UNIVERSITY OF GHANA

STOCHASTIC LOSS RESERVING WITH INDIVIDUAL CLAIM SIZE MODELING

BY

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DECLARATION

Candidate's Declaration

I hereby declare that this submission is my own research work towards the award of the MPhil. degreein actuarial science and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

Signature .... Date ....

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Supervisor's Declaration.

We hereby certify that this thesis was prepared from the candidate's own research work and supervised in accordance with the guidelines of thesis laid down by the University of Ghana.

Signature .... Date ....

(Dr.Anani Lotsi)

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Signature .... Date..............................

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ABSTRACT

This study demonstrated the application of chain ladder, Bornhuetter-Ferguson, Mack model, probability and other statistical models in exploring loss reserving and claim behavior. Secondary data sets from an insurance company in Ghana were used for analysis and deductions. The focus of this research was to determine the probability distribution for the claims, fit the basic reserving methods (Chain-ladder, Bornhuetter-Ferguson) and juxtapose that with the Mack model with the help of basic stochastic assumptions. The research was peaked by determining the suitable model among the three models used for the available data. The chain ladder revealed that an amount of GHC 56,547,882 must be reserved while the Bornhuetter-Ferguson estimated an amount of GHC 230,516. Finally, the Mack stochastic model suggested that the latest payment should be GHC 30,008,300.16 with a development of 20% across the development years. The model also proposed that the ultimate and IBNR claims reserves should amount to GHC 149,758,939.87 and GHC 119,756,639.71 respectively. The behavior of the claim payment was known to follow a log-normal distribution. It was established that the Mack model was very robust as compared to other models. Future research should expand their application to bootstrapping for modeling of various parameters and reserves. A contingency fund must be created by insurance firms to suffice for payment in case there is a catastrophic event.

DEDICATION

This Thesis is dedicated to my beloved parents and Department of Statistics and Actuarial Science.

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LIST OF ABBREVIATIONS

ADR .... Additional Diagonal Risk

BEL .... Best Estimate Valuation of Liabilities

BF .... Bornhuetter Fergusson

BSCR .... Buhlman-Straub Credibility Reserves

CC .... Cape Cod Model

CILI .... Conditional Independent Loss Increment

Dens .... Density

DIC .... Deviance Information Criterion

GDP .... Gross domestic Product

GIMMBS .... Guaranteed Minimum Maturity Benefits

GLM .... Generalized Linear Model

GT .... Generalized-T

HMC .... Hamiltonian Monte Carlo

IAIS .... International Association of Insurance Supervision

IBNR .... Incurred but not reported

IFOA .... Institute and Faculty of actuaries

IBNeR .... Incurred But Not enough Reported

IBNyR .... Incurred but not yet reported

ICP .... Insurance Core Principles

LDFs .... Loss development factors

LSRMs .... Linear Stochastic Reserving Methods

MCMC .... Markov Chain Monte Carlo

MLE .... Maximum Likelihood Estimation

MUM .... Market Value Margin

MSEP .... Mean Square Error Prediction

NIC .... National Insurance commission

ODP .... Over-Dispersed Poisson Model

SCR .... Solvency Capital Requirement

SSNIT .... Social security and national insurance trust

VP .... Vector Projection Method

ZAIG .... Zero-Adjusted Inverse Gaussian

CHAPTER 1

INTRODUCTION

1.1 Background of the study

An insurance policy connotes a contract which comprises of promises made by the insurance company to the insured to settle claims in the future for a forthright premium paid. Correspondingly, policy makers do not have an idea of the up-front cost for their service, but rather depend on the historical data or judgmental analysis to estimate a feasible price for their contract. In Non-Life Insurance or general insurance, such as: property, motor and general insurance, most policies keep running for a time of twelve months. However, the payment of claims can take even decades or quite some number of years. Therefore, many a time, the due date of the insurance contract is not known to the actuary. Perfect example is, losses emanating from casualty insurance that can take a long time to be settled and even at worse cases when the claims are acknowledged, it may take time to establish the extent of the claim settlement cost. Figure 1 summaries the process taken before a claim is paid. The accident date is the date of claim occurrence, the reporting date is strictly on the day the fortuitous event was reported to the

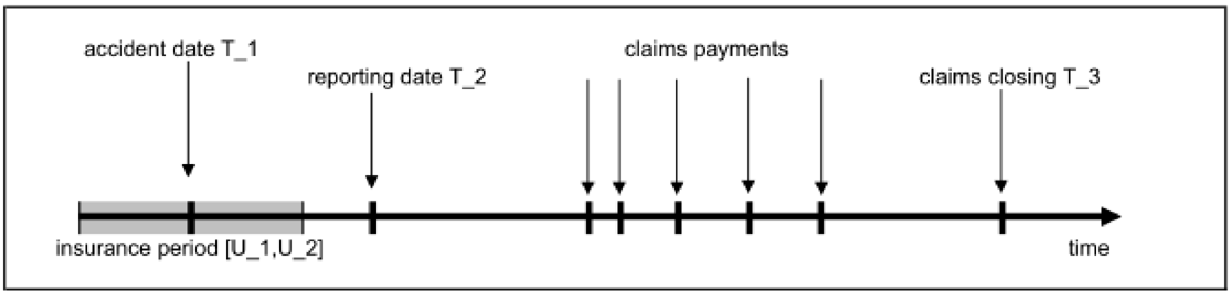

Figure 1.1: Claim process time line

insurance company and the time lag between the day the event happened and the time it was accounted for is known as the delay report. This can be attributed to many reasons. Afterwards, the insurance company starts with investigations and concludes with claim due for payment, that period is also known as the claim payment. It is important to note that claims that are closed have propensity to being reopened. Having this process, we are therefore presented with three significantly arranged dates for every non-life insurance:

$$\text{accident date } T_1 \leq \text{reporting date } T_2 \leq \text{settlement date } T_3$$

In further illustrations, the beginning of insurance U_1 and ending $U_2 > U_1$ with the assumption that $U_2 < \infty$, can only mean that the insurance company is only liable for claims on $T \in [U_1, U_2]$. This situation however creates a big predicament for insurance companies and management. A paper report by the Institute and Faculty of Actuaries (IFoA) estimated that the undaunted cost of United Kingdom mesothelioma-affliated claims to the U.K. Insurance Market between the period 2009 - 2050 could be around £10bn; (Party et al., 2013). In the USA the estimated cost of asbestos-affiliated claims in 2002 was around $120bn.Usually, insurance companies would estimate the total amount of the loss reserve directly. However, the estimation could be done by each component for more accurate results. An insurance company must set aside enough money to pay all claims, present and future, on the policies currently in force. Inadequate reserves can lead to insolvency and over-adequate reserves can lead to under usage of resources that could have paved way for the insurance company to expand and become very competitive. Industry best practice attempts to mitigate or spread risk. therefore, new techniques to accurately predict the necessary reserves must be tested repeatedly. Most insurance companies are unwilling to risk changing their methods. The concern is that new techniques may not account for all the nuances and intangible elements contributing to claims and probably might fail to improve the accuracy of the reserve predictions. This concern discourages companies from testing new techniques due to risk associated with overexposure and this leads to a large proportion of insurance companies continuing to use conventional methods or external consultants when calculating ultimate losses for an accident year and total losses for a line of insurance. The economy of Ghana is on the rise and it is expected to be one of the best performing in Sub-Saharan Africa hence Oscar Akotey and Abor (2013) are of the view that the beginning of the oil and gas industry will expand the interest for insurance. The rising demand of insurance in the country would mean insurers need to be almost certain and ready for payment of claims when the need arises.Consequently loss reserving thus: an estimation of the sum of asset an insurance company needs to put aside to pay for future claims on protection arrangements should be viewed as the number one task to cater for the rising demand of insurance to stay relevant since it helps insurers to have a higher standard of good faith thus (uberrima fidae).

The National Insurance commission (NIC) which is responsible to regulate and supervise insurance activities in the country issued a risk management and governance exercise to educate insurance companies on how to curb risk in the country in 2015. In the year 2016, most of its activities were geared towards ensuring that the principles and core values of risk management and governance was adhered to by the captive insurance firms in the country. To this end, the national insurance commission has organized quite a number of workshops for key stakeholders in the insurance discipline to educate them on what is required to mitigate risk. Apart from the numerous activities, the NIC has worked to ensure that all the registered insurance firms are capitalized as per their requirement. The purpose of this whole exercise is to keep the insurance company solvent so that promised claims can be paid.

1.1.1 Chain ladder dynamics of reserving

The chain ladder method of reserving technique is the most widely used method used by insurance companies for reserving due to its simplicity. It is a specific basic reserving technique used for predicting or projecting ultimate losses. The basic idea behind loss reserving is that: as an insurance firm, an amount of money or assets needs to be set aside to cover your future liability because insurance firms want to remain solvent. If the insurance company is over reserving then it could be a problem because they will be setting aside some amount of money that can be used for other expenses or investment. Again if the insurance company is under reserving, then they will not have enough money to pay their promises to the insured so either way there is an issue. To solve this problem, actuaries in insurance companies would like to get a reserve estimate as right as possible. One of the techniques is the chain-ladder approach also known as the distribution free model which is calculated from the development triangle or reported loss triangle. What are reported losses? These are the case losses + the paid losses. The money paid immediately after a fortuitous event has occurred is the paid losses while the amount of money set aside to be eventually paid to the insured is the case losses. The basic underlying assumption which encapsulates the chain-ladder method is that historical or past payment trends is indicative of the future.The chain ladder is separated by two time axis.

We denote i = Year of accident and j = Developmental period or development year. For engraving motives or reasons let us assume that X_{ij} signifies all the payment in the development period j for the claim in accordance to accident year i, Thus this corresponds to the augmentation of the claim payments for the paid losses which is done in the accounting year i+j. In the loss development triangle of Figure 1.2, accident years are usually on the vertical line and the development years or period on the horizontal line. With this,the loss development triangle is splitted into two parts thus the upper triangle or trapezoid and the lower triangle.The lower triangle is the portion for the estimation of outstanding payment. The diagonals according to Wuethrich and Merz (2008) are the current accounting year payment.

In review, Chain ladder is known to be a pure computational algorithm for the estimation of reserve.With the assumption of independent accident years, the model removes or eliminates the accounting year effect in the claim data which also forms the $C_{io}, C_{i1}, C_{i2}....$

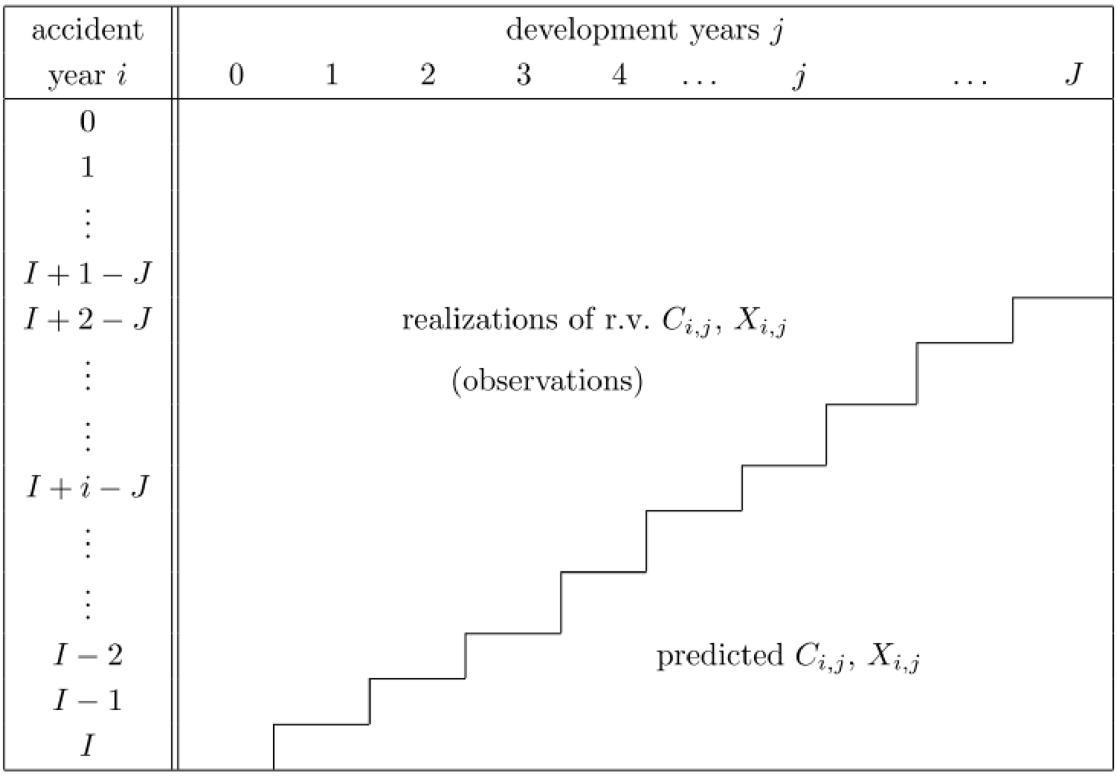

Figure 1.2: Loss development triangle

namely the Markov chains. The age-to-age ratio or link ratio is the bed rock of interest in the chain ladder since it allows and provides the estimation of future liability.

1.1.2 Bornhuetter Ferguson dynamics of reserving

Bornhuetter Ferguson loss reserving technique is another basic method of loss reserving apart from the chain ladder. It was designed to estimate the IBNR by Ronald L. Bornhuetter and Ronald E. Ferguson.Development methods can lead to unrealistic projections. This can understate when losses are large and also overstate when losses are small.The BF method smooths the variance in recent period caused by absence or presence of large claims. The BF works on the principle of earned premium, a priori expected loss ratio and an estimate of the percent unreported claims,usually based on Loss Development factors (LDfs). Actuaries will rely on many techniques to make selections. It is very important to view the range of results provided by various method to determine which method produces the most credible indicator. Apart from the chain-ladder technique,Bornhuetter Ferguson is the second most used reserving technique by actuaries in an insurance company. The BF combines the various stages of the chain ladder method together with expected loss ratio and also assigns weight to the percentage of the paid amounts and incurred losses. Unlike the basic chain ladder which works with the idea of past experience, the BF builds a loss reserving in view of the insurance company's exposure to loss.What the BF technique actually does is to estimate ultimate loss for a particular exposure of risk and again estimates the percentage of ultimate losses that was not reported due to several reasons at the time of occurrence.The BF then calculates the reported loss + IBNR. The IBNR which is calculated as the ultimate loss is multiplied by the percentage that was not reported using the priori estimate losses. The BF method is very significant to the actuary due to reasons such as: It being very easy to calculate, it can also be used with incurred or paid data, current or volatile periods do not significantly distort the estimation and lastly it is very useful when there is little or zero data.Aside the few advantages, it can not be worked on when the development factors are less than 1.

1.1.3 Mack's stochastic reserving

In real world problems,mathematical and statistical models can be broadly categorized as either probabilistic or deterministic.In most situations, the probabilistic models yield a better presentation considering the collection of random variables. A stochastic process is a set of random variables $\{X_t : t \in T\}$, where T, is called the parameter space of the process and t is the indexing parameter. The values assumed by the process are called STATES. In a more simpler way, a stochastic process is a chance or random process indexed by time and space. Using an insurance company which starts with a fixed capital x_0. Assume the state of the insurance company is considered after some time say a year. In the n^{th} period also assume the income of that period is I_n with a total claim C_n. The state of the insurance company after n such periods is

$$X_n = X_0 + (I_1 - C_1) + (I_2 - C_2) + ... + (I_n - C_n) \tag{1.1}$$

It is clear that when $X_n \leq 0$ the insurance company would be insolvent or ruined. In summary, lets assume $\{X_t, T \geq 0\}$ to be the position of the insurance company at time t. Some of the challenges involved in this process are the magnitude of claims of the insured person with contract in force by the insurance company and their arrival to register for a claim. This can be modeled by a probability distribution by making X_t a stochastic process with time parameter t and by making certain assumptions. A stochastic process can be used to study the behavior of claims. A few questions that can be answered are; what is the probability of the insurance company paying an X amount in the next accounting year? A stochastic method of loss reserving as proposed by Mack (2000) enables losses to be estimated with a certain margin of error. This method calculates the process variance,parameter variance and standard error of the reserve estimates.

1.1.4 Individual claim size modeling

Insurance term of contract has many risky components.We can talk of pure randomness when it comes to the outcome of claims and its uncertainty in nature but this can be controlled by the size of the portfolio or a large diversification, another risky component is the model risk. Many a time we attempt to explain or clarify real world behavior with models. This can not always be statistically good since most of the factors change over time (non-stationary). It is therefore necessary to use continuous probability distributions to help in decision making. The determined probability distribution will help the actuary in the insurance firm to determine a good reserve. Again, if the distribution for the historical loss data available is found,so the underlying assumption and properties of that particular distribution and its features would help in prediction of catastrophic events.

1.2 Problem statement

Loss reserving is a double edge sword in the sense that either under or over reserving has consequences on the payment of claims. Various reserving techniques lacks projection error with the estimated reserves. Mack (1993) proposed the error estimation of Chain-ladder and Bornhuetter-Ferguson technique but in spite of the fortuitous events and severity of claims, limited research works have been carried out considering loss reserving, modeling of individuals claims and also the stochastic nature of loss reserving. In addition various studies explore loss reserving using basic or deterministic models devoid of error estimation and individual claim size modeling. Again risk is mostly considered **collective** in most loss reserving research and this does not give insight to the true nature of the historical loss data of the insurance company. This thesis will present a more robust analysis besides the basic or deterministic method with a stochastic analysis (Mack Model) of loss reserving coupled with individual claim size modeling and out of sampling validation to determine the model with a good predictive power.

1.3 Objectives of the study

The main objective of the study is to use a stochastic model (Mack model) to examine the loss reserves and also consider the individual claim size modeling. This is also to check which distribution fits the claims and how "the best fit " distribution would help explain the difference in reserve figures between the deterministic and the stochastic model.

The study specifically seeks;

1. To determine the probability distribution which best fits the individual claims.
2. To fit Chain-ladder and Bornhuetter Ferguson method of reserving to the claim amount.
3. To fit a stochastic model (Mack) in order determine the standard error or variability associated with loss reserving.
4. To determine the model that performs better by using out of sampling technique.

1.4 Research Questions

The study specifically seeks to answer the following:

1. Without the individual claim size modeling, how does the insurance company manage their solvency?
2. What contribution can the individual claim size model add to the decision making of the varied loss reserving method?
3. What will be the reason that makes stochastic modeling of claims better than a deterministic modeling?

4. In spite of various robust methods available, why does an insurance company still prefer the basic methods?

1.5 Significance of the study

The study would be imperative in the following ways;

1. The findings will help policy makers to ascertain the amount of resources that would be invested or set aside for paying claims.

2. The study will also demonstrate the application of Mack modeling. This will be achieved by illustration on its loss reserving as opposed to other deterministic models.

3. The study would help practitioners and non modeling specialists of loss reserving with an alternative method of modeling risk of reserving.

1.6 Scope and Limitations

The study is aimed at exploring different methods of loss reserving for insurance claims. The loss reserving can either be deterministic or stochastic. Further, individual claim size will be modeled to find out which distribution best fit the claim size and how that can be used to explain the variation of difference of conflicting figures for the different method used for the loss reserving. A major limitation is that both the third party and comprehensive claims were all considered in one data, again the date of the fortuitous event occurring was not captured.Life insurance data was also not included because of how different it is to model their data. The data includes the date in which the claim was reported and the paid claims.Due to time and lack of funding,the study will focus on one insurance company in Ghana.

1.7 Thesis Organization

The study was structured in five main chapters. The remaining chapters of this thesis was organized as follows. Chapter Two reviewed previous works on the application of loss reserving techniques and other related works pertaining to this study. Additionally, Chapter Three described Chain-ladder dynamics, Bornhuetter Ferguson technique, Mack model, Individual claim size modeling. Other statistical methods required for further exploration of loss reserving were explained. Chapter Four presented the analysis and results of the study. Finally, Chapter Five highlighted the findings, conclusion and recommendation of the study.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter entails the review of relevant literature and other application of loss reserving techniques in diverse areas of actuarial discipline.First of all, few applications of loss reserving that are not part of the basic reserving techniques would be reviewed; followed by its application to loss reserving. Other statistical methods and probability theorems that would be used for further exploration of loss reserving would be discussed.These other methods include application of individual claim size modeling,Bayesian reserving methods and Poisson model for number of IBNyR claims.

2.2 Historical background of Insurance in Ghana

Non-life insurance became very known after the fire disaster that destroyed many properties in London as at 1966. The disaster is popularly known as "the great fire of London", hence, the need to insure properties started to emerge. The idea of insurance was brought into this country by the British marketers during the 19^{th} century. At that time, there was no provincial legislation ensuring how the business of insurance runs in the country, hence, providing insurance in the country was done by foreign trading companies who also acted as emissaries of the captive insurance companies in UK and many other places around the world. The insurance business transacted was mainly shipment of goods thus; cocoa and gold hence the British insured their companies. Therefore, in the strict sense, there was no insurance market in Ghana at that time, any semblance of an insurance market was an appendage of the British insurance market.

The first local insurance company established was to protect only human life insurance business and that was African life to be precise. Gold coast insurance company, which was established in 1955 commenced life insurance for the Ghanaian people at large. Long before the Gold Coast insurance was created, the insurance companies available then insured only European lives (Bajpai, 2003). Statistical and mortality tables which were used to rate premiums for insurance companies were based on that of the United Kingdom, and adjusted appropriately. Currently there are 29 non-life insurance firms and 22 life insurance firms in the Ghana.

2.3 Application of different techniques to the study of loss reserving

Shi and Frees (2011) employed the use of multiple loss reserving in conjunction with copula regression model to find out how reserves is determined in a present period and this served as a base for managing solvency in a captive insurance company. In the process of modeling these underlying techniques, the copula regression model was used to predict losses that were unpaid for a dependent business line and also to predict the distribution for the individual claims. To determine loss reserving through the copula model, the uncertainty in the parameter was prevented by bootstrapping. It was concluded from this study that, the copula regression model can be used for predicting loss reserves which would be as close as the chain-ladder technique. The association amid paid claims and the predictive probability distribution was achieved. Even though the copula regression model together with bootstrapping is used to estimate loss reserves, care must be taken when applying these models. This is because the bootstrapping simulation of insurance claims can provide unrepresentative results which can make the insurance company to either over reserve or under reserve asset for claim payment.Again, the quantification of risk was considered collective either than individualistic and this did not give proper insight as to how claim payment behaves leading the researcher to obtain a better probability distribution to represent the claims.

Taylor et al. (2008) also applied individual claim loss reserving conditioned by case estimates modeling to forecast loss reserves and error prediction. The basic idea behind this model was to consider individual claims as smaller number of covariates of which they are a function of time. Consequently, it was observed that it would be much likely to achieve a good efficiency of loss reserves with models that have fewer parameters and time covariates. Thus, it was concluded from their study that loss reserving and error prediction associated with it can be attained with good estimation if and only if the models are blended. Even though loss reserving conditioned by case estimates are used to predict reserves, care must be taken when employing these models. This is because, without thorough understanding of the underlying case estimates and improper assumptions the case estimate model would result in biased forecast and error.

Cape Cod model (CC) is also applied to actuarial modeling of loss reserving problems. In this light, the CC model was developed by Bulham and Straub to overcome the challenges of the chain ladder technique owing to its simplicity (Saluz, 2015). The CC method was studied in a stochastic framework hence the properties such as the parameter estimation, loss ration and development pattern which is integral in the loss reserving process were all derived within a stochastic model. This established the calculation of MSEP for the ultimate claim. Although the CC method of reserving is highly established in the actuarial field, the formulas cannot be seen in current writings (Moro and Lo, 2014) hence we mostly refer to the chain ladder method for loss reserving. Again, the one-level premium which is one of the basis for the establishment of CC is not mostly available in an insurance company hence its calculation can be afflicted with unreliability.

Additionally, Milidonis and Grace (2006) described loss reserving by using the concept of tax deductibles and extreme value theory. Due to the high level of insurance claims after a hurricane especially hurricane Andrew. The United States insurance commission allowed insurance companies to practice tax-deferred loss reserves but the interest of such practice waned after a couple of years. The catastrophe of the hurricane event always leaves insurance companies almost insolvent. In sum, extreme value theory was used to predict the tail distribution for losses peculiar to circumstances in a catastrophic event. A major limitation to this study was that, the data were based on simulation and this did not depict the actual value or loss of asset of the event.

Furthermore, Gao and Meng (2018) also proposed the Bayesian-Spline model which incorporates the cubic B-spline in estimating unpaid claims reserves with spirals placed at the k-development period. This model assumes the coefficient of a basis function which is similar to the idea of smoothing parameters. The unpaid claims are considered and modeled as random effects. This facilitates the prediction of unpaid claims and new accident years according to the Hamiltonian Monte Carlo (H.M.C). Programming of H.M.C is very complicated hence inferences is mostly done by Stan thus, sampling through adaptive neighborhood. The Bayesian Spline model is mostly used by insurance companies to estimate reserves of workers compensation. Another contribution is that, it removes the problem of tail factor effects in the chain-ladder model. Even though the Bayesian Spline model is very robust because of its ability to predict unpaid claims it is error inclined to selecting a probability distribution which would best fit the individual claim sizes.

Xia and Scollnik (2015) asserted that financial stability of an insurance company depends on its loss reserving techniques. In an incremental loss triangle, zeros and negative values are always a problem to deal with regardless of the reserving technique used and this is mostly due to principles in insurance such as; subrogation and salvation. Subrogation supports the principle of indemnity where in summary it implies substitution of the insurer in place of the insured for a guaranteed repayment from a third individual for loss covered by insurance. A mixture of models was proposed to deal with such situations hence the Bayesian mixture model for stochastic reserving under this stage was used. Multinomial regression was employed to analyze the sign data while log-normal distribution modeled the loss magnitude of the sign values. Again a Bayesian generalized linear model was fitted and implemented using the Markov Chain Monte Carlo (MCMC) techniques. Even though this method provided a robust tool for stochastic loss reserving with zeros and negatives there was a higher variance of uncertainty due to extra parameters introduced by the mixture of models. Consequently, the plan of a generalized linear model (Klugman et al., 2012) is more broad than regularly observed. The residual is not entirely restricted to following a particular family of a loss distribution but severity, aggregation and frequency of claims can also be explained by the GLM. Usually the GLM works with dependent and independent variables plus a stochastic error term which can be a normal distribution with a single mean zero. This GLM can also allow various transformation of means and their stochastic error into various distribution which will better explain the behavior of loss data or individual claims. Though the GLM provides a robust modeling tool for claim behavior, its approximation of the likelihood function is often better achieved using the various exponential family distributions (Venter, 2007)

Chan et al. (2008) in their paper also used the generalized-t (GT) and the Markov Chain Monte Carlo (MCMC) to estimate loss reserves. They were of the view that various techniques and models inadequately models extreme claims in an insurance company hence the prediction of a closer accurate reserves is still a problem to deal with. The (GT) distribution nest all heavy tailed distribution which includes exponential power distribution and student-t distribution. It was shown that the GT distribution easily detects outliers using mixing parameters. The DIC was used to choose the best model. The GT error distribution performs better than well-known heavy tailed distribution and also provides quite better estimates of loss reserves. This model has a problem modeling low values of claims hence skewed distribution is often applied when such predicament avail itself.

2.4 Application of chain ladder technique to the study of loss reserving

Several researchers have recommended various techniques for loss reserving in insurance.In spite all the various techniques, the chain ladder method headlines all. Gabrielli and Wüthrich (2018) decided to back-test results of reserving method, they sought to assess the performance of the chain ladder method on an historical data. In this peculiar paper, a stochastic scenario generator which is data simulation machine was used to simulate data with similar components for which their lower triangle was already known and based on the simulated results and triangles, the results are juxtaposed with the previous results to ascertain the performance of the chain-ladder technique. This dynamics can be very appropriate when the researcher cannot have access to full data from an insurance company leaving the researcher with the option of simulation. A major limitation with this method is that, it does not consider tail development factors and this is key in the development of patterns. Riegel (2014) also proposed another stochastic model for long tail business line and attrition development in that he presented the IBNR like the chain ladder method which allowed claim payment for larger losses. Mostly in reinsurance market, quota shares is very prevalent in motor insurance where the volume of the ceded insurance is large hence pricing accuracy is very relevant so as to obtain a better cedent reserve. Nevertheless, this method has issues of not being able to calculate the IBNR reserves on incurred triangle of paid claims and subsequently compare them.

Taylor (2017) in his paper tried to investigate the uniqueness of the chain-ladder technique by assuming that claim observation is distributed according to the family of exponential dispersion family. His paper wanted to answer the question relating to why the cross-classified **CL** has different distribution for which the observation were subjected to. The most basic distribution usually attributed to the claims paid is Poisson and the maximum likelihood estimation of parameters. The cross-classified **CL** also known as ANOVA **CL** is another form of loss reserving which has in each cell the mean U_{kj} for the (k,j) cell and this means the product of the column and row effect will be: $U_{kj} = \alpha_k \beta_j$. This establishment makes use of the Lehman-Scheffe theorem which gives under regularity, an unbiased MLE based on a complete sufficient statistic and a minimum variance unbiaseness. In the absence of these qualities, complex methods such as the Tweedie technique are used which may not be easily understood and implemented by other modelers who are not mathematically inclined as opposed to the simple chain ladder.

Verrall and Brydon (2018) studied the chain ladder technique by using recent theories of age-period-cohort model. This approach was developed by Kuang et al. (2009) with the believe that some important implication of the claim reserves and the chain ladder technique would be realized if the calender effect and inflation is incorporated in the chain ladder methodology. The age-period-cohort method was applied together with the over-dispersed Poisson method to give a broader picture and also understand how the chain-ladder deals with issues of inflation and calender year effect on chain ladder reserving. In conclusion, it was disproved that the **CL** technique does not take averages of past values of inflation but rather it uses it as a constant trend for projection and also the chain ladder deals appropriately with calender year effect. In as much as this conclusion was drawn, this method used must be carefully thought through since its method of projection is not reliable and can lead to inappropriate reserves in insurance.

Robert (2013) in his paper opined to how insurance agencies needed to compute the Best Estimate valuation of Liabilities (BEL) and the Market Value Margin (MVM) for non-hedgeable protection specialized dangers. The Cost-of-Capital methodology characterizes the MVM as the present esteem of the current and future Solvency

Capital Requirement (SCR) of the non-hedgeable dangers to ensure against unfriendly advancements in the run-off of the protection liabilities. Mostly the SCR at time t itself relies upon the expansion in the MVM within t and t + 1. Hence there exists an unpredictable circularity reliance between the two amounts. In this paper, he presented a correct and exact surmised investigative equations for MVMs with a Bayesian log-normal chain-ladder system and the simple case where cash flows which are not deducted are considered hence the posterior parameter can be calculated to obtain the BEL and MVM for the lower trapezoid. This method of loss reserving can only deal with cash flows in the reserve triangle that are always positive and makes non-modelers finding it difficult to estimate unbiased reserves

2.5 Application of Bornhuetter Ferguson technique to the study of loss reserving

Originally, other methods gave point estimates of reserves but actuaries did not deem this satisfactory because it was difficult to decide whether they differ significantly or not but taking premium loading as an example, the actuary will have to be certain in his risk calculation. The error of various estimates maybe the standard error of the true estimate.Mack (2008) studied this dynamics and decided to use the Bornhuetter Ferguson (BF) method in conjunction with a proposed error prediction methodology for loss reserving. The **BF** is the next popular claim reserving technique after **CL**. In this paper a stochastic approach for the error prediction of the **BF** technique was determined. The pattern estimate between The **CL** and **BF** differ greatly and this makes the **BF** method to be independent as opposed to other methods which derives its first principle from the chain-ladder technique. The stochastic model argued that the actuary needs to assess uncertainties such as parameters,development pattern and estimated claim priors for a better error prediction in the run-off data for the insurance company. A major limitation is that if the value of the prior estimated value is wrong then the whole reserve would not follow suit and serve the purpose of insurance companies keeping their promise.

Alai et al. (2011) revisited a stochastic approach for the Bornhuetter Ferguson **(BF)** loss reserving. The aim of their research was to derive an estimator using the GLM and MLE techniques to model parameters of the **BF** in order to obtain simpler formula for the mean square error prediction (MSEP). The assumption that the Chain-ladder developmental pattern is used to calculate the **BF** reserves and the idea of considering the data as an Over-dispersed Poisson distribution was maintained. This approach is different from Mack's (2008) idea but very common to practitioners. The MSEP was derived as;

$$msep_{\,Ci,I|D_i}(\widehat{C_{i,I}^{BF}}) = E\left[(\widehat{C}_{i,I}^{BF} - C_{i.I})^2|D_I\right] \tag{2.1}$$

It is clear that the disparity amid these two methods lies on the estimated process variance. Although there is no conclusive way, the method and results of Alai et al. (2011) are in similitude to that of the Chain ladder method.

Saluz et al. (2011) fitted the Bornhuetter Ferguson model of loss reserving for an insurance company in Switzerland. Their main aim was to query how the developmental pattern in the Bornhuetter Ferguson method was estimated and also propose a calculation for the conditional mean square error of prediction (MSEP) for the ultimate claim.This idea was studied from the central idea of the over-dispersed Poisson model and the chain ladder technique. However, the suggested new estimates for the correlation matrix and the conditional (MSEP) was seen to be in accordance with Mack's model. Nevertheless, they discovered in their study that the three distributional models studied to understand the basis of development pattern was in anyway different but the over-dispersed Poisson model (ODP) was presumably inadequate to model most practical cases of loss reserving. A conclusion was arrived for actuaries in insurance companies calculating loss reserving to consider using more than one technique so as to have a fair idea of for the estimates.

England and Verrall (2010) also studied the Bayesian Over-dispersed Poisson model for loss reserving with two different types of prior distributions, thus; the Bayesian predictors and the over-dispersed Poisson parameters. Their study led to two streams with one approaching the Chain-ladder technique and the other for Bornhuetter Ferguson for the other. This research only considered the ODP model with a scale factor which is constant. Again the BF model was used in the domain of the CL model without the consideration of tail factors.

Heberle and Thomas (2016) applied the Bornhuetter Ferguson and the fuzzy method to extrapolate reserves. According to them, the a priori information obtained for the calculation of the ultimate claims are derived from market statistics or organizational data. Mostly, the a priori information given may have vagueness. Likewise the parameters of the claims and the developmental pattern because of the subjective judgment of the actuary. In view of the above predicament. The researchers studied fuzzy numbers to help develop new and fewer parameters for the ultimate claim prediction. It was brought to light through their findings that, models that employ the principle of reserving to minimize risk of fulfilling insurance promise differ a lot in calculation or methodology but almost certain arrive approximately at the same reserve figure at the long run even though the fuzzy set theory offers instrument to model unpredictability, the a priori estimate still comes from an expert knowledge making the fuzzy method almost as reliable as the original BF method.

2.6 Application of Mack's model to the study of loss reserving

Modeling of mean square error and variability has been problematic to most actuaries in a wide range of application areas especially loss reserving. One way of studying and finding the mean square error in recent studies is by assuming the claim payment to be random. This randomness paves way to use a stochastic approach to find the MSEP. Consequently, it helps with inference and diagnostic handling of many forms of loss data. In this light, recent studies sought to apply these models to verify and improve upon the basic methods of loss reserving such as Chain ladder, Bornhuetter Ferguson and others to estimate good reserves. However, for the purpose of this study, few application of stochastic model called the Mack model shall be reviewed.

Dahms (2018) applied a stochastic approach using an expanded group of reserving models called linear stochastic reserving methods (LSRMs). These models focus on the assumption that the conditional changes of the expected claim is a development period and are linearly dependent on the past. This approach is thought of to be more heuristic to the actuary than philosophical because of its practical ability. Moreover, the assumption of independent accident years as done in basic methods is not applied in (LSRMs). Using this framework, many development triangle with different exposures can be coupled and this leads to a new metamorphosed development triangle. The actuary will then use the information of the the whole triangle to make future estimation and error prediction for each of the development triangle. Actuaries are therefore encouraged to look for drivers (exposures) behind claim properties and development of portfolio and if such drivers are obtained heuristically the (LSRMs) can be used for a good reserving.

Bühlmann and Moriconi (2014) enhanced the learning of stochastic loss reserving to the extend of determining the credibility of the estimated reserves by using the Buhlman-Straub credibility reserves (BSCR) with the assumption of conditional independent loss increment (CILI) given a risk parameter $\theta_{i,j}$. This model considers the (CILI) condition where the additional diagonal risk (ADR) is obtained by summing up two component within the model. One of the summation is dependent on the accident year i and the other on the calender year $t = i + j$. The ADR is often tractable in closed form and this provides a credibility formulae for the reserves and the prediction of MSEP. Basically, this model emphasizes the importance a stochastic model plays in the diagonal effect in a loss triangle. Even though the MSEP values can be obtained in such a model, there exist a level of unpredictability from the estimation of the developmental pattern.

The application of stochastic theory to the study of loss reserving is presently seeing great interest from academic communities and the work industry as well. This amplified attention has given birth to a number of different methods that use stochastic approach for loss reserving. It is disputable that the Mack model and other loss reserving techniques are predominantly suited for such predicament due to reasons such as their ability to predict reserves with error estimation. Quantile (2018) evaluated the long and short-term risk for equity-linked insurance products. In this research, specific type of equity product with guaranteed minimum maturity benefits (GMMBs) was used with the assumption that such benefits follows a stochastic volatility model which will intend allow a return of latent volatility component.This return would either be long or short-term memory. The long and short-term memory explicitly form a quantile reserve and its confidence interval. Simulation studies are also performed on the estimated quantile reserve for verification. In conclusion,the factual results showed that the confidence interval for the quantile reserve can be underestimated if the long term memory is ignored.

Several works on loss reserving and risk analysis of claim payment have been carried out since the year 2000 by the application of stochastic methodology. Guszcza and Lommele (2006) were fascinated by the developing nature of actuarial literature devoted to stochastic loss reserving. Such an impressive rate created many gaps for future research and it was decided to subject loss reserving to a claim-level data in which future claim estimate can be predicted using covariate or predictive variables to improve upon the basic methods and then apply the technique of bootstrapping to the claim-level data to estimate reserve variability. However, there are a few weaknesses associated with this method. In that, there are times where better outstanding losses can be estimated by basing our reserve variability estimate on un-summarized claim-level data. In short,claim-level data together with other predictive methods has the potential to improve actuaries estimate of outstanding losses.

Additionally, other researchers applied the theory of position marked Poisson process and statistical tools for recurrent event to analyze liability of claims (Antonio and Plat, 2014). In their study, they found out that using detailed information at the time the claim occurs, the delay between the occurrence date and reporting date to the insurance company, the occurrence and the size of the payments and the final settlement, a parametric approach like the maximum likelihood was used to calibrate the historical data for the purpose of projection for future development open claims. It was deduced from the research that the micro-level stochastic model outperforms the traditional methods of loss reserving in the sense that the loss reserve estimated was close to the median of most predictive distributions like the Burr and Pareto. Though the micro-level stochastic model gives a better estimate, researchers still need to refine the performance of individual probability model for a more detailed conclusion.

Portugal et al. (2018) asserted that many stochastic reserving models in insurance literature is mainly a reproduction of the chain ladder technique for reserve estimation.It ought to be elaborated that the distinction between the incurred claims and the final price makes the selection of reserving strategy among the foremost essential aspects within the insurance business.This method is expounded chiefly to non-life (general) insurance policies, however it may also have an effect on some merchandise of the life assurance business additionally. When the basic assumption of the chain ladder method are not met,higher prediction error might occur hence the need to adopt a stochastic vector projection method. This method uses regression through the approach of Murphy (1994) but in this case it was coupled with a heteroscedastic error which is different from the one used by Mack. In conclusion, the CL and VP approaches were considered to generate higher prediction error with a certain class of data hence in a real world problem, different methodology and approaches should be maintained for verification.

2.7 Application of Individual claim size modeling to the study of loss reserving

Mathematically speaking, one of the goals of this thesis is to fit individual claim size models to non-life insurance data available. This parametric claim size models will aid with prediction certainty aside other methods used for estimating reserves. Parametric support which has unbound support in $\mathbb{R}_+$ would be considered. Most questions we normally ask about probability of claims issues are whether the probability distributions proposed fits the claim data very well, aside that,the question of whether we can assume the properties of such probability distribution such as the mean, confidence interval of their random numbers generated will aid the actuary to have a better estimate of reserves.

Hewitt and Lefkowitz (1979) described the fitting of probability distribution of loss data using the gamma,log-gamma and log-normal +gamma. Their paper also discussed estimation problems in the fitted distribution which encompassed inflation effect of deductibles credit, determination of claim frequency and severity. the aim was to fit the probability distribution to the loss data so as to characterize the properties of the fitted distribution of the data to enable prediction. In conclusion, the log-normal distribution closely approximated loss data which was homogeneous not forgetting the gamma and log-gamma which also gives good fit. In a single insurance loss data fitting, one distribution can not fit an aggregate loss data well as compared to a combination of two or more (compound distribution). Fitting a probability distribution to insurance claims is therefore recommended that the actuary use a combination of distributions.

Hauger (2017) in his thesis assessed the extent to which a six parameter extension of a Pareto distribution can be utilized to predict a claim distribution in a non-life insurance. After fitting the distribution, it was then compared to eight special cases of Pareto distribution which was inline with 99% and 99.5% reserves for the non-life insurance claims. The data used for the research encompassed larger or extreme claims. According to pickand's theorem, those larger claims should be modeled with a two-parameter Pareto distribution and the four parameter extended Pareto distribution. The four parameter extended Pareto distribution was preferred when the actuaries decided to model claims which are not bigger. This research was therefore posited that, there is no difference in the models estimating the 99% and the 99.5% reserves for insurance companies hence there is the need for the actuary to determine the nature of claims and segregate the claim data if possible so that different models can be used to model different sizes of claims.

García et al. (2014) in their paper opined and discussed that heavy tailed distributions are used to fit insurance loss claims data that are skewed to the right. This is of course the situation most intended in practical situations. Using the Marshall and Olkin procedure in their methodology, the log-normal distribution was proposed and used for the analysis. It was finalized that the log-normal distribution in conjunction with other possible distributions which have heavy tail properties should be considered. Consequently (Packová and Brebera, 2015) applied probability modeling to a wide range of insurance practice. Improving or avoiding risk in insurance is an effective tool for the survival of the insurance company because solvency issues is a delicate and also a comprehensive problem.The main objective of their research was to ascertain and present various possibilities as to how insurance claims can be modeled by probability in order to avoid risk. Various tail distributions such as gamma, weibull and Pareto which are known to be appropriate for modeling insurance losses was used but in conclusion, the Pareto distribution was considered to have the best property available for the research data which best fit the tails.This same distribution plays an important role in non-proportional insurance in that it it gives option for a better way of segregation of claims into different layers for claim decomposition. In as much different probability models are used for different forms of data, it is the duty of the actuary to thoroughly find a valid statistical means to pick the right distribution for the claims.

Aside other popular tail probability models, Burr probability model which is mostly used in assessing heavy tails and its impact on data. (Das and Nath, 2016) asserted and fitted the Burr distribution coupled with the integro-differential equation to determine claim severity and ruin of the insurance firm. With this idea, the first two moment determined the claim severity and probability of ruin considering the recursive stable algorithm as an input not forgetting the research assumption was based on a classical risk model. In conclusion, as much as the main aim of the research was to measure the probability of ruin in case of claim severity, little effort was made to compare the methodology used above to the Pollachez Khinchin formula for accuracies which is by far a benchmark.

Vukovic (2015) in his paper endeavored to create and execute the model that can imitate distributions of operational hazard in an insurance firm. It executed summed up Pareto dissemination and Monte Carlo simulation and attempted to copy and develop operational hazard models in insurance. In the meantime, he looked at log-normal, Weibull and log-logistic dissemination and their utilization in the insurance industry. It was realized that operational risk models in insurance are portrayed by extreme tails hence he suggested that the distribution used for the analysis should be done in two part thus, the tail analysis and the body analysis. Separately doing this and using a valid convolution methodology such as the Monte carlo method would provide better insight as to how risk of reserves can be managed in insurance. Having all that done, the student-t-copula and Monte Carlo simulation was used for analysis. Even though this different methods brought a variety in terms of analyzing and computing risk for insurance companies, it is sought after that the actuary should maintain and used valid statistical model selection criteria to select one which would best fit the data available.

Bortoluzzo et al. (2011) used the tweedie and the zero-adjusted inverse gaussian (ZAIG) models to identify various factors that influence probability of claim size. The outcome of the results from the two models used was compared for the purpose of forecast and accuracy. Data on the characteristics of vehicles like the age,origin and territory in which it was used were all considered to distinctly affect claim size and probability. The two models tend to make the insurance company identify clients that are riskier hence the insurance company can propose special package allocation of premium for such clients. Knowing the claim size and probability enables the actuary to plan and determine estimates for reserves. Though claim size and probability is mostly sought by insurers because of its importance, higher computation is usually required and also larger data sets is needed for proper prediction and forecasting. In conclusion, few of the reviewed literature's have questioned the credibility of the reserve estimates from Chain-ladder and Bornhuetter Ferguson so this current this thesis seeks better ways for loss reserving.

CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter focuses on all the statistical methods and models used in the analyses of the data collected for the purpose of the study.It also gives a summary of the data structure considered. The models and statistical methods to be discussed includes pure computational algorithm loss reserving methods such as Chain-ladder and Bornhuetter Ferguson method. Stochastic models such as the Mack method of loss reserving is also considered with probability models which will be used to model individual claims sizes.These models and methods would be needed in satisfying the main and other secondary objectives of the study.

3.1.1 Data source and structures

Loss development triangle was developed by actuaries in an insurance company to track how claims both known and unknown change overtime. Example is a car involving in an accident where no bodily injury occurred but there was a damage to the car, the captive insurance company would then send an adjuster to analyze the damage and this amount will be the books as an opening reserve but when the car gets to the workshop and later it is found out that the engine also got affected or maybe the accident is more extensive than initially thought then the cost of the claim has developed. (changed overtime). A loss development triangle is a way of capturing this changes overtime. Two different secondary data-sets were collected from records at the Donewell insurance (Accra Osu branch) by considering the department in charge of handling data. The specific data requirement needed for loss development triangle regarding the past claim experience includes; the amount of claims that were settled in the past and these claim losses include the claim benefit payment,any indirect expenses that were allocated to the claims, any direct handling expenses and reinsurance recoveries. Reinsurance recoveries result in a reduction in the claims losses (or no change). Now in order to simplify this explanation these potential negative claims are ignored in this thesis. Again to represent the claim loss development triangle or run-off triangle the following additional information is needed.

1. The claim event in respect of which the claim loss is settled.
2. The date of the claim event.
3. The date on which the claim event was reported to the insurer.
4. The date on which the claim loss was settled.

The benefit and expenses in respect of a claim may be paid or allocated over a number of reported periods (Matvejevs et al., 2014).Having all of the above data can allow the actuary to model delays of the occurrence of claims and that of the reporting of event (reporting delays, used in provision for IBNR) for the purpose of the thesis, only paid claims was considered.

Basic Terminology and Definition

For the purpose of this thesis, the following terms are defined as shown by **?**

Calender year

A period of 12 months running from January to December.

Claim Occurrence year

The calender year in which the claim occurred $y \in \{1990; , 1901; ..., Y\}$ where Y represents the most recent complete calender year also called accident year.

Development year, K

The complete number of years that have elapsed between the end of the claims occurrence year and the end of year in which a claim loss amount is settled (partially or fully); $K \in \{0; 1; 2; ...n\}$ where n represents the maximum number of development years observed for any claim occurrence year.

Incremental Claim loss settled,Z(Y, K)

The claim loss amount, for claim occurrence year Y, that is settled in development year K. Clearly $Y + K$ represents the calender year in which the incremental claim loss settlement amount is paid.

Individual Claim Data

The individual claim size data is the singular promised payment made to the insured. The cumulative of the claims gives the paid losses for that particular year. The next data obtained was the assumed expected payment claim. The expected paid claims is used as the prior information for ultimate loss estimation.

3.1.2 Profile of Donewell Insurance

Donewell insurance was established in 1993 by the Methodist Church but it is currently owned by individuals and corporate entities as share holders. Donewell insurance has a branch network of 11 across the 10 regions in Ghana with several other agency offices. The head office location is in Accra, specifically, Osu Kuku Hill.

3.2 Chain-Ladder model

3.2.1 Introduction

The Chain-ladder algorithm is widely the most used reserving technique by actuaries in a captive insurance company.Various literature explains the chain-ladder method as a pure algorithm (in the most basic sense pure algorithm is a couple of instructions that must be followed to achieve a certain result) in the estimation of loss reserves.

Model Assumption

1. The model assumes that there exist development factors $f_0, ..., f_{j-1} > 0$ such that $\forall\ 0 \leq i \leq 1$ and $\forall\ 1 \leq j \leq J$ we will then have

$$E[C_{ij}|C_{io}...C_{ij-1}] = E[C_{ij}|C_{ij-1}] = F_{j-1} \cdot C_{ij-1} \tag{3.1}$$

   coupled with the idea that different accidents are independent.

2. The model assumes that past data pattern is indicative of the future.The Reserves $\mathbb{R}$ is extrapolated into the lower triangle by the observations.

Remarks

- Adding to the assumptions, we can also do stronger assumptions by using the sequence $C_{i0}, C_{i1}, ...$ meaning this forms a Markov chain thus

$$C_{ij} \cdot \prod_{i=0}^{j-1} f_1^{-1} \tag{3.2}$$

  and this forms a martingale for $j \geq 0$

- Factors f_j are called the age-to-age ratio or better still chain ladder factors and it is the pivotal point of interest on the chain ladder model.

Lemma

Assume $D_I = \{C_{ij}; i + j \leq I, 0 \leq j \leq J\}$ be a set of observation in the upper triangle. Using the model assumption in 3.1 $\forall\ I - J + 1 \leq i \leq I$. Thus

$$E[C_{i,J}|D_I] = E[C_{ij}|C_{i,I-i}] = C_{i,I-i} \cdot f_{I-i}...f_{j-1} \tag{3.3}$$

Proof

$$\begin{aligned} E[C_{i,J}|C_{i,J-1}] &= E[C_{i,J}|D_I] \\ &= E[E[C_{i,J}|C_{i,J-1]|D_I}] \\ &= f_{J-1} \cdot E[C_{i,J-1|D_I}] \end{aligned}$$

When the process is iterated until the diagonal $i + j = 1$ is reached then the claim is obtained. The cumulative payment is gotten by

$$C_{i,j} = \sum_{l=0}^{j} X_{i,l} \tag{3.4}$$

Equation (3.4) signifies that we should sum all payment $X_{i,l}$ where $l > 0$ for a fixed accident year i so that we can obtain ultimately $C_{i,J-1} = S_i$ where S_i denotes the total claim. Another idea behind the **CL** is that all the accident years $i \in \{1..., I\}$ have similar behavior and for the cumulative payment approximately we have

$$C_{i,j+1} \approx f_j C_{i,j} \tag{3.5}$$

for any given $f_j > 0$, these are called the **CL** age-to-age factors or link ratio. Structure (3.5) will then provide us with the intuition for projecting the ultimate claim $C_{i,J-1}$ based on the observation of D_I. For every accident year i we choose the observation on the last observed diagonal thus $C_{i,I-i}$ and multiply the observation with the successive **CL** factors $f_{I-i}..., f_{j-2}$. In a whole, the **CL** factors f_j are not known and will have to be estimated and assuming the weighted estimate is

$$\widehat{f}_j^{CL} = \frac{\sum_{i=1}^{I-J-1} C_{i,j+1}}{\sum_{i=1}^{I-J-1} C_{ij}} = \frac{\sum_{i=1}^{I-j-1} C_{ij}}{\sum_{n=1}^{I-J-1} C_{n,j}} \frac{C_{i,J+1}}{C_{i,j}} \tag{3.6}$$

The equation (3.6) means that we have to divide the sum of observed successive columns by each other which is exactly in conformity with the link ration in equation (3.5). The next step is to calculate the development ratios $C_{i,J+1}/C_{ij}$ which has already been observed in D_I. Having been equipped with these necessary steps then we can predict the ultimate claim $C_{i,J-1}$ for $i + J - 1 > 1$ by

$$\widehat{C_{i,J-1}}^{CL} = C_{i,I-i} \prod_{j=I-i}^{J-2} \widehat{f}_j^{CL} \tag{3.7}$$

We can now set

$$\widehat{C_{i,n}}^{CL} = \prod_{j=I-i}^{n-1} \widehat{f}_j^{CL}$$

for $i + n > I$. The chain ladder reserves at time I for accident years $i > I - (J - 1)$ is then given as:

$$\widehat{R_i}^{CL} = \widehat{C_{i,J}}^{CL} - C_{i,I-i} = C_{i,I-i} \left(\prod_{j=I-i}^{J-2} \widehat{f}_j^{CL} - 1 \right)$$

The sum over all the accident years will predict the outstanding loss liabilities by the chain ladder predictor $\widehat{R_i}^{CL} = \sum_{i>1-(j-1)} \widehat{R_i}^{CL}$

3.3 Bornhuetter - Ferguson Algorithm

3.3.1 Introduction

While The **CL** takes the view point that the observation D_I are extrapolated into the lower trapezoid , the Bornhuetter Ferguson **(BF)** uses a different measure by assuming that the lower trapezoid D_I^c is extrapolated independently from D_I using an expert knowledge. It is considered as a very robust technique since it does not consider outliers in the observations.

Model Assumptions

1. Different accident years are independent

2. There exist parameters $\mu_0, ...\mu_I > 0$ and a pattern $\curlyvee_0, ...\curlyvee_J > 0$ with $\curlyvee_J = 1$ such that $\forall\ i \in \{0, ...I\}$, $j \in \{0, ...J-1\}$ and $k \in \{1, ...J-j\}$

$$E[C_{i,0}] = \mu_i \cdot \curlyvee_0,$$

$$E[C_{i,j+k}|C_{i,0,...C_{ij}}] = C_{ij} + \mu_i \cdot (\curlyvee_{j+k} - \curlyvee_j) \tag{3.8}$$

In the above equation, we will have ;

- $E[C_{i,j}] = \mu_i \cdot \curlyvee_j$

- $E[C_{i,J}] = \mu_i$

The sequence $(\curlyvee_j)_j$ denotes the claim development pattern. If $C_{i,j}$ are the cumulative payment, then $\curlyvee_j$ is the expected payout pattern.

Bornhuetter Ideology

Bornhuetter was of the idea that accident years although are independent, they behave similarly and their payment approximately behave as $X_{i,j} \approx \curlyvee_j \cdot \mu_i$ for a given prior information $\widehat{\mu}$ and a development pattern $(\curlyvee_j)_{j=0,...J-1}$ with a normalization $\sum_{j=0}^{j-1} \curlyvee_j = 1$. The prior information value $\widehat{\mu_i}$ should be able to reflect the mean ultimate claim $E[C_{i,J-1}]$ of the accounting year i. It is of the assumption that the prior information value is given by an external expert opinion and in theory this should not be based on D_i. In view of this the only remaining development pattern is $(\curlyvee_j)_j$. In the chain ladder model, one can define the development estimate pattern as

$$\beta_j^{CL} = \prod_{l=j}^{J-2} \frac{1}{\widehat{f_l}^{CL}} = \frac{\prod_{i=0}^{j-1} \widehat{f_l}^{CL}}{\prod_{l=0}^{J-2} \widehat{f_l}^{CL}}$$

According to the estimates in the **CL** pattern the ratio above reflects the proportions paid after the first J development periods hence we obtain:

$$\begin{aligned}
\widehat{\Upsilon_0^{CL}} &= \widehat{\beta_0^{CL}} \\
\widehat{\Upsilon_j^{CL}} &= \widehat{\beta_j^{CL}} - \widehat{\beta_{j-1}^{CL}} \qquad for \quad j = 1, ...J-2 \\
\widehat{\Upsilon_{J-1}^{CL}} &= 1 - \widehat{\beta_{J-2}^{CL}}
\end{aligned}$$

Being equipped with these estimators, we can then predict the ultimate claim $C_{i,J-1}$ for $i + J - 1 > 1$ in the **BF** method by;

$$\widehat{C_{i,J-1}^{BF}} = C_{i,I-i+\widehat{\mu}_i} \sum_{j=I-i+1}^{J-1} \widehat{\Upsilon_j^{CL}} = C_{i,I-i} + \widehat{\mu}_i \left(1 - \widehat{\beta_{I-i}^{CL}}\right) \tag{3.9}$$

Which gives the **BF** reserves at time $i > I - (J-1)$ as

$$\widehat{R_i^{BF}} = \widehat{\mu}_i \sum_{j=I-i+1}^{J-1} \widehat{\Upsilon_j^{CL}} = \widehat{\mu}_i \left(1 - \widehat{\beta_{I-i}^{CL}}\right)$$

and aggregated over all accident years we can predict the outstanding loss liabilities of past claim by

$$\widehat{\mathbf{R}}^{\mathbf{BF}} = \sum_{i>I-(J-1)} \widehat{\mathbf{R}_i}^{\mathbf{BF}}$$

Comparison of CL and BF

The comparison can be taken back to equation (3.7). With modifications it becomes;

$$\widehat{C_{i,J-1}}^{\mathbf{CL}} = C_{i,I-i} + C_{i,I-i} \prod_{j=I-i}^{J-2} \widehat{f_j}^{\mathbf{CL}} \left(1 - \prod_{j-I-i}^{J-2} \frac{1}{\widehat{f_j}^{\mathbf{CL}}}\right)$$

This then gives the following comparison;

$$\begin{aligned}
\widehat{C_{i,J-1}}^{\mathbf{CL}} &= C_{i,I-i} + \left(1 - \widehat{\beta_{I-i}}^{\mathbf{CL}}\right) \widehat{C_{i,J-1}}^{\mathbf{CL}} \\
\widehat{C_{i,J-1}}^{\mathbf{BF}} &= C_{i,I-i} + \left(1 - \widehat{\beta_{I-i}}^{\mathbf{CL}}\right) \widehat{\mu_i}
\end{aligned}$$

The only difference is that the **BF** method uses a prior estimate $\widehat{\mu_i}$ for the ultimate claim while the **CL** method uses the observations based on the estimate of $\widehat{C_{i,J-1}}^{\mathbf{CL}}$

3.4 Mack Model

3.4.1 Introduction

The **CL** and **BL** being the simplest methods used by actuaries in a captive insurance firms are not deemed as random and many are of the view that catastrophic events may not reflect this reserves hence the need to have a stochastic derivation of the **CL** reserves so that unexpected events like the 9/11 disaster in the USA which left many insurance companies having a tough time to pay their promised claims. Mack model measures the reserve uncertainty involved in the data, the standard error in reserves, since it is a specialized case of **CL**, it makes things easier to replace Taylor series of approximation with a more exact procedure and lastly,a more exact process variance is included in the standard error of the estimated reserves and this is important because the claims are seen as random variables. All this can tell whether the difference in reserve estimates between **CL** and other methods is significant.

Model Assumptions

Assuming C_{ik} represents the aggregated claim amount of accident year i, $1 \leq i \leq I$, then the development year k, which is $1 \leq k \leq I$ would be considered as a random variable C_{ik} for which we have an observation if $i + k \leq I + 1$ in the run-off triangle. The main aim is to estimate C_{il} which is the ultimate claim amount,outstanding claim reserve and standard error associated with the reserves R_i obtained.

$$R_i = C_{il} - C_{i,I+1-i}$$

For the accident year $i = 1, 2, ...I$. The assumptions underlying the chain ladder and its development factors $f_1, ..., f_{I-1} > 0$ also includes:

$$E\left(C_{i,k+1}|C_{il}, ..., C_{ik}\right) = C_{ik}f_k, \quad 1 \leq i \leq I, \ 1 \leq k \leq I-1. \tag{3.10}$$

In the chain ladder method we estimate the f_k thus the development factors by

$$\widehat{f}_k = \frac{\sum_{j=1}^{I-k} C_{j,k+1}}{\sum_{j=1}^{I-k} C_{jk}}, \qquad 1 \leq k \leq I-1 \tag{3.11}$$

and the ultimate claim amount C_{il} is also given by:

$$\widehat{C}_{il} = C_{i,I+1-i} \cdot \widehat{f}_{I+1-i} \cdot ...\widehat{f}_{I-1}$$

The next assumption implies that since the chain ladder does not take into account dependencies between accident years then:

$$(C_{iI}, ...C_{iI}, (C_{j1,...C_{jI}}) \qquad i \neq j, \tag{3.12}$$

hence they are independent. This makes it clear that (3.11) and (3.12) are clear assumptions of the **CL**

Theorem 1:

Assume $D_i = \{C_{ik}|i + k \leq I + 1\}$ is the set of observed data. Then under assumptions (3.11) and (3.12) we will have

$$E\left(C_{il}|D_i\right) = C_{i,I+1-i} \cdot f_{I+1-i} \cdot ...f_{I-1}$$

Proof

$$E_i = E(X|C_{il}, ..., C_{i,1+I-i})$$

Then equation (3.12) and repeated application of (3.11) gives

$$\begin{aligned}
E\left(C_{il}|D_i\right) &= E_i\left(C_{il}\right) \\
&= E_i\left(E(C_{il}|C_{i1}, ..., C_{i,I-1})\right) \\
&= E_i\left(C_{i,I-1}f_{I-1}\right) \\
&= E_i(C_{i,I-1})f_{I-1} \\
&= etc. \\
&= E_i(C_{i,I+1-i})f_{I+1-i} \cdot ... \cdot f_{I-1} \\
&= C_{i,I+1-i}f_{I+1-i} \cdot ... \cdot f_{I-1}.
\end{aligned}$$

This equally shows that $\widehat{C_{il}}$ which is the estimator has the same form as $E(C_{il}|D_i)$ meaning it would be the best forecast of C_{il} upon the observation of D_i

Theorem 2:

With the same assumption in equation (3.11) and (3.12), the estimators $\widehat{f_k}$ $1 \leq k \leq I-1$ are uncorrelated and unbiased.

Proof

Assume

$$B_k = \{C_{ij} | j \leq k, i + j \leq I + 1\} \qquad 1 \leq k \leq I$$

, Then equation (3.11) and (3.12) will give:

$$E(C_{i,k+1}|B_k) = E(C_{i,k+1}|C_{il}, ..., C_{ik}) = C_{ik} f_k$$

Therefore

$$E(\widehat{f}_k|B_k) = \frac{\sum_{j=1}^{I-k} E(C_{j,k+1}|B_k)}{\sum_{j=1}^{I-k} C_{ik}} = f_k$$

Which gives the unbiased

$$E(\widehat{f}_k) = E(E(\widehat{f}_k|B_k)) = f_k, \qquad 1 \leq k \leq I-1$$

parameter estimates.Again $\widehat{f}_k$ are uncorrelated because $j < k$.

$$\begin{aligned} E(\widehat{f}_j \widehat{f}_k) &= E(E(\widehat{f}_j \widehat{f}_k|B_k)) \\ &= E(\widehat{f}_j E(\widehat{f}_k|B_k)) \\ &= E(\widehat{f}_j) f_k \\ &= E(\widehat{f}_j) E(\widehat{f}_k) \end{aligned}$$

Calculation of mean square error and standard error

The mean square error $MSE(\widehat{C}_{il})$ of the estimator is given as:

$$MSE(\widehat{C}_{il}) = E((\widehat{C}_{il} - C_{il})^2 | D_i)$$

Where D_i =$\{C_{ik}|i+k \leq I+1\}$ is the data observed. It is worthwhile noting that the $E((\widehat{C}_{il}-C_{il})^2) = E(E((\widehat{C}_{il}-C_{il})^2|D_i)$ thus the unconditional mean square error is not used because it averages all possible data in D_i. Instead we use the conditional mean square error$E((\widehat{C}_{il}-C_{il})^2|D_i)$ which gives the average deviation between $C_{il}$and $\widehat{C}_{il}$ due to randomness only.

Initially,we will see that

$$MSE(\widehat{R}_i) = E((\widehat{R}_i - R_i)^2|D_i) = E((\widehat{C}_{il}-C_{il})^2|D_i) = MSE(\widehat{C}_{il})$$

Next, the general rule $E(X-a)^2 = Var(X) + (E(X)-a)^2$ we would have

$$MSE(\widehat{C}_{il} = Var(C_{il}|D_i) + (E(C_{il}|D_i) - \widehat{C}_{il})^2$$

This tells us the MSE is the sum of the stochastic error (process variance) and the estimation error. Further calculation of MSE needs a formula for C_{ik}. Since $\widehat{f}_k$ is the C_{ik} weighted mean of the individual development factors $\frac{C_{i,k+1}}{C_{ik}}$, $1 \leq i \leq I-k$, we can infer that $Var\left(\frac{C_{i,k+1}}{C_{ik}}|C_{il},...,C_{ik}\right)$ should be inversely proportional to C_{ik}. The variance assumption

$$Var(C_{i,k+1}|C_{il},...,C_{ik}) = C_{ik}\sigma_k^2, \quad 1 \leq i \leq I, \quad 1 \leq k \leq I-1, \tag{3.13}$$

With unknown parameter σ_k^2 can be also be obtained by

$$\widehat{\sigma}_k^2 = \frac{1}{I-k-1}\sum_{i=1}^{I-k} C_{ik}\left(\frac{C_{i,k+1}}{C_{ik}} - \widehat{f}_k\right)^2, \quad 1 \leq k \leq I-2. \tag{3.14}$$

Since σ_k^2 is the unbiased estimator, it still lacks an estimator for σ_{I-1}. Assume $\widehat{f}_{I-1} = 1$ and the claim development will finish after $I-1$ years, then we can have $\widehat{\sigma}_{I-1} = 0$. If this is not the case,we have to extrapolate the exponential decreasing series $\sigma_I, ...\sigma_{I-3}, \sigma_{I-2}$

by an additional member and this requires that

$$\frac{\widehat{\sigma}_{I-3}}{\widehat{\sigma}_{I-2}} = \frac{\widehat{\sigma}_{I-2}}{\widehat{\sigma}_{I-1}}$$

Holds only if $\widehat{\sigma}_{I-3} > \widehat{\sigma}_{I-2}$. This leads to

$$\widehat{\sigma}^2_{I-1} = min\left(\frac{\widehat{\sigma}^4_{I-2}}{\widehat{\sigma}^2_{I-3}}, min\left(\widehat{\sigma^2_{I-3}}, \widehat{\sigma^2_{I-2}}\right)\right) \tag{3.15}$$

Theorem 3:

From the given assumption in (3.11),(3.12),(3.13) the $MSE(\widehat{R}_i)$ can be estimated by

$$\widehat{MSE(R_i)} = \widehat{C}^2_{il} \sum_{k=I+1-i}^{I-1} \frac{\widehat{\sigma^2_k}}{\widehat{f_k}^2} \left(\frac{1}{\widehat{C_{ik}}} + \frac{1}{\sum_{j=1}^{I-k} C_{jk}} \right)$$

With $\widehat{C_{ik}}$ = the estimated future values of C_{ik} and $\widehat{C}_{i,I+1-i} = C_{i,I+1-i}$

Proof:

Using the relations and abbreviations

$$E_i(X) = E(X|C_{i1}, ..., C_{i,I+1-i})$$

$$Var_i(X) = Var(X|C_{i1}, ..., C_{i,I+1-i})$$

We begin from

$$MSE(R_i) = Var(C_{iI}|D_i) + (E(C_{iI}|D_i) - \widehat{C_{iI}})^2$$

Further application of the **CL** assumptions (3.11) and variance assumption (3.13) yields

$$\begin{aligned} Var(C_{il}|D_i) &= Var_i(C_{iI}) \\ &= E_i(Var(C_{iI}|C_{i1},...,C_{i,I-1})) + Var_i(E(C_{iI}|C_{i1},...,C_{i,I-1})) \\ &= E_i(C_{i,I-1})\sigma_{I-1}^2 + Var_i(C_{i,I-1})f_{I-1}^2 \\ &= E_i(C_{i,I-2})f_{I-2}\sigma_{I-1}^2 + E_i(C_{i,I-2})\sigma_{I-2}^2 f_{I-1}^2 + Var_i(C_{i,I-2})f_{I-2}^2 f_{I-1}^2 \\ &= etc \\ &= C_{i,I+1-i}\sum_{k=I+1-i}^{I-1} f_{I+1-i}...f_{k-1}\sigma_k^2 f_{k+1}^2...f_{I-1}^2 \end{aligned}$$

Since the $Var_i(C_{i,I+1-i}) = 0$. .Theorem 1 will also help to obtain the second term of $MSE(R_i)$ thus $(E(C_{iI}|D_i) - \widehat{C}_{iI})^2 = C_{i,I+1-i}^2(f_{I+1-i}\cdot ... \cdot f_{I-1} - \widehat{f}_{I+1-i}\cdot ... \cdot \widehat{f}_{I-1})^2$
In practical aspect, the actuary will have to find estimators for these two terms of $MSE(R_i)$. The first term estimator cab be done by substituting the unkwon parameters thus f_k and σ_k^2 by $\widehat{f}_k$ and $\widehat{\sigma}_k^2$. The variance can then be estimated by

$$\begin{aligned} Var(C_{iI}|D_i) &= C_{i,I+1-i}\left(\sum_{k=I+1-i}^{I-1} \widehat{f}_{I+1-i}\cdot ... \cdot \widehat{f}_{k-1}\cdot \sigma_k^2 \cdot \widehat{f}_{k+1}^2 \cdot ... \cdot \widehat{f}_{I-1}^2\right) \\ &= \widehat{C}_{iI}^2 \sum_{k=I+1-i}^{I-1} \frac{\widehat{\sigma^2}_k/\widehat{f}_k^2}{\widehat{C}_{ik}} \end{aligned}$$

The second term can not be done by this approach since it would yield zero if it is replaced by the estimate of their parameters. In this different approach we can write

$$\begin{aligned} F &= f_{I+1-i}\cdot ... \cdot f_{I-1} - \widehat{f}_{I+1-i}\cdot ... \cdot \widehat{f}_{I-1} \\ &= S_{1+1-i} + ... + S_{I-1} \end{aligned}$$

and

$$S_k = \widehat{f}_{I+1-i} \cdot ... \cdot \widehat{f}_{k-1}(f_k - \widehat{f}_k) f_{k+1} \cdot ... \cdot f_{I-1}$$

means that

$$\begin{aligned} F^2 &= (S_{I+1-i} + ... + S_{I-1})^2 \\ &= \sum_{k=I+1-i}^{I-1} S_k^2 + 2\sum_{j<k} S_j S_k \end{aligned}$$

Replacing S_k^2 with $E(S_k^2|B_k)$ and S_jS_k for $j < k$ with $E(S_jS_k|B_k)$ signifies that S_k^2 and S_jS_k are approximated by averaging over smallest data as possible hence the C_{ik} values possible from the observed data are kept fixed. This in all leads to $E(S_jS_k|B_k) = 0$ for $j < k$ because

$$\begin{aligned} E((f_k - \widehat{f})^2|B_k) &= Var(\widehat{f}_k|B_k) \\ &= \frac{\sum_{j=1}^{I-k} Var(C_{j,k+1}|B_k)}{\left(\sum_{j=1}^{I-k} C_{jk}\right)^2} \\ &= \frac{\sigma_k^2}{\sum_{j=1}^{I-k} C_{jk}} \end{aligned}$$

This results in

$$E(S_k^2|B_k) = \frac{\widehat{f}^2_{I+1-i} \cdot ... \cdot \widehat{f}^2_{k-1} \sigma_k^2 f_{k+1}^2 \cdot ... \cdot f_{I-1}^2}{\sum_{i=1}^{I-k} C_{ik}}$$

Since the sum of all the terms are positive,we can substitute $F^2 = (\sum S_k)^2$ with $E(S_k^2|B_k)$ and then replace f_k and σ_k^2 with their estimates $\widehat{f}_k$ and $\widehat{\sigma}_k^2$ hence $F^2 = (f_{I+1-i} \cdot ... \cdot f_{I-1} -$

$\widehat{f}_{I+1-i} \cdot ... \cdot \widehat{f}_{I-1})^2$ can be estimated by

$$\sum_{k=I+1-i}^{I-1} \left(\widehat{f}^2_{I+1-i} \cdot ... \cdot \widehat{f}^2_{k-1} \cdot \sigma^2_k \cdot \widehat{f}^2_{k+1} \cdot ... \cdot \widehat{f}^2_{I-1} / \sum_{j=1}^{I-k} C_{jk} \right) = \widehat{f}^2_{I+I-i} \cdot ... \cdot \widehat{f}^2_{I-1} \sum_{k=I+1-i}^{I-1} \frac{\widehat{\sigma}^2_k / \widehat{f}^2_k}{\sum_{j=1}^{I-k} C_{jk}}$$

and this leads to the estimator of the second term of $MSE(R_i)$

Remarks

The standard error of the reserves $(\widehat{R_i})$ is defined to be the square root of an estimator of the MSE. Usually the reserves $\widehat{R} = \widehat{R}_2 + ... + \widehat{R}_I$ is of very high interest and in this case we can not sum the values of $(s.e(\widehat{R}_i))$, $2 \leq i \leq I$. Since they are correlated through a common estimator $\widehat{f}_k$ and $\widehat{\sigma}_k$. Proceeding with this idea we obtain

$$\widehat{MSE(\widehat{R})} = \sum_{i=2}^{I} \left\{ (s.e(\widehat{R_i}))^2 + \widehat{C}_{iI} \left(\sum_{j=i+1}^{I} \widehat{C}_{jI} \right) \sum_{k=I+1-i}^{I-1} \frac{2\widehat{\sigma}^2_k / \widehat{f}^2_k}{\sum_{n=1}^{I-k} C_{nk}} \right\}$$

Proof:

$$\begin{aligned} MSE\left(\sum_{i=2}^{I} \widehat{R}_i\right) &= E\left(\left(\sum_{i=2}^{I} \widehat{R}_i - \sum_{i=2}^{I} R_i\right)^2 |D_i\right) \\ &= E\left(\left(\sum_{i=2}^{I} \widehat{C}_{iI} - \sum_{i=2}^{I} C_{iI}\right)^2 |D_i\right) \\ &= Var\left(\sum_{i=2}^{I} C_{iI}|D_i\right) + \left(E\left(\sum_{i=2}^{I} C_{iI}|D_i\right) - \sum_{i=2}^{I} \widehat{C}_{iI}\right)^2 \end{aligned}$$

Since the accident years are independent it yields

$$Var\left(\sum_{i=2}^{I} C_{iI}|D_i\right) = \sum_{i=2}^{I} Var(C_{iI}|D_i),$$

Which the summands have already been proofed and calculated in theorem 3. Further

$$\begin{aligned}\left(E\left(\sum_{i=2}^{I} C_{iI}|D_i\right) - \sum_{i=2}^{I} \widehat{C}_{iI}\right)^2 &= \left(\sum_{i=2}^{I}(E(C_{iI}|D_i) - \widehat{C}_{iI})\right)^2 \\ &= \sum_{i,j}(E(C_{iI}|D_i) - \widehat{C}_{iI}) \cdot (E(C_{ji}|D_i) - \widehat{C}_{jI}) \\ &= \sum_{i,j} C_{i,I+1-i}C_{j,I+1-j}F_iF_j\end{aligned}$$

with

$$F_i = f_{I+1-i}...f_{I-1} - \widehat{f}_{I+1-i}...\widehat{f}_{I-1}$$

observing that $MSE(R_i) = Var(C_{iI}D_i) + (C_{i,I+1-i}F_i)^2$ which in the proof in theorem 3 we saw that

$$MSE\left(\sum_{i=2}^{I}\widehat{R}_i\right) = \sum_{i=2}^{I} MSE(\widehat{R}_I) + \sum_{2\leq i<j\leq I} 2 \cdot C_{i,I+1-i}C_{j,I+1-j}F_iF_j$$

An equivalent strategy for F^2 yields $F_iF_j,\ \ i < j$ the estimator for the proof above hence

$$\sum_{k=I+1-i}^{I-1} \widehat{f}_{I+1-j} \cdot ... \cdot \widehat{f}_{I-i}\widehat{f}^2_{I+1-i} \cdot ... \cdot \widehat{f}^2_{k-1}\widehat{\sigma}^2_k\widehat{f}^2_{k+1} \cdot ... \cdot \widehat{f}^2_{I-1} / \sum_{n=1}^{I-k} C_{nk}$$

This ends the proof.

3.5 Model Accuracy

This is a method of estimating the expected predicted error. It indicates the best fit model and also ensures that the model is not over fitted. If the error rate is low then a good model is built. The sample data is split into two types, thus, the training sample and the hold out sample. The training sample is used to build a model and it is compared to another sample which is not part of the training data. This is then tested for consistency and if that exists then there can be conclusion of a good fit model. This can be achieved my MSE, MAPE,MAD or RMSE.

$MAD = \frac{1}{k}\sum_{k=1}^{k} |actual - forecast|$

$MSE = \frac{\sum_{k=0}^{n} (actual-forecast)^2}{n}$

$MAPE = \frac{1}{k}\sum_{k=1}^{k} \frac{|actual-forecast|}{actual} \times 100\%$

3.6 Individual Claim Size Modeling

3.6.1 Introduction

The individual claim size modeling concentrates on getting the probability distribution that will best fit the claim sizes Y_i. When such objective is obtained, the properties of the distribution would assist in making a prudent decision on which loss reserving method would be most appropriate. Some popular parametric claim size probability distribution are the gamma distribution, Weibull, log-normal, log-gamma and Pareto distribution.

Assumptions

let

$$S = Y_1 + Y_2 + ... + Y_N = \sum_{i=1}^{N} Y_i.$$

With $S=$ Total claims size and Y_i $=$ Individual claim size

1. N is a discrete random variable which takes only values in A$\subset$ N_0
2. $Y_1, Y_2 ... \overset{iid}{\sim}$ G with G(0) $=$ 0
3. N and $(Y_1, Y_2 ...)$ are independent

These standard claim size model assumptions signify that;

- When $N = 0$, then there is no claim in the captive insurance company hence the total amount is zero

- The second assumption explains that the individual claim size are independent and the fact that one insured has a large claim does not mean or give any information that the rest of the insureds will also have large claims.In all the whole portfolio of claims will have the same marginal distribution function G with

$$0 = G(0) = P[Y_1 \leq 0]$$

meaning individual claim size are strictly positive.

- The final assumption explains that both individual claim size and the total claim S are not affected by each other and the the fact that we have many claims at a particular time is no indication whether is it large or smaller claims.

Model Selection

Aside the graphical argumentation of which continuous probability distribution would best fit the insurance claims, the goodness of fit and information criterion test is used to investigate which distribution stands out. This test is well known to investigates whether a given data set $Y_1...Y_n$ fits a particular distribution function G_0 hence we would like to compare the empirical distribution $\widehat{G}_n$ to the samples G_0

Goodness of fit and information criterion

There are different methods which can be used or applied to test choices for a probability distribution. The assumptions usually used are the asymptotic normality. This means the available data must be normally distributed. The goodness of fit test for an example splits the data or the support observation of the null hypothesis distribution function G_0 into k disjoint intervals $I_k = [C_k, C_{k+1})$, k = 1,...K. Further, the supporting data is grouped according to these intervals. The test statistic for the goodness of fit is given by

$$\chi^2_{n,k} = \sum_{k=1}^{k} \frac{(O_k - E_k)^2}{E_k} \tag{3.16}$$

Where: O_k = Count of observed data in the interval I_k.

E_k = Expected value of the observations.

Assuming d parameters were estimated in an underlined distribution then $\chi^2_{n,k}$ = is juxtaposed to a χ^2 distribution with k-1-d degrees of freedom.

3.6.2 Assumption

A better or realistic result will be obtained if $E_k > 4$

Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) is within the framework of maximum likelihood estimation which is used to compare different probability function or densities. These two method work under the assumption of parsimony where the probability with the fewer parameters are chosen or are preferred. Akaike (1974)

The AIC is given by $AIC^i = -2\imath^i_m + 2d^i$ and the $BIC^i = 2\imath^i_m + \log(n)d^i$ where

$\imath$ = log-likelihood function for data m

d^i = The number of parameters

CHAPTER 4

DATA ANALYSIS AND DISCUSSIONS

4.1 INTRODUCTION

This chapter first presents the analysis of the various data obtained from the Donewell Insurance company. Descriptive analysis of the data was done on the claim records to allow the researcher appreciate the nature of the claim records. The researcher juxtaposed a scatter plot of claim sizes and a scatter plot logged claim sizes to have an insight of the preliminary behavior of the claims. The next descriptive analysis was also to describe and distinguish between a histogram of claim sizes and histogram logged claim sizes. Other descriptive analysis such as the box plot, the empirical mean excess plot was also considered. Secondly, the Chain ladder method was used to model the insurance data to obtain reserves, followed by the Bornhuetter Ferguson and the Mack stochastic model respectively. In addition, the probability distribution which fitted the claims records was determined by the researcher using the goodness of fit test. All models as well as various algorithms were done in R.

4.2 Descriptive analysis of claim records

4.2.1 Scatter plots of claim records

The analysis of the claim count data begun by having a scatter plot of the claims (ordered by arrival date). The original scale (lhs) and the log scale (rhs). Although the scatter plot does not give much information about the claims, it helps in aiding the researcher to understand the behavior of the claims. Since both the original and logged scale of the claim count provided an overload of claim count in the scatter plot, This did not show any obvious trend. The reasons attributed to the log transformation of data is to deal with skewness since the transformation can reduce the variability of the data to make it conform more closely to the normal distribution while maintaining its trend or properties. We therefore calculate the sample mean and sample variance of the observation.

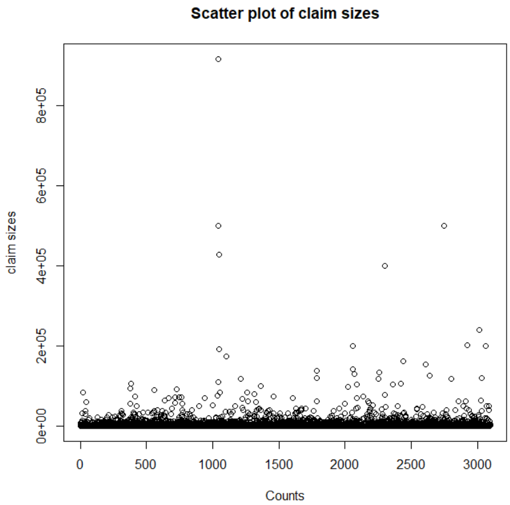

Figure 4.1: Scatter plot of claims

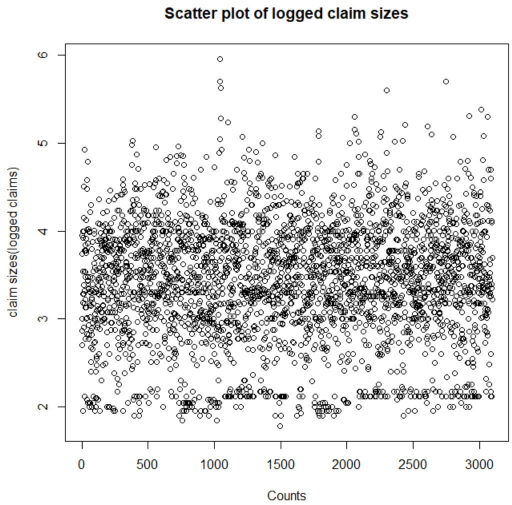

Figure 4.2: Logged scatter plot of claims

$\widehat{\mu}_n = \frac{1}{n}\sum_{1}^{n} Y_i$ and $\widehat{\sigma} = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \widehat{\mu})^2$

From our data set we obtained:

$\widehat{\mu}_n = 8564.718$, $\widehat{\sigma}_n = 28303.82$, $\widehat{Vco}_n = 3.3047$

Since we have had a little fair idea of the claim data from the mean, standard deviation and the coefficient of variation.

4.2.2 Deduction from figure 4.1 and 4.2

Scatter plot is a graph data in points that displays the pictorial scattering of data. The scatter plot usually aim at making certain features of a particular data visible. Features such as patterns,clusters, outliers and whether the scattered data is linear or non-linear.

Looking at figure 4.1, the scatter plot of the claim amount did not show any pattern but rather, it consisted of clusters and outliers. Few claim amount where found to lie outside the cluster of claims. Much information or properties from the scatter plot where not clearly visible hence the need to log transform the data which would still maintain the claim property and also reduce variability or skewness of the data.

Figure 4.2 which is the log transformation of the claim amount showed much overload of the claim amount without any obvious trend. In all, the log transformed data showed a clearer view of outliers

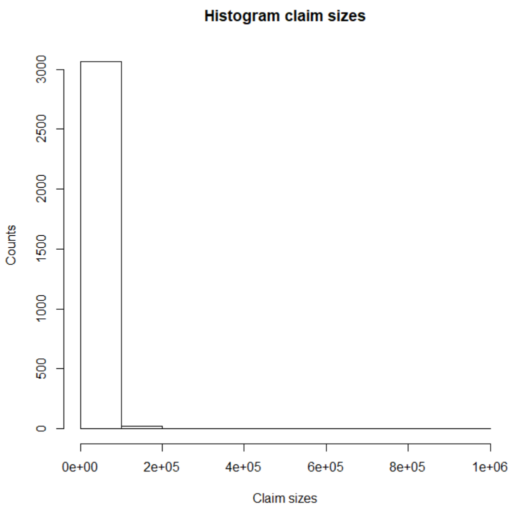

Figure 4.3: Claims sizes

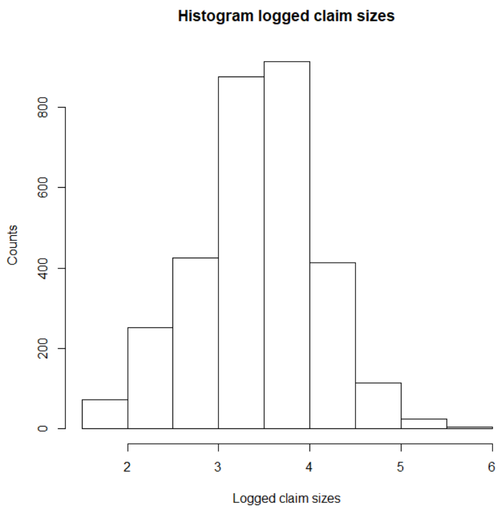

Figure 4.4: Logged claims sizes

Next, a histogram of the original and logged scale will give a clear picture of how the claims are distorted.

From the Figure 4.3 and 4.4, barely few large claim sizes misshape the entire picture and gives us reasons that the histogram isn't much useful. We could therefore use another method by plotting a second histogram while considering only the smaller claims. In figure 4.5 and 4.6 respectively, the corresponding box plot showed positive skewness. The definitive goal is to have a full distribution function $G(y) = P[Y \leq y]$ of the claim data.

4.2.3 Deduction from figure 4.3 and 4.4

Histogram is arguably the most preliminary visualization for quantitative data. It shows the shape of the distribution of values. The shape of the histogram is very vital because it indicates the direction of the skewness of the data. From the figure, it was noticed that only a few large claim distorts the histogram making it right skewed. The skewness of the data is imperative to determining the kind of probability distribution.

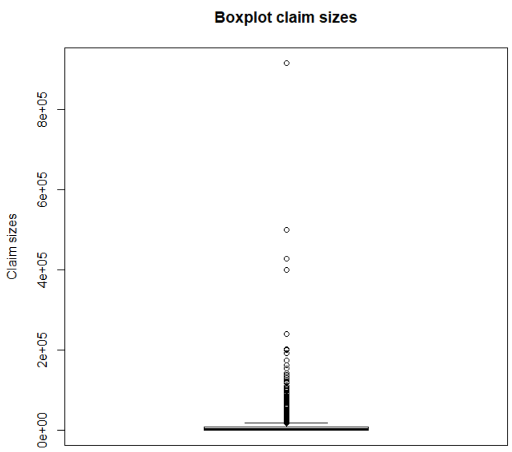

Figure 4.5: Claims sizes

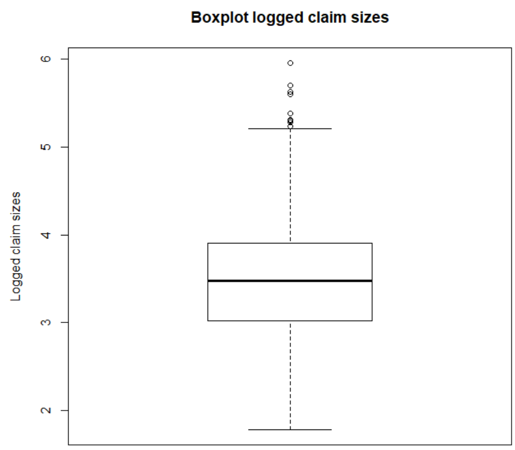

Figure 4.6: Logged claims sizes

4.2.4 Deduction from figure 4.5 and 4.6

Box plot is a very good way to summarize large amount of data because it displays the range and distribution of data along a number line. The five construct thus, lower and upper extreme, lower and upper quartile and the median of the box plot enables the researcher to obtain in-depth understanding of the data. By finding the middle value of the ordered dataset , the data is separated into four equal groups called quartiles. The distance between the points in the box plot tell us about the distribution of the data in the quartiles. A shorter distance between the extreme and the quartiles signifies a bunched data likewise a longer distance in the quartile would mean the data is spread out. looking at these figures, it can be seen that about 75% of the claim amount is more than GHC 3000. This clearly shows the claim amount was spread out with 6 large claims as extreme. The extreme is understood to be 3 times the standard deviation of the data.

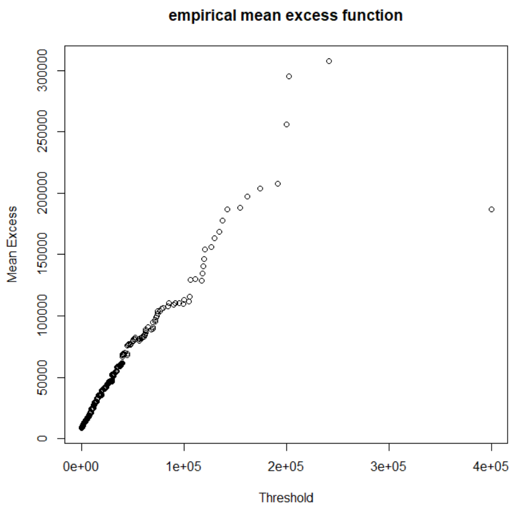

Figure 4.7: Claims sizes

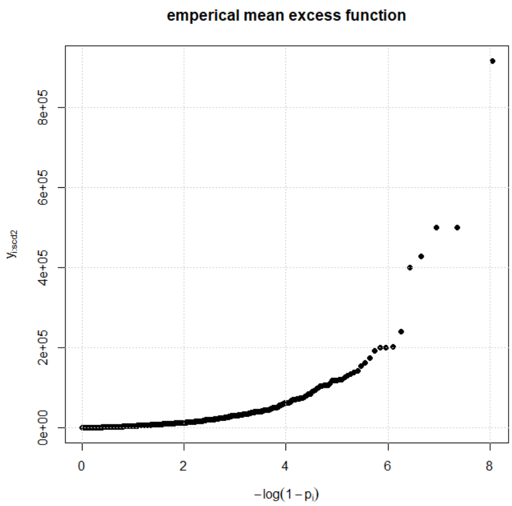

Figure 4.8: Logged claims sizes

Again, considering the empirical distribution function which is given by

$$\widehat{G}_{n(y)} = \frac{1}{n} \sum_{i=1}^{n} 1_{(Y_i \leq y)}$$

The original (lhs) and logged scale (rhs) of the empirical distribution function in figure 4.7 and 4.8 respectively. The sequence of the claim data $Y_1, Y_2 ... Y_n$ we can attribute by the structured sample $Y_1 \leq Y_2 \leq ... Y_n$. This sequence would require the assumption of a finite for the ordered sample. Having obtained that, the loss size index function and the empirical loss size function would be given by

$$I(G(y)) = \frac{\int_0^y ZdG(Z)}{\int_0^\infty ZdG(Z)} \qquad \text{and} \qquad \widehat{I}_n(\alpha) = \frac{\sum_{i=1}^{\lfloor n\alpha \rfloor} Y_{(i)}}{\sum_{i=1}^{n} Y_i}$$

The loss size index function thus, I(G(y)) picks a threshold of the claim sizes and estimate the mean claim among other claims that can be thoroughly explained by the claim sizes up to that particular threshold. Huge claim records can always lead to several modeling impediment. Two very renown and typical plot which explains large claims are the mean excess plot and the log-log plot.

By definition, figure 4.7 and 4.8 can be obtained by denoting the empirical mean excess plot as $u \mapsto e(u)$ and $u \mapsto \widehat{e}_n(u)$. Likewise, the empirical log-log plot is calculated as $y \mapsto (\log y, \log(1 - G(y))$ and $y \mapsto (\log y, \log(1 - \widehat{G}_n(y)))$

4.2.5 Deduction from figure 4.7 and 4.8

There is a linear increase in the mean excess plot and there is also a gradual but not sharp increase in the $\log\log$ plot. This clearly indicates that the data on the claim amount has an interpretation of a heavy tailed distribution coupled with the reason that the survival function $\bar{P} = 1 - P$ is varying regularly at infinity.

4.3 Fitting selected probability distribution to the claim amount

4.3.1 Gamma (α, β)

In most cases the gamma distribution is defined in terms of the mean rate of occurrence of event in a unit of time $\lambda = \frac{1}{\beta}$. The continuous random variable X is said to have a gamma distribution with parameters $\alpha > 0$ and $\beta > 0$ if its probability density function satisfies

$$f(x) = \frac{x^{\alpha-1} \exp^{\frac{-x}{\beta}}}{\Gamma(\alpha)\beta^{\alpha}} \qquad x \geqslant 0$$

where the gamma function $\Gamma(a+b)$ is defined as: $\Gamma(a+b) = \int_0^{\infty} (\frac{x}{\beta}) \exp^{\frac{-x}{\beta}} \frac{dx}{\beta}$

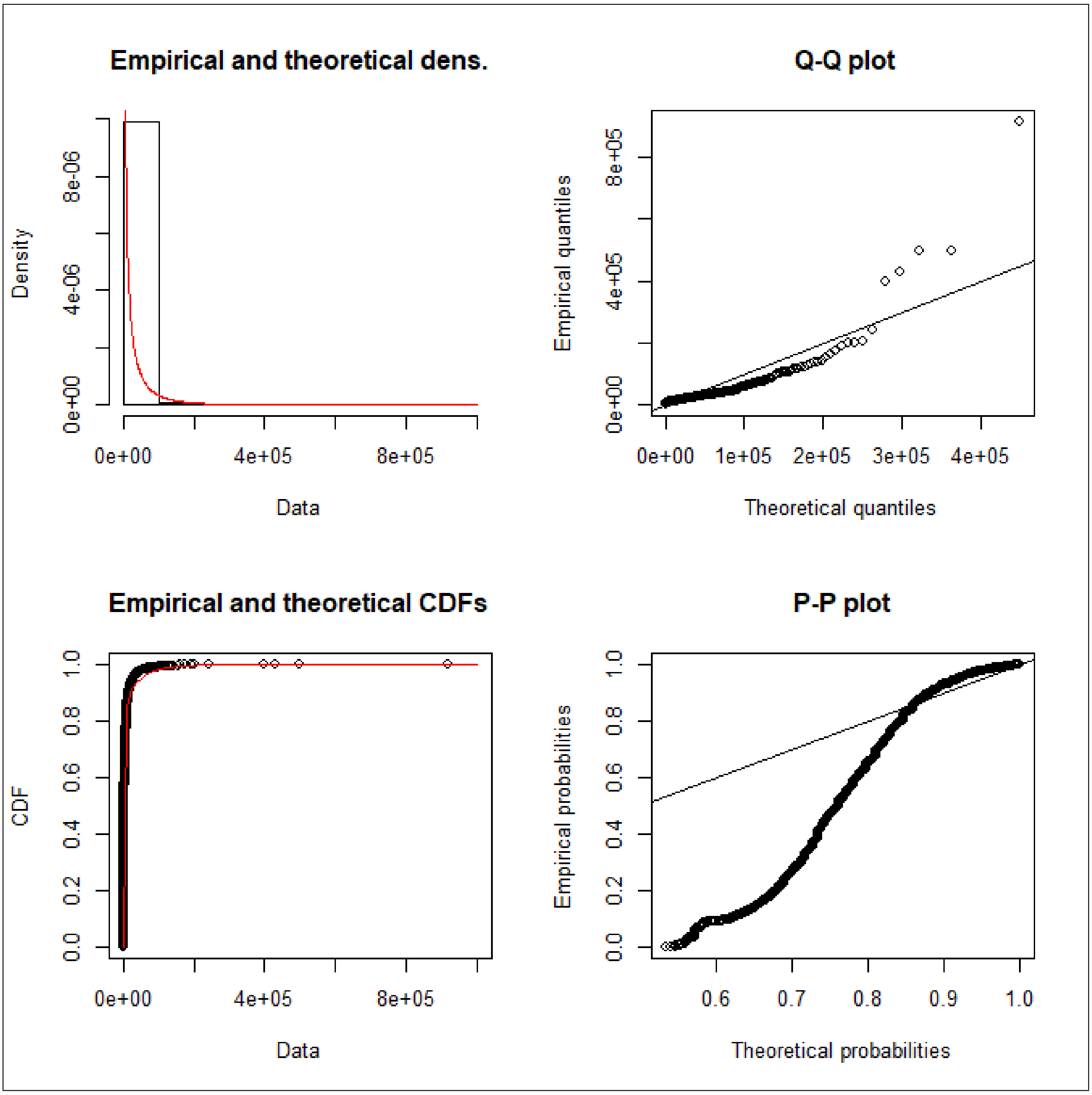

Figure 4.9: Diagnostic fits diagrams

Deductions from figure 4.9

Generally, it can be observed that the probability distribution that best fit the observed claim amount can be supported with argument from figure 4.9. These four diagram all explain the same concept. To decide whether a particular the particular distribution in question duly fit the claim amount, particular attention would be given to the quantile quantile plot (QQ) and the PP plot. For a symmetric distribution, the points on the qq plot should all lie on the 45° line but gauging from the qq plot on the figure, it can be observed that most of the data points did not lie on 45° line. This suggests that this particular distribution, thus, the "gamma" distribution is not a good fit for the observed data. Additionally, the PP plot did show a great variability since most of the data points were found the indication line.and this only confirmed the conclusion the qq plot made.

4.3.2 Log-normal (μ, σ^2)

The normal distribution is the most widely used probability model in statistics and holds central importance in it largely because of the central limit theorem. In summary, the process of making the tail of a probability distribution hefty than the tail of a weibull distribution primes to a log-normal distribution has two parameters μ and σ where $\mu \in \mathbb{R}$ and $\sigma > 0$. Hence the density of the log-normal is given by.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2}$$

where $-\infty < x < \infty, \ -\infty < \mu < \infty, \ \sigma > 0$

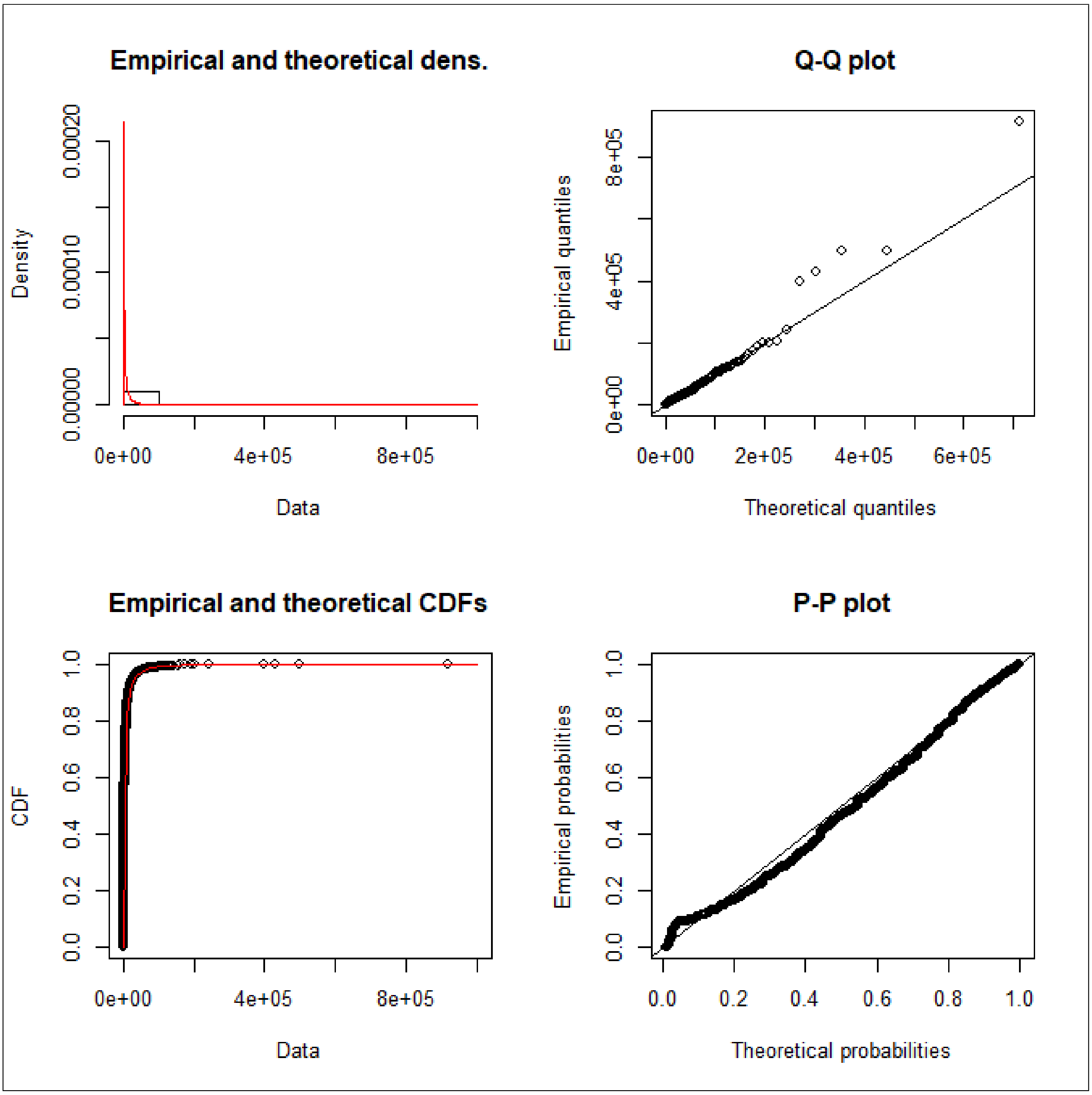

Figure 4.10: Diagnostic fits diagrams

Deduction from figure 4.10

The Log-normal distribution seems to be a good probability fit for the claim amount in the sense that almost all the claim amount with the exception of 5 claim amount falls on the Q-Q and P-P plot with the 45° as a baseline indicating the distribution as an almost near distribution to the claims.

4.3.3 Exponential (λ)

The exponential distribution is used to model the time required to observe the first occurrence of an object of a specified type when events of this type are occurring are occurring randomly with a mean λ. The density function is given by

$$f(x) = \lambda \exp^{-\lambda x}, x > 0$$

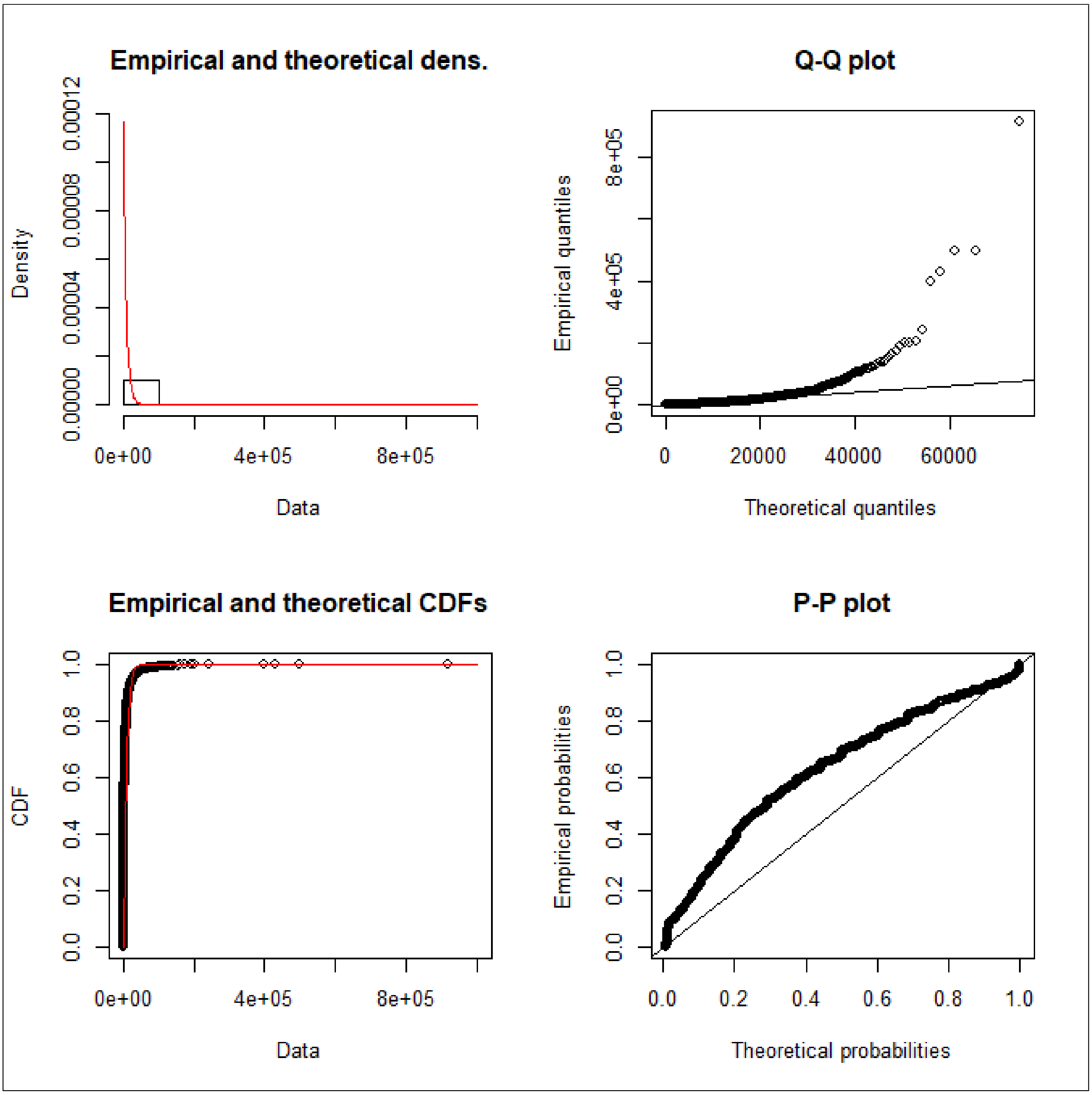

Figure 4.11: Diagnostic fits diagrams

Deduction from figure 4.11

Clearly, it can be seen from the the diagram that the claim amount did not fall in the 45° line. Almost all the claims deviate from the line indicating that the exponential distribution is not a good probability distribution for the claim amount.

Estimating comparative analysis of statistical test

Statistical Test	AIC	BIC
Gamma Distribution	65978.14	65990.22
Log-normal distribution	60414.56	60426.64
Exponential distribution	62265.08	62271.11

Table 4.1: Parameter comparison

Deduction from table 4.1

Akaike information criterion (AIC) and Bayesian information criterion (BIC) are two popular criteria in statistics in terms model selection. These two criterion strikes a balance between models that are complex and models that are good at explaining a required data. Starting with a simple model and adding parameters to it, the model will get better at explaining the data but also becomes more complex. Often after reaching the tipping point where the benefit of increasing expansionary power by adding a new parameter are offset by the increase in model complexity. Again,when it comes to these criterion, the smaller the number of parameters, the better (Parsimony). It is important to note that AIC and BIC cannot tell how well a particular model explains a data. It can only tell if the model strikes a better balance between model complexity and explanatory power than other models. Log-normal distribution had the lowest AIC hence it was deemed to be the distribution for the claim amount. This again recognizes the statistical distribution for large volume of claims and the random outcome consequences of Donewell insurance company. The actuary will understand the amount of money to pay in case there is a catastrophe. Again, the log-normal distribution would be informative to guide decisions such as premium loading, expected profit and also the impact deductibles and reinsurance play in reserving.

4.4 Chain Ladder modeling

4.4.1 Model building

4.4.2 Incremental loss payment by development years

Let the development period of the chain ladder model range from $1, 2, ...11$

Let the origin represent the payment of losses from 2008 to 2018. The main objective of this chain ladder reserving exercise is to forecast development claims beyond age 11 at the bottom right corner of the triangle below. The chain-ladder develops the loss reserve in a three step process. First, single age-to-age column factors are chosen to model the loss development indicated by existing experience data. The selected patterns of loss development are then projected to create the lower half of the loss-development triangle, so the model can be used to estimate the expected ultimate payments less payments-to-date that represents the reserve requirement

origin	dev 1	2	3	4	5	6	7	8	9	10	11
2008	21016.6	242362.7	327145.9	392124.5	469060.7	573796.2	706117.7	842153.4	980314.9	1083291	1217170
2009	260937.3	399436.4	529477.7	943149.1	1301312	1445461	1602888	1763146	3130876	3439854	NA
2010	217682.3	468232.3	759557.1	1040173	1243560	1574103	1943199	2111670	2315224	NA	NA
2011	156986.2	339339.4	548288.3	666743.9	852052.8	3288128	3486031	3795327	NA	NA	NA
2012	324158.6	515585.6	923503.3	1283198	1643247	2081274	2291556	NA	NA	NA	NA
2013	360524.4	678377.8	869321.2	1161223	1595018	5428159	NA	NA	NA	NA	NA
2014	349314.5	923898	1133603	1462172	1823949	NA	NA	NA	NA	NA	NA
2015	448567.7	1539019	2084648	2748265	NA	NA	NA	NA	NA	NA	NA
2016	1229789	2319610	2751325	NA	NA	NA	NA	NA	NA	NA	NA
2017	179659	1900596	NA	NA	NA	NA	NA	NA	NA	NA	NA
2018	2296876	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

Table 4.2: Chain ladder representation of claim loss

4.4.3 Cumulative loss payment through development years

origin	dev 1	2	3	4	5	6	7	8	9	10	11
2008	21016.6	242362.7	327145.9	392124.5	469060.7	573796.2	706117.7	842153.4	980314.9	1083291	1217170
2009	260937.3	399436.4	529477.7	943149.1	1301312	1445461	1602888	1763146	3130876	3439854	NA
2010	217682.3	468232.3	759557.1	1040173	1243560	1574103	1943199	2111670	2315224	NA	NA
2011	156986.2	339339.4	548288.3	666743.9	852052.8	3288128	3486031	3795327	NA	NA	NA
2012	324158.6	515585.6	923503.3	1283198	1643247	2081274	2291556	NA	NA	NA	NA
2013	360524.4	678377.8	869321.2	1161223	1595018	5428159	NA	NA	NA	NA	NA
2014	349314.5	923898	1133603	1462172	1823949	NA	NA	NA	NA	NA	NA
2015	448567.7	1539019	2084648	2748265	NA	NA	NA	NA	NA	NA	NA
2016	1229789	2319610	2751325	NA	NA	NA	NA	NA	NA	NA	NA
2017	179659	1900596	NA	NA	NA	NA	NA	NA	NA	NA	NA
2018	2296876	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

Table 4.3: Cumulative payment

These cumulative payment can now be analyzed for development patterns. From the cumulative payments, we calculate the age-to-age loss-development factors (sometimes called link ratios) where each entry is the ratio of successive development year cumulative payment.

4.4.4 Development patterns in chain ladder

Kuzmin (2013) is of the view that a thorough and deep understanding of the patterns in the whole insurance business process starting from underwriting to payment of claims equips the actuary a better statistical basis to predict sure reserves.Figure 4.12 and table 4.4 illustrates the cumulative claim development and incremental claim development by the origin year. Figure 4.12 showed that the incremental payment were not well behave as the graph indicated series of higher and lower payment which are haphazardly arranged. Obtaining a pattern from the graph would be extremely difficult but in table 4.4, it was observed that some part of 2013, 2017 and 2018 appeared to be extremely higher as compared to the rest of the development years. The claim payment in 2008 stood out a bit well as it was the lowest claim payment in the said development period.

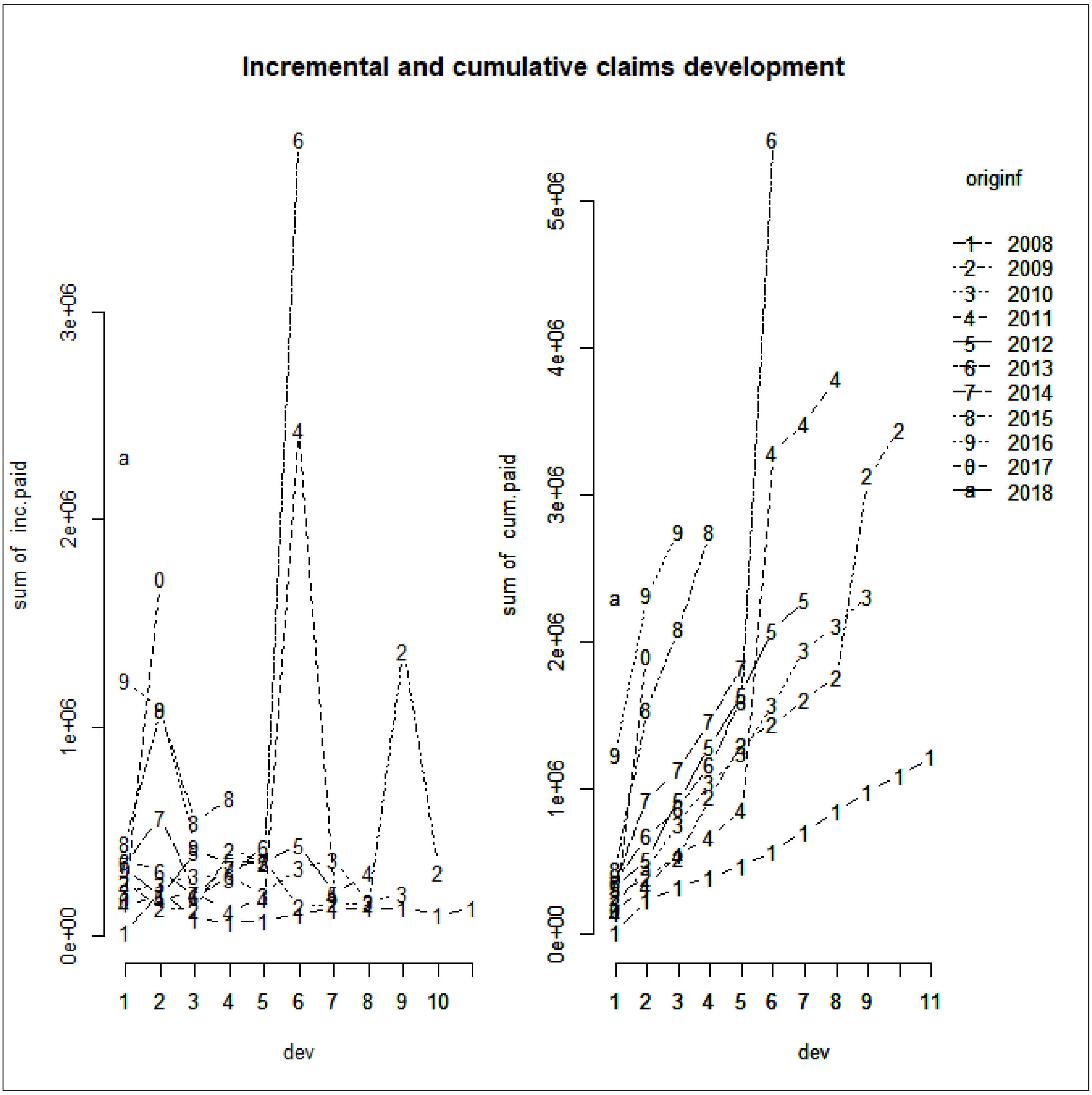

Figure 4.12: Incremental payment

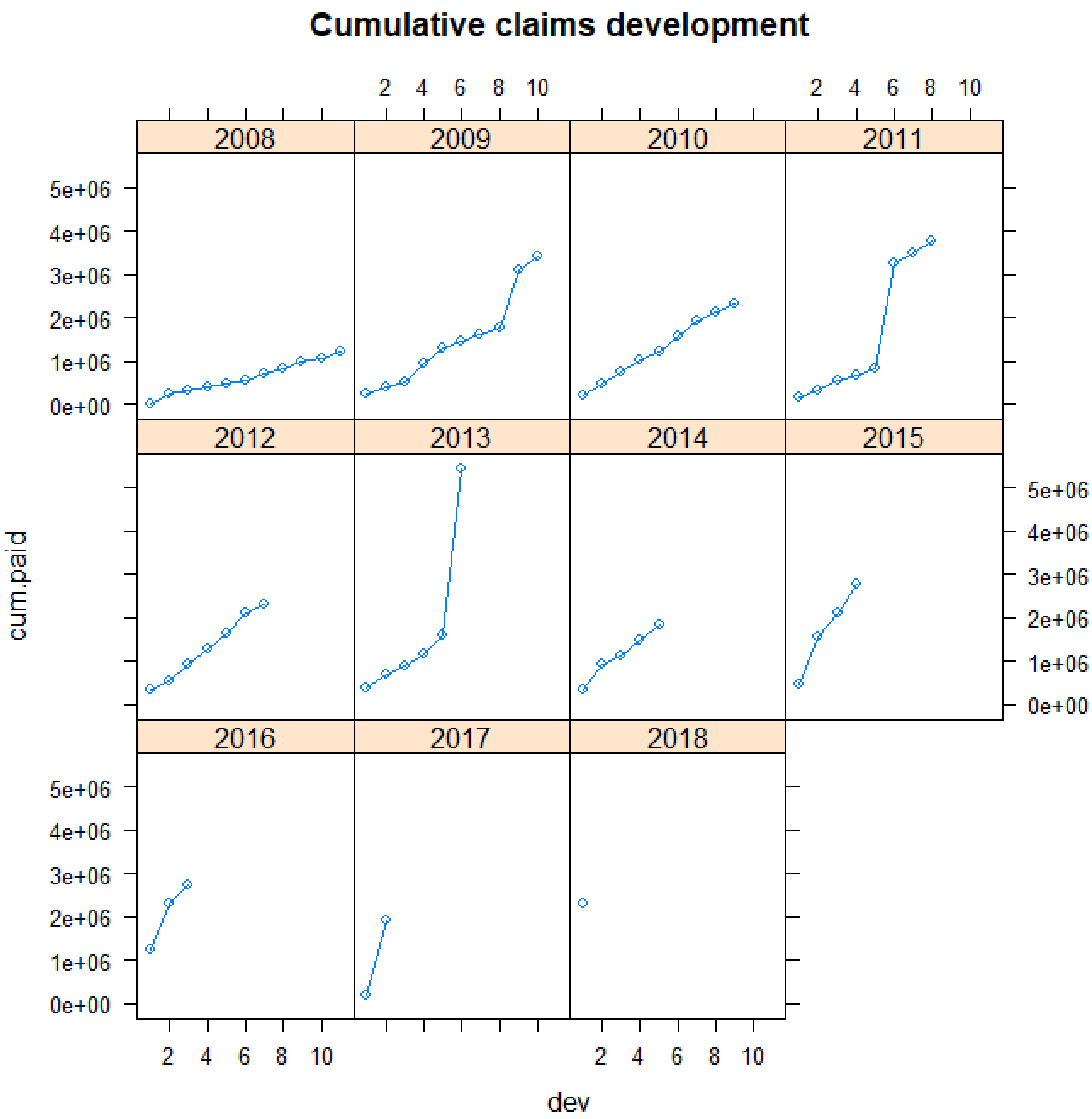

Figure 4.13: Cumulative claim development

4.4.5 Age-to-Age paid loss-development factors based on cumulative payment

origin	dev 2/1	3/2	4/3	5/4	6/5	7/6	8/7	9/8	10/9	11/10
2008	11.53197	1.34982	1.198623	1.196204	1.223288	1.230607	1.192653	1.164057	1.105044	1.123585
2009	1.530775	1.325562	1.781282	1.379752	1.110773	1.108911	1.099981	1.775733	1.098687	NA
2010	2.150989	1.62218	1.369447	1.195532	1.265804	1.23448	1.086698	1.096395	NA	NA
2011	2.161587	1.615752	1.216046	1.277931	3.859066	1.060187	1.088724	NA	NA	NA
2012	1.590535	1.791174	1.389489	1.280587	1.266562	1.101035	NA	NA	NA	NA
2013	1.881642	1.281471	1.335781	1.373567	3.403197	NA	NA	NA	NA	NA
2014	2.644889	1.226978	1.289845	1.247424	NA	NA	NA	NA	NA	NA
2015	3.430962	1.354531	1.318335	NA	NA	NA	NA	NA	NA	NA
2016	1.886185	1.186116	NA	NA	NA	NA	NA	NA	NA	NA
2017	10.57891	NA	NA	NA	NA	NA	NA	NA	NA	NA
Average	**2.628181**	**1.336797**	**1.351402**	**1.284858**	**2.025678**	**1.119051**	**1.100031**	**1.362403**	**1.100203**	**1.123585**

Table 4.4: Ratio of successive development years

Several observations about the above tables are required. The triangles presented are based on paid loss data but it is impossible to represent the incurred loss data in an identical format. Hence the need to create both paid-loss and incurred-loss development triangles. The paid loss data are purely objective,representing actual payment with no subjective reserve estimate. Generally, it is a good idea to do both a paid-loss and incurred-loss development analysis. In theory, the values of expected ultimate payment and expected ultimate incurrals should be the same,since once all claims are completely paid, and there is no reserve. More explicitly, in practice, the values derived from the two bases will differ, and the reconciliation of the difference assist to define the reserve estimate.

In terms of statistical modeling techniques, the chain ladder method is open to criticism. For example, assume we can obtain complete loss-development data for n accident years. This will produce a loss-development triangle with $n-1$ columns. Assume also that the method chosen to derive the lower half of the loss-development triangle uses one factor to model each column as illustrated above. These factors will then be applied to the last n payment points, one for each accident year (thus the diagonals in table 4.3), to estimate the expected ultimate payment per accident year. The chain ladder method provides a model with $2n - 1$ parameters.

4.4.6 Estimated paid losses and loss reserves by accident year,based on average paid loss-development factors

origin \ dev	1	2	3	4	5	6	7	8	9	10	11	Estimated ultimate losses	Paid-to-date	Estimate loss Reserve
2008											1217170	1217170	1217170	0
2009										3439854	3864968	3864968	3439854	425114
2010									2315224	2547216	2862014	2862014	2315223.5	546790.5
2011								3795327	5170766	5688894	6391954	6391954	3795326.7	2596627.3
2012							2291556	2520782	3434322	3778452	4245412	4245412	2291556.2	1953855.8
2013						5428159	6074389	6682014	9103598	10015809	11253610	11253610	5428159.2	5825450.8
2014					1823949	3694732	4134595	4548181	6196458	6817363	7659885	7659885	1823948.5	5835936.5
2015				2748265	3531129	7152931	8004497	8805192	11996224	13198285	14829391	14829391	2748264.9	12081126.1
2016			2751325	3718148	4777291	9677254	10829343	11912610	16229780	17856057	20062794	20062794	2751325.4	17311468.6
2017		1900596	2540711	3433524	4411589	8936459	10000356	11000699	14987389	16489175	18526985	18526985	1900596	16626389
2018	2296876	6036606	8069718	10905436	14011934	28383667	31762780	34940032	47602417	52372335	58844758	58844758	2296876	**56547882**

Table 4.5: Reserve estimate

It is the target of a reserving exercise to forecast the longer term claims development in bottom right corner of the Triangle and potential developments after age 11. With this regard, the estimated loss reserves for the years 2009-2018 increases every year and this could be due to reasons such as the long tail of payment. Table 4.6 predicted that an amount of GHC 56,547,882 should be set aside by Donewell insurance company at the end of 2018 so that they will be able to pay all promises of benefit made by the insurance company to their respective clients. The tail factors for the above reserves will also help with estimation and the correlation between the reserves in the years considered.

4.4.7 Tail Factors

The tail factors basically talks about the time range the insurance company uses to pay their clients. A long tail would signify that the insurance company takes longer time to pay their promises likewise short tail. Apart from figure 4.12 and table 4.4 no sense check have been conducted but the ratio analysis presented in figure 4.14 below suggests that the insurance company should expect another 2.5% claim development after 11 years.

This also means that the insurance company should considered increasing their reserves from GHC 56,547,882 to GHC 57,961,579.05.

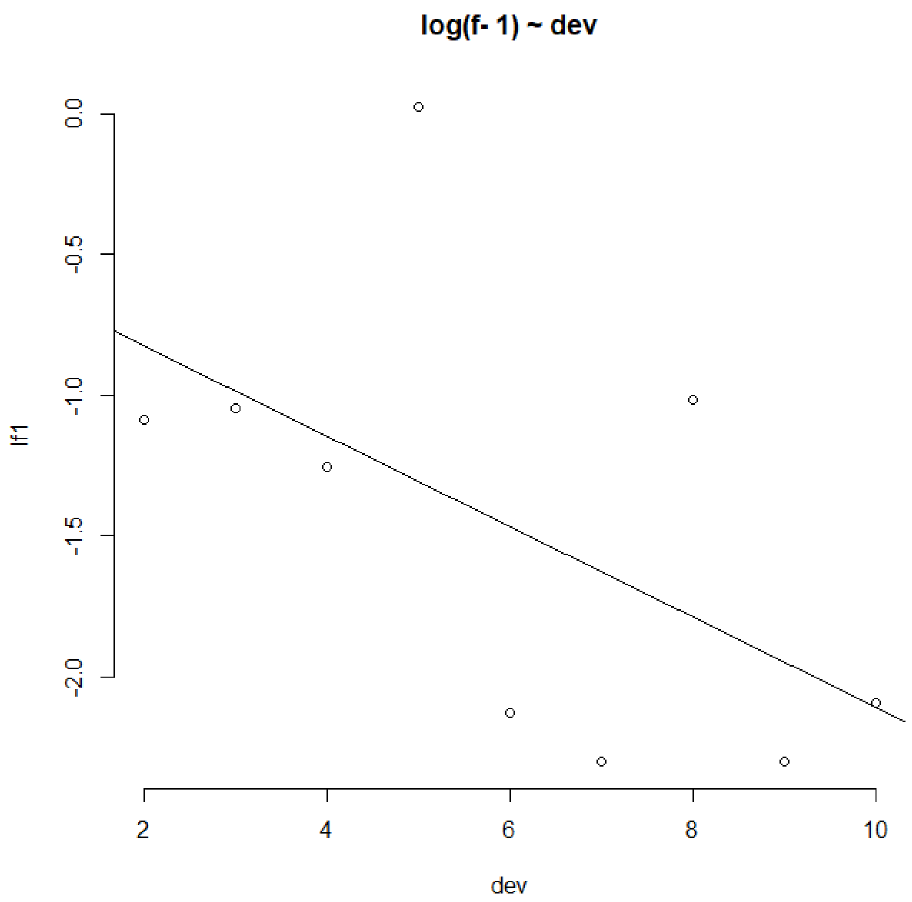

Figure 4.14: Reserve estimate

4.5 Bornhuetter Ferguson Reserves

This model formally combines the expected loss ratio and chain-ladder methods of loss reserving as just described. This method is meant to be a stabilizer for long tail lines or immature data. It only uses initial loss ratio expectation to the extent losses are not paid or reported. in addition, it assumes that past experience is not fully representative of the future. In this way, we arrive at the actuarial reserve using the Bornhuetter-Ferguson method as discussed in chapter three is GHC 2305106.

4.6 Mack Stochastic Loss Reserving

As the provision for outstanding claims is commonly the largest item on the liabilities facet in captive insurance record, it's necessary not solely to estimate the mean however additionally the uncertainty of the reserve.

Through out the previous years, there has been many statistical models and techniques developed to fix loss reserving in a stochastic phenomenon or framework. The basic idea is to regard the loss payment not as absolute values but rather random variables and, the loss payment values can allow various statistical techniques to explicitly test modeling assumptions that will not only estimate the mean reserves but the volatility of the reserves. Mack model provides a stochastic framework for the chain-ladder method and allows us to estimate the mean squared error of future payments.

4.6.1 Mack development by origin period

The standard error in the Mack model in figure 4.15 below is shown by the dotted lines across the link ratios. It can be deduced that in 2008, which is the origin year, to 2010 has basically minimal volatility. The dispersion started to become visible at the development period 2011 to 2015 also, the development period from 2016 to 2018 made reserving matter worse since the disparity from the normal deviation was very huge. This opined that the reserves estimates in the basic Chain-ladder and the Bornhuetter Ferguson although can be reliant upon but careful consideration and expert opinion must be considered because of the huge nature of the standard error. This finding is in assonance to (Alai et al., 2011) whose major findings concluded that a stochastic approach to the various loss reserving models is better since the researcher has the chance to estimate the MSEP of the reserves and this indeed provides guidelines as to which reserve estimate to be used. Thus, it simply suggests that if a captive insurance wants to get a good reserve estimate, several methods needs to be used including the assumption that the loss payment are random so that the MSEP can be estimated to aid on determining which reserve estimate to used.

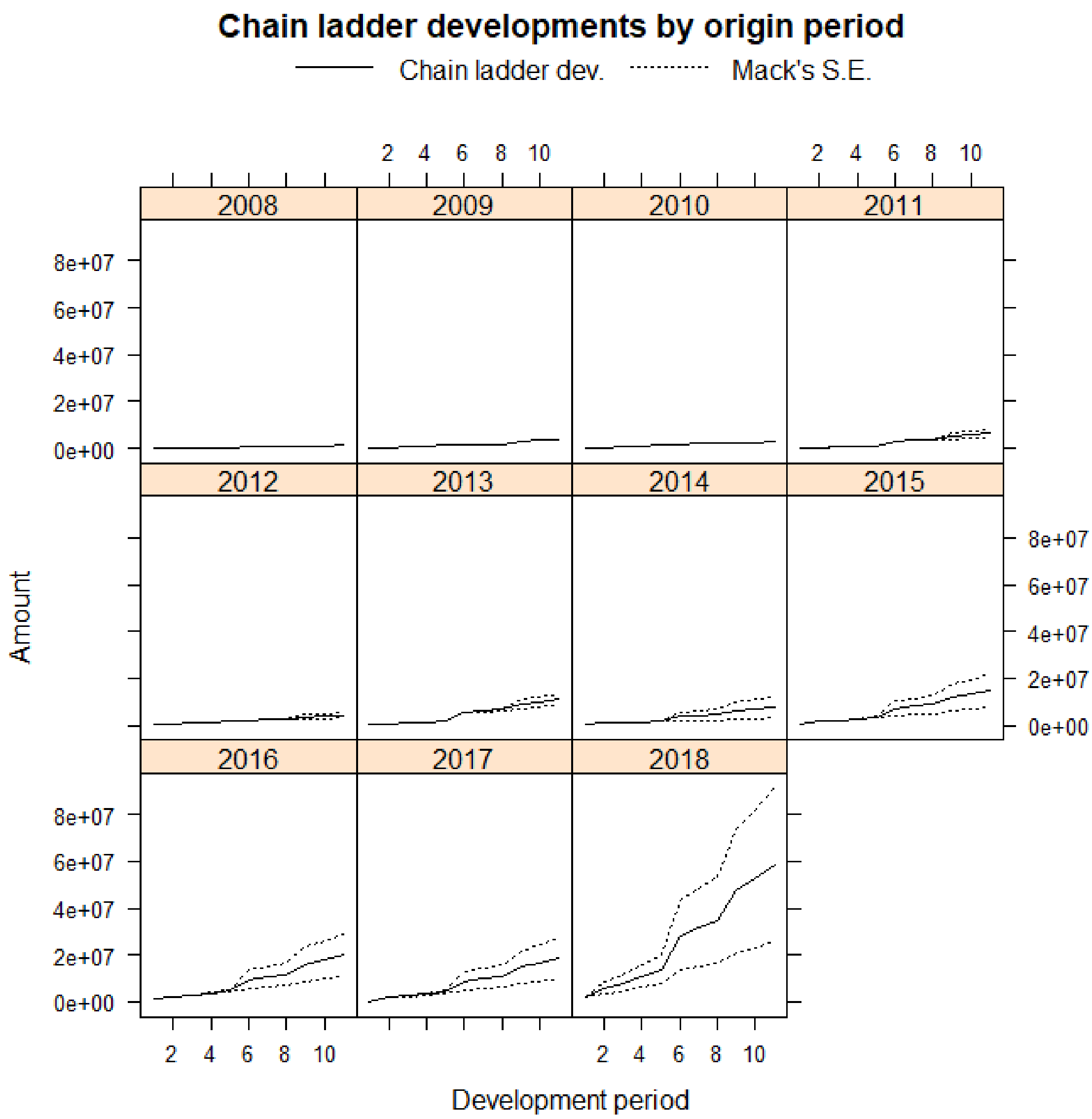

Figure 4.15: Development period

4.6.2 Mack Forecast

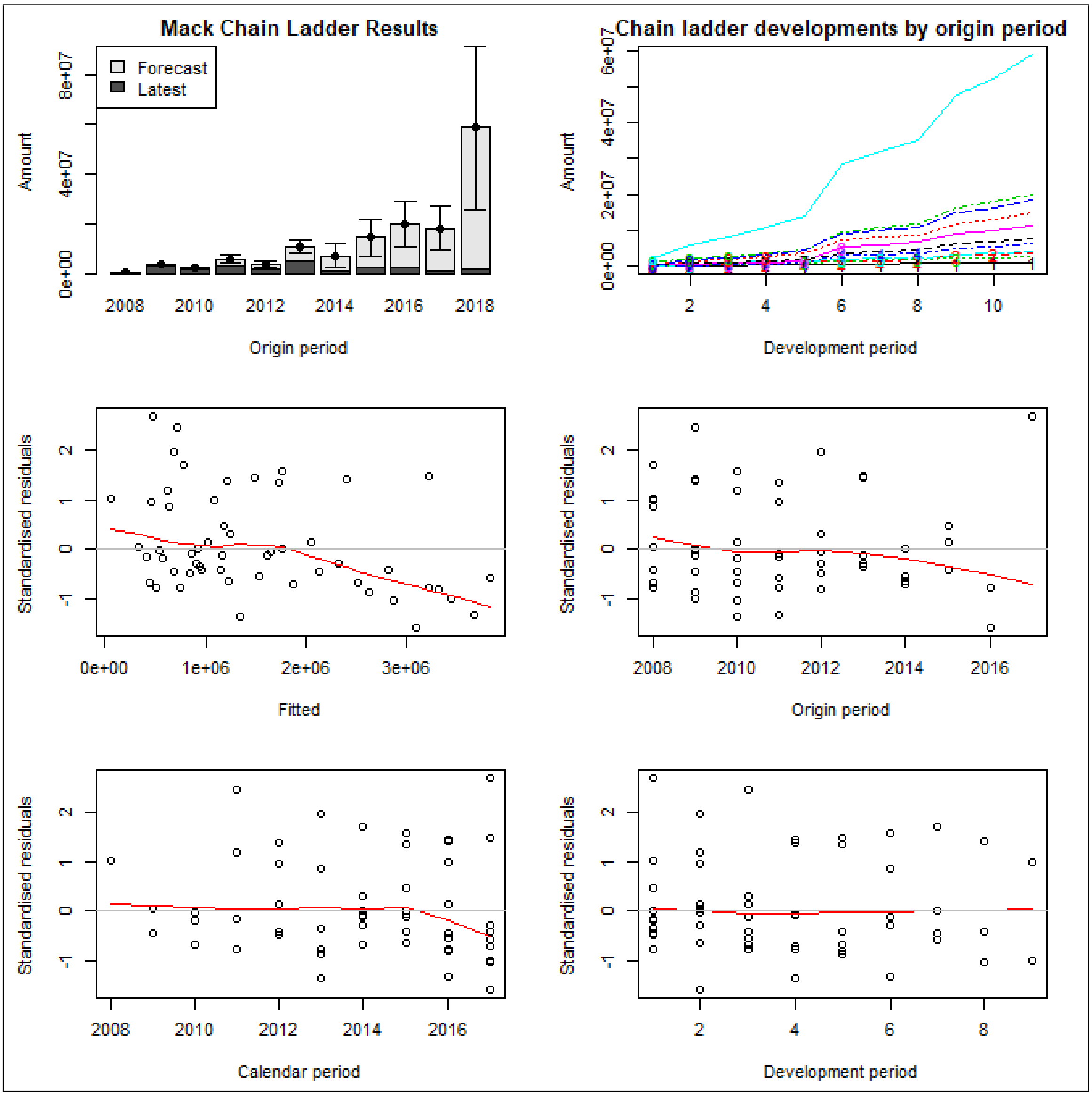

Figure 4.16: Mack forecast

Deductions from figure 4.16

Generally,it can be observed that the top most left figure indicates the actual positions and latest year forecast that are stacked on top and the standard error is depicted by the whiskers.The top right corner in this figure also represents the claim development for each origin year and clearly, it can be seen that as the years developed, the amount of claims of Donewell insurance proliferated. The next four plots in this figure also explains the residuals in terms of how the standardized residuals performs against the fitted values,origin, development period and the calender year. Obviously, the residual plot did not show any trend but according to (Mack, 1993), this model is strictly applicable and works best when about 95% of the standardized residuals are contained in the range of -2 to 2 and gauging from the four plots almost all the residual falls within that range. hence it can be inferred that the Mack model reserve estimate should be highly considered in this reserving disparity.

4.6.3 Mack Reserve Estimate

Deduction from table 4.6

The results in table 4.6 below gives immediate access to the statistic of the stochastic Mack model. This also includes the future payment forecast which is also called the incurred but not reported (IBNR) and the mean square error associated with the individual loss payment across the various years. The mean square error of the mack model was ±41%. This means that the predicted reserves or the future payment is encapsulated with ±41% errors

MackChainLadder(Triangle= cum.triangle, Weights =1 Alpha = 1
est.sigma = "Mack")

Latest	Dev.To.Date	Ultimate	IBNR	Mack.S.E	CV(IBNR)
2008: 1,217,170	1.000	1,217,170	0	0	NaN
2009: 3,439,854	0.890	3,864,968	425,114	232	0.000547
2010: 2,315,224	0.809	2,862,014	546,790	11,742	0.021474
2011: 3,795,327	0.594	6,391,954	2,596,627	1,591,251	0.612815
2012: 2,291,556	0.540	4,245,412	1,953,855	1,203,815	0.616123
2013: 5,428,159	0.482	11,253,610	5,825,451	2,518,260	0.432286
2014: 1,823,949	0.238	7,659,885	5,835,937	4,599,330	0.788105
2015: 2,748,265	0.185	14,829,391	12,081,127	7,133,703	0.590483
2016: 2,751,325	0.137	20,062,794	17,311,468	8,962,387	0.517714
2017: 1,900,596	0.103	18,526,985	16,626,389	8,679,071	0.522006
2018: 2,296,876	0.039	58,844,758	56,547,882	32,548,942	0.5756

Totals
Latest: 30,008,300.16
Dev: 0.20
Ultimate: 149,758,939.87
IBNR: 119,750,639.71
Mack.S.E 48,598,404.98
CV(IBNR): 0.41

Table 4.6: Mack reserves

4.7 Model Accuracy

The table below showed that the Mack stochastic model as compared to chain ladder has a better predictive power according to the available data.

ACTUAL	MAE	MAPE	RMSE
Chain-ladder	0.8277627	77.22046	0.9098146
Mack model	0.6328042	59.30379	0.7954899

Table 4.7: model accuracy

From the table above, values for MAE, MAPE and RMSE made us know that the Mack model had a better predictive power with lesser errors as compared to Chain-ladder model.

CHAPTER 5

DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

This chapter discusses the major findings, conclusion made and recommendation drawn from the study about stochastic loss reserving with individual claim size modeling for Donewell insurance company in Ghana.

5.2 Discussion

It was revealed that Mack chain ladder was more robust and had a better predictive power than chain ladder and Bornhuetter Ferguson within the data considered. This findings agrees with the outcome from other reserving surveys.

Taylor et al. (2008), opined from his study that even though other reserving techniques can be used to predict or forecast good reserves, models that have error prediction associated with it can produce trusted estimated reserves. In addition, (Saluz et al., 2011) agreed to the reasoning that a stochastic approach to loss reserving yields better reserves. This was explained when they conducted a survey in Switzerland with an insurance company to question the development pattern in the Bornhuetter Ferguson method and also propose a calculation for the MSEP. It was observed that the results was in similitude to that of Mack stochastic loss reserving technique.

This can be explained from the view point of MSEP calculation that, models with predictive abilities most often have error calculation associated with it so that practitioners can always come to terms with surety the kind of decision to be certain on Having knowledge of many reserving technique will be of good help because there is no single reserving technique which can be thought of as being super robust in terms of reserving. The actuary will have to investigate across various methods and also include his expertise or knowledge to determine which reserving method or technique would suit that particular data for the insurance company.

Getting a reserve technique right will not only solve the insurance company solvency problems but it can help the insurance company to make enormous profit regardless of the any risky investment they will undergo. This is because a good reserving technique can give suggestion on the amount of premium to take and also the basic underwriting process to add.

In the investigation of loss reserving, prior knowledge of previous payment or information is very key to calculate ultimate claim. Various market statistics can also help to estimate a good reserve. This reasoning is in consonance with (Heberle and Thomas, 2016) who studied fuzzy numbers to help develop new and fewer parameters for the ultimate claim prediction and they concluded that even with the vagueness of prior information, the long run methods usually provide the same reserve figure.The Mack's model of loss reserving also revealed that modeling of a mean square error and variability has been problematic to most actuaries and one way to counter that is to assume the claim payment to be random in order to use a stochastic methodology to calculate the error.

Finding from (Dahms, 2012) coincided with that of Mack stochastic loss reserving in that, many different triangle with different random exposures are metamorphosed to create a new triangle and this leads the actuary to use the new triangle with the various stochastic assumptions to calculate the MSEP and reserves. In addition, (Bühlmann and Moriconi, 2014) assessed the credibility of reserves using the Buhlman-Straub credibility reserves and this is to conclude that reserving in insurance should be a whole discipline on its own since the chain ladder and other developed methods is somewhat cumbersome and would require rigorous training to master the various techniques.

According to (Quantile, 2018), this high attention given to to a stochastic process has given birth to a number of different methods that use stochastic approach for loss reserving. Moreover,certain data available may require the use of Bornhutter-Ferguson because of its tail effect, hence, different methodology and approaches should always verified and cross-checked for verification. Model accuracy was used in conjunction with development factors to find model accuracy and also compare between the chain-ladder and the Mack model. The mean absolute percentage (MAPE), mean absolute deviation (MAD) and mean square error (MSE) was used to compare between the two predictive models. It was revealed that the Mack model per the data available was a better model to predict the chain reserves.

According to (Mack, 2008), if 90% of the loss data points falls between -2 to 2 in the graph of mack model, that data is a good fit to apply a stochastic method for a good estimate of reserve. It was seen that not only 90% of the data fell in that category but rather about 95% hence aside the model accuracy method, the loss data point aligned itself best to the Mack model.
Individual claim size modeling also play a major role in loss reserving process. Various probability distribution was fitted to the claim data and it was found out that the log-normal distribution was the best fit for the data. Having found the appropriate distribution for the claims, the actuary can then obtain additional knowledge of the behavior of the claims or the paid losses. According to (Bortoluzzo et al., 2011), if an insurance company determines the probability distribution of their claims, they can

estimate better reserves and also the insurance company can be able to foretell the amount of money in case there is a catastrophic event. Catastrophic situation has left many insurance company insolvent and even big companies hardly survive hence the need to know and prepare for such situations.

5.3 Conclusion

The main conclusion drawn from the findings of this study were that; The Mack model revealed that loss reserving was very delicate hence serious attention must be given to that domain to make the insurance company stable in terms of solvency. Overtime, the two basic methods thus; chain ladder and Bornhuetter-Ferguson method will not always be a reference point since many reserving techniques are emerging. Example is the likes of tweedie Poisson reserving process.

With the study from 2008 to 2018 from (Donewell insurance), the chain-ladder proposed a reserve estimate of GHC 56,547,882, while Bornhuetter's Method suggested an amount of GHC 2,305,106. Finally, the Mack stochastic method suggested that the latest payment should be GHC 30,008,300.16 with a development of 20% across the development years. The model again did propose that the ultimate and the incurred but not reported claims (IBNR) should be GHC 149,758,939.87 and GHC 119,756,639.71 respectively. The main aim of this research aside finding or estimating the loss reserving amount was also to find out the standard error of the reserves and per the Mack model, it was estimated that the loss claim payment reserves should be GHC 48,598,404.98.

Among other conclusions, about 95% of the loss claim payment data was found to lie between -2 to 2 on the graph of Mack model indicating that the model was suited for this data aside other methods. In addition, the concluded probability distribution that suited the loss claim payment was the log-normal distribution. This in another sense means that, the insurance company payment of their claims follows a log-normal distribution. Having know this, the standard deviation, kurtosis and other various statistic concerning such a distribution can be easily obtained for inclusion into reasoning and decision making.

5.4 Recommendations

From the findings of the study and discussions, the following recommendations were made.

- Mack model can be used by actuaries and other modeling specialists in estimating the standard error of reserves and developmental percentages However future studies can consider bootstrapping in conjunction to compare between models and forecast of future reserve errors.

- National insurance commission and other insurance agency must give adequate attention and interest to loss reserving since it is the backbone for the survival of insurance firms. Again, every insurance company must establish or create a reserving department so that insurers and other workers can be trained and equipped with the necessary reserving skills.

- Public education on insurance must be organized for both the insured and the insurer. This should be centered on educating people on the need to report for claim payment after a fortuitous event at the early stages rather than waiting till the fall outside the term of insurance. Doing this, insurance firms will be able to eradicate the IBNR and easily determine the amount to be reserved and invested.

- A contingency fund must be created by insurance firms to suffice for payment in case there is a catastrophic event. Such monies must be invested in accordance to the standard of the social security and national insurance trust (SSNIT) national investment policies act,2008.

- This research can be expanded for future studies by modeling claim payment from different insurance companies across the country and also Bootstrap chain ladder should be considered to estimate parameters and reserves.

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APPENDIX

R CODES

```
loadingpackageforthethesis
library(evir)
library(latticeExtra)
library(actuar)
library(ChainLadder)
library(TSA)
library(fitdistrplus)
sc < −read.csv(”mep.csv”, header = T)
head(sc)
tail(sc)
as.vector(sc)
scd < −as.vector(sc)
??meplot
mean(scd[, 1])
hist(scd[, 1])
hist(log10(scd[, 1]))
plot(scd[, 1])
plot(log10(scd[, 1]))
boxplot(scd[, 1])
boxplot(log10(scd[, 1]))
??ecdf
ecdfplot(scd[, 1])
ecdfplot(log10(scd[, 1]))
meplot(scd[, 1])
```

$scd1 < -sort(scd[,1])$

$scd2 < -length(scd1)$

$i < -c(1 : scd2)$

$fun < -function(x) - log(1 - x)$

$Q < -fun(i/(scd2 + 1))$

$plot(Q, scd1, pch = 19, xlab =$

$expression(-log(1 - p[i])), ylab = expression(y[i : scd2]))$

$grid()$

$?plot$

$plot(x = scd[,1], xlab = "Counts", ylab = "claimsizes",$

$main = "Scatterplotofclaimsizes")$

$plot(x = log10(scd[,1]), xlab =$

$"Counts", ylab = "claimsizes(loggedclaims)",$

$main = "Scatterplotofloggedclaimsizes")$

$hist(x = scd[,1], xlab = "Claimsizes", ylab = "Counts",$

$main = "Histogramclaimsizes")$

$hist(x = log10(scd[,1]), xlab = "Loggedclaimsizes", ylab = "Counts",$

$main = "Histogramloggedclaimsizes")$

$boxplot(x = scd[,1], ylab = "Claimsizes",$

$main = "Boxplotclaimsizes")$

$boxplot(x = log10(scd[,1]), ylab = "Loggedclaimsizes",$

$main = "Boxplotloggedclaimsizes")$

$ecdfplot(x = scd[,1], xlab = "numberofclaims", ylab = "empiricallosssizeindex$

$function",$

$main = "empiricallosssizeindexfunction")$

$ecdfplot(x = log10(scd[,1]), xlab =$

$"Loggedclaims", ylab = "empiricaldistribution",$

$main = "empiricaldistribution")$

$grid()$

$meplot(scd[,1], main = "empiricalmeanexcessfunction")$

$scd1 < -sort(scd[,1])$

$scd2 < -length(scd1)$

$i < -c(1 : scd2)$

$fun < -function(x) - log(1 - x)$

$Q < -fun(i/(scd2 + 1))$

$plot(Q, scd1, pch = 19, main =$

$"empericalmeanexcessfunction", xlab = expression(-log(1 - p[i])), ylab =$

$expression(y[i : scd2]))$

$grid()$

$?fitdistr$

$fw < -fitdist(sort(scd[,1]), distr = "lnorm")$

$summary(fw)$

$mean(scd[,1])$

$sd(scd[,1])$

$Importingdata$

$ClaimsData = read.csv(file.choose(), header = T)$

$attach(ClaimsData)$

$?fitdist$

$NB : Theestimationcoulddoneusingothermethodstoo$

$?fitdist$

$FittingGammadistribtiontoclaimdata$

$Claims_Gamma = fitdist(scd[,1], distr =$

$"gamma", method = c("mme"), discrete = F)$

$summary(Claims_Gamma)$

$plot(Claims_Gamma)$

$FittingLogNormaldistribtiontoclaimdata$

$Claims_LogNormal = fitdist(scd[, 1], distr = "lnorm", method = c("mme"), discrete = F)$

$summary(Claims_LogNormal)$

$plot(Claims_LogNormal)$

$FittingExponentialdistribtiontoclaimdata$

$Claims_Exponential = fitdist(scd[, 1], distr = "exp", method = c("mme"), discrete = F)$

$summary(Claims_Exponential)$

$plot(Claims_Exponential)$

$Fittinglogisticdistribtiontoclaimdata$

$Claims_logistic = fitdist(scd[, 1], distr = "logis", method = c("mme"), discrete = F)$

$summary(Claims_logistic)$

$plot(Claims_logistic)$

$FittingNormaldistribtiontoclaimdata$

$Claims_normal = fitdist(scd[, 1], distr = "normal", method = c("mme"), discrete = F)$

$summary(Claims_normal)$

$plot(Claims_normal)$

$Fittingweibulldistribtiontoclaimdata$

$Claims_weibull = fitdist(scd[, 1], distr = "weibull", method = c("mme"), discrete = F)$

$summary(Claims_weibull)$

$plot(Claims_weibull)$

$FittingBurrdistribtiontoclaimdata$

$Claims_pareto = fitdist(scd[, 1], distr = "pareto", method = c("mme"), discrete = F)$

$summary(Claims_pareto)$

$plot(Claims_pareto)$

$Chainladder$

$n < -11$

$Claims < -data.frame(originf = factor(rep(2008 : 2018, n : 1))$

$dev = sequence(n : 1),$

$inc.paid =$

$c(21016.6, 221346.1, 84783.18, 64978.59, 76936.19, 104735.5, 132321.5,$

$+ 136035.7, 138161.5, 102976.5, 133878.3, 260937.3, 138499.1, 130041.3,$

$+$

$413671.4, 358162.5, 144149.7, 157426.4, 160258.1, 1367730, 308978.3,$

$+$

$217682.3, 250550, 291324.8, 280616, 203387, 330543.2, 369095.3, 168471,$

$+$

$203553.9, 156986.2, 182353.2, 208948.9, 118455.6, 185308.9, 2436075,$

$+$

$197902.9, 309296, 324158.6, 191427, 407917.7, 359694.3, 360048.9, 438027.3,$

$+ 210282.4, 360524.4, 317853.4, 190943.4, 291901.6, 433794.8, 3833141.6,$

$+ 349314.5, 574583.5, 209704.6, 328569.5, 361776.4, 448567.7, 1090451,$

$+ 545629.3, 663616.9, 1229789, 1089821, 431715.4, 179659, 1720937, 2296876))$

$(inc.triangle < -with(Claims,$

$M < -matrix(nrow = n, ncol = n$

$, dimnames = list(origin = levels(originf), dev = 1 : n))$

$M[cbind(originf, dev)] < -inc.paid$

$M))$

$(cum.triangle < -t(apply(inc.triangle, 1, cumsum)))$

$(latest.paid < -cum.triangle[row(cum.triangle) == n - col(cum.triangle) + 1])$

$Claimscum.paid < -cum.triangle[with(Claims, cbind(originf, dev))]$

$op < -par(fig = c(0, 0.5, 0, 1), cex = 0.8, oma = c(0, 0, 0, 0))$

```
with(Claims,
interaction.plot(x.factor = dev,
trace.factor = originf, response = inc.paid,
fun = sum, type = "b", bty = "n", legend = FALSE); axis(1, at = 1 : n)
par(fig = c(0.45, 1, 0, 1), new = TRUE, cex = 0.8, oma = c(0, 0, 0, 0))
interaction.plot(x.factor = dev, trace.factor = originf, response = cum.paid,
fun = sum, type = "b", bty = "n"); axis(1, at = 1 : n)
)
mtext("Incrementalandcumulativeclaimsdevelopment",
side = 3, outer = TRUE, line = −3, cex = 1.1, font = 2)
par(op)
library(lattice)
xyplot(cum.paid dev|originf, data = Claims, t = "b", layout = c(4, 3),
as.table = TRUE, main = "Cumulativeclaimsdevelopment")
f < −sapply((n − 1) : 1, function(i)
sum(cum.triangle[1 : i, n − i + 1])/sum(cum.triangle[1 : i, n − i])
)
tail < −1
(f < −c(f, tail))
full.triangle < −cum.triangle
for(kin1 : (n−1))full.triangle[(n − k + 1) : n, k + 1] < −full.triangle[(n − k + 1) : n, k] ∗ f[k]
full.triangle
(ultimate.paid < −full.triangle[, n])
(ldf < −rev(cumprod(rev(f))))
(dev.pattern < −1/ldf)
(reserve < −sum(latest.paid ∗ (ldf − 1)))
sum(ultimate.paid − latest.paid)
a < −ultimate.paid
```

$(b < -c(dev.pattern[1], diff(dev.pattern)))$

$(X.hat < -a\% * \%t(b))$

$BornhuetterFergusson$

$(BF2018 < -ultimate.paid[n] * dev.pattern[1] +$

$8564.718 * (1 - dev.pattern[1]))$

$TailFactors$

$dat < -data.frame(lf1 = log(f[-c(1, n)] - 1), dev = 2 : (n - 1))$

$(m < -lm(lf1\ dev, data = dat))$

$plot(lf1\ dev, main = "log(f - 1)\ dev", data = dat, bty = "n")$

$abline(m)$

$sigma < -summary(m)sigma$

$extrapolation < -predict(m, data.frame(dev = n : 100))$

$(tail < -prod(exp(extrapolation + 0.5 * sigma^2) + 1))$

$MackStochasticlossreserving$

$(mack < -MackChainLadder(cum.triangle, weights = 1, alpha = 1,$

$est.sigma = "Mack"))$

$MackChainLadder(Triangle = cum.triangle, weights = 1, alpha = 1,$

$est.sigma = "Mack")$

$plot(mack, lattice = TRUE, layout = c(4, 3))$

$plot(mack)$

$accuracy("forecast", mack)$

$accuracy(scd[, 1], mack, threshold = 20)$