

**NUCLEAR DESIGN OF A SUBCRITICAL ASSEMBLY DRIVEN  
BY ISOTOPIC NEUTRON SOURCES**

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DECLARATION

I declare that, except for references to other people's work, this thesis is the result of my own research and that it has neither in part nor in whole been presented elsewhere for another degree.



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## **DEDICATION**

**This work is dedicated to my father and mother  
for their love and care**



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**ABSTRACT**

A feasibility study of a conceptual nuclear design of a facility capable of producing high thermal neutron fluxes using isotopic neutron sources in a multiplying medium was carried out. The one-dimensional multigroup neutron diffusion equation was solved using the finite difference technique. A computer code SUNDES was written in FORTRAN 77 programming language and used to study the effect of reflectors, shielding materials and strength of isotopic neutron sources on the production levels of neutron fluxes. The neutronic calculations showed that with a homogeneous mixture of 20% enriched  $\text{UO}_2$  and Be as multiplying medium, BeO as reflector, Al as cladding and concrete as shield, thermal neutron fluxes as high as  $1 \times 10^7 \text{ n/cm}^2\text{-s}$  could be produced. Simultaneous irradiation of samples in two different regions at different fluxes is possible in the assembly. The analysis also revealed that different orders of thermal neutron fluxes could be produced depending on the strength of the driving isotopic neutron sources.

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**CHAPTER ONE****INTRODUCTION**

Neutron activation analysis (NAA) is one of the most powerful nuclear techniques for multi-element analysis. Though the first activation analysis in history was performed in 1936 [1], The technique was not widely developed until the 1960s when Ge(Li) detectors with high resolution and efficiency were developed. The technique has since become an important means for the super-trace, trace, semi-micro and normal analysis. Developing countries with small laboratories cannot explore this technique fully due to their inability to acquire nuclear reactors and generators to produce high neutron fluxes for the purpose. Such laboratories most often use isotopic sources such as Am/Be, Pu/Be etc in a non-multiplying medium. These devices are not capable of producing high thermal neutron fluxes for the analysis.

In this study a computer model for the nuclear design of a subcritical assembly driven by isotopic neutron sources in a multiplying medium (neutron multiplier) which is capable of producing thermal neutron fluxes of the order of  $10^7 \text{ n/cm}^2\text{-s}$  is provided. It is hoped that this device will be a substitute for high neutron flux generating devices for certain neutron activation analysis.

In the neutronic calculation for the design of the subcritical assembly, the one-dimensional multi-group neutron diffusion equation is solved. It is a conservative equation which takes account of both neutrons gained in the assembly through

fission, isotopic sources, scattering processes and those lost as a result of diffusion and structural absorption. Several methods of simulations such as the Monte Carlo method, numerical method, analytical method etc are available [2]. The Monte Carlo method is rather too complex for this analysis. It is difficult to use the analytical method to solve practical problems for multi-region geometries. The numerical method transforms the analytical equations characterizing the system into a set of algorithms and numerical equations. Based on these algorithms and numerical equations, computer programmes can then be developed.

Two well known approaches are available in the literature for numerical simulation; the finite element method and the finite difference method. The finite element method uses a triangular or tetrahedral element to set up a variational formulation of the problem which is then solved by optimization technique [3]. The finite difference method covers the region under consideration by a mesh consisting of horizontal and vertical lines and seeks the approximate values of the solution at the intersection [4]. For this work, it was found that the finite difference technique is more appropriate for the analysis.

In Chapter Two of this work, a review of the basic nuclear reactor physics and methods used for calculations will be presented. Special reference will be on critical and subcritical assemblies. The analytical and numerical methods of solution to the neutron diffusion or transport equations are discussed. The early works on subcritical assemblies and their importance are

also presented. The basic principles of neutron activation analysis, its applications and description of a typical neutron activation analysis experimental set-up located at the National Nuclear Research Institute of Ghana Atomic Energy Commission are discussed. Last but not the least, the aims and objectives of this work are outlined.

In Chapter Three, the mathematical model developed using the multi-group diffusion equation is presented. The computational flowchart and algorithms based on the finite difference technique are presented. A description of the input and output of the code are also given.

Chapter Four discusses the results of the preliminary investigations of the nuclear design. This is followed by a description of the neutron multiplier. Finally, a description of the mechanical features of the neutron multiplier and its mode of operation are presented.

The conclusions and recommendations on the work are contained in Chapter Five.

**CHAPTER TWO****LITERATURE REVIEW****2.1 INTRODUCTION**

Since the advent of nuclear reactors, neutron activation analysis has become a powerful tool for multi-element analysis. This nuclear technique has however been an elusive expedition for most developing countries and small laboratories. They cannot afford nuclear reactors which produce high neutron fluxes. Other high neutron flux generating devices such as accelerators are equally expensive. Alternative sources must therefore be sought for the purpose. It is envisaged that a form of subcritical assembly could be designed to achieve thermal fluxes greater than  $1 \times 10^7 \text{ n/cm}^2\text{-s}$  for neutron activation analysis.

The area of computational reactor physics is extensively studied in various nuclear institutes. A wealth of literature exists and can be found in reactor physics books [2,6,7,8]. In the present review, not all the detailed information in the literature will be considered. A review of the nuclear reactor theory and methods used for reactor physics calculations for the determination of neutron fluxes in nuclear reactors with special reference to critical and subcritical assemblies is first presented. The analytical and numerical methods and techniques that are normally used to provide solutions to neutron diffusion or transport equations are also discussed. This is followed by a discussion of the early works on subcritical assemblies and their importance. Next is a

discussion on the principles underlying neutron activation technique and a description of a typical neutron activation analysis experimental equipment using Am/Be source immersed in a pool of de-ionized water located at the National Nuclear Research Institute, Kwabenya-Accra.

Finally, the aims and objectives of the conceptual nuclear design of a subcritical assembly driven by isotopic neutron sources are stated.

## **2.2 Basic Reactor Physics**

Basic nuclear reactor physics is a very broad field of physics covering cross sections, transport theory, diffusion theory, reactor kinetics, multi-group theory etc [6, 7].

For the purpose of this work, emphasis will be on the mathematical methods for analyzing the behaviour of neutrons namely the transport theory and the diffusion theory. The analytical and numerical methods of solutions will also be discussed.

The detailed behaviour of neutrons in a nuclear reactor may be formulated mathematically by considering the production and collisions of neutrons of a particular energy moving in a particular direction and then integrating over all directions and energies. Neutrons are born at fast energy and slowed down to thermal energy as a result of a large number of collisions. Some are absorbed and others diffuse out of the reactor. Their distribution pattern can

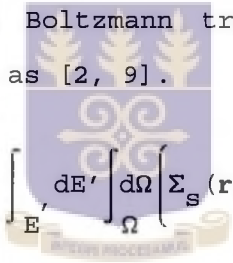
thus be predicted by mathematical models based on either the transport theory or the diffusion theory.

## 2.2.1 Reactor Physics Equations

### 2.2.1.1 Neutron Transport Equation

The neutron transport equation is a linear equation. It is fundamental and exact in describing the neutron population in nuclear reactors. Though the method is expensive to solve on digital computers, it is often used in areas near boundaries of reactors or near highly absorbing materials such as fuel rods or coolant elements where the diffusion equation is inaccurate.

The energy dependent Boltzmann transport equation may be written for a steady state as [2, 9].



$$\Omega \cdot \nabla \phi(\mathbf{r}, E, \Omega) + \Sigma_t \phi(\mathbf{r}, E, \Omega) = \int_E^\infty dE' \int_{\Omega'} d\Omega \left[ \Sigma_s(\mathbf{r}, E' \rightarrow E, \Omega' \rightarrow \Omega) \phi(\mathbf{r}, E', \Omega') \right] + S(\mathbf{r}, E, \Omega) \quad (2.1)$$

where  $\phi(\mathbf{r}, E, \Omega)$  is the number of neutrons of energy  $E$  per unit energy crossing a unit surface at position  $\mathbf{r}$  per unit time going in a unit solid angle centered in the direction  $\Omega$ .  $\Sigma_s(\mathbf{r}, E' \rightarrow E, \Omega' \rightarrow \Omega)$  is the probability per unit path length that a neutron at  $\mathbf{r}$  and going in a direction  $\Omega'$  with energy  $E'$  is scattered into a unit solid angle centered at  $\Omega$  and a unit energy interval centered at

E.  $S(\mathbf{r}, E, \Omega)$  is the number of neutrons with energy  $E$  created per unit volume at position  $\mathbf{r}$  going in a solid angle centered at  $\Omega$ .

The multi-group equations can be cast by dividing the energy range into  $G$  groups of arbitrary width  $\Delta E_g = E_g - E_{g-1}$ . For simplicity let the initial and final energies be labelled  $E_0$  and  $E_g$  respectively. Let also the thermal group be indexed by  $G+1$ . Equation (2.1) can thus be integrated from  $E_{g-1}$  to  $E_g$  and simplified to obtain

$$\Omega \cdot \nabla \bar{\phi}^g(\mathbf{r}, E, \Omega) + \bar{\Sigma}_t^g(\mathbf{r}, \Omega) \bar{\phi}^g(\mathbf{r}, \Omega) = \bar{Q}^g(\mathbf{r}, \Omega) + \bar{S}^g(\mathbf{r}, \Omega) \quad g = 1, 2, \dots, G \quad (2.2)$$

Where

$$Q(\mathbf{r}, E, \Omega) = \int dE' \int d\Omega' \Sigma_s(\mathbf{r}, E' \rightarrow E, \Omega' \rightarrow \Omega)$$

and the group cross sections averaged over the appropriate flux is:

$$\bar{\Sigma}_t^g(\mathbf{r}, \Omega) = \frac{\int_{E_{g-1}}^{E_g} dE \Sigma_t(\mathbf{r}, E) \phi(\mathbf{r}, E, \Omega)}{\int_{E_{g-1}}^{E_g} dE \phi(\mathbf{r}, E, \Omega)}$$

and

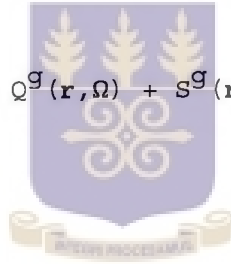
$$\overline{Q^g}(\mathbf{r}, \Omega) = \frac{\int_{E_{g-1}}^{E_g} dE Q(\mathbf{r}, E, \Omega) \phi(\mathbf{r}, E, \Omega)}{\int_{E_{g-1}}^{E_g} dE \phi(\mathbf{r}, E, \Omega)}$$

The thermal equation is formulated to read

$$\Omega \cdot \nabla \phi^{G+1}(\mathbf{r}, \Omega) + \Sigma_t^{G+1}(\mathbf{r}) \phi(\mathbf{r}, \Omega) = Q^{G+1}(\mathbf{r}, \Omega) + S^{G+1}(\mathbf{r}, \Omega) \quad (2.3)$$

Using equations (2.2), (2.3) and boundary conditions the desired multi-group transport equations are approximated as

$$\Omega \cdot \nabla \phi^g(\mathbf{r}, \Omega) + \Sigma_t^g(\mathbf{r}) \phi(\mathbf{r}, \Omega) = Q^g(\mathbf{r}, \Omega) + S^g(\mathbf{r}, \Omega), \quad g = 1, 2, \dots, G+1 \quad (2.4)$$



### 2.2.1.2 Neutron Diffusion Equation

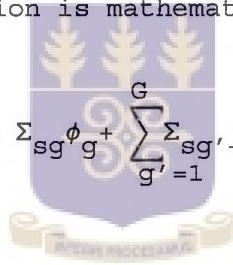
The neutrons in a reactor have energies spanning the range from 10MeV down to less than 0.01eV. The neutron-nuclear cross sections are sensitively dependent on the incident neutron energy. The neutron energies are discretized into energy intervals or groups. A multi-group diffusion equation can be successfully arrived at by adopting the concept of neutron balance for any given energy group. An account is taken of the way neutrons can

enter or leave this group. For a typical group,  $g$  the neutron balance is represented as:

$$\left( \begin{array}{l} \text{Time rate of change} \\ \text{of neutrons in group } g \end{array} \right) = - \left( \begin{array}{l} \text{Change due} \\ \text{to leakage} \end{array} \right) - \left( \begin{array}{l} \text{Absorption} \\ \text{in group } g \end{array} \right) + \left( \begin{array}{l} \text{Source} \\ \text{neutrons} \\ \text{appearing} \\ \text{in group } g \end{array} \right) - \left( \begin{array}{l} \text{Neutrons} \\ \text{scattering} \\ \text{out of group } g \end{array} \right) + \left( \begin{array}{l} \text{Neutrons} \\ \text{scattering} \\ \text{into group } g \end{array} \right) \quad (2.5)$$

This neutron balance equation is mathematically represented as:

$$\frac{1}{v_g} \frac{\partial \phi}{\partial t} = \nabla \cdot D_g \nabla \phi_g - \Sigma_{ag} \phi_g + S_g + \Sigma_{sg} \phi_g + \sum_{g'=1}^G \Sigma_{sg' \rightarrow g} \phi_{g'} \quad g=1, 2, \dots, G \quad (2.5)$$



where the source term due to fission is given by the expression

$$S_g = \chi_g \sum_{g'=1}^G \nu_{g'} \Sigma_{fg'} \phi_{g'} + S_g^{\text{ext}} \quad (2.7)$$

To satisfactorily account for the average behaviour of neutrons in each group, the energy-dependent diffusion equation

$$\frac{1}{v} \frac{\partial \phi}{\partial t} - \nabla \cdot D \nabla \phi + \Sigma_t \phi(r, E, t) = \int_0^{\infty} dE' \Sigma_s(E' \rightarrow E) \phi(r, E', t) + S^{\text{ext}}(r, E, t)$$

$$+ \chi(E) \int_0^{\infty} dE' \nu(E') \Sigma_f(E') \phi(r, E', t) \quad (2.8)$$

is integrated over a given energy group,  $E_{g-1} < E < E_g$ . Other energy-dependent equations could also be used as a starting point for the development of the multi-group diffusion equation [9, 10]. Further simplification gives the multi-group diffusion equation as:

$$\frac{1}{v_g} \frac{\partial \phi_g}{\partial t} = \nabla \cdot D_g \nabla \phi - \Sigma_{tg} \phi_g + \sum_{g'=1}^G \Sigma_{sg' \rightarrow g} \phi_{g'} + \chi_g \sum_{g'=1}^G \nu_{g'} \Sigma_{fg'} \phi_{g'} + S_g \quad (2.9)$$

$g = 1, 2, \dots, G$

If it is assumed that neutrons can only scatter to the lower groups, then the scatter term may be simplified to read:

$$\sum_{g'=1}^G \Sigma_{sg' \rightarrow g} \phi_{g'} = \sum_{g'=1}^{G-1} \Sigma_{sg' \rightarrow g} \phi_{g'} + \Sigma_{sg \rightarrow g} \phi_g \quad (2.10)$$

and hence the multi-group equation is rewritten as

$$\begin{aligned} \frac{1}{v_g} \frac{\partial \phi_g}{\partial t} = & \nabla \cdot D_g \nabla \phi - \Sigma_{tg} \phi_g + \sum_{g'=1}^{G-1} \Sigma_{sg' \rightarrow g} \phi_{g'} + \Sigma_{sg \rightarrow g} \phi_g + S_g \\ & + \chi_g \sum_{g'=1}^G \nu_{g'} \Sigma_{fg'} \phi_{g'} \quad g = 1, 2, \dots, G \end{aligned} \quad (2.11)$$

For a steady state the LHS of equation (2.11) becomes zero thus,

$$0 = \nabla \cdot D_g \nabla \phi_g - \Sigma_{tg} \phi_g + \sum_{g'=1}^{G-1} \Sigma_{sg' \rightarrow g} \phi_{g'} + \Sigma_{sg \rightarrow g} \phi_g + S_g + \chi_g \sum_{g'=1}^G \nu_{g'} \Sigma_{fg'} \phi_{g'} \quad (2.12)$$

In practice, only a certain number of groups are dealt with in reactor physics calculations. The most commonly used is the two multi-group calculations. In collapsing the groups for group constants calculations, the particular type of reactor being analyzed and its operating conditions with respect to fuel loading, isotopic composition, temperature and coolant conditions are considered. Analytical and numerical schemes can be employed to provide a solution to the multi-group equation. A brief outline of the analytical and numerical methods of solution to the neutron diffusion equation is presented. Emphasis will be placed on the finite difference method, a numerical technique which is of interest in this work.

## 2.2.2 Methods of Solution of Diffusion Equation

### 2.2.2.1 Analytical Method

Analytical solutions to neutron diffusion equation are provided in the literature. In this work, the solution as related to subcritical reactor physics will be discussed.

A feasibility study using a neutron source located at the

center of a multiplying medium for different fuel to moderator ratio was carried out by Akaho and Danso [11]. The Fermi age theory [12] was used to study the flux production levels in unreflected subcritical assemblies. Solid moderators as Be, BeO and graphite were used with different fuel enrichments.

For a homogeneous mixture of fuel and moderator in a cylindrical assembly the time dependent thermal equation is written as

$$\frac{\partial n(r, z, t)}{\partial t} = DV^2 \phi_T(r, z, t) - \Sigma_a \phi(r, z, t) + S(r, z, t) \quad (2.13)$$

where  $n(r, z, t)$  is the number of thermal neutrons and is related to the thermal flux by the equation

$$\phi(r, z, t) = \frac{2vn(r, z, t)}{\sqrt{\pi}} \quad (2.14)$$



Substituting equation (2.14) into (2.13) gives the expression

$$\frac{\sqrt{\pi}}{2v} \frac{\partial \phi(r, z, t)}{\partial t} = DV^2 \phi(r, z, t) - \Sigma_a \phi(r, z, t) + S(r, z, t) \quad (2.15)$$

Further simplification of equation (2.15) gives:

$$L_T^2 V^2 \phi(r, z, t) - \phi(r, z, t) + \frac{p}{\Sigma_a} q(r, z, \tau_T, t) = t_d \frac{\partial \phi(r, z, t)}{\partial t} \quad (2.16)$$

where  $q(r, z, \tau_T, t)$  is the slowing down density in the absence of

resonance absorption. The number of neutrons which become thermalized is  $pq(r, z, \tau_T, t)$ . By the dictates of Fermi age theory the equation

$$\frac{\partial^2 q}{\partial r^2}(r, z, \tau_T, t) = \frac{\partial q}{\partial \tau}(r, z, \tau_T, t) \quad (2.17)$$

must be satisfied. It will be assumed here that the neutrons from fission and isotopic sources are emitted mono-energetically at zero lethargy ( $\tau = 0$ ). In the slowing down model, neutrons start to slow down soon after being emitted from the sources. The source condition satisfied at  $\tau = 0$  is:

$$q(r, z, 0, t) = \frac{S\delta(r)\delta(z)}{\pi r l} + \eta f \epsilon \Sigma_a \phi(r, z, t) \quad (2.18)$$

The total number of fission neutrons produced per  $\text{cm}^3$  at the point  $(r, z)$  is accounted for by the second term on the RHS of equation (2.18). Substituting  $k_\infty/p$  for  $\eta f \epsilon$  in equation (2.18) gives:

$$q(r, z, 0, t) = \frac{S\delta(r)\delta(z)}{\pi r l} + \frac{k_\infty \Sigma_a \phi(r, z, t)}{p} \quad (2.19)$$

Assume that in a cylindrical geometry

$$\phi(r, z, t) = \sum_{n=\text{odd}} A_{mn}(t) J_0\left(\frac{x_n r}{R}\right) \cos\left(\frac{n\pi z}{H}\right) \quad (2.20)$$

and

$$q(r, z, \tau, t) = \sum C_{mn}(\tau, t) J_0 \left( \frac{X_n r}{R} \right) \cos \left( \frac{n\pi z}{H} \right) \quad (2.21)$$

$$\text{where } C_{mn}(\tau, t) = T_{mn} e^{-B_{mn} \tau}$$

For an infinite cylinder of radius  $R$  and height  $H$ , the infinite source at  $r = 0$  is given by the equation:

$$S(r, z) = \frac{S \delta(r) \delta(z)}{\pi R^2 H} \sum_{n=1}^{\infty} \frac{J_0(X_n r/R)}{J_1^2(X_n)} \sum_{n=\text{odd}} J_0 \left( \frac{X_n r}{R} \right) \cos \left( \frac{n\pi z}{H} \right) \quad (2.22)$$

Substituting equations (2.20), (2.21) and (2.22) into (2.19) gives the following expression:

$$\begin{aligned} \sum_{n=\text{odd}}^{\infty} T_{mn}(t) e^{-B_{mn} \tau} J_0 \left( \frac{X_n r}{R} \right) \cos \left( \frac{n\pi z}{H} \right) &= \frac{2S}{v} \sum_{n=1}^{\infty} \frac{J_0(X_n r/R)}{J_1^2(X_n)} \sum_{n=\text{odd}} J_0 \left( \frac{X_n r}{R} \right) \cos \left( \frac{n\pi z}{H} \right) \\ &+ \frac{k \Sigma_a}{\rho} \sum_{n=\text{odd}} A_{mn}(t) J_0 \left( \frac{X_n r}{R} \right) \cos \left( \frac{n\pi z}{H} \right) \end{aligned} \quad (2.23)$$

From the neutron diffusion equation for a cylindrical geometry

$$\nabla^2 \phi = -B_{mn}^2 \phi \quad (2.24)$$

where  $B_{mn}^2 = \left( \frac{X_n}{R'} \right)^2 + \left( \frac{n\pi}{H'} \right)^2$ .  $R'$  and  $H'$  are the extrapolated radius

and height of the cylinder respectively. Substituting equation (2.20), (2.23) and (2.24) into (2.16) and replacing  $\tau$  with  $\tau_T$  gives the expression

$$\begin{aligned}
 & -L_T^2 \sum_{n=\text{odd}} B_{mn}^2 A_{mn}(t) J_0 \left( \frac{X_n r}{R'} \right) \cos \left( \frac{n\pi z}{H'} \right) - \sum_{n=\text{odd}} A_{mn}(t) J_0 \left( \frac{X_n r}{R'} \right) \cos \left( \frac{n\pi z}{H'} \right) \\
 & + \frac{p}{\Sigma_a} \sum_{n=\text{odd}} \left( \frac{2S}{v} \sum_{n=1}^{\infty} \frac{1}{J_1^2(X_n)} + \frac{k_{\infty} \Sigma_a A_{mn}(t)}{p} \right) e^{-B_{mn}^2 \tau_T} J_0 \left( \frac{X_n r}{R'} \right) \cos \left( \frac{n\pi z}{H'} \right) \\
 & = t_d \sum \frac{dA_{mn}}{dt} J_0 \left( \frac{X_n r}{R'} \right) \cos \left( \frac{n\pi z}{H'} \right) \quad (2.25)
 \end{aligned}$$

Simplifying equation (2.25) further gives the expression:

$$\begin{aligned}
 & -A_{mn}(t) \left( 1 - \frac{k_{\infty} e^{-B_{mn}^2 \tau}}{1 - L_T^2 B_{mn}^2} \right) + \frac{2PS}{\Sigma_a k_{\infty} v} \sum_{n=0}^{\infty} \frac{J_0(X_n r/R')}{J_1^2(X_n)} \frac{k_{\infty} e^{-B_{mn}^2 \tau}}{(1 + L_T^2 B_{mn}^2)} \\
 & = \frac{t_d}{(1 + L_T^2 B_{mn}^2)} \frac{dA_{mn}}{dt} \quad (2.26)
 \end{aligned}$$

$$\text{Let } k_{mn} = \frac{k_{\infty} e^{-B_{mn}^2 \tau_T}}{(1 + B_{mn}^2 L_T^2)} \quad \text{and } t_{mn} = \frac{t_d}{(1 + B_{mn}^2 L_T^2)} \quad \text{then equation (2.26)}$$

becomes

$$t_{mn} \frac{dA_{mn}}{dt} = (k_{mn} - 1) A_{mn} + \frac{2PS}{\Sigma_a k_{\infty} v} \left( \sum \frac{J_0(X_m r/R')}{J_1^2(X_n)} \right) k_{mn} \quad (2.27)$$

Integrating equation (2.27) subject to the boundary conditions  $\phi(0) = 0$  and  $J_0 = 1$  yields:

$$A_{mn} = - \frac{2PS}{\Sigma_a k_{\infty} v} \left( \frac{k_{mn}}{k_{mn} - 1} \right) \sum_{n=1}^{\infty} \frac{1}{J_1^2(X_n)} \left( e^{(k_{mn} - 1) \frac{t}{t_{mn}}} - 1 \right) \quad (2.28)$$

The expression for the thermal flux is thus obtained by substituting equation (2.28) into (2.19) to give:

$$\phi(r, z, t) = \frac{2PS}{\Sigma_a k_{\infty} v} \sum_{m=1,2} \sum_{n=1}^{\infty} \left( \frac{k_{mn}}{k_{mn} - 1} \right) \left( e^{(k_{mn} - 1) \frac{t}{t_{mn}}} - 1 \right) x \quad (2.29)$$

$$J_0 \left( \frac{X_m r}{R'} \right) \cos \left( \frac{n\pi z}{H'} \right)$$

The time behaviour of the flux following the insertion of the source depends on the magnitudes of  $k_{mn}$ . The eigenvalue of  $B_{mn}$  increases monotonically while  $k_{mn}$  decreases monotonically ie  $k_{m1} > k_{m3} > k_{m5} \dots$ . In subcritical situations,  $k_{mn} < 1$  thus all the time dependent exponents in equation (2.29) will be negative and the flux then approaches a steady state as:

$$\phi(r, z) = \frac{2PS}{\Sigma_a k_\infty v} \sum_{m=1,2}^{\infty} \sum_{n=1}^{\infty} \left( \frac{k_{mn}}{1-k_{mn}} \right) \frac{1}{J_1^2(x_n)} J_0 \left( \frac{x_m r}{R'} \right) \cos \left( \frac{n\pi z}{H'} \right) \quad (2.30)$$

For  $m = n = 1$

$$\phi(r, z) = \frac{2PS}{\Sigma_a k_\infty v} \left( \frac{k_{eff}}{1-k_{eff}} \right) \frac{1}{J_1^2(2.405)} J_0 \left( \frac{2.405}{R'} \right) \cos \left( \frac{\pi z}{H'} \right) \quad (2.31)$$

$$\text{where } k_{eff} = \frac{k_\infty e^{-B^2 \tau_T}}{1-B L_T} \quad \text{and } B^2 = \left( \frac{2.405}{R'} \right)^2 + \left( \frac{\pi}{H'} \right)^2$$

For equation (2.31) to be solved, the resonance escape probability  $p$ , and infinite multiplication factor  $k_\infty$  etc must be known for the fuel-moderator system. These parameters are evaluated from the following expressions [13-19].

$$p = \exp \left( - \frac{N_{238} I_{eff}}{\xi \Sigma_s} \right), \quad (2.32)$$

where  $I_{eff}$  is the effective resonance integral and is given by the expression:

$$I_{eff} = 3.9 \left( \frac{N_m \sigma_s^m}{M} \right)^{0.415} \quad (2.33)$$

The infinite multiplication factor is given by the expression:

$$k_\infty = \epsilon \eta p f, \quad (2.34)$$

where the symbols have their usual meanings.

The fast fission factor  $f$  is given by

$$f = \frac{\sigma_a^{235} + t\sigma_a^{238}}{\sigma_a^{235} + t\sigma_a^{238} + (r/a)\sigma_a^m} \quad (2.35)$$

$$r = \frac{N_m}{N_u} \quad \text{and} \quad t = \frac{N^{238}}{N^{235}} \left( \frac{1 - e}{e} \right), \quad \text{where } e \text{ is the enrichment.}$$

Table 1 [11, 20] contains the values  $\Sigma_a$ ,  $L_T^2$  and  $\tau_T$  for various moderators required to obtain the flux from equation (2.31)

Table 1: Thermal Neutron Diffusion Parameters of Moderators.

| Moderator | $\Sigma_a \text{ (cm}^{-1}\text{)}$ | $L_T^2 \text{ (cm}^2\text{)}$ | $\tau_T$ |
|-----------|-------------------------------------|-------------------------------|----------|
| Be        | $1.04 \times 10^{-3}$               | 480                           | 102      |
| BeO       | $6.00 \times 10^{-4}$               | 790                           | 100      |
| Graphite  | $2.40 \times 10^{-4}$               | 3500                          | 368      |

Different moderators, fuel enrichment, moderator to fuel ratio and source strengths were used on the basis of the above analysis to conclude that Be-moderator for 20% enriched uranium has the highest infinite multiplication factor. The variation of the infinite multiplication factor with different moderator to fuel

ratios for 20% enriched uranium is shown in Figure 2.1.

This technique of analysis, however, is applicable to source(s) placed at the center of the assembly not at various regions. The analytical solution is expected to be difficult in analyzing multiple source problems in the multi-regions because as given in equation (2.28) the source problem is not easily defined.

A review of the numerical approach to the solution of diffusion equation which is practical is presented in the next section.

#### **2.2.2.2 Numerical Method**

Both analytical and numerical methods are used in solving the neutron diffusion equation. Basically, the analytical solution is suitable for one-speed ie neutrons diffusion in homogeneous reactors. In real reactors, however, calculations are based on the heterogeneous nature of the core. Not only must one consider non uniformities corresponding to fuel pellets, cladding materials, moderator, coolant element, but spatial variation as well [6]. These complex situations cannot be catered for by the analytical method and hence must be discarded in favour of a direct numerical solution of the diffusion equation.

In the use of the numerical method , the differential diffusion equation is rewritten in finite difference form and the resulting difference equations are solved using digital computers. Difference equations can be formulated for plane, cylindrical and spherical geometries. There are other numerical methods such as

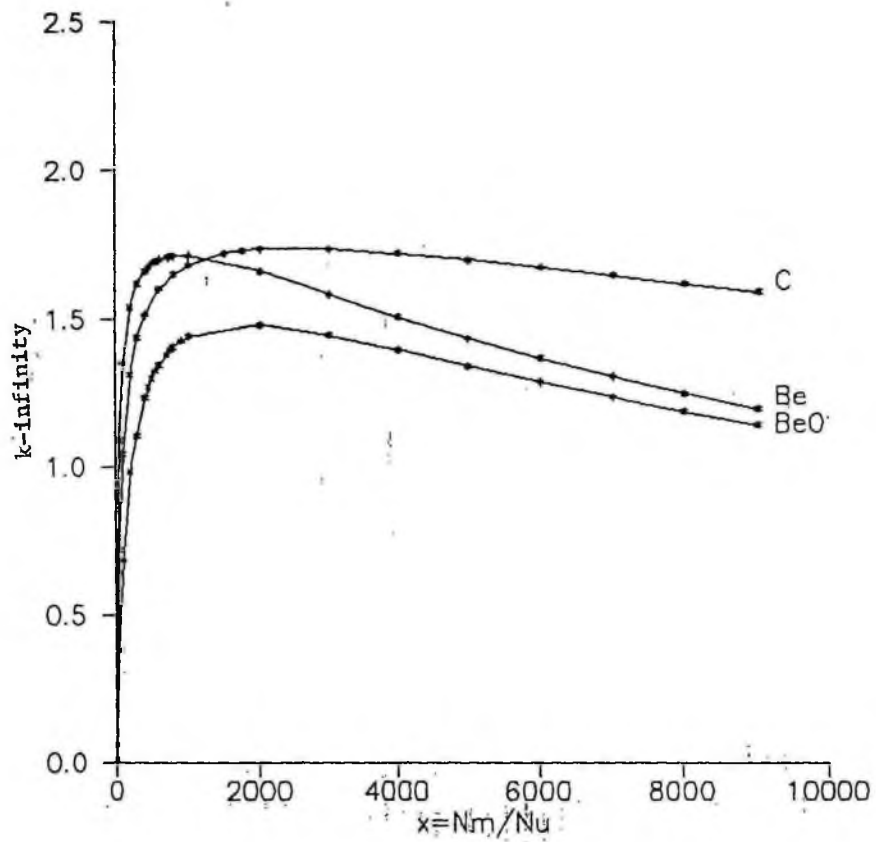


Figure 2.1: Variation of the factor  $k_{\infty}$  with Different  $N_m/N_u$  values for 20% Enrichment of Uranium, [11].

the finite element method, Monte Carlo method etc for solving diffusion equations. Details of these methods can be found in references [2, 3]. Only the finite difference method shall be discussed.

### 2.2.2.3 Finite difference method

For simplicity, assume the differential diffusion equation

$$D \frac{d^2 \phi}{dx^2} + \Sigma_a \phi(x) = S(x) \quad (2.36)$$

is to be solved numerically subject to the boundary condition for a finite slab of width  $a$ ,  $\phi(0) = \phi(a) = 0$ . But for real reactors, it is assumed that group constants  $D(x)$  and  $\Sigma_a(x)$  for the various subregions are constant. Suppose the spatial variable  $x$ , is divided into a set of  $N+1$  discrete points as shown in Figure 2.2

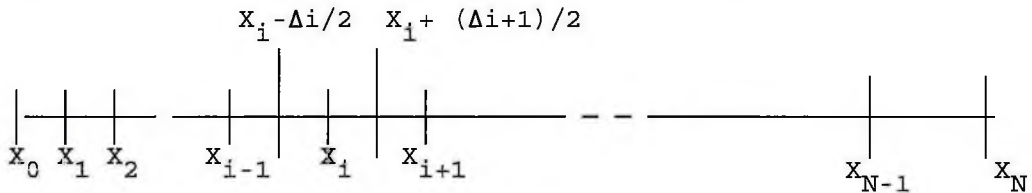


Figure 2.2 Mesh Points with intervals

There exists a number of schemes that can be used to generate a difference equation representing equation (2.36) on this mesh. Taylor series expansion can be used to derive a central difference formula for  $d^2 \phi / dx^2$ . However, the commonly used scheme is to

integrate the original difference equation over any arbitrary mesh interval  $X_i - \Delta i/2$  and  $X_i + (\Delta i + 1)/2$  say, where  $\Delta i$  is the interval between mesh points. Thus integrating equation (2.36) term by term yields

$$\int_{X_i - \Delta i/2}^{X_i + (\Delta i + 1)/2} dx \Sigma_a(x) \phi(x) \approx \Sigma_{ai} \phi_i \left( \frac{\Delta i}{2} + \frac{\Delta i + 1}{2} \right) \quad (2.37)$$

$$\int_{X_i - \Delta i/2}^{X_i + (\Delta i + 1)/2} dx \frac{d}{dx} D(x) \frac{d\phi}{dx} = D(x) \frac{d\phi}{dx} \Big|_{X_i - \Delta i/2}^{X_i + (\Delta i + 1)/2} \quad (2.38)$$

and

$$\int_{X_i - \Delta i/2}^{X_i + (\Delta i + 1)/2} dx S(x) \approx S_i \left( \frac{\Delta i}{2} + \frac{(\Delta i + 1)}{2} \right) \quad (2.39)$$

Equation (2.38) is simplified further by applying a simple two - point difference formula on  $\frac{d\phi}{dx}$  to give

$$\left. \frac{d\phi}{dx} \right|_{X_i + (\Delta i + 1)/2} \approx \frac{\phi_{i+1} - \phi_i}{\Delta i + 1} \quad (2.40)$$

$$\left. \frac{d\phi}{dx} \right|_{X_i - \Delta i/2} \approx \frac{\phi_i - \phi_{i-1}}{\Delta i} \quad (2.41)$$

and using the centered average for D;

$$D(X_i + (\Delta i + 1)/2) = \frac{1}{2} \left( D_{i+1} + D_i \right) \approx D_{i,i+1} \quad (2.42)$$

$$D(X_i - \Delta i/2) = \frac{1}{2} \left( D_{i-1} + D_i \right) \approx D_{i,i-1} \quad (2.43)$$

Substituting equations (2.37) (2.43) into (2.36) a set of difference equations are obtained as:

$$a_{i,i-1} \phi_{i-1} + a_{i,i+1} \phi_{i+1} = S_i \quad i = 1, 2, \dots, N-1 \quad (2.44)$$

where

$$a_{i,i-1} = \left( \frac{D_i + D_{i-1}}{\Delta i} \right) \frac{1}{\Delta i + \Delta i + 1}$$

$$a_{i,i} = \Sigma_a + \left( \frac{D_{i+1} + D_i}{\Delta i + 1} + \frac{D_{i-1} + D_i}{\Delta i} \right) \frac{1}{\Delta i + \Delta i + 1}$$

and

$$a_{i,i+1} = - \left( \frac{D_{i+1} + D_i}{\Delta i} \right) \frac{1}{\Delta i + \Delta i + 1}$$

Similarly, equations of the type as in Eq.(2.44) with different coefficients can be derived for two and three dimensional problems [6]. The essential task involved in solving the 3-point finite difference equation (2.44) is to select a suitable algorithm for

its solution on digital computers. It can be solved either by the Gaussian elimination or by any other iterative method as the Gauss-Seidal method.

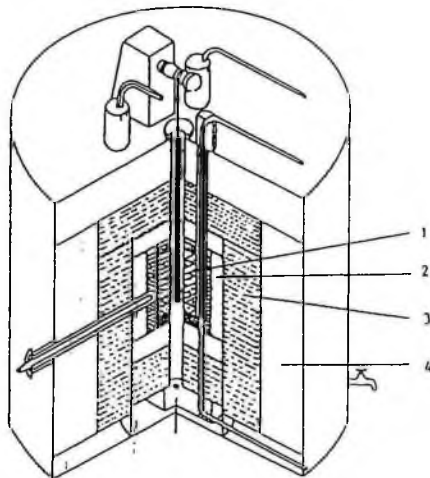


### 2.3 Subcritical assemblies.

Subcritical assemblies were designed for the investigation of definite problems dealing with neutron physics and core technology [21-23]. In these devices a chain reaction cannot be sustained without the introduction of external neutron sources. The introduction of these sources offset losses due to diffusion through the sides of the assembly and structural absorption. Such an assembly can be obtained by simply surrounding a neutron source with a moderator and enriched uranium. Subcritical assemblies are smaller than critical assemblies and consequently requires less material. Other advantages are that no complicated and expensive control mechanisms and instrumentation systems are required. The low flux levels at which these facilities operate present no problems with respect to radiation protection. Subcritical facilities are much cheaper to design and construct as compared to nuclear reactors and are particularly attractive for training reactor physics students. A typical schematic drawing of a subcritical assembly based on this principle is shown in Figure 2.3. Some critical assemblies also use isotopic sources to drive them in the presence of a multiplying medium. A typical type of such a reactor using 20% enriched compressed mixture of  $U_3O_8$  in

Fig.2.3 Schematic drawing of subcritical uranium assembly

RS-1<sup>55</sup> [26]



- (1) Active zone, containing granular  $\text{UO}_2$  (U enriched to 36% in  $\text{U}^{235}$ ) mixed with polyethylene (660g  $\text{U}^{235}$ );
- (2) lead shield;
- (3) shield of paraffin loaded with 5% boron carbide;
- (4) water shield.

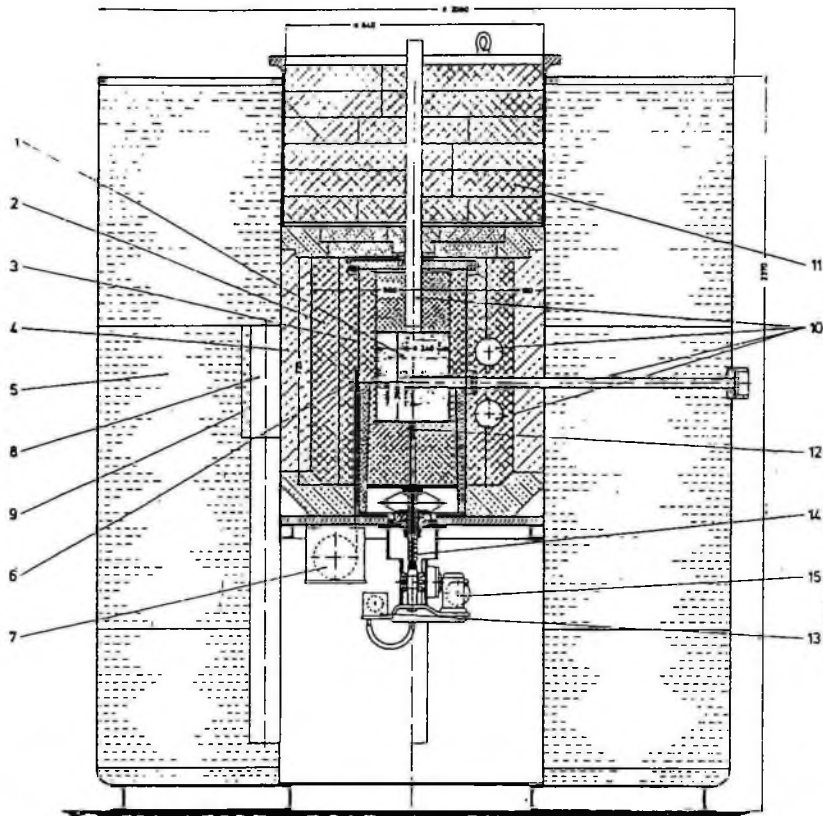
polyethylene powder is shown in Figure 2.4 and the neutron flux distribution within it is presented in Figure 2.5.

The degree of subcriticality of an assembly is measured by the value of the effective multiplication factor,  $K_{eff}$  [24]. The effective multiplication factor is defined as the ratio of the number of neutrons at a given time to that of the preceding time [14-16]. This is mathematically expressed as

$$K_{eff} = \frac{\text{number of neutrons in one generation}}{\text{number of neutrons in the preceding generation}} = \frac{n(t)}{n(t-1)} \quad (2.45)$$

This parameter can also be used to define criticality and supercriticality. The assembly is said to be subcritical if the effective multiplication factor is less than unity and critical when it is unity. If the value is greater than unity then the assembly is supercritical.

The usefulness of subcritical assemblies cannot be over emphasized. They serve as research as well as teaching tools and can be used to predict the optimal arrangement, fuel proportion and spacing and for obtaining other important data relating to the design and construction of a proposed full-size operating reactor [24, 25]. Works by Kerten and Went [25] on subcritical aqueous suspension reactors have shown that the concentration of fuel as well as the enrichment of the fissile components can be adjusted to the experimentally desired values. The PUK subcritical facility designed to have  $K_{eff}$  equal to 0.95 [20, 25] is capable of



- |                           |                                 |
|---------------------------|---------------------------------|
| 1 Core                    | 9 Reflector of Neutron Detector |
| 2 Reflector (Graphite)    | 10 Experimentation Channel      |
| 3 Inner Reactor Vessel    | 11 Thermal Column               |
| 4 Lead Shielding          | 12 Neutron Source               |
| 5 Water Shielding         | 13 Drive for Neutron Source     |
| 6 Control Plate           | 14 Hoist for Reactor Core Half  |
| 7 Drive for Control Plate | 15 Motor for Hoist              |
| 8 Neutron Detector        |                                 |

Figure 2.4 Reactor SUR [23]

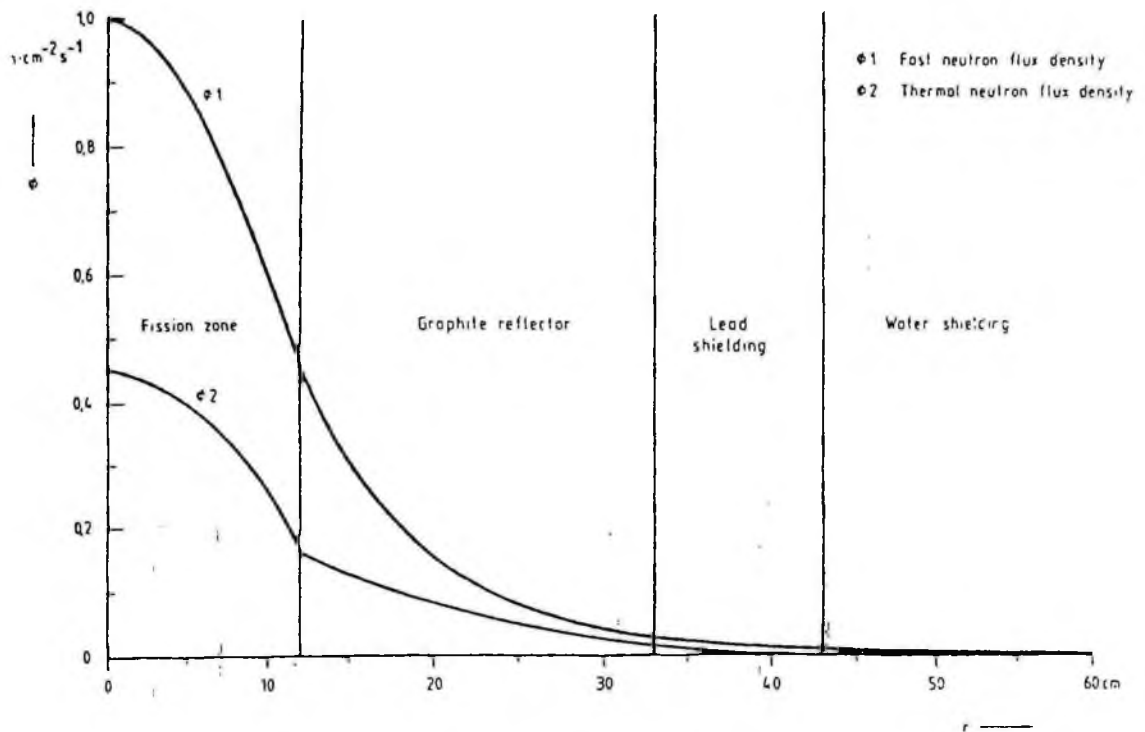


Figure 2.5: Relative Neutron Flux Distribution [23].

measuring the effect of water gaps in a lattice. It was used to study methods for preventing flux peaking and to supply experimental data for testing calculational techniques. Researchers utilized subcritical assemblies and other facilities to measure the effective multiplication factor of damaged cores such as TMI-2 and also to estimate its infinite multiplication factor distribution [25]. Further work [21, 23, 25] have also been carried out in connection with the use of subcritical assembly for specific reactor physics problems.

As stated above subcritical and critical assemblies played complimentary roles for nuclear reactors. They can be designed to find application in neutron activation analysis technique. Before presenting work in this area in Chapter Three, the principles underlying neutron activation analysis and the equipment presently in use in most scientific research laboratories are very briefly discussed.

## **2.4 Principles of Activation Analysis**



Neutron activation analysis was proposed since the advent of nuclear reactors . G. von Hevesy and H. Levy used the technique for the analysis of dysprosium in rare earths using an isotopic neutron source [26]. It has since developed and become a powerful nuclear analytical technique for multi-element analysis. The technique is very sensitive and more advantageous when the analysis involve trace elements and also when neutron flux of the order

$10^{12}$  n/cm<sup>2</sup>-s is used. Moreover, the technique is non-destructive and hence the original sample may be recovered after analysis. Small quantities of the samples are required for analysis and the results are not influenced by the chemical state of the elements being investigated [26-28]. Additionally, the samples are not contaminated after irradiation which is of prime concern in trace elements analysis.

The high neutron fluxes produced by nuclear reactors are particularly useful for the determination of extremely small quantities of a large number of elements using neutron activation analysis. The technique is highly sensitive for trace elements in a variety of matrices in many fields of study such as:

- (i) high purity materials
- (ii) biological
- (iii) environmental
- (iv) geological
- (v) forensic science etc [23, 26-29].

When the sample is exposed to high flux of thermal neutrons, some elements in the sample absorb the neutrons, become unstable and disintegrate giving off characteristic rays. Each element gives off a unique characteristic rays. Thus the element can be identified and quantified if possible. This is done by using Si(Li), Ge(Li) or NaI detectors to detect the emitted rays. The signals from the detector are amplified in a spectroscopy amplifier and passed on to a multichannel analyzer which converts the analog signals to digital for analysis.

#### 2.4.1 Neutron Activation Analysis Equipment

Various experimental equipment using radioisotopes for neutron activation analysis are being used in many laboratories. A schematic drawing of one such facility is shown in Figure 2.6. The experimental equipment of the Ghana Atomic Energy Commission (G.A.E.C) is discussed here.

The G.A.E.C radioisotope experimental facility is a typical example of such facilities for NAA and is shown in Figure 2.7. It consists of a 20Ci Am/Be radioisotope neutron source, a pneumatic transfer system, a High Purity Germanium detector (HPGE), a Canberra spectroscopy amplifier, an oscilloscope, a Canberra series 30 Multichannel Analyzer (MCA), an APPLE II PLUS Micro-computer and a printer.

The 20Ci Am/Be radioisotope source is cylindrical in shape and has a source strength of  $10^5$  n/s. The source is centrally located in a fibre glass tank filled with de-ionized water (non-multiplying medium) which serves as moderator. As an additional measure to prevent any radiation hazards through leakage from the source the fibre tank is surrounded with concrete blocks.

Samples are transferred to and from the source in rabbits by means of the pneumatic transfer system. It is operated at a pressure of 15psi and has a sample travel time of 3 seconds [30,31]. The HPGE detector is maintained at liquid nitrogen temperature (77°K) in a dewar flask to keep down dark currents. It is operated at a bias voltage of 3,000V and has a resolution of 2.25KeV for

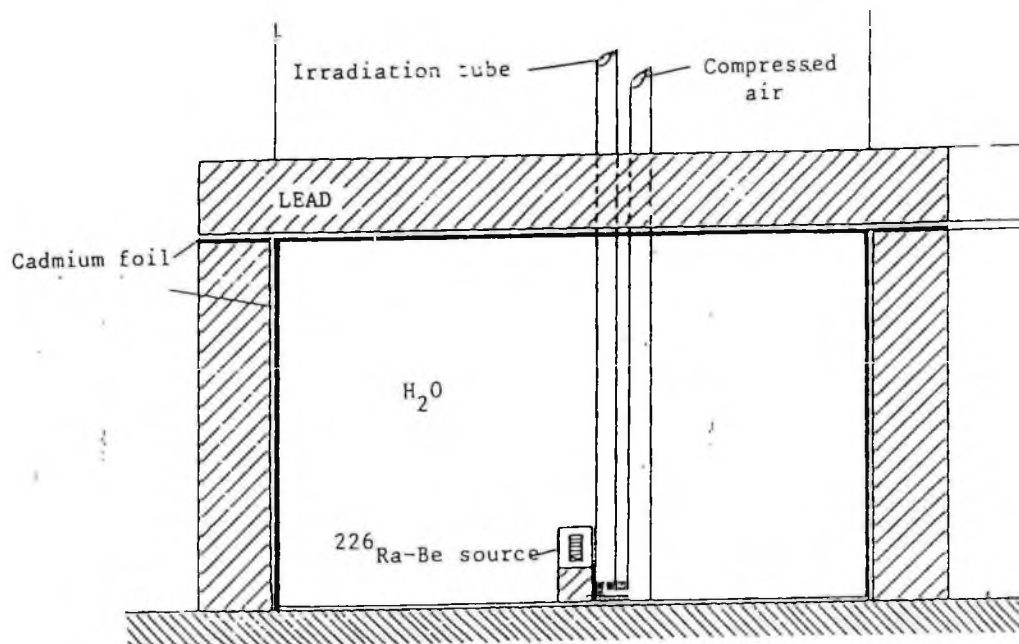


Figure 2.6: <sup>228</sup>Ra-Be Source Arrangement [26].

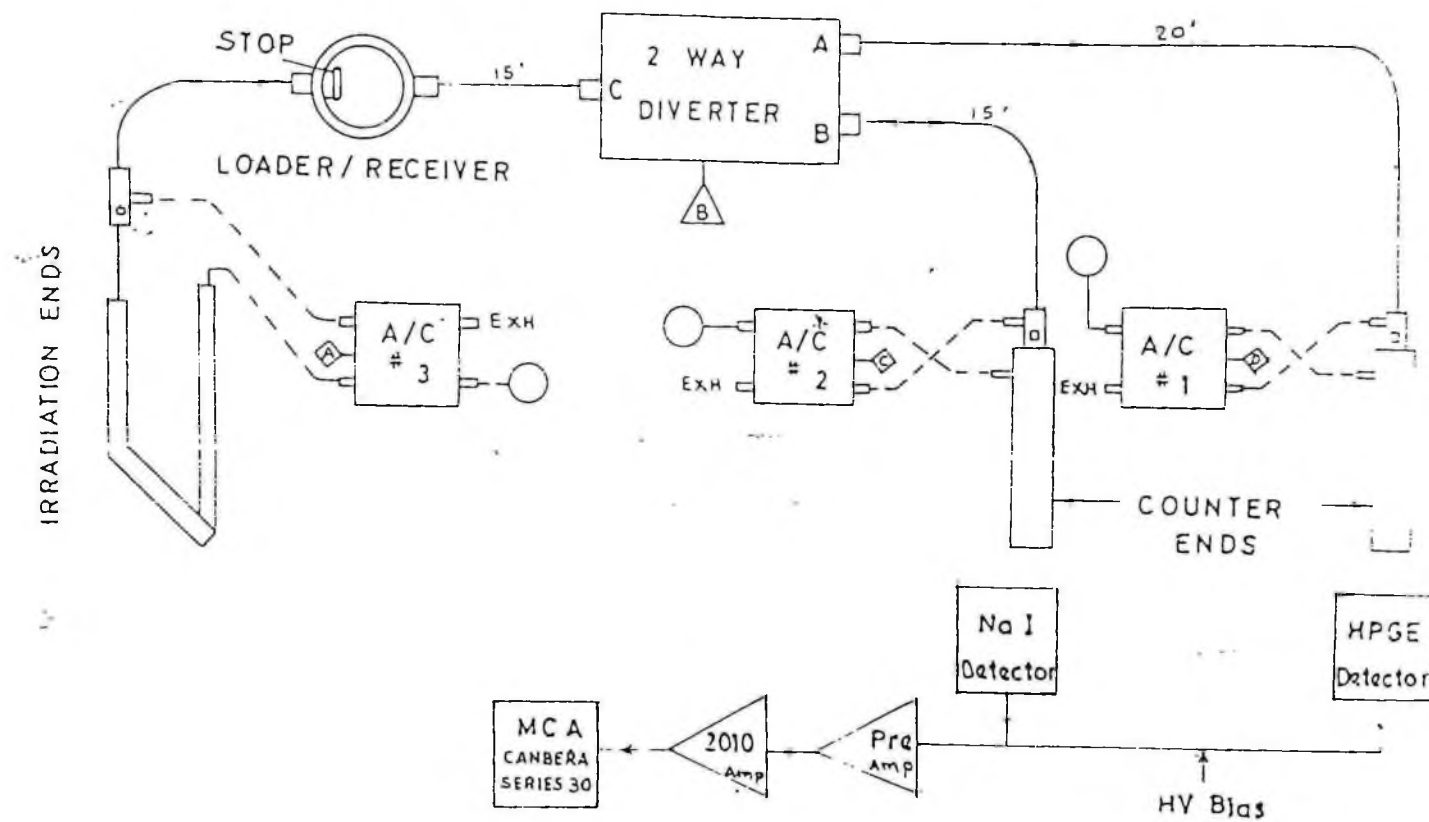


Figure 2.7: PNEUMATIC TRANSFER SYSTEM  
SHOWING TUBE CONNECTIONS [30,31]

the 1332KeV photo peak of Co-60. Characteristic rays from the irradiated sample are detected as electrical signals, amplified by the Canberra spectroscopy amplifier and processed in the Canberra series 30 multichannel analyzer. Since these electrical signals are analog, they are often converted to digital for easy analysis. An APPLE II PLUS micro-computer coupled to the MCA is used for analyzing the data.

#### 2.4.2 Isotopic Sources

The major source of obtaining high thermal neutron flux for activation analysis is from nuclear reactors. There are however other neutron sources such as accelerators, generators, and isotopic sources. The most commonly available are the isotopic sources. The choice of a particular source is determined by factors such as price, half-life, neutron yield, required shielding from gamma rays, toxicity, small size and of course availability [26, 28, 31].

There are three main types of isotopic neutron sources namely: alpha-emitters, spontaneous fission sources and gamma-emitters. Alpha-emitters produce neutrons through an  $(\alpha, n)$  reaction. These radioactive alpha emitters can introduce  $(\alpha, n)$  reactions on some target nuclides. A typical reaction using beryllium as target material is  ${}^9\text{Be}(\alpha, n) {}^{12}\text{C}$ . Table 2 [26] shows a number of target nuclides with their Q-values and neutron yields using  ${}^{210}\text{Po}$  alpha-emitters as neutron source. Beryllium gives the highest Q-

and neutron yield values and is generally preferred for neutron activation analysis purposes.

| Target Nuclide    | Q-value | Neutron Yield per $10^6$ alphas |
|-------------------|---------|---------------------------------|
| ${}^7\text{Li}$   | -2.79   | 2.6                             |
| ${}^9\text{Be}$   | 5.70    | 80                              |
| ${}^{10}\text{B}$ | 1.06    | 13                              |
| ${}^{11}\text{B}$ | 0.16    | 26                              |
| ${}^{13}\text{C}$ | 2.22    | 10                              |
| ${}^{18}\text{O}$ | -0.70   | 29                              |
| ${}^{19}\text{F}$ | -1.95   | 12                              |

In the case of spontaneous fission, some transuranium elements disintegrate not only by the  $\alpha$ -decay, but also by spontaneous fission. This process releases several neutrons. The most frequently used of these transuraniums is  ${}^{252}\text{Cf}$ . It is small in size, has similar neutron spectrum with fission uranium but has a relatively short half-life of 2.65 years and also very expensive. For gamma-emitters,  $(\gamma, n)$  reactions occur only when the incident  $\gamma$ -energy exceeds the binding energy of the neutron in the nucleus. Most nuclei have very high binding energies, thus in practice only  ${}^{124}\text{Sb}$  is used as a gamma source because of its low binding energy. The neutron flux energy obtained is very high but serious handling problems arise as the  $\gamma$ -dose rate is also quite high.

## 2.5 Concluding Remarks

The reviewed research work as related to subcritical assemblies has revealed that most developing countries and small laboratories cannot fully explore the activation technique because they cannot afford nuclear reactors. They therefore depend on facilities which use radioisotopes in non-multiplying medium for the technique. The thermal neutron flux emanating from such facilities is low. With the low flux very small fractions of samples are normally activated. The result is that photo peaks are not so much different from the background. This presents a problem when trace elements are to be analyzed. Poor detection limits are therefore associated with these devices. Samples requiring high fluxes take longer time to irradiate. Another major problem associated with these facilities is that only one irradiation channel is available. Samples cannot therefore be irradiated simultaneously. This, coupled with the low flux makes the technique time consuming.

It will be of interest therefore to provide a conceptual nuclear design of a multi-region subcritical assembly driven by isotopic neutron sources in a multiplying medium (neutron multiplier) for neutron activation analysis. The envisaged multi-region assembly will be capable of providing thermal neutron flux as high as  $1 \times 10^7 \text{ n/cm}^2\text{-s}$ . It is to be realized by surrounding radioisotope neutron source(s) with a multiplying medium. In this work the multiplying medium to be applied is  $\text{UO}_2$  (20% enriched).

Multiple isotopic neutron sources will also be introduced to drive the multi-region assembly. This will increase the neutron flux and also provide for the possibility of creating more than one irradiation site in different regions. This will permit simultaneous irradiation of samples. Attention will therefore be focused on the selection of a homogeneous fuel-moderator system, reflecting material, shielding materials and isotopic source strengths to determine optimal conditions for the achievement of neutron flux of the order of  $10^7 \text{ n/cm}^2\text{-s}$  for neutron activation analysis and other reactor physics experiments.

Neutronic calculations for the design of the subcritical assembly will involve solving the one-dimensional (1-D) steady state multi-group neutron diffusion equation. The multi-groups will be collapsed to two energy groups, fast and thermal. A numerical scheme based on finite difference method is to be used to develop a computer code. The code will solve the 1-D multi-group neutron diffusion equation with the possibility of sources placed at different positions. It will be written in FORTRAN 77 programming language for IBM PC and will have the capability of investigating the following:

- (i) choice of reflector
- (ii) effect of multiple sources
- (iii) choice of shielding materials.

**THEORY OF THE CODE SUNDES****3.1 INTRODUCTION**

In the concluding remarks of Chapter Two, it was stated that there is the need for the design of a multi-region subcritical assembly using multiple sources to achieve high fluxes for NAA. The neutronic design of nuclear reactors, subcritical assemblies and other nuclear devices must necessarily go with the ability to predict how neutrons are distributed throughout the system. It is necessary for one to solve a modified multi-group diffusion equation. Detailed calculations based on transport theory are very complicated and difficult to solve. The present analysis has therefore been restricted to the solution of two group, one-dimensional diffusion equation.

In this chapter, the mathematical model, numerical methods and computational procedures used in the development of the **SUNDES** code which solves the neutron diffusion equation in the presence of isotopic sources is discussed.

**3.2 MATHEMATICAL MODEL**

For a steady state to be maintained in a subcritical assembly, external neutron sources must be introduced. The neutrons from the external sources, fission and scattering processes offset losses from the system through diffusion and structural absorption. A spectrum of neutrons,  $\eta(r,E)$  is emitted by the sources and are

elastically scattered in the assembly and slowed down, part of these are absorbed by the multiplying medium to cause fission. As a result of the fission process a new generation of neutron spectrum,  $\chi(r, E)$  is produced. In the presence of the external (isotopic) sources, equation (2.6) is thus modified and is presented mathematically as

$$\begin{aligned}
 -D(r, E) \nabla^2 \phi(r, E) + \Sigma_t(r, E) \phi(r, E) = & \int_0^\infty \Sigma_s(r, E' \rightarrow E) \phi(r, E) dE' \\
 + \int_0^\infty \chi(r, E') \nu_g \Sigma_{fg}(r, E') \phi(r, E') dE' + \sum_{n=1}^N \int_0^\infty & \eta^n(r, E') q^n(r, E') \delta(r \rightarrow r_n) dE'
 \end{aligned} \quad (3.1)$$

Since there is a wide range of energy groups in the assembly, equation (3.1) may be written in the form of multi-group diffusion equation for any energy group,  $g$  and space point,  $k$  in the form;

$$\begin{aligned}
 -D_{g,k} \nabla^2 \phi_{g,k} + \Sigma_{tg,k} \phi_{g,k} = & \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg' \rightarrow g,k} \phi_{g',k} + \sum_{g=1}^G \chi_{g',k} \nu_g \Sigma_{fg',k} \phi_{g',k} \\
 + \sum_{n=1}^N \sum_{g'=1}^G \eta^n_{g',k} q^n_{g',k} \delta(r \rightarrow r_n) & \quad (3.2)
 \end{aligned}$$

where  $\delta(r \rightarrow r_n)$  is the Kronecker delta function which is defined as

$$\delta(r \rightarrow r_n) = \begin{cases} 1 & \text{for } r = r_n \\ 0 & \text{for } r \neq r_n \end{cases}$$

The differential operator for the leakage term may be written as

$$-D_{g,k} \nabla^2 \phi_{g,k} = -D_{g,k} \frac{1}{r} \frac{d}{dr} \left[ r^\alpha \frac{d\phi_{g,k}}{dr} \right] \quad (3.3)$$

where

$$\alpha = \begin{cases} 0: & \text{for plane geometry} \\ 1: & \text{for cylindrical geometry} \\ 2: & \text{for spherical geometry} \end{cases}$$

and  $r$  a variable, is defined for the respective geometries as;

$$\begin{cases} 0 \leq r \leq H : & \text{slab thickness} & (\alpha=0) \\ 0 \leq r \leq R_C : & \text{radius of cylinder} & (\alpha=1) \\ 0 \leq r \leq R_S : & \text{radius of sphere} & (\alpha=2) \end{cases}$$

Let the local variable  $r$  be divided into  $K+1$  intervals (not necessarily equally spaced) as depicted in Figure.3.1. Whence the following terms are defined;

$$r_{k-1/2} = \frac{1}{2}(r_k + r_{k-1})$$

$$r_{k+1/2} = \frac{1}{2}(r_k + r_{k+1})$$

$$\Delta k = (r_k - r_{k-1})$$



$$\Delta k_+ = (r_{k+1} - r_k)$$

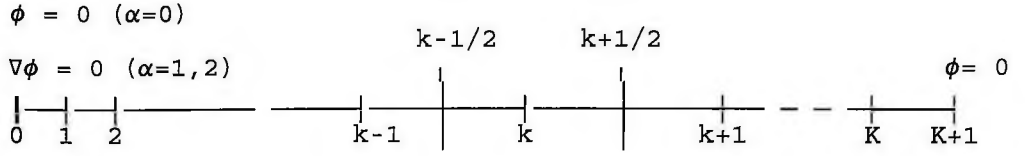


Figure 3.1: Mesh points with intervals

Assuming that the current is continuous at  $r_k$  and then integrating the differential equation (3.2) between the limits  $r_{k-1/2}$  and  $r_{k+1/2}$  the leakage term is obtained as follows;

$$\begin{aligned} -D_{g,k} \int_{r_{k-1/2}}^{r_{k+1/2}} \frac{d}{dr} \left( r^\alpha \frac{d\phi_{g,k}(r_k)}{dr} \right) dr &= D_{g,k+} r_{k-1/2}^\alpha \frac{d}{dr} \left( \phi_{g,k+}(r_{k-1/2}) \right) \\ &+ D_{g,k+} r_{k+1/2}^\alpha \frac{d}{dr} \left( \phi_{g,k+}(r_{k+1/2}) \right) \end{aligned} \quad (3.4)$$

where the quantities on the positive and negative sides of  $r_k$  are denoted by + and - respectively. From the terms defined earlier;

$$\frac{d\phi_{g,k-}(r_{k-1/2})}{dr} = \frac{\phi_{g,k}(r_k) - \phi_{g,k}(r_{k-1/2})}{\Delta k_-} \quad (3.5)$$

and

$$\frac{d\phi_{g,k}(r_{k+1/2})}{dr} = \frac{\phi_{g,k}(r_{k+1}) - \phi_{g,k}(r_k)}{\Delta k} \quad (3.6)$$

Substituting equations (3.5) and (3.6) into (3.4) gives

$$\begin{aligned} -D_{g,k} \int_{r_{k-1/2}}^{r_{k+1/2}} \left[ r^\alpha \frac{d\phi_{g,k}(r_k)}{dr} \right] dr &= \frac{D_{g,k} r_{k-1/2}^\alpha}{\Delta k} \left[ \phi_{g,k}(r_k) - \phi_{g,k}(r_{k-1}) \right] \\ &\quad - \frac{D_{g,k} r_{k+1/2}^\alpha}{\Delta k} \left[ \phi_{g,k}(r_{k+1}) - \phi_{g,k}(r_k) \right] \end{aligned} \quad (3.7)$$

In the next integration the last term on the LHS and the first term on the RHS of equation (3.2) are combined, i.e.

$$\begin{aligned} &\int_{r_{k-1/2}}^{r_{k+1/2}} \left[ \sum_{t=g,k} \phi_{g,k}(r_k) \sum_{\substack{g'=1 \\ g' \neq g}}^G \sum_{s=g' \rightarrow g,k} \phi_{g',k}(r_k) \right] r^\alpha dr \\ &= \int_{r_{k-1/2}}^{r_k} \left[ \sum_{t=g,k} \phi_{g,k}(r_k) \sum_{\substack{g'=1 \\ g' \neq g}}^G \sum_{s=g' \rightarrow g,k} \phi_{g',k}(r_k) \right] r^\alpha dr \end{aligned}$$

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$$+ \int_{r_k}^{r_{k+1/2}} \left( \Sigma_{tg, k+} \phi_{g, k+}(r_k) - \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg' \rightarrow g, k} \phi_{g', k+}(r_k) \right) r^\alpha dr \quad (3.8)$$

Simplifying equation (3.8) further it becomes

$$\begin{aligned} & \left( \frac{r_k^{\alpha+1}}{\alpha+1} - \frac{r_{k-1/2}^{\alpha+1}}{\alpha+1} \right) \left( \Sigma_{tg, k-} \phi_{g, k-}(r_k) - \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg' \rightarrow g, k-} \phi_{g', k-}(r_k) \right) \\ & + \left( \frac{r_{k+1/2}^{\alpha+1}}{\alpha+1} - \frac{r_k^{\alpha+1}}{\alpha+1} \right) \left( \Sigma_{tg, k+} \phi_{g, k+}(r_k) - \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg' \rightarrow g, k} \phi_{g', k+}(r_k) \right) \end{aligned} \quad (3.9)$$

If the fission neutrons term and isotopic neutrons term in Eq.(3.2) are combined and integrated an expression of the form below is obtained;

$$\left( S_{g', k}(r_k) + Q_{g', k}(r_k) \right) \int_{r_{k-1/2}}^{r_{k+1/2}} r^\alpha dr = \left( S_{g', k} + Q_{g', k} \right) \left( \frac{r_{k+1/2}^{\alpha+1}}{\alpha+1} - \frac{r_{k-1/2}^{\alpha+1}}{\alpha+1} \right) \quad (3.10)$$

where

$$S_{g', k}(r_k) = \sum_{g'=1}^G \chi_{g', k} (\nu \Sigma_f)_{g', k} \phi_{g', k} \quad (3.11)$$

and

$$Q_{g', k} = \sum_{n=1}^N \sum_{g'=1}^G \eta_{g', k}^n q_{g', k}^n \quad (3.12)$$

Using equations (3.7) (3.10), the simplified integrated equation becomes:

$$\begin{aligned}
 & D_{g,k-\frac{r_{k-1/2}^\alpha}{\Delta k-}} \left( \phi_{g,k}(r_k) - \phi_{g,k}(r_{k-1}) \right) - D_{g,k+\frac{r_{k+1/2}^\alpha}{\Delta k+}} \left( \phi_{g,k}(r_{k+1}) - \phi_{g,k}(r_k) \right) \\
 & + \left( \frac{r_k^{\alpha+1} - r_{k-1/2}^{\alpha+1}}{\alpha+1} \right) \left( \Sigma_{tg,k-} \phi_{g,k-}(r_k) \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg' \rightarrow g,k-} \phi_{g,k-}(r_k) \right) \\
 & + \left( \frac{r_{k+1/2}^{\alpha+1} - r_k^{\alpha+1}}{\alpha+1} \right) \left( \Sigma_{tg,k+} \phi_{g,k+}(r_k) \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg' \rightarrow g,k+} \phi_{g,k+}(r_k) \right) \\
 & = \left( \frac{r_{k+1/2}^{\alpha+1} - r_{k-1/2}^{\alpha+1}}{\alpha+1} \right) \left( S_{g',k} + Q_{g',k} \right) \tag{3.13}
 \end{aligned}$$

To simplify work, denote the following by;

$$\tau_k = \frac{r_{k-1/2}^\alpha}{(r_k - r_{k-1})}$$

$$\rho_k = \frac{r_{k+1/2}^\alpha}{(r_{k+1} - r_k)}$$

$$\mu_k = \frac{(r_k^{\alpha+1} \quad r_{k-1/2}^{\alpha+1})}{(\alpha+1)}$$

$$\beta_k = \frac{(r_{k+1/2}^{\alpha+1} \quad r_k^{\alpha+1})}{(\alpha+1)}$$

$$\xi_k = \frac{(r_{k+1/2}^{\alpha+1} \quad r_{k-1/2}^{\alpha+1})}{(\alpha+1)} = \mu_k + \beta_k$$

$$U_k = \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg' \rightarrow g, k-} \phi_{g', k-}(r_k)$$

and

$$W_k = \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg' \rightarrow g, k+} \phi_{g', k+}(r_k)$$

thus substituting the above definitions into equation (3.13) gives

$$\begin{aligned} & D_{g, k-} \tau_k \left( \phi_{g, k}(r_k) - \phi_{g, k}(r_{k-1}) \right) \quad D_{g, k+} \rho_k \left( \phi_{g, k}(r_{k+1}) - \phi_{g, k}(r_k) \right) \\ & + \mu_k \left( \Sigma_{tg, k-} \phi_{g, k-}(r_k) \quad U_k \right) + \beta_k \left( \Sigma_{tg, k+} \phi_{g, k+}(r_k) \quad W_k \right) \\ & = \xi_k \left( S_{g', k} + Q_{g', k} \right) \end{aligned} \quad (3.14)$$

Simplifying equation (3.14) further gives the expression

$$-D_{g,k-\rho_k} \phi_{g,k}(r_{k+1}) + \left( D_{g,k-\tau_k} + D_{g,k+\rho_k} + \mu_k \Sigma_{tg,k-} + \beta_k \Sigma_{tg,k+} \right) \phi_{g,k}(r_k)$$

$$\begin{aligned} -D_{g,k-\tau_k} \phi_{g,k}(r_{k-1}) &= \xi_k (S_{g',k} + Q_{g',k}) + \mu_k \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg' \rightarrow g,k-} \phi_{g',k}(r_k) \\ &\quad + \beta_k \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg' \rightarrow g,k+} \phi_{g',k}(r_k) \end{aligned} \quad (3.15)$$

If it is still assumed that the material properties remained the same within the delta neighbourhood of  $r_k$  then the following approximations are made;

$$\Sigma_{sg',g,k-} = \Sigma_{sg',g,k+} = \Sigma_{sg',g,k}, \quad D_{g,k-} = D_{g,k+} = D_{g,k}$$

$$\text{and} \quad \Sigma_{tg,k-} = \Sigma_{tg,k+} = \Sigma_{tg,k}$$

Using the above approximations, equation (3.15) is rewritten as a finite difference equation in the form

$$\begin{aligned} -D_{g,k} \rho_k \phi_{g,k}(r_{k+1}) + \left( D_{g,k} \tau_k + D_{g,k} \rho_k + \xi_k \Sigma_{tg,k} \right) \phi_{g,k}(r_k) \\ -D_{g,k} \tau_k \phi_{g,k}(r_{k-1}) = \xi_k \left( S_{g',k} + Q_{g',k} + \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg' \rightarrow g,k} \phi_{g',k} \right) \end{aligned} \quad (3.16)$$

For brevity and without loss of generality let us designate

$$\phi_{g,k}(r_{k+1}) = \phi_{g,k+1} , \quad \phi_{g,k}(r_k) = \phi_{g,k} \quad \text{and} \quad \phi_{g,k}(r_{k-1}) = \phi_{g,k-1}$$

then the finite difference equation (3.16) for energy group  $g$ , and position  $k$ , on the radius becomes a nonhomogeneous simultaneous linear equation of the form

$$-a_{k,k+1}\phi_{k+1} + a_{k,k}\phi_k - a_{k,k-1}\phi_{k-1} = \sigma_k \quad (3.17)$$

The subscript  $g$  has been omitted for simplicity and the coefficients are given by

$$a_{k,k+1} = D_{g,k}\rho_k ,$$

$$a_{k,k-1} = D_{g,k}\tau_k$$

$$a_{k,k} = a_{k,k+1} + a_{k,k-1} + \xi_k \Sigma_{tg,k}$$

$$\sigma_{g,k} = \xi_k \left( S_{g',k} + Q_{g',k} + \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg' \rightarrow g,k} \phi_{g',k} \right)$$

Collapsing the multi-group analysis to two (fast and thermal) groups and further assuming that neutrons can only scatter from higher energies to lower energies and that all neutrons are born at fast energy then

$$\sigma_{g,k} = \begin{cases} \xi_k (S_{1k} + Q_{1k}) & \text{for fast group } (g=1) \\ \xi_{k1} \phi_{1k} & \text{for thermal group } (g=2) \end{cases} \quad (3.18)$$

where

$$S_{1k} = \sum_{g=1}^G \chi_{g',k} (\nu \Sigma_f)_{g',k} \phi_{g',k}$$

and

$$Q_{g',k} = \sum_{n=1}^N \sum_{g'=1}^G \eta_{g',k}^n q_{g',k}^n$$

To evaluate  $Q_{g',k}$ , the expression for  $q_{g',k}$  must be obtained.  $q_{g',k}$  is dependent on the type of source and its location in the assembly. For the purpose of this work, three types of sources are considered:

- (a) a cylindrical source located at the center of the assembly
- (b) concentration of point sources at the center and at any other position
- (c) the combination of a cylindrical source at the center and placement of point sources at any other part.

For case (a), if the source strength is  $q_0$ , then the expression for  $q_{1k}$  everywhere in the assembly is of the form

$$q_{1k} = \frac{q_o}{\pi r z} = \frac{q_o}{\pi r_{k1}^2 h} J_0(x_k) / J_1^2(y) \quad (3.19)$$

where  $x_k = 2.405r_k/r_{k1}$ , while  $r_{k1}$  and  $h$  are the radius and height of the assembly respectively. In the evaluation of  $q_{1k}$  for the mesh points  $k$ , Bessel functions [6, 33, 34] for small  $x_k$  and  $y$  are used:

$$J_0(x_k) = 1 - \frac{x_k^2}{4} + \frac{x_k^4}{64} - \frac{x_k^6}{2304} + \dots \quad (3.20)$$

$$J_1(y_k) = \frac{y_k}{2} - \frac{y_k^3}{16} + \frac{y_k^5}{384} - \dots \quad (3.21)$$

For the case of point sources concentrated at the center of the assembly  $q_{1k}$  is expressed simply as

$$q_{1k} = \begin{cases} q_o & 0 \leq r \leq r_1 \\ 0 & r_k > r_1 \end{cases} \quad (3.22)$$

and any other place apart from the center of the assembly as

$$q_{1k} = \begin{cases} q_{o,n} & r_k = r_j \\ 0 & r_j < r_k \leq r_{kz} \end{cases} \quad (3.23)$$

where  $j$  and  $kz$  are the first and last mesh points of any particular zone.

Apart from the source conditions, boundary conditions are

often applied in arriving at reasonable numerical solutions to differential equations. Equations of the type (3.17) have infinite number of solutions but every physical case has a single function representing the flux. In order to obtain such a function, some restrictions or boundary conditions must be imposed upon the general solution. These conditions are dictated by the nature of the problem. In the derivation of equation (3.17) it was assumed that (i) the current is continuous between  $r_{k-1/2}$  and  $r_{k+1/2}$  and (ii) the neutron flux must be finite and non-negative in the region  $r_k$  and zero at the outermost boundary. Apply the boundary condition that the flux is maximum at the center ie  $\nabla\phi = 0$ , the first equation of the series is given by

$$\begin{aligned}
 & -D_{g,1}r_{3/2}^{\alpha} \left( \frac{\phi_{g,2} - \phi_{g,1}}{r_2 - r_1} \right) + \Sigma_{ag,1}\phi_{g,1} \left( \frac{r_{3/2}^{\alpha+1} - r_1^{\alpha+1}}{\alpha + 1} \right) \\
 & = \left( \frac{r_2^{\alpha+1} - r_1^{\alpha+1}}{\alpha + 1} \right) \left( \sum_{\substack{g'=1 \\ g' \neq g}}^G \Sigma_{sg' \rightarrow g,1} \phi_{g',1} + S_{g,1} + Q_{g,1} \right) \quad (3.24)
 \end{aligned}$$

where the subscript 1 denotes the mesh point at center of the assembly and the coefficients  $a_{1,1}$  and  $a_{1,2}$  are respectively defined as:

$$a_{1,1} = D_{g,1}r_{3/2}^{\alpha} + \Sigma_{ag,1} \left( \frac{r_{3/2}^{\alpha+1} - r_1^{\alpha+1}}{\alpha + 1} \right)$$

and

$$a_{1,2} = -D_{g,1} r_{3/2}^{\alpha+1}$$

Also imposing condition (ii), the last equation of the series for the last mesh point is thus obtained as

$$\begin{aligned} & \left( D_{g,k1} \tau_{k1} + \sum_{ag,k1} \mu_{k1} \right) \phi_{g,k1} - \tau_{k1} D_{g,k1} \phi_{g,k1-1} \\ & = \mu_{k1} \left( \sum_{\substack{g'=1 \\ g' \neq g}}^G \sum_{sg' \rightarrow g,k1} \phi_{g',k1} + S_{g,k1} + Q_{g,k1} \right) \end{aligned} \quad (3.25)$$

where the coefficients are fixed by virtue of the condition as

$$a_{k1,k1-1} = -D_{g,k1} \tau_{k1}$$

and

$$a_{k1,k1} = D_{g,k1} \tau_{k1} + \sum_{ag,k1} \mu_{k1}$$

Several other conditions can be imposed but are however ignored at this particular stage of development of the code SUNDES.

### 3.2 1 Matrix form of multi-group equations.

In order to simplify the equations, it is convenient to consider the matrix form of the multi-group equations. The equation

(3.17) obtained for any group  $g$  and position  $k$  in the assembly can be transformed into a matrix equation of the form

$$\mathbf{A}_{g,k} \phi_{g,k} = \xi_k \left( \mathbf{B}_{g,k} + \sum_{\substack{g'=1 \\ g' \neq g}}^G \mathbf{C}_{g' \rightarrow g,k} \phi_{g',k} \right) \quad (3.26)$$

where

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & & & \\ a_{21} & a_{22} & a_{23} & & \\ & & & & \\ & & & a_{kl-1,kl-2} & a_{kl-1,kl-1} & a_{kl-1,kl} \\ & & & & a_{kl,kl-1} & a_{kl,kl} \end{pmatrix}, \quad \phi_{g,k} = \begin{pmatrix} \phi_{g,1} \\ \phi_{g,2} \\ \vdots \\ \phi_{g,kl-1} \\ \phi_{g,kl} \end{pmatrix}$$

$$\mathbf{C}_{g' \rightarrow g,k} = \begin{pmatrix} c_1 & & & & \\ & c_2 & & & \\ & & c_3 & & \\ & & & \ddots & \\ & & & & c_{kl-1} \\ & & & & & c_{kl} \end{pmatrix}, \quad \mathbf{B}_{g,k} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{kl-1} \\ b_{kl} \end{pmatrix}$$

The matrix  $\mathbf{C}_{g' \rightarrow g,k}$  represents the scatter term whilst  $\mathbf{B}_{g,k}$  is for the sum of fission and source terms. The solution to this matrix equation provides the solution for the multi-group diffusion equation. The numerical method and techniques applied in its solution are presented below.

### 3.2.2 Computational Procedure

A computer code named SUNDES; an acronym for SUBcritical Nuclear DESign was developed to solve the matrix equation (3.26). The steps providing a suitable solution to the matrix equation as solved using the code are as follows;

(a) Guess the initial values of  $\phi_{1,k}$  and  $\phi_{2,k}$  Calculate the fission density, S and source density, Q.

(b) determine the coefficients of the matrix  $A_{1,k}$

$$\text{and } \sigma_{1k} = \xi_k \left( S_{1,k} + Q_{1,k} \right)$$

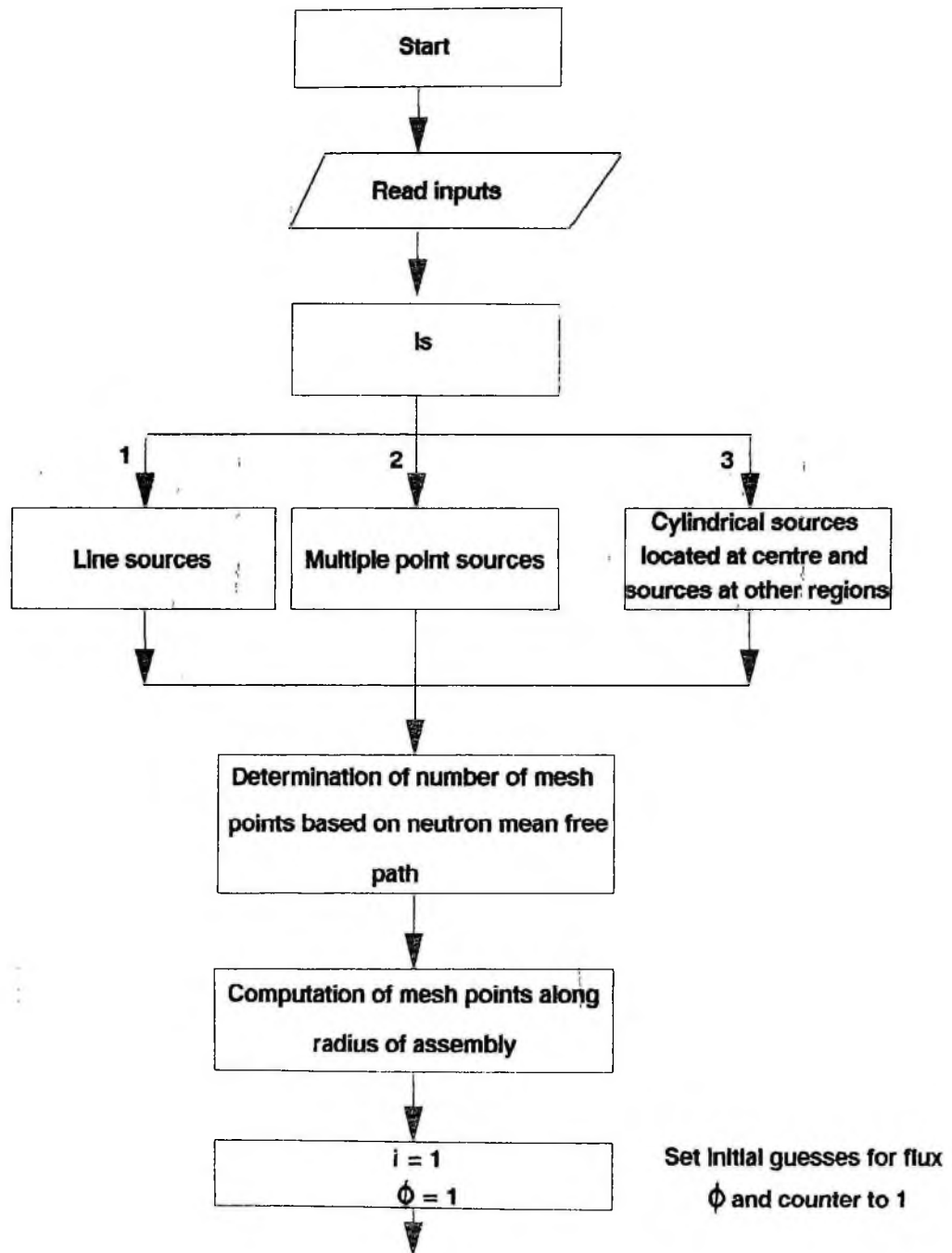
(c) Solve for new values of  $\phi_{1,k}$  using Gauss-Seidal method

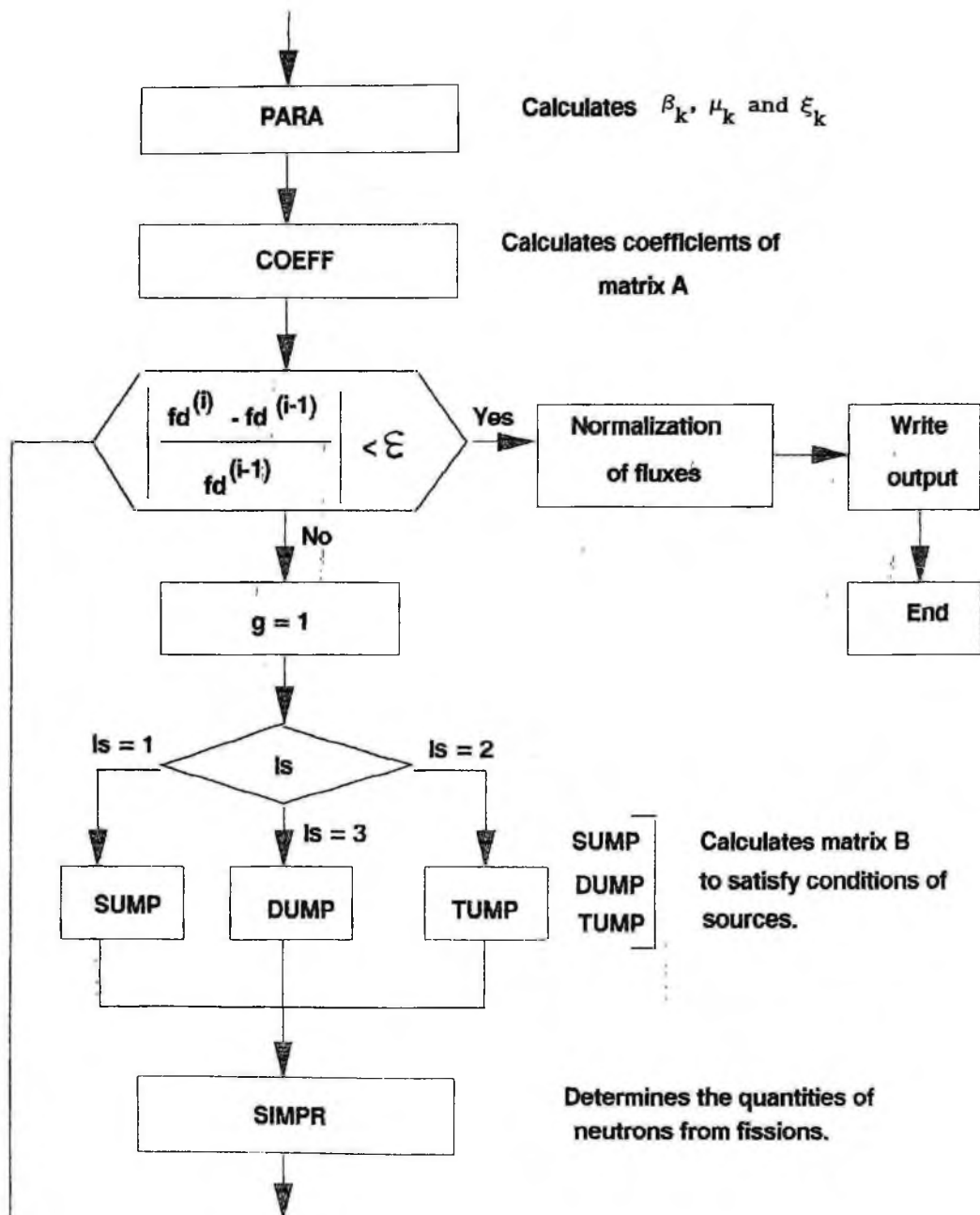
(d) Introduce  $\phi_{1,k}$  obtained in (b) and determine the coefficients for the second group and solve for  $\phi_{2,k}$

(e) Calculate the approximate value for fission density, fd

(f) Introduce the values of  $\phi_{1,k}$  and  $\phi_{2,k}$  and repeat steps (b) through (d) until convergence is achieved in  $\phi_{1,k}$ ,  $\phi_{2,k}$  and fd.

The flow chart of iterative procedure of the code is presented in Figure 3.2. First, the input data consisting of geometry





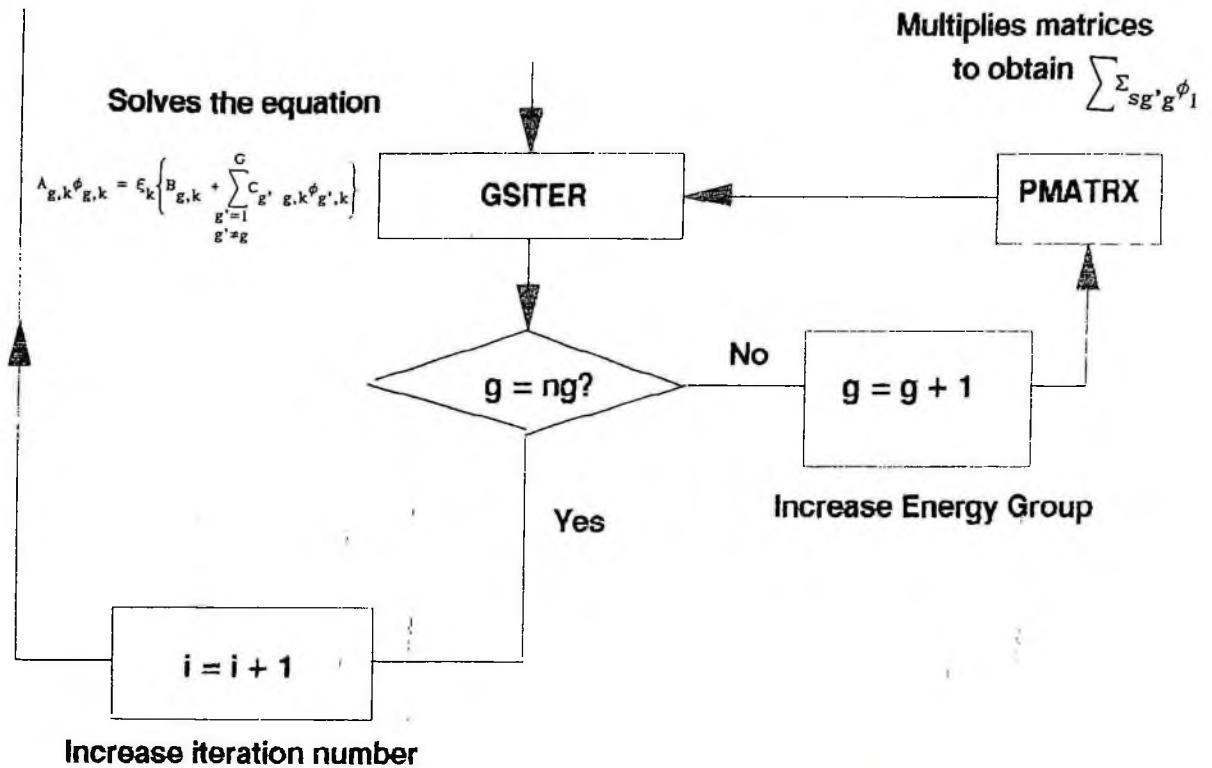


Figure 3.2 Computational Flowchart for SUNDES



parameters, macroscopic group constants etc. are read. There are three types of sources and are discussed in section 3.1. The required source is selected and the number of mesh points determined based on neutron free path. Mesh points along the radius of assembly are computed followed by setting the counter for the fluxes.

Subroutine PARA calculates the parameters  $\beta_k$ ,  $\mu_k$  and  $\xi_k$  while COEFF calculates the coefficients of the matrix A. The convergence criterion is applied to test for convergence and if that is achieved the normalized fluxes are written out and programme terminated. For non-convergence and depending on the type of source, there are three subroutines SUMP, DUMP and TUMP to calculate matrix B to satisfy source conditions. Subroutine SIMPR, determines the quantities of neutrons from fission. Another subroutine GSITER solves the matrix equation (3.26) and the flux is tested for convergence. If convergence is still not achieved, the energy groups are increased. The multiplication of matrices when the energy groups are increased is performed by a subroutine PMATRIX and recycled through GSITER until convergence is achieved.

For an outer iteration loop to begin, the previous outer iteration fluxes are examined and the fission density computed from the equation

$$fd^{(i)} = \sum_{g=1}^G (\nu \Sigma_{fg})^{(i)}_{g,k} \phi^{(i)}_{g,k} \quad (3.27).$$

The computation of effective multiplication factor will not be of

concern here since a fixed source problem is being considered. The group fluxes are rather simply allowed to find their own levels until convergence is achieved, i.e.

$$\left| \frac{f_d^{(i)} - f_d^{(i-1)}}{f_d^{(i)}} \right| \leq \varepsilon \quad (3.28)$$

where the limit of  $\varepsilon$  has been chosen to be as small as  $10^{-5}$ .

### 3.2.3 Inner iteration

The neutron diffusion equation represented by equation (3.26) is solved in the inner loop for any group  $g$ . Gauss-Seidal method [6] is used to solve the nonhomogeneous linear equation. A general expression for the solution of the simultaneous equations is given by

$$\phi_i^{(m)} = \frac{1}{a_{ii}} \left[ b_i - \sum_{j=1}^{i-1} a_{ij} \phi_j^{(m)} - \sum_{j=i+1}^{i-1} a_{ij} \phi_j^{(m)} - \sum_{j=i+1}^n a_{ij} \phi_j^{(m-1)} \right] \quad (3.29)$$

where the subscripts denote the iteration numbers.

The test for convergence for the iteration method is by comparing the changes in the  $\phi_i$  values between successive iterations ie

$$\frac{\sum_{i=1}^n |\delta\phi_i|}{\sum_{i=1}^n |\phi_i^{(m)}|} < \varepsilon_2 \quad (3.30)$$

where  $\delta\phi_i = \phi_i^{(m)} - \phi_i^{(m-1)}$  and  $\varepsilon_2$  is a tolerance.

There are several acceleration schemes to increase the convergence, such as coarse mesh rebalance, Chebyshev, synthetic method and overrelaxation [35]. For this work, the rate of convergence is improved by applying the technique of overrelaxation which is easily programmable. Equation (3.29) can be used to estimate new values of  $\phi_i^{(m)}$ , which, provided the process is convergent, are closer to the required solutions than the previous  $\phi_i^{(m-1)}$ . Estimation of the new values of  $\phi_i^{(m)}$  becomes

$$\phi_i^{(m)} = \phi_i^{(m-1)} + \omega\delta\phi_i \quad (3.31)$$

where 
$$\delta\phi_i = \frac{1}{a_{ii}} \left( b_i - \sum_{j=1}^n a_{ij}\phi_j \right) \quad (3.32)$$

and  $\omega$  is the overrelaxation factor. Varga [36] has exhaustively discussed the theory of predicting its optimum value. It is found to be within a range  $1 < \omega < 2$ . An optimum value of 1.5 was chosen to provide convergence to a tolerance limit of  $10^{-6}$  for 20 iterations in this work.

### 3.2.4 Normalization of Fluxes

The code SUNDES provides for the normalization of neutron fluxes to desired power levels in the assembly. The power equation is given by

$$P_x = \zeta \int \sum_{V_A, g=1}^G \phi_{g,k} \Sigma_{fg} dV_k^\alpha \quad (3.33)$$

where  $V_A$  is the volume of the assembly,  $\zeta = 3.2 \times 10^{-11}$  watts/sec per fission and

$$dV_k^\alpha = \begin{cases} 1 & \alpha=0 \text{ (slab)} \\ 2\pi r_k & \alpha=1 \text{ (cylinder)} \\ 4\pi r_k^2 & \alpha=2 \text{ (sphere)} \end{cases}$$

If the fission cross section is considered a constant in each of the multi-regions  $l$  of total number of regions,  $nl$ , then the power equation (3.33) may be rewritten as

$$P_x = \zeta \sum_{g=1}^G \sum_{l=1}^{nl} \Sigma_{fg,l} \bar{\phi}_{g,l} \Delta V_l^\alpha \quad (3.34)$$

$$\text{where } \Delta V_l^\alpha = \begin{cases} (R_{l+1} - R_l)HL & \text{for } \alpha=0 \text{ (slab)} \\ \pi (R_{l+1}^2 - R_l^2)H & \text{for } \alpha=1 \text{ (cylinder)} \\ \frac{4}{3} (R_{l+1}^3 - R_l^3) & \text{for } \alpha=2 \text{ (sphere)} \end{cases}$$

For  $\alpha = 0$ ,  $H$  and  $L$  are the transverse dimensions of the slab. For  $\alpha = 1$ ,  $H$  is the height of the cylinder.

Having solved the matrix equation (3.26), the fluxes should be normalized to the total power of the subcritical assembly. This requires a normalizing constant  $\phi$  such that  $\phi_{g,k} = \phi \phi_{g,k}$  and relates the power through the equation

$$\phi = \frac{P}{\zeta \int_V \sum_{g=1}^G \Sigma_{fg,k} \phi_{g,k} dV_k^\alpha} = \frac{P}{P_x} \quad (3.35)$$

### 3.2 5 Input Description

The input data for the code SUNDES consists of two sections

- (i) the geometry and convergence criterion
- (ii) nuclear properties of macroscopic group constants, buckling and fractions of fission and isotopic neutrons in energy group  $g$ .

SUNDES.INP file is defined on UNIT 1 through which all the input data enter the main program SUNDES.FOR (see Appendix A). SUNDES.INP is written according to a specified format and in the order listed below.

**Section (i)**

| Card | Format | Parameter | Mathematical<br>symbol | Description                                       |
|------|--------|-----------|------------------------|---------------------------------------------------|
| 2    | i4     | ig        | $\alpha$               | geometrical factor<br>( $\alpha=0,1,2$ )          |
| 3    | i4     | ng        | G                      | total number of<br>energy groups                  |
| 4    | i4     | nl        | nl                     | total number of<br>homogeneous zones              |
| 5    | i4     | nmax      | nmax                   | maximum number of<br>iterations                   |
| 6    | f8.5   | erl       | $\epsilon$             | convergence crit-<br>erion for fission<br>density |
| 7    | f8.5   | P         | P                      | input power,watts                                 |

**Section (ii)**

The macroscopic group constants for the various zones starting from the center of the assembly for energy group, g denoted  $k_g$  is as follows

| Card | Format   | Parameter   | Mathematical<br>Symbol                      | Description                                   |
|------|----------|-------------|---------------------------------------------|-----------------------------------------------|
| 8    | nl(f8.3) | sigf(kg,l)  | $\nu_g \Sigma_{fg}(\mathbf{r}, E)$          | macroscopic fission<br>cross section          |
| 9    | nl(f8.3) | sigag(kg,l) | $\Sigma_{ag}(\mathbf{r}, E)$                | macroscopic absorp-<br>tion cross section     |
| 10   | nl(f8.3) | d(kg,l)     | $D_g(\mathbf{r}, E)$                        | diffusion constant                            |
| 11   | nl(f8.3) | sigs(kg,l)  | $\Sigma_{sg}(\mathbf{r}, E' \rightarrow E)$ | macroscopic scatter-<br>ing cross section     |
| 12   | nl(f8.3) | b(kg,l)     | $B_g(\mathbf{r}, E)$                        | transverse buckling                           |
| 13   | nl(f8.3) | q(kg,l)     | $q_g,$                                      | quantity of neutrons<br>from isotopic sources |
| 14   | nl(f8.3) | fq(kg,l)    | $\chi_g(\mathbf{r}, E)$                     | fraction of fission<br>neutrons               |
| 15   | nl(f8.3) | fkg(kg,l)   | $\eta_g(\mathbf{r}, E)$                     | fraction of isotopic<br>neutrons              |

### 3.2.6 Output Description

Output from the code SUNDES.FOR can be obtained on screen or on printed paper. The output file, SUNDES.OUT is written in UNIT 6. The information displayed by the code is in the following order: first the name of the code then the geometrical characteristics of the assembly and other important input parameters. Group constants of all the zones are also printed in this order: fission

cross section,  $\nu\Sigma_f$  absorption cross section,  $\Sigma_a$  scattering cross section,  $\Sigma_{s1\rightarrow2}$  diffusion constant,  $D$  transverse buckling  $B$ , fission spectrum,  $\chi$  isotopic spectrum,  $\eta$  and the quantity of neutrons from the isotopic sources,  $q$ .

The other portion of the output consists of the iteration number,  $i$  and the fission density,  $fd$ . Fast and thermal flux distributions at the various mesh points starting from the center of the assembly are also printed.

### 3.3 Concluding remarks.

The computer code SUNDES, which has been developed based on the mathematical model discussed above will be used for the design of the neutron multiplier. Preliminary test cases will be conducted using the code. These test cases ultimately enable a complete realization of the design of the neutron multiplier.

## **NUCLEAR DESIGN OF THE NEUTRON MULTIPLIER**

### **4.1 INTRODUCTION**

Feasibility studies are normally carried out before embarking on detailed construction. Computer codes are employed to perform neutronic, thermal hydraulics, structural calculations etc. Additionally, various processes associated with the functioning of the nuclear equipment are thoroughly investigated. The control rod worth, fuel burn up, dose rate, radiation shielding etc are studied by the use of computer codes. The trend of results and values of neutron fluxes are normally used to select the geometry of the nuclear device.

In this chapter, the SUNDES code is tested by using nuclear data generated from WIMSPC for single and two region problems. This will be followed by its application to multi-region problems to obtain maximum thermal neutron fluxes generated by the neutron multiplier. A description of the mechanical aspects and operating characteristics of the device is also presented.

### **4.2 APPLICATION OF THE CODE.**

#### **4.2.1 Single and Two-Region Problem.**

Geometrical features and nuclear properties of macroscopic

group constants are needed in neutronic modelling. For the verification of the code SUNDES, macroscopic group constants ( $D_1, D_2, \Sigma_{a1}, \Sigma_{a2}, \nu\Sigma_{f1}, \nu\Sigma_{f2}$  and  $\Sigma_{s1, 2}$ ) were generated using WIMSPC [37], a PC version of Winfrith Improved Multi-group Scheme (WIMS). [38-40]. WIMS is a general lattice code based on transport theory. It contains a library of elements with specific identification numbers from which elements of the assembly are selected. Data cards are available through which geometry specifications and material compositions enter the code. The programme then computes macroscopic cross sections for each homogeneous region.

Macroscopic cross sections were generated for a single homogeneous region of 20% enriched  $UO_2$  and Be moderator. With an isotopic source located at the center and/or at different positions of the region the neutron flux distribution was studied. Another case study was a two region problem which consists basically of the initial single homogeneous region and then a water region as reflector.

For the one homogeneous region with the presence of a source of strength  $8.45 \times 10^9$  n/s at the center, the flux distributions of both fast and thermal neutron is as shown in Figure 4.1. As the position of the source is shifted to 6cm and 12cm away from the center, the trend of the neutron flux is observed to change and is presented in Figure 4.2. The effect of maintaining point sources of equal strengths at the center, followed by one at 9cm and then

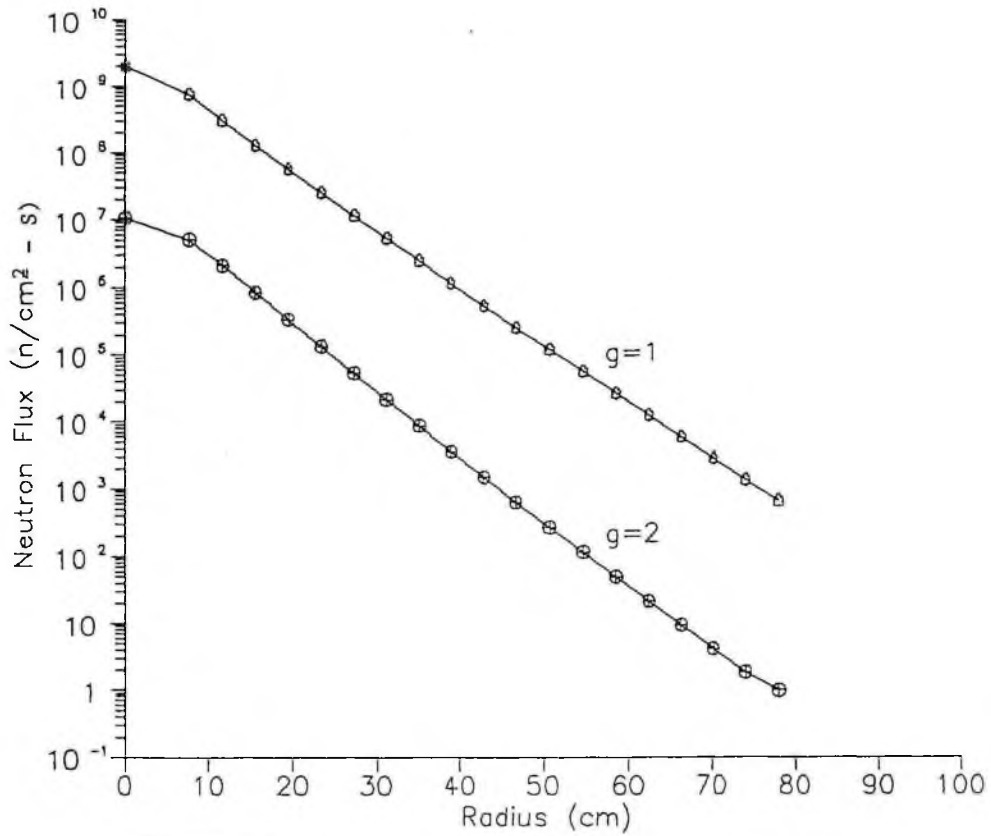
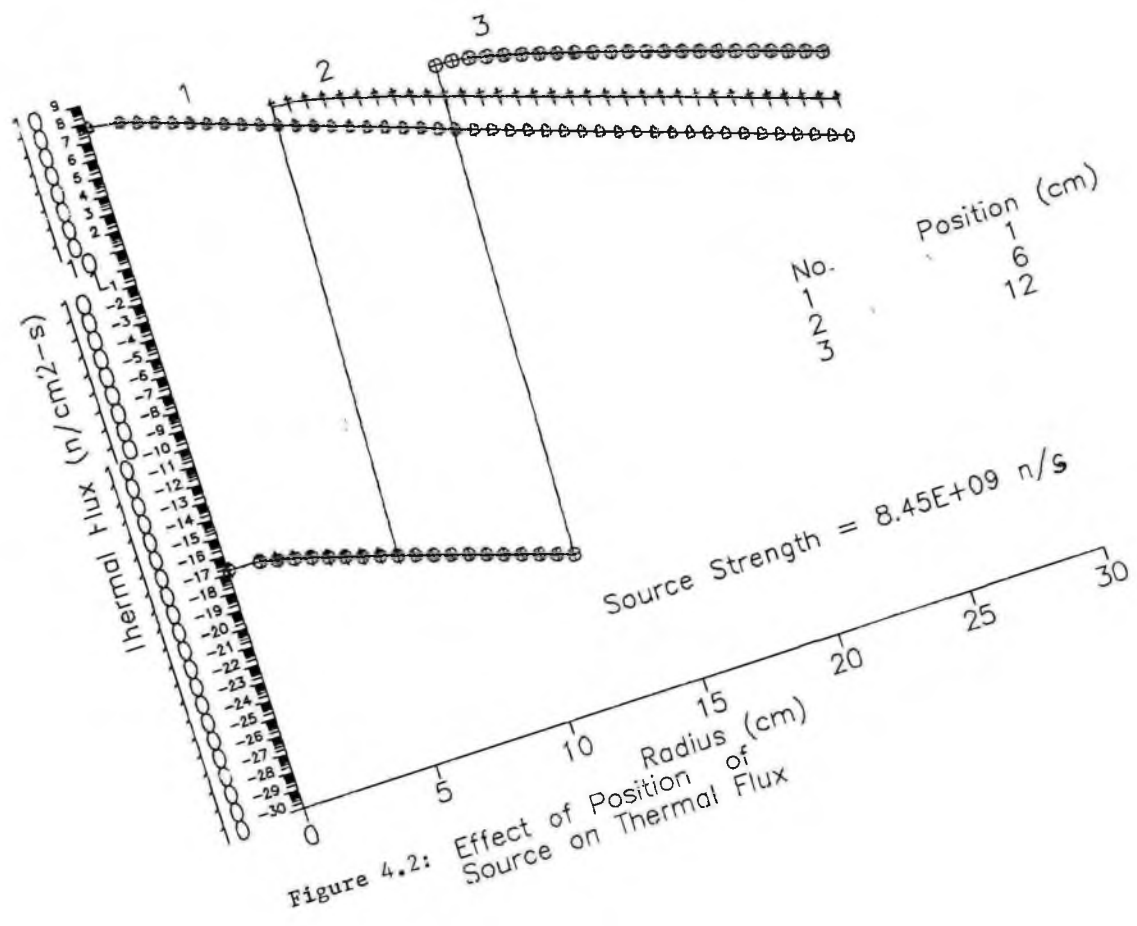


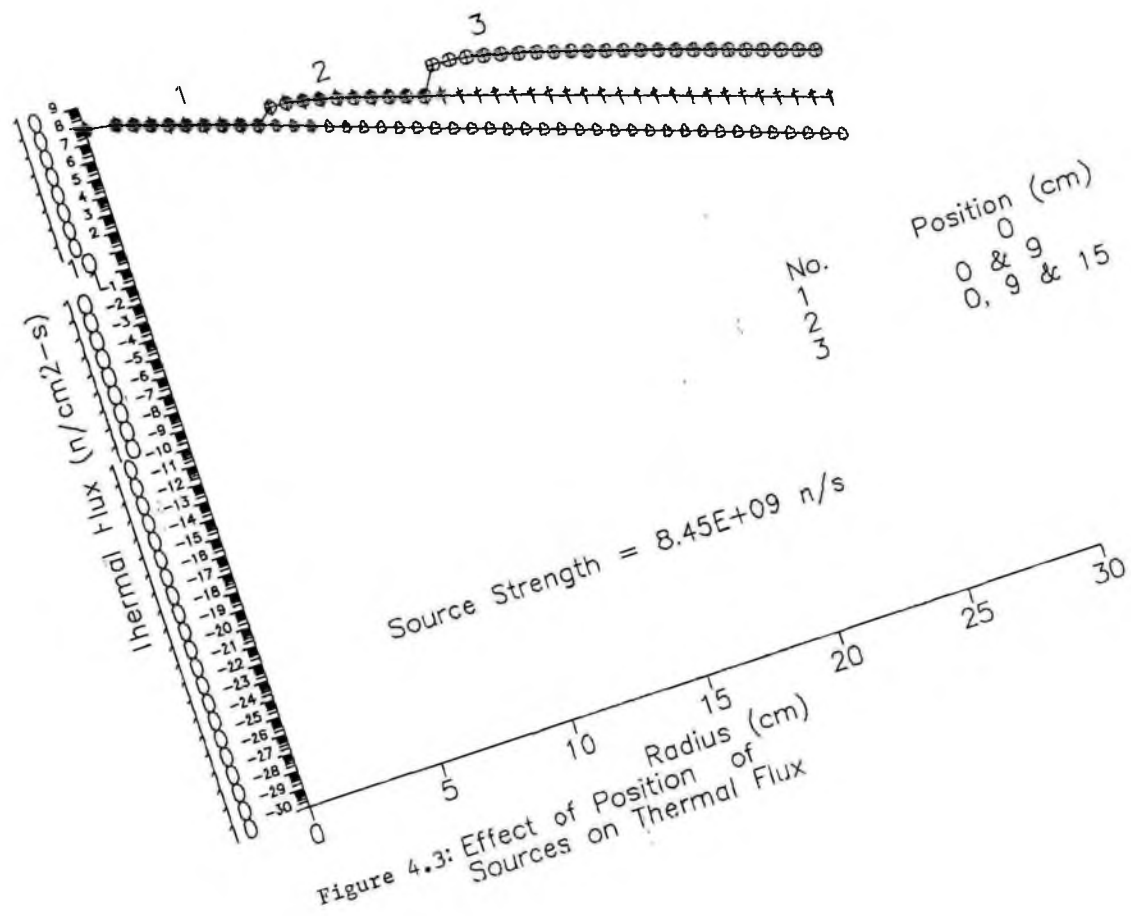
Figure 4.1: Variation of Neutron Fluxes in a  
Homogeneous Mixture of 20% UO<sub>2</sub> + Be.



another at 15cm is presented in Figure 4.3. Figure 4.4 represents the behaviour of the neutron fluxes when equal strength of sources are kept at 0, 6, 12, and 18cm from the center of the homogeneous mixture. It can be seen from the plots that the neutron fluxes are influenced by the position of the source. In the two region problem where water ( $H_2O$ ) is used as a reflector there is thermalization of neutrons in this region. In the thermalization region the collision takes place effectively with the  $H_2O$  molecules and some neutrons are scattered back into the fuel region. This results in a peaking in the thermal flux. This observation as seen in Figure 4.5 is consistent with reactor physics predictions for a fuel region surrounded by a reflector. For practical purposes, a nuclear device will have multi-regions of fuel, moderator, coolant, reflector and shield. In the next section the code will be applied to various multi-regions with the ultimate aim of achieving high flux of thermal neutrons in the neutron multiplier.

#### **4.2.2 Application to Multi-regions.**

As stated, for the complete realization of the aim of this work, multi-region problems were considered. A multi-region consisting of nine (9) distinct homogeneous regions was adopted for the conceptual nuclear design of the neutron multiplier. The nine concentric radial rings are hereby denoted as A,B,C,D,E,F,G,H



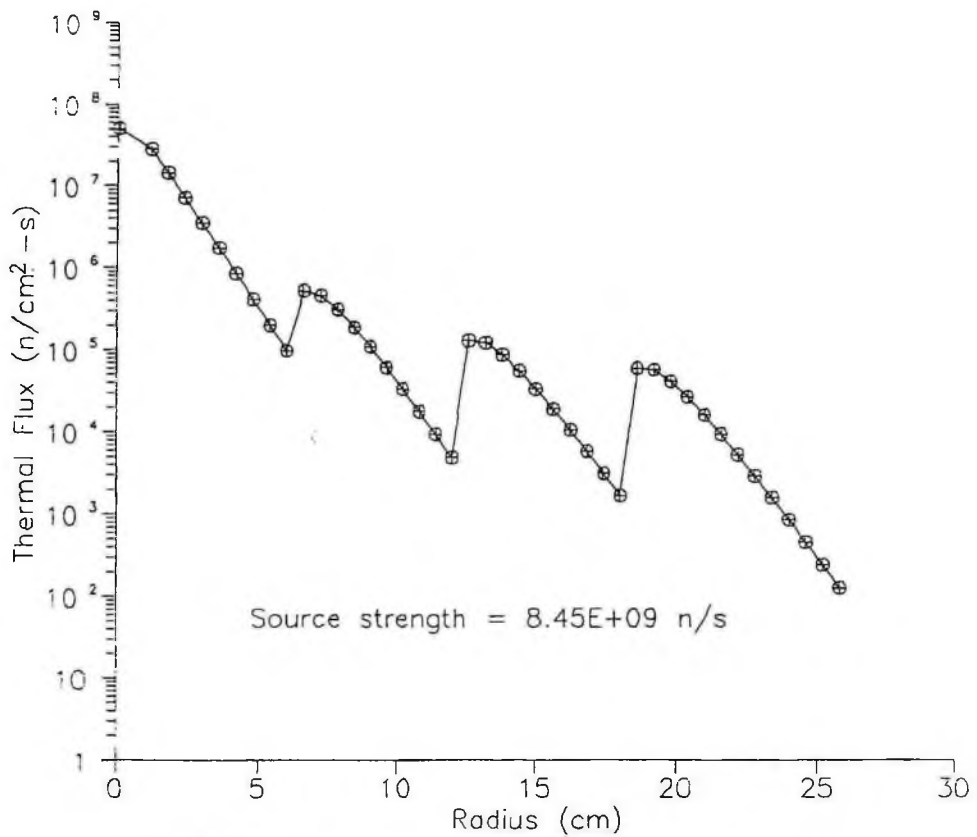


Figure 4.4: Effect of Source Located at 1, 6, 12 and 18cm on Thermal Flux.

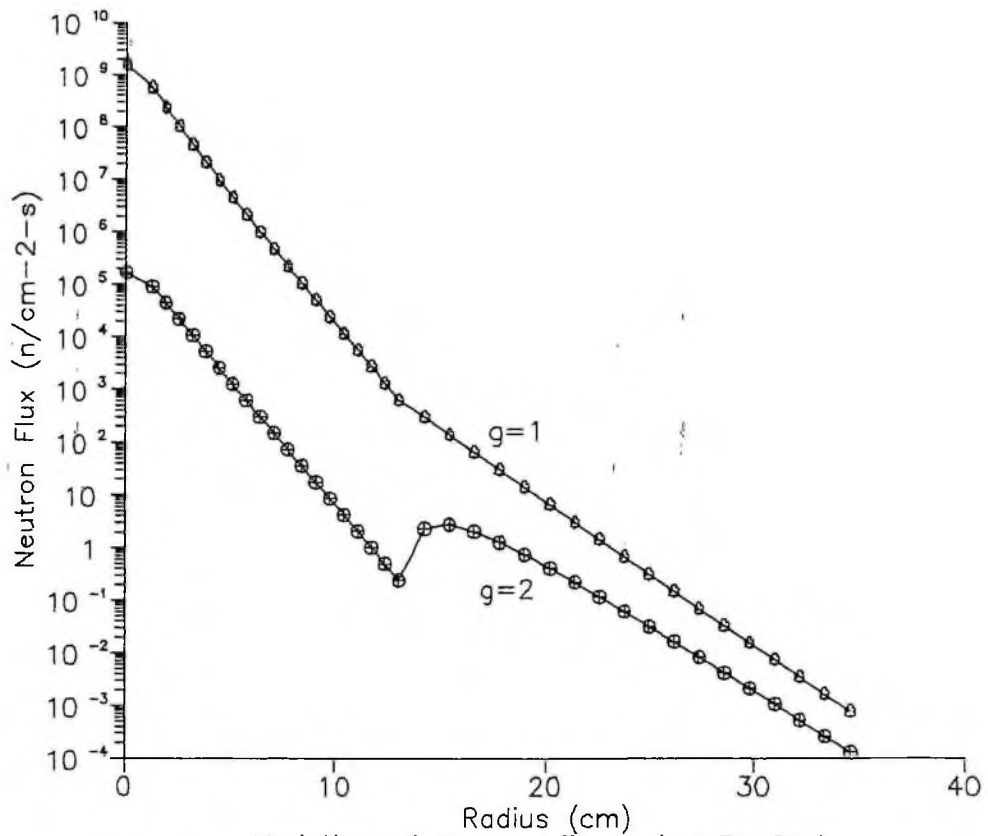
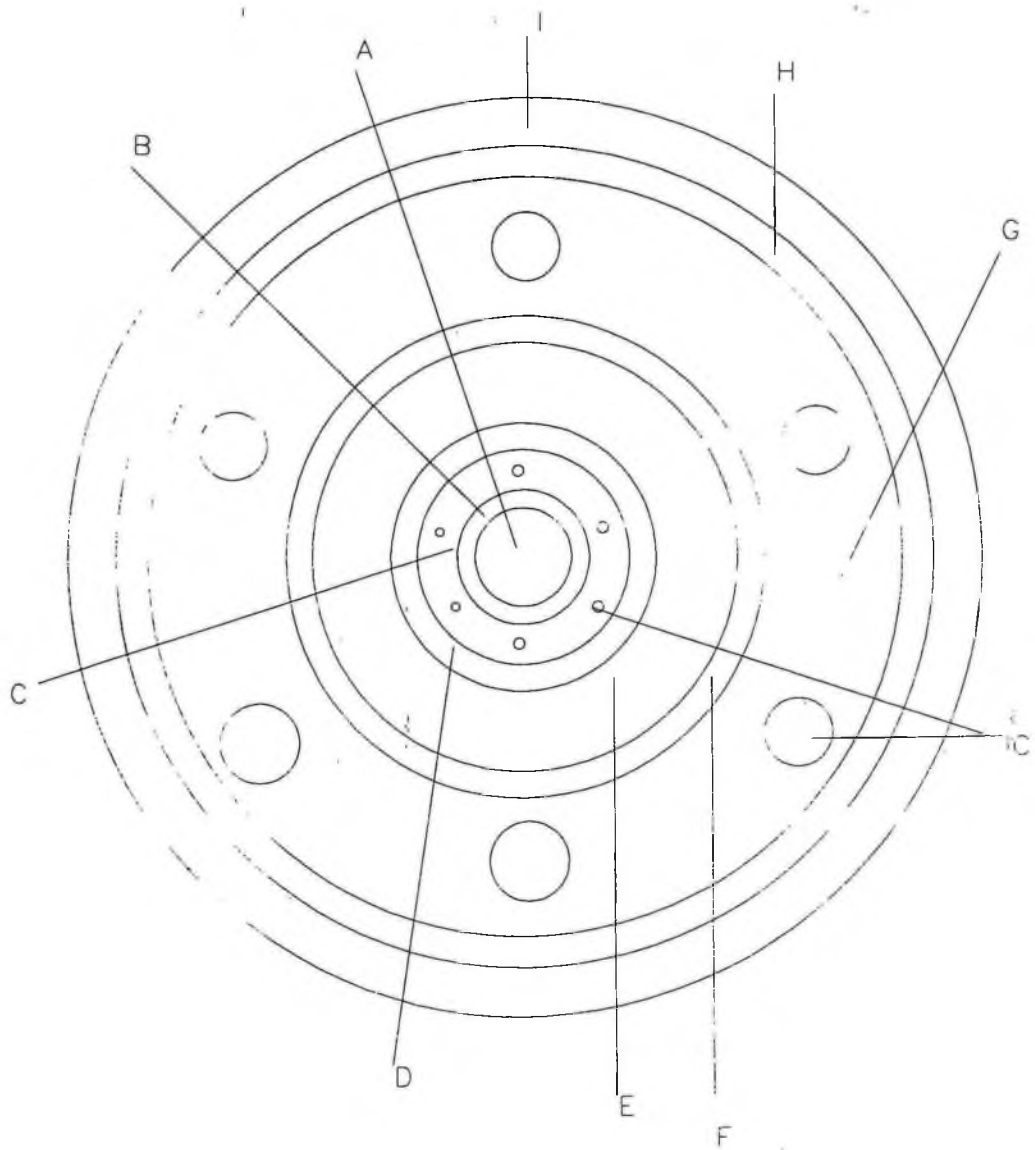


Figure 4.5: Variation of Neutron Fluxes in a Two-Region Homogeneous Mixture of 20% UO<sub>2</sub> + Be.

and I starting from the center. A horizontal cross section of the geometry is shown in Figure 4.6.

The first concentric ring A is occupied with a homogeneous mixture of fuel and moderator (20% enriched  $\text{UO}_2$  + Be). Regions B,D and F consist of Al material as cladding which separate regions A, C and E from each other. Region C is occupied by a moderator. Again in region E there is a homogeneous fuel-moderator mixture to cause more fission in the system. Regions G and H constitute the reflector and shielding materials respectively. Finally, region I is ordinary concrete to act as biological shield.

A study was carried out to determine the choice of a reflector, shield and source strength to optimize neutron flux yield. The effect of different solid reflectors such as Be, BeO, C and liquid reflector ( $\text{H}_2\text{O}$ ) in region G on neutron fluxes was thoroughly investigated. Macroscopic cross sections are first generated for the various homogeneous zones of the multi-region problem. A typical input data for WIMS for this geometry is listed in Table 4.1. These two-group constants computed by the lattice code form part of the input data for SUNDES as listed in Table 4.2. A comparison of the variation of thermal flux with radius of assembly for the four different reflectors is presented in Figure 4.7. It is observed that Be as reflector produced the highest flux though not very much different in value from that of BeO. The closeness in their values could be attributed to the fact that they



A ——— UO<sub>2</sub> + Moderator (Be)

B ——— Al Cladding

C ——— C Reflector

D ——— D Al Cladding

E ——— UO<sub>2</sub> + Moderator (Be)

F ——— Al Cladding

G ——— Reflector

H ——— Al Shield

I ——— Concrete

IC ——— Irradiation Channels

Figure 4.6: Horizontal Cross Section of Neutron Multiplier.

```

CELL 7
SEQUENCE 1
NMESH 21
NGROUP 28
NMATERIAL 5
NREGION 9 2
PREOUT
INITIATE
* SUPERCELL CALN FOR SOURCE + BeO (SURO) *
FEWGROUP 3 5 9 14 21 25 26 27 28 29 30 32 33 35 $
38 40 41 43 45 47 49 52 55 58 60 63 66 69
MESH 2 1 3 1 4 1 4 1 4
MATERIAL 1 -1 303.0 1 235.4 1.39889E-05 2238.4 2.657891E-4 $
16 5.315782E-04 9 1.231023E-01
MATERIAL 2 2.7 303.0 2 $
27 0.9947 29 0.16E-2 55 0.01E-2 11 0.0001E-2 56 0.32E-2 $
58 0.03E-2 7 6.0E-6 63 0.012E-2 112 1.0E-6
MATERIAL 3 1 303.0 3 2001 0 111111 16 0.888889
MATERIAL 4 2.95 303.0 2 9 0.3600 16 0.64
MATERIAL 5 2.3 303.0 2 56 .014 2001 .01 16 .531 29 .35 $
23 .016 27 0.034 12 .001
ANNULUS 1 3.0 1
ANNULUS 2 4.0 2
ANNULUS 3 9.0 4
ANNULUS 4 10.0 2
ANNULUS 5 32.0 1
ANNULUS 6 33.0 2
ANNULUS 7 40.0 4
ANNULUS 8 41.0 2
ANNULUS 9 43.0 5
ARRAY 1 (1 4 6.0 0.0)
ARRAY 2 (1 6 36.5 0.0)
RODSUB 1 1 1.30 3
RODSUB 1 2 1.85 2
RODSUB 2 1 1.30 3
RODSUB 2 2 1.85 2
*BELL 1.15581
POWERC 0 0
BEGINC
THERMAL 10
REGION 11 11
LEAKAGE 7
BUCKLING 1.33445E-03
BEGINC

```

Table 4.1: WIMS Input Data for (20% enriched  $\text{UO}_2+\text{Be}$ )

```

2
1
22.00
0.2
2
9
14
68
0.20
20
.00001
3.00000 1.00000 5.00000 1.0000 22.00000 1.00000 7.00000 1.00000 2.00000
.01549 .00000 0.00000 0.0000 0.01413 0.00000 0.00000 0.00000 0.00000
.01089 .00000 0.00000 0.0000 0.00619 0.00000 0.00000 0.00000 0.00000
.00869 .01160 0.01825 0.0112 0.00795 0.01139 0.01846 0.01179 0.00795
.00592 .00816 0.01340 0.0053 0.00381 0.00746 0.01469 0.00986 0.00592
.00238 .00018 0.16572 0.0003 0.37762 0.00057 0.37776 0.00014 0.01320
.43375 3.48285 0.15976 3.4989 0.43862 3.49181 0.15732 3.47456 0.48911
.53685 3.36493 0.35474 3.3572 0.56610 3.36958 0.31216 3.42266 0.64401
.00134 0.00134 0.00134 0.00134 0.00134 0.00134 0.00134 0.00134 0.00134
.00134 0.00134 0.00134 0.00134 0.00134 0.00134 0.00134 0.00134 0.00134
.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000
.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000
0.82e+14 .82e+00 .82e+00 .82e+00 .82e+00 .82e+00 .82e+00 .82e+00 .82e+00
.77e+14 .77e+00 .77e+00 .77e+00 .77e+00 .77e+00 .77e+00 .77e+00 .77e+00
8.45e+09 .00e+00 .00e+00 .00e+00 .00e+00 .00e+00 .00e+00 .00e+00 .00e+00
.00e+00 .00e+00 .00e+00 .00e+00 .00e+00 .00e+00 .00e+00 .00e+00 .00e+00
1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000
1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000
1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000 1.00000
0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000

```

Table 4.2: SUNDES.INP Input Data for Multigion Problem

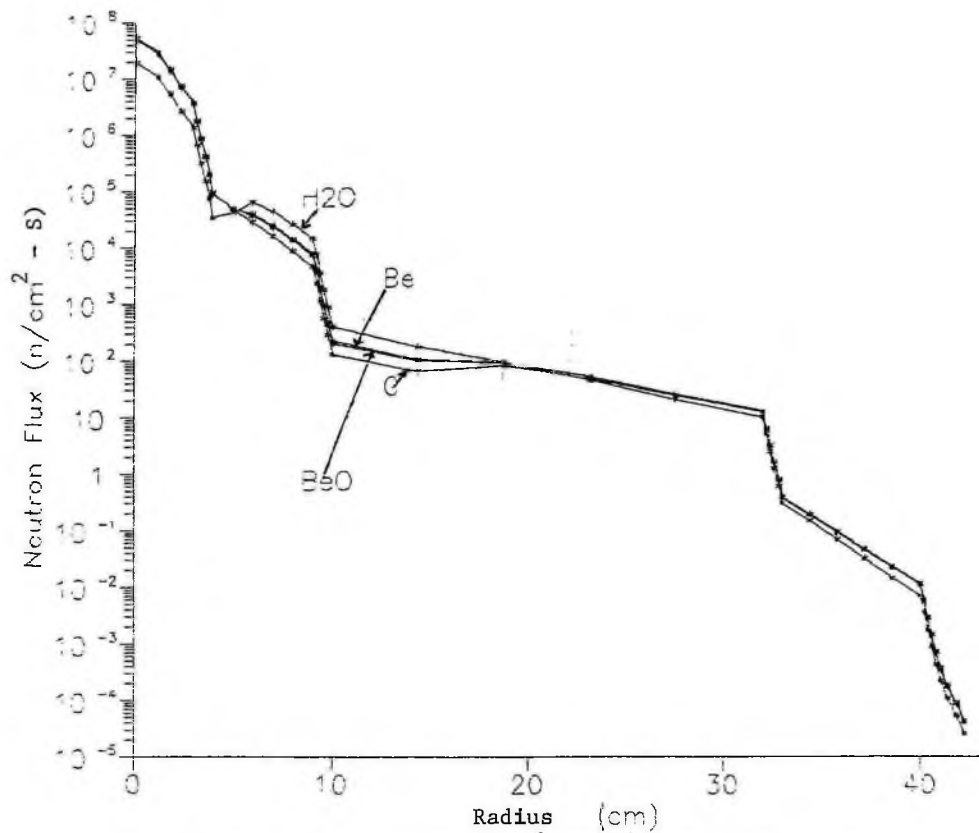


Figure 4.7: Comparison of Thermal Neutron Fluxes For Four Different Reflectors.

both have the same microscopic absorption cross sections but Be has a slightly higher microscopic scattering cross section as in Table 4.3. From the plot, it can be observed that  $H_2O$  has the lowest value of flux in the central region A, but rather tends to have the highest values from region C. This is not surprising because  $H_2O$  as reflector has the highest microscopic scattering cross section as compared with the rest of the tested reflectors. By scattering collision, more neutrons are returned into the fuel region thus producing additional fissions. Although water is the cheapest among the tested materials and it produced the highest flux, it was not selected as reflector for region G for the present analysis in order to avoid leakage and possible contamination. Graphite produced the least flux and could therefore not be a possible choice as reflector. The choice of either Be or BeO as reflector was not too obvious as a result of their closeness in flux. An advantage of using either Be or BeO is that reactivity will increase with the presence of fast neutrons available from  $(n, 2n)$  and photo neutrons from  $(\gamma, n)$  reactions. However, BeO was preferred to Be because it is much less expensive and easier to fabricate.

Investigations were also conducted to select shielding material among aluminium (Al), lead (Pb) and stainless steel (SS) for region H. It was observed that Pb was the most efficient, followed by SS and then Al as seen in Figure 4.8. However, the

| <b>Element or<br/>Molecule</b> | <b>Microscopic<br/>Absorption<br/>Crosssection (barns)</b> | <b>Microscopic<br/>Scattering<br/>Crosssection (barns)</b> |
|--------------------------------|------------------------------------------------------------|------------------------------------------------------------|
| <b>Be</b>                      | <b>0.0095</b>                                              | <b>7.0</b>                                                 |
| <b>BeO</b>                     | <b>0.0095</b>                                              | <b>6.8</b>                                                 |
| <b>C</b>                       | <b>0.0034</b>                                              | <b>4.8</b>                                                 |
| <b>H<sub>2</sub>O</b>          | <b>0.664</b>                                               | <b>103</b>                                                 |

**Table 4.3: Nuclear Properties of Be, BeO, C and H<sub>2</sub>O**

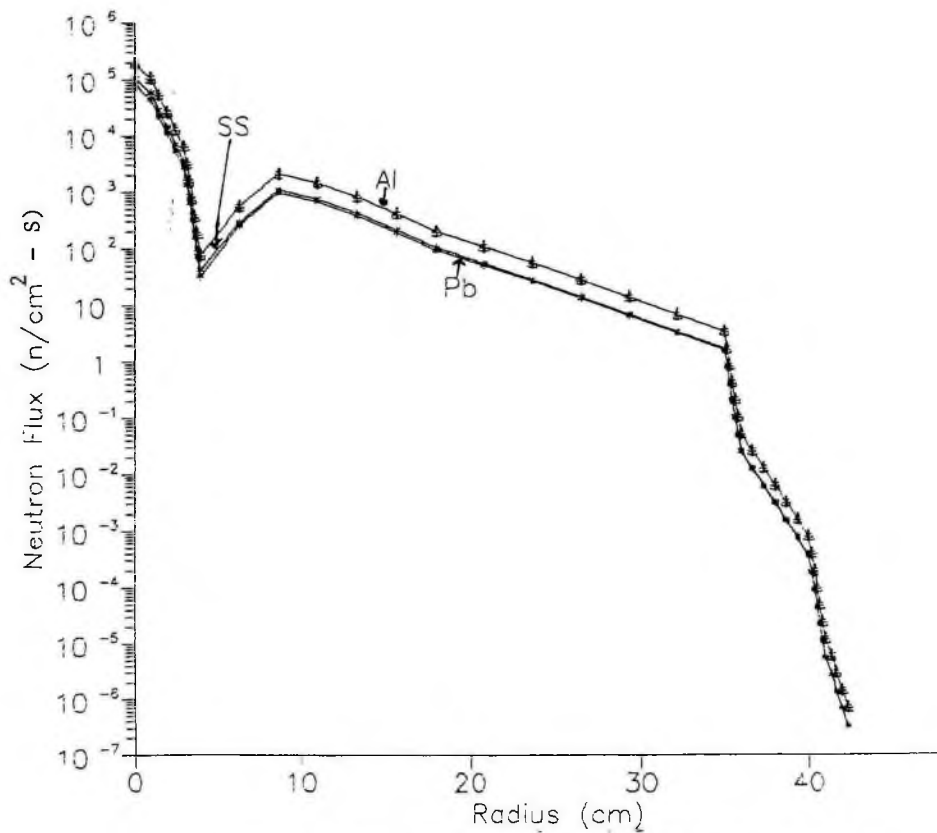


Figure 4.8: Effect of Al, SS, and Pb Shields in Region H on Thermal Flux.

differences in neutron attenuation of these materials were not significant. Al was therefore selected because it is light in weight and less expensive. Its choice is also consistent with the low power of two milliwatts produced in the multiplier since it has found wide application in the construction of low power research reactors with less effect of radiation damage.

Multiple source effect was investigated by placing sources of different strengths in various zones of the neutron multiplier. The effect of the presence of source in the fuel-moderator region E can be seen in Figure 4.9. Its effect in region G is illustrated in Figure 4.10. The effect of placing sources in both regions E and G was also studied and the trend is shown in Figure 4.11. It was observed that the fluxes increased generally in the assembly and became more pronounced in the regions where the sources were located. Different source strengths have different effects on the values of thermal fluxes produced in the regions E and G. This can be seen in Figures 4.12 and 4.13 respectively. The two plots reveal that the values of the thermal fluxes achieved in the neutron multiplier depended on the source strength.

Based on the results obtained, it was decided that the configuration with BeO in region G and Al as shield in region H could be the accepted geometry of the assembly. Figure 4.14 presents the variation of neutron flux (fast and thermal) with radius of the assembly.

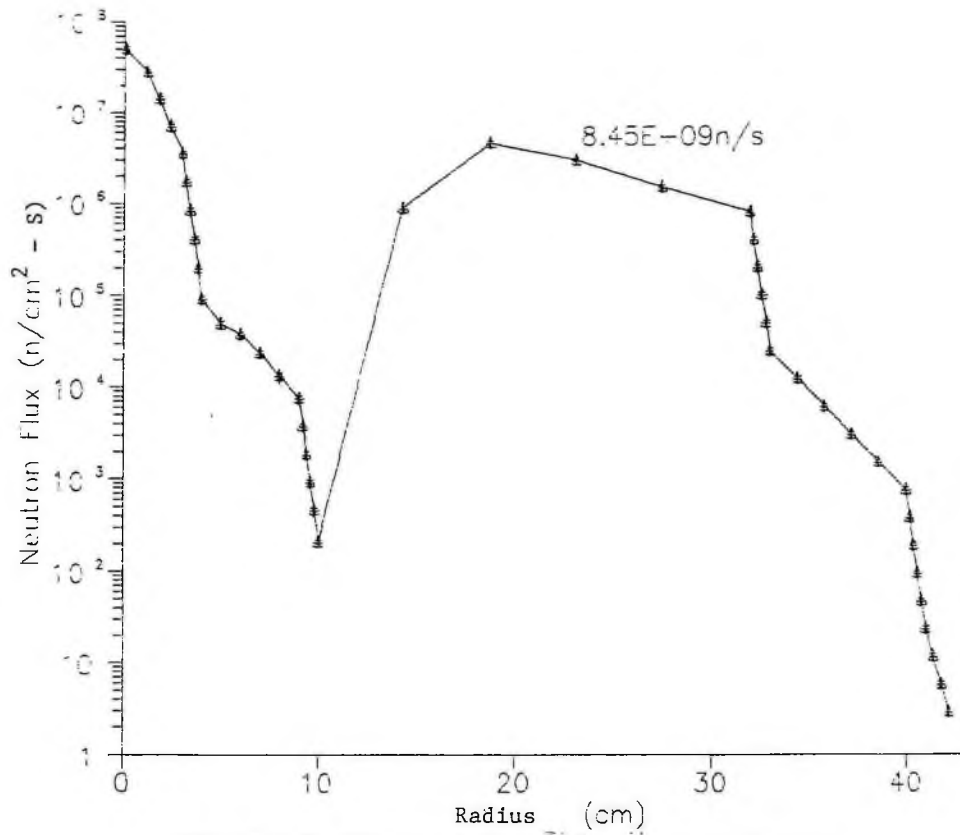


Figure 4.9: Effect of Source Strength on Thermal Neutron Flux in Region E.

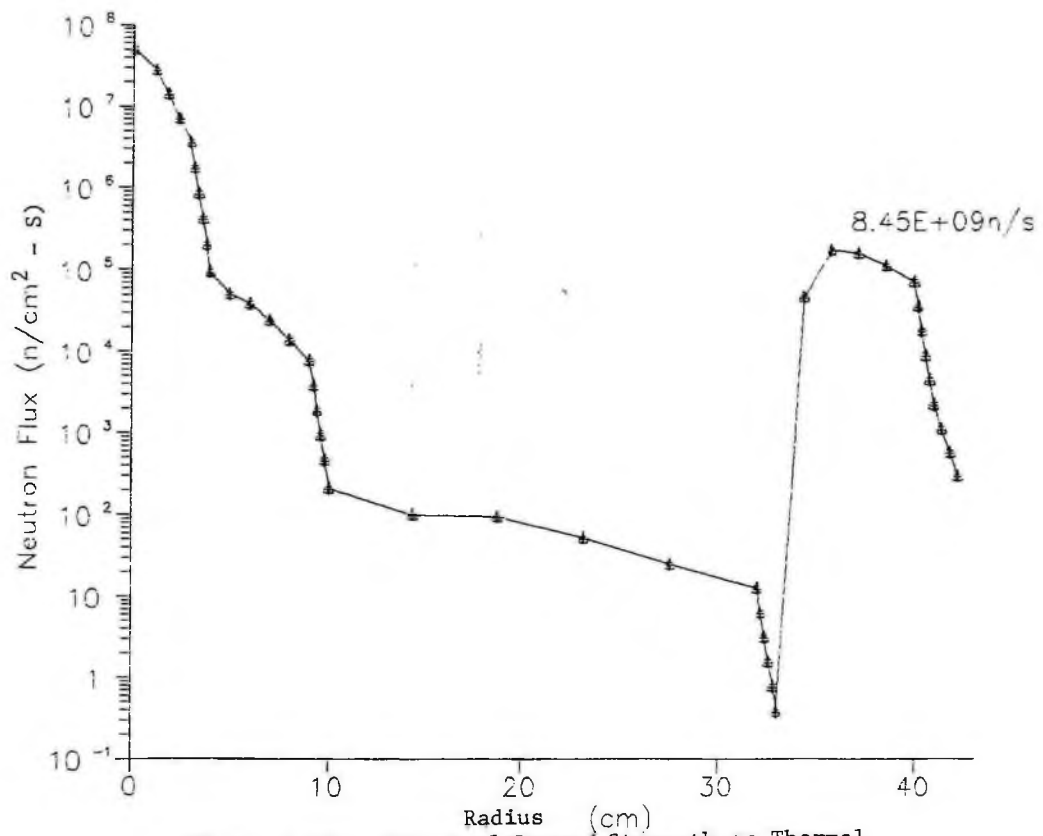


Figure 4.10: Effect of Source Strength on Thermal Neutron Flux in Region G.

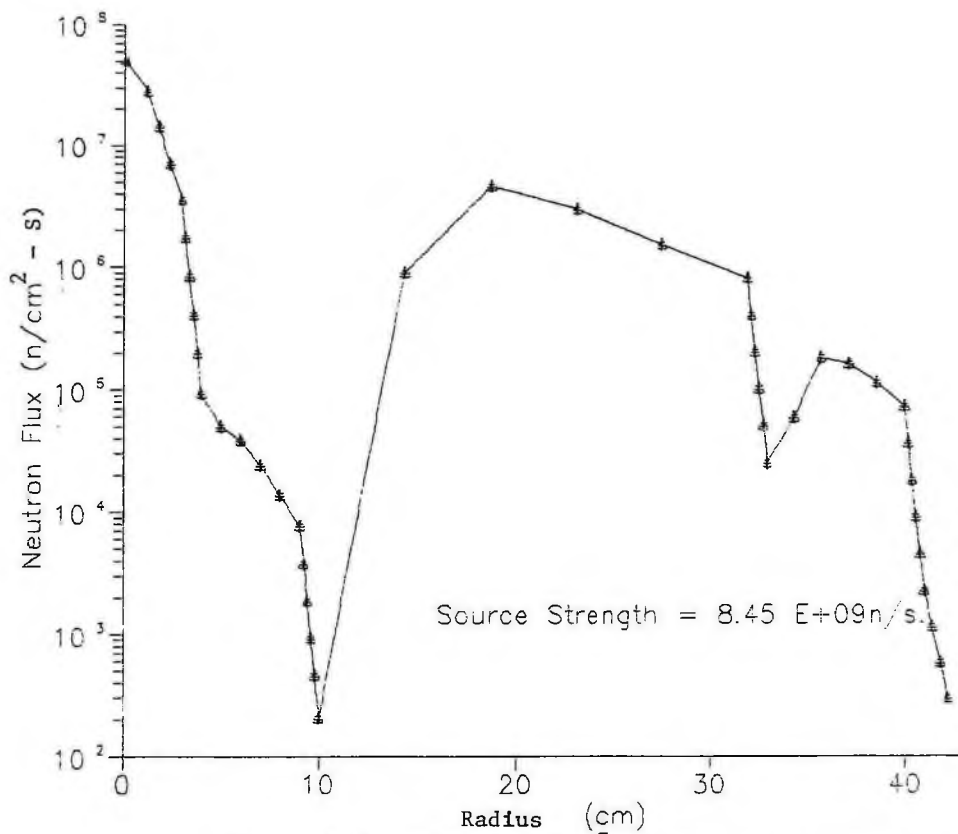


Figure 4.11: Effect of Source Strengths on Thermal Neutron Flux in Regions E and G.

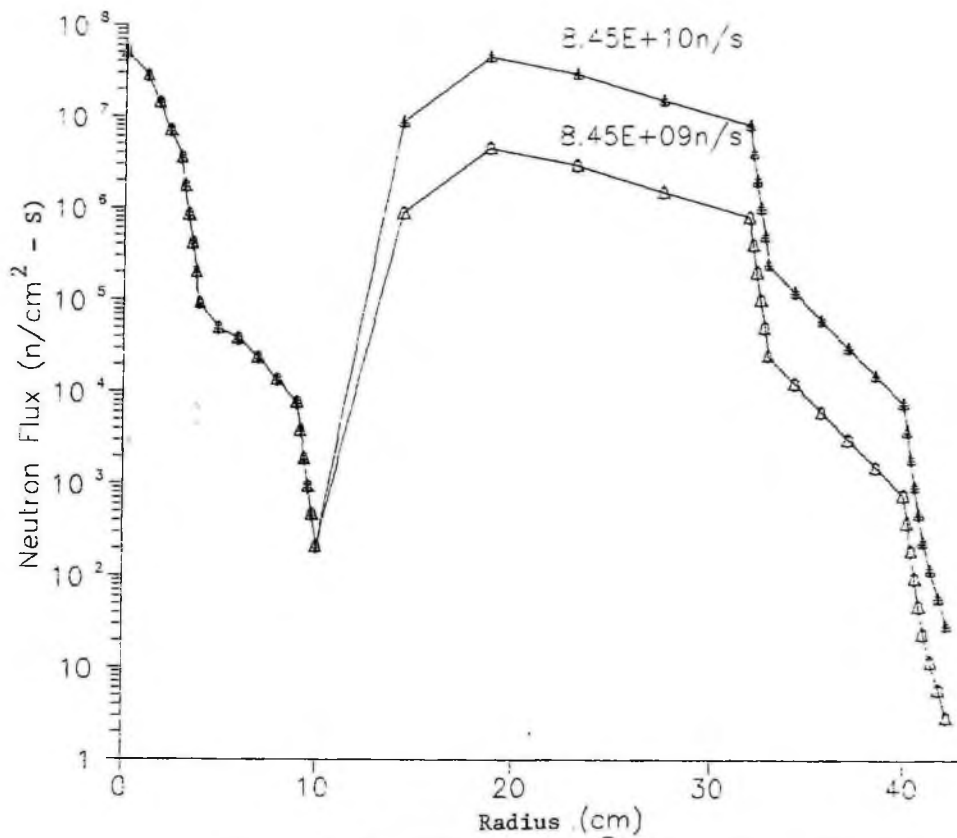


Figure 4.12: Effect of Source Strengths on Thermal Flux in Region E.

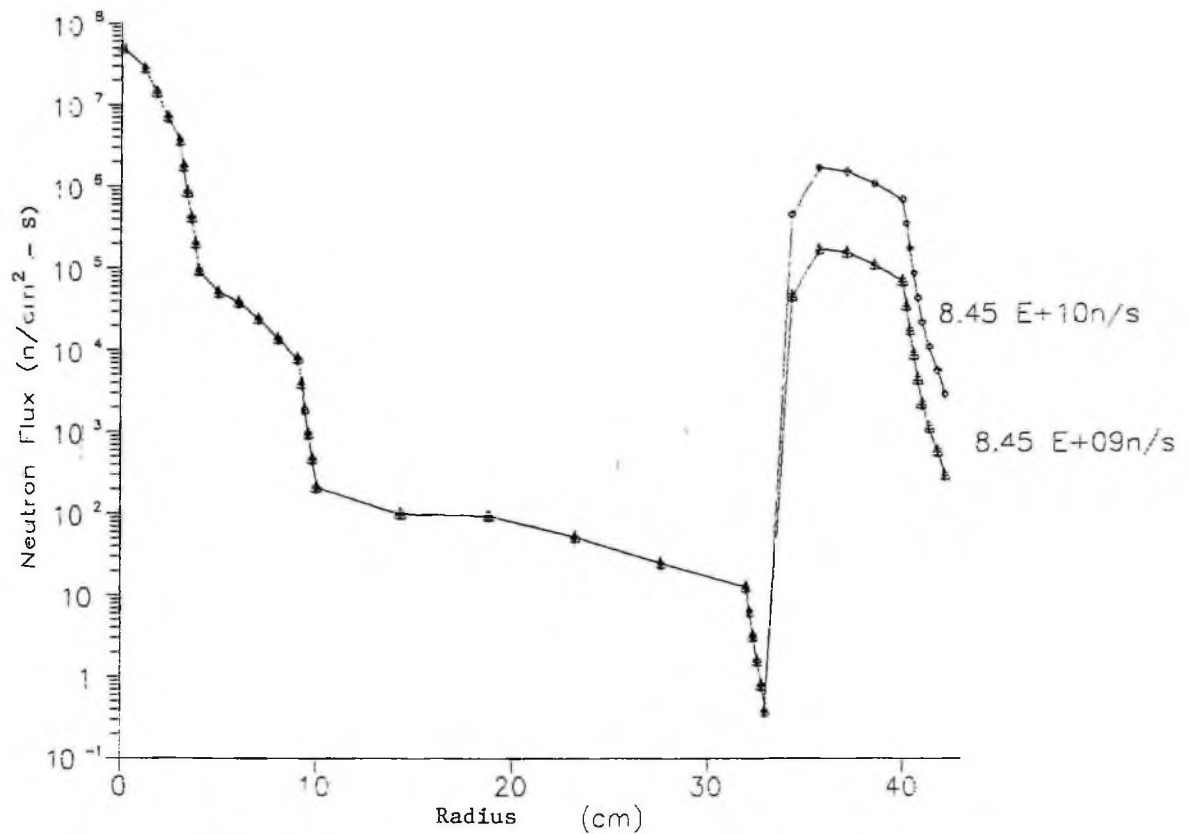


Figure 4.13: Effect of Source Strengths on Thermal Flux in Region G.

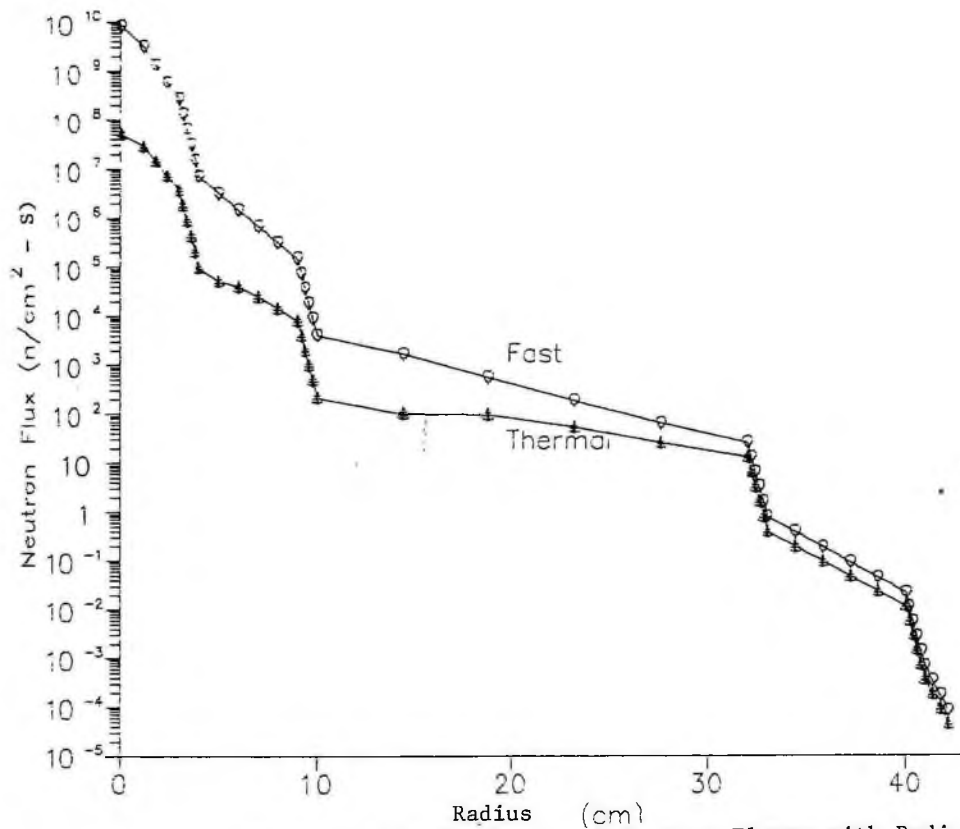


Figure 4.14: Variation of Neutron Fluxes with Radius.

#### 4.2.3 Description of Accepted Geometry.

The selected assembly which is cylindrical in shape has height 86cm and radius 43cm; the height to diameter ratio being unity. A radioisotope source of strength  $8.45 \times 10^9 \text{ n/s}$  is embedded in the homogeneous mixture of fuel and moderator. Region B is constructed of Al cladding of thickness 1cm which separates the fuel-moderator mixture in A from a 5cm water region, C. Region C contains six inner irradiation channels each of diameter 26mm. A 1cm Al cladding in region D also separates the water in C from another fuel-moderator mixture of thickness 22cm. Region G is BeO reflector of thickness 7cm. This is separated from the fuel-moderator mixture in E by yet another 1cm thick Al cladding in region F. The BeO reflector has six outer irradiation channels each of radius 35mm for samples irradiation at different flux. The last two regions, H and I are 1cm Al cladding and 2cm thick concrete respectively which serve as shield for preventing leakage of neutrons from the assembly.

Thermal neutron fluxes greater than  $1 \times 10^7 \text{ n/cm}^2\text{-s}$  in regions of irradiation which is the main objective of the study were achieved in the assembly.

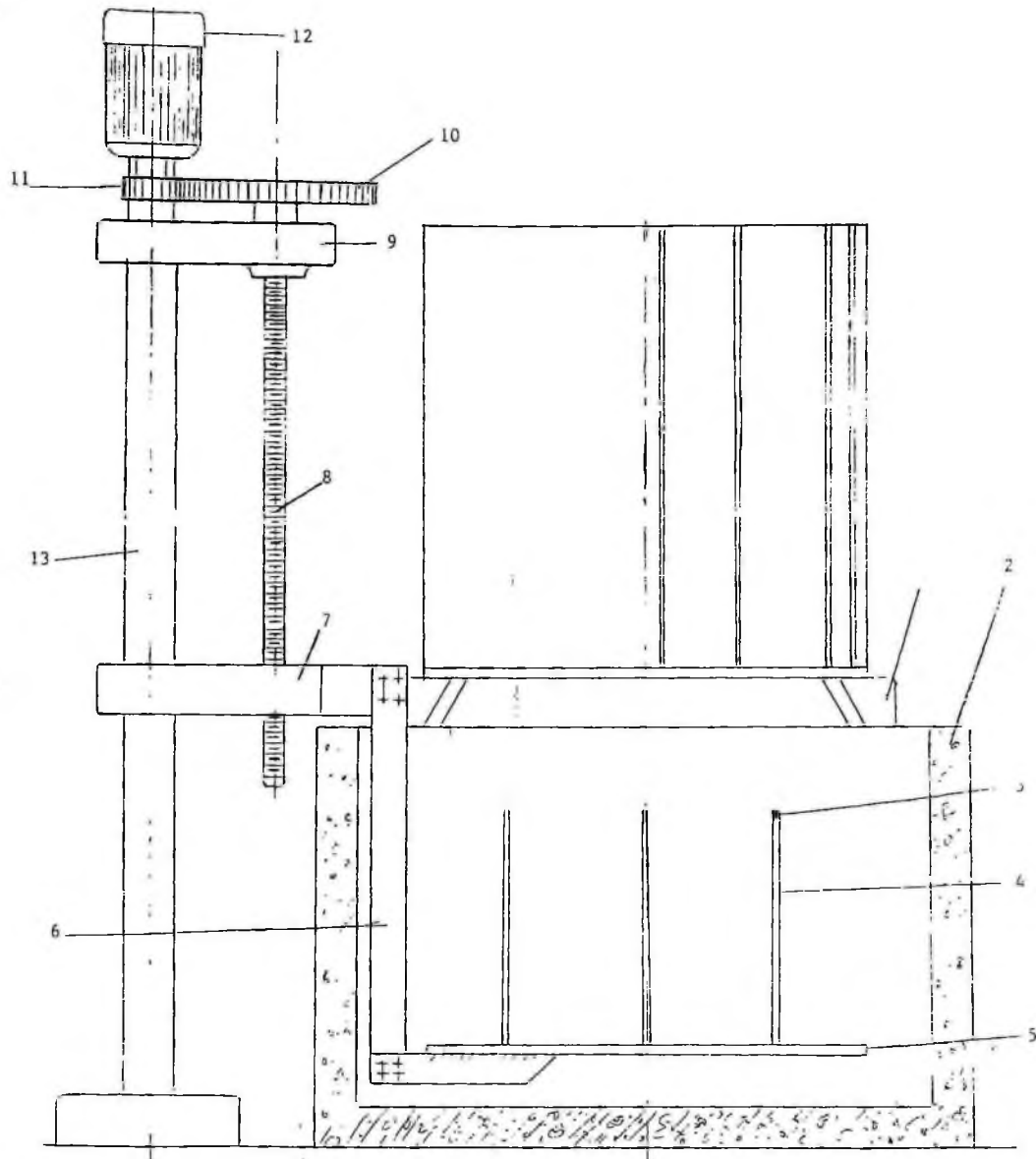
#### 4.3 DESCRIPTION OF MECHANICAL FEATURES OF THE NEUTRON MULTIPLIER.

The mechanical aspect of the multiplier is basically a lifting device for introducing or withdrawing neutron sources in and out of the multiplier. The vertical cross section is shown in Figure 4.15 . The mechanical engineering design features are presented in reference [42]. For this work, only its technical features and operational characteristics are presented.

Four supports for the multiplier (1) bear it from underneath. The supports are bolted onto a concrete tank (2) which contains water into which the neutron sources (3) are submerged when outside the multiplier. The tank with water also serves as a shield against the radiation from the neutron sources and  $\gamma$ -emitting sources from fission fragments when out of the multiplier.

The sources are mounted on thin metal rods (4) which are fitted vertically onto a metal plate (5). The rods and plate materials are made of Al. At such low powers of operation it is expected that Al will be resistant to heat, corrosion and radiation while ensuring adequate strength for reliability.

The plate is bolted rigidly to an arm (6) which is also connected rigidly to an extension of a movable nut (7). A screw (8) runs through the nut and is supported in thrust bearing (9) at the upper end. The screw is keyed to a spur gear (10) at its upper end. The spur gear is meshed with a spur gear pinion (11) which



- |                      |                      |
|----------------------|----------------------|
| 1. Supports          | 8. Screw             |
| 2. Concrete tank     | 9. Thrust bearing    |
| 3. Neutron source(s) | 10. Spur gear        |
| 4. Thin Metal rods   | 11. Spur gear pinion |
| 5. Metal plate       | 12. Electric motor   |
| 6. Arm               | 13. Steel pillar     |
| 7. Movable nut       |                      |

Figure 4.15: Vertical Cross Section of Neutron Multiplier

is keyed to an electric motor (12) mounted against a vertically supported cylindrical steel pillar (13). The pillar also serves as a guide for the nut and provides a counter moment against that set up by the weight of the plate and arm, tending to bend the screw inwards towards the pillar.

A projection on the movable nut actuates an electrical switch at the two extreme ends of its movements up and down the screw. This switch is one of two alternative switches for switching the motor on or off. The second switch is located in the control room. When one switch is on the other must be off in order to close the circuit and vice versa.

The switch in the control room also has a provision to change the polarity of the motor. At the same time this switch is kept 'on' or 'off', it reverses the polarity of the motor. The reversal of the polarity of the motor is necessary to change the direction of the rotation of the motor and hence to raise or lower the nut for the purpose of introducing the sources into or out of the multiplier.

When the sources are in their lowest position in the water, the first switch would have been in the 'off' position. At the same time the second switch would be off. To raise the sources into the the multiplier, the second switch is kept in the 'on/up' position with the polarity of the motor automatically reversed. The motor is thus started and the direction of its rotation is such as would



raise the nut and consequently the sources upwards. Near its highest position 'on' its travel upwards, the projection on the nut would actuate the first switch to put it in the on position. This will open the circuit and thus put off the motor. The nut will remain at its highest position and will not overhaul downwards by virtue of its design.

In lowering the sources into the water, the second switch in the control room is put off and the polarity of the motor would be automatically reversed. The motor is thus started and the direction of its rotation is such as would lower the nut together with the sources. Near its lowest position, when the sources would have been totally immersed in the water, the projection on the nut will actuate the first switch putting it off. This will open the circuit and cut off power supply to the motor. The nut together with the sources would then remain in their lowest positions.

In effect, the movement of the sources is simply controlled in the control room by the second switch by putting it in the 'on/up' or 'off/down' position as the situation demands.

**CHAPTER FIVE.****CONCLUSIONS AND RECOMMENDATIONS.**

With reference to the primary objectives of this work as outlined in Chapter Two, a finite difference scheme was used to obtain a numerical solution to the one-dimensional neutron diffusion equation. A computer programme dubbed 'SUNDES' written in FORTRAN 77 programming language for an IBM.PC was developed based on the numerical scheme. With the aid of the computer package an extensive study was carried out to present a nuclear design of the neutron multiplier.

The trend of neutron flux distribution was found to conform to a large extent to reactor physics predictions. It can be said that with an isotopic neutron source placed at the center of the assembly and with the appropriate materials used in the various regions as presented in the design description, thermal neutron fluxes higher than  $1 \times 10^7 \text{ n/cm}^2\text{-s}$  were obtained in the assembly for NAA and other reactor physics experiments. This is the main objective of the study and this clearly shows that it is technically feasible to achieve such levels of fluxes in a neutron multiplier.

The results revealed that different fluxes are produced depending on the source strength. It is therefore very satisfying to mention here that since different geological and biological

samples require different fluxes, this design would be appropriate. The desired fluxes were obtained by introducing the appropriate source strength. With such high fluxes obtained in the neutron multiplier, the efficiency of NAA can be improved. Samples will be sufficiently exposed to the neutron flux thus increasing the probability of excitation and hence good detection limits. The design has provided for simultaneous irradiation of samples at the same flux and also at different fluxes in different regions. This will ultimately mitigate the time consuming nature of the medieval radioisotope source in a non-multiplying medium used for NAA.

The mechanical design of the neutron multiplier is simple, affording the possibility of manufacturing most of the components locally. It will be easy to install and operate. Exposure of personnel and/or the general public to radiation is minimized by the shielding and cement materials. Another advantage of the design is the inherent ability of the screw not to overhaul. This makes it possible to maintain the sources at rest anywhere between and at the extreme ends of its travel.

From the foregoing results and discussions, a conceptual nuclear design of a subcritical assembly driven by isotopic neutron sources (neutron multiplier) has been achieved with great success. It may be recommended however that, for actual realization of the design, construction and installation of the subcritical assembly the following should be considered;

- (1) The neutron multiplier is compact and the possibility exists for the leakage of neutrons in the axial direction. A detailed nuclear design in two-dimensions ( $r$ ,  $z$ ) will be required to correctly account for the leakage. In case there is much leakage, then reflectors would be needed for the bottom and top of the assembly. The author is not aware of the presence of any computer package available in literature developed for subcritical reactor physics analysis employing multiple sources to drive such assemblies. There is therefore the need to develop another version of SUNDES for this aspect of neutronic design.
- (2) A comprehensive thermal and structural analysis be carried out for the assembly.
- (3) In addition to a remote control device to be incorporated in the mechanical design, a design of irradiation systems for transfer of rabbit capsules for activation analysis must be provided.
- (4) Instrumentation to detect parameters such as temperature, water level in the tank and neutron flux levels within the regions of interest such as near irradiation sites are needed. Protection limits for flux and temperatures should be incorporated in the design so that the system could be shutdown if limits are exceeded.
- (5) Radiological consequences must be determined to ascertain the

safety of the neutron multiplier after it has aged. Procedures for decommissioning and waste disposal must be studied and subjected to approval by the competent national authority.

- (6) Finally, an exercise to determine an economic feasibility with respect to cost of fuel, moderator, reflector, structural materials and operating cost must be carried out and compared with the cost of low power research reactors.

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```
C -----
C The code SUNDES solves one-dimensional two-group neutron
C diffusion equation for a sub-critical assembly driven by
C isotopic neutron sources. Multiplying medium is U-235.
C -----
C For enquiries: Reactor Simulation Group
C                 National Nuclear Research Institute
C                 Ghana Atomic Energy Commission
C                 P.O.Box 80
C                 Legon, Accra
C                 Ghana, West Africa
C -----
C
C Options:
C -----
C (a) Choice of three geometries: Slab, Cylinder and Sphere
C
C (b) Determination of keff.
C
C (c) Variation of neutron flux with radius of assembly.
C -----
C List of parameters used in the program.
C -----
C r--radius along assembly
C l--homogeneous zone for material
C nl-total number of zones
C fn-fraction of fission spectrum in neutron energy groups
C fq-fraction of source spectrum for for neutron energy groups
C b--transverse buckling
C kg-energy group
C ng-total number of energy groups
C h--interval
C k--position on radius of assembly
C kl-number of intervals
C ka-number of intervals for multiplying region
C nmax-maximum number of allowed iterations
C ig-type of geometry(0 for slab, 1 for cylinder and 2 for sphere)
C sigf-macroscopic fission cross-section
C siga-macroscopic absorption cross-section
C sigs-macroscopic scattering cross-section
C scf--ratio of neutrons released per fission to the thermal
C ----energy released per fission(n/kWs)
C sigt-total neutron cross-section (cm-1)
C d--diffusion coefficient(cm-1)
C tau-parameter reqd. for calculating coefficient for matrix
C rf--radius of assembly(cm)
C p--power within the assembly(kW)
C erl-minimum allowed value for convergence criterion
```

```

-----
Main Program
-----

dimension phi(2,100), siga(2,10), d(2,10), b(2,10)
dimension scf(2,10), sigt(2,10), sigs(2,2,10)
dimension ml(10), dv(100), f(200), sigma(2,100)
dimension scm(50), dd(50), gam(50), fi(50)
dimension c(100,100), phi1(100), sx(100), fl(2,100)
dimension g(100,100), xn(100), gx(100), ax(100), ek(50)
dimension a(2,100,100), bb(2,100), xx(2,100)
dimension delr(10), noptsr(100), sp(2,10)
dimension sff(2,30), sa(2,30), sgs(2,2,30), df(2,30), buk(2,30)
dimension fc(2,30), st(2,30)

common /mk/ hr(10,60)
common /ff/ fn(2,10), fq(2,10)
common /fq/ sigf(2,10), q(2,10)
common /xy/ x(100), y(100), w(100)
common /sq/ s(100), se(100), qq(100), qj(100)
common /aa/ a(2,100,100)
common /rr/ r(100)
common /tt/ tau(100), beta(100), rho(100)
common /ue/ u(100)
common /et/ eta(100)
common /gt/ sr(2,100), sm(2,100), sd(2,100)
common /dw/ ak(50)
common /fr/ sdx(2,100), sdm(2,100)
common /bb/ bb(2,100), xx(2,100)

open(unit=1, file='sundes.inp', status='old')
open(unit=6, file='sundes.out', status='new')

-----
input for geometrical data for subassembly
-----

read(1,19) jn
read(1,20) ig
read(1,21) rf
read(1,22) p
read(1,23) ng
read(1,24) nl
read(1,25) jl
read(1,25) kl
read(1,26) ka
read(1,27) h
read(1,28) nmax
read(1,29) erl
read(1,34) (delr(1), l=1, nl)

```

```
-----
input for nuclear data for the assembly
-----
```

```
read(1,30) ((sigf(kg,1),l=1,nl),kg=1,ng)
read(1,31) ((siga(kg,1),l=1,nl),kg=1,ng)
read(1,32) ((sigs(ng,kg,1),l=1,nl),kg=1,(ng-1))
read(1,33) ((d(kg,1),l=1,nl),kg=1,ng)
read(1,34) ((b(kg,1),l=1,nl),kg=1,ng)
read(1,34) ((sp(kg,1),l=1,nl),kg=1,ng)
read(1,35) ((scf(kg,1),l=1,nl),kg=1,ng)
read(1,36) ((q(kg,1),l=1,nl),kg=1,ng)
read(1,37) ((fn(kg,1),l=1,nl),kg=1,ng)
read(1,38) ((fq(kg,1),l=1,nl),kg=1,ng)
```

```
determination of number of mesh points along the radial dire
```

```
s1=0.
```

```
do 44 kg=1,ng
```

```
do 45 l=1,nl
```

```
if(kg.gt.1) then
```

```
s1=s1+sigs(ng,kg,1)
```

```
sigt(kg,1)=siga(kg,1)+s1+(d(kg,1)*(b(kg,1)**2))
```

```
else
```

```
sigt(kg,1)=siga(kg,1) + (d(kg,1)*(b(kg,1)**2))
```

```
end if
```

```
45 continue
```

```
44 continue
```

```
do 2 l=1,nl
```

```
es=0.0
```

```
do 1 kg=1,ng
```

```
1 es=es+siga(kg,1)+sp(kg,1)
```

```
noptr(1)=delr(1)*es+1.0
```

```
number of points to be less than 20 per region
```

```
if(noptr(1).lt.20)noptr(1)=20
```

```
2 continue
```

```
sumr=0.0
```

```
do 3 l=1,nl
```

```
3 sumr=sumr+noptr(1)
```

```
if(sumr.lt.50)go to 5
```

```
flt2r=sumr
```

```
do 4 l=1,nl
```

```
flt1r=noptr(1)
```

```
flt3r=flt1r/flt2r*50
```

```
4 noptr(1)=flt3r
```

```
5 continue
```



```

c
c      computation of mesh spacings for radial direction
c
      j=0
      do 6 i=1,nl
      j=j+noptr(i)
      if(i.eq.1)jf1=j
      if(i.eq.(nl-2))jf2=j
6      continue
      nr=j+1
      jsr=ng*nr
      sumr=0.0
      do 7 i=1,nl
      noptr=noptr(i)
c
      if(i.gt.1) sum=sum+noptr(i-1)
      do 7 j=1,noptr
      do 7 kg=1,ng
      der=delr(i)
      l=sumr+j
      sff(kg,l)=sigf(kg,i)
      sa(kg,l)=siga(kg,i)
      sgs(ng,kg,l)=sigsg(ng,kg,i)
      df(kg,l)=d(kg,i)
      buk(kg,l)=b(kg,i)
      fc(kg,l)=fn(kg,i)
      st(kg,l)=sigt(kg,i)
c
      hr(i,l)=der/noptr(i)
c
c
c      write(*,*)kg,i,noptr(i)
c
7      continue
      jr=1
      hr(nl,nr)=hr(nl,nr-1)
c
      sumd=0.000
      k=1
c      do 227 k=1,nr
      do 337 l=1,nl
      do 447 m=1,noptr
c      write(*,*)m,l,k
      sumd=sumd+hr(l,m)
      r(k)=sumd
c      write(*,*)k,r(k)
      k=k+1
447      continue
337      continue
      kl=k-1
c      write(*,*)kl
      ka=kl-2

```

```

c      orelax=1.00
c      nc=20
c      kl=n1*j1
c      -----
c      subroutine PARA to calculate radii at positions k ,tau,rho,
c      beta
c      -----
c
c      write(*,*) k1
c
c      call para(noptr,n1,kl,ig)
c
c      pi=3.142
c      term=1.
c
c      do 51 i=1,n1
c      ml(i)=i
51      continue
c
c      summation of isotopic sources in the system
c
c      smq=0.
c      do 404 kg=1,ng
c      do 405 l=1,n1
c      smq=smq+q(kg,l)
405      continue
404      continue
c
c      guess of initial fluxes for g=1,...,ng
c
c      do 52 kg=1,ng
c      do 53 k=1,kl
c      phi(kg,k)=term
53      continue
52      continue
c
c      coefficients for zone 1
c
c      do 333 kg=1,ng
c      kg=1
c
c      l=1
c      k=1
c
c      r(k)=hr(1,1)
c      r(k)=.070
c      r32=.5*(r(k+1)+r(k))
c      dl=((r32*(ig+1))-r(k)**(ig+1))/(ig+1)
c      bx=((r(k+1)-r(k))/(r32**ig)*d(kg,l))
c      tx=r(k)*bx*((signt(kg,l)*d(kg,l))**.5)
c      tx=0.
c      a(kg,k,1)=(1.+(signt(kg,l)*dl*bx)+tx)

```

```

c      a(kg,k,2)=-1.
c      write(*,*)j1,kl
c      do 334 m=3,kl
334      a(kg,k,m)=0.
c      continue
c
c      coefficients for points k=2,...,kl
c
c      do 336 l=1,nl
c
c      if(l.eq.1)then
c      j=2
c      kz=noptr
c
c      else
c      j=kz+1
c      kz=kz+noptr
c      if(l.gt.1)write(*,*)l,j,kz
c      end if
c
c      call coeff(j,kz,kg,l,kl,d,sigt,a)
c
336      continue
c
c      l=nl
c      kl1=kl-1
c      kl2=kl-2
c      kl3=kl-3
c
c      uk=((r(kl)**(ig+1))-(rke**(ig+1)))/(ig+1)
c      tk=(rke**ig)/(r(kl)-r(kl1))
c      vk=((siga(kg,l)*d(kg,l))*0.5)
c      a(kg,kl,kl1)=-(tk*d(kg,l))
c
c      do 338 m=1,kl2
c      a(kg,kl,m)=0.
338      continue
c
c      a(kg,kl,kl)=(d(kg,l)*tk)+(siga(kg,l)*uk)+(vk*(r(kl)**ig))
c
c      333      continue
c
c      set iteration i=1
c
c      i=1
c
c      summation of fission sources and isotopic sources
c
c
778      do 88 l=1,nl
c
c      if(l.eq.1) then
c      j=1
c      kz=noptr

```

```

c
      else
      j=kz+1
      kz=kz+noptr
      end if
c
      ak(1)=1.
      ak(2)=1.
c
      -----
c      caln. of values using subroutine SUMP,DUMP and TUMP
c      -----
c      test for the kind of source used
c
      if(jn.eq.1) go to 305
      if(jn.eq.2) go to 306
      if(jn.eq.3) go to 307
305    call sump(i,j,kz,1,ng,kl,ig,pi,scf,phi,dv)
      go to 88
306    call dump(j,kz,1,ng,ig,pi,scf,phi,dv)
      go to 88
307    call tump(j,kz,1,ng,kl,ig,pi,scf,phi,dv)
c
88      continue
c
      m=kl-1
c
c
c      -----
c      for fission sources in the entire volume
c      of the assembly
c      -----
c
      call simpr(x,kl,m,h,smf)
c
      fi(i)=smf
c
c      for isotopic sources in the assembly
c      call simpr(w,kl,m,h,smw)
c
c      calculation of subcritical multiplication,m
c
      scm(i)=(smq+smf)/smq
c
c      calculation of factors for estimation of keff
      tt=smf/smq
      gam(i)=1./scm(i)
      dd(i)=smf/smq
c
      spq=0.0
      srp=0.0
      do 876 k=1,(kl)
      spq=spq+x(k)
      srp=srp+se(k)
876    continue

```

```

px=3.2e-11*smf/2.5
ax(i)=p/px
if(i.eq.1) go to 779
c
c
c      check for convergence
c
gt=fi(i-1)/fi(i)
if(i.gt.2) ak(i)=ak(i-1)*(fi(i-1)/fi(i))
c
c      calculation of matrix B for g=1 and C for g=2,...,ng
c
779      kg=1
c
c
c      do 102 k=1,k1
sigma(kg,k)=sr(kg,k)
102      continue
c
c      if(kg.eq.1) go to 500
c
777      kkg=kg-1
do 55 kg=1,kkg
c
c      do 888 l=1,n1
if(l.eq.1) then
j=1
kz=noptr
c
c      else
j=kz+1
kz=kz+noptr
c
c      end if
c
c      computation of coefficients for matrix C
c
c      do 222 k=j,kz
phii(k)=phi(kg,k)
do 223 m=1,k1
mk=m-k
c
c      if(mk.eq.0) then
c(k,m)=sigs(ng,kg,l)
else
c(k,m)=0.
c
c      end if
c
223      continue
222      continue
c
c
888      continue

```

```

c -----
c product of matrix C and phi1
c -----
c call pmatrx(c, phi1, k1, sx)
c
55 continue
do 809 k=1, k1
sigma(kg, k)=eta(k)*sx(k)
809 continue
c
500 neqn=k1
c
do 7009 k=1, k1
phi(kg, k)=0.
7009 continue
do 718 k=1, k1
sigma(kg, k)=sigma(kg, k)/eta(k)
a(kg, k, m)=a(kg, k, m)/eta(k)
718 continue
c
call gsiter(a, sigma, phi, kg, k1, orelox, nc)
c
kg=kg+1
if(kg.le.ng) go to 777
c
if(kg.gt.ng) i=i+1
if(i.eq.nmax) go to 667
go to 778
c
667 do 234 kg=1, ng
do 244 k=1, k1
fl(kg, k)=ax(i-1)*phi(kg, k)
244 continue
234 continue
c
str=0.
do 235 kg=1, ng
do 236 k=1, k1
str=str+sigf(kg, 1)*fl(kg, k)
236 continue
235 continue
c
srq=0.
do 407 k=1, k1
srq=srq+sd(1, k)/eta(k)
407 continue
sdq=(srq+str)/srq
19 format(i4)
20 format(i4)
21 format(f8.3)
22 format(f8.3)
23 format(i4)
24 format(i4)

```

```

c 25      format(i4)
26      format(i4)
27      format(f8.3)
28      format(i4)
29      format(f8.3)
c
30      format(9f8.5)
31      format(9f8.5)
32      format(9f8.5)
33      format(9f8.5)
34      format(9f8.5)
35      format(9e8.2)
36      format(9e8.2)
37      format(9f8.5)
38      format(9f8.5)
c -----
c      output of nuclear and geometrical data of the assembly
c -----
c
write(6,551)
write(6,552)
write(6,553)
write(6,554)
write(6,555)
write(6,556)
write(6,557)
write(6,558)
write(6,559)
write(6,660)
write(6,661)
write(6,662)
write(6,663)
write(6,664)
write(6,665)
c
if(ig.eq.0) write(6,771)
if(ig.eq.1) write(6,772)
if(ig.eq.2) write(6,773)
c
write(6,774)
c
write(6,775) rf
write(6,776) ng
write(6,777) nl
write(6,778) kl
c write(6,779) erl
write(6,800)
c
write(6,801)
write(6,802) (ml(i),i=1,nl)
write(6,803) ((sigf(kg,l),l=1,nl),kg=1,ng)
write(6,804) ((siga(kg,l),l=1,nl),kg=1,ng)
write(6,805) ((sigs(ng,kg,l),l=1,nl),kg=1,(ng-1))

```

```

C      write(6,806) ((d(kg,1),l=1,nl),kg=1,ng)
      write(6,807) ((b(kg,1),l=1,nl),kg=1,ng)
      write(6,808) ((scf(kg,1),l=1,nl),kg=1,ng)
      write(6,8099) ((q(kg,1),l=1,nl),kg=1,ng)
      write(6,810) ((fn(kg,1),l=1,nl),kg=1,ng)
      write(6,811) ((fq(kg,1),l=1,nl),kg=1,ng)
      write(6,812)
C
C      -----
C      results of calulation of keff and neutron fluxes
C      -----
C
      write(6,813)
      write(6,814)
C
551  format(' *****
552  format(' *
553  format(' *
554  format(' *
555  format(' *
556  format(' *
557  format(' *
558  format(' *
559  format(' *
660  format(' *
661  format(' *
662  format(' *
663  format(' *
664  format(' *
665  format(' *****
C
771  format(//,          Slab          Geometry          ' //)
772  format(//,          Cylindrical Geometry          ' //)
773  format(//,          Spherical   Geometry          ' //)
C
774  format(' -----
C
775  format(5x,'Radius of assembly(cm)          ',f8.3)
776  format(5x,'Total number of energy groups          ',i4)
7771  format(5x,'Total number of material zones          ',i4)
7781  format(5x,'Number of finite difference points          ',i4)
C7791  format(5x,'Convergence criterion allowed for keff. ',f10.5)
C
800  format(' -----
C
801  format(//,5x,'          Nuclear Data          ' //)
802  format(5x,'l:',5i10)
803  format(5x,'sigf:',9f8.5)
804  format(5x,'siga:',9f8.5)
805  format(5x,'sigs:',9f8.5)
806  format(5x,'d:',9f8.5)
807  format(5x,'b:',9f8.5)
808  format(5x,'scf:',9e8.2)
8099  format(5x,'q:',9e8.2)
810  format(5x,'fn:',9f8.5)

```

```

c 811 format(5x,'fq: ',9f8.5)
812 format(5x,'-----')
c
813 format(/,5x,' i          d          x          m          keff',
814 format(5x,' --          --          --          --          ',
c
c -----
c calculated values of subcrit. mult., m , keff & neutron fluxes
c -----
      do 817 i=2,(nmax-1)
      write(6,818) i,dd(i),gam(i),scm(i),ak(i)
818 format(6x,13,8x,5e10.3)
817 continue
c
      do 819 kg=1,ng
c
      if(kg.eq.1) write(6,820)
820 format(/,15x,'Mat. Zone      Radius      Fast Neutron Fluxes',/)
      write(6,821)
821 format(5x,' -----      -----      -----')
c
      if(kg.eq.2) write(6,822)
822 format(/,5x,'Mat. Zone      Radius      Thermal Neutron Fluxes
      write(6,823)
823 format(6x,' -----      -----      -----')
c
      do 824 k=1,kl
      l=(k+noptr-1)/noptr
      write(6,825) l,r(k),phi(kg,k),f1(kg,k)
825 format(10x,13,10x,f7.3,15x,2e15.5)
824 continue
819 continue
c
      close(1)
      close(6)
c
      end
c
c -----
c subroutine Para determines parameters for the
c evaluation of coefficients for the matrix A
c -----
c
c subroutine para(noptr,nl,kl,ig)
c
c common /mk/ hr(10,60)
c common /rr/ r(100)
c common /tt/ tau(100),beta(100),rho(100)
c common /ue/ u(100)
c common /et/ eta(100)
c
c
c
      kl1=kl+1

```

```

k=1
r32=0.5*(r(k+1)+r(k))
eta(k)=(r32** (ig+1))-(r(k)** (ig+1))/(ig+1)

k=2
do 14 l=1,n1

do 15 m=1,noptr

tau(k)=(0.5*(r(k)+r(k-1))**ig)/hr(l,m)
rho(k)=(0.5*(r(k+1)+r(k))**ig)/hr(l,m)
u(k)=(r(k)** (ig+1))-(0.5*(r(k)+r(k-1))** (ig+1))/(ig+1)
beta(k)=(0.5*(r(k+1)+r(k))** (ig+1))-(r(k)** (ig+1))
* / (ig+1)
eta(k)=u(k)+beta(k)

k=k+1
m=m+1
15 continue
14 continue

rkl=0.5*(r(kl)+r(kl-1))
eta(kl)=(r(kl)** (ig+1))-(rkl** (ig+1))/(ig+1)
eta(kl)=eta(kl-1)

return
end

-----
subroutine Coeff calculates the coefficients for tridiagonal
matrix A for point k=2,... (kl-1)
-----

subroutine coeff(j,kz,kg,l,kl,d,sigt,a)

dimension a(2,100,100)
dimension sigt(2,10),d(2,10)

common /aa/ a(2,100,100)
common /tt/ tau(100),beta(100),rho(100)
common /ue/ u(100)
common /et/ eta(100)

do 1 k=j,kz
do 2 m=1,kl

mk=m-k

lower coefficients,m=k-1

if(mk.eq.-1) then
a(kg,k,m)=-d(kg,l)*tau(k)

```

```

c0      upper coefficients m=k+1
c
c      else if (mk.eq.1) then
c        a(kg,k,m)=-d(kg,1)*rho(k)
c
c      diagonal coefficients
c
c      else if (mk.eq.0) then
c        a(kg,k,m)=+d(kg,1)*tau(k)+(d(kg,1)*rho(k))
*      +(eta(k)*sig(kg,1))
c
c      for any other coefficients
c
c      else
c        a(kg,k,m)=0.
c
c      end if
c
c      2      continue
c      noptr=7
c      if (l.eq.1) a(kg,k,m)=a(kg,noptr,m)
1      continue
c
c      return
c      end
c
c      -----
c      subroutine Sump determines the summation of fission sources
c      and isotopes. It also calculates points for integration by
c      Simpson's Rule
c      -----
c      subroutine sump(i,j,kz,l,ng,kl,ig,pi,scf,phi,dv)
c
c      dimension dv(100),phi(2,100),scf(2,10)
c
c      common /rr/ r(100)
c      common /fq/ sigf(2,10),q(2,10)
c      common /ff/ fn(2,10),fq(2,10)
c      common /xy/ x(100),y(100),w(100)
c      common /sq/ s(100),se(100),qq(100),qj(100)
c      common /et/ eta(100)
c      common /gt/ sr(2,100),sm(2,100),sd(2,100)
c      common /fr/ sdx(2,100),sdm(2,100)
c      common /dw/ ak(50)
c
c      do 22 k=j,kz
c        if (ig.eq.0) dv(k)=1.
c        if (ig.eq.1) dv(k)=2.*pi*r(k)
c        if (ig.eq.2) dv(k)=4.*pi*(r(k)**2)
22      continue
c
c      initialise values to zero
c      do 222 k=j,kz

```

```

cc      sb=0.
        ss=0.
        sq=0.
        sw=0.
        sz=0.

c
do 66 kg=1,ng
c
    if(i.gt.1)then
    sb=sb+(fn(kg,1)*sigf(kg,1)*phi(kg,k))
    else
    sb=sb+(fn(kg,1)*sigf(kg,1)*phi(kg,k))
    end if
    ss=ss+(fq(kg,1)*q(kg,1))
c    write(6,*)kg,1,k,phi(kg,k)
    sq=sq+(((sigf(kg,1)*phi(kg,k))/scf(kg,1)))
    if(i.gt.1)then
    sw=sw+((fn(kg,1)*sigf(kg,1)*phi(kg,k)))
    else
    sw=sw+((fn(kg,1)*sigf(kg,1)*phi(kg,k)))
    end if
    sz=sz+(fq(kg,1)*q(kg,1))
c
66      continue
c
    x(k)=sb*dv(k)
    y(k)=ss*dv(k)
    w(k)=sq*dv(k)
    s(k)=sw*eta(k)
    se(k)=sw
    qq(k)=sz

c
sm(1,k)=s(k)

c
c    the case of infinite line source(ii>0)
c
yy=2.405
aj1=(yy/2.)-(yy**3)/16.+(yy**5)/384.
bb=(pi*(r(kl)**2)*2.*r(kl)*(aj1**2))

c
xx=((2.405*r(k))/r(kl))
aj0=(1.-((xx**2)/4.)+(xx**4)/64.)-(xx**6)/2304.)

c
sd(1,k)=eta(k)*((qq(k)*aj0)/bb)

    if(sd(1,k).lt.0.)sd(1,k)=0.0

c
sr(1,k)=sd(1,k)+sm(1,k)

c
222    continue
c
    return
    end

```

```

-----
subroutine DUMP calculates only point source problem(jj>0)
-----
subroutine dump(j,kz,l,ng,ig,pi,scf,phi,dv)

dimension dv(100),phi(2,100),scf(2,10)

common /rr/ r(100)
common /fq/ sigf(2,10),q(2,10)
common /ff/ fn(2,10),fq(2,10)
common /xy/ x(100),y(100),w(100)
common /sq/ s(100),se(100),qq(100),qj(100)
common /et/ eta(100)
common /gt/ sr(2,100),sm(2,100),sd(2,100)

do 22 k=j,kz

  if(ig.eq.0) dv(k)=1.
  if(ig.eq.1) dv(k)=2.*pi*r(k)
  if(ig.eq.2) dv(k)=4.*pi*(r(k)**2)
22 continue

do 222 k=j,kz

  sb=0.
  ss=0.
  sq=0.
  sw=0.
  sz=0.

  do 66 kg=1,ng
    sb=sb+(fn(kg,1)*sigf(kg,1)*phi(kg,k))
    ss=ss+(fq(kg,1)*q(kg,1))
    sq=sq+(((sigf(kg,1)*phi(kg,k))/scf(kg,1)))
    sw=sw+((fn(kg,1)*sigf(kg,1)*phi(kg,k)))
    sz=sz+(fq(kg,1)*q(kg,1))
66 continue

  x(k)=sb*dv(k)
  y(k)=ss*dv(k)
  w(k)=sq*dv(k)
  s(k)=sw*eta(k)
  se(k)=sw
  qq(k)=eta(k)*sz
  sm(1,k)=s(k)

  if(k.eq.j) then
    sd(1,k)=qq(j)
  else
    sd(1,k)=0.
  end if

  write(6,*)l,k,sd(1,k)

```

```

c      sm(1,k)=s(k)
c
c      zz=2.405
c      aj2=(zz/2.)-(zz**3)/16.+(zz**5)/384.
c      cc=(pi*(r(k1)**2)*(aj2**2))
c      vv=((2.405*r(k))/r(k1))
c      aj3=(1.-((vv**2)/4.)+(vv**4)/64.)-(vv**6)/2304.)
c
c      if(r(k).lt.r(k1)) then
c        sdx(1,k)=((qj(k)*aj3)/cc)
c      else
c        sdx(1,k)=0.
c      end if
c
c      if((1.gt.1).and.(k.eq.j)) then
c        sdm(1,k)=qq(j)
c      else
c        sdm(1,k)=0.
c      end if
c
c      sr(1,k)=sdx(1,k)+sdm(1,k)+sm(1,k)
222  continue
c
c      return
c      end
c
c      -----
c      subroutine Simpr subprogram for Simpson's Rule
c      -----
c      subroutine simpr(f,ndim,nstep,h,sint)
c
c      dimension f(ndim+1)
c
c      apply Simpson's rule
c
c      sint=0.
c
c      do 2 j=2,nstep,2
2      sint=sint+f(j-1)+4.*f(j)+f(j+1)
c      sint=sint*h/3.
c
c      return
c      end
c      -----
c      subroutine Pmatrx calculates the product of matrices
c      -----
c      subroutine pmatrx(g,xn,ni,gx)
c
c      dimension g(100,100),xn(100),gx(100)
c
c      multiplication of matrices
c
c      do 2 i=1,ni
c      gg=0.

```

```

c      do 1 j=1,ni
c      gg=gg+g(i,j)*xn(j)
1      continue
c      gx(i)=gg
2      continue
c
c      return
c
c      end
c
c      -----
c      subroutine Gsiter for solving simultaneous linear equations
c      using Gauss-Seidel Method
c      -----
c
c      subroutine gsiter(a,bb,xx,kg,meqn,orelax,nc)
c
c      dimension bb(2,100),xx(2,100)
c
c      dimension a(2,100,100)
c
c      neqn=meqn
c
c      set up iteration loop
c
c      do 5 iter=1,nc
c
c      sumx=0.
c      sumdx=0.
c
c      obtain new estimate for each unknown in turn
c
c      do 3 i=1,neqn
c      deltax=bb(kg,i)
c      do 2 j=1,neqn
2      deltax=deltax-a(kg,i,j)*xx(kg,j)
c      write(*,*)kg,i,a(kg,i,i)
c      deltax=deltax/a(kg,i,i)
c
c      sumdx=sumdx+abs(deltax)
c      xx(kg,i)=xx(kg,i)+deltax*orelax
3      sumx=sumx+abs(xx(kg,i))
c      return
5      continue
c
c      return
c      end

```