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AN INVESTIGATION INTO PUPILS' EXPERIENCES OF SOLVING NON-ROUTINE MATHEMATICS PROBLEMS: A CASE STUDY OF GHANA INTERNATIONAL SCHOOL

BY

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THIS THESIS IS SUBMITTED TO THE UNIVERSITY OF GHANA, LEGON, IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE AWARD OF MPHIL IN EDUCATION DEGREE

FEBRUARY 2020

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DECLARATION

I do hereby declare that this work is the result of my own research and has not been presented by anyone for any academic award in this or any other university. All references used in the work have been fully acknowledged.

I bear sole responsibility for any shortcomings.

…… ……

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CERTIFICATION

I hereby certify that this thesis was supervised in accordance with procedures laid down by the University.

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DEDICATION

I dedicate this work to the Lord through whom I am continuously endowed with wisdom.

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LIST OF ACRONYMS

TIMMS	Trends in International Mathematics and Science Study
BECE	Basic Education Certificate Examination
WASSCE	West Africa Senior School Certificate Examination
COAG	Council of Australian Governments
NNO	National Numeracy Organisation
NEA	National Education Association
NCTM	The National Council of Teachers of Mathematics
OECD	The Organisation of Economic Co-operation and Development
AMP	African Mathematics Programme
DE	Department of Education
IMU	International Mathematical Union
EGRA	Early Grade Reading Assessment

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ABSTRACT

The main purpose of the study was to investigate pupils' experiences when working on non-routine mathematics problems. This was deemed imperative to discover learners' reactions to problems of this kind, investigate their competencies and skills necessary for successfully working out non-routine exercises and also delving deeper into how pupils felt whenever they were tasked to work on complex type problems. Again, the study sought to unearth how pupils were benefitting from useful classroom practices such as collaborative and co-operative learning, as far as the solving of non-routine problems was concerned. The research was carried out at Ghana International School (G.I.S) where a total number of 144 pupils, from Year Four to Year Six, were selected using the simple random sampling technique. With respect to the research design, the sequential mixed method was adopted where emphasis dwelt on the use of assessments and interviews as the main instruments for data collection. The data gathered was analysed statically using simple frequency tables and the responses from the participants were also transcribed into thematic areas. The results from the study revealed that most pupils exhibited inappropriate comprehension of problem solving and computational processes including inaccurate choices of strategies. Again, learners grappled with incompetence at thinking mathematically and lacked the much needed creative knowledge of how to work purposefully on non-routine problems. Some of the recommendations included teachers benefitting from more in service training, pupils making more progress through collaborative and co-operative learning and educational authorities creating more opportunities for teachers to build on their competence through more Professional Development packages in the area of mathematics.

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CHAPTER ONE

INTRODUCTION

1.1 Background to the Study

The significance of mathematics education cannot be over emphasised. From my years of experience as a primary school teacher, there is clear attestation that the efficient delivery of the curriculum seeks to equip learners with an array of benefits. In my opinion, the enhancement of pupils' reasoning abilities, critical thinking skills and competence at problem solving are few of the several existing advantages associated with the in-depth study of Mathematics. In fact, Legner (2013) upholds the view that the basic arithmetic required for many jobs will be largely acquired by the end of primary school thus laying more emphasis on the essence of its fundamental teaching and learning. Based on this opinion and the research of other scholars (Kavkler, Magajna and Babuder, 2014) who have, over time, unravelled the benefits of the subject, it is prudent to say that offering young learners a meaningfully concrete mathematical knowledge remains paramount and must be the duty of all educators within this field.

Across the globe today, it is therefore not surprising that governments' commitment to effective teaching and learning of mathematics continue to initiate several reforms within the area. Kilpatrick (1997) believes that such reform impulse especially within the United States has, over the past years, heated up so much and been led by professional organisations under the banner of raising expectations and providing mathematical literacy for all. Such growth in reforms is certainly prevalent in other developed countries; Australia's reform for Mathematics Year 1-10, which was implemented in 2007, was an example of such excellent initiatives which aimed at representing a significant shift in the teaching of the subject from the previous syllables

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(Department of Education, 1987). Bruniges (2005) believes that such rapid reforms, apart from development in teaching and learning, characterise a feature of education due to several context related developments including globalisation, new technology, knowledge of the economic and cultural diversity.

Perhaps these factors account for a number of laudable strides which have, over time, also seeped into Africa. For example, the African Mathematics Project (AMP) pursued a policy of bringing together African American and British educators in English speaking African countries to influence Mathematics education (Mereku, 1999). South Africa, for instance, has also had a taste of several radical curriculum change particularly due to the need for social, economic and political transformation; and in countries like Malawi, Ethiopia, Ghana and Nigeria, there have been other advances within the area of Mathematics (Parker, 2006; IMU, 2014). Evidently, such transformational developments remain positive leaps particularly towards the enhancement of Mathematics Education and the good news according to White, Bloomfield and Cornu (2010) is that, these contexts related developments promise opportunities for change in primary mathematics education, thus shaping the curriculum reform agenda of 'productivity, participation and quality.' Highlighting further on the importance of the subject, they also comment that the proper grounding of one's knowledge in the area of Mathematics education comes with great benefits for the future. For instance, an excellent achievement in the subject often prepares the individual adequately for one's contribution to economic productivity. The need for such global reform is therefore in the right direction.

Despite these global strides, however, pupils' performance in Mathematics continues to be worrying. In a recent 2015 publication from Trends in International Mathematics and Science Study (TIMMS) the 2016 report disclosed that though countries like Russia, Kazakhstan and

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Northern Ireland saw some improvements in the performance of students, there still appeared to be significant gaps between high performing countries and others. A good example was the gap between the United States and Singapore, the highest performer in the world (Simms, Gilmore, Sloan and McKeaveney, 2017). This was worrying considering the fact that America, for example, continues to spend more than most countries in preparing students for such external tests yet the country recurrently attains poor results from students, with 12 percent performing below the expected levels in the subject (OECD, 2012). The situation is similar in the United Kingdom where a large percentage of children fail to meet the expected levels in mathematics by the end of primary school (DE, 2015). It is also worth noting that just like the United States, the UK government also incurs a loss to poor numeracy skills which is estimated at 20.2 billion a year (Simms, Gilmore, Sloan and McKeaveney, 2017).

International Mathematics tests richly focus on non-routine problems (Marchis, 2012); this category of Mathematics is what Liljedah (2008) defines concisely: 'Tasks that cannot be solved by direct effort and will require some creative insight to arrive at an answer.' Lester and Kehle (2003) therefore suggest that reasoning or higher order thinking must occur during such process. Oviedo (2005) admonishes that in this peculiar task, one should be able to read, interpret and transform the stated words within their contexts into a symbolic form, before embarking on a search for manipulative or computational strategies. Unfortunately, pupils lack such skills to work capably on non-routine exercises thus aggravating cases of mathematical deficiencies among learners today. Dowker (2009) believes that such low attainment in mathematics have negative effects which are not only restricted to a small percentage of learners identified with problems such as Dyscalculia but goes beyond to include a wider proportion of children who also fail to attain the requisite mathematical skills they need to grasp for everyday life. These poor numeracy

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skills, often detected at an early stage, have over years contributed to a 'viscous cycle of disadvantage and a poverty of opportunity' (Northern Ireland Audit Office, 2013). These revelations are indeed disturbing and stress the fact that it is high time educators adequately supported pupils to become confident and capable 'problem-solvers' when working on non-routine exercises.

If global scores in International mathematics tests, which are mostly non-routine problems, are not too pleasing, then there is obviously a clear indication that pupils struggle particularly in this area. Romanian pupils, for instance, have high scores, above international average on routine problems but they obtain lower scores than the average on non- routine problems (Marchis, 2012). The poor attainment in International Mathematics Assessments are perhaps even more worrying in Africa where the lukewarm performance of pupils raises questions about the effectiveness of the periodical curriculum and educational reforms in most African countries (Ndlovu and Mji, 2012). Even in countries where English is considered the official language, pupils still do not have the luxury of excelling when presented with mathematical problems in the language they know best. In the English-speaking country of Kenya, for example, where pupils and students are assessed in the official language, mathematics education has faced various challenges which have incredibly influenced poor performance in national examinations (Ndlovu and Mji, 2012). This gives an indication that to be competent especially at non-routine problems and the effective use of problem solving skills, the need to possess significant mathematical competence is necessary other than language.

Griffin and Jitendra (2009) unearth a number of these skills as one's knowledge of sentence structure, mathematical relations, basic numerical skills and mathematical strategies to solve a number of reasoning type mathematical problems. In fact, critics like Njagi (2015) believe that

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many language challenges have contributed to dismal performance such as poor mastery of specialised mathematical symbols, use of terms that have different meanings, linguistic barrier that leads to poor communication in classrooms and language challenges in solving non-routine problems. Highlighting more on the latter factor, a report by COAG (2008) acknowledged that language can provide a formidable barrier to both the understanding of mathematical concepts and the provision of students' access to assessment items aimed at eliciting mathematical understanding. Judging from this statement, learners' challenges at non-routine problems which Milgram (2007) explains as problems one has no straightforward solution to, could be linked to language deficiency and no wonder Moeller, Klein and Nuerk (2011) state that linguistic complexity contributes to difficulty for learners and this has over the years, contributed to pupils' inability to work competently on such problems.

Pupils' general poor performance on such problems have compelled experts to carry out extensive works within the area. However, most of these significant researches have been set within Western contexts thus signaling the fact that a lot more needs to be done in Africa. In Ghana, for instance, mathematics performance is poor among learners in both Primary and Junior High School, particularly due to learners' problem solving inabilities (Mereku and Anamuah-Mensah, 2005; Adu, Acquaye, Buckle and Quansah,2007) and, obviously, the situation needs redress.

1.2 Statement of the Problem

The importance of mathematical literacy in this technological age is recognised universally, for it is a tool for developing a rational personality (Kavkler, Magajna and Babuder, 2014). Mereku (1992) believes that the subject occupies a privileged position in the school curriculum because the ability to cope with more of it improves one's chances of social development. Undoubtedly, Mathematics education is of essence and it is not surprising that in our modern society, there are

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growing needs for mathematical skills and proficiency because students must master advanced skills to stay on track for promising careers (Njagi, 2015). The desire among educators to see pupils becoming mathematically able is therefore of immense relevance though this quest remains far-fetched.

In Ghana where this study is set, pupils' attainment in mathematics is generally low (Mereku, 2003) and the area of non-routine problems is not an exception. Evidences of such poor performances are enshrined in various reports focussing on outcomes from national assessments (NEA, EGRA & EGMA), national examinations (BECE & WASSCE) including international examinations (Mereku, 2012). Debilitating standards and records of worrying grades attained by some BECE candidates, for instance, indicate that most students underperform in the subject area; a challenge mainly caused by lack of basic concepts (Mills & Mereku, 2016). For a developing nation like Ghana which requires citizens well-grounded in numerical skills to contribute towards economic productivity, such challenges raise thought provoking questions: Are we, indeed, churning out students fit to be considered nation builders? If students continue to show such significant deficiencies, how ready is the next generation's preparedness to cope with everyday life which undoubtedly requires a good knowledge of mathematics?

Probing further into the country's state of affairs as far as Mathematics education is concerned, it is worth commenting on the findings of a recent study carried out by Mills and Mereku (2016). This research was to investigate whether JHS 2 students within the Efutu municipality will attain the national minimum set by the end of basic education and the revelation was quite disappointing; 8 out of the total 19 content standards (i.e. 42 percent) were challenging for the participants. Again, it was disclosed that the overall number of students who attained the proficiency mean score of at least 65 percent was 30 percent of the pupils while another 10 percent of them fell below the

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minimum competency level, signalling the fact that these students, who were just left with one year before writing their BECE papers, were operating at minimum competency levels as far as mathematics was concerned. This is indeed a reflection of what is happening in most Ghanaian schools, emphasising the point that students' handicap in the area of mathematics has become a menace continuously hindering learners from becoming able mathematical problem solvers.

Ghana's first participation in the 2003 Trends in International Mathematics and Science Study officially announced some of these struggles being faced to the world. Clutching the 45th position out of the 46 countries which participated, it was an undeniable fact that mathematics was indeed a problem for most of the Ghanaian students assessed (Anamuah-Mensah,Mereku and Asabere-Ameyaw, 2004). No wonder due to such low results, other African countries outweighed the nation in terms of performance (Anamuah-Mensah et al, 2004). These poor results, obviously, require immediate redress for according to Onivehu and Ziggah (2004), no nation can enjoy technological advancement without an effective mathematics education programme since the subject plays a key role in all areas of human activity. Going by this statement, it is a fact that Ghana is behind as far as mathematics education is concerned, for instead of churning out students who are mathematically knowledgeable, critical and creative, we, unfortunately, end up with masses of learners absolutely crippled in the subject area.

Across developed countries, the curricular for mathematics undergo regular review aimed at making the process of teaching and learning more effective. Again, through such frequent amendments, more opportunities are provided to equip learners with adequate skills such as deepening their knowledge of problem solving, enabling them to be more creative at non-routine work and guiding them to work encouragingly when confronted with difficult tasks while showing sound knowledge of methodologies appropriate for a particular type of problem. In Ghana,

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however, the aggravating problem of underperformance is also attributed to the curriculum which lays more emphasis on number work, knowledge of facts and procedures (Anamuah-Mensah and Mereku, 2005). This, unfortunately, denies pupils from grasping the required fundamental skills needed to work innovatively on non-routine type of problems, thus preventing them from broadening their scope on these kinds of problems.

Over the years, a number of concerned scholars, well vexed in the subject area, have carried out extensive researchers to find some solutions to the current predicament. For example, Mereku and Cofie (2008) have looked at how overcoming language difficulties can help minimise mathematical difficulties among students. Anamuah-Mensah and Mereku (2005) also analysed the report of 2003 TIMMS report, highlighting some struggles and challenges educators have to pay attention to if learners have to do better on non-routine problems. Atteh, Andam,Obeng-Dneteh, Okpoti and Johnson (2014) also looked at how the constructivist approach could enhance students' competence in problem solving. Nyala, Assuah, Ayebo and Tse (2016) have also looked at the prevalent rate of problem solving approach in teaching mathematics in Ghanaian basic schools. Armah (2015) delved into problem solving from the perspective of teachers' beliefs, intentions and behaviour. Finally, Atteh, Andam and Obeng-Denteh (2017) in their contribution towards the enhancement of problem solving approach looked specifically at a four step framework to foster students' progress.However, the experiences of learners themselves when working on non-routine problems have not been given much attention in Ghana. Furthermore, with majority of these studies set within the public domain, most of these researches often aim at confronting teaching and learning difficulties in the government schools. It is in light of this that the researcher aims to expand studies in the area of students' experiences in solving non-routine mathematics problems but this time, with focus on a privately owned school.

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1.3. Objective of the Study

- This research is intended to unravel the realities in connection with pupils' difficulty when solving non-routine problems.
- To unearth pupils' experiences of solving such complex problems.
- Again, it is to identify the shortfall or lapse within the learning process which often handicaps learners from working competently on non- routine exercises.
- Finally, the study also aims at unravelling reasons why pupils work better on routine mathematical problems as opposed to questions testing their conceptual understanding of topics.

1.4. Research Questions

This research aims at addressing a key question: What are learners' experiences towards the solving of non-routine mathematical problems?

Other related research questions include:

1. What characterises pupils' experiences of solving non-routine mathematical problems?
2. What explains, if any, the differences between pupils understanding of routine and non-routine mathematics problems?
3. What opportunities are pupils given to engage in collaborative and co-operative learning?
4. What challenges do pupils encounter when conceptualising their understanding of non-routine mathematical problems?

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1.5 Significance of the Study

The significance of the study cannot be over emphasised. Among several of its importance will be its benefit to policy makers and authorities within the domain of education who will fall on useful data and information to improve on areas which require redress as far as the teaching and learning of non-routine mathematics problems are concerned. For example, this research will propel them to structure regular in service training for teachers, develop mathematics manuals to focus on effective teaching techniques and organise mathematics conferences where priority will be given to sharing of best practices among teachers of mathematics. With respect to its significance for teachers, the research seeks to equip them with more insight into effective problem solving teaching approaches to foster learners' progress, offer them insight into strategies that can be adopted to build pupils' confidence and knowledge when working on non-routine problems and enable them to become conversant with key methodologies they could adopt or consider when delivering the curriculum to make the process of Teaching and Learning worthwhile. Finally, it will add to existing literature creating more opportunities for individuals, particularly in the field of mathematics education, to develop sound knowledge and understanding of how pupils work on non-routine problems and measures to support them adequately in the learning of mathematics.

1.6 Organisation of the Study

Chapter One examines the background study, statement of the problem, objectives of the study and research questions. The organisation of the study and its significance will also be touched on. The emphasis in Chapter two will dwell on adequate review of related literature highlighting on non- routine mathematical problems as the thematic focus. In addition to this, the theoretical frame work will also fall under this section. Chapter three will give attention to the research methodology

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I intend to use for this study. This will cover the research design, study population, sample and sampling procedures, sources of data, data collection instrument, field study and data processing and analysis.

Chapter four will focus on the analysis and interpretation of data. It will also highlight the main findings of the study and the experiences of pupils when working on non-routine exercises. Chapter five, the final chapter, will summarise the main findings, draw conclusions from the findings and end with some recommendations.

1.7 Chapter Summary

Chapter one examined the background study, the statement of the problem, the objective of the study, research questions, the significance of the study including the organisation. In the next chapter, adequate literature focusing on relevant thematic areas will be reviewed including the theoretical frame work.

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CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.1 Introduction

Every research is underpinned by extensive literature including insightful theories. In view of this, the chapter will review relevant literature related to the study variables and the relationship between them. In this study, the constructs examined are pupils' experiences towards non-routine problems. Areas that will therefore be given much attention will include an observation into pupils' perceptions of non-routine problems, an observation into learners' attitudes and competence at problem solving and an investigation into their reasoning and logical capabilities. In all, 3 categories will be highlighted on; these will focus on definition of concepts, a review of empirical literature and the proposed conceptual framework. The definition of concepts will throw more light on what non routine problems and problem solving skills entail while the empirical section will also review appropriate literature bothering on the thematic areas. Finally, the theoretical frame work will delve deeper into some theories relevant to the study.

2. 2 Definition of Non-Routine Mathematics Problems

The definition of non-routine problems continues to be tagged with several variations highlighting the fact that as at yet, scholars continue to find difficulties in coming up with a common explanation to mathematical problems of this nature (Mamona-Downs and Downs, 2005). Milgram (2007), for example, believes that non- routine problems differ from an exercise in that the problem solver does not have an algorithm that, when applied, will certainly lead to a solution. Certainly, this suggests that particularly for complex tasks of this kind, it is expected that

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individuals embed their own originality and creativity when working out problems. This is quite similar to the explanation provided in the TIMMS 2011 framework where non-routine problems in general are termed unfamiliar exercises or tasks to the student who is expected to solve the task (Mullis, Martin, Ruddock, O'Sullivan and Preuschoff, 2009). They make cognitive demands over and above those needed for solution of routine problems, even when the knowledge and skills required for their solution have been learned' (Mullis et al., 2009, p. 45).

Another interesting definition is what has been provided by Gilfeather and Regato (1999); these two looked at non-routine problems as heuristics or procedures where solutions do not necessarily lead to answers but on the other hand provide clues which could lead to new discoveries. Their definition provides more insight into non-routine problems as tasks which have no clear or prescribed methods leading to a solution. This gives a clear indication that these kind of problems are always shrouded in uniqueness where learners often have no straight forward paths to obtain answers. Additionally, the latter definition brings out the fact that non-routine problems often engage students' in the area of critical thinking, for they are compelled to make attempts at developing meaningful strategies to obtain solutions to problems. If this is the case, then pupils must have a better understanding of non-routine tasks as problems where answers to questions can vary. For example, while all answers are expected to be the same, students may provide procedures or methods that could vary and therefore, it is worth noting that some solutions will be more complicated than others. Adding further to previous explanations, some problems are regarded as non-routine simply based on the audience. An example of this follows: One morning, a farmer looks out in the barnyard and sees a number of pigs and chickens. The farmer says to her daughter, "I count 24 heads and 80 feet. How many pigs and how many chickens are out there?" (Bassarear, 2007, p. 7).

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The latter's explicit example makes it clear that regarding non-routine exercises, a particular option, strategy or method cannot be adopted as a fixed approach to arrive at a solution. Embedding my own example in here, we can conclude that tasking Year 3's (pupils aged approximately 8 years) to add up single digit numbers may not be too much of a challenge for most of them. For example, a greater number of them could impressively find answers to questions such as the following: 8+4; 6+4; 7+2 etc. It is, however, likely that they may not show the same competence at addition problems tweaked in a non-routine form which may require some level of critical thinking. For these children, questions such as (A) _____ + 4 = 15 or (B) _____ + _____= 29 may seem somehow challenging. With particular reference to Question A, they may not be able to understand the inverse relationship between addition and subtraction to be able to solve this problem accurately. With respect to Question B, their knowledge of number concepts may be limited thus hindering their ability to work on this problem.

Similarly, Year 6's (pupils aged approximately 11 years) may find the identification of 2D and 3D shapes manageable if they have been well taught. In view of this, they may be able to recognise shapes such as cones, pyramids, triangles, cuboids etc. and possibly be able to link these shapes to everyday objects around them. However, providing them with riddle-like puzzles may compel some pupils to throw in the towel. A typical example could be this: '*I am a shape with three rectangles. My top and bottom are triangular bases, and I have a total of five faces. Who am I*? For some pupils, linking these clues to the physical look of a particular shape may be a struggle thus rendering them unable to identify this shape as a Triangular Prism. At both levels of learning, the latter questions require pupils to engage in some level of thinking in order to deduce right answers; and it must be stated that for pupils to work accurately on such examples of non-routine

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problems they must be adequately taught, in the first instance, how to recognise complex type problems and their requirements.

Learners ought to have a better understanding of non-routine problems in order to enhance their competence at solving such peculiar problems; having an in-depth knowledge of its definition is paramount if pupils are expected to develop sound knowledge of how to go about such problems.

2.3 The Characteristics of Non Routine Mathematics Problems

No matter the above scholarly variations in the definition of non-routine problems, one thing remains a fact; non-routine problems require learners to delve deeper into the realm of logical reasoning. Based on this, Lester and Kehle (2003) reiterate that reasoning and higher order thinking are inevitable processes when solving questions which are non-routine in nature. This suggests that for pupils to work competently on such problems, they must be conversant with the peculiar characteristics of non-routine problems, for this will enable them to easily detect complex type questions from routine types and respond to them more appropriately.

A dominant characteristic of non-routine problems, for example, focuses on the challenge they offer learners to think along logical paths. Again, Lester and Kehle (2003) admonish that processes such as deep thinking processes are unavoidable when working on such exercises. This suggests that for pupils to work excellently on these problems, they must be absolutely prepared to engage in in-depth critical thinking processes that often lead to solutions. To enhance this process effectively, there is also the need for learners to think outside the box using applied knowledge, thus compelling pupils to conceptualise what has been taught earlier.

Another characteristic of non-routine problems centers on the fact that they offer learners opportunities to us an array of strategies during the problem solving process. This brings in the

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idea of diversity when working on non-routine problems and it is, therefore, no wonder that Baroody (2003) believes that this flexibility is what enables one to know that what has been learnt in one situation and what has been applied to one problem will not necessarily fit another situation or be appropriate for another problem. Digging deeper into this characteristic, Demetriou (2004) lays further emphasis that when it comes to these exercises, the use of flexibility cannot be ignored as it appears to be strongly interconnected with problem solving and performance. Similarly, Civil (2007) and Gustein (2006) hold similar views and reiterate the fact that teachers can give more focus to non-routine tasks via problem solving approaches by engaging students in understanding mathematics more deeply and personally by connecting to, including and building on students' community mathematics knowledge. This may mean spending longer time to teach them the skill of the solving process itself and taking students through several exercises which will provide them more opportunities to practice non-routine questions.

In a Ministry of Education publication in Jamaica (2010), other key characteristics of non-routine problems were categorised as open, rich in critical thinking processes and novel. These problems were characterised as 'open' considering the fact that either the approach to be taken to solve the problem or the answer to be obtained at the end of the solution process can differ from one student to the next. Secondly, these were considered critical thinking questions based on the fact that learners are required to invent reasonable strategies to arrive at answers. Clearly, this implies that students are often put in ambiguous situations when they are presented with non-routine problems. Mathematics problems of this nature are also characterised as novel for they are not always linked to content in the same way that regular routine problems are. For example, a child may solve a non-routine problem without realising that he or she is applying one of the standard topics in the curriculum.

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O'Brien and Moss (2007) also unravel another key characteristic of non-routine problems based on their puzzle-like nature. From their perspectives, these problems come under cognitive activities that require an insightful approach to the problem situation and strategic thinking. This obviously suggests that non routine exercises entail more than a direct application of an algorithm, formula or procedure.

2.4 The Role and Attitude of the Learner when solving Non-Routine Mathematics Problems

In facilitating one's understanding of non-routine problems, the duty falls on the learner to play an effective role throughout the process. This responsibility can be carried out effectively by challenging the individual to make meaningful interpretations in order to have a better understanding of ways to solve the problem itself.

Francisco and Maher (2005) stress the fact that non-routine problem solving values the students' construction of knowledge which automatically challenges him or her to tow along paths of exploration while questioning his or her own choices of methodologies, solutions and inventions in the most reasonable and appropriate ways. The role of the learner when solving non-routine problem also focuses on another key point suggested by Lesh and Zawojewski (2007). They believe that for learners to work authentically on non-routine problems, multiple iterations of the problem must be attempted for a successful solution. This is laudable in the sense that with non-routine problems, there is often the need to use procedures in which repetition of a sequence of operation could yield the required results and enable the learner to get closer to the desired result. However, if learners need to assume this role more efficiently, Weber (2005) admonishes that it is prudent to give learners a longer period of time to try out a multiplicity of procedures some of which may call for repetition of processes. This will enable learners to work accurately within a more reasonable time frame.

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Aside giving learners adequate time to work on non-routine problems, one must bear in mind that the attitude of the learner committed to the task is also paramount and this is what has been defined as 'a disposition to respond favourably or unfavourably to an object, person, institution, or an events' (Azjen,2014,p.3). Taking Ajzen's definition into consideration, it is clear that learners with a positive attitude, commitment and zeal towards work may tend to work better at non-routine problems as opposed to learners with ill feelings towards mathematics, particularly non-routine exercises.

According to Gerrig and Zimbardo (2002), attitudes are based on beliefs and may be either conscious or unconscious. In view of this, even with more willing participants, one cannot ask for attitudes about certain subjects. Instead, one must ask questions about beliefs, which, when combined, give some information about these attitudes (Gerrig and Zimbardo, 2002).

It must be noted that attitudes affect both attention and behaviour (Delamateur and Myers, 2011). If for example, a group of pupils perceive non-routine questions as challenging, they may not fully exert themselves in a typical problem-solving class. On the other hand, if pupils show more commitment and passion for complex type exercises, they will automatically show readiness to work on such problems and be prepared to engage in more critical thinking and reasoning processes. According to both scholars, attitudes consist of three parts: "(1) beliefs or cognitions, (2) an evaluation, and (3) a behavioural disposition" ((Delamateur and Myers, 2011, p. 145).They note that frequently, the beliefs involved cannot be proven either true or false.

The authors go on to say that the evaluation consists of "a direction (positive or negative) and an intensity (ranging from very weak to very strong)" ((Delamateur and Myers, 2011, p. 145). This could be simplified to a rating, where attitudes are ranked from 1 to 9, with 1 being a strong negative feeling, 5 being neutral, and 9 being a strong positive feeling. The behavioural disposition

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includes seeking out or avoiding activities associated with that attitude. It may be important to determine not only the content of students' attitudes, but also the intensity of those attitudes since a learner with a strong attitude might be more likely to display behaviours stemming from those attitudes which could affect others (Holland, Verplanken, and Van Knippenberg, 2002).

Centingoz and Ozkal (2009) also perceive attitude as what really affects how pupils interact with the people around them and these could include friends, families, class mates and even lessons. They go further to state that in view of this, students' attitudes towards non-routine problems remain vital as this can determine one's success or failure when confronted with mathematical problems of a challenging nature. Unfortunately, most pupils do not show positive attitudes towards this aspect of mathematics and it is based on this that Effandi and Namah (2009) strongly advocate that the prevalence of negative attitudes among pupils must be overcome. This may imply that when students are taught to develop positive attitudes towards non-routine problems, underperformance in this area will certainly improve. This is because pupils will cultivate better working habits that will boost their self-esteem and commitment.

Strand and Mills (2014) are also of the belief that teachers who teach non-routine problems must first be grounded in the delivery of the curriculum due to the fact that it is their priority to build a solid foundation on which students can later construct a deeper understanding of how to go about complex problems independently. If teachers are not so confident executing this part of their duty, it cripples learners from fully exhibiting positive attitudes.

2.5. Meaning and Processes of Problem Solving

Problem solving cannot be ignored when touching on the subject of non-routine mathematics problems. This is to say that the essence of problem solving when working on non-routine

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questions cannot be over emphasised. It is therefore not surprising that this area of mathematics education has, over years, been given in depth attention compelling more researches to dig deeper and identify the processes involved in teaching students how to solve problems more accurately. Simply put, problem solving focuses on a mathematical question to which the solution is unknown (Callejo and Vila, 2009).

D'Ambrosio (2003) provides an excellent definition describing problem solving as the process that often provides the learner with an environment to work on their own while they discover accurate mathematical paths to answers. The avenue a learner takes when working out the problem can therefore be termed as the process of problem solving. In literal terms, this suggests that problem solving begins the moment learners are tasked to work on either routine or non-routine problems. It must be noted, however, that to be able to execute an effective problem solving process, there is the need for the learner to have a commendable level of mental agility for if the learner is less endowed with this skill, there will, obviously, be gross deficiency in the problem solver's ability to work accurately (D'Ambrosio, 2003). He further believes that the process of problem solving offers learners opportunities to use their existing knowledge to solve problems and believes that this process is essential in that learners are able to construct new knowledge and new understanding during the process. We can conclude from his perspective that problem solving as a process is a driving force for developing deeper understanding of mathematical ideas and processes. In view of this, when students are presented with either routine or non-routine tasks, reading the problem carefully for effective comprehension of questions is absolutely important; this may mean breaking down important information to have a better insight into the choice of appropriate strategies to adopt.

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To enable learners to be more successful during the problem solving stage, a few researchers have developed a number of effective processes deemed imperative for helping learners to tackle complex problems with more capability and independence. Against such background, Bruder and Collet (2011) believe that learning how to solve problems can be established as a long-term teaching and learning process which basically should encompass four phases. These include: intuitive familiarization with heuristic methods and techniques, making students aware of special heurisms by means of prominent examples (explicit strategy acquisition), short conscious practice phase to use the newly acquired heurisms with differentiated task difficulties and expanding the context of the strategies applied.

Similarly, Joseph (2011) emphasised on an eight step problem solving procedure. Here, he stressed on a number of key facts including: carefully reading out the questions, making note of the necessary information, looking out for the underlining goal of the problem, organizing one's plan for working out the problem and after arriving the answer, going back to recheck if the solution is right. According to him, these are necessary processes learners should follow in their thinking and problem-solving processes. Particularly for young learners, it is vital that teachers adopt some of these models and present them in child-friendly ways so that learners will have a fair idea of how to go about their work. Ang (2009) further argues that if we aim to follow modelling as a process for enabling students get better at problem solving, we should bear in mind that it must be seen as a process where one must be encouraged to work step by step on an array of exercises with the view of coming up with reasonable answers.

On the other hand, Cirillo, Pelesko, Fellon-Koestler and Rubel (2016) hold the opinion that mathematical modeling involves the construction of mathematical structures and concepts with mathematical representatives such as counting blocks, counting scales, fraction cards etc. and used

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to visualize and reduce the complexity of mathematical structures. Kaur and Dindyal (2010) are also of the view that despite these differing opinions, the process of modelling mathematical tasks unearth a distinct and common characteristic which is peculiar in all the various beliefs and that truly, mathematical modelling is associated with real life problems. It is on this judgment that Blum (2002) believes that through the act of modelling learners benefit better from mathematical concepts.

With less modelling opportunities for students to have a better knowledge of problem solving processes, it remains an undeniable fact that learners continue to face challenges when confronted with problems. McGinn and Boote (2003) in their study also stressed on four basic factors that had to do with one's perception about the difficulty of problems:

- Categorisation- recognizing the category a problem or question appropriately fits.
- Interpretation of aims – Identifying the best solution that one can use for solving a problem from start to finish.
- The importance of resources– Figuring out the relevance of resources in the problem solving process.
- Complicated nature of problem – carrying out a number of problem solving processes to derive an answer.

Mc Ginn and Boote (2003) further stated that the nature of problems in terms of its difficulty always depended on the belief of the learner. For instance, how well the problem has been placed as in its level of difficulty. For example, Singaporean studies carried out by Yeo (2009) indicate that Singaporean learners' problem-solving difficulties were categorised into the following: lack

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of understanding of questions, inability to use appropriate strategies and difficulty translating problems into mathematical states.

Francisco and Maher (2005) also looked into the learner's mathematical thinking skills when solving problems and concluded that in successfully promoting reasoning in problem solving, we have to provide students with the opportunity to work on complex tasks as opposed to simple tasks that are crucial for stimulating their mathematical reasoning. To this end, Wilburne (2006) stated that the best mathematical problems one can employ in the classroom are non-routine mathematical problems that encourage rich and meaningful mathematical discussions, those that do not exhibit any obvious solutions, and those that require the student to embed different strategies when working. Cai (2003) laid more emphasis on the fact that during the process of problem solving the invented strategies could spark the mathematical understanding of learners but was quick to point out that in helping the student to develop effective strategies when solving problems, he or she had to be properly guided.

It is also worth adding that the process of problem solving in itself has a key aim of underpinning the teaching and learning of mathematics (NCTM, 2000). This is because the process of finding solutions to problems offers learners an opportunity to engage in more detailed problem solving activities. This is why it is believed by some scholars that the process of working out problems also develops pupils' problem solving skills and strategies (Lesh and Zaworjewski, 2007). Through the use of these skills, learners are able to recognise and solve problems using their innate critical thinking and creative abilities, engage more actively in group work and come to the realization that problem solving contexts do not exist in isolation (Department of Basic Education, 2011). According to Brenner, Herman, Ho and Zimmer (1999), one of the reasons why learners from Singapore do excel in international mathematics tests is as a result of their brilliant problem

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solving skills which is a deficiency among most students in other countries. To enable more learners to actively engage in the process of problem solving, Lesh and Zaworjewski (2007) believe that every student is capable of developing the skill due to the fact that skillfulness at problem solving is highly connected with the social environment. This follows that the more problem solving is done through interactive teaching, learners get conversant with the skill itself thereby attaining higher levels of achievement (Rigelman, 2007).

2.6 Strategy Thinking Processes in Problem Solving

Strategy, according to Fülöp (2015) is the thinking aspect of organising a war, of winning a game, or of keeping a business organisation moving in a deliberately chosen direction by laying out goals and ideas. There is remarkably little agreement on what strategy in mathematics problem solving is. In mathematical problem solving, it is worth noting that strategy use plays a crucial role in enabling learners to make head way during the problem solving process. This suggests that the choice of strategy one adopts when solving a particular problem is of prime essence. Numerous studies in mathematics education hold strategy as central to processing mathematical problems (Pape and Wang, 2003). In view of this, it is imperative for learners or problem solvers to have in depth knowledge of a number of potent strategies that can facilitate one's understanding when working on non-routine problems. By investigating the learning of strategy thinking, we can see that strategy thinking is an ability that can be developed over time and that at least three factors are important in this development (Sloan, 2006; Cassey and Gold man, 2010). Sloan (2006) sees strategy thinking as a mental process and highlights on the importance of dialogue for the creative and cognitive learning processes. This also unearths the importance of peer learning, coaching and the promotion of dialogue among students where they will be able to think through problems carefully and be able to adopt the best methods to solve them.

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Goldman (2012) describes thinking as an activity which should focus more on instructiveness and dynamism as well as practicality; after all, practice makes perfect. This belief of his ties in perfectly with the variation theory which reveals that there is a difference between being informed of something and expressing the very thing (Marton and Tsui, 2004). Naturally, the learning environment of teaching through problem solving fosters a positive setting where children through social interactions can make more progress at their learning and therefore if they are offered more opportunities to engage in regular group discussions, their understanding of how to competently tackle complex problems will improve. To foster effective learning of strategy thinking, we must bear in mind that the appropriateness of tasks for learners is also essential because this can lead to excellent experiential mathematical learning or creative dialogues (Fülöp, 2015). This means that a problem should be appropriate for the age of the learner and also be able to challenge the minds of the problem solver to a fair extent. According to Gravemeijer and Doorman (1999), 'well chosen context problems offer opportunities for the student to develop informal highly context specific solution strategies.'

What is true for solving problems in general also applies to mathematical problem solving (Kolovou, 2011). Numerous studies in mathematics education hold strategy use central to processing mathematical problems (Pape and Wang, 2003). This literally suggest that the choice of strategies plays a crucial role when working on complex problems and there is, therefore, the need for the problem solver to have an inner conviction regarding the appropriateness of strategies one adopts.

The selection of one's strategy should also promote flexibility especially where problem-solving is concerned (Kolovou (2011). He believes that people are able to work in a flexible way and can modify their behaviour according to changing situations and conditions. This may suggest that the

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strategies used by learners could be one that creates room for alteration as and when necessary. Demetriou (2004) emphasised that more flexible thinkers can develop more refined concepts that are better adjusted to special features of the environment and produce more creative and appropriate solutions to problems. In ensuring that one is on the right track, with regards to accuracy and competence at problem-solving, it is necessary to consider the strategies to be incorporated into the solution or working out process. Cai (2003) is convinced that success in solving a mathematical problem is positively related to students' use of problem-solving strategies. This calls for a proper understanding of the problem in order to ponder over the most effective strategies available. In doing so, learners should, once again, consider flexibility in this situation.

2.7 Developing Mathematical Knowledge through a Problem Solving Approach.

It is important for students to develop sound mathematical knowledge through problem solving.

This will enable learners to be able to explain procedures, strategies and methodologies when working on problems. When learners are presented with such tasks which challenge them in this regard, they are compelled to think mathematically, think critically and think of best procedures to adopt including being able to explain processes. In depth understanding of mathematical facts must therefore be the main aim of all students (Eduafo, 2014). However, for students to enhance their knowledge of mathematics, the teacher needs to play important role. For students to have a better understanding of the subject through Problem Solving, the teacher must see to it that there is a fair balance in terms of how students are engaged. According to Eduafo (2014) to make this possible, the selection of tasks should be interesting and engaging and of immense importance for the learner. Marcus and Fey (2003) further state that the choice of these engaging activities that will enable learners to be actually occupied is usually easy to do but ensuring that these tasks lead to effective learning of mathematics is much more difficult. It is on this basis that researchers say

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that finding the most appropriate tasks that engage pupils is even more challenging for teachers to do. To enable the selection of these tasks to be successful, Marcus and Fey (2003) provide four key guides teachers could be asking themselves.

1) Will the choice of tasks promote teacher's understanding of mathematical facts and strategies?

2) Will the chosen exercises occupy learners well enough while they find the questions manageable?

3) Will working on the questions enable students to broaden their mathematical thinking?

4) Will working on these exercises build students' understanding and connection of mathematical topics?

According to Blair and Haesbroeck (2000) problem solving should be seen as the successful completion of a process of solving a math problem whether it is a pure mathematics problem or a problem with practical application and should be subject to a positive way of thinking of the student engaged in a successful project. Jacobs, Hiebert, Givvin, Hollingsworth, Garnier and Wearne (2006) are of the view that deep understanding will develop over time when we give them such opportunities. However, for learners to develop an understanding of mathematics through a problem solving approach, the teacher's role is crucial, for good guidance facilitates understanding and puts the learner in a better position to take charge of his or her learning. Unfortunately, students tend to struggle when required to transfer knowledge from one situation to the other. In view of this, if we aim to help them have mastery in the process of solving problems, then we must offer them the best support where they will gradually acquire the skill of knowledge transfer. This should involve teaching them about problem-solving and guidance about the problem-solving process and

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instruction associated with a variety of problem-solving strategies. These may also include developing a good understanding of the questions, planning an appropriate solution to the problems and evaluating the solution. Using such approaches also provide learners with more opportunities to discover various pathways which lead to desired answers because they are able to try out a variety of problems solving strategies which enable them to arrive at successful outcomes. Vygotsky (in Slavin, 2011:58) states that a person will be able to solve problems that the level of difficulty is higher than his or her basic ability after he or she gets assistance from someone who is more capable or more competent. Such assistance may be through instructions, encouragement or breaking down problems into solving stages or giving an example. Such assistance is referred to as scaffolding. The scaffolding used in this study refers to 3 levels of scaffolding that Anghileri (2006) has described: (1) environmental provisions; (2) explaining, reviewing, restructuring; And (3) developing conceptual thinking.The disadvantage here is that there is a possibility that this can reduce problem-solving to an activity rather than a process (Anderson, 2000). For learners to develop an understanding of mathematics through a problem solving approach, the teacher's role should include ensuring a balance when engaging learners in solving challenging problems, examining better solution methods and providing information for learners just at the right time (Hiebert, Carpernter, Fenema, Fuson, Human, Muray Oliver and Wearne, 1997). It is therefore essential for teachers to use excellent methodologies to develop learners' skills at problem solving. This is because the appropriate approach a teacher uses for a problem goes a long way in helping students develop a better understanding of concepts.

2.8 The Teacher's Role and Beliefs about Problem Solving

Anderson (2000) believes that teachers' problem-solving beliefs and practices vary considerably. He is of the view that teachers hold a variety of beliefs about the term problem and problem solving

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and also about problem-solving instructional practices. According to him, among teachers, there is consistency between reported beliefs and practices and also between reported practices and what actually occurs in the classrooms. However, for some teachers, reported practices are not readily observed; a situation that suggests either lack of reflection or the possibility of constraining factors that might be adversely influencing teachers' plans. If this is the case, then it literally means that when it comes to the subject matter of problem-solving, most teachers have varying perceptions. Funkhouser (1993) in his attempt to find out teachers' opinion about problem-solving sought to interview a number of them who gave various responses. According to him, two-thirds of the responses were categorised as vague, for example: 'Problem-solving is finding a solution to a problem and problem-solving is using thinking skills'. According to him, only one-third of teachers were able to give a precise definition that involved either reference to strategies or skills such as 'problem-solving is identifying the problem issue, determining the steps, then solving the problem.' Grouws, Good and Dougherty (1990) also sought to interview teachers to find out their perceptions of problem-solving and just like Funkhouser, various ideas were captured. These ranged from the following: Problem-solving is solving problems, problems solving is solving practical problems and problem-solving is solving thinking problems. It was noted that when it came to the definitions, teachers gave their views based on what was happening in their classrooms.

Phipps (2007) holds the view that teachers' beliefs about teaching and learning have a strong effect on their pedagogical decisions. In view of this what they consider right or appropriate is sharply linked to everyday practices within the classroom. Haflu (2008) states that teachers should play an important solving disposition and in doing so, they must choose problems that engage students to this end. If this classroom practice is carried out well, the phobia often associated with the process

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of problem solving particularly when working on non-routine exercises will be embraced with readiness and much commitment from pupils. Teachers need to also create a favourable environment that sparks learners' interests to explore, take risks, share failures and successes and question one another (Haflu, 2008). It is believed that in such a classroom, students are more able to build on their self-esteem and confidence while developing keenness to explore problems and the ability to make adjustments in their problems solving strategies (NCTM, 2004). Again, if as part of a teacher's belief, he or she believes in the development of problem solving skills, then he or she will be more tempted to provide more open ended questions.

Furthermore, a teacher's belief about the importance and skills of implementing problem solving teaching approaches have significant effect on students' achievement and problem solving behaviour (Thompson, 1992). If teachers hold the belief that problem solving is teaching, the classroom environment will be tagged by an effective teaching of the problem solving as a process, particularly through heuristics which are general rules of the thumb and procedural skills lines that help the problem solver to understand and find solutions for a given problem (Good and Brophy, 1990).It is however worth noting that teachers beliefs and classroom practices stem from the teacher's own belief of the process.

During the process of problem-solving, it is also imperative that teachers assume more responsible roles to facilitate pupils' understanding of methodologies and processes. One effective way where teachers will become more useful is when there are more opportunities for professional growth, development and training. Kaur (2001) believes that increasing the level of support for teachers through appropriate learning experiences is one strategy that may address the lack of problem-solving teaching approaches. He reveals that with particular reference to Singapore, where educators concentrate a lot on problem solving, the government ensures that teachers get hundred

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hours of in-service training each year to ground them solidly in the delivery of the curriculum. Evidently, this puts the teachers in a good position to be more confident in the classroom and obviously have more in-depth knowledge in the area of choosing friendly approaches to solve seemingly complex problems.

The teacher's role in helping students to develop the needed problem-solving skills also focuses on another key area which is the ability of facilitators to delve into what Jaworski (1994) described as investigative teaching. This she also called 'teaching triad' describing it as teaching that offers learners the chance to explore problems through effective student interaction.

2.9 The Importance of Problem-Solving for Learners

Problem Solving offers a number of benefits for the learners particularly when students get the opportunity to work on non-routine problems which, per their nature, set a fine pace for the development of learners' mathematical skills. Primarily, the presentation of these tasks creates a favourable platform for pupils to explore and invent new methodologies of solving problems. D'Ambrosio (2003) believes that teaching through problem solving should be targeted at encouraging students to apply their knowledge of concepts meaningfully. For instance, students should be able to use previous knowledge acquired to invent answers to new problems. When they are offered opportunities to work on word problems, for instance, there is often the use of an array of methods and strategies they apply in the process. This promotes a learning environment where there is always a search for new pathways. It is, however, important to note that the teacher's guidance is required to make students' quest to discover new routes absolutely meaningful. Apart from concentrating on an appropriate length of time suitable enough to engage students in discussions, the teachers must also place importance on some key areas. These include aspects of the questions that require highlighting, how to execute a working out plan , what questions to post

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to students bearing in mind differentiated questioning strategies, and also guiding students excellently without depriving them of their own ability to think critically (NCTM, 2009). Ball and Bass (2000), however, believe that there are no specific research based guidelines that teachers can use to achieve the appropriate balance between teacher directed and teacher guided instruction. Nonetheless, there is the need for teachers to have specific ideas on how to play their roles, but they also need concrete examples to guide their practice.

Problem Solving also plays a vital role in augmenting students' abilities to test their own ability to reason. In view of this, students are able to take more charge of their learning by being able to try out new methods, determine the appropriateness of strategies adopted and probe further into their own choice of strategies and become more accountable in the process. Large and small group discussions also afford learners opportunities to exercise their thinking skills. For those students who benefit greatly from peer coaching, small group tasks and discussions of this nature deepen their understanding of concepts. Furthermore, pupils become more accountable as they have to work hand in hand in coming up with appropriate solving approaches in order to derive answers.

Problem Solving also plays a key role in the initial learning of mathematical concepts and skills and according to Schen and Charles (2003), it helps develop the understanding of students which is a prerequisite for broadening one's scope of mathematical ideas. This is because it introduces concepts and skills in problems solving contexts which further evokes students' thinking and reasoning of mathematical concepts.

Problem Solving also fosters another benefit for learners and this focuses on its ability to initiate excellent focussed classroom conversation (Lester and Charles, 2003). Quite often, the opportunity given to learners to share how they arrived at answers or the strategies they incorporated in the

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working out process opens up positive discussions that aim at solidifying students' understanding of concepts.

According to Lesh and Zawojewski (2007), another importance of problem solving in school mathematics is related to the connection between school curriculum and students' levels after school. The two have stated that 'there is growing recognition that a serious mismatch exists between the low level skills emphasised in test driven curriculum materials and the kind of understanding and abilities that one needs for success beyond school'(Lesh and Zawojewski, 2007, p. 764). They therefore believe that problem solving provides enrichment in the area of creativity, flexibility and metacognitive control of thought that do align with projections and post-secondary demands. In view of this, by studying problem solving, students become more prepared for many aspects of their lives after school especially in careers and trades.

Many studies have attributed detailed discussions to the enhancement of learners ability to test their own mathematical competence, for through such fruitful exchange of ideas, students are given the chance to build a solid understanding of what they are studying(Corden, 2001; Nystrand, 1996; Weber, Maher, Powell and Lee, 2008). Discussions in both large and small groups also pave the way for students to persevere and put in their best effort when working out tasks tasks (Corden, 2001; Matsumara, Slater and Crosson, 2008).

During the process of problem-solving, pupils' engagement in detailed discussions and interaction often expands their knowledge as they learn from one another and are able to have a better understanding of the processes involved when working on a particular task.

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2.10 Difficulties in Mathematics Problem Solving for Learners

Problem Solving can only be an efficient process if the necessary support systems, imaging, mathematics notation, planning and organisation are critical aspects in problem solving (Goldin, 1998). In most cases, students do not have a good acquisition of these skills hence they are unable to diagnose meaningful solutions that will help attain desired results. For example learners should be taught how to be proficient on number facts, tables, mathematics principles, arithmetic skills; that is accuracy and logarithms in computational and mathematical working procedures; information skill that is expertise to connect information to a concept, operational and experience and language skill (Tambychik and Meerah,2010).

Geary (2004) also attributes difficulties to problem solving to weaknesses in the area of conceptual understanding and procedural knowledge which are essential skills in problem solving. He believes that these skills should be backed by cognitive systems that control focus and the processing of information. Zalina and Norlia (2005) links difficulties in mathematics Problem solving to the fact that most learners do not understand problems the way they ought to be understood. He believes that first of all, questions must be thoroughly understood but unfortunately because of the long sentences and information often involved questions are really less meaningful to students. For example, they could have problems with an excellent comprehension of language and the terms used.

The entire problem solving process can also be daunting for most students especially when they are tasked to work on complex problems. Whereas most of them can impressively follow accurate pathways to achieve answers for routine problems, not most of them can show the same competence when deciding on problem solving approach to solve tricky tasks. MohdNizam and Rosaznisham (2004) believe that though several mathematics skills are involved in problem-

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solving, large numbers of students have still not acquired the basic skills they need in mathematics. Hill (2008) believes that these poor skills often exhibited by the students vary and can therefore be grouped into some key areas. These include deficiencies in the area of number facts, arithmetic, information skill, language skill and visual spatial skill. As a result, many students face difficulties in mathematics particularly in the area of problem-solving (Tay Lay Heong, 2005). If this is the case, then it is indeed vital to augment effective teaching skills to boost the learning skills of students. This will enable them to develop a better understanding of problem solving processes especially suitable for working out non-routine problems. Whereas most pupils can impressively use appropriate problem solving processes to work on routine problems many of them fail to use excellent approaches towards non-routine problems which often test for their conceptual understanding.

Stendall (2009) states that the ability to use cognitive abilities in learning is crucial for meaningful learning to take place. If pupils and students are therefore failing in the use of their cognitive abilities, then it is worth concluding that many are not exerting themselves as much as possible Tarzimah (2005) holds the view that such learners face difficulties in making accurate perceptions and interpretations, memorising and retrieving facts, giving concentrations and using their logic. It is evident that the lack of mathematics skills continue to cause difficulties in solving problems for they basically fail to apply and integrate mathematical concepts and skills during the process of making decision and solving problems (Tambychik and Meerah,2010).

Adding more to this, Garderen (2006) states that deficiency in visual-spatial skill might cause difficulty in differentiating, relating and organising information meaningfully. According to him, this could cause students to experience an array of hindrances such as incomplete mastery of number facts, weaknesses in computational exercises, inability to connect conceptual aspects of

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maths, inefficiency to transfer knowledge, difficulty to make meaningful connections regarding information, incompetency to transform information mathematically, incomplete mastery of mathematical terms, incomplete understanding of mathematical language and difficulty in comprehending and visualising mathematical concepts.

2.11 Teaching and Learning approaches in Problem Solving

In the process of helping students learn and develop problem-solving skills, teachers often adopt a number of approaches they deem vital in enhancing pupils' mathematical skills. Siemon and Booker (1990) have highlighted three approaches to teaching problem solving which includes: teaching for, teaching about and teaching through problem-solving. Teaching for problem-solving most closely resembles a traditional approach to teaching mathematics where students learn mathematical content so that they can apply it to problems related to that area (Anderson,2000). This may literally suggest that for pupils to be able to work well on problem-solving, there is the need for them to first, have a thorough understanding of topics. This requires the teacher to go step by step and gradually immerse the learner into more complex problems. It is therefore believed that the acquisition of such basic knowledge will strengthen and help the learner to apply the concept learned. He further believes that a key challenge for teachers when using this approach is that a problem must have a blockage and this may be removed if students are able to follow a rehearsal procedure. However, Tripathi (2008) admonishes that when the teacher assumes a more effective role as a facilitator, he or she will be able to challenge learners better at critical thinking without making answers obvious for them. According to him, the teacher could ask questions that help students review their knowledge and construct new connections. Teaching about problem-solving also includes guidance about the problem-solving process itself and instructions about a variety of problem-solving strategies. These may include developing a good understanding of the

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questions, planning an appropriate solution to the problem and evaluating the solution. The disadvantage here is that there is a possibility that this can reduce problem-solving to an activity rather than a process (Anderson, 2000). According to Stacey and Chick (2004), in order to overcome these possible occurrences, giving learners explicit critical thinking opportunities during the fundamental stage of their learning is necessary. This implies that rather than students being taught new ideas and skills with some applications and problems added on at the end, problems are used as the vehicle for learning, or as the context in which the learning of mathematical ideas take place. With this approach, students take a more active role as they are made to have a better sense of mathematical procedures needed to solve the problem. This requires the student to plunge deeper into breaking down the question and engaging in more critical thinking processes to find a solution.

According to Hiebert and Wearne (1996) classrooms with a Primary focus on teaching through a problem-solving approach used fewer problems and spent more time on each of them compared to those classrooms with a primary focus on problem-solving. This unravels the fact that in a typical classroom where learners are presented with more non-routine exercises, the teacher tends to spend longer time on detailed explanations to learners thus giving them more room to understand the processes involved in solving a task. Learners in this typical class can benefit from such teaching opportunities enabling them to develop more of their mathematical competence.

Teaching mathematics through a problem-solving approach provides a learning environment for learners on their own to explore problems and to invent ways to solve the problems (Eduafo, 2014). According to D'Ambrosio (2003), proponents of teaching mathematics through problem-solving base their pedagogy on the notion that learners who encounter problematic situations use their existing knowledge to solve those problems and in the process of solving the problems, they

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construct new knowledge and new understanding. Learners are therefore compelled to develop their knowledge by extending the routes to solve the problem at hand. Furthermore, he illustrates how learning mathematics through a problem-solving approach has been put into practice with three examples: using elementary, middle and secondary school learners. It is vital to note that in the study all learners did not have formal instructions to follow before solving the problems. However, they were ready to think mathematically and explore all possible routes to arrive at answers. In this regard, one can conclude that they were able to delve deeper into mathematical thinking and also challenged their respective abilities and creativity to a large extent. The study shows that teaching through problem-solving approach offers learners a platform to make progress in their learning. Undoubtedly one can say that presenting students with problem-solving tasks offers students the opportunity to be independent problem solvers couching right pathways to solutions and helping them to develop a deeper understanding of mathematical processes

2.12 Fostering Understanding of Problem-Solving through Co-operative and Collaborative Learning

21st Century teaching and learning strongly advocate good practices such as the creation of a learning environment that gives learners opportunities to engage in co-operative and collaborative tasks. According to Siegel (2005), co-operative learning is where learners are put into groups and challenged to find solutions to given tasks. When students 'work on high level co-operative tasks, they demonstrate higher level reasoning and problem solving discourse' (Gillies, 2014, p.134). The teacher has a role of ensuring conditions are set up for higher level thinking problem solving to take place and roles and expectations are established. He found that establishing a strong cooperative learning environment allows for less discipline issues. The positive gains for students who are given opportunity to interact , listen, share ideas and question one another are much higher

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than traditional 'sit and get'. Gillies (2002) believes that co-operative learning shows that learners benefit academically and socially from co-operative, small group learning. In view of this, learners who do not show progress in a larger group will find learning co-operatively more useful. These learners will be able to find learning more meaningful and worthy as opposed to being in a larger group. Johnson, Johnson and Stanne (2000) therefore believe that co-operative learning can produce positive effects on student achievement.

Gillies and Ashman (1998) also believe that learning in smaller groups also promotes some social benefits which include more on-task behaviours and helping interactions with group members. Others include promoting one's self esteem, making more friends within the learning environment, enabling learners to be more involved in classroom activities and improving attitudes towards learning (Lazarowitz, Baird and Bolden, 1996). This can, therefore, support other learners who otherwise may have struggles working on their own to have a better sense of belonging within the learning environment. Those with higher strengths at mathematics, for instance, may help low achievers to make some level of progress through explanations, sharing of ideas and suggestions. From the perspective of Johnson and Johnson (1999), there is enough proof that students who regularly engage in collaborative work reap a number of benefits. A good example is the high levels of achievements they are able to clutch and again, more progress is made in their learning. This stems from the fact that they are able to render effective support for one another. In a study carried out by Gokhale (1995) to find out the efficacy of individual learning as opposed to collaborative learning, it was discovered that students who engaged a lot more in collaborative learning, performed better. His findings suggested that in co-operative learning, students are able to work purposefully in groups and this ensures more accountability as each child is expected to

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play an effective role. Ccollaborative learning on the other hand engages co-operative groups to discover pathways to mathematical problems.

2.13 Using Assessment Strategies to foster the Problem Solving Process

Assessments are an integral part of the teaching and learning process. They discover both the strengths and weaknesses of learners and dictate the next steps forward. Particularly for the teacher, it also ensures a period of self-reflection and provides some key insights into what has to be done, for example, regarding the effectiveness of one's use of teaching strategies. Anderson (2000) believes that regardless of the problem-solving tasks and approaches that teachers use in the classrooms, there is evidence to suggest that unless problem-solving forms part of the assessment procedures, students will not value problem-solving tasks and therefore will not make a commitment to such tasks. This may mean that for students to be more challenged and committed to problem-solving tasks, there is the need for some level of communication where they will be briefed about areas they will be assessed on. For example, if it is a word problem, teachers could inform pupils that their working out procedure, accurate computations and presentations will be assessed. This will put learners in a better position to be more devoted to classroom tasks. Tsamir, Tirosh, Tabach and Levenson (2010) also hold the view that assessment strategies should be carefully considered especially where mathematical problems and questions have more than one outcome. Also, if problem solving is to be valued by students as well as teachers, it may need to form part of classroom practices as well as more formal school and state assessment procedures. To make this more effective, assessing pupils' knowledge of problem solving competence should therefore be more frequent to create familiarity among learners, especially when it comes to the skills they will most likely be tested on at the external level.

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2.14 Supporting Problem Solving Skills with On Line Game Generated Feedback.

In Primary school mathematics, the development of basic skills especially in the area of being able to engage in fundamental computations remains the focus. However, other researches (Goldenberg, Shteingold and Feuzerg, 2003) have stressed the fact that the study of the subject at the basic level should also involve innovative thinking, help pupils to explore patterns and support them in inventing ways to solve non-standard problems. Offering pupils non-routine exercises provide that avenue for them to exert these skills to a considerable degree. For example, presenting them with questions which do not have straightforward answers would compel them to explore, seek solutions and help them to try out several patterns or pathways to derive answers. Kaput, Carraher and Blanton (2007) are convinced that algebraic activities could offer pupils in primary school opportunities for developing more sophisticated thinking skills. This is because, with the provision of such complex exercises, learners will be compelled to engage more in depth logical reasoning while finding a route or solution to an answer. In view of this, it is right to state that the incorporation of algebra in the curriculum at the elementary level is necessary. According to Kilpatrick, Swafford and Findell (2001) algebra builds on students' proficiency in arithmetic and develops it further. Along these lines, the National Council of Teachers of Mathematics (NCTM) suggested that algebra is a strand that permeates all levels of schooling from prekindergarten through grade 12 (NCTM, 2000). For example, the concept of function can build on experiences with numerical patterns in the primary school (Ibid). Carraher and Schliemann (2007) also believe that rich problem-solving contexts play an indispensable role in bringing about these experiences as reasoning in particular situations may support students in generating abstract knowledge. Over the years, function tables (Schliemann, Carraher and Brizuela, 2001) or function machines (Warren, Cooper and Lamb, 2006) and patterning activities (Moss and Beatty, 2006) have been

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successfully implemented to support this reasoning in primary school students but new technologies might bring in further improvements for stimulating the development of algebraic concepts. Roschelle, Pea, Hoadley and Means (2000) are of the view that the incorporation of computers in algebra work can carry out calculations and diminish the routine workload so that students can be more focused on exploring the relationships between the quantities in a problem. Furthermore, computer tools can surpass the constraints of paper and pencil activities by creating a dynamic environment that provide instant feedback. This therefore will take away the load of engaging in extra tasks so that the learner may give full attention and time to the aspect of work which requires critical reasoning.

2.15 Using ICT to enhance Problem Solving Skills

Undeniably, problem-solving is a major goal of mathematics education and an activity that can be seen as the essence of mathematical thinking (NCTM, 2000). This suggests that the area of non-routine under the curriculum area is of importance in that it challenges pupils' critical thinking abilities and provides an opportunity for them to think more in depth. Despite the growing body of research in the area of problem-solving (Lesh and Zawojewski, 2007) there is still much that we do not know about how learners attempt to tackle mathematical problems and how to support students in solving non-routine problems. However, a considerable body of research has shown that computers can support children in developing higher order mathematical thinking (Clements, 2002). Logo Programming, for instance, has the ability to create a rich learning atmosphere for learners where students themselves can engage in deeper reflections of their own respective abilities (Clements, 2000). Additionally, the array of learning softwares can, in modern times, support learners to explore their learning and become more creative problem solvers (Ibid). Based on the underlining beliefs and principles of the National Council of Teachers of Mathematics

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(NCTM, 2000) technology is believed to further support decision making, reflection, reasoning and problem solving. This may mean organising or developing technology based on mathematical exercises to be more appealing. This may include child friendly exercises which are interactive and would naturally engage learners, even learners who are often less attracted to mathematics. It could also be in the form of a challenge, for instance, where the ability to solve a problem accurately may come with awarding of points. Clements (2000) believes that among the unique contributions of computers is the fact that they also provide students with an environment for testing their ideas and giving feedback. For instance, in such a technology-based environment, there is the likelihood that learners will readily know their performance immediately after an exercise. Haitie and Timperly (2007) express that feedback that is often computer based supports students to know their errors better, thus aiding them to develop a better understanding of mathematics. Again based on the fact that most of these computer based applications are interactive in nature, students find it more appealing thereby engaging them more in their learning (Rochelle, Pea,Hoadley, Gordin and Mean, 2000). Wegerif and Dawes (2004) are also of the opinion that the effect of this, is an enhanced interaction among peers and also one that also supports students' skilfulness at problem solving. A number of scholars have also declared that embedding technology or the use of computers offers teachers a better idea of how to track processes learners adopt when solving problems (Bennet and Persky, 2002).

2.16 Theoretical Framework

The theoretical framework will largely dwell on the constructivism where two belief systems will be highlighted on: the radical and social constructivism. The radical constructivism is often associated with Jean Piaget (1896-1980). From the perspective of Von Glaser field (1990) Piaget suggested that individuals construct their own knowledge through the process of accommodation

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and assimilation. In this regard, radical constructivists believe that learning is a process whereby the learner constructs or develops new ideas or concepts based on current and past knowledge experience.

Social constructivism is regarded by many scholars as a further stretch of what is often described as individual learning. It is therefore not surprising that social constructivism believe that true knowledge is what has been carved by the individual himself within a social environment of excellent interaction (Jones, 1996). In adding further to this, Ernest (1992) believes that knowledge is always interspersed with culture and society. Undoubtedly, these theories suggest that individuals are able to make their learning more fulfilling when they are to construct new knowledge through their interaction with others. The implication here for teachers is that the act itself should provide learners an opportunity to engage in both collaborative and co-operative learning.

2.17 The Belief of Radical and Social Constructivists

Radical and social constructivism do have a number of overlaps. The only exception is that particularly for social constructivism there is a lot of concentration on students interacting more with one another. Owing to this, we can conclude that learning is done typically through social interactions.

In the radical constructivist classroom, learning is more student cantered and therefore based on the active participation of learners. In this classroom, for example, learners are challenged to explore and find new pathways of going about questions. Students therefore have the opportunity to use inquiry methods to ask questions, investigate a topic and use a variety of resources to find

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solutions and answers. They are also able to explain their answers, processes of working and challenge their own thinking and that of others.

In a radical constructivist classroom the role of the teacher is to challenge students' thinking and present them with problems that enable them to solve tasks from different angles and also offer them the chance to work on exercises that are engaging in nature. The role of the teacher as a guide or facilitator is therefore of essence.

The social constructivist classroom on the other hand is characterised by more peer interaction structured in a purposeful manner which makes every student accountable. Through group participation, students build on their existing knowledge through interactions; here, students share ideas, engage in deep discussions and promote group or community participation.

In this typical class, the teacher serves as a guide or facilitator to ensure the process of collaborative learning is moving successfully. The role of the student here is to play an active role by engaging in co-operative learning with other students who come together to find solutions to problems.

The adoption of both theories is deemed appropriate owing to the fact that they inform the study in a number of ways. When students are tasked to work on non-routine problems, the radical constructivist classroom fosters an atmosphere where students are able to engage themselves independently and purposefully. In the research to be carried out, I therefore expect to witness such an environment teeming with pupils who fully own their learning. Additionally, I wish to see pupils showing more capability at using previous knowledge to explore new routes to more complex problems and being more accountable and productive at finding innovative and meaningful approaches to work. Again, the benefits of both collaborative and co-operative learning cannot be over emphasised. It is therefore not surprising that more scholars continue to

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advocate that for students to show more progress on non-routine problems, there is the need for the promotion of such social interactions. During the study, the use of interviews will enable me to find out how frequent pupils engage in such fruitful discussions, geared towards solidifying their mathematical understanding.

Again, the choice of both theoretical foundations has a good connection with the research questions raised and through the data analysis, one will realise, to a large extent, the effect of both radical and social constructivism when students are confronted with non-routine problems.

2.18 Chapter Summary

The literature review under this chapter has probed into the roles of both the teacher and learner in the problem solving classroom. For instance, learners should be able to develop their creativity and become more analytical in the process of working out mathematical problems which seem unfamiliar or complex in nature. The teacher on the other hand should be able to create a favourable platform for learners to have a sound understanding of problem solving exercises by modeling exercises, choosing problems judiciously and guiding learners on the right paths. Again it has unravelled a number of key elements which facilitate one's competence at solving problems and these include the incorporation of technology, the use of games and effective teaching and learning approaches. It has also looked at the concept of problem solving itself including its key features. Based on the literature review, many have also stated that to make the process of problem solving more beneficial, the adoption of methodologies should be more socially child friendly, thus supporting the belief system of the constructivists that a learner constructs his or her own knowledge by interacting co-operatively and collaboratively in both small and large groups. Finally, the theoretical frame work has given attention to radical and social constructivism as the baseline theories for this study.

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CHAPTER THREE

METHODOLOGY

3.1 Introduction

This chapter seeks to discuss, in detail, the research methods and methodology adopted for this study. Focus will primarily dwell on the research design, the research matrix, the area of study, research methods, population, sampling strategy, instrument design, methods of data collection and data analysis including the ethical considerations.

3.2 Research Design

This research makes use of the sequential mixed method which Creswell (2003) highlights as a two phased design. Creswell (2011) unearths a number of purposes underpinning the use of this mixed method approach. For instance, he believes that its use is appropriate when either the qualitative or quantitative research is insufficient to fully understand the problem. Other reasons he provides include the need to explore before administering instruments, the need to explain one's statistical results, the need to see if our quantitative and qualitative results match, the need to enhance our experiments by talking to others and the need to develop new instruments by gathering qualitative data. With regards to this study, therefore, the adoption of the sequential mixed method is deemed fit in order to attain a systematic build-up of results commencing from the quantitative phase which will examine pupils' knowledge and understanding of non-routine problems. In doing so successfully, the selected participants, who will be chosen from Year 4 to Year 6, will be tested on an array of story problems levelled to suit each Year Group. The findings will then inform the next phase of the research which is the qualitative stage. Here, focus will dwell on gathering in

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depth information on pupils' perceptions regarding challenges they often face when confronted with non-routine problems. Again, the choice of research design had some ontological and epistemological underpinnings. Crotty (2003) defines ontology as the study of being and this encourages one to explore and investigate what kind of world we live in. Linking this view to the study, the use of both qualitative and quantitative designs offered opportunities for students to come up with their own thoughts, perceptions, stance and express their own views as far as their experiences with non-routine questions were concerned. With regards to epistemology, I based the design on constructionism which Crotty (2003) expresses as meaning that is constructed based on the human being's interaction with the world. For the qualitative aspect of this study some questions were therefore targeted at helping students to express how they benefitted from peer interaction, collaborative and co-operative learning.

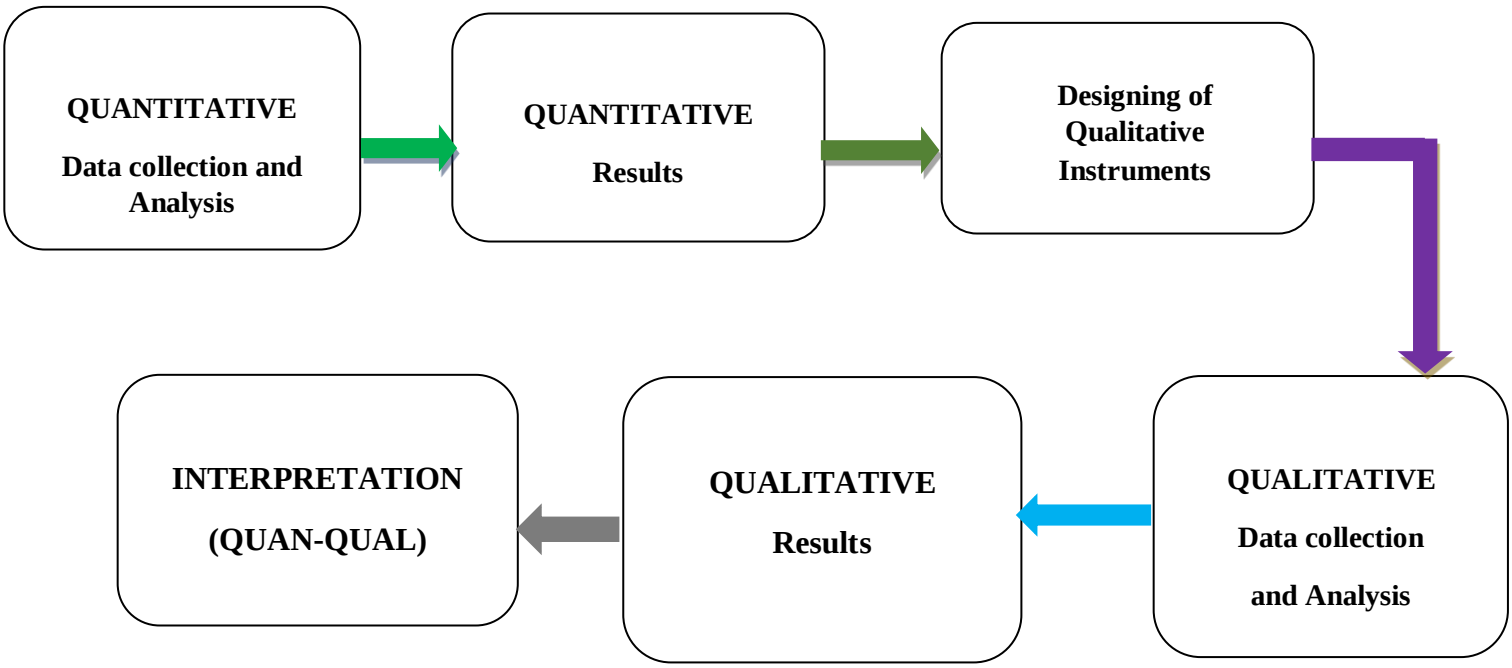

Figure1: Sequential Mixed Method Design

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The research design will begin with quantitative data collection through the provision of non-routine assessments. The results will inform the development of the qualitative data which will mainly focus on interviews. Thereafter, the findings from both the quantitative and qualitative data will be analysed, followed by a final integration of both data.

Table 1: Research Design Matrix

Research	Type of Data	Instrument
What characterises pupils' experiences of working on non-routine mathematics problems?	Quantitative & Qualitative	Non-Routine Assessment Tool & Structured Interview
What explains, if any, the difference between pupils' understanding of routine and non-routine mathematics problems?	Qualitative	Structured Interview
What opportunities are pupils' given to engage in collaborative and co-operative learning?	Quantitative	Non-Routine Assessment Tool
What challenges do pupils encounter when conceptualising their understanding of mathematical concepts?	Quantitative & Qualitative	Non-Routine Assessment Tool & Structured Interview

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3.3 Area of the study

The area of study for this research is the Junior section of Ghana International School. Located at Cantonments, the establishment has for decades provided world class holistic education to children between the ages of 3 and 18 across its four sections: Infant school, Junior school, Lower secondary and Upper secondary. It offers a curriculum modelled along the English National curriculum and the Cambridge international examinations, IGCSE and GCE Advanced levels with English as the main language of instruction. With respect to the study which lays emphasis on pupils' experiences of non-routine mathematics problems, I aim to carry out the research at the Primary section of the school where I can easily obtain the adequate number of pupils fit for the study.

3.4 Methods of Data Collection

With reference to this study, two methods of collecting data will be considered. These included the use of both assessment tools and structured interviews.Prior to the collection of data, some ethical procedures will be duly followed and these include seeking approval from the Vice Principal, briefing participants on their expectations while assuring them of confidentially. Consent from parents will also be sought.

Assessment Tool

The assessment tool is a set of printed worksheets distributed to all 144 selected participants across the chosen Year Groups, in this case Year 4 to 6. In all, selected pupils will have an hour to work on ten structured questions and these are typically story problems with child friendly themes for each Year Group. The varied nature of questions will provide opportunities for pupils to be tested on key areas including their understanding of mathematical concepts, their ability to read, comprehend and use suitable mathematical operations when working and their ability to adopt

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unique ways of solving non-routine problems. Additionally, these questions are well differentiated thus providing pupils with opportunities to work on questions which progress steadily in terms of their level of difficulty.

Design of Assessment Tool

The design of assessment tool will focus on key areas: presentation, nature of problems, differentiation and a well-developed marking scheme. The presentation of word problems will in printed format with a bold font making them visual friendly and this will be in three different versions to suit the ability of children from Year 4 to 6. In all, there will be ten questions for each Year Group and the nature of these questions will also be based on child friendly themes so participants can relate better with the questions and find them more engaging. In view of this, the story problems for Year Four pupils will center on a 'birthday' theme while Year Five and Six pupils will have questions focusing on zoo animals and the waterpark respectively. The design will also factor in differentiation so for this reason, pupils will commence their work steadily, starting from easy to more difficult problems. A fully developed marking scheme will also be developed for each Year Group; each correct answer will attract two marks, while a mark of one will be awarded for a good working out process with an incorrect answer. A full score will therefore be out of 20 marks for each Year Group.

Structured Interviews

Interviews are often used as complementary research methods in the social sciences because they give the opportunity for a more in depth, open discussion, and more informal, free interaction between the interviewer and the interviewee (Potter, 2002; Winchester,1999; Sarantakos,2013). With reference to the study, one on one interviews will be structured enabling pupils to answer

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questions generated from the results obtained at the quantitative phase of the study. In all, 18 pupils carefully drawn from the sample will take part in the process and this selection will mainly depend on their earlier scores. In view of this, 6 pupils will be chosen to represent each Year Group and their marks should fall within the range of Excellent (17-20 marks),Good (16-11 marks) and Poor(10-0 marks). This mode of selection is deemed imperative in order to attain differing views from participants while outlining the variations and commonalities in their responses.

Development of interviews

The design of interview questions will be based on the outcome of the quantitative results. This is to say that the interview questions will be informed by the challenges pupils experience during the assessment. The interview schedule will be in two parts with the initial section focussing on the background information of the participants. The second will center on interview questions which hover around the following areas: pupils' perceptions of complex problems, the level of mathematical support rendered to pupils, the creation of co-operative and collaborative learning opportunities for pupils and learners' beliefs about non-routine and routine problems. Complete scripts for both the assessments and interviews are available in the Appendices.

3.5 Population of the study

The target population of the study will comprise all pupils across the Junior school, from Year 3 to Year 6, totalling 444 in number. These are pupils with different social backgrounds and religious beliefs and out of this number, the accessible population that will be considered for the study are 332 pupils from the Upper Primary, precisely Year 4 to Year 6, who all stand an equal chance of being randomly selected. Chang (2002) accounts students' challenges in the area of non-routine problems to lack of metacognitive skills and links it to their inability to attain the expected

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levels even at international assessments. If this is the case, then equipping Upper Primary pupils with these skills at the fundamental stage of their learning remains vital to prepare them fully for higher learning. It is in this regard that the study seeks to concentrate on pupils from the Upper Primary level.

Table 2: Accessible Population of the Junior School

Year Group	Class Name	Class Name	Class Name	Class Name
Year 4	**4 Tamarix**	**4 Sequoia**	**4 Maple**	**4 Eucalyptus**
	28 Pupils	29 Pupils	28 Pupils	28 Pupils
Year 5	**5 Oak**	**5 Meranti**	**5 Euptelea**	**5 Willow**
	26 Pupils	28 Pupils	26 Pupils	28 Pupils
Year 6	**6 Mahogany**	**6 Lotus**	**6 Flamboyant**	**6 Baobab**
	27 Pupils	28 Pupils	29 Pupils	28 Pupils

3.6 Sampling Technique

Sampling for Quantitative Phase of the study

The simple random sampling approach will be adopted for this study. This is appropriate so as to ensure fairness while enabling every child within the accessible population to stand an equal chance of being selected. These will include all pupils from Year 4 to Year 6 and considering the fact that there are four streams of classes in each Year Group, the number of classes involved will

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be 12 in all. Each pupil will have a likelihood of being picked in each class and therefore each pupil will be given a special identification number. Out of this lot, 12 pupils will be randomly selected from every class. The total number of pupils therefore chosen for the initial quantitative stage of the research will stand at 144.

Sampling for Qualitative Phase of the study

A total number of 18 pupils will be considered for the interview stage suggesting a selection of 6 pupils from each Year Group. These are pupils who will also take part in the initial mathematics assessment. The selection of these pupils will be done through purposive sampling which will enable the researcher to select two pupils from each score range: the high ability group, middle ability group and low ability group to enable more diverse responses.

Table 3: Score Range

Range	**Marks**
Excellent	17-20 Marks
Good	16 – 11 Marks
Poor	10 – 0 Marks

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3.7 Reliability and Validity

A pilot study will be conducted with the goal of enabling the researcher to decide how best to conduct the intended large scale research. Another dominant reason for conducting the research is to ascertain the reliability and suitability of the adopted measuring scales (i.e. data collection measures). Additionally, it is to curb all possible errors prior to the start of the main study. Fifteen pupils across the three Year Groups will be selected for the pilot study and these will be participants not included in the main survey. In terms of analysing the data, it is hoped that the Cronbach's alpha's for the various scales will be above the value of .70, suggesting they will be reliable and acceptable as posited by Sekeran (2003). Lincoln and Guba (2007) are of the view that trustworthiness of a research cannot be over emphasised. In their opinion, these include credibility, transferability, dependability and confirmability. Particularly for this research, I bore a number of things in mind to achieve a high level of trustworthiness. For instance, for credibility I ensured prolonged engagement of the participants during both assessments and interviews; again I chose the simple random technique to ensure fairness in my selection, information was treated with confidentiality. In terms of dependability, for example, the adoption of the sequential mixed method aided a systematic tracking down of students' experience as I was able to merge both findings to arrive at a useful and detailed conclusion. I must also state that in the area of confirmability, I made a conscious effort not to be swayed by biases and for this reason, I remained as neutral as possible.

3.8 Methods of Data Analysis

The results of the mathematics assessment administered will be analysed using version 20 of the SPSS package. The interviews will also be transcribed and using content analysis, the responses provided by pupils will be put into thematic areas.

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3.9 Ethical considerations

There are several types of ethical issues, which the researcher has to take into consideration for this study. The most important one is related to the informed consent of the participants. All the participants including the4445 Vice Principal and Year Co-ordinators will be informed in advance about the purpose of this research and following an explicit explanation, one hopes that they will give their informed consent. The identity of children will also be kept in strict confidentiality, thus meeting the requirements of the code of ethics of the University. For instance children will be given special codes to identify each one of them rather than having them to write down their names on the worksheets.

In addition, the privacy and confidentiality policy of Ghana International School (G.I.S) also had to be taken into consideration. Again, regarding the fact that the participants involved in this study were minors, parents were also contacted to give their approval before allowing children to engage in any part of the study. Consent forms are attached in the Appendices. Finally, all the information collected in the course of this thesis has been used only for the purposes of the study and will be kept confidential and I must add that prior to carrying out this research, permission was fully granted by the Ethics Committee of the University of Ghana.

3.10 Chapter summary

This chapter outlines and justifies the research methodology implemented in this thesis. Due to the nature of the study, the researcher will opt for the sequential mixed method design. The key research tools are assessment sheets supplemented by interviews and the participants will be carefully selected through a simple random sampling technique. The results of both the

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quantitative and qualitative stages will be analysed and the major results and findings of this work will be discussed in the following chapter.

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CHAPTER FOUR

DATA ANALYSIS AND PESENTATION OF DATA

4.1 Introduction

This chapter discusses the data analysis and findings from the assessments completed by 144 pupils across the Junior section of Ghana International School (GIS). This assessment was structured as part of the study to answer the following research questions:

- What characterises pupils' experiences of solving non-routine mathematical problems?
- What explains, if any, the differences between pupils understanding of routine and non-routine mathematics problems?
- What opportunities are pupils given to engage in collaborative and co-operative learning?
- What challenges do pupils encounter when conceptualising their understanding of non-routine mathematical problems?

Pupils aged between 8-11 years had to sit for a 60 minute assessment which basically focused on non-routine problems, precisely story problems. In all, the 48 pupils selected from each Year Group had 10 problems to solve and these ranged from simple to more challenging questions.

4.2 Background characteristics of participants

The assessment was completed by pupils from Year 4 to Year 6 who were chosen through the simple random sampling method. Below is a table which highlights the gender of the selected group.

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Table 4: Gender of Participants

Class	Number of Females	Number of Males
Year 4	20	28
Year 5	27	21
Year 6	22	26

4.3 Summary of Quantitative and Qualitative Findings

The following gives a detailed overview of quantitative and qualitative findings and how they answered the various research questions. With reference to the quantitative findings, graphs have been used to give visual images of students' performance based on the lower, medium and higher order questioning. A smaller sample drawn from students who took part in the assessment also gave varied responses to the interview questions which have been transcribed in this section.

Research Question 1: What characterises pupils' experiences of solving non-routine mathematical problems?

Quantitative & Qualitative

The above research question was answered by both quantitative and qualitative results. Since this section focuses on the findings of the quantitative analysis, emphasis will, first, dwell on answering the question quantitatively using the outcome of the assessments which aimed at testing pupils' varying abilities on non-routine problems. Areas the researcher showed interest in included learners' use of problem solving strategies, their understanding of non-routine problems, their adoption of appropriate computational methods and accuracy at problem solving. Below are

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displays of bar graphs showing their performance as well as a thorough description and analysis of the results obtained.

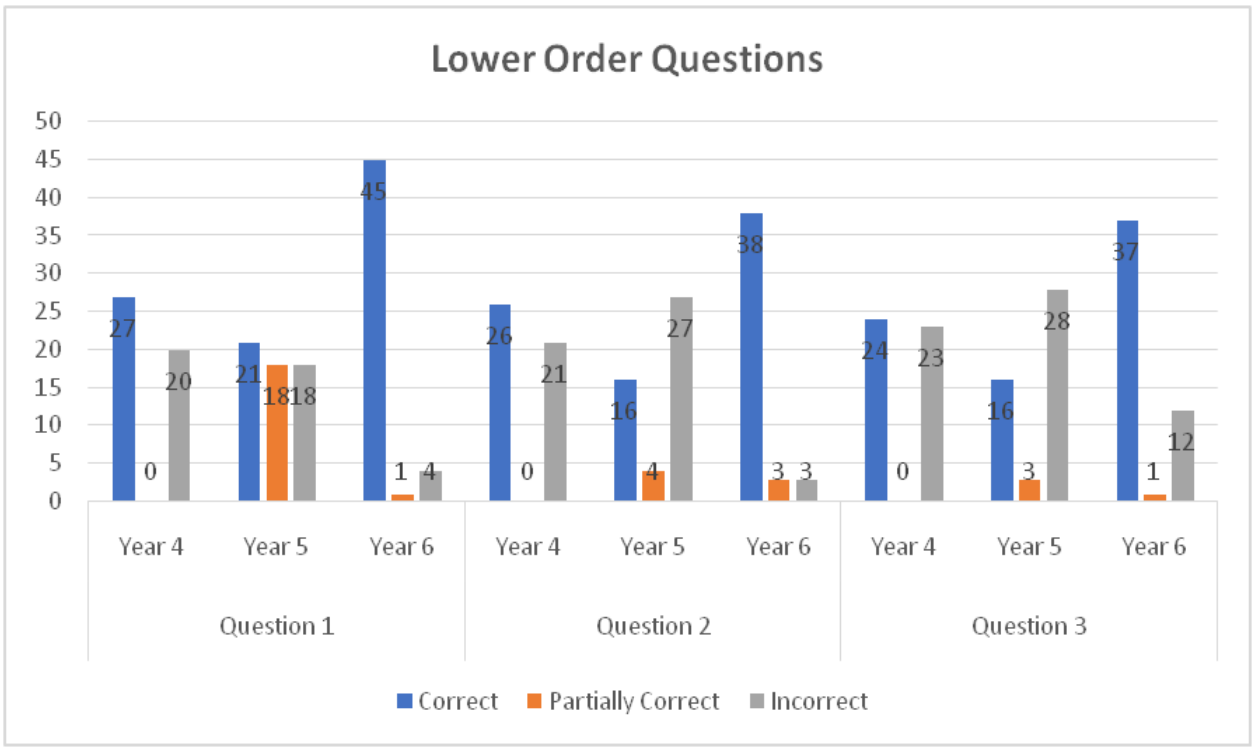

Figure 2: Pupils Performance on Lower Order Questions

4.3.1 Outcome of Pupils' Performance on Lower Order Questions

The questions set for each Year Group were categorised under three main areas based on their levels of difficulty. For every Year Group, the lower order questions were from questions 1-3. For Year Four, there were 27 pupils with correct answers for **Question 1**, there were no partial marks and 20 pupils had incorrect answers. For Year Five, 21 pupils had correct answers, 18 pupils had partial marks and another set of 18 pupils had incorrect answers. For Year Six, 45 pupils obtained correct answers, 1 pupil had a partial mark and 4 pupils had incorrect answers.

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For **Question 2**, 26 pupils from Year 4 obtained correct answers. There was, however, no partial mark for any Year Four child and regarding inaccurate answers, there were 21 pupils. In Year Five, 16 pupils had the question right while 4 pupils had partial marks. For incorrect answers, there were 27 pupils. For Year Six, 38 pupils had the question right while 3 pupils attained partial marks with another set of 3 pupils obtaining inaccurate answers.

For **Question 3**, there was a total of 24 pupils who had correct answers from Year Four with no one attaining a partial mark. For incorrect answers 23 was recorded. For Year Five, 16 pupils had the answer right, 3 had partial marks and 28 pupils had incorrect answers. For Year Six, 37 pupils had correct answers, 1 pupil had a partial mark and 12 pupils had incorrect answers.

4.3.2 Examination of results for Lower Order Questions

The general performance of pupils on the lower order problems was disappointing regarding the fact that as these were the easiest category of questions, one expected to see a greater number of pupils scoring more impressively. A critical examination into the results, however, proved otherwise. Across the Year Groups, mind blockage hindered pupils from meaningfully understanding the questions set. For this reason, though pupils were able to read questions competently, an in depth comprehension of what each question required and how to arrive at answers were not forthcoming. This unearthed the fact that most learners lacked concrete reasoning skills which remain vital in one's ability to work accurately on non-routine problems. For example in Year Four and Five, the number of incorrect answers were significant, pointing to the fact that pupils only showed more capability at reading through questions but exhibited less strengths at reasoning and application of knowledge.

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It is worth stressing that pupils in Year Four also portrayed the least commitment towards the assessment. For this reason, pupils did not fully exert themselves, thus failing to show expertise in the sound use of methodologies, computational skills and appropriate choice of procedures relevant enough to attract even half marks. Based on this shortcoming, it was not surprising that for this particular Year Group, there were no records of partial marks.

Positive attitude and commitment when working on non-routine problems have, over time, been linked to the enhancement of learners' confidence and self-esteem. This was obviously not the case during the assessment, owing to the fact that very little positive attitude was shown among pupils. Even in Year Six where pupils had more correct answers on the lower order questions, there was the realisation that as the questions progressed, the number of accuracies decreased pointing to the fact that the attitudes among pupils in this Year Group could have been better.

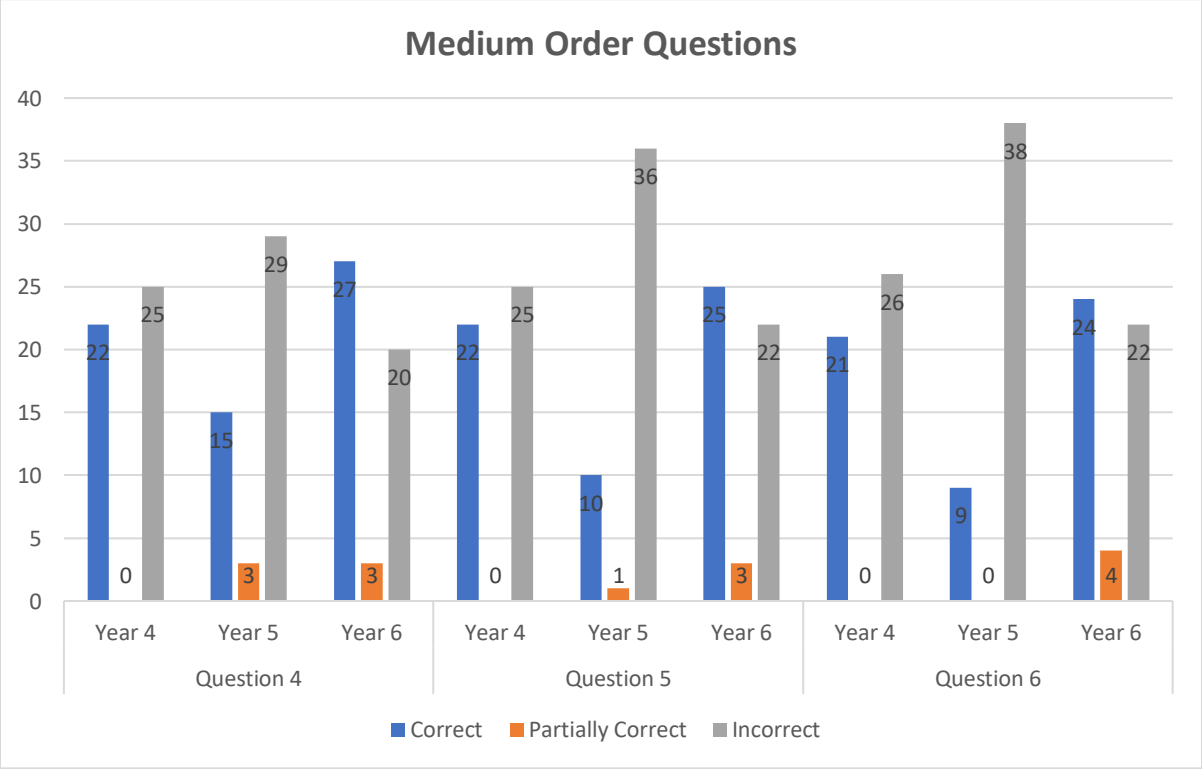

Figure 3: Pupils Performance on Medium Order Questions

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4.3.3 Outcome of Pupils' Performance on Medium Order Questions

The questions set as medium order problems, for each Year Group, were from Questions 4 to 6. Regarding **Question 4**, here are the details: for correct answers there were 22 pupils from Year Four, there was no record of a partial mark and 25 pupils had incorrect answers. For Year Five, there were 15 pupils with correct answers, 3 pupils with partial marks and 29 pupils with incorrect answers. For Year Six, 27 pupils attained correct answers, 3 pupils had partial marks and 20 pupils had incorrect answers.

For **Question 5,** 22 pupils from Year 4 obtained correct answers. There was however no partial mark for any Year Four child and regarding inaccurate answers, there were 20 pupils. In Year Five, 10 pupils had the question right while 1 pupil had a partial mark. For incorrect answers, 36 pupils had inaccuracies. For Year Six, 25 pupils had the question right while 3 pupils attained partial marks with 22 pupils obtaining inaccurate answers.

For **Question 6**, there was a total of 21 pupils who had correct answers from Year Four, with no one attaining a partial mark. For incorrect answers, 26 pupils attained that. For Year Five, 9 pupils had the answer right, no partial marks were recorded and 38 pupils had incorrect answers. For Year Six, one recorded 24 pupils with correct answers, 4 pupils with partial marks and 12 pupils with incorrect answers.

4.3.4 Examination of Results for Medium Order Questions

The overall performance of pupils on the medium order problems was worrying; across the Year Groups, inaccurate answers soared as opposed to answers which were recorded as accurate. An examination of the marked sheets unravelled pupils' challenge at conceptualising knowledge and their clear unfamiliarity regarding the nature of non-routine problems. Owing to this, most pupils

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could not examine questions in detail to unlock the fact that non-routine problems were different from the everyday routine questions most of them were used to. If they had had a better understanding of the nature of these problems, they would have realised that such peculiar questions always required more commitment, focus and zeal in order to discover tricky bits. This struggle therefore saw many of them simply putting down answers without carefully thinking through problems and using correct methods to get answers. Again due to this deficiency, pupils failed to recognise that particularly for word problems, there was the need to highlight key words which always provided great hints towards accurate problem solving paths. Due to this, pupils read questions without paying keen attention to some useful information that could have helped them to have a better idea of how to solve problems the right way. It was therefore not surprising that the inaccuracies recorded for these set of questions were even more significant this time.

Pupils' lack of problem solving skills were also evident across the Year Groups. In light of this, pupils failed to show meaningful working out processes; hence, most of the answers written down could not be linked to sound written methods. It was also clear that pupils were not too conversant with the process of problem solving itself which required problem solvers to think through questions more critically and couch right routes to derivation of answers. In view of this, most of the pupils worked without showing systematic approaches or step by step procedures that could have lead them to their final answers; and even for those who made an effort to show some knowledge of problem solving ability, most of their methods were meaningless, having no bearing with questions.

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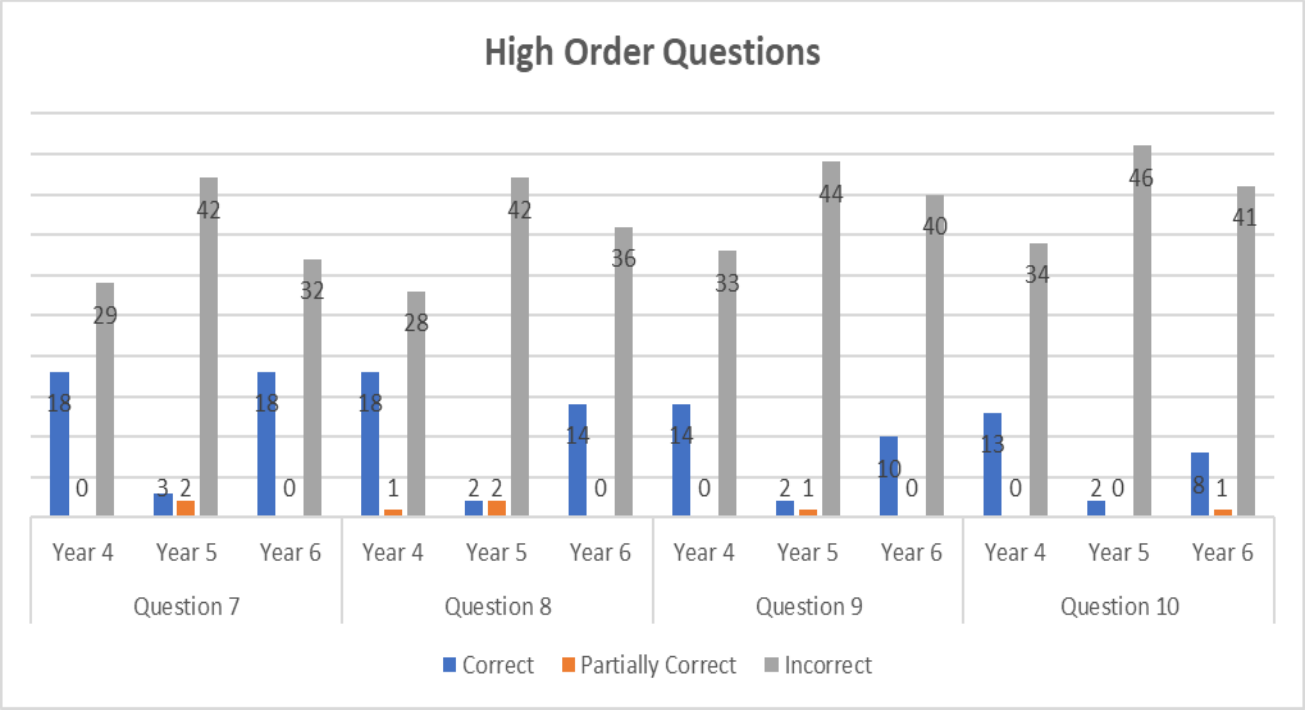

Figure 4: Pupils Performance on High Order Questions

4.3.5 Outcome of Pupils' Performance on High Order Questions

Questions described as high order problems for each Year Group were from **Questions 7 to 10.** For **Question 7**, 18 pupils from Year Four had accurate answers, no record of a partial mark was observed and 29 pupils had incorrect answers. For Year Five, one had 3 pupils with correct answers, 2 pupils with partial marks and 42 pupils with incorrect answers. For Year Six we had 18 pupils attaining correct answers, no record of a partial mark and 32 pupils with incorrect answers.

For Question 8, we had 18 pupils from Year 4 obtaining correct answers. There was 1 partial mark recorded and regarding inaccurate answers, there were 28 pupils. In Year Five, 2 pupils had the question right while 2 pupils had partial marks. For incorrect answers, one recorded 42 pupils. For Year Six, 14 pupils had the question right while there was no partial mark attained. For inaccurate answers the record was 36 pupils.

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For **Question 9,** there was a total of 14 pupils who had correct answers from Year Four with no pupil attaining a partial mark. For incorrect answers 33 was recorded. For Year Five, 2 pupils had the answer right, 1 pupil had a partial mark and 44 pupils had incorrect answers. For Year Six, we had 10 pupils with correct answers, no pupil with a partial mark and 40 pupils with incorrect answers.

For **Question 10**, there was a total of 13 pupils who had correct answers from Year Four with no pupil attaining a partial mark. For incorrect answers 34 was recorded. For Year Five, 2 pupils had the answer right, no partial mark was recorded and 46 pupils had incorrect answers. For Year Six, we had 8 pupils with correct answers, 1 pupil with a partial mark and 41 pupils with incorrect answers.

4.3.6 Examination of Results for High Order Questions

Generally, pupils' performance on this category of questions fell drastically with more inaccuracies recorded across each Year Group. Though the questions within this level were high order problems it was anticipated that since they had been set appropriately to match the ability of each Year Group, more pupils could have exerted themselves in thinking critically to find the best procedures for a considerable number of questions. However, it was evident that pupils struggled on these problems. Again, the graph showed that Years Six and Four had the least marks for partial marks suggesting that most pupils in Year Six this time, failed to exert themselves as compared to their earlier performance. The graph also showed that inaccurate answers recorded were usually obtained by more than 30 pupils in each Year Group while minimal values (single digits) were recorded for correct answers across the Year Groups. This brought to bare the limited opportunities pupils are given to work extensively on non-routine problems hence it was clear that as problems got more complex, pupils showed less zealousness at working. One can also conclude that because

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they have not trained and challenged themselves to explore suitable routes to obtain answers for non-routine problems and again, use their creativity to uncover the 'mystery' enshrined in non-routine, there was enough evidence that they struggled even more on these set of questions. Failure to apply critical thinking skills was also lacking and based on this, pupils showed less competence at understanding questions from a conceptual angle. Lacking this skill, most pupils left spaces blank and showed a challenge at application of knowledge.

The findings from the quantitative study provided a good response to Research Question One by clearly unearthing some factors which characterise pupils' experiences of solving non-routine questions; a number of them included their lack of problem solving skills, their lack of conceptual understanding, their lack of critical and creative thinking abilities, poor attitudes towards work and challenges at reasoning mathematically.

4.3.7 Qualitative Results

The second phase of the study highlighted on interviews. For this process, 18 participants were drawn from the sample (6 pupils drawn from each Year Group) to respond to questions which were carefully selected to address the research questions earlier stated in the introduction of this chapter. In the subsequent paragraphs, a thorough examination of the responses were given a broader outlook.

One of the common themes captured in the responses focused on pupils' lack of confidence when solving complex type problems. For these individuals, being presented with problems of such nature had an impact on their self-esteem and for this reason, their confidence levels waned anytime they were confronted with exercises which were complicated in nature. For example, Jean (Year 4) expressed that whenever she had to work independently on tasks which were complicated

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in nature, her confidence level fell. Ekow (Year 5) also chipped in a similar view, stating categorically that complicated problems always deflated his self esteem. In the earlier assessment carried out at the quantitative stage, these revelations were translated in a number of ways. For example, most of the pupils with very little determination for solving the assigned problems did not engage themselves actively during the problem solving process. In view of this, either spaces were left blank or pupils literally failed to attach any importance to the problem solving process. It was therefore evident that the keenness, resilience, positive attitude and commitment one requires when solving such tricky problems were non-existent and this was particularly due to the lack of confidence exhibited by pupils. Again, the expression of lack of confidence which ran through the responses across the various Year Groups signalled the fact that this was indeed a problem not centered on only one particular Year Group but across the Upper Primary.

Another relevant point disclosed by the pupils dwelt on the fact that most of them struggled anytime they had non-routine questions to tackle. In other words, steps that had to be taken such as reading the questions well and finding accurate problem solving paths to successfully derive answers remained challenges and therefore, solving some of these tasks became a struggle. It was therefore clear that when it came to working competently on non-routine exercises, pupils did not obviously see such questions as easy but rather questions tagged with difficulty. Linking this to the attitude shown by most pupils during the assessment period, it is relevant to state that for most of them, there was simply no motivation, for they virtually threw in the towel to the extent that it became a struggle for them to think along accurate paths and adopt right methodologies for solving questions. Kirsten (Year 4) reiterated that Mathematics was literally a difficult subject and especially when working on complicated questions she always struggled. Jael (Year 6) also had a general phobia for anything which involved critical thinking hence she struggled with how to

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figure out answers. In view of this, being asked to solve non-routine questions was always a requirement she often felt uneasy about.

Other themes focused on the emotional state of pupils when confronted with challenging problems. For instance, some of them commented on the fact that when they were assessed on non-routine exercises, they became 'frightened' and uncomfortable. In such a state of being fearful, at the sight of non-routine questions, it is obvious that the required preparedness, motivation and ability to think within the right frame of mind often disappears, compelling the learner to struggle. Again, it must be added that such presence of 'fright' creates an environment of nervousness crippling the learner from working to the best of his or her ability due to blockage of ideas.

Two participants also gave responses pointing to the fact that though problems of this nature were difficult, such tasks always gave them opportunities to try their best. Their responses obviously revealed their determination, readiness and positive commitment towards work and it is therefore not surprising that these two obtained marks which fell within the higher ability group. That notwithstanding, most of the pupils confessed that what characterises their experiences when solving non-routine problems included waned confidence, discomfort, lack of self-esteem, fright and even feelings of failure.

RESEARCH QUESTION 2: What explains, if any, the differences between pupils understanding of routine and non-routine mathematics problems?

Qualitative Results

Pupils revealed that when working on maths problems that were simple and straightforward in nature, they become very confident. These responses were in sharp contrast to the previous responses they provided when asked about their feelings towards non-routine problems. One

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therefore perceives that when pupils have problems that are manageable, they have a boost in their esteem and are more committed towards work. Again, they are not confronted by a sense of failure knowing that they can stand up to the task. Additionally, this revelation showed that for most of these pupils, they had over the years acquired more opportunities to work on routine problems which often do not require so much utilisation of critical thinking skills hence, their natural response to questions of such nature remained positive. For this reason, it was evident from the answers provided that when they are presented with routine questions a comfort zone is naturally created and their familiarity with such straightforward problems automatically enables them to find quick solutions to problems unlike non-routine problems where they often struggle. Another point worth noting is the fact that for these routine problems, they do not have to call on adults for support. This literally brings out the fact that for most pupils in the school, their preference in terms of maths problems was more titled towards routine tasks which they saw as less stressful, less time consuming and manageable; and therefore, calling on teachers for prompts and guidance was not so much required in this case. Jean (Year 4), Meher (Year 4), Ekow (Year 5),Zoe (Year 6), Sean (Year 5) and Kirsten (Year 4) were among some of the pupils who vehemently stressed that anytime they had routine problems to work on, their confidence levels soared owing to the fact that they were positive they could work excellently on such problems.

Others revealed that they get excited when presented with problems of this nature in that they believe they can work accurately on them. This clearly brings to light the perception learners have on routine problems. They consider them as problems one can easily find solutions to and for this reason, they believe that when they are presented with tasks of such nature, they do not have to put in too much effort to find answers or solutions to. For most of these pupils who disclosed that they felt excited when asked to work on routine problems, another key point they talked about was

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the fact that they regarded such problems as easy and for this reason, they were convinced they could have full scores or get all answers correct when asked to work on them. This revelation makes it clear that most pupils are more comfortable with mathematics problems that are not too involving, too cumbersome and complex. These revelations, however, are worrying especially in our world today where educators are being challenged to engage the minds of learners along critical thinking paths and help students to develop their aptitude and skills in the area of in-depth reasoning and analysis. If therefore we have learners who are not ready to challenge themselves, own their learning, 'crack' their brains and engage in more productive problem solving processes, then our quest to nurture confident, committed, resilient, innovative and critical thinkers for the future will be a dream far-fetched. Again, we will produce learners who will only be satisfied with mediocrity with no conviction to aspire or exert themselves in their learning.

Richard (Year 4), Nathaniel (Year 6) and Bridget (Year 4) when interrogated expressed that such problems in their opinion were always easy exercise to handle and for this reason, they always thought they would attain full scores.

Another interesting point made by one respondent revealed that when she is tasked to work on routine questions, she normally gets bored. This discloses the point that not all pupils love to work on routine problems. In the school where the study was carried out, it was obvious that although most pupils preferred to work on routine problems, a few of them still loved to be challenged and therefore they preferred being presented with non-routine questions as opposed to routine problems. This particular pupil obtained a score that was within the high range which definitely points to the fact that the problems assigned were questions she had a liking for. In view of this, she had a positive attitude towards the assessment and strove hard to work purposefully on each task.

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Again this draws one's attention to the fact that as teachers, we need to find opportunities that will break the boredom in class especially for the gifted and talented pupils who often wish to be challenged more than other groups of learners. It is therefore important, that for such individuals we carefully differentiate exercises that are challenging and appropriate for their level as they always show readiness to engage in critical thinking processes.

Finally one respondent also declared that when presented with routine questions, he always had the premonition that he would not get all the questions right. This was a relevant point in the sense that most often pupils do not pay particular attention to these routine problems and in view of this encounter errors. This was also a good point to highlight the fact that though one could be presented with routine questions, there could be instances where these questions could be tricky in nature and therefore not carefully following the right procedures could give one an inaccurate answer. Personally, I was convinced that this particular pupil was talking from experience and perhaps stressing that he did not do too well on some routine questions in the past because he did not pay too much attention to detail. It is therefore worth noting that even with presumably 'simple' questions such as these, teachers have to prompt pupils that during the problem solving process, they should be careful in the procedures they adopt while checking for accuracy all the time.

Without a doubt, it was obvious that pupils had a good understanding of what routine problems were and therefore they could literally express how they felt when presented with routine questions.

It was clear from their responses that most pupils preferred these questions, loved to work confidently on such problems which looked manageable in nature and often relied on adult support when problems were tricky. The assumption one therefore had from the responses was the fact that most pupils preferred more of routine questions as opposed to non-routine problems.

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However, the answers they provided when asked about their understanding of non-routine questions revealed some surprising facts with majority of pupils disclosing they liked mathematics problems which made them think and gave varying reasons for their choices; out of the 18 respondents who were interviewed, 16 of them confessed they liked maths problems which compelled them to engage in deep thinking.

Jean (Year 4), Ekow(Year 5),Jael (Year 6),Maya (Year 4), Sean (Year 5) and Eric (Year 6) similarly stated that they had a liking for complex type problems or non-routine problems due to the fact that it challenged their ability, enabled them to put their brains to use and also offered them an opportunity to engage in more rigorous problem solving processes.

In the area of mathematics, non-routine problems are often the type of problems that offer opportunities for learners to engage in more in depth critical thinking and therefore if the same pupils who earlier disclosed their affection for routine problems revealed, at the latter stage, they also had a passion for mathematics problems that made them think, then there are obviously some interesting facts and questions to answer.

- Do most pupils really know what non-routine questions are?
- Do most pupils really know what to look out for when working on non-routine problems?
- Have pupils been well taught how to recognise non-routine problems?

These should serve as guiding questions for teachers when they are delivering lessons which focus on non-routine problems for if pupils lack a proper understanding of what these type of problems are, then we do not expect them to work competently on such questions. This is to say that teachers owe it a duty to properly introduce pupils to non-routine problems right from the fundamental stage. These include giving them a solid understanding of what these problems are, guiding them

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to identify questions that are non-routine in form and taking them through some relevant steps or processes that serve as eye openers when handling such complex problems. It was clear that though these pupils had a good understanding of routine problems, some possibly lacked an understanding of what non-routine questions were.

Nonetheless, there were two respondents who expressed their dislike for questions of this nature stating that they spent too much time on them, hence they were honest enough to highlight earlier that they preferred to answer questions that were routine in nature.

The results from the findings of both the quantitative and qualitative studies point to the fact that most pupils faced challenges on non-routine problems and had over time not been given opportunities to get accustomed to problems of this nature. Their handicap and unfamiliarity with excellent problem solving skills for working on non-routine exercises undoubtedly curtailed them from working to the best of their ability and these challenges were further envisaged in the responses they provided during the interview session. Based on these findings, it is a fact to state that non routine problems were indeed a struggle for most of the pupils assessed.

RESEARCH QUESTION 3: **What opportunities are pupils given to engage in collaborative and co-operative learning?**

Qualitative Results

The responses provided by pupils pointed to an interesting fact that most pupils were not given the privilege to work with others when presented with difficult maths problems. The genuineness in the answers provided clearly suggested that across the Upper Primary, more attention was given to individual centered work and owing to this, there were limited avenues for pupils to work together. For this reason, in the event pupils found questions challenging, their chances of breaking

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through mainly depended on adult support. For other children, who were not so comfortable to draw the attention of teachers or Teaching Assistants for help, their alternative was either to crack their brains in a bid to find solutions to problems or simply give up on questions, as was evident in the attitude of most of the pupils earlier assessed. Ekow (Year 5), for example, said he could not recall being given the opportunity to work with others although it had happened once or twice in his previous class. Jael (Year 6) also expressed a similar view and further added that she wished practices of this nature existed where children could work with one another or in a group. Sean (Year 5) stated that collaborative or co-operative engagement among pupils was never 'heard' of, signalling the fact that on a typical day in his classroom, all tasks assigned to pupils, in the area of mathematics, were often independently structured. Owing to this, pupils were not offered the chance to deliberate on questions in groups or through partnership.

Others revealed that sometimes, they were given opportunities to work with one another but it was clear that this was not a frequent practice. If this was the case therefore, then it was an undeniable fact that even in most of these classes where children were given opportunities to work with one another, more concentration was still offered to independent work, most of the time. One can therefore conclude that in the Upper Primary, pupils greatly missed out on good practices such as co-operative and collaborative learning which have proven to adequately support the headway pupils make especially in their understanding of non-routine problems. It is also worth noting that in a typical problem solving class where the teacher's focus is more on pupils making progress on non-routine exercises, more opportunities are offered to pupils to engage in collaborative and co-operative work and this is always at the heart of affairs; unfortunately, the pupils assessed had no taste of these all important practices. Still on the benefits of collaborative and co-operative learning, it is relevant to note that pupils benefit greatly from peer centered teaching at a more

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child friendly level and are able to explain concepts to one another in a less tensed atmosphere hence pupils are able to put their thoughts together and delve deeper into finding appropriate solutions to problems. In this situation where pupils, on a regular day, got very limited chances to engage collaboratively, it was clear that their reasoning and creative abilities were being underutilised. The old saying which states: Two heads are better than one probably endorses such good practices highlighting more on the fact that when pupils come together to work collaboratively, varying ideas are brought to the table with each child being accountable. Therefore, in this scenario where pupils are constantly being given limited offers to delve deeper into co-operative learning, pupils who work well or learn better with their peers are being hindered from accomplishing their full potential. Again, it also points out the fact that in most of these classrooms, traditional practices or methods of teaching and learning are still being encouraged thus hindering the successful implementation of these 21st century practices which have been tried, tested and seen to aid pupils' understanding of non- routine problems. Furthermore, pupils' lack of opportunities to engage in collaborative learning also hinders the progress of some able pupils who grasp concepts better when they take charge of explaining methods and processes to their peers.

Evidently, it became obvious that most pupils across the Upper Primary are often denied co-operative and collaborative learning practices which, to a large extent, promote pupils' understanding of non-routine problems and offer more avenues for pupils to engage in paired work, group activities and presentations which aim at boosting their knowledge of mathematical concepts particularly in the area of non-routine tasks.

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RESEARCH QUESTION 4: What challenges do pupils encounter when conceptualising their understanding of non-routine mathematical problems?

Qualitative & Quantitative Results

A number of respondents revealed that in the event where they encountered difficult maths problems, they called for assistance mainly from the teacher or the Teaching Assistant. Most often, drawing the attention of these adults enabled them to have further explanations to questions, more clarification on how to approach problems, quick prompts on what they needed to consider in ensuring accuracy and guidance on how to solve specific problems. *Jean (Year 4) stated that 'most often, she would raise her hand and ask the teacher for help or would try very hard to recall what a teacher had said previously about a problem'*. Eric (Year 6) also expressed similar views stating that when struggling, he would always ask the teacher for help. Again, these responses were similar to answers other participants provided across the Upper section thus denoting the fact that for most of them, anytime they were presented with non-routine questions, they counted on teachers for more ideas on how to answer such questions. It is clear that the lack of adult support during the assessment contributed considerably to pupils' inability to actively engage in the process. More so, having been used to adult support on a regular basis, it became evident that they had become overly reliant on their teachers, hence it became difficult for them to own their learning and put in their best efforts. From this observation, we can conclude that having been used to regular guidance and support from teachers anytime they got stuck on mathematical problems, the assessments thrusted them into a state of limbo.

Another section of pupils felt more relaxed relying on peer assistance. For these individuals, whenever they encountered difficult problems, their immediate call for help had to be a friend they believed could offer them the required support. This pointed to a key fact that pupils do observe

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the competencies and strengths of their peers and therefore know who to call on in the event of any challenge. *Nathaniel (Year 6), for instance, said' that his best friend was better at Maths than he was and since they were both in the same class, he could call on him for help or ask him for clarification anytime he was stuck on a problem'. Rossa (Year 6) also stated that 'she felt more comfortable asking a friend for help than the teacher simply because drawing the teacher's attention too much would suggest that she hardly paid attention in class'*. These revelations point to the fact that regarding non-routine problems, it is likely that pupils may prefer to learn better and make headway through peer coaching or support because a more relaxed environment is created where the child being peer coached can easily ask for clarification on problems that seem complex in nature. This literally throws more light on the fact that pupils work differently, have different learning styles and specific learning needs and therefore teachers ought to observe their pupils and offer them opportunities that serve as gateways to their progress. For instance, if a child works better by partnering with another pupil, there is the need for such an individual to have more of such working relations and this could be through paired work or group activities. While this is ongoing, the adults in class could facilitate the process by guiding the effectiveness of such peer support. It is relevant to state here that in the school where this study was carried out, though peer opportunities are given to some extent, the emphasis is often on independent work thus giving less opportunities to pupils who otherwise benefit from peer support.

There were other responses which brought out the fact that for some pupils, encountering a difficult problem meant failure in the sense that these were pupils who would not call for assistance whatsoever. In view of this, they preferred to struggle on questions without any attempt to try their best. Often times, these typical group of learners have an innate 'phobia' for Mathematics and as such would not want to exert themselves or call for support when the need be. In the earlier

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assessment, a number of pupils who fell within this group visibly conveyed this message through body languages which suggested unpreparedness, sheer laziness, lack of zeal and incompetence.

Again, a minimal number of pupils also dwelt on the fact that anytime they were confronted with difficult maths problems, it gave them an opportunity to engage in more critical thinking processes while striving relentlessly to find solutions to answers. For these pupils, their priority therefore did not focus on being so reliant on adult support but rather believing in themselves and attaching a high level of commitment to the work assigned them and for this reason, they had a positive attitude towards solving non routine questions. Again for these individuals, encountering difficult problems did not connote failure or a limitation to what they could do. It rather offered them a privilege to be committed to the problem solving process by using all means possible to tackle and find possible solutions to questions. Unfortunately, from the assessment carried out, the performance of pupils revealed the fact that those with such great self-esteem, resilience, determination and focus numbered only a few and these were the pupils who obtained marks which fell within the high scoring range.

From the diverse answers provided, it was clear that pupils when confronted with non- routine questions of a seemingly challenging nature often failed in their bid to apply their creativity, imagination and critical thinking abilities to solve questions. A number of pupils therefore, unable to conceptualise their understanding of complex questions often resort to adult support or simply fail to engage in effective logical reasoning processes.

4.4 Chapter Summary

The chapter analysed the data based on the objectives of the study. At the quantitative stage, for example, the assessments administered enabled the researcher to identify the specific challenges

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that pupils encounter when required to work on non- routine questions. Again the responses from the participants at the qualitative stage offered learners the opportunity to express their views regarding their perceptions, beliefs and attitudes towards problems of this nature. The findings generated at both levels of the study were in sync and dominantly bothered on the fact that as far as the solving of non-routine problems was concerned, pupils' skills at working accurately were challenged. In the next chapter, the findings generated will be thoroughly discussed as well as highlights on some effective recommendations.

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CHAPTER FIVE

DISCUSSIONS, CONCLUSIONS AND IMPLICATIONS

5.1 Introduction

This section forms the final chapter of the study and here, attention will be given to discussions pertaining to the findings of the study which were presented in Chapter Four. The significance of this part of the study therefore seeks to spark discussions on how the findings from the study answer the research questions posed in chapter one and of course, how the comprehensive study positively adds to existing literature.

The chapter will be divided into five sub sections and this will include revisiting the purpose of the study and research questions, summarising the key findings from the research, discussing possible contributions in the area of Mathematics education with special emphasis on non-routine problems, discussions on the implications of the findings from the study and finally a chapter summary.

5.2 Revisiting the Purpose of the Study and Research Questions

The need for pupils to be competent at solving non-routine mathematics problems remains a dream far-fetched. As a Primary School teacher with significant length of years in the classroom, I have over time being convinced that most pupils continue to underperform in this area of mathematics which usually requires application of knowledge, logic and reasoning. These shortfalls continue to be worrying and with no signs of improvement from learners, several studies have been carried out aimed at finding the way forward. It is against this background that my interest was sparked to carry out a research in the area of non-routine problems and plunge deeper into why pupils fail to

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make progress when confronted with complex type problems. The study therefore sought to find answers to the following research questions:

1. What characterises pupils' experiences of solving non-routine mathematical problems?
2. What explains, if any, the differences between pupils understanding of routine and non-routine mathematics problems?
3. What opportunities are pupils given to engage in collaborative and co-operative learning?
4. What challenges do pupils encounter when conceptualising their understanding of mathematical concepts?

5.3 Summary of the Findings

Pupils' Experiences with Non-Routine Problems

5.3.1 Unfamiliarity with the nature of non-routine problems

Based on the findings, pupils' lack of familiarity with the nature of non-routine problems restrained them from working competently on questions. For this reason, pupils treated the tasks as everyday routine problems thus showing less commitment; many of them failed to read the questions meaningfully, failed to process information accurately, failed to adopt sound methods in the process of working out and failed to use their critical thinking abilities to find solutions to problems. Based on this deficiency, learners also spent too little time working out problems for they lacked knowledge of the very nature of non-routine problems which require more in-depth thinking and detailed problem solving processes before answers can be derived. It is based on this expectation that Lester and Kehle (2003) reiterate that non-routine problems involve higher order thinking processes and for that matter, as per their nature, learners are required to engage in deep

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reasoning. This requires learners to be more analytical and engage themselves in series of critical thinking processes in order to solve problems of this nature.

Again, the findings showed that there was very little evidence of pupils engaging in the use of meaningful approaches to find solutions to answers. In view of this, pupils failed in their attempt to develop strategic pathways that were accurate enough to possibly lead them to right answers. Many of them, therefore, showed no effort in the working out stage and could not spell out their methods on paper and for this reason, some decided to simply write down wrong answers, with absolutely no clue of how they arrived at their final destination. Additionally, the assessment itself was structured in a way that learners could have attained marks for partially accurate answers but due to the fact that pupils lacked the ability to engage in strategic thinking processes, most of them did not put in any effort to adopt routes which could lead them to accuracy. This weakness further attests to the fact that pupils lacked familiarity with the nature of non-routine problems, for if they had, first of all, been able to detect that the word problems they were working on required unlocking of mathematical 'puzzles,' they would have paid more attention to detail while engaging in fiercer strategic reasoning processes to discover answers. O'Brien and Moss (2007) describe non-routine problems with reference to their puzzle like nature. This hammers on the fact that learners should envisage non-routine exercises as puzzles being carefully put together in order to have an excellent finished piece. During the process of problem solving, therefore, there is the need for the learner to carry out some strategic thinking and in the study, there was very little evidence of this.

Limited use of mathematical strategies

Pupils showed less knowledge of appropriate strategies during the assessment stage which is a great requirement if we expect pupils to be good at non-routine problems. This prevented them

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from being able to try out a number of methods in the event their initial choice of strategy proved unsuitable and in light of this, the use of diverse methods was a missing ingredient. Masses of pupils, therefore, struggled in their quest to develop strategies for solving problems and when a child got stuck on a specific question, the drive to adopt other meaningful approaches became a challenge. Demetriou (2004) states categorically that non-routine problems come with the use of varied strategies presenting the problem solver with opportunities to include diverse strategies during the problem solving stage. To add more, majority of the pupils fell short of certain pertinent qualities required of a determined learner such as being poised to conquer all odds to own his or her learning. In view of this, many of them were simply not resilient, creative in their thinking and could not challenge their mental agility to an appreciable level. Again, they showed less willingness to own their learning and work independently. Francisco and Maher (2005) hold the view that problem-solving recognises the power of children's construction of their own personal knowledge and lay emphasis on minimal interventions in the pupils' mathematical activity which always offers him or her an invitation to explore patterns, make conjectures, test hypotheses, reflect on extensions and applications of learnt concepts, explain and justify their reasoning.

Non Flexible Use of Methods

The findings revealed that there were quick spillage of answers that were mostly inaccurate and not backed by written methods or procedures. Again, unfamiliarity with flexible working methods was also evident due to pupils' difficulty to think outside the box and for this reason, they could not devise meaningful processes tailored towards attaining solutions to problems. Additionally, during the assessment, pupils were crippled in the use of flexible methods thus showing less competence at trying out methods which were good enough to help them obtain answers. It was therefore evident that pupils had a blockage in ideas thus restraining them from adopting an array

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of flexible problem solving processes. What was even more worrying was the fact that pupils did not commit to the process of engaging in logical processes and reasoning excellently and for this reason, many of them left questions unanswered, in the event of difficulty. Kolovou (2011) lays emphasis on the fact that being able to ensure flexibility while working on problems is paramount and according to Demetriou (2004) by showing such competence at flexibility, learners can develop more refined concepts and appropriate solutions to problems. Cai (2003) is convinced that when working on non-routine problems, one's success largely depends on one's choice of flexible methods. In view of this, a proper understanding of questions remains vital for it promotes a fine terrain for learners to adopt the best methods when working.

Negative Attitude of Pupils towards Non-Routine Mathematics Problems

Based on the findings, a few pupils who had challenging behaviour and for this reason portrayed negative attitudes during the assessment had some commendable marks making them as equally strong as those limited pupils who generally excelled due to their commitment. In order to excel at non-routine mathematics problems, more researchers keep hammering on the fact that attitudes among students must change for the better. Ajzen (2005) believes that a learner will automatically do well on non-routine problems if he or she has the right attitude and commitment towards work. In like manner, a negative attitude will be detrimental for the learner because he or she will not be so committed in the process of solving problems. In the study carried out, however, it was obvious that this belief did not hold in that some pupils who did not really show much enthusiasm and commitment for the assessment had some good scores. To add to this, there were also some pupils who worked determinedly and seemed to have the right attitude towards work but unfortunately, performed poorly. This is to say that positive attitude having a correlation with excellence especially when working on non-routine problems is not always the case.

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Inability to Order Information

The results showed that pupils lacked the ability to infer meanings from questions and owing to this, they could not highlight important information, key words, discover clues and show the ability to organise facts accordingly. For this reason, pupils rushed through work committing less to the problem solving process. This was due to the fact that they lacked the ability to adequately create meaningful and creative approaches to work. Additionally in the study carried out, learners speedily wrote down answers as if the word problems they were being assessed on were like any 'ordinary' questions most of them were used to; Çelik and Güler (2013) believe that primary school pupils make unpardonable errors because they solve non-routine problems like routine ones and based on the findings, I do emphatically agree with his observation. Failure to read questions thoroughly, think through problems carefully and couch suitable approaches to derive answers were therefore the weaknesses of many of the pupils. When solving non-routine problems, there is the need to inculcate not only operational skills but also show knowledge of how to organise, classify and adapt data to real life (Yazgan and Binta, 2005). This literally suggests that simply being knowledgeable of the use of operational signs is woefully inadequate; the learner must show further competence at being able to own his or her learning the creative way and being able to have excellent conceptual understanding of concepts. This means the learner should be able to make meaning out of questions by processing all necessary information accurately and being able to apply his or her knowledge of concepts the right way. This also involves using the most appropriate approach or strategy to arrive at answers and the need to show a high level of knowledge of how to figure out relevant facts from which the learner can develop better clues to arrive at a solution.

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Lack of familiarity with Problem Solving Teaching and Learning Approach

It was evident through the research that pupils lacked this skill of adopting right problem solving approaches thus signalling the fact that learners were not given opportunities to engage in such regular processes. Owing to this, most of them were challenged when it came to the choice of suitable methods and strategies to adopt for working out a particular type of problem. This resulted in working out processes that were disjointed, muddled up, not precise and which obviously lacked substance. Again failure to use accurate mathematical approaches saw most of the pupils deciding to write down answers which had no bearing with questions and to make matters worse, many of them failed to support their answers with sound written methods. Additionally, one could tell that their unfamiliarity with problem solving approaches also hindered them from reasoning to a considerable extent. In light of this, though they could read questions independently, they unfortunately could not make meaning out of these questions, and therefore, their low reasoning abilities curtailed their competence at using methods that were meaningful enough to help them arrive at expected answers. If most pupils had been benefitting from frequent problem solving practices especially on non-routine problems, their performance could have been better. The need to enhance pupils' abilities of non-routine problems is to create regular opportunities whereby learners will continuously have access to engage in rigorous problem solving processes. Through such frequent practice, learners' knowledge of useful approaches to adopt when working on non-routine will be broadened. Siemon and Booker (1990) suggest that to do this effectively, teachers should ensure that Problem solving approaches are taught religiously. They therefore prescribe three significant approaches to teaching which includes teaching for, teaching about and teaching through problem-solving. According to both scholars, these approaches, for example, enable learners to build on their learning, starting from very simple processes to more complex ones.

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Again, using such approaches in teaching problem solving, make the learning more picturesque and orderly for the learner who, later on, will rely on such models or approaches to develop more complex ideas. This may obviously require the teacher to go step by step and gradually aid the pupil to develop expertise when working on even more complicated problems. Again, they believe that the acquisition of such basic knowledge will strengthen and help the learner to apply the concept learned. That notwithstanding, there were few pupils naturally gifted at solving non-routine problems who worked to the best of their abilities during the assessment. This revealed an interesting fact compelling the researcher not to fully believe that in order for pupils to understand and work better at non-routine tasks, the onerous fell on the teacher to be able to teach the skill of problem solving itself. In my view it also largely depends on the innate ability and determination of the individual.

5.4 The Contributions of this Research

Over the years, several scholars have probed into the teaching and learning of mathematics, carrying out significant studies in diverse areas. However, having acquainted myself with a number of works done in the field, I have realised that in helping learners to work better at non-routine tasks, more attention has been given to the promotion of best practices such as the adoption of effective teaching methodologies, review of the curricular, helping pupils to overcome language difficulties, having a better look into problem solving approaches, among others. Very little, however, has been done in dissecting the learner's self to unearth his or her experiences when solving non-routine mathematics. After all, the concern of most researchers in the field of mathematics education is to see learners excel particularly in the area of non-routine problems and if this is the case, being able to have a solid understanding of how these pupils learn, behave and approach complex type problems remain steps in the right direction to enable us sympathise with

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them and come to terms with their emotions, perceptions and the experiences they go through when tasked to work on complicated questions.

Another significant contribution centers on the setting of the research. Though most studies in the area of non-routine problems continues to be carried out to depict the plight of learners' on non-routine problems, it must be stated that the setting for most of these researches are usually in the government schools and therefore extending the study to focus on an international school like G.I.S. has provided a good opportunity to announce that weak standards in the field of non-routine problems are not only peculiar to public schools but also prevalent in top rated privately owned schools where the environment is often favourable in terms of teaching and learning.

Over the years, researchers, in mathematics education, have contributed in making many to believe that most learners do prefer routine problems to non-routine exercises. Following this, there have been several researches aimed at enabling learners to overcome their phobia for these type of problems. However, a major contribution of this research aims at demystifying such scholarly pronouncements. During the interviews, it was revealed that a greater number of pupils stated categorically that though they liked routine problems, they also had a passion for non-routine problems in that it made them utilise their brains; an indication that learners do not loathe complex problems after all, as it has been portrayed.

Another area most researchers continue to look at focuses on positive attitude and its correlation with excellence on non-routine problems. It is believed that when pupils have the right attitude towards work such as being resilient, determined, focussed, able to engage in creative and critical thinking processes, they attain high scores on non-routine problems. This research, however, has debunked this assertion owing to the fact that during the assessment, a few pupils who did not exhibit right attitudes to work ended up having some good scores. On the contrary, some pupils

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who glaringly focused and showed positive attitudes did not do too well on the questions. This unearths the fact that having the right to work when working on non-routine problems is not a total guarantee for one's success; a pupil could be naturally gifted at the subject and excel due to his or her inherent ability and not always a matter of attitude.

5.5 Limitations of the research

A major limitation of the study was the sample size used which was relatively small considering the fact that this was a case study which focussed specifically on G.I.S. The inclusion of more private schools would have drawn a larger proportion of participants which would have provided a more representative view, which could have been generalised to a wider population.

Another limitation of the study focussed on my own research biasness which stemmed from my own experience as a classroom teacher; having acquainted myself with the attitudes of pupils especially when tasked to work on mathematics problems in general, I had to make a conscious effort to be as objective as possible especially during the process of data collection, analysis and dissemination of findings.

5.6 Recommendations

The limitations of the study, however, did not hinder the significance of the study. It brought to bare some useful information and insights necessary to promote effective teaching and learning of non-routine problems. For example, the findings admonish teachers to develop the problem solving skills of learners through effective teaching of problem solving processes to broaden students' confidence and competence when working out problems which fall under both categories, that is both routine and non-routine problems.

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Again, the results of the findings further charges teachers to be more explicit when teaching students about non-routine problems. For example, in order for pupils to make head way, there is the need for teachers to go through some of the characteristics and nature of such problems with their students so that whenever they are required to work on them, such questions will not be treated as mere 'routine' problems but problems which rather require time, effort, reasoning, the use of appropriate strategies and some level of critical thinking.

In our contemporary world today, where there is so much advocacy for best practices to promote the effective teaching and learning of mathematics; the findings give some useful information to curriculum developers, policy makers and other authorities within the field of education by pointing to the fact that in reality, what is being preached is not being practiced. For instance, most pupils still do not benefit from adequate collaborative and co-operative learning opportunities which often boost mathematical understanding. This may therefore draw, for instance, the attention of the Ministry of Education and the Ghana Education Service to organise more intensive in service trainings for teachers and also taking a critical look at the curriculum to see how well it encourages collaborative and co-operative learning among students.

Finally, since this study adds to existing literature, researchers, teachers and those within the field of education can fall on it to enhance their knowledge of best practices including ways students can be supported to work better on non-routine problems.

5.7 Conclusion

In conclusion, the study has provided some useful information regarding students' experiences when solving non-routine problems and has hammered on some positive practices essential in aiding pupils to make better progress at non-routine problems and how they can overcome the fear

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often associated with these tasks. Particularly for stakeholders, this urges them to consider the organisation of enhanced workshops and trainings to equip teachers with more competent skills necessary to deliver the curriculum effectively and enhance the entire teaching and learning process, particularly in the area of non-routine problems.

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APPENDIX A

ASSESSMENT FOR YEAR 4

LOW ORDER QUESTIONS

1) There are 75 boys and 21 girls doing the 'Shoo-Shoo' dance. If 11 of them decide to rest after a while, how many children will be left on the dancing floor?

2) Amy's party has just started. So far, Tim has wolfed down 2 pieces of chicken. Lucy intends to have double of this number every hour. If she decides to spend 7 hours at the party, how many pieces of honey coated chicken will she enjoy?

3) Amy is in a floral dress. Her sister, Yuki, has a similar one. If granny bought these dresses for them at a cost of $24.99 each, how much did both dresses cost?

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ASSESSMENT FOR YEAR 4

MEDIUM ORDER QUESTIONS

4) There are 45 dancers at Amy's party. If each of them will be given 8 packets of ginger biscuits, how many of these will be distributed altogether?

5) 334 children have already started swimming in Amy's pool. If this number is expected to triple after three hours. What will be the expected number?

6) Three of Amy's friends are surprising her with 5 books from her favourite author, Roald Dahl. If each book costs $3.17, what is the total cost of the book?

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ASSESSMENT FOR YEAR 4

HIGH ORDER QUESTIONS

7) Amy has invited 256 of her classmates to her birthday party. If each pupil enjoy 9 of cold lemonade, how many cups of this drink will be served in total?

8) Superstar, Kylie Holmes, is singing at Amy's party. She is wearing a pair of boots which she bought for $34.87. She also got her sunglasses for $56.77 and her skirt for $44.33. How much did she spend in total?

9) There are 528 guests seated at the forecourt. If every waiter has been designed to serve 4 guests at this section, how many of them will be required to do the serving?

10) At one o'clock (1 p.m.) 35 children will be allowed to get onto a bouncy castle. These kids will have the privilege of bouncing for 30 minutes. If the supervisor, however, intends to give them more time to bounce until three o'clock (3 p.m.), how many minutes of play will they have in all?

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ASSESSMENT FOR YEAR 5

LOW ORDER QUESTIONS

1) The guards have decided to show us Old Rompti. Born in 1967, he is the oldest male chimpanzee in the zoo. How old will he be in the year 2040 if he is still alive?

2) A lioness is fed with 517 of mutton a day and there are 25 of them in the zoo. How many kilograms of meat will be needed to feed them daily?

3) There's a splendid cafeteria at the zoo and here, a meal of fish and chips is sold for $7.39. This excludes drinks. At dinner time, this is slightly cheaper. It costs $7.23. If Jane decides to buy two packs of fish and chips at 1 p.m. How much will she pay in total?

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ASSESSMENT FOR YEAR 5

MEDIUM ORDER QUESTIONS

4) A porcupine has been trapped somewhere in the zoo. Currently, there are 9 zoo keepers looking frantically for the haired animal. If they are successful, they will share a sum of $4,707. How much money will each zoo keeper receive?

5) All 108 year 6 pupils will be visiting the zoo next week. On arrival, they will be divided into two groups. If the first group expected to be 1/9 of the total number of students', how many pupils will be in the second group?

6) Alfred realizes that he has $38.50 left after paying his zoo pass. If he intends to feed 16 monkeys with bananas which cost $5 a bunch, how many bunches of bananas can he buy with the money left?

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ASSESSMENT FOR YEAR 5

HIGH ORDER QUESTIONS

7) 15 girls and 27 boys are gazing excitedly at 67 wolves. If these animals are provided with 255 large bones every 3 days, how many of these will they require in 30 days?

8) Parrots at the zoo rattle 117 words every 48 hours. How many words will they rattle in 192 hours?

9) A male lion requires 675 kilograms of meat a day while a cub needs ½ of this quantity. How many kilos of meat do both animals require to survive in 24 hours?

10) Year 5 pupils from Hemshead Primary School have arrived at Hunterland zoo. There are 3 streams within this year group and each class is made up of 27 pupils. If each year 5 child paid $5.75 for a zoo pass, how much was the total charge?

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ASSESSMENT FOR YEAR 6

LOW ORDER QUESTIONS

1) Thames Company usually makes a yearly profit of $17,924. Last year, it made its highest hopes to triple the amount which generated last year. What is the expected profit Thames wishes to make?

2) The are 8 gates which lead to Thames Water park. A total of 122,432 visitors can pass through them at time. Round this figure to the nearest 1000.

3) Sue can't find her tube of sun block. Thankfully, her friend Trix has one to spare which she bought for $23.34. If Sue's costs 9 times more, how much did she buy her cream?

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ASSESSMENT FOR YEAR 6

MEDIUM ORDER QUESTIONS

4) Booking a ticket to Thames Water Park costs $40.00 excluding a compulsory tax rate of 4%. How much does Simon have to pay altogether if he wishes to buy himself a ticket and two more for his brothers?

5) Swimsuits are sold in pairs for $78.50. If the price for a pair costs 4 times this amount last year, how much did Katie pay if she bought her swimsuits 12 months ago?

6) After a ride, one can enjoy a litre of orange juice for $4.50. If Ceece has just drank 2.25 litres. How much is this in millilitres?

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ASSESSMENT FOR YEAR 6

HIGH ORDER QUESTIONS

7) While waiting to get onto a ride, visitors can perch in the tents mounted all over. 15,000 of these tents really need to be cleaned. If the workmen intend to thoroughly scrub 3,456 tents every day, how many days will they require to clean all the tents?

8) Frank Reynolds Company produces swimming goggles for Thames Water Park. In the year 2008, they produced 876,419 goggles every month from March to October. How many goggles did they supply within this period?

9) Booking a ticket to Thames Water Park costs $40.00 excluding a compulsory tax rate of 4%. How much does Simon have to pay altogether if he wishes to buy himself a ticket and two more for his brothers?

10) Only 6 children can get into a wave pool at the same time. If there are only 14 of these pools here, how many more will be required to cater for all the 5,257 children here today?

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APPENDIX B

TRANSCRIBED RESPONSES FROM 18 PARTICIPANTS

1) How do you feel when working on Maths problems that are complex in nature?

Jean (Year 4):I struggle a bit because I know that they are difficult questions to answer and if you're not so smart you may end up getting all of them wrong.

Ekow (Year 5): I feel scared because I can't get a full score. Again, they are not so easy to find answers to so you only have to crack your brains and try different methods to find answers which is not so easy.

Jael (Year 6): I feel it's an opportunity to show my teacher that I can do well no matter what so even when I am struggling I like to try very hard and work on the questions I am given. Sometimes, they are difficult but I try my best.

Maya (Year 4): I struggle a lot and I get frustrated sometimes because they are very complex and so most of the time I do not have answers for these questions and I have to spend so much time on them. Sometimes, I even leave the questions blank.

Sean (Year 5): I feel that I am going to fail no matter how hard I try because mathematics is difficult and some are so challenging that you really feel less confident when you are working on them.

Rossa (Year 6): When I work on such maths problems I feel good because I use my brain to think.

Kirsten (Year 4): I do not feel confident in myself because I cannot easily find an answer to a problem.

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Nevin (Year 5): It makes me have no confidence in myself because I feel that I am clueless and I cannot make my grades.

Nathaniel (Year 6):I feel like I have an opportunity to try and work it out because since these questions are complex I believe I have to try my best and get some questions right.

Richard (Year 4): I feel sometimes they are hard and not so easy to find an answer.

Alex (Year 5):I become scared.

Zoe (Year 6): feel a bit scared that I can't work the answer out.

Bridget (Year 4): I don't feel confident because they are not so simple to discover the answers and sometimes I have to think very hard yet I end up not being able to answer them.

Maxine (Year 5):I feel a bit scared sometimes because I know that I am not going to do well on the questions and that means I am going to fail.

Eric (Year 6):I feel uncomfortable because these are problems that I do not like to solve because they are so difficult and no matter how you try, you always get stuck.

Kweku (Year 4): I am not confident when I have complex problems to work on because they are difficult.

Meher (Year 5): I struggle a lot because sometimes I do not a clue of the strategies to use and I end up wasting too much time on questions.

Joshua (Year 6): I struggle on complex problems because sometimes my mind goes blank when I have to work on difficult problems.

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2) How are you supported when you encounter a difficult maths problem in class?

Jean (Year 4): I raise my hand and ask the teacher for help or sometimes I recall what my teacher has said about the problem.

Ekow (Year 5): I ask a friend who knows to help me understand. For example, my friend Brad is good at Maths and I sit close by him so when I don't understand something, he helps me out.

Jael (Year 6): I think hard on my own to find a solution but sometimes my answers are wrong. Sometimes too, I am able to get a few answers correct.

Maya (Year 4): I struggle on difficult problems and when I feel that the questions I am answering are too difficult, I try and try hard to see if I can come up with something.

Sean (Year 5): I feel that I am not going to do well so I become very upset with myself because I become aware that I am not going to pass.

Rossa(Year 6): When I work on such maths problems I feel good because I use my brain to think and it is always a good time for me to show my teacher that I can solve problems on my own.

Kirsten (Year 4): I ask a teacher to explain it to me.

Nevin (Year 5): I ask a teacher to help me to understand what I don't understand. Sometimes, my teacher reminds me of something she said in class and based on that I can try and work on my own.

Nathaniel (Year 6): Sometimes I am supported a bit by the Teaching Assistant who takes her time to remind me of what we have done in class. Sometimes too, she tells me that I should try and solve the problems on my own.

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Richard (Year 4): Anytime I don't understand a problem, the teacher helps me out. Sometimes, the teacher gives me some hints which help me to carry out the work I am resented.

Alex (Year 5): The teacher prompts me to read the questions carefully to be able to answer the exercises I am given. Other times, the teacher can use a few examples to help me to remember what I have to do.

Zoe (Year 6): I call for help from either the Teacher or the Teaching Assistant in class.

Bridget (Year 4): I skip it and come back to it. Sometimes when I come back to the question I am able to solve it. Other times, no matter how hard I try I fail to get the answer right.

Maxine (Year 5):I ask the teacher for some explanations. But I only ask the teacher when I know that I cannot do the work.

Eric (Year 6): Teachers and friends try and help me when I am stuck on a question.

Kweku (Year 4): My teacher and TA help me. Sometimes they tell me to try my best by reading the questions carefully to see what strategies I have to use.

Meher (Year 5):I ask the teacher or T.A for help.They often give me a clue on how to work but not all the time.

Joshua (Year 6):I get a bit of help from the adults in the class.

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3)Are you given any opportunities to work with others when presented with some difficult maths problems?

Jean (Year 4): No, because it will make noise and disturb the class. My teacher always likes the class to be very quiet so I do not like to work when she is around.

Ekow (Year 5): No we are often told that the work will be recorded on Ed-Admin as part of our assessments so we have to work independently.

Jael (Year 6): No, we are not often given the chance to work with others. Usually we work on own and find solutions to problems.

Maya (Year 4): Not that much. Most of the time we are told to work on our own.

Sean (Year 5): No, we are hardly given that opportunity to work with other. I wish I could work with my friends sometimes because I am not that great at Maths.

Rossa (Year 6): No, not always.Quite often, my teacher wants to know what I can do so she tells the class that we should work on our own.

Kisten (Year 4): Sometimes, but not often because most of the work we do in class, my teacher expects us to work on our own.

Nevin (Year 5): No, we are not expected to work with one another most of the time.

Nathaniel (Year 6): Not all the time in class. Most of the time we are expected work on our own because the work we do is recorded.

Richard (Year 4): No. We always have to do independent work which out teacher marks and records the marks.

Alex (Year 5): No because we do independent work.

Zoe (Year 6): Not a lot. Most of the time we're asked to work individually.

Bridget (Year 4): No because it's independent work.

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Maxine (Year 5): Only sometimes.

Eric (Year 6): Not at all. We are made to work on our own because most of our work is recorded.

Kweku (Year 4): No. I always have to solve questions on my own.

Meher (Year 5): Not as much. Most of the time, we work individually but you can always call for help from the teacher when you are stuck.

Joshua (Year 6): No. We always have to be able to read the questions and find solutions to them on our own.

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4)How do you feel when working on maths problems that are simple and straightforward?

Jean (Year 4): I feel confident that I will have everything right because I find them easy and manageable.

Ekow (Year 5): I feel okay that I do not have to work too hard, just with a little effort I know I can get all answers right.

Jael (Year 6): I feel happy and confident because they are questions that I know I can handle.

Maya (Year 4): I feel that I can get all correct.

Sean (Year 5): I feel excited that I can get everything correct to make my teacher proud of me.

Rossa (Year 6): I normally feel a bit bored because when the questions are too easy, I finish very quickly and sometimes there is nothing much to do.

Kirsten (Year 4): I feel confident in myself because when I am asked to work on easy questions I know I can get all my answers right.

Nevin (Year 5): I become so confident in myself because I can capably work on my own.

Nathaniel (Year 6): I feel confident that I can get everything right and again I do not have to spend so much time on questions.

Richard (Year 4): I feel it's a piece of cake because they are questions that when I take my time,I can get all my answers correct.

Alex (Year 5): I feel confident that no matter what I will do well because even if I don not get a full score, at least I will not do badly.

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Zoe (Year 6): I feel excited and relieved that I can answer some questions.

Bridget (Year 4): I feel so confident that I can work on my own without calling on anyone for support.

Maxine (Year 5): I feel that I can easily get it wrong.

Eric (Year 6):I feel confident and comfortable.

Kweku (Year 4): I feel confident that I will get a full score.

Meher (Year 5): I feel confident that I will do well because it is always a delight to work on easy questions

Joshua (Year 6): I feel confident in myself because these are questions that I usually do not spend too much time on.

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5)Do you like Maths problems that make you think?

a) Yes **b)No**

Jean (Year 4): Yes because I find it to be a challenge so I know I have to try my best to get solutions to the questions I am asked.

Ekow (Year 5): Yes because it helps me to get better at Maths. The more difficult I find questions, the more I am challenged to work at them.

Jael (Year 6): Yes it helps my brain to grow because I always have to try every method possible to find answers.

Maya (Year 4): Yes, they challenge me and make me think but sometimes, you keep trying so hard and you still find out that the solutions are wrong.

Sean (Year 5): Yes because when you try hard you can get the answer. But not in all the case though as some questions are too hard enough to solve.

Rossa (Year 6): Yes, I feel it's an opportunity to use my brain.

Kirsten (Year 4): Yes, it makes you get more knowledge because thinking hard is always good for you to learn more things.

Nevin (Year 5): No, I like straight forward questions because when questions are too difficult, you keep spending too much time on them.

Nathaniel (Year 6): They help me to learn and put my brain to use.

Richard(Year 4): No, I waste a lot of time on such questions.

Alex (Year 5): Yes, I like to try because it is always good to think very hard and do your best.

Zoe(Year 6): Yes, it makes me feel challenged and I feel like I can solve tricky questions.

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Bridget(Year 4): Yes, it challenges me to do my best and think very hard to get my answers.

Maxine(Year 5): Yes, they help me to understand maths better.

Eric(Year 6): Yes because it helps me to know what I know and also work on my weaknesses. For instance, if I am stuck on a problem because of the times tables I know I have to work harder on them.

Kweku (Year 4): Yes, it makes my brain sharper because you have to put your mind to work.

Meher (Year 5): Yes, it helps my brain to be challenged.

Joshua(Year 6): Yes, it helps me to work harder at maths and I know that when I try my best I will get some of the questions right.

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APPENDIX C

QUANTITATIVE ANALYSIS GENERATED BY SPSS

class/level * Question1

Crosstab

Count

	question1			
	correct	partially correct	Incorrect	
year 4	24	0	23	47
year 5	2	0	45	47
year 6	38	3	9	50
Total	64	3	77	144

Symmetric Measures

		Value	Asymp. Std. Error[a]	Approx. T[b]	Approx. Sig.
Interval by Interval	Pearson's R	-.243	.083	-2.983	.003[c]
Ordinal by Ordinal	Spearman Correlation	-.250	.091	-3.079	.002[c]
N of Valid Cases		144			

a. Not assuming the null hypothesis.

b. Using the asymptotic standard error assuming the null hypothesis.

c. Based on normal approximation.

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class/level * QUESTION 2

Crosstab

Count

	QUESTION2			
	correct	partially correct	Incorrect	
year 4	22	0	25	47
year 5	10	1	36	47
year 6	37	1	12	50
Total	69	2	73	144

Symmetric Measures

		Value	Asymp. Std. Error[a]	Approx. T[b]	Approx. Sig.
Interval by Interval	Pearson's R	-.239	.081	-2.939	.004[c]
Ordinal by Ordinal	Spearman Correlation	-.243	.084	-2.984	.003[c]
N of Valid Cases		144			

a. Not assuming the null hypothesis.

b. Using the asymptotic standard error assuming the null hypothesis.

c. Based on normal approximation.

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class/level * QUESTION 3

Crosstab

Count

	QUESTION3			
	correct	partially correct	Incorrect	
year 4	18	0	29	47
year 5	16	3	28	47
year 6	10	0	40	50
Total	44	3	97	144

Symmetric Measures

		Value	Asymp. Std. Error[a]	Approx. T[b]	Approx. Sig.
Interval by Interval	Pearson's R	.165	.081	1.992	.048[c]
Ordinal by Ordinal	Spearman Correlation	.165	.080	1.995	.048[c]
N of Valid Cases		144			

a. Not assuming the null hypothesis.

b. Using the asymptotic standard error assuming the null hypothesis.

c. Based on normal approximation.

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class/level * **QUESTION 4**

Crosstab

Count

	QUESTION4			
	correct	partially correct	Incorrect	
year 4	14	0	33	47
year 5	9	0	38	47
year 6	8	1	41	50
Total	31	1	112	144

Symmetric Measures

		Value	Asymp. Std. Error[a]	Approx. T[b]	Approx. Sig.
Interval by Interval	Pearson's R	.127	.083	1.522	.130[c]
Ordinal by Ordinal	Spearman Correlation	.120	.084	1.441	.152[c]
N of Valid Cases		144			

a. Not assuming the null hypothesis.

b. Using the asymptotic standard error assuming the null hypothesis.

c. Based on normal approximation.

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class/level * **QUESTION 5**

Crosstab

Count

	QUESTION5			
	correct	partially correct	Incorrect	
year 4	22	0	25	47
year 5	16	4	27	47
year 6	27	3	20	50
Total	65	7	72	144

Symmetric Measures

		Value	Asymp. Std. Error[a]	Approx. T[b]	Approx. Sig.
Interval by Interval	Pearson's R	-.088	.084	-1.055	.293[c]
Ordinal by Ordinal	Spearman Correlation	-.090	.085	-1.082	.281[c]
N of Valid Cases		144			

a. Not assuming the null hypothesis.

b. Using the asymptotic standard error assuming the null hypothesis.

c. Based on normal approximation.

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class/level * **QUESTION 6**

Crosstab

Count

	QUESTION6			
	correct	partially correct	Incorrect	
year 4	27	0	20	47
year 5	2	1	44	47
year 6	25	3	22	50
Total	54	4	86	144

Symmetric Measures

		Value	Asymp. Std. Error[a]	Approx. T[b]	Approx. Sig.
Interval by Interval	Pearson's R	.029	.089	.350	.727[c]
Ordinal by Ordinal	Spearman Correlation	.019	.095	.231	.818[c]
N of Valid Cases		144			

a. Not assuming the null hypothesis.

b. Using the asymptotic standard error assuming the null hypothesis.

c. Based on normal approximation.

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class/level * **QUESTION 7**

Crosstab

Count

	QUESTION7			
	correct	partially correct	Incorrect	
year 4	21	0	26	47
year 5	2	2	43	47
year 6	18	0	32	50
Total	41	2	101	144

Symmetric Measures

		Value	Asymp. Std. Error[a]	Approx. T[b]	Approx. Sig.
Interval by Interval	Pearson's R	.073	.093	.869	.386[c]
Ordinal by Ordinal	Spearman Correlation	.070	.095	.831	.408[c]
N of Valid Cases		144			

a. Not assuming the null hypothesis.

b. Using the asymptotic standard error assuming the null hypothesis.

c. Based on normal approximation.

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class/level * QUESTION 8

Crosstab

Count

	QUESTION8			
	correct	partially correct	Incorrect	
year 4	26	0	21	47
year 5	15	3	29	47
year 6	24	4	22	50
Total	65	7	72	144

Symmetric Measures

		Value	Asymp. Std. Error[a]	Approx. T[b]	Approx. Sig.
Interval by Interval	Pearson's R	.025	.084	.296	.768[c]
Ordinal by Ordinal	Spearman Correlation	.022	.085	.258	.797[c]
N of Valid Cases		144			

a. Not assuming the null hypothesis.

b. Using the asymptotic standard error assuming the null hypothesis.

c. Based on normal approximation.

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class/level * QUESTION 9

Crosstab

Count

	QUESTION9			
	correct	partially correct	Incorrect	
year 4	13	0	34	47
year 5	3	2	42	47
year 6	14	0	36	50
Total	30	2	112	144

Symmetric Measures

		Value	Asymp. Std. Error[a]	Approx. T[b]	Approx. Sig.
Interval by Interval	Pearson's R	-.007	.093	-.088	.930[c]
Ordinal by Ordinal	Spearman Correlation	-.009	.093	-.106	.916[c]
N of Valid Cases		144			

a. Not assuming the null hypothesis.

b. Using the asymptotic standard error assuming the null hypothesis.

c. Based on normal approximation.

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class/level * **QUESTION 10**

Crosstab

Count

	QUESTION10			
	correct	partially correct	Incorrect	
year 4	18	1	28	47
year 5	21	8	18	47
year 6	45	1	4	50
Total	84	10	50	144

Symmetric Measures

		Value	Asymp. Std. Error[a]	Approx. T[b]	Approx. Sig.
Interval by Interval	Pearson's R	-.455	.069	-6.091	.000[c]
Ordinal by Ordinal	Spearman Correlation	-.454	.070	-6.073	.000[c]
N of Valid Cases		144			

a. Not assuming the null hypothesis.

b. Using the asymptotic standard error assuming the null hypothesis.

c. Based on normal approximation.