return data.

After obtaining the non-parametric measure of long range memory property in the time series using the Hurst exponent, the next stage of the analysis is to fit the autoregressive fractionally integrated moving average ARFIMA model to the time series of the stock return for those that showed long range memory property to include the short range memory in the analysis. The study demonstrates how the parameters in the ARFIMA model for two of the selected equities that had evidence of long memory as estimated by the Hurst exponent are obtained. The results of the remaining models for the equities are presented in the appendix.

Before the start of the analysis, the time series, autocorrelation and partial autocorrelation functions of the equities under study are visualized for further understanding of the data. Figure 3 shows graph of UNIL (Unilever Ghana Limited) data. The share prices grew exponentially for the first 190 weeks under review and dropped sharply thereafter. The plot of the return looks stationary with a constant variation around the point zero. A steady look at the ACF plot shows that, the first lag is Significant but there are higher order lags that are significant suggesting the presence of a long memory process. Even though the ACF decays to zero, the decay is quit slow giving a signal of long range memory process. The Partial autocorrelation function tails off after the first two lags but there are still some higher order lags that are significant with no clear pattern. This suggests an AR of order 1 and MA of order 1 however, an AR of order zero was also analysed and the best model was taken based on AIC model selection criterion.

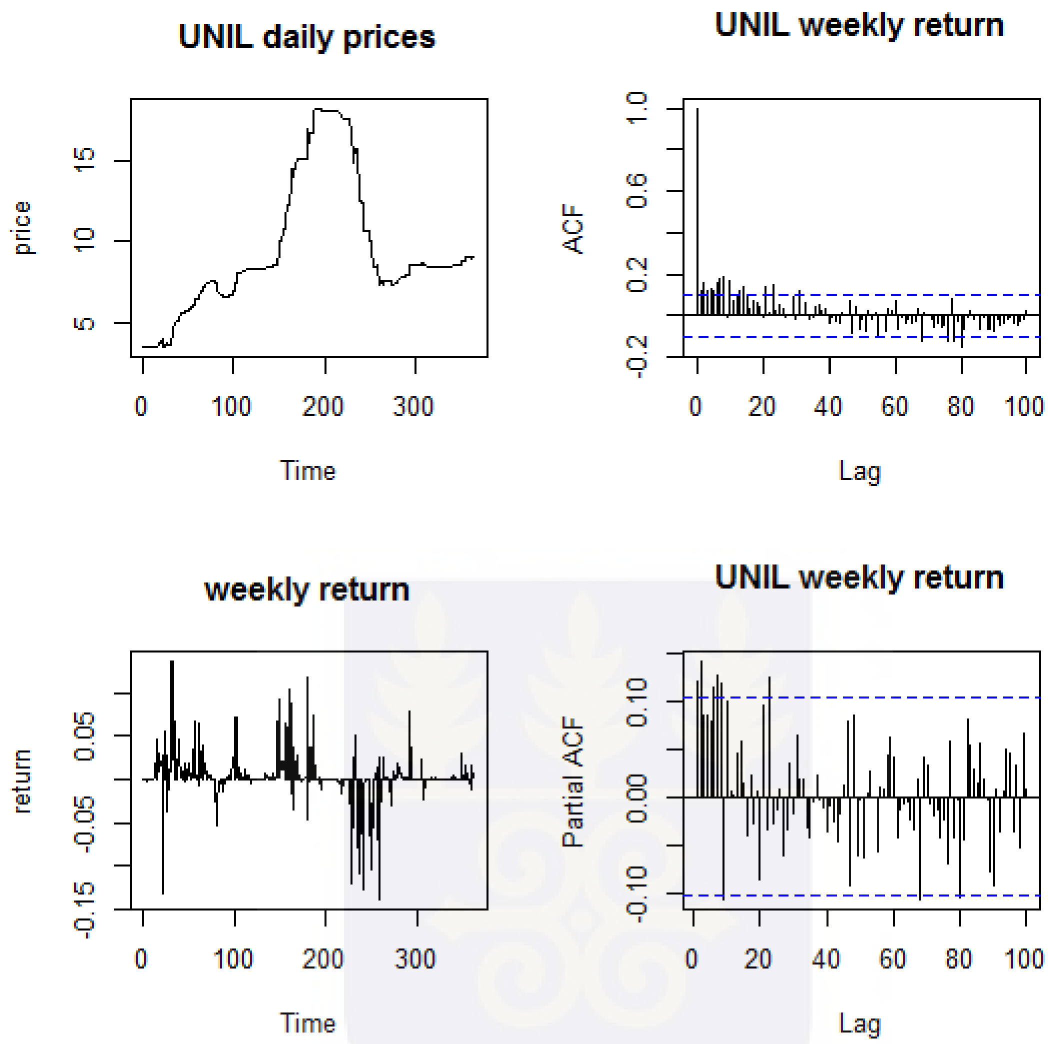

Figure 3: Plot of time series and its ACF PACF and Returns of UNIL

The same process is carried out for the GCB data. From Figure 4, the price of the equity was moving in an upward trend from January-2010 for 95 weeks and dropped sharply within three weeks and started moving upwards again. The price series showed upward and downward movement. However, the weekly return for GCB looks stationary with more variation within the first 60 weeks followed by 70 weeks of very little variation in stock return. The remaining weeks of the series showed higher variation similar to the first 60 weeks. The ACF plot shows that the first two lags are significant with few high order lags also being significant. Even though a look at the ACF of the GCB data may not suggest the presence of a long range memory property, the GCB return data will be fitted to a long range memory method due to the Hurst exponent estimated which was greater than 0.5 and then a comparison will be made with its short range memory counterpart. From Figure 4, an AR of order one and MA of order two for the ARFIMA (1, d, 2) was used from which the fractional difference parameter d was estimated using the QMLE method.

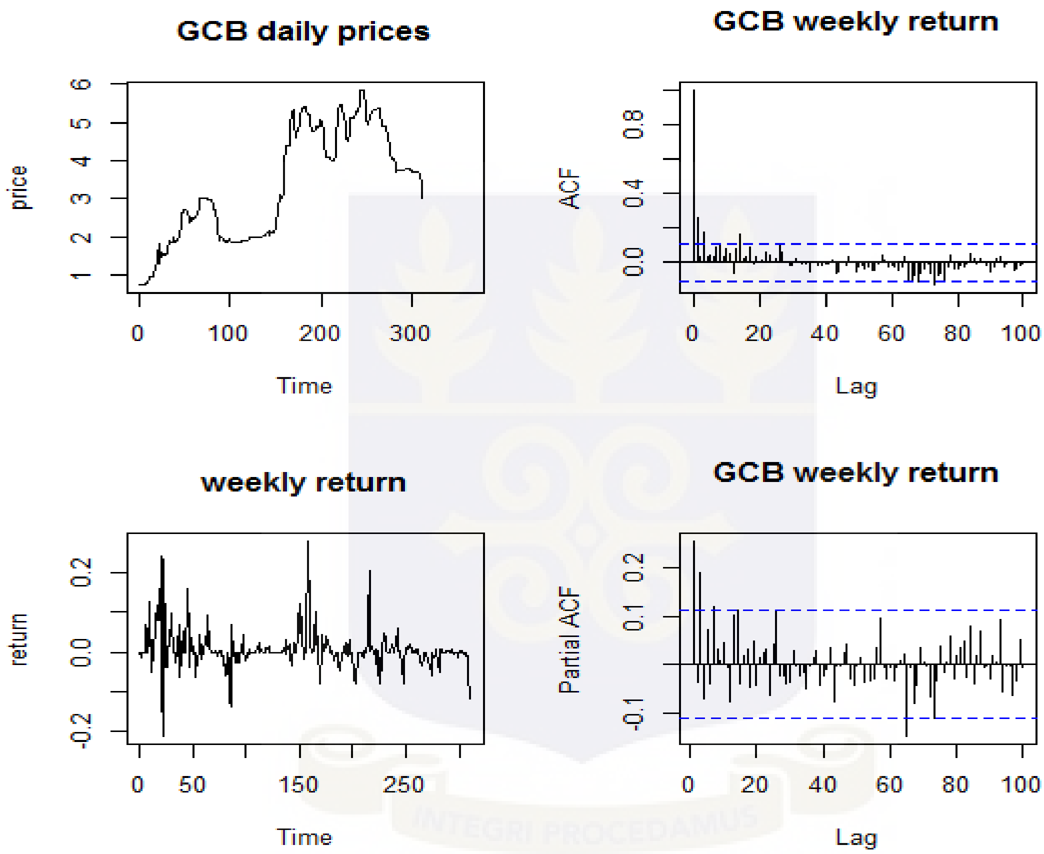

Figure 4: Plot of time series and its ACF PACF and Returns of GCB

Where the choice of the order of AR or MA lags are not straight forward from the ACF and PACF plots, for example the case of GCB, the best model was selected based on the AIC by varying the AR and the MA lags.

At this point where the order of the AR and MA are chosen, the fractional difference parameter d is estimated by using the ARFIMA function in R which implement the Quasi

Maximum likelihood estimation method to obtain the parameters of the ARFIMA model. This function implement the Quasi Maximum likelihood estimation of (Haslett & Raftery, 1989).

Table 3 shows a summary of the ARFIMA fit for UNIL. The estimated long range parameter d was 0.347 which is a very strong sign of long memory process. The long memory parameter d is significantly different from zero which is indeed an indication of a long memory when the AR and the MA lags to model the short range memory or dependence in the time series were accounted for. However, a comparison between the ARFIMA (1, d, 1) and ARFIMA (0, d, 1) in terms of the AIC selection criteria was made. The ARFIMA (0, d, 1) was a better fit in terms of the AIC. The result of the ARFIMA (1, d, 1) for UNIL is given in Table 3b in the appendix.

Table 3a: Summary results for UNIL ARFIMA (0, 0.273, 1).

	Estimate	Std. Error	Std. Err.	z-value	P-value
theta(1)	0.224353	0.106958	0.100499	2.09758	0.035942
d	0.273236	0.079981	0.080418	3.41626	0.000635
Fitted mean	0.003239	0.005521		0.58672	0.557395

sigma^2 estimated as 0.000673143; Log-likelihood = 1322.66; AIC = -2637.31; BIC = -1197.31

Similarly, Table 4 shows the summary results for the ARFIMA model fitted to the GCB stock return data. At first, an ARFIMA (1, d, 2) was fitted to the GCB data and the parameters were estimated. The long range memory parameter for the model was significantly different from zero at a 5% significance level indicating the presence of a long memory in the data. However, the second coefficient for the moving average parameter was found to be insignificant. ARFIMA (1, d, 2) was compared to ARFIMA (1, d, 1) on the bases of the AIC selection criteria. The ARFIMA (1, d, 1) had smaller AIC compared to ARFIMA (1, d, 2) so ARFIMA

(1, d, 1) was selected as the best model for the GCB return data. The result of the ARFIMA (1, d, 2) for GCB is given in Table 4b in the appendix.

Table 4a: Summary result of GCB ARFIMA (1, d, 1).

Parameters	Estimate	Std. Error	Std. Err.	z-value	P-values
phi(1)	-0.63211	0.112517	0.120706	-5.61792	1.93E-08
theta(1)	-0.82755	0.075333	0.081523	-10.9853	2.22e-16
D	0.127596	0.050587	0.052517	2.52233	0.011658
Fitted mean	0.005279	0.005518		0.95664	0.338747

sigma^2 estimated as 0.00185235; Log-likelihood = 976.489; AIC = -1942.98; BIC = -402.978

At this stage, the error terms of the model will be investigated and corrections will be made if necessary. By using visual inspections, the ACF and PACF of the residuals including plots of residuals for the two time series data are investigated for any violation of the assumption. Given that the model fits the data well, the error terms are expected to be serially uncorrelated and there should be no systematic pattern in the data. The graph of the ACF and PACF function for the UNIL data shows that the residuals are more of a white noise process which is a good indication that, all systematic relations in the data has been captured by the fitted model. Also, time series plot of the residuals showed no significant pattern in the residuals. Figure 5 is the residual plot, ACF and PACF of the residual terms of the model for UNIL. A formal test to check serial correlation was performed by using Box-Ljung which concluded that the residuals are serially uncorrelated. The test statistic and the p-values is given in Table 5.

Table 5: Box Ljung test of correlation.

Equity	Test statistic	P-value
GCB	1.3423	0.7191
UNIL	0.8077	0.8476

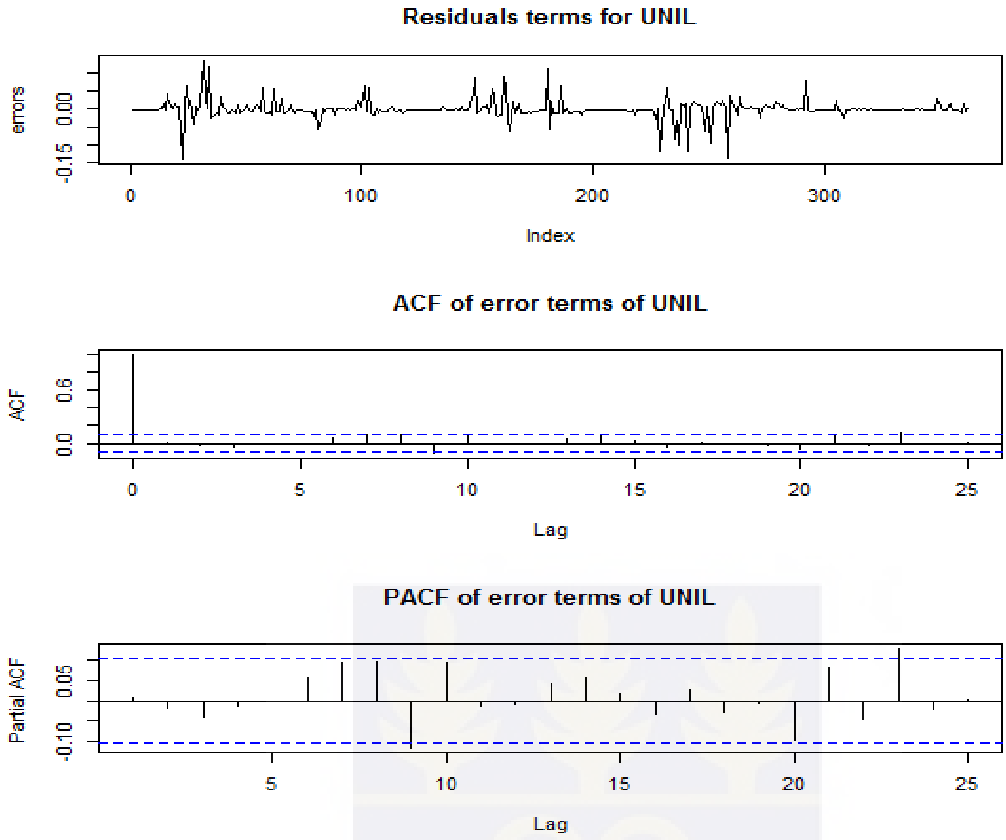

Figure 5: Diagnostic plots of residual for ARFIMA model for UNIL

Figure 6 is the graph of ACF, PACF and residual plots for model checking for GCB. Again, a look of the ACF and PACF plot of the residuals show that the error terms follow a white noise process, showing no systematic pattern in the residual plot. Again, the Box-Ljung test concluded that there was no serial correlation. Table 5 gives the summary table of the Box-Ljung test for the error terms for the model.

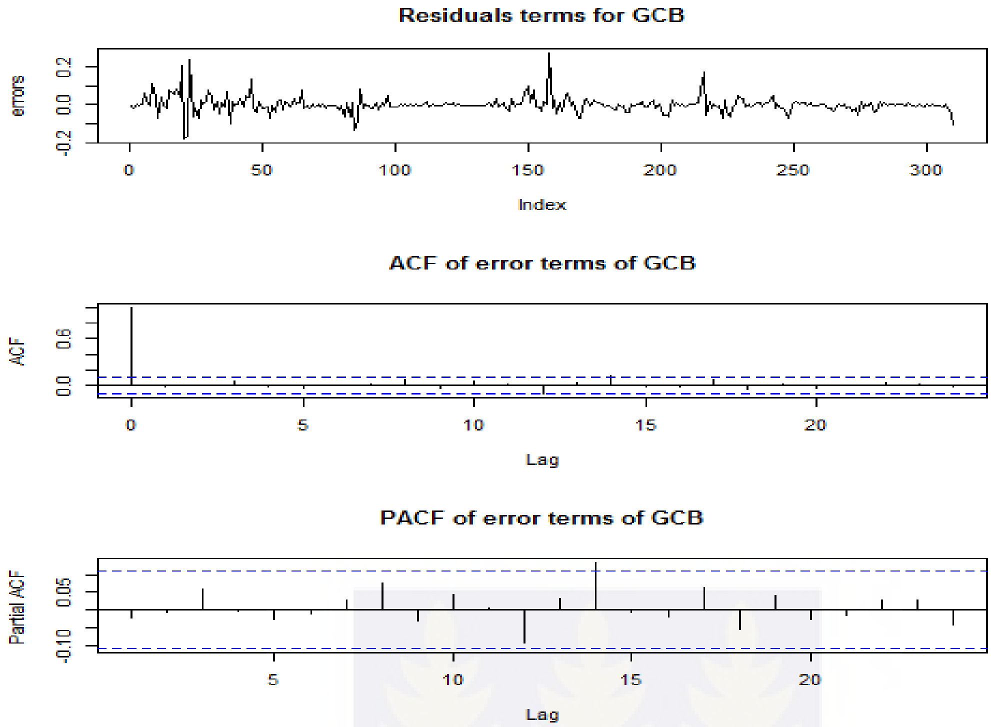

Figure 6: Diagnostic plots of residual for ARFIMA model for GCB

This stage of the analysis involves a graphical display of how well the model fits the data. Generally, the fitted values mimicked the original data and was able to reproduce significant pattern in the data.

A graph of the fitted and the original return of UNIL time series is given in Figure 5.

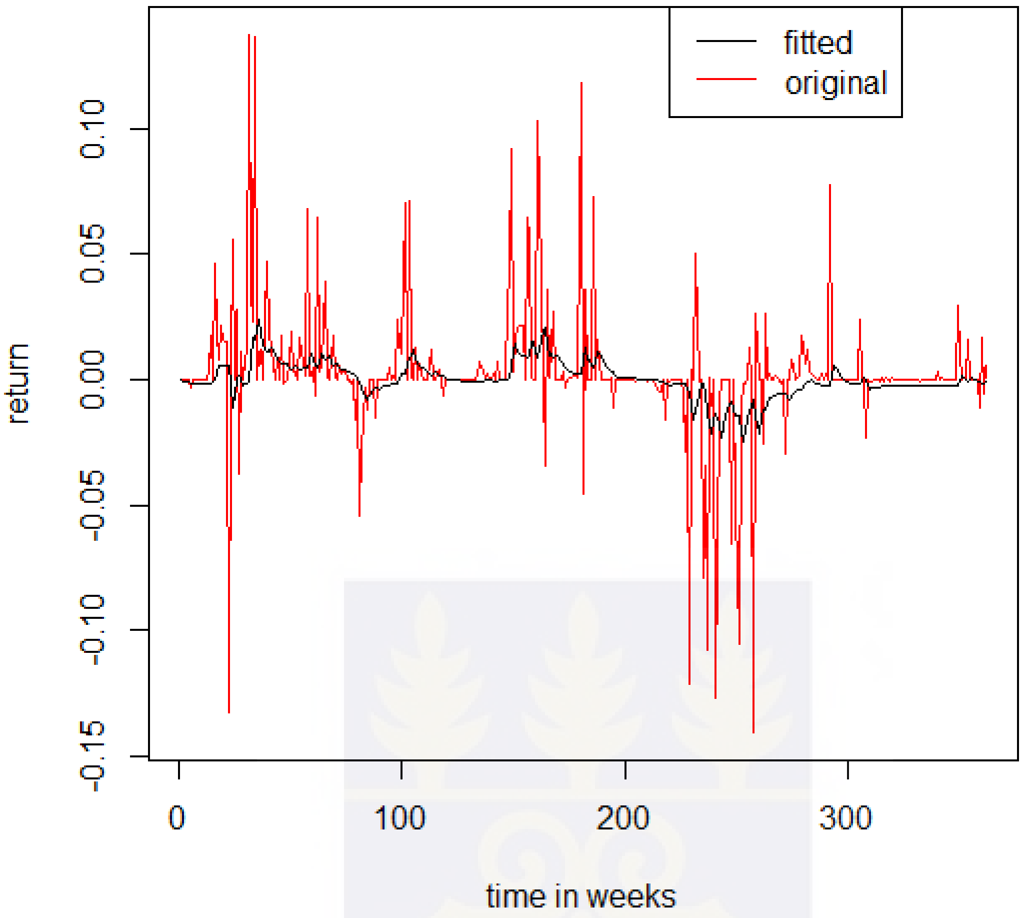

Figure 7: Graph of fitted and original time series of return for UNIL

A similar graph is produced for the GCB data also indicating a good fit. The predicted values reproduced the significant pattern in the data.

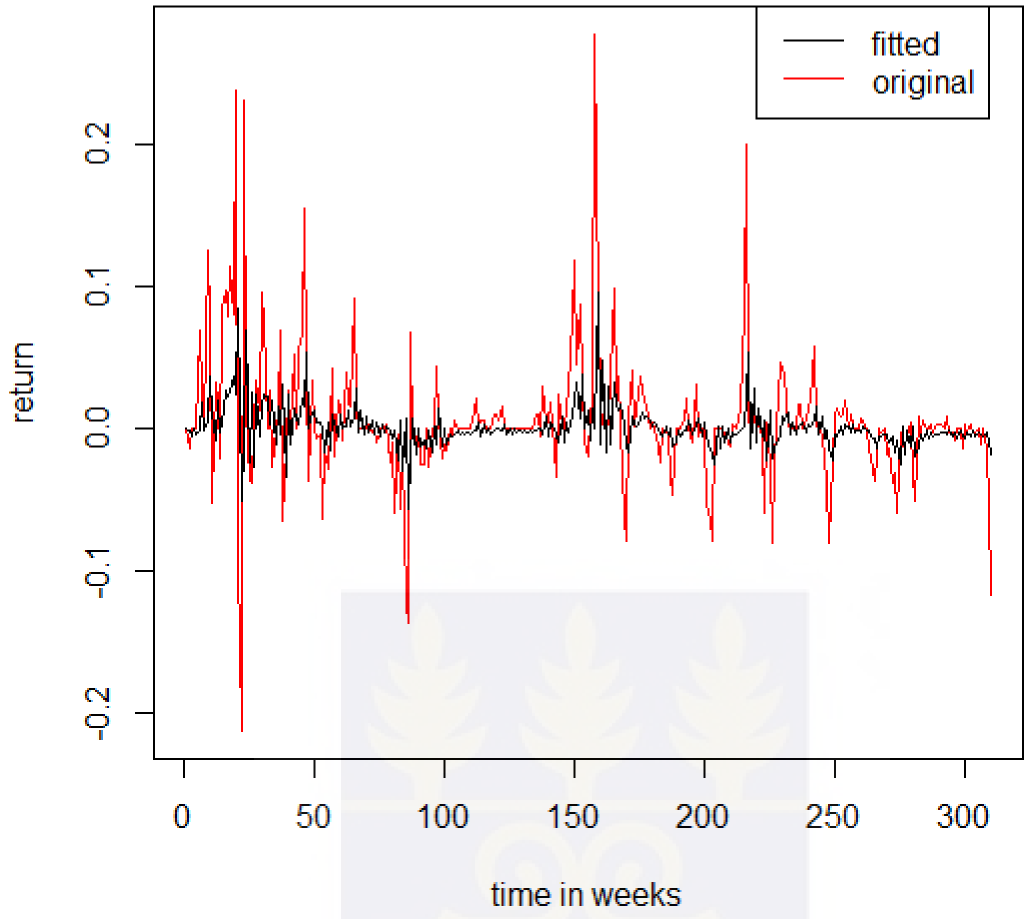

Figure 8: Graph of fitted and original time series of return for GCB

The main aim of this research is to look for long range memory property in the selected equities which can be used as an arbitrage by researchers and investors to maximize their profit at no further risk. In spite of the evidence of long range memory in some of the selected time series data, it is important to compare the long range memory model to its counterpart short range model to check if the presence of the long range memory will be useful in getting good future predictions. The comparison will be made in terms of the ability of the model to predict with smaller standard error or confidence interval. Also, the Mean Absolute Percentage Error (MAPE) was computed to compare the results from the two types of models. The short range memory model employed in this study is the popular Autoregressive integrated Moving Average (ARIMA). For each time series data, the full details of the

ARIMA model is not included but the summary of the model is presented in the appendix. The best ARIMA model was obtained based on the Box-Jenkins method. Table 6 shows the summary results of the ARIMA model for the two equities that were analysed in this chapter.

Table 6: Summary results of ARIMA for UNIL and GCB

GCB ARIMA(1,1,2)				UNIL ARIMA(0,1,1)
Parameter	AR(1)	MA(1)	MA(2)	MA(1)
Coefficients	-0.5378	-0.1569	-0.7476	-0.9119
s.e	0.1112	0.0861	0.0752	0.0243
AIC	-1057.22			-1604.51

Table 7a: 6 weeks predicted values for UNIL time series data

ARFIMA	SD	Observed	ARIMA	SD
0.00304	0.02614	0	0.00321	0.02624
0.00265	0.02617	0	0.00321	0.02634
0.00261	0.02634	-0.0111	0.00321	0.02644
0.00258	0.02645	0.01685	0.00321	0.02654
0.00255	0.02653	-0.0055	0.00321	0.02665
0.00252	0.02659	0.00556	0.00321	0.02675

Table 7b: 6 weeks predicted values for GCB time series data.

ARFIMA	SD	Actual	ARIMA	SD
0.00359	0.04293	0	-0.002	0.0432
0.00234	0.04498	-0.011	-0.005	0.0451
0.00372	0.045	0	-0.0034	0.0452
0.00322	0.04531	-0.014	-0.0043	0.0455
0.00382	0.04531	-0.053	-0.0038	0.0455
0.00368	0.04539	-0.117	-0.004	0.0458

From table 7, the standard deviation (SD) of the ARFIMA models were mostly smaller compared to its short term memory counterpart (ARIMA model) which is a desired characteristics for any model. However, the Mean Absolute Percentage Error (MAPE) of the short term model GCB was 3.04%. This outperformed the long rage memory model which had MAPE of 3.82%. For the UNIL data, the short range memory model had an MAPE of 4.58% which was outperformed by the long range memory model whose MAPE was 4.48%.

Table 8 is shown in appendix and it gives the AIC for both Long and Short memory models fitted to the data. The number of parameters that were estimated and the level of fractional integration in the case of ARFIMA are also given.

Due to the small level of fractional integration in the return series, the absolute returns and the square returns were also analysed using the same procedure. Table 9 gives the summary of the stationarity test and the Hurst exponent for the absolute return whereas Table 10 gives the same information for the square returns. From Table 9, the ADF test rejected the null hypothesis of an $I(1)$ series in favour of $I(0)$ for all the equities whereas the KPSS rejected the null of $I(0)$ for five absolute returns of the equities. The estimated Hurst Exponent were all above 0.5 suggesting presence of long memory in the absolute return of the equities. When accounted for short memory, the fractions difference parameter d had larger values compared to the return series with GCB having 0.3276 which was the highest level of fractional integration in the absolute returns. MLC had a fractional integration level of 0.0641 which was the least among the selected equities.

The ADF test for the squared return series also rejected the null hypothesis of an $I(1)$ process in favour of an $I(0)$ process for all the selected equities. However, the KPSS test of stationarity rejected the null of an $I(0)$ for four of the equities. The Hurst exponent for most of the squared returns were above 0.5 with UTB having the highest level of fractional integration with $H = 0.668$ and TOTAL having a Hurst exponent of 0.5 and fractional integration of zero.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

This chapter presents the summary of findings form the study and gives recommendation to stake holders and investors who trade on the Ghanaian stock market.

The main purpose of the work was to investigate for the presence of long range memory in the equities listed on the Ghanaian stock market so that traders and investors can use it as a form of arbitrage in their trading strategy. The study considered time series data of weekly returns of 16 equities listed on the Ghanaian stock market ranging from January-2010 to May 2017 for all the selected equities except SOGEGH for which only 192 weeks of data point was obtained.

5.2 SUMMARY

A brief description of the empirical distribution of the selected stock equities showed that, the expected return for most of the selected equities were positive except MLC, SIC, PBC and UTB for which their mean return were negative. Also, all the equities were found to be leptokurtic having fat tails ranging from FML having the maximum of about 130 to ETI having the minimum of 2.78. In terms of skewness, five equities were positively skewed whereas four were negatively skewed with the remaining having skewness very close to zero. The empirical distribution of the weekly returns were investigated using the Shapiro-Wilks test of normality and the Kolmogorov-Smirnov test of normality. In all the tests, all selected equities showed non-normality in distribution for the weekly return data.

Investigation of the time series in terms of stationarity is important especially for fractionally integrated processes. The ADF test and the KPSS test were employed to investigate presence of unit root in the time series. The ADF test concluded that, all the weekly returns are stationary $I(0)$ process since the null of an $I(1)$ process was rejected however, the KPSS test of null $I(0)$ against an alternative $I(1)$ concluded that four of the equities GCB, GGBL, TOTAL and UNIL are not $I(0)$ and for the remaining equities, there was no statistical significant evidence against an $I(0)$ process. Combining the conclusion from the ADF and KPSS, there is an indication that four of the equities are fractionally integrated process as was stated by Baillie, Chung and Tieslau (1996) thus equites that had the null hypothesis for both test being rejected and the remaining are $I(0)$ process. All test were conducted at 5% significance level. However the presence of long memory in the remaining equites were investigated using other methods to establish the existence of long range memory.

The presence of long range memory was investigated using the non-parametric modified rescaled Range statistic (M-R/S). The result from the M-S/R showed that most of the analysed equities possessed long range memory with Hurst exponent bigger than 0.5 and GGBL having the highest level of fractional integration except SOGEGH and SIC whose Hurst exponent were 0.48 and 0.49 respectively showing no evidence of long memory. Thus the results implies that the SIC and SOGEGH equities could best be modelled with non-long memory models. A plot of the spectra density was also used as a graphical approach of investigating the presence of long memory. From the density plots, SGEGH, ETI, PBC, SIC and SCB had no poles at the origin whereas 11 of the remaining equities had poles at the origin which indicate the presence of long range memory. After the presence of long range memory was investigated, 14 of the 16 equities were found to be possessing long range memory based on the estimated Hurst exponent and were modelled using the ARFIMA models.

The parametric ARFIMA was implemented using the (Haslett & Raftery, 1989) method to simultaneously estimate the short term memory parameters and the long range memory parameters for the stock return data. However the parametric method of estimation using the ARFIMA model found that, most of the equities had very small level of fractional integration when the short memory of the ARMA process was accounted for. UNIL had a fractional difference of order 0.273236 which was the highest among all the series analysed whereas ETI SCB and UTB had fractional difference very close to zero. The ARFIMA and the ARIMA models were compared in terms of the AIC. The AIC for the ARFIMA models were smaller compared to the ARIMA models.

Out of sample prediction for GCB and UNIL return were obtained using the ARIMA and ARFIMA model. This was to check if the use of long memory model gives good prediction of stock returns. The predicted values were compared in terms of the standard errors and the MAPE. The ARFIMA model predicted values with smaller standard error and confidence interval when compared to the ARIMA models. However, the MAPE for the short memory model for GCB was smaller than that of ARFIMA even though the AIC of ARIMA was larger than that of the ARFIMA. The UNIL long memory model had smaller MAPE and AIC compared to the short memory model.

Also the same procedure was repeated for the squared return and absolute return to investigate the presence of long memory in the data. The Hurst estimate for the squared and absolute were all above 0.5. When the ARFIMA model was fitted to the square and absolute return, the level of fractional integration was found to be higher than the return data.

5.2 CONCLUSION.

The study found evidence of long memory in the stock returns, absolute return and the square returns of the equities that were analysed. This means that the stochastic process that describes the price movement is not a semi-martingale and therefore there is an approximate arbitrage opportunity in the market. The level of fractional integration in the return data were very small compared to that of the squared and the absolute return. The study also found that, even though the level of fractional integration (long range memory) were small in the return of the equities, the ARFIMA model which uses the long range memory fitted the data well in terms of the AIC compared to the short memory model (ARIMA).

5.3 IMPLICATION TO INVESTORS

The study found very little level of fractional integration in most of the weekly stock return data of the equities that were selected for analysis. However, due to the model used by Hedge fund managers which rely heavily on good prediction of volatility, the higher level of fractional integration in the squared return and absolute return gives investors who manage Hedge funds an advantage.

5.4 LIMITATION

The study did not consider structural breaks in the analysis. However, the presence of structural breaks may inflate the long range memory parameter.

REFERENCES

Adu, D. (2012). Effect of Macroeconomic Variables on Stock Market Returns Ghana: An Analysis Using Arbitrage Pricing Model: Institute of Distance Learning, Kwame Nkrumah University of Science and Technology, Kumasi.

Ariyo, A. A., Adewumi, A. O., & Ayo, C. K. (2014). *Stock price prediction using the ARIMA model.* Paper presented at the Computer Modelling and Simulation (UKSim), 2014 UKSim-AMSS 16th International Conference on.

Aye, G. C., Balcilar, M., Gupta, R., Kilimani, N., Nakumuryango, A., & Redford, S. (2014). Predicting BRICS stock returns using ARFIMA models. *Applied Financial Economics, 24*(17), 1159-1166.

Bachelier, L. (1900). *Théorie de la spéculation*: Gauthier-Villars.

Baillie, R. T. (1996). Long memory processes and fractional integration in econometrics. *Journal of econometrics, 73*(1), 5-59.

Baillie, R. T., Chung, C.-F., & Tieslau, M. A. (1996). Analysing inflation by the fractionally integrated ARFIMA--GARCH model. *Journal of applied econometrics*, 23-40.

Baillie, R. T., & Chung, S.-K. (2002). Modeling and forecasting from trend-stationary long memory models with applications to climatology. *International Journal of Forecasting, 18*(2), 215-226.

Barkoulas, J. T., Baum, C. F., & Travlos, N. (2000). Long memory in the Greek stock market. *Applied Financial Economics, 10*(2), 177-184.

Beran, J. (1994). *Statistics for long-memory processes* (Vol. 61): CRC press.

Bernard, V. L. (1994). Accounting-based valuation methods, determinants of market-to-book ratios, and implications for financial statement analysis.

Bhansali, R., & Kokoszka, P. (2003). Prediction of long-memory time series. *Theory and applications of long-range dependence*, 355-367.

Booth, G. G., Kaen, F. R., & Koveos, P. E. (1982). R/S analysis of foreign exchange rates under two international monetary regimes. *Journal of monetary economics, 10*(3), 407-415.

Box, G. E., & Jenkins, G. M. (1976). *Time series analysis: forecasting and control, revised ed*: Holden-Day.

Cheung, Y.-W., & Lai, K. S. (1995). A search for long memory in international stock market returns. *Journal of International Money and Finance, 14*(4), 597-615.

Costa, R. L., & Vasconcelos, G. (2003). Long-range correlations and nonstationarity in the Brazilian stock market. *Physica A: Statistical Mechanics and its Applications, 329*(1), 231-248.

Crato, N. (1994). Some international evidence regarding the stochastic memory of stock returns. *Applied Financial Economics*, *4*(1), 33-39.

Crato, N., & Ray, B. K. (2000). Memory in returns and volatilities of futures' contracts. *Journal of Futures Markets, 20*(6), 525-543.

Davidson, J. (2004). Convergence to stochastic integrals with fractionally integrated integrator processes: theory, and applications to fractional cointegration. *Preprint, University of Exeter*.

De Jong, R. M., & Davidson, J. (2000). The functional central limit theorem and weak convergence to stochastic integrals I. *Econometric Theory*, *16*(5), 621-642.

Dickey, D. A., & Fuller, W. A. (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American statistical association, 74*(366a), 427-431.

Dickey, D. A., & Fuller, W. A. (1981). Likelihood ratio statistics for autoregressive time series with a unit root. *Econometrica: Journal of the Econometric Society*, 1057-1072.

Diebold, F. X., & Rudebusch, G. D. (1991). On the power of Dickey-Fuller tests against fractional alternatives. *Economics letters, 35*(2), 155-160.

Ding, Z., & Granger, C. W. (1996). Modeling volatility persistence of speculative returns: a new approach. *Journal of econometrics, 73*(1), 185-215.

Ding, Z., Granger, C. W., & Engle, R. F. (1993). A long memory property of stock market returns and a new model. *Journal of empirical finance, 1*(1), 83-106.

Durbin, J. (1960). The fitting of time-series models. *Revue de l'Institut International de Statistique*, 233-244.

Fama, E. F. (1965). The behaviour of stock-market prices. *The journal of business, 38*(1), 34-105.

Figurska, M., & Wisniewski, R. (2016). Fundamental Analysis–Possiblity of Application on the Real Estate Market. *Real Estate Management and Valuation, 24*(4), 35-46.

Fox, R., & Taqqu, M. S. (1986). Large-sample properties of parameter estimates for strongly dependent stationary Gaussian time series. *The Annals of Statistics*, 517-532.

Geweke, J., & Porter-Hudak, S. (1983). The estimation and application of long memory time series models. *Journal of time series analysis, 4*(4), 221-238.

Giraitis, L., Kokoszka, P., Leipus, R., & Teyssière, G. (2003). Rescaled variance and related tests for long memory in volatility and levels. *Journal of econometrics, 112*(2), 265-294.

Granger, C. W., & Joyeux, R. (1980). An introduction to long-memory time series models and fractional differencing. *Journal of time series analysis, 1*(1), 15-29.

Haslett, J., & Raftery, A. E. (1989). Space-time modelling with long-memory dependence: Assessing Ireland's wind power resource. *Applied Statistics*, 1-50.

Hassler, U. (1993). Regression of spectral estimators with fractionally integrated time series. *Journal of time series analysis, 14*(4), 369-380.

Hassler, U., & Wolters, J. (1994). On the power of unit root tests against fractional alternatives. *Economics letters, 45*(1), 1-5.

Henry, O. T. (2002). Long memory in stock returns: some international evidence. *Applied Financial Economics, 12*(10), 725-729.

Hipel, K. W., & McLeod, A. I. (1978). Preservation of the rescaled adjusted range: 2. Simulation studies using Box-Jenkins Models. *Water Resources Research, 14*(3), 509-516.

Hosking, J. R. (1981). Fractional differencing. *Biometrika, 68*(1), 165-176.

Hurst, H. E. (1951). Long-term storage capacity of reservoirs. *Trans. Amer. Soc. Civil Eng., 116*, 770-808.

Jasemi, M., Kimiagari, A. M., & Memariani, A. (2011). A modern neural network model to do stock market timing on the basis of the ancient investment technique of Japanese Candlestick. *Expert Systems with Applications, 38*(4), 3884-3890.

Jensen, M. J. (1999). Using wavelets to obtain a consistent ordinary least squares estimator of the long-memory parameter.

Kallah-Dagadu, G. (2013). *Modelling Ghana Stock Exchange Indicies and Exchange Rates with Stable Distributions.* University of Ghana.

Kasman, A., Kasman, S., & Torun, E. (2009). Dual long memory property in returns and volatility: Evidence from the CEE countries' stock markets. *Emerging Markets Review, 10*(2), 122-139.

Kiliĉ, R. (2004). On the long memory properties of emerging capital markets: evidence from Istanbul stock exchange. *Applied Financial Economics, 14*(13), 915-922.

Kumar, A. (2004). *Time Series Behaviour of Stock Trading Volume: An Evidence from Indian Stock Market.* Paper presented at the Econometric Society 2004 Far Eastern Meetings.

Kuwornu, J. K., & Owusu-Nantwi, V. (2011). Macroeconomic variables and stock market returns: full information maximum likelihood estimation. *Research Journal of Finance and Accounting, 2*(4), 49-63.

Kwiatkowski, D., Phillips, P. C., Schmidt, P., & Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of econometrics, 54*(1-3), 159-178.

Levinson, N. (1947). A heuristic exposition of Wiener's mathematical theory of prediction and filtering. *Studies in Applied Mathematics, 26*(1-4), 110-119.

Lo, A. W. (1989). *Long-term memory in stock market prices*.

Lo, A. W., & MacKinlay, A. C. (1989). The size and power of the variance ratio test in finite samples: A Monte Carlo investigation. *Journal of econometrics, 40*(2), 203-238.

Lobato, I. N., & Robinson, P. M. (1998). A nonparametric test for I (0). *The review of economic studies, 65*(3), 475-495.

MacKinnon, R. (1991). Determination of the subunit stoichiometry of a voltage-activated potassium channel. *Nature, 350*(6315), 232.

man Lui, K., & Chong, T. T. (2013). Do technical analysts outperform novice traders: Experimental evidence. *Economics Bulletin, 33*(4), 3080-3087.

Mandelbrot, B. (1972). Statistical methodology for nonperiodic cycles: from the covariance to R/S analysis *Annals of Economic and Social Measurement, Volume 1, number 3* (pp. 259-290): NBER.

McKenzie, M. D. (2001). Non‐periodic Australian Stock Market Cycles: Evidence from Rescaled Range Analysis. *Economic Record, 77*(239), 393-406.

McMillan, D. G., & Thupayagale, P. (2008). Efficiency of the South African equity market. *Applied Financial Economics Letters, 4*(5), 327-330.

Mondal, P., Shit, L., & Goswami, S. (2014). Study of effectiveness of time series modeling (ARIMA) in forecasting stock prices. *International Journal of Computer Science, Engineering and Applications, 4*(2), 13.

Moulines, E., & Soulier, P. (1999). Broadband log-periodogram regression of time series with long-range dependence. *The Annals of Statistics, 27*(4), 1415-1439.

Murphy, J., & Kalayjian, S. (2015). Methods and system for visually designing trading strategies and execution thereof: Google Patents.

Nabeya, S., & Tanaka, K. (1988). Asymptotic theory of a test for the constancy of regression coefficients against the random walk alternative. *The Annals of Statistics*, 218-235.

Ngene, G., Tah, K. A., & Darrat, A. F. (2017). Long memory or structural breaks: Some evidence for African stock markets. *Review of Financial Economics*.

Nortey, E. N., Asare, K., & Mettle, F. O. (2015). Extreme value modelling of Ghana stock exchange index. *SpringerPlus, 4*(1), 696.

Omari-Sasu, A. Y., Frempong, N. K., Boateng, M. A., & Boadi, R. K. (2015). Modeling Stock Market Volatility Using GARCH Approach on the Ghana Stock Exchange. *International Journal of Business and Management, 10*(11), 169.

The Origins of Technical Analysis on the Foreign Exchange Market. (2017). Retrieved from https://www.forexcharts.net/the-origins-of-technical-analysis-on-the-foreign-exchange-market/.

Osei, V. (2005). Does the stock market matter in Ghana. *A granger-causality analysis: Working*.

Palma, W. (2007). *Long-memory time series: theory and methods* (Vol. 662): John Wiley & Sons.

Panas, E. (2001). Estimating fractal dimension using stable distributions and exploring long memory through ARFIMA models in Athens Stock Exchange. *Applied Financial Economics, 11*(4), 395-402.

Parke, W. R. (1999). What is fractional integration? *The review of economics and statistics, 81*(4), 632-638.

Pérez-Rodríguez, J. V., Torra, S., & Andrada-Felix, J. (2005). Are Spanish Ibex35 stock future index returns forecasted with non-linear models? *Applied Financial Economics, 15*(14), 963-975.

Peters, E. E. (1991). Chaos and Order in the Capital Markets. A New View of Cycle, Prices, and Market Volatility. *J. WILEY*.

Peters, E. E. (1994). *Fractal market analysis: applying chaos theory to investment and economics* (Vol. 24): John Wiley & Sons.

Pham, D. T., & Le Breton, A. (1991). Levinson-Durbin-type algorithms for continuous-time autoregressive models and applications. *Mathematics of Control, Signals, and Systems (MCSS), 4*(1), 69-79.

Rege, S., & Martín, S. G. (2011). Portuguese Stock Market: A Long-Memory Process? *Business: Theory and Practice/Verslas: Teorija ir Praktika, 12*(1), 75-84.

Robinson, P. M. (1994). Semiparametric analysis of long-memory time series. *The Annals of Statistics*, 515-539.

Sadique, S., & Silvapulle, P. (2001). Long-term memory in stock market returns: international evidence. *International Journal of Finance & Economics, 6*(1), 59-67.

Samuelson, P. A. (1965). Proof that properly anticipated prices fluctuate randomly. *IMR; Industrial Management Review (pre-1986), 6*(2), 41.

Sowell, F. (1992). Maximum likelihood estimation of stationary univariate fractionally integrated time series models. *Journal of econometrics*, *53*(1-3), 165-188.

Stoffer, D. S., & Shumway, R. H. (2000). Time Series Analysis and Its Applications. *With R Examples*.

Wafi, A. S., Hassan, H., & Mabrouk, A. (2015). Fundamental Analysis Models in Financial Markets–Review Study. *Procedia Economics and Finance*, *30*, 939-947.

Whitcher, B., & Jensen, M. J. (2000). Wavelet estimation of a local long memory parameter. *Exploration Geophysics*, *31*(1/2), 94-103.

APPENDIX

Complete table of summary test statistics.

Equity	ADF Statistics	ADF p-value	KPSS statistic	KPSS p-values	Hurst exponent	Shapiro statistics	Shapiro P-value	K-S statistics	K-SP p-value	Kurtosis	Skewness	mean	SD
BOPP	-6.227	0.01	0.2302	0.1	0.5771	0.6623	3.39E-26	0.4421	0	19.054	2.8405	0.0068	0.0539
Calbak	-6.997	0.01	0.2475	0.1	0.5645	0.8775	2.32E-16	0.4478	0	7.2046	1.4222	0.005	0.054
ETI	-6.774	0.01	0.1707	0.1	0.5489	0.9039	2.23E-14	0.4276	0	2.7455	0.1514	0.0014	0.0641
FML	-6.593	0.01	0.0807	0.1	0.5358	0.3966	6.08E-33	0.4615	0	126.37	-8.454	0.0046	0.0532
GCB	-5.418	0.01	0.6429	0.01	0.5751	0.7768	2.90E-20	0.452	0	10.205	1.4635	0.0055	0.0457
GGBL	-6.093	0.01	0.6087	0.02	0.6674	0.6324	3.94E-27	0.4525	0	12.271	-1.201	0.0008	0.0311
Goil	-6.964	0.01	0.1906	0.1	0.6037	0.832	3.67E-19	0.4526	0	10.883	0.6457	0.0068	0.0422
HFC	-6.096	0.01	0.3146	0.1	0.6233	0.7868	1.98E-21	0.442	0	4.6471	-0.199	0.0003	0.0423
MLC	-6.505	0.01	0.2924	0.1	0.6125	0.7033	8.41E-25	0.4414	0	5.7105	0.6783	-6E-04	0.0466
PBC	-7.645	0.01	0.148	0.1	0.5349	0.7389	1.82E-23	0.42	0	7.124	0.1445	-0.002	0.0674
SCB	-7.682	0.01	0.066	0.1	0.5194	0.4106	1.19E-32	0.4512	0	78.638	-4.522	0.002	0.0653
SIC	-8.21	0.01	0.1711	0.1	0.4918	0.737	1.53E-23	0.4349	0	24.764	2.5734	-4E-04	0.0654
SOGEGH	-5.719	0.01	0.0338	0.1	0.4761	0.7611	2.27E-16	0.4393	0	7.333	1.0544	0.0006	0.0484
TOTAL	-6.995	0.01	0.5124	0.03	0.5926	0.4197	1.84E-32	0.4424	0	129.98	-8.715	0.0009	0.0593
UNIL	-4.096	0.01	0.895	0.01	0.65	0.6073	7.16E-28	0.4563	0	12.146	-0.344	0.0031	0.0269
Utb	-6.351	0.01	0.1717	0.1	0.558	0.8183	2.89E-19	0.4138	0	7.4027	0.015	-0.002	0.0787

Table 3b: Summary results for UNIL ARFIMA (1, d, 1).

Parameters	Estimate	Std. Error	z-value	Pr(>|z|)
phi(1)	0.210203	0.154758	1.35827	0.174378
theta(1)	0.502962	0.198997	2.52748	0.011488
D	0.347004	0.112250	3.09136	0.001993
Fitted mean	0.003274	0.007762	0.42182	0.673156

sigma^2 estimated as 0.000672704; Log-likelihood = 1323.08; AIC = -2636.16; BIC = -836.165

Table 4b: Summary result of GCB ARFIMA (1, d, 2).

Parameters	Estimate	Std. Error	z-value	Pr(>|z|)
phi(1)	-0.51817	0.166313	-3.11566	0.001835
theta(1)	-0.62895	0.209332	-3.00456	0.002660
theta(2)	0.126393	0.117729	1.07360	0.283002
d	0.192453	0.087584	2.19735	0.027996
Fitted mean	0.005188	0.007165	0.72406	0.469030

sigma^2 estimated as 0.00185104; Log-likelihood = 977.033; AIC = -1942.07; BIC = -94.0663

Table 8: Fitted models for return equities.

		ARFIMA				ARIMA		
Equity	d	MA	AR	AIC	AR	d	MA	AIC
BOPP	0.15535	0	0	-1098.59	1	0	1	-1096.63
calbak	0.019813	0	0	-1083.23	0	0	0	-1083000
ETI	4.58E-05	0	0	-958.890	0	0	0	-958.890
FML	0.028408	0	0	-1094.36	0	0	0	-1093.90
GGBL	0.269169	0	0	-1536.82	3	1	1	-1526.01
Goil	0.188101	3	3	-1282.64	3	0	1	-1287.22
HFC	0.102897	0	0	-1267.62	0	0	0	-1267.62
MLC	0.064116	0	0	-1192.72	0	0	0	-1189.67
PBC	0.002724	0	0	-922.650	0	0	0	-922.650
SCB	4.58E-05	0	0	-945.450	0	0	0	-945.450
TOTAL	0.092159	0	0	-1020.72	5	1	0	-970.660
Utb	4.58E-05	0	0	-760.560	0	0	0	-760.560

Table 9: Summary results of absolute returns

Equity	ADF statistics	ADF P-value	KPSS statistic	KPSS P-values	Hurst exponent	d	SD
BOPP	-6.541	0.01	0.289	0.1000	0.580	0.1923	0.047108
calbak	-7.120	0.01	0.803	0.0100	0.652	0.228	0.040775
ETI	-6.344	0.01	0.079	0.1000	0.562	0.232	0.048172
FML	-7.130	0.01	0.760	0.0100	0.604	0.128	0.049633
GCB	-4.468	0.01	0.800	0.0100	0.650	0.327	0.034898
GGBL	-5.233	0.01	0.125	0.1000	0.603	0.276	0.026385
goil	-4.680	0.01	1.069	0.0100	0.657	0.220	0.032884
HFC	-5.813	0.01	0.207	0.1000	0.617	0.187	0.035149
MLC	-7.012	0.01	0.183	0.1000	0.582	0.064	0.04121
PBC	-4.591	0.01	0.172	0.1000	0.633	0.230	0.056347
SCB	-5.872	0.01	0.170	0.1000	0.592	0.181	0.059882
SIC	-5.910	0.01	0.439	0.0601	0.606	0.230	0.053414
SOGEGH	-4.949	0.01	0.312	0.1000	0.653	0.195	0.040197
TOTAL	-5.840	0.01	0.241	0.1000	0.610	0.049	0.055691
UNIL	-4.565	0.01	0.420	0.0681	0.606	0.137	0.024118
Utb	-5.252	0.01	1.225	0.0100	0.638	0.185	0.062969

Table 10: Summary results of squared returns.

Equity	ADF statistics	ADF P-value	KPSS statistic	KPSS P-values	Hurst exponent	d	SD
BOPP	-6.81	0.01	0.253	0.1000	0.527	0.05	0.013765
calbak	-7.16	0.01	0.486	0.0446	0.601	0.11	0.009022
ETI	-6.24	0.01	0.113	0.1000	0.584	0.18	0.008754
FML	-6.93	0.01	0.303	0.1000	0.506	0.00	0.031685
GCB	-5.42	0.01	0.519	0.0372	0.588	0.21	0.007184
GGBL	-5.24	0.01	0.141	0.1000	0.607	0.21	0.003496
goil	-5.81	0.01	0.814	0.0100	0.581	0.08	0.006477
HFC	-5.91	0.01	0.289	0.1000	0.588	0.12	0.004545
MLC	-6.80	0.01	0.126	0.1000	0.543	0.05	0.006007
PBC	-4.66	0.01	0.303	0.1000	0.638	0.34	0.01251
SCB	-6.23535	0.01	0.129476	0.100000	0.545620	0.06	0.038034
SIC	-6.26639	0.01	0.224731	0.100000	0.542888	0.10	0.021869
SOGEGH	-5.08277	0.01	0.393146	0.080110	0.626124	0.20	0.006922
TOTAL	-6.3981	0.01	0.099485	0.100000	0.507336	0.00	0.040304
UNIL	-5.13552	0.01	0.238059	0.100000	0.561255	0.06	0.002694
Utb	-4.64009	0.01	1.535405	0.010000	0.667884	0.20	0.017972

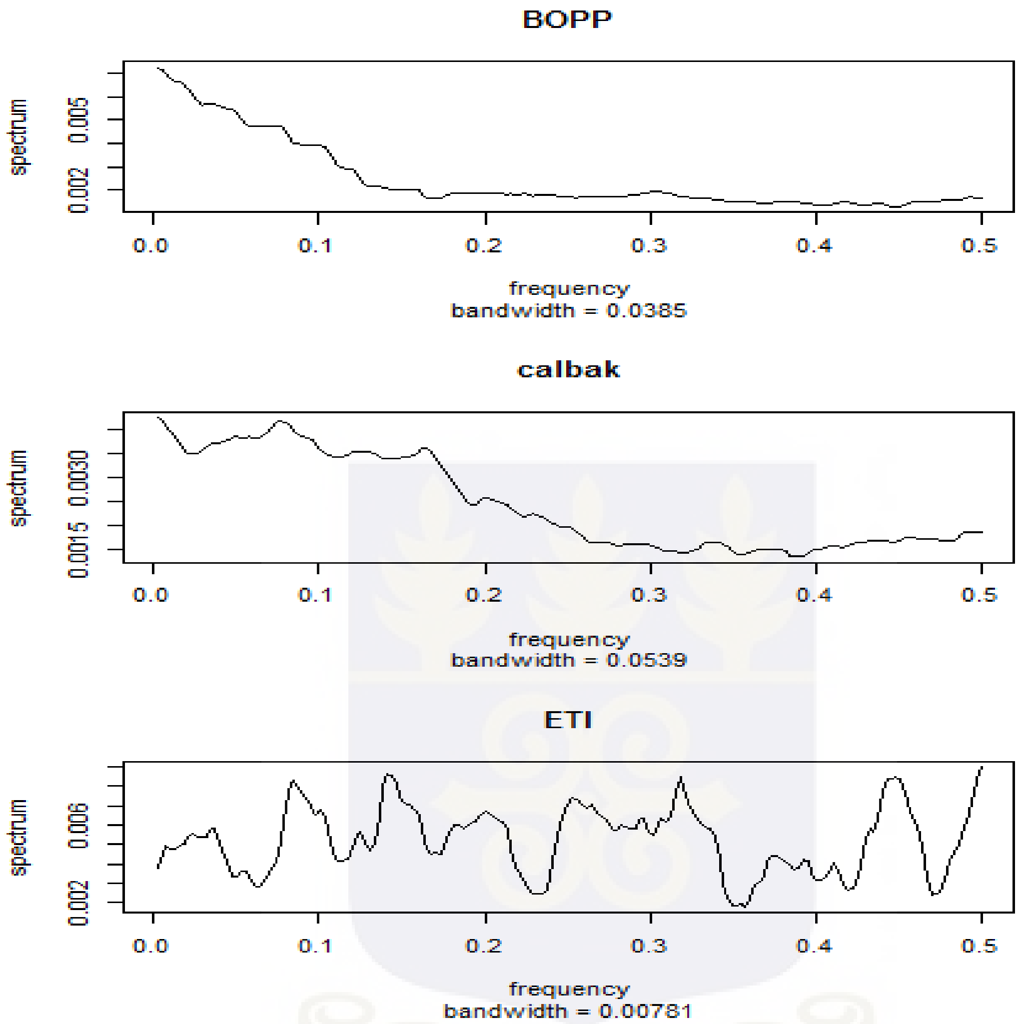

Figure 9: A spectra density graph of BOPP, Calbank and ETI.

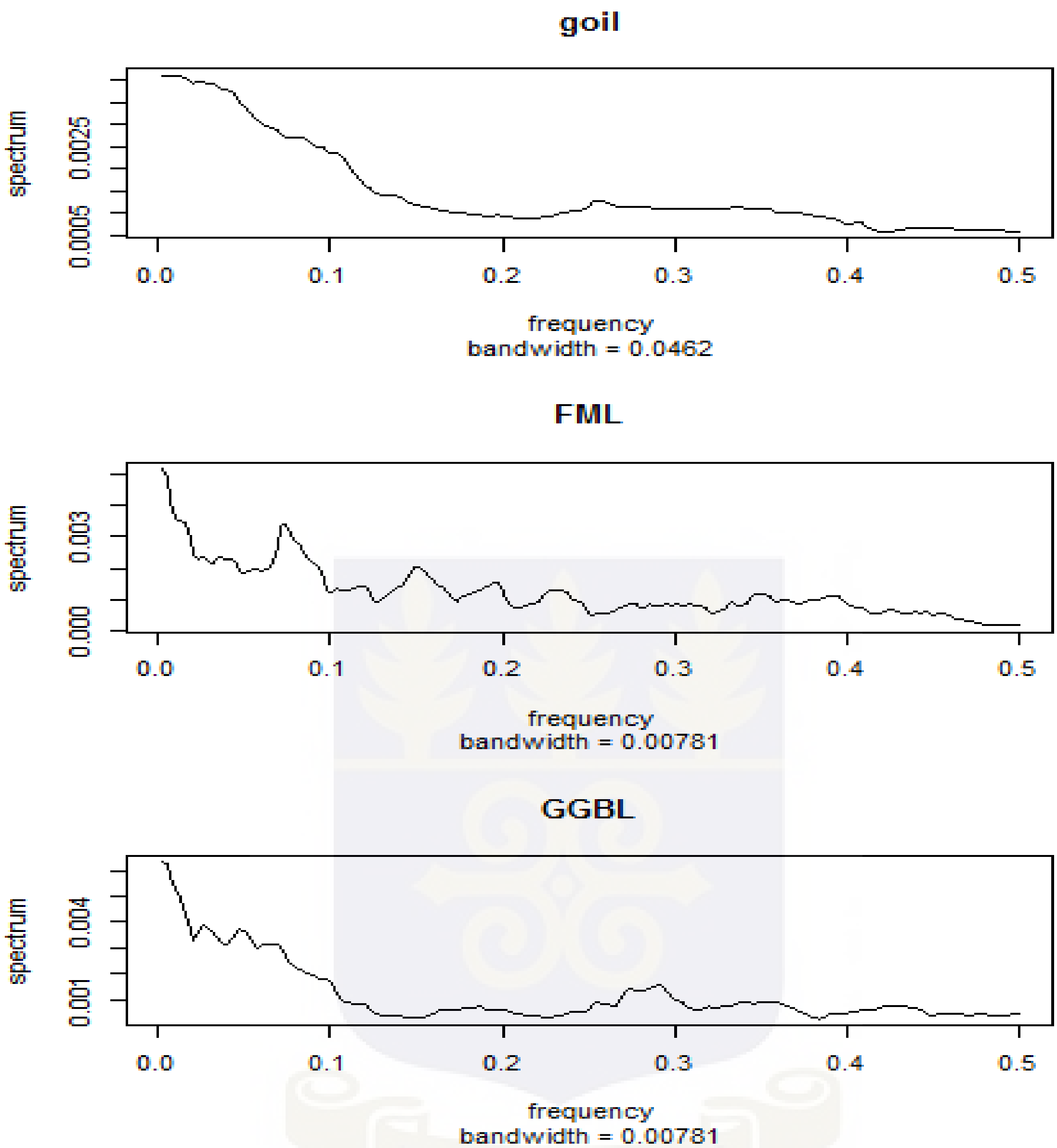

Figure 10: A spectra density graph of Goil, FML and GGBL.

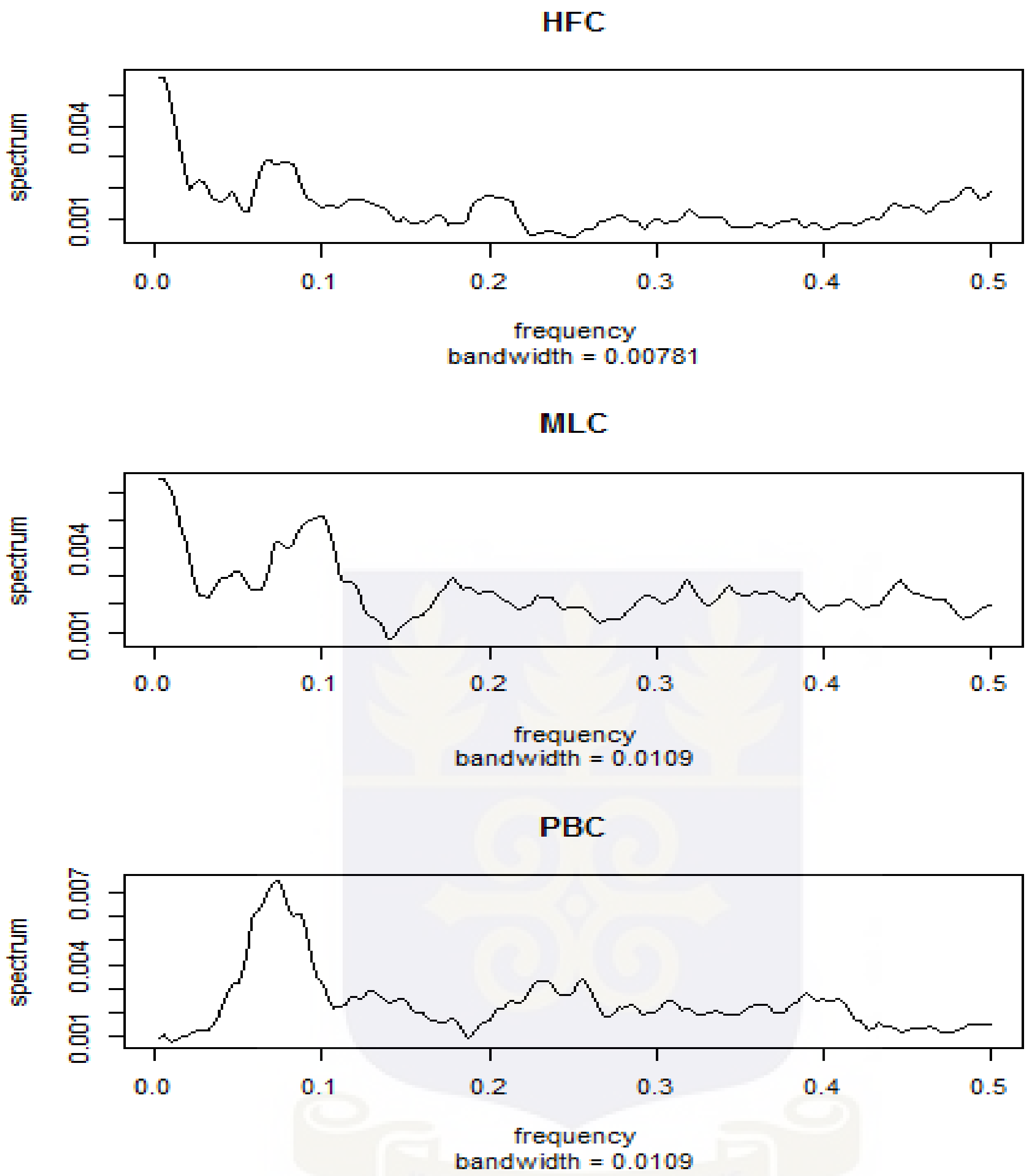

Figure 11: A spectra density graph of HFC, MLC and PBC.

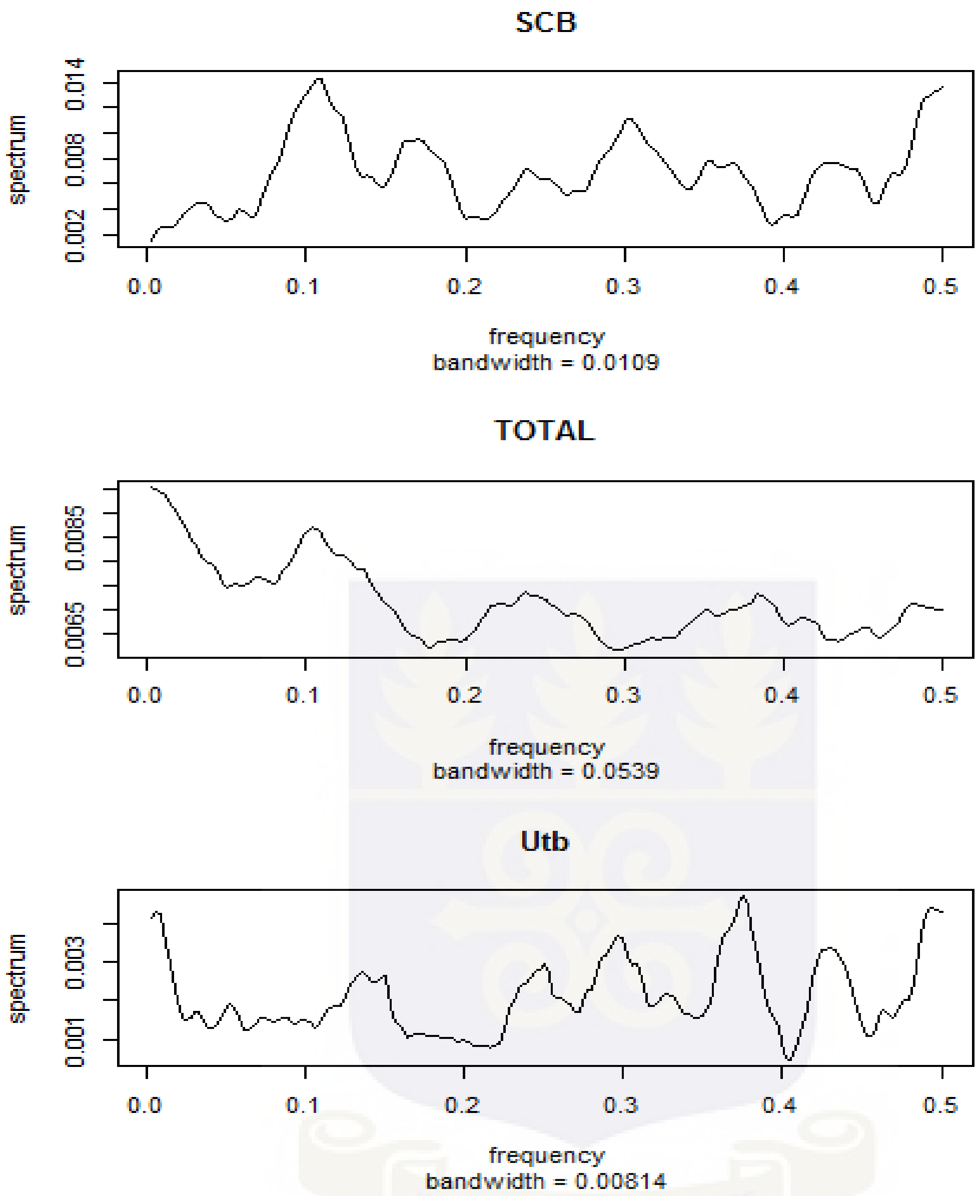

Figure 12: A spectra density graph of SCB, TOTAL and UTB.