NUMERICAL MODELLING OF SUSPENDED RADIOACTIVE SEDIMENT TRANSPORT IN A STREAM USING MATLAB

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DECLARATION

This thesis is the result of research work by Linda Sarpong from the Department of Nuclear Engineering, Graduate School of Nuclear and Allied Sciences, University of Ghana, under the supervision of Dr. Barnabas Amisigo and Dr. S.Yamoah.

Sign …… Date ……

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Sign …… Date ……

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Sign …… Date ……. ……...

Dr. S.Yamoah

DEDICATION

To the Almighty God be the glory and honour for His love, care, protection and guidance in the course of this work. I dedicate this work to my family and friends for their sacrifices, understanding and support which have contributed immensely to the success of this work. May the almighty God bless you all.

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I would like to use this opportunity to appreciate my abled supervisors, Dr. Amisigo and Dr. Yamoah for their immerse support and guidance through this research work. I s say thank you to them for their patience and tolerance. Though a lot of challenges were encountered, their hard work and good supervision encouraged me a lot in completing the work.

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TABLE OF CONTENT

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

The use of materials that contain radioactive substances has gained grounds in Ghana due to numerous benefits derived from them. These radioactive materials can be found in the areas of medicine, agriculture and industries such as mining. Though there are strict measures to ensure such material do not find its way into the environment, improper management of the waste poses a threat to the environment. To be able to understand the impact the radioactive material has on the environment, mathematical models play a very relevant role in tracking the level of pollution in any medium. This thesis was concerned with the numerical modelling for the transport of the radioactive solute material that suspends in a stream using Matlab at different velocities as a result of flooding or an accident for research purposes. The modelling was done by using partial differential equations describing relevant physical processes evolution which includes water level, dissolved and suspended substances concentration and velocities. The equation system basis are the mass conservation and momentum laws, state equation and state transport equations. The implicit finite difference scheme was used to evaluate the transport equation, Advection-Dispersion Equation (ADE) with respect to time and space. Solution algorithms for Matlab programming were developed and implemented for generating results for analysis. The results obtained showed that the model was able to simulate accurately the various levels of suspended radioactive sediment concentration changes in the flowing stream longitudinally.

CHAPTER 1

INTRODUCTION

The main reason for this chapter is to give a description of the introduction of this research work beginning from background knowledge. Concept of sediment transport modes in streams are briefly explained with emphasis on suspended sediment transport. The precise problem to be solved is clearly stated with justification as well as the aims and objectives. Finally the overall structure of this thesis is provided at the end of this chapter.

1.1 Background of Research

In developing countries such as Ghana, disposal of waste due to many human activities into streams is causing environmental pollution. Some of the sources of these wastes include agricultural, industrial and domestic activity. Substances from these wastes that contain radioactive particles could also be transported in streams. Radioactive wastes are wastes that contain radioactive material. They are usually by-products of nuclear power generation and other applications of nuclear fission or nuclear technology, such as research and medicine into the environment (IAEA, Safety guide series No. NS-G-3.2,Vienna. Pp 1-15 2005). However in this current growing urbanization and industrialization, radioactive materials are found in agriculture, medicine and industrial waste disposed of into the environment. Radioactive waste can contain radionuclides of very light elements, such as radioactive hydrogen (tritium), or of very heavy elements, such as uranium. Radioactive waste is classified as high, intermediate, or low level. Depending on the radionuclides contained in it, a waste can remain radioactive from seconds to minutes, or even for

millions of years. Some of the naturally occurring radioactive elements includes; Rubidium – 87, Uranium – 238, Thorium – 232. The following are some of other radioactive elements that could be exposed through human activities; Polonium, Astatine, Plutonium, Radon, Actinium, Technetium, Neptunium, Caesium, Cobalt, Tritium, and Indium.

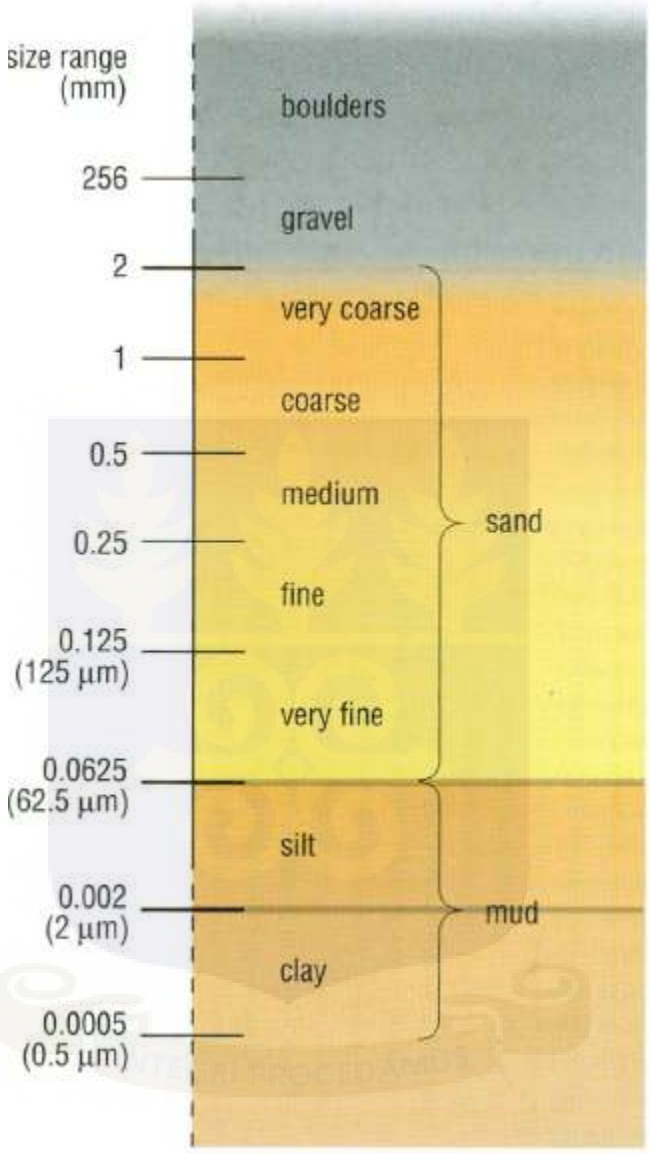
Everything on Earth is exposed to radiation. However, exposure to radiation at levels greater than natural background radiation can be hazardous. Exposure to certain high levels of radiation, such as that from high-level radioactive waste, can even cause death. Radiation exposure can also cause cancer, birth defects, skin burns, nausea, vomiting, diarrhoea, hair loss, general weakness and other abnormalities, depending on the time of exposure, amount of radiation, and the decay mechanism (http://www.pollutionissues.com). High-level radioactive waste from nuclear reactors can be hazardous for thousands of years. Radioactive waste can cause imbalance aquatic ecosystem which can result in danger to natural life. The biological effects of radiation from these wastes on aquatic life are mortality, pathophysiological, reproductive, developmental and genetic changes. Since human beings depend on aquatic life, these changes in their system can be transferred to man when consumed including the effects mentioned above damaging man's natural life (Kaur et al, 2012). Even the water that these wastes find itself is used for irrigation and drinking and all living things are affected. Radioactive waste can be categorized by its source or point of origin. Because of this, the governments of many nations have developed waste classification systems to regulate the management of radioactive waste within their borders. The proper treatment, storage, and disposal of radioactive waste are prescribed based on the waste classification system defined in a nation's laws, rules, and regulations (http://www.pollutionissues.com). In

rivers or streams, as they flow along its path from upstream to downstream, it affects residents along the stream and therefore it is necessary to study its transport.

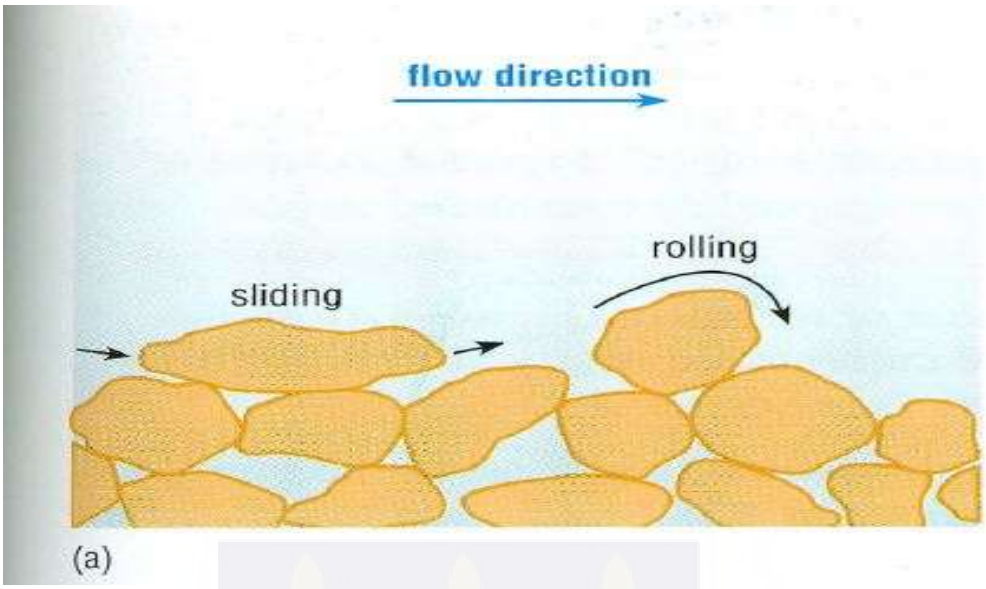
1.2 Sediment transport modes in streams

Sediments are moved about more in shallow water than in the deep sea; however the principles governing these processes are as valid in the deep ocean as they are in shallow water or anywhere else where there is moving water. When water flows over a surface very fast, sediment particles on the surface are picked up and transported, and is deposited again immediately the speed diminishes. One of the most important parameters controlling the transport and deposition of sediment is grain size which is shown in the table below (Rijn, 2007).

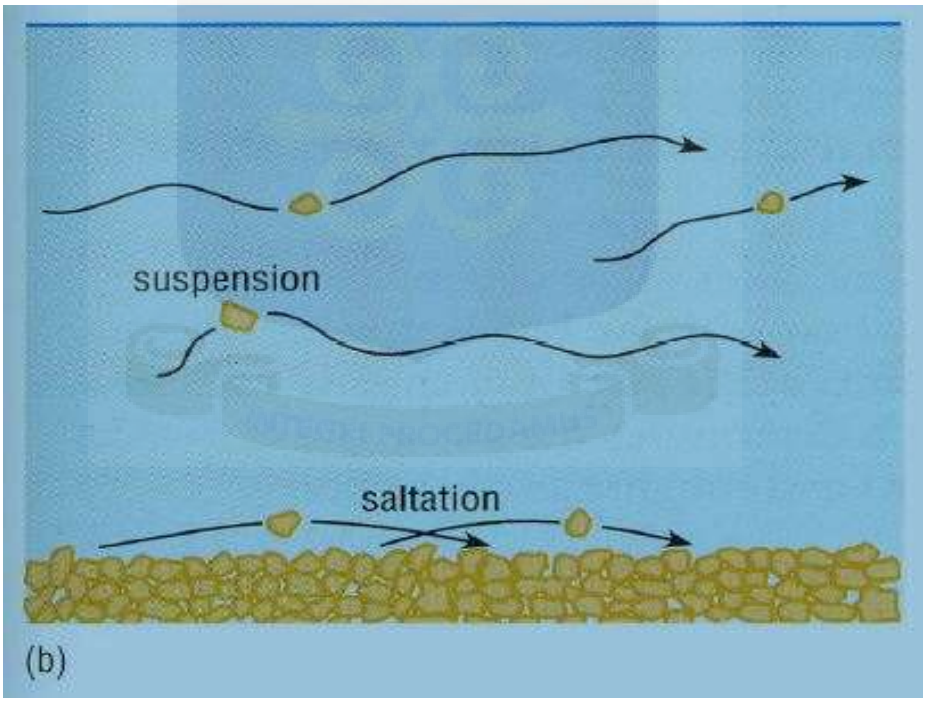
Table 1.1: The classification of sedimentary particles according to size (modified from Wentworth scale by Rijn, 2007)



Particle size and shape determine their transport behavior. Particles too large to be lifted into suspension may be rolled along the bed by the water flow.



(a) Sliding and Rolling of Sediment



(b) Saltation and Suspension of Sediment

Figure 1.1: modes of sediment transport in water (a) sliding and rolling(b) saltat ion and suspension (Rijn, 2007).

Sediment is transported by the flow in one of these three principal modes: as bedload transport, suspended-load transport or as dissolved-load transport. Dissolved load is very important in sediment studies, especially when the total mass of material is being exported from a river system; however in river geomorphology it is much less important than the particulate load. Some scientists find it useful to think of it in terms of an additional mode of transport, transitional between that moved as bedload and that forming the suspended load, called the saltation load as shown in Figure 1.2. Although it will be convenient at times to use this term, it does not represent a separate process.

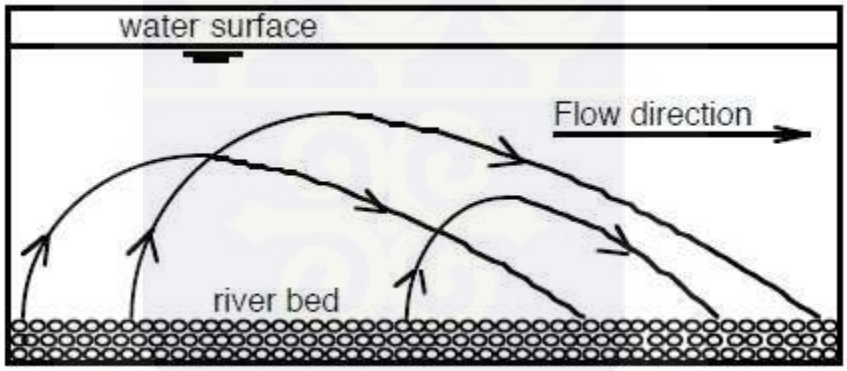


Figure 1.2: The asymmetrical trajectory of sediment grains in intermittent suspension (saltation) (Hickin, 2005)

Bedload transport refers to the particles or grains of sediment moved along the bed of a river which is at all times wholly supported by the bed itself. In other words, bedload is a material that moves by sliding and rolling, mostly as a result of shear stress exerted on the boundary of the flowing water. Bedload consists largely of coarser fraction, the sand and gravel of the sediment available to the river (Hickin, 2005). The focus of this thesis is on the transport of suspended material in rivers or streams.

Suspended-sediment transport refers to the particles or grains of sediment moved along a river within and wholly supported by the flow. In order for sediment grains to remain in suspension the upward-directed forces associated with turbulence in the flow must be strong enough to overcome the downward force of gravity acting on the grains.

As physical reasoning implies, the suspended-sediment load consists largely of the finer fraction, the fine sand, silt and clay, of the sediment available to the river. Because turbulence is generated at the channel boundary and is most intense there, suspended sediment tends to have higher concentrations and involve coarser material near the boundary and both sediment size and concentration decline as we move up through the water column towards the surface of the flow. This concept or principle definitely aligns with how the radioactive sediment is expected to behave in the flow. In most rivers the bulk of transported sediment, often 90 per cent or more, moves as suspended-sediment load. The suspended load also includes the wash load of the flow. Wash load differs from the rest of the suspended load in that its suspension is not dependent on the forces of turbulence associated with flow. Rather it can be kept in suspension for a very long time by the fine-scale turbulence associated with molecular agitation (or Brownian motion) of the water. This motion continues even if the flow ceases as it might do where a river enters a slough or lake. The wash load is confined to the finest component (clay) of the available sediment (Hickin, 2005).

1.3 Research Problem Statement

There is increasing rate of radioactive material use by medical centres, research institutes, industrial facilities, mining operations and research reactors from many developing countries such as Ghana in this present generation due to its enormous benefits.

Currently Ghana has; a 30 kW Research Reactor, 1 Gamma Irradiation Facility, 185PBq 60Co Gamma Source for Radiation Processing, 18.5PBq 60Co Gamma Source for radiation therapy at Korle Bu Radiotherapy Centre, 18.5TBq 137Cs Gamma Source for brachytherapy at Korle Bu Radiotherapy Centre, 22.2PBq 60Co Gamma Source for radiation therapy at Komfo Anokye Radiotherapy Centre, 27TBq 192Ir Source for non-destructive testing, 37TBq 60Co Gamma Source for destination inspection at Tema port, and 37TBq 60Co Gamma Source for destination inspection at Takoradi port.

It is important to state that there has not been any reported case of radioactive waste disposal into the environment in the country since there are strict regulations regarding the management of these wastes. However in the event of an accident such as flooding in the waste disposal site or the facilities that house these radioactive materials, a certain quantity of radioactive material from these facilities can get into streams and flow with the stream in suspension or on the other hand, the flood water becomes contaminated which eventually discharges into streams or rivers. As they move along in suspension in the stream they pollute the water making it unsafe for drinking and other uses such as irrigation for farmlands and other industrial purposes. It is important, to be able to predict with sufficient accuracy the flow, transport, and fate of the contaminant that have been discharged to the water course. The level of concentration determines how harmful these radioactive

sediments could be. To determine radionuclide concentrations, the river channel and flow characteristics (such as river discharge, velocity, and width and dispersion coefficients) must be determined (IAEA, Safety Reports Series No. 19. Vienna. Pp 34-36, 2001).

For proper planning to prepare for such situation, managers and/or decision makers need reliable support tools for assessment and to predict consequences of their decisions. Computational estimation of concentration of radioactive material in a stream or river can be used to address this issue. There is therefore the need to develop a computational model capable of simulating the transport of a radioactive material as it moves in suspension from the upper section of a stream or river to its mouth to understand the behaviour of the radioactive material in the stream in order to predict both its temporal and spatial distribution to provide information for proper remedial action. This is the main focus of the thesis.

1.4 Relevance and Justification of the Research

Records indicate that about 15 institutions such as the Kole-Bu teaching hospital, Komfo Anokye teaching hospital, Ghana Atomic Energy (GAEC), and mining companies such as Goldfields and AngloGold Ashanti use radioactive materials. These materials contain traces of carbon-14, caesium-137, cobalt-60, tritium-3, indium-125, etc. which have the potential of getting into streams if they are not properly managed or in the event of an accident or incident such as flooding.

Both short-lived radioactive elements, such as iodine-131, and longer-lived elements — such as cesium-137, with a half-life of 30 years — can be absorbed by phytoplankton,

zooplankton, kelp, and other marine life and then be transmitted up the food chain, to fish, marine mammals, and humans (Grossman, 2011). It has been recognized that exposure to high levels of radiation can cause damage to the tissues of the human body and that exposure to radiation has the potential for the induction of hidden destruction.

It is therefore essential that activities involving radiation exposure, such as the transport and disposal of radioactive material, be subjected to certain standards of safety in order to protect those individuals exposed to radiation and the environment in general. For radioactive material in stream, field measurements of concentration and velocity at every point will be tedious. Hence modelling will provide unlimited degree of analysis to help predict future events. Research in the field of computational estimation of in-stream pollutant concentration has progressed. Among the reasons are: better technological support for monitoring; capability of data processing tools to deal with impressive quantity of data; and enhanced and accurate modelling methods. Nonetheless, there still are important issues that require attention, for instance the wider applicability and transferability of mathematical models or the use of new software. This research project will develop a model to predict radioactive concentration in stream in case of an accident. Predicting the concentration of radioactive sediment in streams with time will provide necessary information for proper remedial actions to help conserve water quality and its safe use.

1.5 Research Aim and Objectives

Serious pollution may happen if the capacity of a stream to transport and disperse contaminant is overestimated or underestimated if valuable resources are not optimally used, which would result in unnecessary expenditure on treatment facilities. Therefore in solving water pollution problems, studies on dispersion processes in rivers are widely used by environmental scientists/engineers, hydrologists and hydro-dynamicists. It is established that no water quality management study which is aimed at achieving optimum usage of a stream can bypass the need for a reliable means of predicting the dispersion characteristics of the water body hence the time of travel of a pollutant in a stream, the rate at which the pollutant spreads out, the decrease in peak concentration and the resulting concentration patterns of pollutant are the important variables that must be properly understood (Sooky 1969).

Even though there have been many studies on longitudinal dispersion of pollutants in rivers and natural streams, the overall aim of this thesis is to model the transport of radioactive sediment suspended in stream flow. The specific objectives of the research are:

- To develop and test the 1D numerical modelling of flow of suspended radioactive sediment transport in stream
- To determine the concentration of the radioactive material as it is transported in the stream.
- To contribute to the understanding of the hydrodynamics of radioactive sediment transport suspended in stream.

1.5 Structure of the Thesis

This thesis is organized in 5 Chapters with references listed at the end.

Chapter 1 gives introduction to this thesis providing an overview understanding of the concept of this research. Also presented here is explanation for the various modes of sediment transport defining the scope; following is the problem statement, justification and objectives and outline of the structure of this thesis.

Chapter 2 presents the literature review where various modes of transport of solute are explained with emphasis on advection dispersion. Numerical and analytical approaches to modelling a sediment transport also elaborated. Work done involving simulation of solute transport was also mentioned.

Chapter 3 which describes the method used in solving the problem presented in chapter one includes the description of the problem, mathematical modelling, streamflow routing, flow chat algorithm and concluding remarks for this chapter.

Chapter 4 presents the results and discussion of the numerical simulations. This chapter is divided into two parts. The first part presents results obtained in this research work. The second part however presents a model verification study by comparing with an analytical solution.

In summary the conclusion and recommendations are finally presented in chapter 5.

CHAPTER 2

LITERATURE REVIEW

This chapter is divided into three (3) sections. The first section involved the mechanism of radioactive solute transport where the possible modes of transport which include advection, diffusion, dispersion or convection were explained and briefly explained further the dispersion coefficient. The second section directly focused on numerical methods and section went further to describe the One-Dimensional Transport with Inflow and Storage (OTIS). The last section which is solute transport models presented some work done by various scientists and engineers related to this thesis.

2.1 Transport of solute in a stream

2.1.1 Mechanism of solute transport in water

Several mechanisms exist for the transformation and transport of contaminants in water. Within the water body, the possible modes of transport include advection, diffusion, dispersion or convection.

Advection occurs when a contaminant moves as a result of fluid motion where concentration usually remains unchanged. Advection can be referred to as a flow that is unidirectional and does not change the identity of the substance being transported. Basic examples of advection are the downstream transport of a contaminant in a river or through a conduit (Chapra, 1997).

Diffusion on the other hand occurs when a contaminant is transported along a concentration gradient from a region of high concentration toward a region of lower concentration

(Tchobanoglous and Schroeder, 1985). Diffusive transport occurs because of random mixing or motions in the water. Diffusion can be modeled by using Fick's First Law:

$$J_x = -D\left(\frac{dc}{dx}\right) \quad \text{.....2.1}$$

where: J_x is the mass flux in the x direction, D is the diffusion coefficient which coefficient is used to quantify the rate of the diffusive process and dc/dx is the concentration gradient.

Dispersion is a third possible mode of transport within a water body. Dispersion and diffusion are similar because during the flow, the contaminant mixes in the water body. However, whereas in diffusion mixing of contaminants in water is due to random motions in the water over time, mixing of contaminants in water as a result of dispersion is rather due to velocity differences in a given space (Williamson et al, 2001; Chapra, 1997).

Dispersion is the major mechanism responsible for the broadening of a cloud of dissolved substance as the center of the cloud is being carried along a stream by advection. Mechanical dispersion is mixing that occurs as a consequence of local variations in velocity around some mean velocity of flow. The mechanical dispersion is calculated as the water velocity multiplied with the dispersivity. The dispersivity is normally expressed as a length (e.g. meters or centimetres). The dispersivity in the direction of flow (longitudinal dispersivity) is typically larger than the dispersivities in the directions perpendicular to the flow direction. Dispersion is a process that occurs to all constituents dissolved or suspended in water. Dispersion occurs due to the flow and hence only in flowing water, while diffusion occurs irrespective of whether the water is flowing or not. Taylor in 1953 and 1954 defined dispersion as the mixing process due to the interaction between lateral diffusion and velocity gradient. With this background information from Taylor, Elder

(1959) derived a value for the longitudinal dispersion coefficient for an infinitely wide steady turbulent open-channel flow. His value however was found to be too low comparatively to actual measurements in natural streams (Fischer et al. 1979). To account for the transverse variation of velocity, Fischer (1967) continued Elder's analysis to 3D channels and obtained results in better agreement with measurements (Fischer 1973). Also, Liu (1977) developed an expression for the longitudinal dispersion coefficient in rivers and streams where the dispersion is dominated by lateral gradient rather than vertical gradient of velocity based on Fischer's research. Sayre (1975) applied numerical solutions to Aris' method of moments (Aris 1956) to study dispersion of sediment particles in a 2D channel. Many others have also studied effects of bends, channel irregularities, dead zones, and flow unsteadiness (Fischer et al. 1979). Particularly, Liu and Cheng (1980) presented a modified Fickian model based on a time dependent dispersion coefficient and a timescale that essentially absorbs the effects of channel irregularity and dead zones. In natural streams the non-uniformity in cross section and bottom slope, and the meandering nature of the streams can cause a much larger longitudinal dispersion coefficient than given by Elder (1959) and Fischer (1967).

This thesis focuses on tracking the concentration of the radioactive sediment in suspension due to velocity differences in a given space and time in the stream which is more of dispersion. Solute transport is mathematically described by the advection dispersion equation:

$$\frac{\partial c}{\partial t} = -\frac{\partial}{\partial x_i}(cv_i) + \frac{\partial}{\partial x}\left(D_{ij}\frac{\partial C}{\partial x_j}\right) + R_c \quad \text{i,j=1,2,3}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.2$$

where: c is the concentration of the solute, R_c is the sources or sinks, D_{ij} is the dispersion coefficient tensor, and v_i, the velocity tensor.

2.1.2 Radionuclide transport in surface water

Many important physical and chemical processes affect radioisotope contaminant behaviour and transport. These include wind and stream flow/water movement, chemical-specific factors affecting partitioning between solid and dissolved phases, interactions between the dissolved and particulate phase, radioactive material within sediment, water column or biota and chemical partitioning or transformation in sediment, water, or biota (Chapman, 1992). Some of the ways of radionuclide transfer in water and sediments are as follows:

- Dispersion of dissolved radionuclides by water flow. This process is driven by the flow hydraulics which includes advection and turbulent diffusion.
- Dispersion of dissolved radionuclides inherent to suspended sediment. This process is also driven by the suspended sediment transport in the river flow. A rough quantitative estimation of the ratio of the flux of radionuclides carried by the suspended sediment to the horizontal flux of radionuclides in dilute can be done by the formula

R= $\sum_{t=1}^{n} K_{di}\ S_i$ ……(2.3)

Where n is the number of typical grain size fractions of the suspended sediments by which the suspended sediment fractional distribution (i.e.. Clay, silt, sand) could be represented. S_i [Kg/m^3]-the concentration of i-fraction of suspended sediment in the river water;

K_{di} [m^3/Kg]-the equilibrium distribution coefficients for i-fraction of suspended sediments. For instance , in plain rivers a typical range of the total suspended sediment concentration is

10^1 to 10^2[Kg/m^3]. It is clear that radionuclide suspended sediment transport is very important for Caesium-137 (typical K_{di} value range is $1^{-10}m^3$/Kg).

2.2 Numerical Methods

Analytical methods can fail if: (1) The PDE is not linear and cannot be linearized without any serious affect on the result, (2) The solution region is complex, (3) The boundary conditions are of mixed types, (4) The boundary conditions are time-dependent and (5) The medium is inhomogeneous or anisotropic (Clough R.W., 1980)

Several approximate numerical analysis methods have evolved over the years; a commonly used method is the finite difference scheme. Other methods include finite elements, finite volumes, or spectral methods. The familiar finite difference model of a problem gives a pointwise approximation to the governing equations. This model (formed by writing difference equations for an array of grid points) is improved as more points are used. Finite difference methods are one means of obtaining approximate solutions to ordinary or partial differential equations (Glasstone and Sesonske, 1981). Most finite difference methods are somewhat limited in that they require structured grids, which restrict the class of mechanics problems which can be discretized and thus solved (Everstine, 2010). Nevertheless, there are still many problems which could be solved with finite differences. Furthermore, finite differencing lends itself quite easily to Taylor series analysis of the truncation error, and this fact is exploited in the high-order compact (HOC) method to construct rigorous high-

order accurate approximations (Chapman, 1992). The finite difference techniques are based upon the approximations that permit replacing differential equations by finite difference equations. These finite difference approximations are algebraic in form, and the solutions are relation to grid points. Thus, a finite difference solution basically involves three steps:

- Dividing the solution into grids of nodes.
- Approximating the given differential equation by finite difference equivalence that relates the solutions to grid points.
- Solving the difference equations subjects to the prescribed boundary conditions and/or initial conditions.

Agusto and Bamigbola (2007), in the study of contaminant transport model used finite difference by complementing the Crank-Nicolson numerical scheme, which was found to be simple and accurate. The finite element method is a numerical analysis technique for obtaining approximate solutions to a wide variety of engineering problems. Although originally developed to study stresses in complex airframe structures, it has since been extended and applied to the broad field of continuum mechanics. Because of its diversity and flexibility as an analysis tool, it is receiving much attention in engineering. With finite difference techniques we can treat some fairly difficult problems; but, for example, when we encounter irregular geometries or an unusual specification of boundary conditions, we find that finite difference method, which envisions the solution region as built up of many small, interconnected sub regions or elements. A finite element model of many small, interconnected sub regions or elements. A finite element model of a problem gives a piecewise approximation to the governing equations. The basic premise of the finite

element method is that a solution region can be analytically modeled or approximated by replacing it with an assemblage of discrete elements (White, 2004). Since these elements can be put together in a variety of ways, they can be used to represent exceedingly complex shapes (Zienkiewicz and Cheung, 1982).

2.2.1 One-Dimensional Transport with Inflow and Storage (OTIS)

The OTIS solute transport model was developed to simulate the transport of solutes in streams and rivers in which one-dimensional transport may be assumed (Runkel et al, 1998). Advection, the downstream transport of solute mass at a mean velocity, and dispersion, the spreading of solute mass due to shear stress and molecular diffusion, are considered in most mechanistic models of stream water quality and solute transport. Consideration of these important mechanisms leads to the familiar advection dispersion equation. Within the OTIS model, additional terms are added to the advection-dispersion equation to account for the effects of transient storage and lateral inflow. Transient storage has been noted in many streams, where solutes may be temporarily detained in small eddies and stagnant pools of water that are stationary relative to the faster moving water near the center of the channel (Haggerty and Gorelick, 1995).

In addition, significant portions of the flow may move through the coarse gravel of the streambed and the porous areas within the stream bank. The travel time for solutes carried through these porous areas may be significantly longer than that for solutes traveling within the water column. Lateral inflow is any water that is added to the stream due to ground-water inflow, interflow, or small springs (Avetisov et al, 2001). These flows act to dilute

(or concentrate solutes in the stream channel if they carry solute concentrations that are lower (or higher) than the stream-solute concentration (Harleman, 1986).

The OTIS model is formed by writing mass balance equations for two conceptual areas: the main channel and the storage zone. The main channel is defined as that portion of the stream in which advection and dispersion are the dominant transport mechanisms.

The storage zone is defined as the portion of the stream that contributes to transient storage; that is, stagnant pools of water and porous areas of the streambed (IAEA Safety Guide Series No. NS-G-2-3. Vienna. Pp 5-20,2005).

Water in the storage zone is considered immobile relative to water in the stream channel. The exchange of solute mass between the main channel and the storage zone is modeled as a first-order mass transfer process (White, 20004)

Consideration of the hydrologic processes discussed above gives rise to mass conservation equations for the main channel and the storage zone as (Runkel et al, 1998):

1D, 2D, and 3D models are widely used to analyze flow hydrodynamics and sediment transport. The choice of a particular model dimension depends on the type of problem being analyzed. For example, 2D or 3D models are applied in the investigations of flow problems such as scour and structural protection, navigation channels, intake structures, river restoration etc. that require detailed knowledge of flow, sediment transport and channel evolution. 1D models are used to study sediment transport, scour and deposition in channels where the lateral variations of hydraulic and sediment conditions can be ignored, and have wide application in the simulation of morphological changes that typically occur in a one year or three period (Minesota dept. of transportation, 2013).It is also noted that

the capacity to consider longitudinal flows using 1-D modeling approach gives ability to perform well with limited topographic data and time efficient solution algorithm making 1 D modeling approach a favorable choice for real-time operational models (Ahmad, 1999). Thus, in this thesis, due to limitations posed by the available data, a 1D model of the transport of radioactive sediment suspended in a stream is presented.

2.3 Solute Transport Models

Mathematically models have been derived to simulate circulation and dissolved substance transport in water bodies such as rivers, coastal waters and estuaries. Usually these models comprise of one or more partial differential equations which describes relevant physical processes evolution (velocities, dissolved substances concentrations and water level) occurring in water bodies. The mass conservation and momentum laws and state equations and constituents transport equations are the fundamental equations upon which these models are based (Meier ***et al***, 2003).

Advection-diffusion equation (ADE) has been used to describe solute transport due to combined effect of diffusion and convection in the medium (Boris et al., 2001). It is a partial differential equation of parabolic type, derived on the principle of conservation of mass using Fick's Law (Kumar et al., 2009). Generally, radionuclide transport and its behaviour in surface water can be expressed in the following three dimensional advection-diffusion equations (IAEA, Safety Guide Series No. NS-G-2-3. Vienna. Pp 5-20, 2005).

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = Ex\frac{\partial^2 C}{\partial x^2} + Ey\frac{\partial^2 C}{\partial y^2} + Ez\frac{\partial^2 C}{\partial z^2} - \lambda iC + S \ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

Where C is radionuclide concentration (Bq/m^3); u, v, w are flow velocities in the x, y and z directions, respectively (m/s); S is radionuclide addition or subtraction, for example production of a daughter product (Bq/m^(-3) s^(-1)); t is time (s); x, y, z are longitudinal, lateral and vertical directions, respectively, in Cartesian coordinates (m); Ex, Ey, Ez are dispersion coefficients in the x, y and z directions, respectively, , (m^2/s); λ is radionuclide decay constant (s^{-1}).

Equation 2.4 which represents the Advection Dispersion Equation explains the behaviour of the radioactive material. There is a fundamental law used in the development of the contaminant model for streams, which is the conservation of mass along the longitudinal axis (flow direction) of a surface water body, such as stream and rivers. The stream surface water is represented by one-dimensional (longitudinal) discretization for mass balance, where mass concentrations are assumed to be uniform across the width and depth of the stream. A constituent mass balance is performed for the water column and sediment bed. The bed is treated as a single active layer. The bed layer for each longitudinal segments interacts only with the water column immediately above the bed segment (Fant *et al*, 2009).

Analytical and numerical solutions along with initial and boundary conditions help to understand the contaminant or pollutant concentration distribution behaviour through an open medium like rivers on the basis of which remedial processes to reduce or eliminate the damages may be enforced. Analytical solutions are obtained by using transformed variables; the Laplace transform, the Fourier transform and Bessel functions (Toride, et al., 1995).

For unsteady water flow conditions, spatial and temporal variability of river properties, or complicated initial and boundary conditions are considered in such model. In solving these equations, numerical approaches could be used. In numerical approaches, spatial and temporal domains of the problem are treated as a mesh of discrete points or cells. The concentration for each discrete node or cell can then be calculated at each discrete time step by applying numerical techniques (Everstine, 2010).

Nasir Hassan, 1992 did a research on longitudinal dispersion of pollutants in natural streams using the aggregated dead-zone approach. He finally made it clear that amongst the most successful of these alternative models and the most recent is the aggregated dead-zone model (ADZ) developed at that time by Young (1982), Beer & Young (1983), Young (1983) and Young & Wallis (1986). The model argues that, while Fickian-type dispersion is probably taking place to some extent, most of the dispersion observed in natural streams arises because of the dead-zone effects caused by irregularities in river beds and banks. In other words, it is assumed that the cumulative effects of the dead-zones often dominated the observed dispersion and they could be integrated in their total effect to yield an "Aggregated Dead-Zone (ADZ)" whose "residence time" then defines the dispersion properties associated with the stretch of river being studied.

Hassan 1992, made the following conclusions that recent approach to water-quality modelling reviewed in his work was based on the assumption that the longitudinal transportation and dispersion of pollution were governed not by the classically assumed mechanisms of the distributed parameter, Fickian-Diffusion model, but by the aggregative effects of the dead-zones in the river whose "residence time" then defined the dispersion properties. These dead zones, which arose from various factors associated with the non-

uniformity of the river, tend to be characterized by natural channels and lead to transient retention of a pollutant and increase of the dispersion processes in the stream. The resultant ordinary differential equation, Aggregated Dead-Zone (ADZ) model could have been simplified into discrete form, where the dispersive characteristics could be described by parameters and the presence of the non-integral time delay or additional parallel dead zones relevant parameters and simulated using scientific computing skills which this thesis is focused on.

The results from Hassan in Figure 2.1, Yorkshire confirmed when ADZ model was fitted to data from a wide variety of river channels in the UK (Young & Wallis 1986) indicated that the parameters of the ADZ model (namely the advective transport time delay b, the residence time T and the ADZ volume V.) were all relatively smooth functions of stream discharge Q. The results also suggested that the measure of the dead-zone effects in relation to the total water volume in the reach at any time, appeared to be relatively independent of discharge (Hassan, 1992)

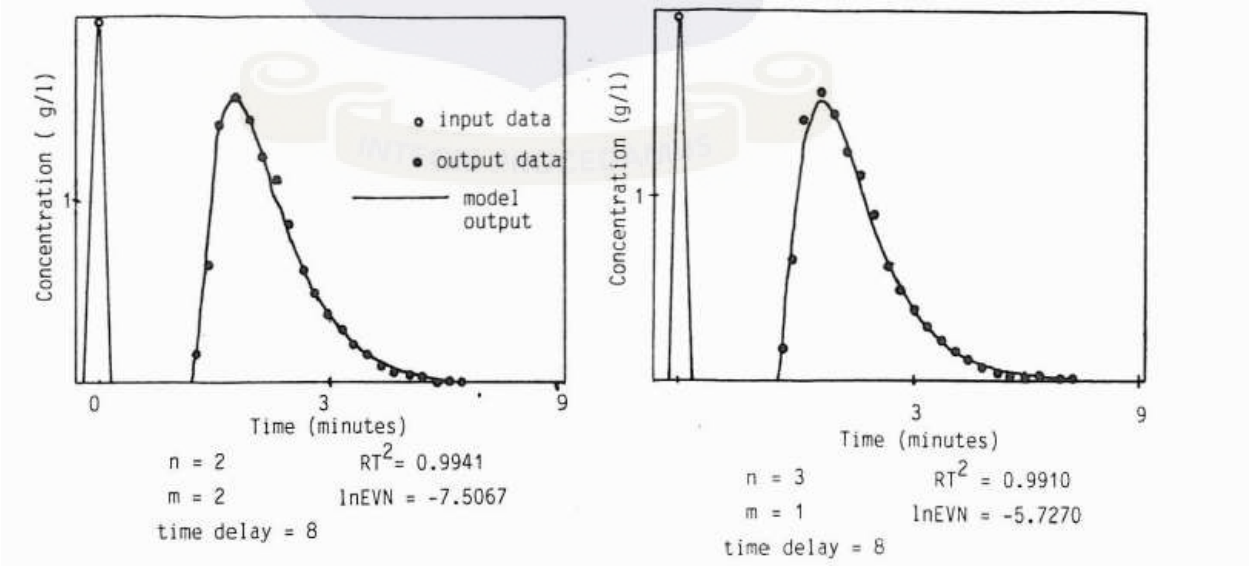


Figure 2.1: Modelling results for crimple beck

Ramireddygari et al. (1996) did a research on surface water transport of trace elements and described contaminant transport processes over land surfaces in water which they considered degrading and non- degrading materials based on the application of Advection Dispersion Equation. This was feasible because the transport equation could easily be extended to represent reactive and decaying constituents (Langevin *et al*., 2005). However this thesis focuses on suspended none reactive and decaying constituents.

A non-degrading heavy metal was used and a physically-based model was developed for rainfall, runoff, erosion, and solute transport processes on land surfaces under time-varying rainfall events. The diffusion wave approximation to the full Saint-Venant equations for overland flow was used to describe the overland flow dynamics. It was assumed that the soil chemicals are uniformly distributed along the slope and that chemical flux from the soil surface to the overland flow is uniform (Ramireddygari et al., 1996).

Results obtained (Figure 2.2 and 2.3) for the contaminant transport indicated that concentration hydrographs increased gradually with flow depth during rising stage but gradually with a faster rate during the falling stage of overland flow(Ramireddygari et al., 1996). Though the model performed well when compared to other existing results in literature, it was developed for rainfall-runoff hence restricted to overland flow but not

streams and rivers (Ramireddygari et al)

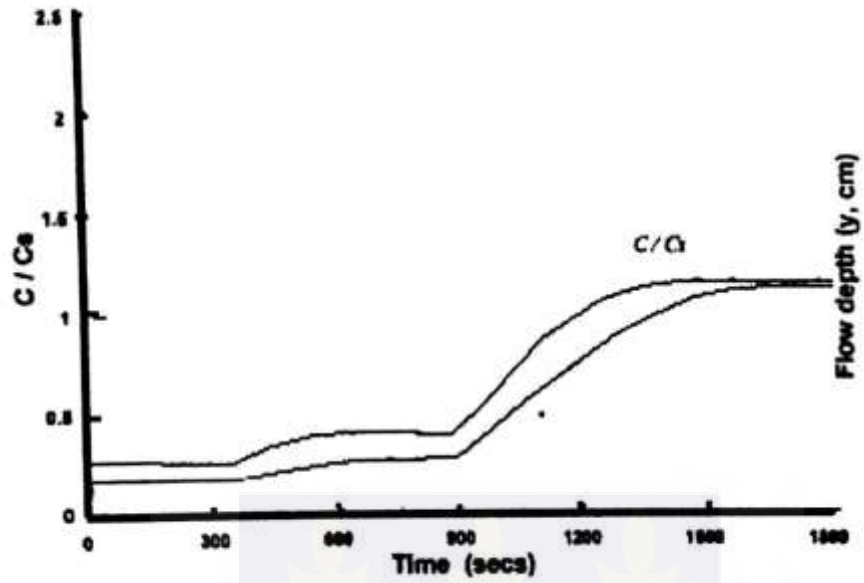


Figure 2.2 : Overland flow depth and chemical hydrographs at the outlet (Ramireddygari et al., 1996)

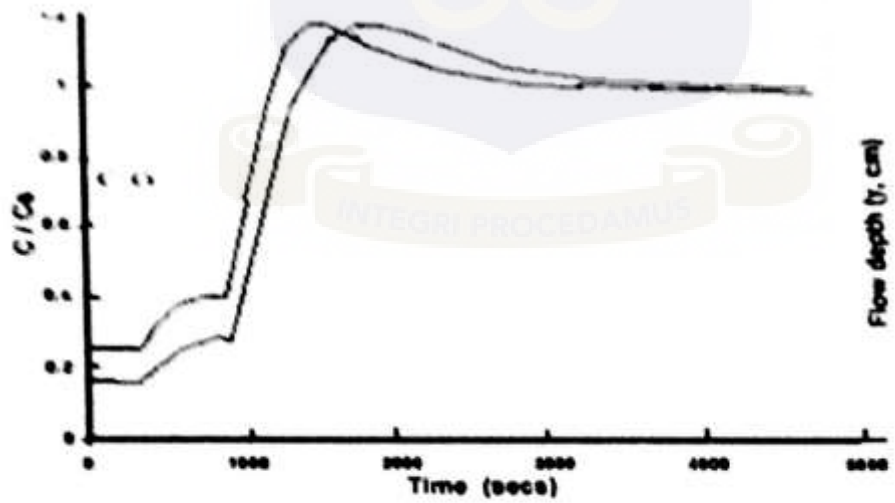


Figure 2.3: Overland flow and chemical hydrographs results up to t = 5,000 secs (Ramireddgyari et al., 1996).

Kachiashvilli ***et al***, 2006 researched on mathematical modelling and computer simulation of diffusion and transport of chemicals in rivers. They considered the transport of NO3 and PO4, in two rivers in western Georgia flowing into the black sea. River khobistskali subject to pollution sources schkhomuri and chanistskali, river choga polluted with NO3 were the two rivers considered. Their main objective was to put together in one effort the full cycle of mathematical modelling, numerical methods and approximations, computer implementation, and simulation of time dependent transport of substances in a water media during the eutrophication process.

The software they developed was based on modelling distribution of the concentration of polluting substances in rivers due to multiple sources and polluting substances. Also, one of their main goal was to give the practitioners in environmental and in particular in water protection agencies a user friendly, accurate and relatively simple software tool hence their developed code provided a reliable tool for quick analysis of water quality in rivers. Figures 2.4 and 2.5 were some of the simulated results.

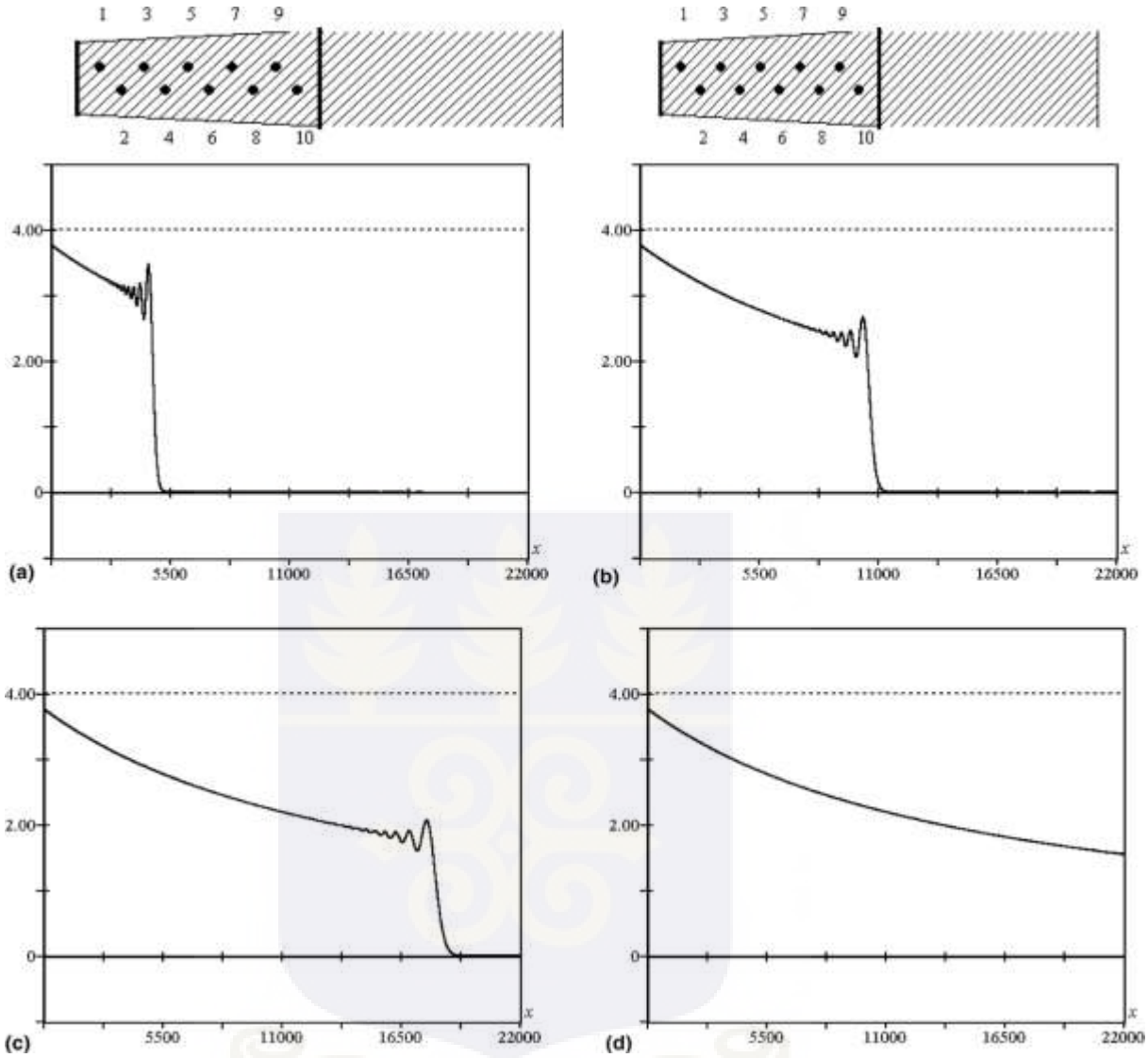


Figure 2.4. Plots of concentration of polluting substance in the river Choga depending from longitudinal coordinate ξ at 1D model: (a) t = 2 h13 min 36.02 s; (b) t = 4 h 27 min 12.03 s; (c) t = 6 h 40 min 48.05 s; (d) t = 8 h 54 min 24.06 s.

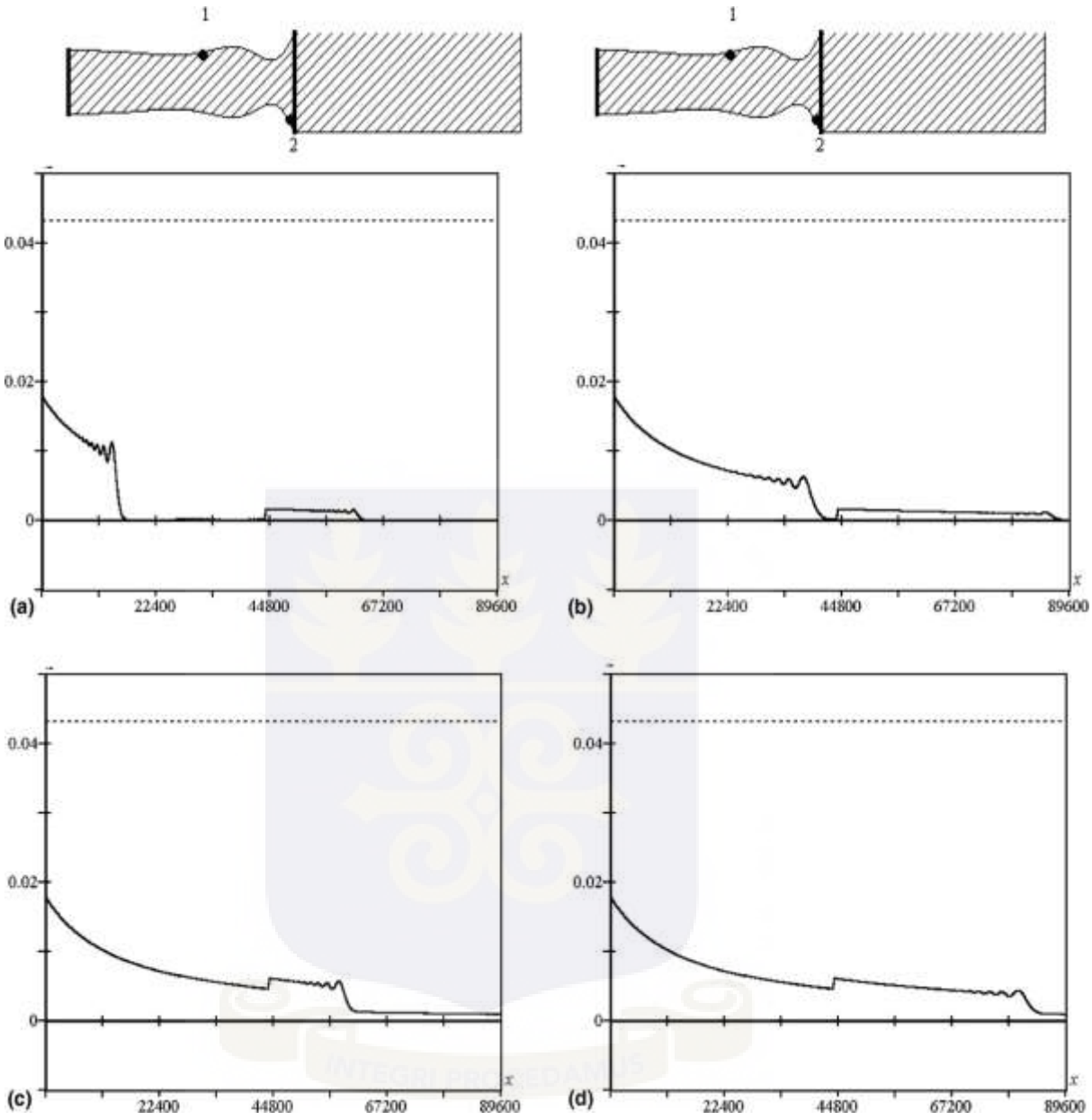


Figure 2.5. Plots of concentration of polluting substance in the river Khobistskali depending from longitudinal coordinate ξ at 1D model: (a)t = 5 h 40 min 49.25 s; (b) t = 11 h 21 min 38.50 s; (c) t = 17 h 02 min 27.75 s; (d) t = 22 h 43 min 17.00 s.

The results concluded that, because the pollution sources were spread at 1000 m of each other their influence on the river was relatively weak (the volume of waste waters and the pollution level were small in comparison with the water volume of the river). On the other hand, the increase in the water flow rate of the river is enough so that the effect of these sources on the river is not accumulated.

In summary from Kachiashvilli ***et al***, the modeling results were used to estimate the degree of influence of agricultural activities along the river banks on the pollution of Choga and Khobistskali in a particular interval. The obtained results showed the good quality of the applied mathematical models and the software package for its practical application for simulation of the pollution processes in rivers due to multiple sources. Hence this thesis also considers multiple sources of pollutant to understand the behavior of each pollutant at the same length of stream within the same time.

Jaiswal et al, 2009 also presented a work on analytical solutions for temporally and spatially dependent solute dispersion of pulse type input concentration in one dimensional semi-infinite media. It was observed that the temporal dependence of increasing nature caused faster solute transport through the medium than that of decreasing nature. Similarly the effect of the inhomogeneity of the medium on the solute transport was studied with the help of a function linearly interpolated in a finite space domain. Some obtained results are presented in figure 2.6

a.

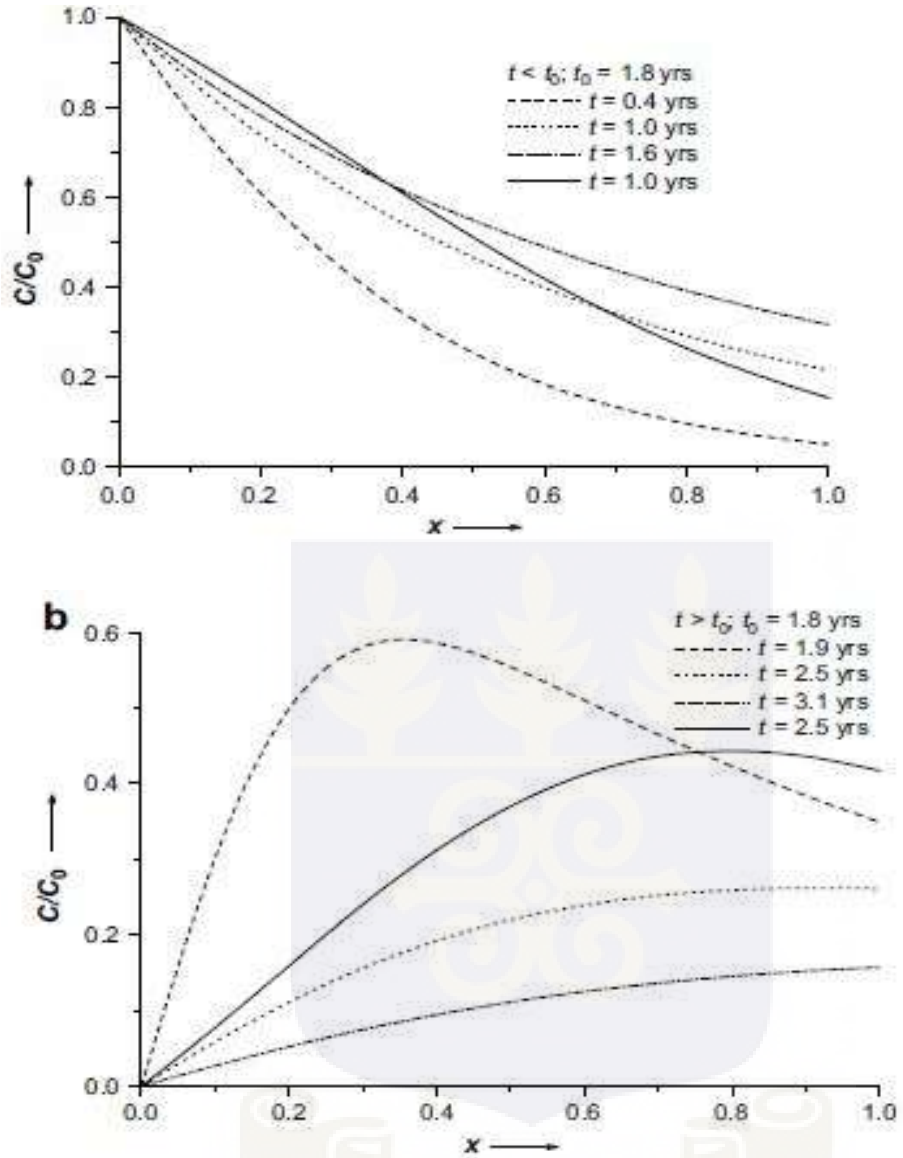


Figure 2.6a and b: show the temporal dependent concentration dispersion pattern along uniform flow(Jaiswal *et al*, 2009)

Though a lot of research works have been done in this area, literature indicates gaps that need to be addressed when determining the concentration of radioactive contaminant transport in surface water.

One of the major drawbacks involves the complete disregard for the effect of concentration of the contaminant in the river due to velocity. Velocity at any particular point along a stream contributes to the concentration depending on what it is also carrying or its concentration. Chapter three of this thesis presents the methodology for the analysis of the transport of suspended radioactive sediment in surface water with the drawback taken care of by employing OTIS in the procedure for the modeling.

Equation (2.2) will be used for the analysis of the transport of the radioactive material in the stream flowing in a longitudinal direction

CHAPTER 3

METHODOLOGY

In this Chapter, the detailed numerical approach to model the transport of radioactive sediment in suspension in a stream is presented. The modelling procedure, which includes the formulation of the problem, verification and calibration are explicitly explained. The Advection dispersion equation (ADE) which governs the domain of this work was solved using the implicit finite difference scheme. The numerical solution from the finite difference scheme was written in an algorithm and implemented in Matlab script.

3.1 Modelling Procedure

In order to simplify the numerical procedure for modelling the transport of radioactive sediment suspended in a stream or river and to better understand the phenomenon of such processes, the following steps were used, namely, (1) formulation of the problem to be solved, (2) verification of the problem, that is translation of the mathematical formulation into an algorithm, (3) calibration of the algorithm and finally, (4) model analysis and evaluation. These processes are presented in Figure 1.

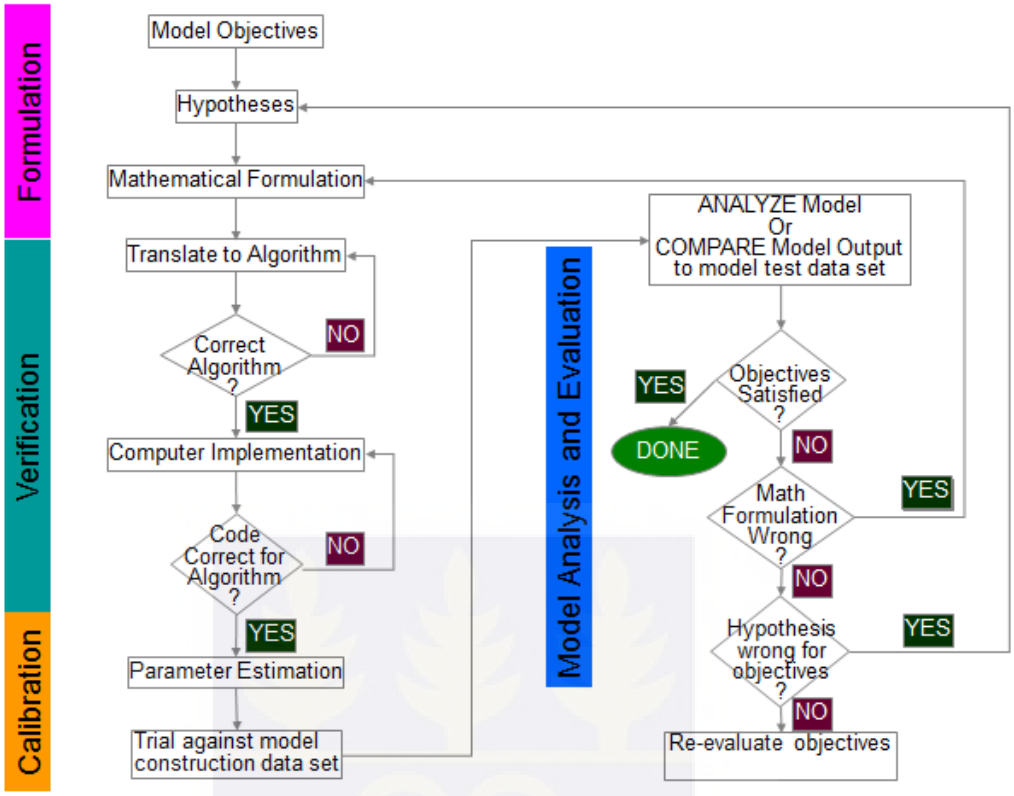


Figure 3.1: Modelling Process (Beveridge, 2012)

3.2 Formulation of the problem

3.2.1 Description of the physical problem

The main purpose of this thesis is to model and monitor the concentration of radioactive sediment suspended in stream flow in order to

- Track the longitudinal concentration of the suspended radioactive sediment from the upper section of the stream to its mouth

- Track at any point in time the concentration of the suspended radioactive material as it decays along the reach of the stream

In Figure 3.2, a simplification of the domain of interest is presented.

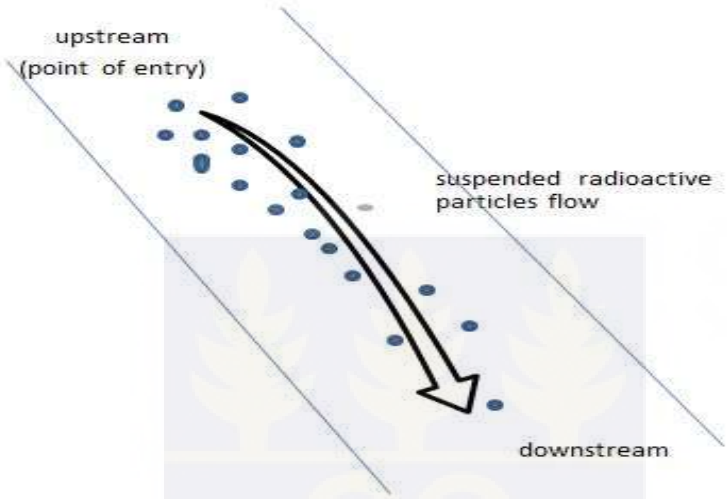


Figure 3.2: Radioactive particles flowing in a stream

- In this domain, a reach of stream with given flow velocity is considered.
- It is assumed that a radioactive material from runoff gets into the upper section of the stream a constant rate
- It is further assumed that the material flows with the stream in suspension
- The task is to simulate (monitor) the concentration of the material as it moves in suspension from the upper section of the river to its mouth.

To give a conceptual view mathematically, figure 3.3 is presented to describe the physical system

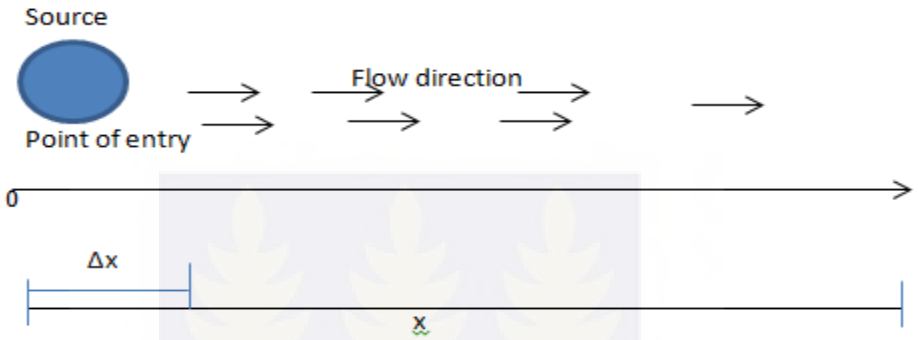


Figure 3.3: Physical system depicting the problem.

3.2.2 Mathematical formulation of radioactive sediment transport equations

The modelling and simulation of the suspended radioactive sediment transport in a stream was done by formulating a one dimensional advection-dispersion transport equation in a longitudinal direction with time and space. To ensure the correctness and accuracy of the model, the suspended radioactive sediment concentration profile in the stream was modelled based on the following assumptions:

- The water is incompressible and homogeneous, i.e., without significant variation in density.

- The bottom slope is small since it is a hypothetical stream with a rectangular cross sectional shape.
- The dispersion coefficient is insignificant due to velocity gradient being approximately zero.
- The direction of flow of the river is considered only longitudinally which means at a given cross section there is no suspended gradient laterally and along depth, vertically.

These assumptions ensure that the flow in all direction can be regarded as having a direction parallel to the bottom.

The one-dimensional advection-dispersion equation which describes the transient transport of suspended sediment is presented as

$$\frac{\partial c}{\partial t}(x,t) = -\frac{\partial}{\partial x_i}(cv_i)(x,t) + \frac{\partial}{\partial x}\left(D_{ij}\frac{\partial C}{\partial x_j}\right)(x,t) - -\lambda C(x,t) \ldots \ldots$$

3.1

where $c(x,t)$ is the radioactive sediment concentration of the stream (ML^{-3})at point $X_0 \leq x \leq L$ and time t, v_i is velocity of the stream (LT^{-1}) in the x direction , D dispersion coefficient (L^2T^{-1}) in the x direction, and λ Decay rate of radioactive particle (T^{-1}), X_0 Initial point of entry of radioactive particle, L is the total distance covered by the radioactive particle in meters, T is the total time covered by the radioactive particle in days.

Figure 3.4 presents a flow chart describing the processes the advection dispersion equation 3.1 went through to produce the simulated results in the next Chapter. From the flow chart

(Figure 3.4), the advection dispersion equation was solved by a numerical method, namely,the finite difference method (FDM) using a set of algebraic equations (discrete model) with appropriate initial and boundary conditions. By direct or iterative solution methods (Euler) an approximate numerical solution (only at discrete grid points) was arrived. For better results analysis the numerical solution obtained and analytical solution from literature which has been solved using calculus techniques (transformation) are compared to verify the numerical solution.

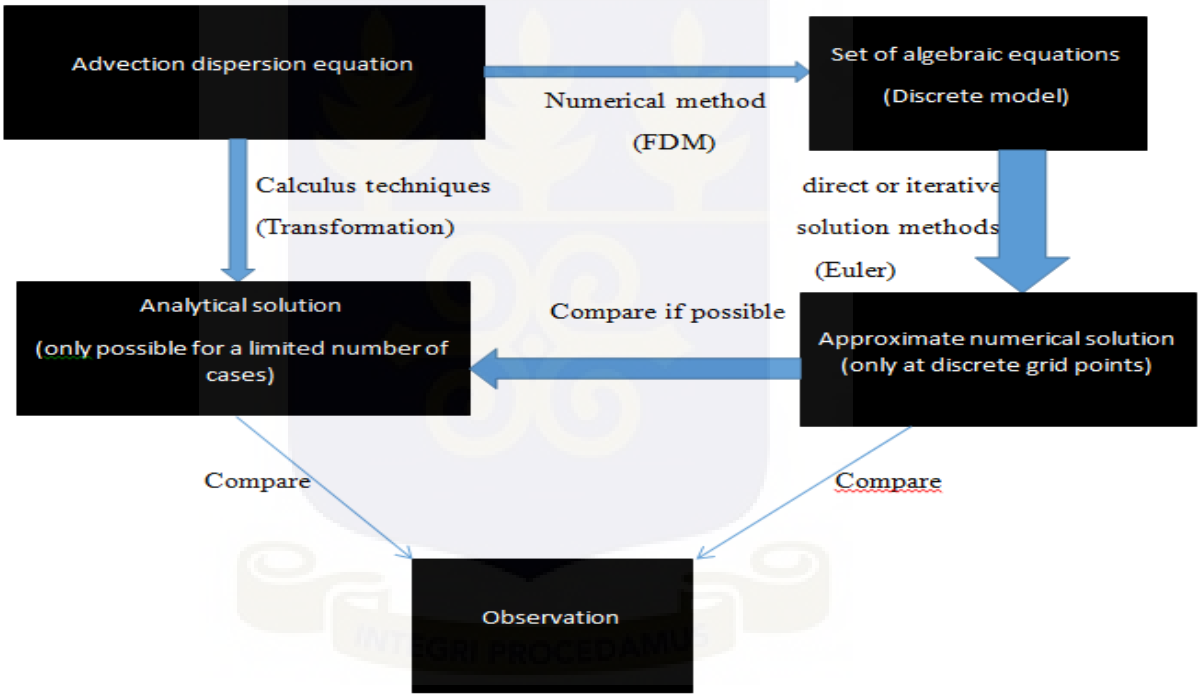


Figure 3.4 A flow chart showing the modelling is executed.

To determine the concentration of the radioactive sediment in suspension in stream by using the advection-dispersion equation, the velocity, v(x,t) of the radioactive particle is required for the substitution into the Advection-Dispersion Equation. Since the radioactive

particle is considered not to dissolve but only suspended, the longitudinal flow of the fluid will affect the movement of the particle. Assumed velocity is substituted into the Advection-Dispersion Equation (ADE) to determine the concentration of the radioactive particle in suspension with time and space.

3.2.2.2 Numerical Approach to the Solution of the Advection-Dispersion Equation

3.2.2.2.1 Finite difference (FTCS) method

Finite difference schemes involve calculating approximate values of the unknown function at a finite number of mesh or grid points in the domain. In this thesis, in x −domain, $0 = x_0 \leq x \leq x_{np} = L$ are the grid points. The time was divided into equal step size $is\ \Delta t$, with time $is\ 0 \leq t \leq T$. A forward difference estimate for the time derivative (FT), and a central difference approximations for the space derivatives (CS) hence the acronym FTCS.

The physical model for the relative concentration distribution in the stream was discretised into grid points using space and time steps of Δx and Δt respectively as shown in Figure 3.5.

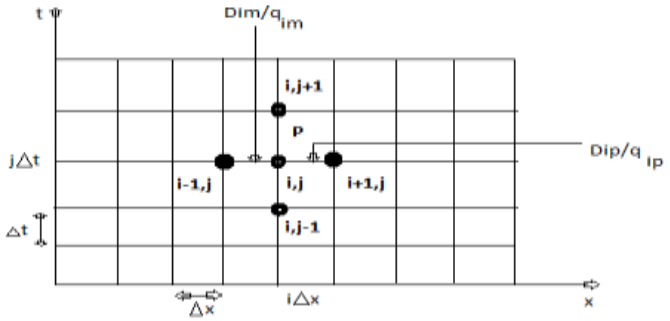


Figure 3.5: Representation of concentration profile in time (t) and space(x) coordinates

In order to solve the Advection-Dispersion Equation, the following initial and boundary conditions were specified

C(x, 0) =C_0 , 0≤x≤L, t=03.2

C(x,t)= C_T x=0, t≥0 3.3

C(x,t) =C_{X_L} x=L, t≥0 3.4

Converting the Advection-Dispersion Equation (3.1) into Finite differences (FTCS) the Forward difference estimate for the time derivative (FT) becomes:

$$\frac{\partial c}{\partial t} = \frac{C_i^{j+1} - C_i^j}{\Delta t} \quad 3.5$$

Central difference approximations for the space derivatives (CS) become:

$$\frac{\partial}{\partial x_i}(cv_i) =$$

$$\frac{v_i C_{i+1}^{j+1} - v_i C_{i-1}^{j+1}}{2\Delta x_i} \qquad 3.6$$

$$\frac{\partial}{\partial x}\left(D_{ij}\frac{\partial c}{\partial x_j}\right) = D_{ij}\frac{C_{i+1}^{j+1} - 2c_i^{j+1} + C_{i-1}^{j+1}}{\Delta x^2} \qquad 3.7$$

And the decay component becomes:

$$\lambda C = \lambda C_i^j \qquad 3.8$$

Substituting equation (3.5), (3.6), (3.7), and (3.8) into equation (3.1), the Advection-Dispersion Equation can be transformed as:

$$\frac{C_i^{j+1} - C_i^j}{\Delta t} = -\frac{v_i C_{i+1}^{j+1} - v_i C_{i-1}^{j+1}}{2\Delta x_i} + D_{ij}\frac{C_{i+1}^{j+1} - 2c_i^{j+1} + C_{i-1}^{j+1}}{\Delta x^2} - \lambda C_i^j \qquad 3.9$$

Multiplying through (3.9) by Δt gives:

$$C_i^{j+1} - C_i^j = -v_i\frac{\Delta t}{2\Delta x_i}(C_{i+1}^{j+1} - C_{i-1}^{j+1}) + D_{ij}\frac{\Delta t}{\Delta x^2}(C_{i+1}^{j+1} - 2C_i^{j+1} + C_{i-1}^{j+1}) -$$

$$\Delta t\lambda C_i^j \qquad 3.10$$

By letting $r_x = \frac{\Delta t}{2\Delta x}$, and $r_{xx} = \frac{\Delta t}{\Delta x^2}$ equation (3.10) can be rewritten as:

$$C_i^{j+1} - C_i^j = -v_i r_x (C_{i+1}^{j+1} - C_{i-1}^{j+1}) + D_{ij} r_{xx} (C_{i+1}^{j+1} - 2C_i^{j+1} + C_{i-1}^{j+1}) - \Delta t \lambda C_i^j \quad 3.11$$

Further simplification of equation (3.11) was done by multiplying through each group by their coefficient to give:

$$C_i^{j+1} - C_i^j = -v_i r_x C_{i+1}^{j+1} + v_i r_x C_{i-1}^{j+1} + D_{ij} r_{xx} C_{i+1}^{j+1} - 2D_{ij} r_{xx} C_i^{j+1} + D_{ij} r_{xx} C_{i-1}^{j+1} - \Delta t \lambda C_i^j \quad 3.12$$

Grouping like terms of eqn(3.12) for simplification: $-(r_{xx} D_{ij} + r_x V) C_{i-1}^{j+1} +$

$$\left(1 + r_{xx} D_{ij} + r_{xx} D_{ij}\right) C_i^{j+1} + \left(-r_{xx} D_{ij} + r_x V\right) C_{i+1}^{j+1}$$

$$= (1 - \lambda \, \Delta t) C_i^j \quad 3.13$$

Equation. (3.13) presents the general finite difference solution for the Advection-Dispersion Equation as presented in equation 3.1

Finally, equation (3.13) can be put in the form:

$$aC_{i-1}^{j+1} + bC_i^{j+1} + dC_{i+1}^{j+1} = eC_i^j \quad 3.14$$

where

$$-r_{xx} D_{iJ} - r_x V = a \quad 3.15$$

$$1 + r_{xx} D_{iJ} + r_{xx} D_{iJ} = b \quad 3.16$$

$$-r_{xx} D_{iJ} + r_x V = d \quad 3.17$$

$$1 - \lambda \Delta t = e \qquad 3.18$$

Applying the dirichlet boundary condition at the upper boundary because it specifies the value of the function,

For i=1, which is the first point of entry equation (3.18) reduces to:

$$aC_0^{j+1} + bC_1^{j+1} + dC_2^{j+1} = eC_1^j \qquad 3.19$$

Applying the upper boundary condition (C(x, 0) =C_0 , x=0, t>0) was known and eqn(3.19) becomes:

$$bC_1^{j+1} + dC_2^{j+1} = eC_1^j - aC_0^{j+1} \qquad 3.20$$

Since at point 1 the concentration $C_0$is assumed to be known,

$$C_1^{j+1} = C_i^j = C_0$$

Which makes eqn 3.20 become:

$$bC_1 + dC_2^{j+1} = eC_1 \qquad 3.21$$

Finding the concentration from i=2 to i=np to form a matrix,

For i=2 equation (3.18) becomes

$$aC_1 + bC_2^{j+1} + dC_3^{j+1} = eC_2^j \qquad 3.22$$

For i=3 equation (3.18) becomes

$$aC_2^{j+1} + bC_3^{j+1} + dC_4^{j+1} = eC_3^j \qquad 3.23$$

For i=np equation (3.18) becomes

$$aC_{np-1}^{j+1}+bC_{np}^{j+1}+dC_{np+1}^{j+1}=eC_{np}^{j} \qquad 3.24$$

The above set of equation can be expressed in a tridiagonal matrix notation as follows

$$\begin{bmatrix} b & d & 0 & 0 & 0 \\ a & b & d & 0 & 0 \\ 0 & a & b & d & 0 \\ : & : & : & : & : \\ 0 & a & b & d & 0 \\ 0 & 0 & a & b^1 & d^1 \\ 0 & 0 & 0 & 0 & b \end{bmatrix} \begin{Bmatrix} C_1^{j+1} \\ C_2^{j+1} \\ C_3^{j+1} \\ : \\ C_{np-1}^{j+1} \\ C_{np}^{j+1} \\ C_{np+1}^{j+1} \end{Bmatrix} = \begin{Bmatrix} eC_1^{j} \\ eC_2^{j} \\ eC_3^{j} \\ : \\ eC_{np-1}^{j} \\ eC_{np}^{j} \\ eC_{np+1}^{j} \end{Bmatrix} \qquad 3.25$$

where: $\begin{bmatrix} b & d & 0 & 0 & 0 \\ a & b & d & 0 & 0 \\ 0 & a & b & d & 0 \\ : & : & : & : & : \\ 0 & a & b & d & 0 \\ 0 & 0 & a & b^1 & d^1 \\ 0 & 0 & 0 & 0 & b \end{bmatrix}$ =A, $\begin{Bmatrix} C_1^{j+1} \\ C_2^{j+1} \\ C_3^{j+1} \\ : \\ C_{np-1}^{j+1} \\ C_{np}^{j+1} \\ C_{np+1}^{j+1} \end{Bmatrix}$ =g and $\begin{Bmatrix} eC_1^{j} \\ eC_2^{j} \\ eC_3^{j} \\ : \\ eC_{np-1}^{j} \\ eC_{np}^{j} \\ eC_{np+1}^{j} \end{Bmatrix} = C$

3.3 Numerical Solution

A structured algorithm and flow chart were developed for the numerical solution of the discretised one dimensional advection-dispersion equation using the implicit finite difference method. A tri-diagonal matrix was generated for each radioactive sediment by solving the ADE (3.1) with the implicit finite difference method and applying the defined

initial and boundary conditions (eqn 3.2, 3.3, and 3.4). An algorithm based on the flow chart (Figure 3.6) to solve the tridiagonal matrix of the suspended radioactive sediment transport in stream were developed into computer programs, which were implemented and simulated using Matrix Laboratory (MATLAB) software package (version R2013a) with symbolic math Toolbox. The MATLAB code was developed to simulate the concentration profiles of an assumed radioactive material of half-life 1-hr and Chromium-51 with half-life of 27.7 days. The programs were executed using Lenovo thinkPad PC of processor Ci5 E -420 APU with HD Graphics, installed memory – 4.00GB, x64 – based processor, 500GB Hard disk, and 64 bit windows 7 operating system.

3.3.1 Definition of Parameters and Initialisation

T- maximum time covered by the radioactive sediment

L- length of stream

dt-change in time=1 day

dx- change in distance=1m

D- dispersion coefficient

v-velocity of stream

λ- decay of the radioactive sediment

np- number of nodes

np+1-maximum number of nodes

m-number of time steps

m+1-maximum number of time steps

3.3.2 Simulation of Suspended Radioactive Sediment Concentration Profiles

The simulations were done using an initial time and space of t=0 and x=0, respectively.

The maximum simulations time was 31 days and the maximum distance or space was 10000m (10 km). Space and time step of 1m and 1 day respectively were used for the simulations of the radioactive sediments.

3.3.3 Model Analysis of Suspended Radioactive Sediment Concentration Profiles

Spatial and temporal analyses of the radioactive sediment concentration in the stream were done. Plots of radioactive sediment concentration against distance and time were obtained at a time 31 days and at a distance or length of stream 10 km respectively.

Due to unavailability of field data in the study area, the numerical transport models developed were verified by comparing the results with an analytical solution of similar work done by Jaiswal et al, 2009.

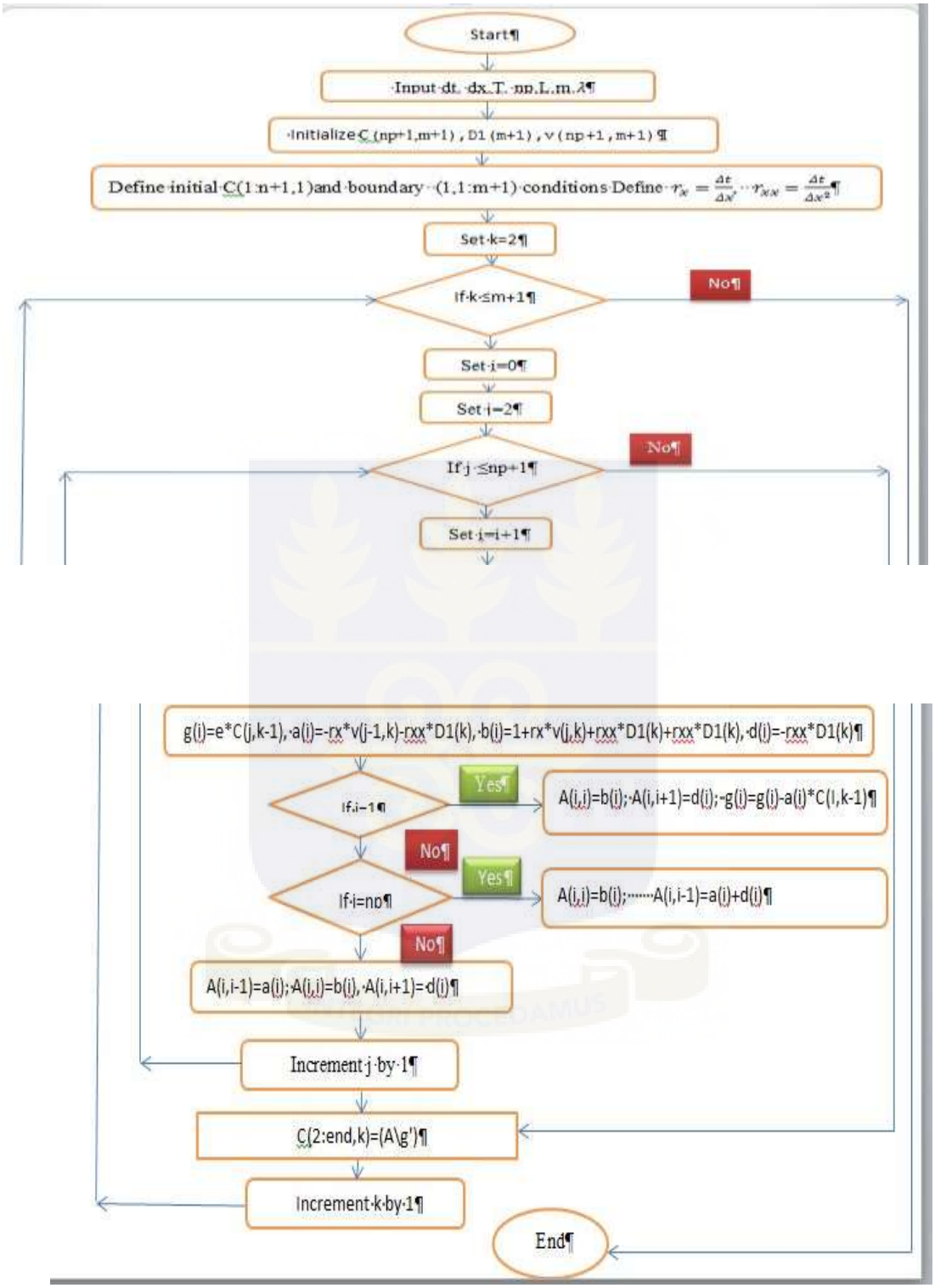


Figure 3.6: Flow chart algorithm for estimating concentration distribution of suspended radioactive sediment in stream.

3.3.4 Numerical Error, Convergence and Stability

Two types of numerical errors encountered in the numerical model were Truncation errors and Round-off errors. Truncation errors resulted from the truncation of the Taylor's series used to defined the finite difference scheme, and the truncation error was of order x^2 . Round-off errors were encountered due to finite arithmetic of conducting mathematical computation using computer.

The consistency of the numerical solution was tested by varying the step size (Δx) and was observed that as the step size (Δx) approached zero, the numerical solution approached the analytical solution by Jaiswal ***et al***, 2009 and therefore the implicit finite difference numerical solution was consistent.

The stability of the numerical solution was tested and was observed that the errors in the numerical solution does not grow without bound but rather the errors grew for some time and started to decreased, therefore the implicit finite difference numerical solution was stable.

In summary, numerical model using finite difference scheme has been developed in this thesis. A structured program has been developed and implemented into MATLAB program. The results obtained from implemented program are presented in the next chapter.

CHAPTER FOUR

Results and Discussions

In this chapter, the results for the fully implicit finite difference numerical solution of suspended radioactive sediment transport for simulation of the concentration distribution in a stream with respect to time and space is presented. In the determination of the distribution of concentration of the radioactive sediment source, short-lived radioactive sediment contaminant of half-life one hour and long-lived Ra-226 of half-life 1600 years were simulated. From Chapter one it has been clearly stated that velocity is one of the important factors controlling the transport of sediment suspended in the stream, hence the presentations of the results for the individual sources were grouped considering:

- Steady uniform streamflow velocity and
- Unsteady uniform streamflow velocity.

A standard unit amount of suspended radioactive sediment sources was kept constant at the point of entry into the stream for all the simulations over a period of thirty-one (31) days. The radioactive sediment sources were transported by the stream through a length of 10 km. For the spatial variation of the concentration distribution, the results show how after two days the relative concentration distribution within the stretch of the stream looks like from the first day of entry into the stream as the suspended radioactive sediment decays with distance. Further results were provided for after five days, after ten days and finally the thirty-one day to give a general outlook at each point for different days.

In addition, temporal variation results for the concentration distribution were also presented for each radioactive sediment.

These results also show how the radioactive sediment is behaving in the stream after 3.3 km from the initial point of entry. More results were provided for after 5.8 km, 8.3 km, 9.2 km and at the length of the stream which is 10 km. These temporal variation results show how at these various distances simulated, the relative concentrations of the radioactive sediment behaved with time within the thirty-one days as it decays along the stream.

The profiles of the velocities used in this thesis is presented in Figure 4.1

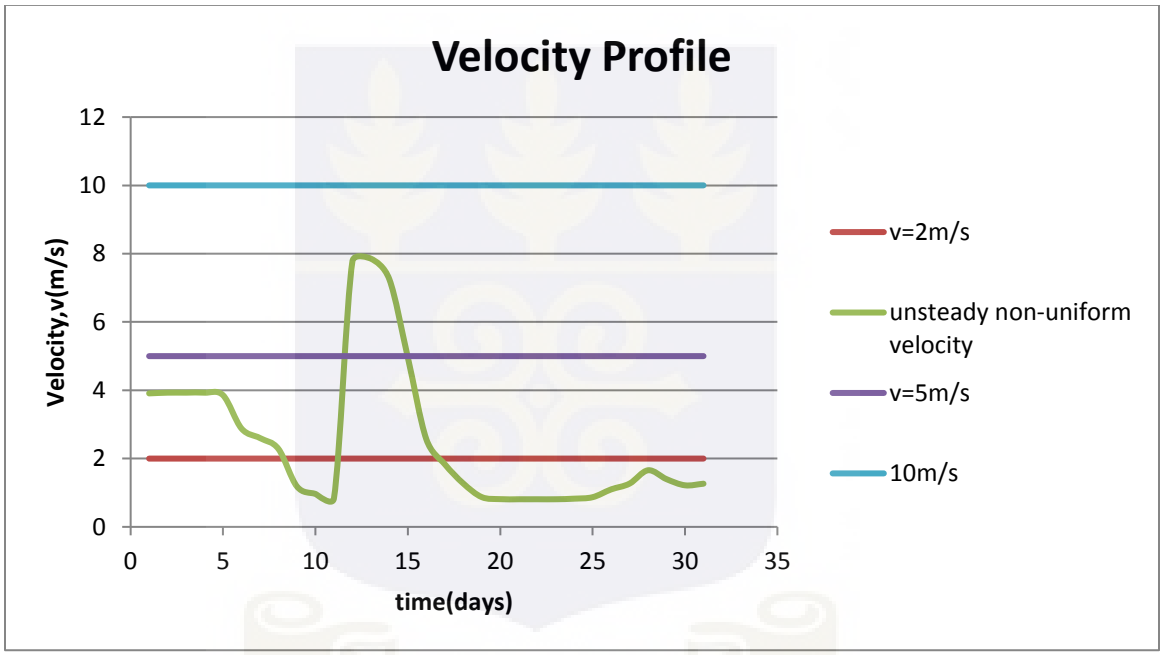


Figure 4.1: Plot of various Velocity Profile

From Figure 4.1,$v = 2m/s$, $v = 5m/s$(c) , $and\ v = 10m/s$ maintains a constant velocity hence it is a steady uniform flow . From the unsteady non-uniform velocity, there was rise in the first few steps to maximum and then gradually fell to minimum.

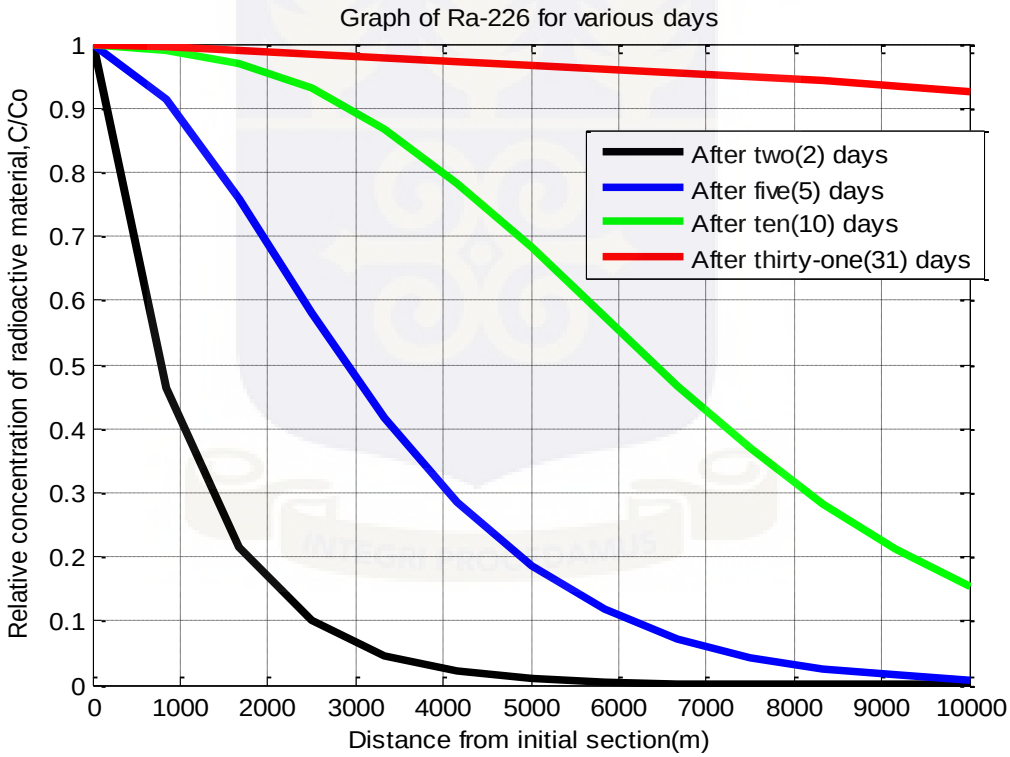
Finally, this Chapter validates the output based on analytical solution by Jaiswal et al, 2009.

4.1 Spatial Variation of Radioactive Sediment Concentration in Stream

The results of concentration distribution of two radioactive sediments in the stream with respect to distance at different velocities are presented.

Figure 4.2 , 4.3 and 4.4 show results of simulations done for decaying 1hour, and non-decaying Ra-226 suspended radioactive sediment over a time period, T=31days within the length of the stream

(a)



(b)

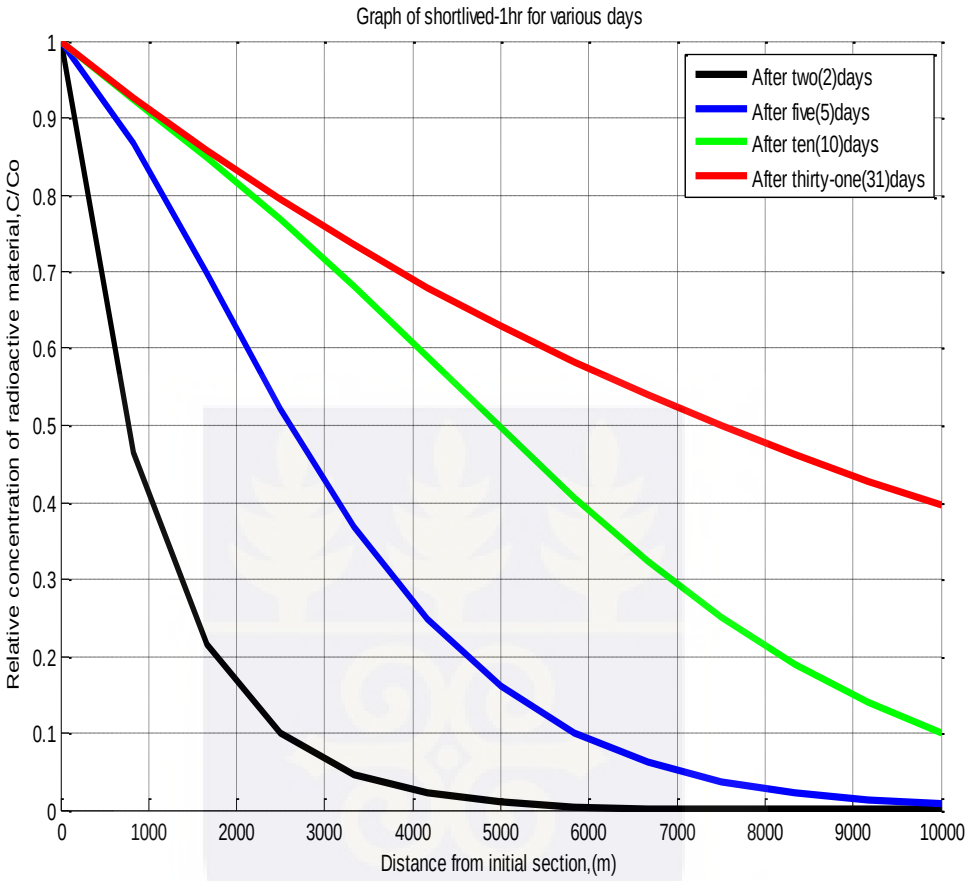


Figure 4.2: Simulation of (a) non decaying Ra-226 and (b)decaying 1hr concentration distribution with respect to distance from initial section of stream for duration of thirty-one days (31) at (a)$v = 2m/s$

Table 4.1 Relative concentration reading of (a) non decaying Ra-226 and (b) decaying 1hr, C(x,t) at 0≤t≤31 days (a)$v = 2m/s$

(a)

AFTER TWO DAYS		AFTER FIVE DAYS		AFTER TEN DAYS		AFTER THIRTY-ONE DAYS	
D (km)	C/Co	D (km)	C/Co	D (km)	C/Co	D (km)	C/Co
2	0.18	2	0.7	2	0.96	2	0.99
5	0.01	5	0.19	5	0.68	5	0.98
10	0	10	0.01	10	0.15	10	0.94

(b)

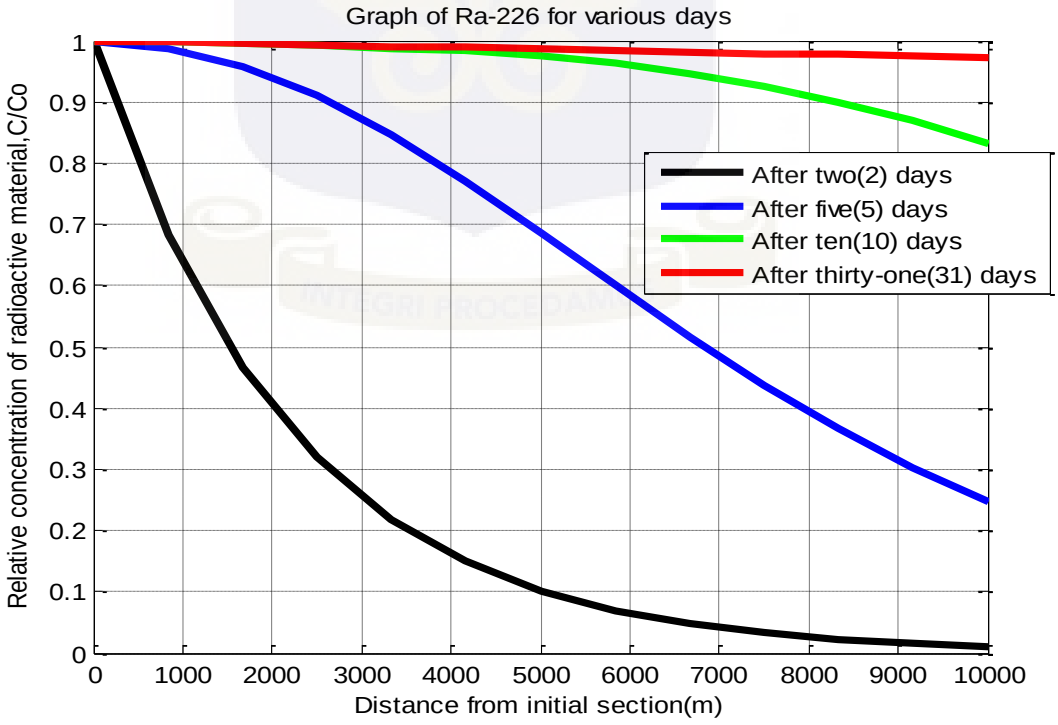
AFTER TWO DAYS		AFTER FIVE DAYS		AFTER TEN DAYS		AFTER THIRTY-ONE DAYS	
D (km)	C/Co	D (km)	C/Co	D (km)	C/Co	D (km)	C/Co
2	0.18	2	0.61	2	0.8	2	0.83
5	0.01	5	0.15	5	0.5	5	0.63
10	0	10	0.01	10	0.1	10	0.4

From Figure 4.2 (a), table 4.1(a), after two days the long lived suspended radioactive sediment experienced a sharp decrease in concentration. At about 6 km, the relative concentration had approached zero. This means there was decay. Since it took a short period to decay the material did not appear after 6km to the end of the 10 km length of stream because of the velocity used. After five days and ten days the relative concentration had also decreased within the length of stream but the ten days had a substantial amount

left more than the five days at the end of 10 km. after thirty one days the radioactive sediment had decrease in relative concentration however it was not substantial.

Figure 4.2(b) and table 4.1(b) the short-lived suspended radioactive sediment also decayed at about 6 km within the stretch of the stream after two days. After five days the relative concentration had decreased and the remains at 10 km were not substantial however for the ten and thirty-one days though they decreased, it remained a substantial amount at 10 km.

(a)



(b)

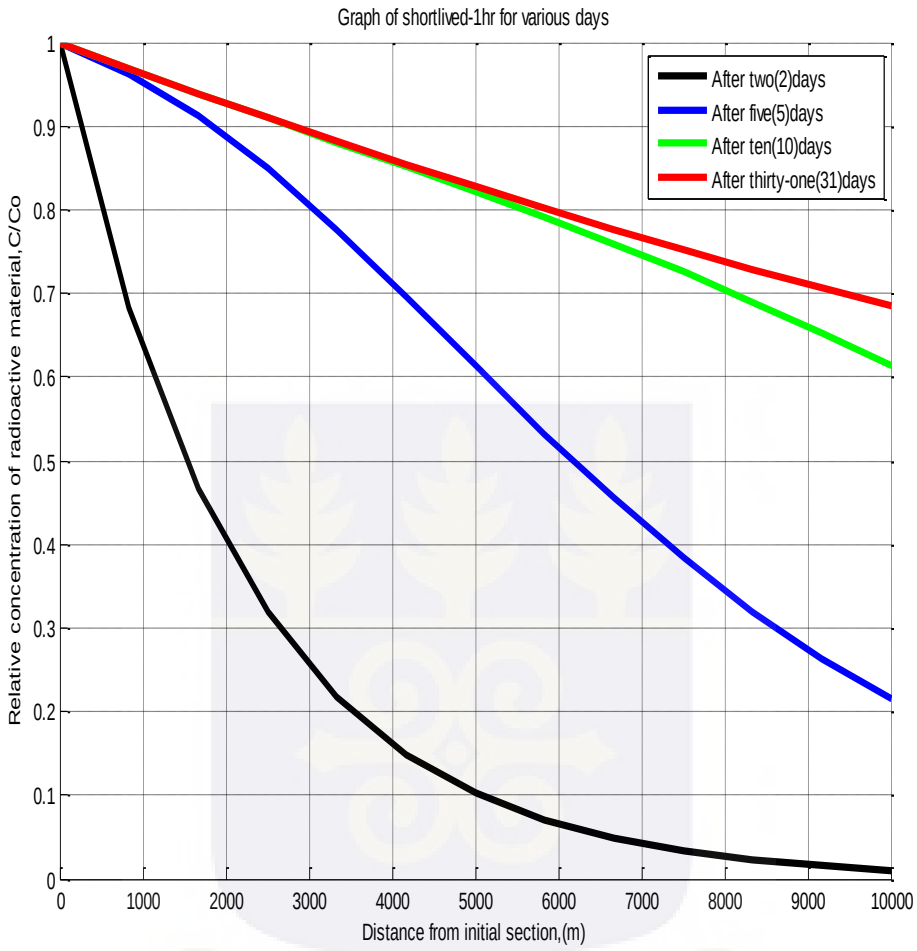


Figure 4.3: Simulation of (a) non decaying Ra-226 and (b)decaying 1hr concentration distribution with respect to distance from initial section of stream for duration of thirty-one days (31) at (a)$v = 5m/s$

Table 4.2 Relative concentration reading of (a) non decaying Ra-226 and (b) decaying 1hr, C(x,t) at 0≤t≤31 days (a)$v = 5m/s$

(a)

AFTER TWO DAYS		AFTER FIVE DAYS		AFTER TEN DAYS		AFTER THIRTY-ONE DAYS	
D (km)	C/Co	D (km)	C/Co	D (km)	C/Co	D (km)	C/Co
2	0.4	2	0.95	2	1	2	1
5	0.1	5	0.68	5	0.98	5	0.99
10	0.01	10	0.25	10	0.82	10	0.98

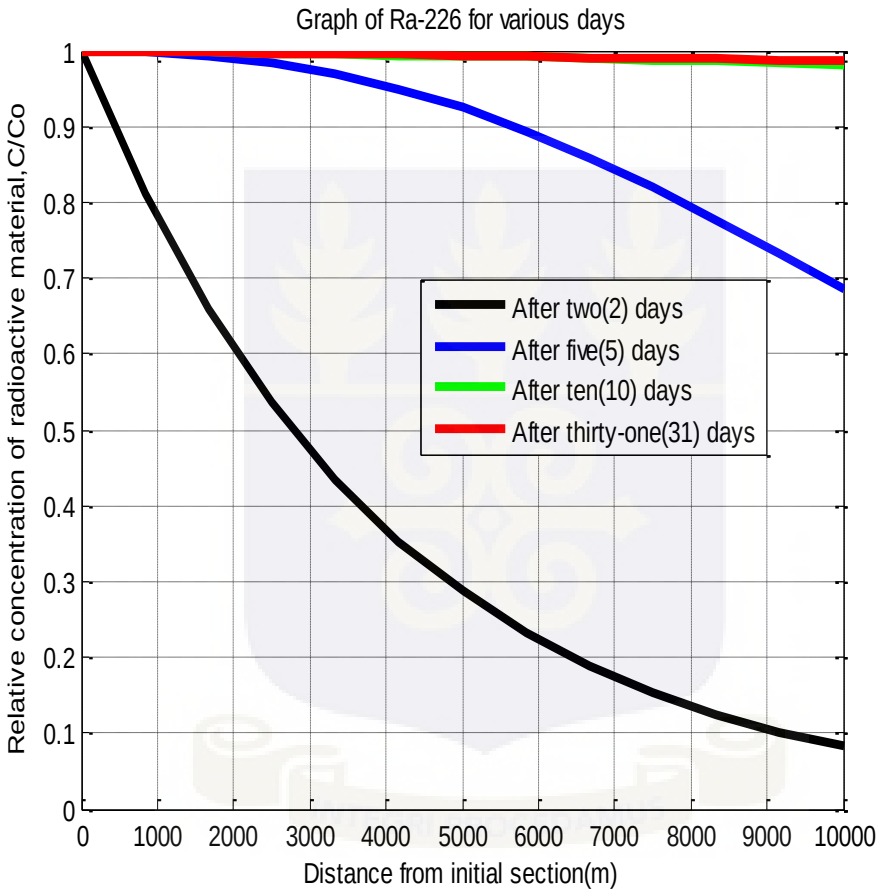
(b)

AFTER TWO DAYS		AFTER FIVE DAYS		AFTER TEN DAYS		AFTER THIRTY-ONE DAYS	
D (km)	C/Co	D (km)	C/Co	D (km)	C/Co	D (km)	C/Co
2	0.4	2	0.95	2	0.97	2	0.98
5	0.1	5	0.6	5	0.82	5	0.83
10	0.01	10	0.22	10	0.62	10	0.69

From Figure 4.3 (a), table 4.2(a) the long lived suspended radioactive sediment experienced a sharp decrease in concentration after two days. The relative concentration did not approach zero compared to figure 4.2(a) due to change in velocity. After five days, ten days and thirty one days the relative concentration decreased within the length of stream but relative to each other the thirty one days was smallest followed by the ten days then finally the five days. This means the material did not decay at all for all the days within the 10 km.

Figure 4.3(b) and table 4.2(b) the short-lived suspended radioactive sediment did not also decay for all the days. However compared to figure 4.3(a) they all decreased lower than figure 4.3 (b) at the end of 10 km.

(a)



(b)

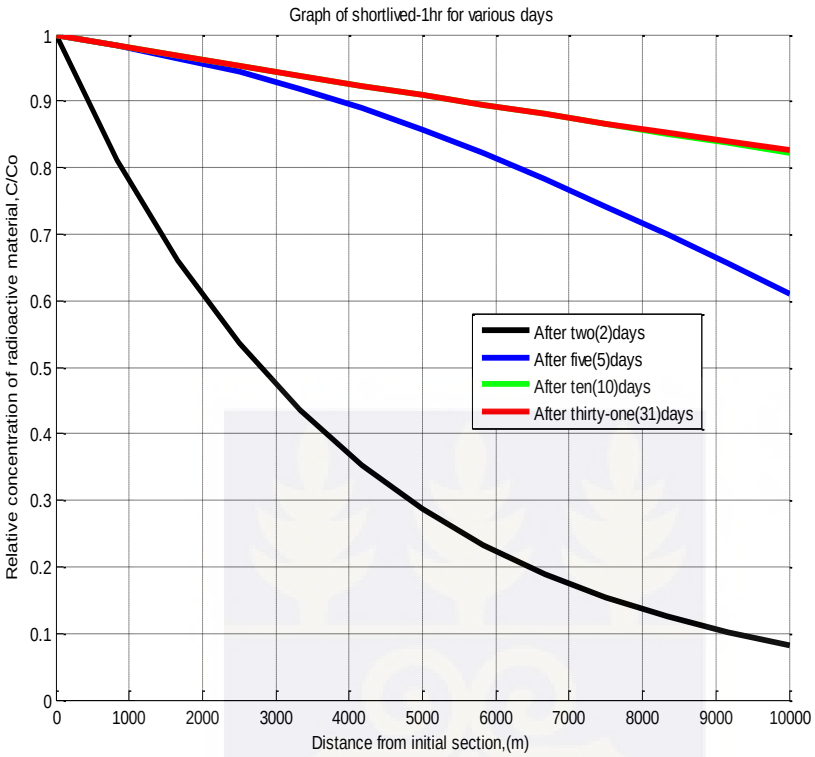


Figure 4.4: Simulation of (a) non decaying Ra-226 and (b)decaying 1hr concentration distribution with respect to distance from initial section of stream for duration of thirty-one days (31) at (a)$v = 10m/s$

Table 4.3 Relative concentration reading of (a) non decaying Ra-226 and (b) decaying 1hr, C(x,t) at 0≤t≤31 days (a)$v = 10m/s$

(a)

AFTER TWO DAYS		AFTER FIVE DAYS		AFTER TEN DAYS		AFTER THIRTY-ONE DAYS	
D (km)	C/Co	D (km)	C/Co	D (km)	C/Co	D (km)	C/Co
2	0.6	2	0.99	2	1	2	1
5	0.3	5	0.94	5	0.99	5	1
10	0.09	10	0.69	10	0.98	10	0.99

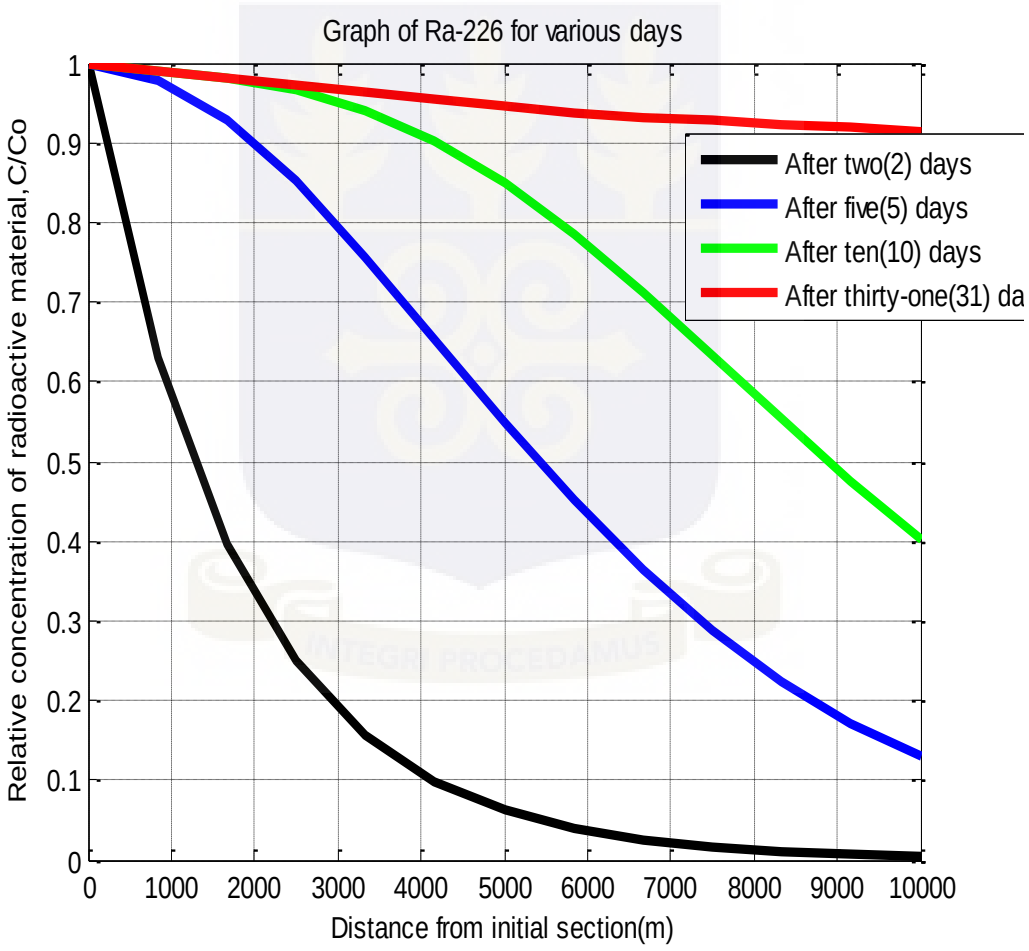
(b)

AFTER TWO DAYS		AFTER FIVE DAYS		AFTER TEN DAYS		AFTER THIRTY-ONE DAYS	
D (km)	C/Co	D (km)	C/Co	D (km)	C/Co	D (km)	C/Co
2	0.6	2	0.97	2	0.98	2	0.99
5	0.29	5	0.85	5	0.91	5	0.92
10	0.09	10	0.61	10	0.83	10	0.84

From Figure 4.4 (a), table 4.3(a) the long lived suspended radioactive sediment experienced decrease in concentration after two days. The relative concentration amount at 10 km was appreciable. After five days the relative concentration also decreased at 10 km but the amount was more substantial than after two days. After ten days and thirty one days did not have much difference in the behavior of the reduction in relative concentration. The amount was also more substantial than after five days at 10 km.

Figure 4.4(b) and table 4.3(b) the short-lived suspended radioactive sediment experienced decrease in concentration after two days. The pattern for the five days, ten days and thirty one days were similar to the figure 4.4(a) however the amount left at 10 km were relatively smaller to the figure 4.4(a)

(a)



(b)

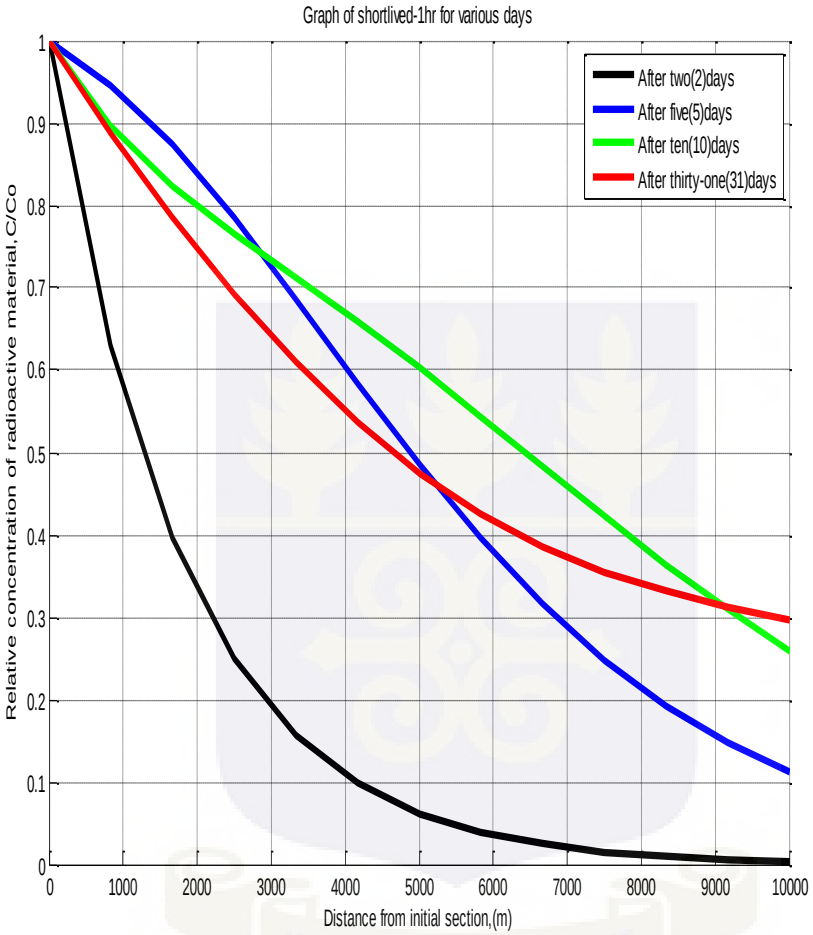


Figure 4.5: Simulation of (a) non decaying Ra-226 and (b)decaying 1hr concentration distribution with respect to distance from initial section of stream for duration of thirty-one days (31) at (a)$v = unsteady\ non - uniform$

Table 4.4 Relative concentration reading of (a) non decaying Ra-226 and (b)decaying 1hr, C(x,t) at 0≤t≤31 days (a)$v = unsteady\ non - uniform$

(a)

AFTER TWO DAYS		AFTER FIVE DAYS		AFTER TEN DAYS		AFTER THIRTY-ONE DAYS	
D (km)	C/Co	D (km)	C/Co	D (km)	C/Co	D (km)	C/Co
2	0.34	2	0.9	2	0.98	2	0.99
5	0.06	5	0.55	5	0.85	5	0.96
10	0	10	0.14	10	0.4	10	0.92

(b)

AFTER TWO DAYS		AFTER FIVE DAYS		AFTER TEN DAYS		AFTER THIRTY-ONE DAYS	
D (km)	C/Co	D (km)	C/Co	D (km)	C/Co	D (km)	C/Co
2	0.34	2	0.84	2	0.8	2	0.75
5	0.06	5	0.5	5	0.6	5	0.48
10	0	10	0.12	10	0.26	10	0.3

From Figure 4.5 (a), table 4.4(a) the long lived suspended radioactive sediment experienced decrease in concentration after two days. The relative concentration was very small. After five days the relative concentration also decreased at 10 km but the amount was more substantial than after two days. After ten days and thirty one days the relative concentration also decreased but the thirty one days had a substantial amount at the end of the simulation than the five days.

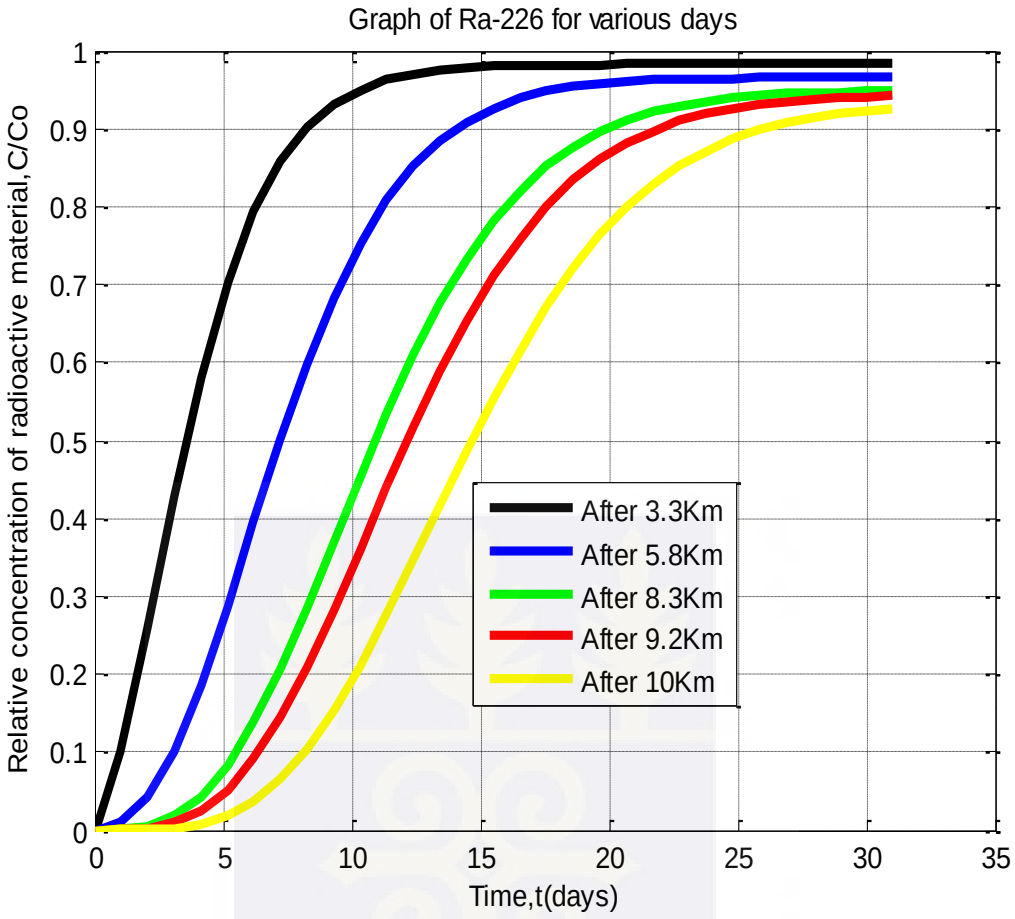
Figure 4.5(b) and table 4.4(b) the short-lived suspended radioactive sediment decayed at about 9 km from the initial point of entry after two days. This is because the relative concentration after two days approached zero. After five days, ten days and thirty one days, figure 4.5(b) gradually decreased in relative concentration. Physically the differences exist because of the unsteady non uniform velocity used.

This concludes that long-lived radioactive sediment were generally not decaying due to the substantial amount of the relative concentration left at the end of 10 km and the short period of thirty one days considered for this thesis. This means it will take longer time to decay.

4.2 Temporal Variation of Radioactive Sediment Concentration in Stream

The results of concentration distribution of three radioactive sediments in the stream with respect to distance are presented. Figure 4.2.1, 4.2.2 and 4.2.3 show results of simulations done for decaying 1hour and non-decaying Ra-226 suspended radioactive sediment over a distance , L=10Km of the stream within thirty-one days.

(a)



(b)

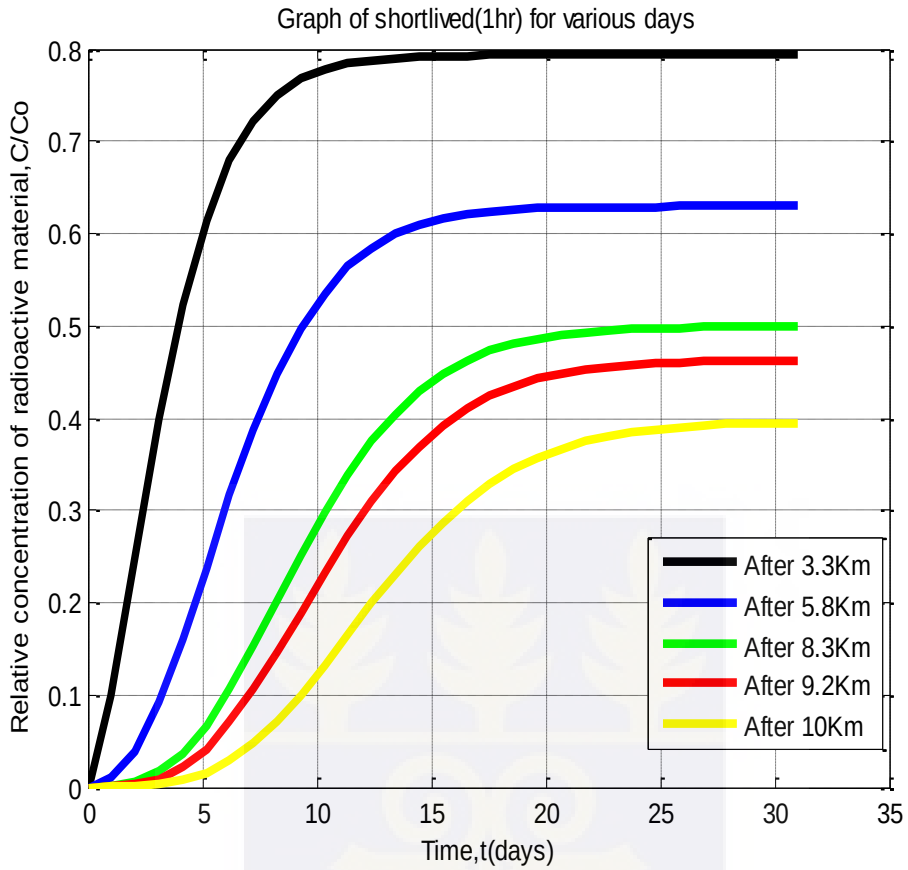


Figure 4.6: Simulation of (a) non decaying Ra-226 and (b) decaying 1hr sediment concentration distribution with respect to time in days for distance of 10Km distribution at $v = 2m/s$

Table 4.5: Relative concentration reading of (a) non decaying Ra-226 and (b) decaying 1hr, C(x,t) at 0≤x≤10Km (a)$v = 2m/s$

(a)

AFTER 3.3 km		AFTER 5.8 km		AFTER 8.3 km		AFTER 9.2 km		AFTER 10 km	
T	C/Co	T	C/Co	T	C/Co	T	C/Co	T	C/Co
5	0.7	5	0.3	5	0.09	5	0.05	5	0.02
15	0.98	15	0.92	15	0.76	15	0.7	15	0.52
31	0.99	31	0.98	31	0.95	31	0.94	31	0.93

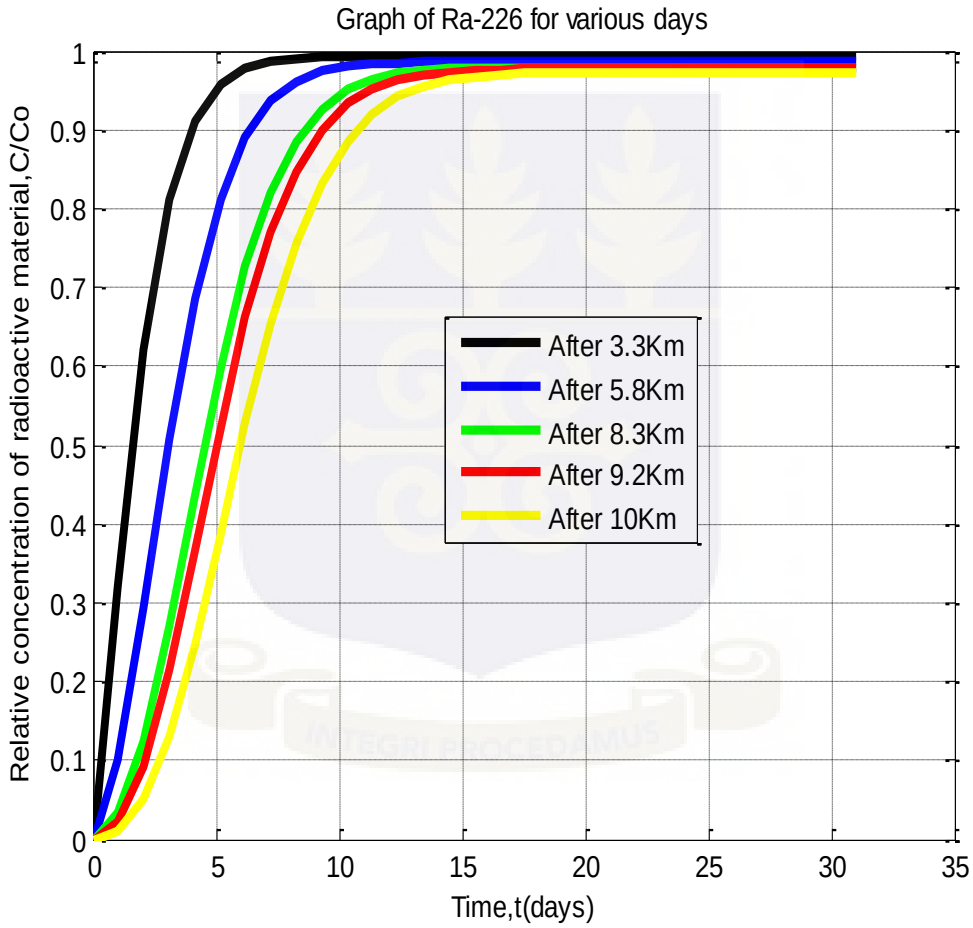
(b)

AFTER 3.3 km		AFTER 5.8 km		AFTER 8.3 km		AFTER 9.2 km		AFTER 10 km	
T	C/Co	T	C/Co	T	C/Co	T	C/Co	T	C/Co
5	0.6	5	0.25	5	0.06	5	0.04	5	0.1
15	0.79	15	0.61	15	0.45	15	0.38	15	0.28
31	0.8	31	0.62	31	0.5	31	0.48	31	0.4

Figure 4.6 (a) and table 4.5 (a) showed the concentration long-lived radioactive sediment a general increased in relative concentration of the material from the initial point of entry with time for all the specified distances. The simulation for 3.3 km almost reached the maximum concentration at end of the thirty one days. The simulations for after 5.8 km 8.3 km, 9.2 km and 10 km distances from initial point of entry also increased in relative concentration and gradually approached the maximum at the end of thirty one days. They maintained constant relative concentration of 0.98, 0.95, 0.94 and 0.93 respectively.

Figure 4.6 (b) and table (b) also showed an increase in relative concentration however figure 4.6(a) was closed to the maximum than figure 4.6(b). This means the relative concentration with time was physically not decreasing significantly for figure 4.6(a) compared to figure 4.6(b).

(a)



(b)

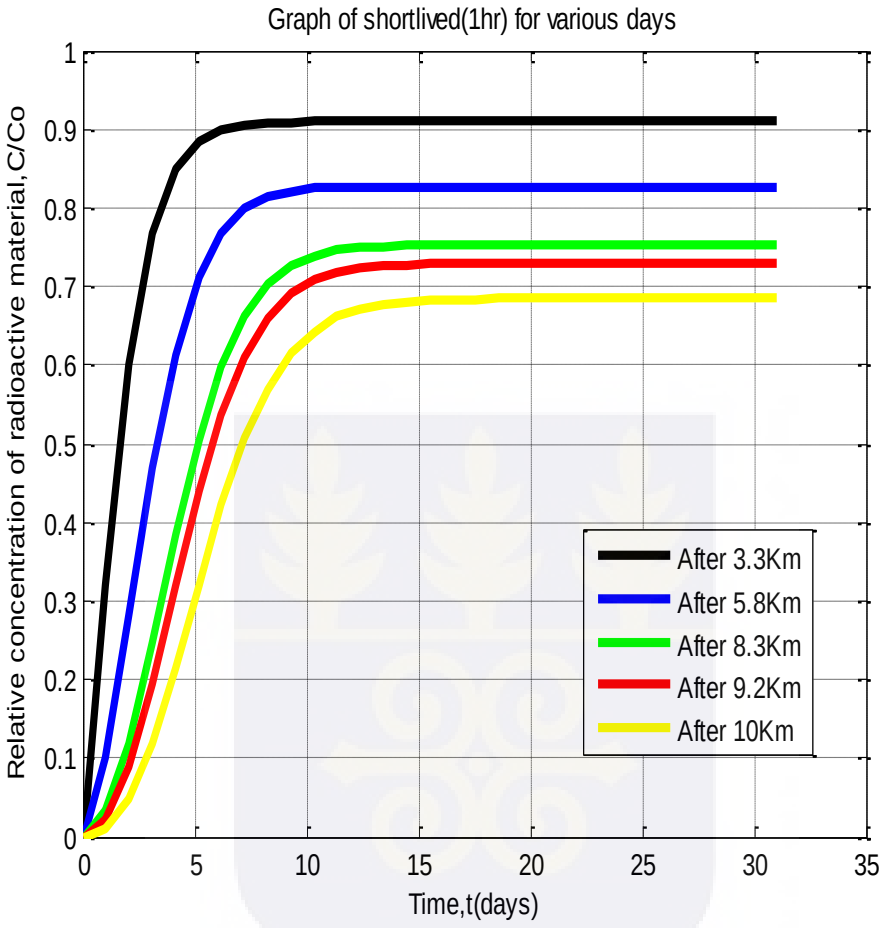


Figure 4.7: Simulation of (a) non decaying Ra-226 and (b) decaying 1hr sediment concentration distribution with respect to time in days for distance of 10 km distribution at $v = 5m/s$

Table 4.6: Relative concentration reading of (a) non decaying Ra-226 and (b) decaying 1hr,, C(x,t) at 0≤x≤10 km , $v = 5m/s$

(a)

AFTER 3.3 km		AFTER 5.8 km		AFTER 8.3 km		AFTER 9.2 km		AFTER 10 km	
T	C/Co	T	C/Co	T	C/Co	T	C/Co	T	C/Co
5	0.97	5	0.8	5	0.6	5	0.5	5	0.39
15	0.99	15	0.98	15	0.96	15	0.97	15	0.96
31	0.99	31	0.99	31	0.97	31	0.98	31	0.97

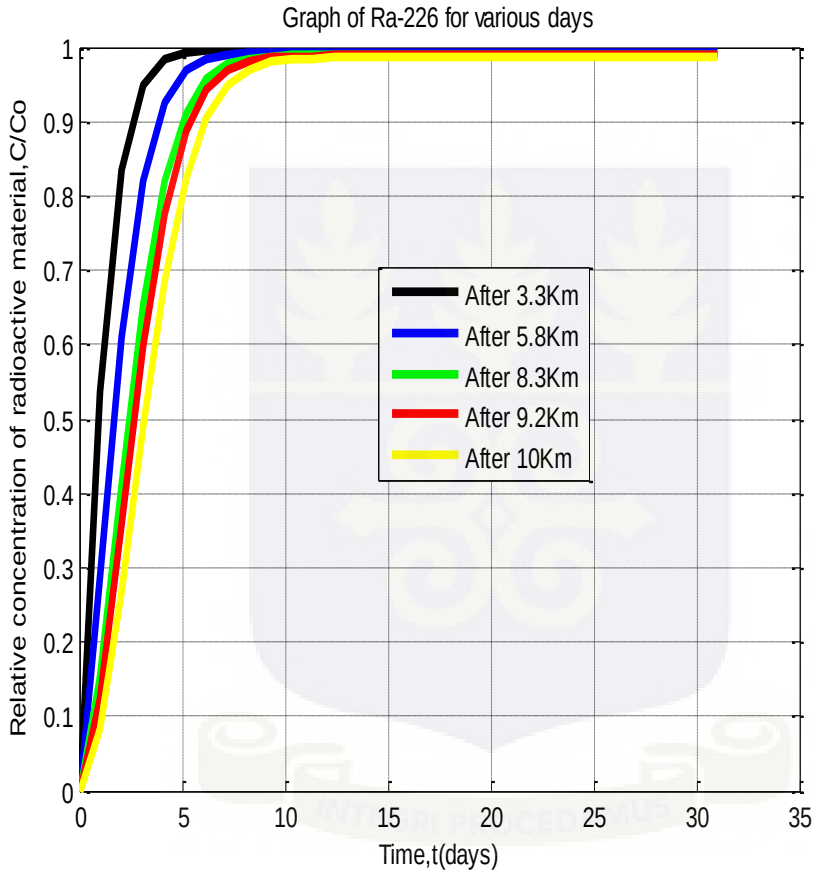
(b)

AFTER 3.3 km		AFTER 5.8 km		AFTER 8.3 km		AFTER 9.2 km		AFTER 10 km	
T	C/Co	T	C/Co	T	C/Co	T	C/Co	T	C/Co
5	0.89	5	0.7	5	0.5	5	0.4	5	0.3
15	0.92	15	0.84	15	0.76	15	0.74	15	0.68
31	0.92	31	0.84	31	0.76	31	0.74	31	0.68

For Figure 4.7 (a) and table 4.6 (a) showed the concentration long-lived radioactive sediment a quickly increased in relative concentration of the material from the initial point of entry with time for all the specified distances. The simulation for 3.3 km reached the maximum concentration at end of the thirty one days. It maintained a constant relative concentration after reaching maximum within the thirty one days. The simulations for after 5.8 km, 8.3 km, 9.2 km and 10 km were almost at maximum.

Figure 4.7 (b) and table (b) also showed an increase in relative concentration however figure 4.7(a) was at the maximum while figure 4.7(b) was close to the maximum concentration of 0.9 .

(a)



(b)

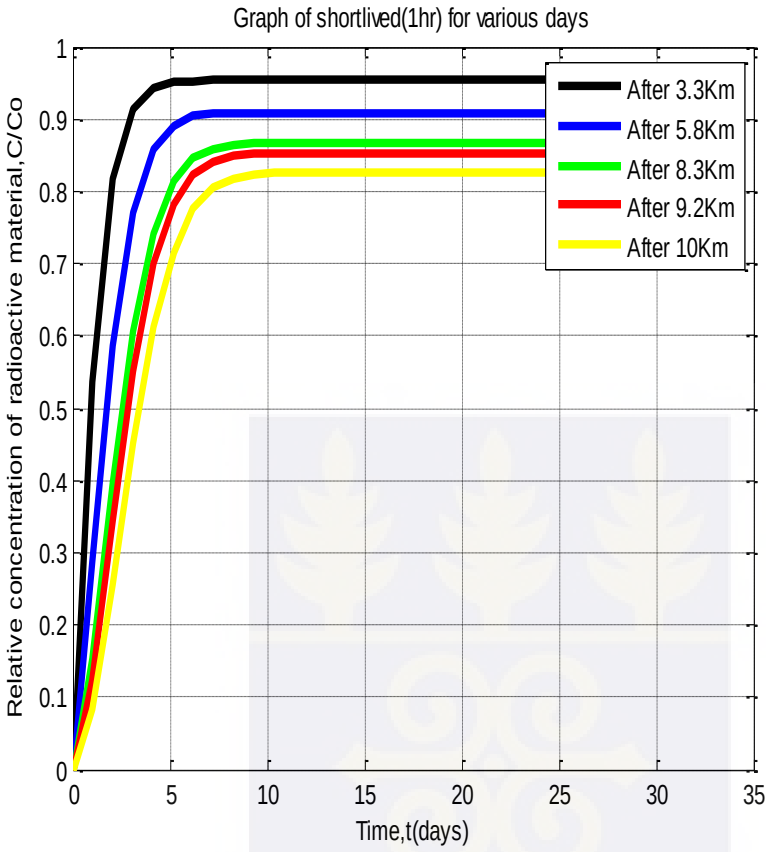


Figure 4.8: Simulation of (a) non decaying Ra-226 and (b) decaying 1hr sediment concentration distribution with respect to time in days for distance of 10 km distribution at $v = 10m/s$

Table 4.7: Relative concentration reading of (a) non decaying Ra-226 and (b) decaying 1hr, C(x,t) at 0≤x≤10 km ,$v = 10m/s$

(a)

AFTER 3.3 km		AFTER 5.8 km		AFTER 8.3 km		AFTER 9.2 km		AFTER 10 km	
T	C/Co	T	C/Co	T	C/Co	T	C/Co	T	C/Co
5	0.99	5	0.98	5	0.9	5	0.89	5	0.8
15	1	15	1	15	1	15	1	15	0.99
31	1	31	1	31	1	31	1	31	0.99

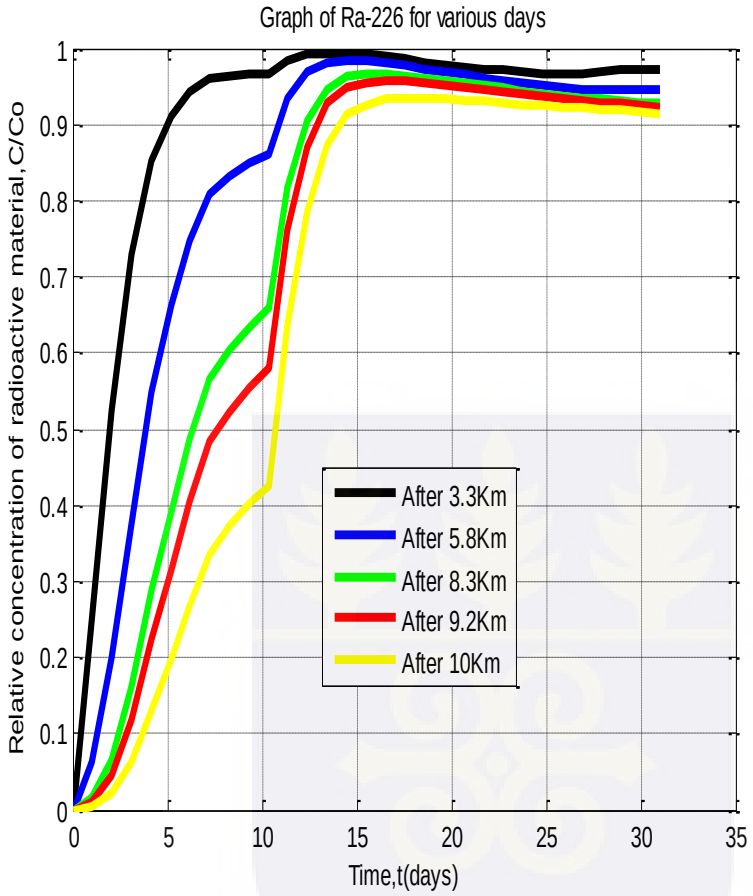
(b)

AFTER 3.3 km		AFTER 5.8 km		AFTER 8.3 km		AFTER 9.2 km		AFTER 10 km	
T	C/Co	T	C/Co	T	C/Co	T	C/Co	T	C/Co
5	0.96	5	0.9	5	0.8	5	0.78	5	0.7
15	0.97	15	0.91	15	0.87	15	0.85	15	0.84
31	0.97	31	0.91	31	0.87	31	0.85	31	0.84

For Figure 4.8 (a) and table 4.7 (a) also showed the concentration long-lived radioactive sediment a quickly increased in relative concentration of the material from the initial point of entry with time for all the specified distances. The simulations reached the maximum concentration.

Figure 4.8 (b) and table 4.7(b) also showed an increase in relative concentration however figure 4.8(a) was at the maximum while figure 4.8(b) was close to the maximum concentration of 0.95

(a)



(b)

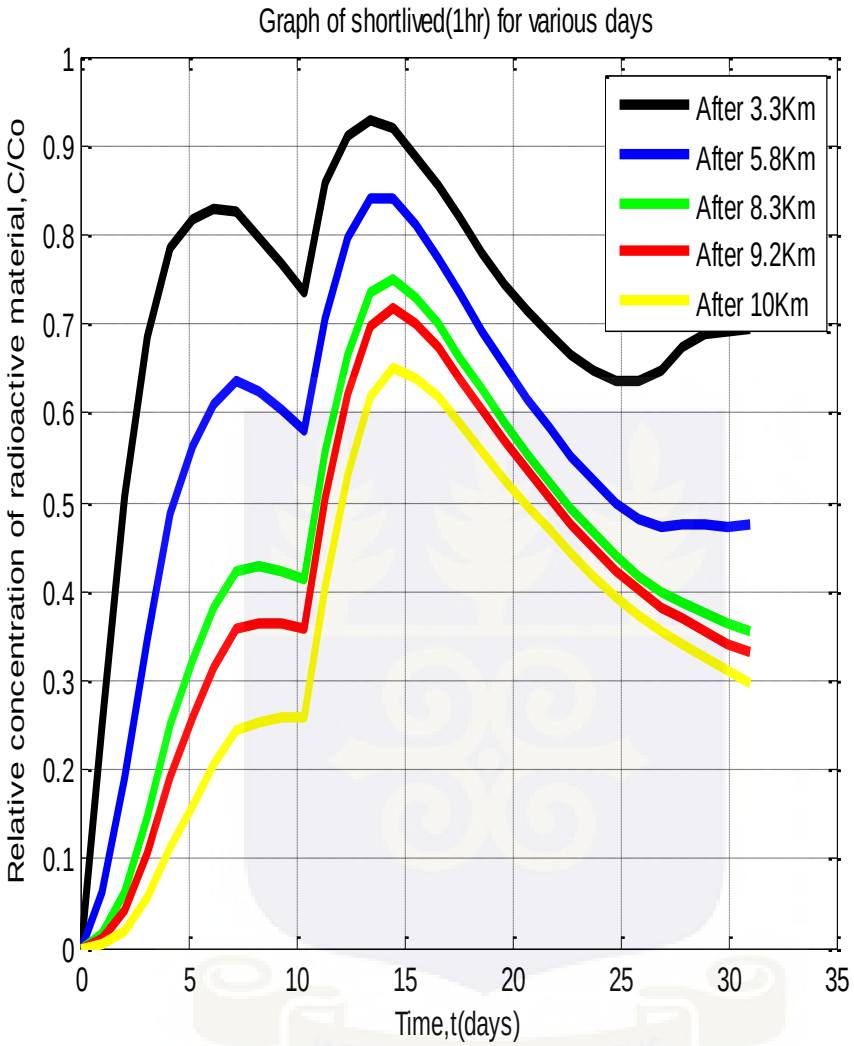


Figure 4.9: Simulation of (a) non decaying Ra-226 and (b) decaying 1hr sediment concentration distribution with respect to time in days for distance of 10 km distribution at $v = unsteady\ non - uniform$

Table 4.8: Relative concentration reading of (a) non decaying Ra-226 and (b) decaying 1hr, C(x,t) at 0≤x≤10Km (a)$v = unsteady\ non - uniform$

(a)

AFTER 3.3 km		AFTER 5.8 km		AFTER 8.3 km		AFTER 9.2 km		AFTER 10 km	
T	C/Co	T	C/Co	T	C/Co	T	C/Co	T	C/Co
5	0.9	5	0.65	5	0.4	5	0.3	5	0.2
15	1	15	0.99	15	0.97	15	0.96	15	0.92
31	0.98	31	0.95	31	0.94	31	0.93	31	0.92

(b)

AFTER 3.3 km		AFTER 5.8 km		AFTER 8.3 km		AFTER 9.2 km		AFTER 10 km	
T	C/Co	T	C/Co	T	C/Co	T	C/Co	T	C/Co
5	0.82	5	0.55	5	0.3	5	0.25	5	0.15
15	0.9	15	0.82	15	0.74	15	0.7	15	0.65
31	0.7	31	0.48	31	0.36	31	0.34	31	0.3

For Figure 4.9 (a) and table 4.8 (a) showed the concentration long-lived radioactive sediment a quickly increased in relative concentration of the material from the initial point of entry with time for all the specified distances. The simulations reached the maximum concentration for the 3.3 km after 15 days. However the simulation gradually reduced to 0.92 due to the unsteady non uniform nature of the velocity. The same pattern followed for the rest of the specified distances simulations.

Figure 4.9 (b) and table 4.8(b) also showed similar pattern physically like figure 4.9(a) however the output for figure 4.9(b) was more unsteady.

4.3 Model Validation

Due to unavailability of field data in this study area, the model was compared with similar work done by Jaiswal ***et al***, (2009) who used analytical method to simulate concentration for temporal and spatial dependent solute dispersion of pulse type input. A unit mass of the contaminant was used by considering different times for t=1.8 years, 0.4 years, 1.0 years, and 1.6 years shown in Figure 4.3.1

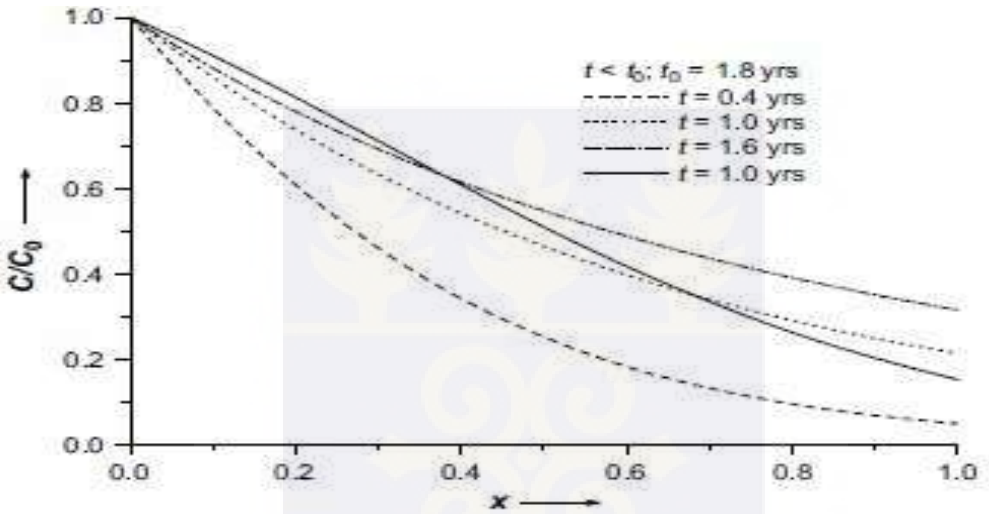


Figure 4.3.1: show the temporal dependent concentration dispersion pattern along uniform flow(Jaiswal *et al*, 2009)

Comparing the profile of Figure 4.3.1 to Figure (4.2-4.5), the shapes of the graph are similar. As time goes by the concentration levels decrease. Though the time covered by the material in literature according to Jaiswal ***et al***, 2009 is in years and the length of stream was not same,

the pattern was similar as shown in Figure (4.2-4.5) hence the model developed in this thesis can be used for actual monitoring of suspended radioactive sediment concentration in streams.

CHAPTER 5

Conclusion and Recommendation

5.1 Conclusion

In this thesis, numerical modeling of suspended radioactive sediments in stream was done to predict the concentration distribution from its point of entry to the length of stream. The research focus was centered on the relative concentration of the radioactive sediment at every point with time as it decays to the end of stream to provide better understanding of its behavior. Below are some conclusions drawn from the results of this thesis conducted on the deterministic modeling of the radioactive sediment transport in stream:

- Implicit finite difference solution of the Advection Dispersion Equation was developed to give the numerical solution of the concentration profile of the radioactive sediments in the stream.
- Solution algorithm structure program, computational flow chart and numerical coding for Matlab programming were developed and implemented to simulate the relative concentration distribution of the radioactive sediments in suspension in the stream.
- Deterministic models for predicting the concentration distribution of the suspended radioactive sediment were developed.
- To understand the hydrodynamics of radioactive sediment in stream the half-life of the material and velocity are important factors within a length of stream.
- The one-Dimensional numerical model results showed reasonable agreement between predictions from analytical model (Jaiswal ***et al***, 2009) for the various concentration distributions in stream.

5.2 Recommendations

Based on the outcome of this thesis the following are recommendations for consideration:

- Field work for various hydrological stations needs to be conducted to provide experimental results to validate the computational models.
- Discharge of radioactive material into stream which would suspend needs to be avoided by institutions.
- Level of radioactive pollution needs to be checked periodically and regularly by authorities in the various streams to ensure residents around these streams and users are safe.

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Appendix A

Table A. Unsteady Non-Uniform Velocity data, v1

time(days)	velocity(m/s)
1	3.909
2	3.93
3	3.93
4	3.93
5	3.855
6	2.879
7	2.6
8	2.281
9	1.182
10	0.965
11	0.835
12	7.775
13	7.855
14	7.251
15	4.963
16	2.568
17	1.835
18	1.271
19	0.872
20	0.812
21	0.809
22	0.809
23	0.809
24	0.826
25	0.869
26	1.097
27	1.267
28	1.658
29	1.397
30	1.217
31	1.264

Appendix B

```
%MATLAB Code for numerical modeling of suspended radioactive sediment
transport
%at every space and time is in streams by Linda Sarpong
%the model uses the one dimensional advection dispersion equation
%*****************************************************************
clear;
clc
% define and input parameters
L=10000; %distance or length of stream in meters
np=12;% Number of nodes
x=linspace(0,L,np+1);%number of grid points along the time axis
dx=x(2)-x(1);%space step
T=31*24*3600;%number of days(time)in seconds
oneday=24*3600;%converting one day to seconds
m=30;  %Number of time steps
tt=linspace(0,T,m+1)/oneday;% number of time steps
dt=3600*0.1;% time step
%halflife=3*24*3600; %halflife of radioactive sediment,Ga-67
halflife=1*3600; %halflife of radioactive sediment,-1hr
decay=log(2)/halflife;%decay rate of the transport
C=ones(np+1,m+1)*.000;%initialising C
A=zeros(np,np);%initilising A
%v=ones(np+1,m+1)*10;
D1=ones(m+1,1)*0.5;

%*****************************************************************
*****
%concentration of the the radioactive material
%setting the boundary condition
C(1,1)=1;%initial condition
C(1,:)=1;%boundary condition
D=0.5;%Dispersion coefficient in m^2 per s
rx=dt/dx;
rxx=dt/(dx^2);
e=1-decay*dt;
v1= xlsread('C:\Users\lenovo\Desktop\project\vel_exll.xlsx','B4:B34')
v=repmat(v1',np+1,1);
%v=[v1'];

for k=2:m+1 %TIME LOOP
    i=0;
    for j=2:np+1 %space loop
        i=i+1;
        %g(i)=e*C(i,k-1);
        g(i)=e*C(j,k-1);
        %a(i)=-rxx*D;
        %b(i)=1-rx*v(j,k)+rxx*D+rxx*D;
        %d(i)=rx*v(j,k)-rxx*D;
        a(i)=-rx*v(j-1,k)-rxx*D1(k);
        b(i)=1+rx*v(j,k)+rxx*D1(k)+rxx*D1(k);
        d(i)=-rxx*D1(k);
        if i==1
            A(i,i)=b(i);A(i,i+1)=d(i);
```

```
            g(i)=g(i)-a(i)*C(i,k-1);
        elseif i==np
               A(i,i)=b(i);A(i,i-1) = a(i)+d(i);
        else
                A(i,i-1) = a(i),A(i,i)=b(i),A(i,i+1)=d(i);
        end % end if

      end % end i
%       z=length(k)

   C(2:end,k)=(A\g');
    cc=length(C)
    xx=length(x)



end  % for k

figure
h=plot(x,C(:,2)' ,'-k')
hold on
r=plot(x,C(:,5)' ,'-b')
s=plot(x,C(:,10)' ,'-g')
%w=plot(x,C(:,20)' ,'-r')
f=plot(x,C(:,31)' ,'-r')
%h=plot(tt,C(4,:)' ,'-k')
%hold on
%r=plot(tt,C(7,:)' ,'-b')
%s=plot(tt,C(10,:)' ,'-g')
%w=plot(tt,C(11,:)' ,'-r')
%f=plot(tt,C(13,:)' ,'-y')
%set(u,'linewidth',2)
set(h,'linewidth',3)
set(r,'linewidth',3)
set(s,'linewidth',3)
%set(w,'linewidth',3)
set(f,'linewidth',3)
grid on
%xlabel('Time,t(days)')
xlabel('Distance from initial section(m)')
ylabel('Relative concentration of radioactive material,C/Co')
title('Graph of Ra-226 for various days')
%title('Graph of shortlived(1hr) for various days')
%legend('After 3.3Km','After 5.8Km','After 8.3Km','After 9.2Km','After
10Km')
legend('After two(2) days','After five(5) days','After ten(10)
days','After thirty-one(31) days')
```