UNIVERSITY OF GHANA

EFFECTS OF DEPENDENT CLAIMS ON THE PROBABILITY OF RUIN, THE TIME TO RUIN GIVEN RUIN OCCURS

BY

SEFA MICHAEL ADDO

(10308147)

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DECLARATION

Candidate's Declaration

I, Michael Addo Sefa hereby declare that apart from references to other people's publications, which have been duly acknowledged, this thesis is a result of my independent ideas, thought, deliberations and has not been submitted for the award of any degree at this institution and other universities elsewhere.

SIGNATURE: …………………………………… DATE: ………………

MICHAEL ADDO SEFA

(10308147)

Supervisors' Declaration

We hereby certify that this thesis was prepared from the candidate's own work and supervised in accordance with guidelines on supervision of thesis laid down by the University of Ghana.

SIGNATURE: …………………………………… DATE: ………………

PROF. KWABENA DOKU-AMPONSAH

(PRINCIPAL SUPERVISOR)

SIGNATURE: …………………………………… DATE: ………………

DR. E. N. N. NORTEY

(CO-SUPERVISOR)

ABSTRACT

Ruin basically occurs to an insurance company when the claims paid out supersedes its initial capital and total premiums accumulated. In the classical theory of risk, the surplus is a significant model that deals with how long an insurance company's capital or surplus evolves. The first time ruin occurs is very crucial and the business must try to prevent it from happening again because it makes the business inefficient and inoperable. The time to ruin is so much a function of the initial capital and the way in which the insurance company's business books are priced. An insurance company has no control over how claims are issued, can attempt and handle its excess to assess the number of claims that will arise over time. The hypothesis that individual claims occur separately is one of the assumptions of the excess method. But this premise of independence is no longer true and sensible, as individual risks are generally homogeneous and share comparable features that claims from one eventuality could cause claims from another. In this thesis, the impacts of dependent claims on the likelihood of ruin and the time-to-ruin was examined also proposed premium adjustment when there is claim dependence. the objectives of this study is to determine how dependent claims, affect the likelihood of ruin and time to ruin of insurance companies in Ghana. And also to determine the time left for an insurance company to go to ruin when the assumption of dependency claims hold. The study employed Pollaczek-Khinchine formula, which was used to estimate probability of ruin and time to ruin, copulas and Pearson correlation was used to determine claims dependence. The study found out that the moment of ruin occurs more quickly in the presence of dependent claims. Therefore, the study recommended an adjustement of premims to hedge against negative cash flows in the assumption of claims dependent.

DEDICATION

This study is devoted to my parents and all the lecturers at the Department of Statistics at the University of Ghana.

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INTRODUCTION

1.1 Background of the study

Insurance firms are risk-takers. Thus, they are in the business of pooling people's risk together to create a lesser shock for their customers in times of loss. They basically are into the business of creating a cushion to relieve its customers of unfortunate outcomes when they occur (Charpentier, 2010). These customers (insured) are rewarded to decrease their economic burden by insurance companies. To simplify this, an insurance contract shall grant the policyholder the right to claim all or part of the loss if certain unforeseen events happen. The policyholder agrees in writing to pay a stipulated quantity called premium. In order to make this agreement binding, the insurer is required to honour its commitments when such events occur (Dickson, 2016).

To ensure that they can pay as specified in the contract, the insurance company sets aside a reserve or surplus amount from which it can withdraw when the claims are due. This surplus, set aside by the insurance company, does not arise in a day, but takes a long time from possible excesses of premiums over claims paid. Additional sources of excess accumulation, such as investment revenue, are accessible, but the traditional strategy to risk theory has been to disregard interest effects, although literature on this topic is increasing. A very significant stochastic structure for knowing how the capital or surplus of the insurance company evolves over time is the surplus process (Bladt, Nielsen & Samorodnitsky, 2015). To begin with the first surplus $\boldsymbol{\mu_0}$ which is added to the total premiums obtained until the time limit t $\pi(t)$ and the overall claims $S(t)$ subtracted to ascertain the reserve or surplus $\mu(t)$.

One of the main objectives of an insurance company is the so-called likelihood of ruin or probability of ruin. This gives a measure of how certain the insurance company can support its business books. The time to ruin is described as the first moment the surplus becomes negative.

In classical model, the aggregate claims process and the total amount of claims paid from 0 to 1 represent the claim number process, and this is because the claim amount is generally assumed to be independent and identically random variables. This study examines the deviation from the assumptions and their impact on the likelihood of ruin by the insurer. The latest discussion of concern has proven or validated that the independence no longer holds. The rise in interest is understandable because it no longer seems realistic to assume that risks are independent. In practical terms, risk dependence exists in several circumstances. For instance, a unique disastrous event such as an earthquake or fire outbreak can influence the risk of an entire portfolio. Another instance is that an insurance portfolio can cover the life of people of common interest (insurable interest) such as a family or group (religious community members, company staff) whose mortality is to some extent dependent.

This study aims to evaluate the probability of ruin of the company when using the hypothesis of dependence in the event of claims. Assuming that, this is represented by adding each claim as a product of a Bernoulli claim indicator and the size of the ruin should be acknowledged

within a period that a company has fixed N policyholders. The researcher assumed that the incidence of claims among policyholders is no longer independent but dependent. We also assumed that the claim sizes are independent when claims are made.

1.2 Statement of the Problem

Insurance companies anticipate the claim size of policyholders due to the assumption of independence of claims (Kasumo, Kasozi, & Kuznetsov, 2018). Due to the assumption, insurance companies can indicate the time to ruin emphatically. Since the claims are unrelated, the severity of the claims paid is easily absorbed by accumulated premiums most often than not. This problem makes it unlikely for most insurance firms to go into ruin (Milevsky, 2016). But in reality, independent claims are not realistic. An insurance portfolio can cover the life of people of common interest whose mortality depends on a particular case of event happening. For instance, when there is a fire outbreak, the individuals affected who have fire policies will all go for claims. In actual sense, these claims are dependent but it's the claim sizes that are independent. With the assumption of independency, they record a bigger reserve or surplus amount which in the actual sense is not feasible.

In this light, this study aims at analysing the likelihood of ruin when removing the hypothesis of independence in the case of claims. Therefore, the study proposed adjusted premiums process to accommodate claims dependency against ruin occuring.

1.3 Research Questions

The study is guided by the following questions

- What is the exact time of the insurance company's ruin indicator?
- What is the impact of dependency claims on insurance companies ' reserves?
- What is the size of premiums accumulated to dependent claims?

1.4 Objectives of the study

The primary goal is to investigate the effect of dependent claims on the likelihood of ruin and the time-to-ruin that occurs given ruin.

The specific objectives are;

- To determine how dependent claims, affect the likelihood of ruin and time to ruin of insurance companies in Ghana.
- To determine the time left for a given insurance company to go to ruin when the assumption of dependency of claims hold.
- To determine how premiums can be adjusted to accommodate claims dependency.

1.5 Significance of the study

Ruin probability is a key characteristic for an insurance company. However, due to low level of expectations of dependent claims, many insurance companies do not anticipate ruin before it occurs.

- To inform policy makers to factor in strategies to capture and manage dependent claims.
- To also inform management of insurance companies the actual value of the reserve or surplus considering dependent claims.
- The study will serve as a literature in research areas of actuarial, pension and insurance.

1.6 Limitations of the Research

Despite the efforts to minimize all limitations, there were certain constraints within which the research was done. The main limitation was the data collections, the researcher was only privileged to obtain annual data of the selected

1.7 Organisation of the study

The first section is the introduction of the research focusing on the background, problem statement, study questions, objectives and significance of the study. The second chapter gives an examination of a relevant literature of dependent claim, ruin theory and the surplus theory. Chapter three discusses the research methods which includes the data sources and structures and also specified the models used. Chapter four provides the analysis and discussion and the study's conclusion and recommendations are presented in chapter five.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This section provides an examination of the study's literature. It includes concept and definition of ruin probability, aggregate claims process, joint distribution of claim occurrence, and previous studies on dependent claim and ruin time.

2.1 Definition and Concept of Ruin Probability

Mathematical models portray the vulnerability of an insurer to insolvency in actuarial science and applied probability ruin theory (sometimes called risk theory). Key number of interests in such models are the likelihood of ruin, excess allocation immediately before ruin, and deficit at ruin moment.

In 1903, the Swedish actuary Filip Lundbeg introduced the theoretical basis of the ruin hypothesis known as the Cramer-Lundberg model or classical compound Poisson model (Panjer, & Willmott, 2015). The model depicts an insurance company with two adverse cash flows; incoming premiums and outgoing claims. Premiums come from clients at a constant pace $c > 0$ and claims arrive according to an intensity Poisson method and are independent and identically distributed non-negative random variables with F distribution and mean μ (they form a Poisson compound process). So, for an insurer that begins with the original capital surplus X

$$X_t = X + ct - \sum_{i=1}^{Nt} \xi_i \quad \text{for } t \geq 0 \qquad (2.1)$$

The main purpose of the model is to study whether the excess level of the insurer will eventually fall to zero (which will make the company bankrupt). This amount is known as the probability of ultimate ruins is defined as

$$\psi(x) = p^x\{r \ < \ \infty\} \qquad (2.2)$$

where the time to ruin is

$$\gamma = inf\{t > 0 : x(t) < 0\} \qquad (2.3)$$

with the convention that $inf\phi = \infty$.

The Pollaczek-Khinchine Formula can be used to compute this precisely in line with the tail function of the stationary waiting time distribution in $M \ / \ G1$ queue.

$$\psi(x) = \left(1 - \frac{\lambda u}{c}\right) \sum_{n=0}^{\infty} \left(\frac{\lambda u}{c}\right)^n \left(1 - F_l(x)\right) \qquad (2.4)$$

Where $F_l(x) = \frac{1}{\mu} \int_0^x (1 - F(u)) \, du$.

If the claim sizes are distributed exponentially, this will be simplified to

$$\psi(x) = \frac{\lambda u}{c} \ e^{-(\frac{1}{\mu} - \frac{\lambda}{c})x} \qquad (2.5)$$

2.2 The Aggregate Claims Process

This chapter provides an alternative representation of the method of aggregated claims. Similar to the risk model for each person, each claim must be expressed as the product of a claim and the quantity connected with a claim. It enables us to impose on the occurrence of the claim the dependence framework. Consider $n(t)$ risk portfolio $Y_1, Y_2 \ldots, Y_n(t)$ for $[\,0, t]$ period. The sum of these claims is the amount of the overall claims as:

$$S(t) = Y_1 + Y_2 + \cdots + Y_{n(t)} = \sum_{k=1}^{n(t)} Y_k \ = \sum_{k=1}^{n(t)} I_k B_k \qquad (2.6)$$

It is apparent that $n(t)$ refers to the complete amount of exposures to claims and that $n(1) \leq n(2) \leq \cdots \leq n(t)$ for any t, with severe disparities primarily due to fresh policies that have been in place over the era. Therefore, each insurance risk can be described as the product of the indicator in a typical set-up of the individual risk model,

if claim occurs $$I_k = \begin{cases} 1, & if \ claim \ occurs \\ 0, & otherwise \end{cases} \quad (2.7)$$

.

and the benefit level, denoted by B_k., if there is a claim. We suppose that *I* and *B* are autonomous, for simplicity and for reasons we believe that is realistic. I_k has a Bernoulli distribution with the random variable indicator,

$$Prob(I_k = 0) = p_k \text{ and } Prob(I_k = 1) = q_k = 1 - p_k. \quad (2.8)$$

The gain level Bk is assumed to have a distribution function

$$FBk\ (b) = Prob(Bk \leq b). \quad (2.9)$$

We will also suppose that its generating function exists and is

$$M_{B\ k}(t) = E(e^{B_x{}^t}) \quad (2.10)$$

and its mean and variance are

$$\mu_k = E(B_k) and\ Var(B_k) = \sigma_k^2 \quad (2.11)$$

respectively. It is straightforward to find the mean and variance of the aggregate claims:

$$
\begin{aligned}
E\left[S(t)\right] &= \sum_{k=1}^{n(t)} E(Y_k) \\
&= \sum_{k=1}^{n(t)} E\left[E\left(Y_k \mid I_k\right)\right] \\
&= \sum_{k=1}^{n(t)} E\left(Y_k \mid I_k = 1\right) q_k \\
&= \sum_{k=1}^{n(t)} q_k \mu_k
\end{aligned}
\tag{2.12}
$$

and

$$
Var\left[S(t)\right] = \sum_{k=1}^{n(t)} Var\left(Y_k\right) + 2\sum\sum_{i<j} Cov\left(Y_i, Y_j\right) \tag{2.13}
$$

where,

$$
\begin{aligned}
Var\left(Y_k\right) &= E\left[E\left(Y_k^2 \mid I_k\right)\right] - \left[E\left(Y_k\right)\right] \\
&= E\left[q_k B_k^2\right] - \left(q_k \mu_k\right)^2 \\
&= q_k E\left(B_k^2\right) - q_k^2 \mu_k^2 \\
&= q_k \sigma_k^2 - q_k\left(1 - q_k\right)\mu_k^2
\end{aligned}
\tag{2.14}
$$

$$
\begin{aligned}
Cov\left(Y_i, Y_j\right) &= E\left(I_i B_i I_j B_j\right) - E\left(Y_i\right)E\left(Y_j\right) \\
&= E\left(I_i I_j\right)\mu_i \mu_j - q_i \mu_i q_j \mu_j \\
&= \left[Cov\left(I_i, I_j\right)\right]\mu_i \mu_j \quad for\ i \neq j.
\end{aligned}
\tag{2.15}
$$

Likewise, the moments generating function of Y_k can be obtained as follows.:

$$
\begin{aligned}
M_{Y_k}(t) &= E\left(e^{Y_k t}\right) = E\left[E\left(e^{Y_k t} \mid I_k\right)\right] \\
&= E\left[p_k + q_k e^{B_k t}\right] \\
&= p_k + q_k M_{B_k}(t) \\
&= M_{I_k}\left[\log\left(M_{B_k}(t)\right)\right],
\end{aligned}
\tag{2.16}
$$

where MI_K is the moment generating function of the indicator I_k. The moment generating function of the sum can also immediately be evaluated using

$$M_{S(t)}(u) = E\left(e^{S(t)u}\right) = E\left[\exp\left(\sum_{k=1}^{n(t)} Y_k u\right)\right]. \qquad (2.17)$$

Equations (2.6) to (2.16) holds despite if the individual risks are not independent. The findings are already well known in the event of autonomy. For instance, see Bowers al. (1997), & Klugman al. (1998). It is well-known that the sum of independent Bernoulli trials has a binomial distribution.

2.3 Background of Insurance claims in Ghana

Claims settlement goes way back as the history of insurance. In the actual sense, it is the ultimate reason customers buy insurance products. The very first insurance was in the field of marine. History shows that modern insurance was practiced as early as in the year 1347. In this era, ships with cargo would be pledged against a loan and given the condition that should the vessel not successfully reach its destination, the loan would not be refunded. (Irukwu, 1977).

An ancient maritime practice that made it through generations literally unchanged is that of "general average".

Its operation involved when a cargo is thrown overboard during a journey in an attempt to save the voyage. If the journey is successful, the owners of the cargo that was not thrown overboard would receice a claim (Fisher, et al, 2005).

In recent times, Ghana has a bit of growth in the insurance industry seeking to the needs of the local and foreign stakeholders, thus the need to uphold the customer interest and the need to attend to their requirements with speed and efficiency. Insurance has even been hailed as a possible solution to the catastrophic food crises affecting third world nations like Ghana. Insurance claims arise from the occurrence of an event insured against in a policy taken by an insurance policyholder. Customers of insurance companies tend to be delayed in receiving their claims due. Thus, when an event insured for in a policy has happened, insurance companies delay in the payment. In Ghana, to the layman insurance companies are seen as profit making venture who are in the system to take people's money and not return them. This notion should be reversed because the idea of insurance is not to defraud but to create a cushion of relief for their customers who have suffered a loss. Claims payment in Ghana is also seen as a tussle between the insurance company and the customers. This is due to the requirements needed to issue a claim, which include a police report, a driver's licence or birth certificate in the case of a general insurance. In the case of life insurance, a doctor's report and a death certificate is needed to receive a claim. This may take about two weeks to a month to receive settlement. Most claims are settled by means of negotiation between the parties without the need of any formal procedures as litigation. This is, of course, the fastest and most economical method of adjustment. In most claims, there may be nothing over which to negotiate and the claim may be paid almost immediately. When negotiation does not work out, the contract may itself prescribe some other procedure to be followed to be followed, such as arbitration and litigation. When activities with adjustment of the loss are completed and the amount is determined and agreed upon, the insured is entitled to receive payment. There are at least four methods of payment, which insurers can employ in providing claim settlements. They are as follows; Cash, Repairs, Replacement and Reinstatement.

The method of payment is normally given to the insured by writing in the policy. In spite of the above and all other policyholders who have contributed to the fund. Although the insured is entitled to be paid in accordance with the promise of the insurance contract, the fund should be protected against payment of unearned claims. There are certain prohibiting factors like Average and Excess/ franchise/ deductibles inherent in the practice of the insurance that makes it possible for clients not to receive their full payment.

Average is a condition in the policy which provides that the amount of premium paid by the insured is only for a smaller proportion of the total value at risk, since that is what has been disclosed by the insured , any claims settlement under this policy will recognize this fact and the amount payable to the insured will be proportionately reduced.

Excess/Franchise/Deductibles are amounts of money (decided at inception of the policy) that are subtracted from each claim to be settled. It is immediately observed that, should the total amount of the claim be less than the amount of the excess, the claim will not be paid (Wildman, et al, 2005).

2.4 The Image of the claims in the Insurance Industry

It is commonly recognized even among insurance professionals throughout Western African nations that today, unlike in other areas of the globe, the insurance sector does not enjoy a favourable government picture. Insurance individuals are considered fraudsters in some fields who exploit society without providing much in exchange, except for the occasional allegations they are forced to pay either out of fear of losing their clients to another business. (Irukwu, 1977)

To all intends and purposes, the claim department can be seen as the shop window of the insurance company. It does not matter how cheap an insurance company's premiums are, or how efficient they conduct their underwriting administration, if a claim is not properly and fairly dealt with, this is where an insurer will be judged (Roff, 2004).

Lijadu (2002), said the insurance industry is conscious of the insurance sub-sector's misconduct picture of the public. He stressed this by stating that, the insurance sector is viewed as fast collecting premium, slow paying claims, using tiny prints to confuse you, offering bad services, and participating in sharp procedures.

It is important to note that, in the minds of the Ghanaian people this perception is still perceived while the tale stays different for the other areas of the globe.

2.5 The Evolution of Claims

Over the years, claims have evolved. The Chartered Insurance Institute Claims Faculty reports, the way claims are handled, regarded, and managed has changed over latest years. These modifications are incremental and accumulative. It is agreed that claims are much nearer to the industry's core than ever before and are thought to be the greatest trigger for profit and loss from an organization in many instances. So, it's not surprising that a claim has a higher presence in this ever-competitive sector. (Faculty of Claims, CII 2007).

2.6 Claims Administration, Payment and E-Commerce

The involvement of new and enhanced technology, better processes and administrative services all contributed to the development of claims. The internet plays a major role in reducing costs. It also provides better customer services to their customers.

An instance is internet claims and fast communication through standardized formats to request for data. It allows insurers to distinguish their brand from their rivals and provide their clients with better service (Tovstik, 2013).

The processing of claims is programmed to guarantee that claims are registered in the documents of the insurer and to establish a reserve as rapidly and as easily as appropriate for the prospective liabilities involved in the claims. A feature that has been solely manual for many years, one or one that had the help of certain billing devices. The dependence on technology has been greatly enhanced by the emergence of technology where telephone dependence exists and the need for fast and effective service is of paramount significance (Rudź, 2015).

The e-business revolution is gaining converts among the growing number of insurers and brokers of insurance companies around the world. According to Milevsky(2016), a study undertaken on more than 60 major insurance and financial organisations in the UK, a staggering 90 percent of the business processing recently stays based on paper. The later price of this with others, amounted to as much as 35% of the net premium, mainly owing to administrative duplication between sides.

2.7 The Outsourcing of Claims service

More insurers in the developed world now place their claims service in the hands of third parties more than ever before. According to Dickson (2016), this is a revolution in the handling of claims. Claims management is regarded to be one of any insurer's most precious property and the issue to be asked is whether he will be willing to trust it to an outsider. It seems that many insurers are willing to do just that, as they enable their clients to be cared for outside businesses.

Claims outsourcing means an insurance company uses an external company to handle its claims. The outsource company is paid a fee and can handle the claim from start to finish but customer loyalty is the key to insurer. It costs far more to win business than to retain an existing client. So, is outsourcing really such a good idea, and why is it growing so rapidly?

Cost is a significant factor, according to Yang, J (2015). In her words, using outside companies is far cheaper than employing large numbers of individuals in-house involving wage bills and overheads. But it's not the bottom line alone. The outsource suppliers would agree that they can deliver better service, leading to higher customer loyalty.

While outsourcing allegations are deemed viable, there is still some caution when it comes to signing cheques. While aid firms say they are willing to accept validation of claims, insurers are unwilling to give up control.

There is always the fear of paying dubious claims. And at the same moment, customers loyalty could be at risk if an outsource business were to become too hard when negotiating a claim settlement.

Therefore, outsourcing seems to be an unstoppable force in the insurance industry, as it becomes essential to survive the need to cut expenses and deliver enhanced services. For the insurance sector in West Africa, with specific regard to Ghana, the outsourcing of claims is an area that still needs to be considered for broad implementation

2.8 Insurance claims fraud

Insurance fraud remains a problem, but while it becomes more effective in detecting the cheats, it is not a picture that it wants to project**.**

The British Insurers Association report, June 2003, estimated that such costs of fraud in the insurance industry exceed £ 1bn per year. The study says that insurance fraud occurs not only when people make false claims for a non-occurring accident, but also when people inflate claims for real occurrences. This highlights the importance of putting in place effective strategies to detect fraud but also raises some interesting issues around prevention. (Stears, 2003).

Financial crime costs at least £ 13.9 billion, up to £ 20 billion when account is taken of income tax and EU-related fraud. For every man, this amounts to £ 330; female and baby in the UK. Individuals lost at least £ 2.75 billion while businesses lost £ 3.7 billion in 2005. In 2005-2006, at least £ 6.8 billion was defrauded from the public sector. (Police Officers Association, March 2007).

Fraud is everybody's a concealed tax. It raises the price of products and services, impoverishes both tiny and corporate shareholders, hits the future of private pensioners, and jeopardizes employment and saps faith in the distinctive position of the country at home and overseas. It damages the development and investment of businesses. (Bulletin of fraud, June 2006).

Fraud-dominated countries ' governance defines transnational organized crime and fraud as a threat to national security, reflected in the cabinet declaration below; Potential impacts include: undermining lawful cross-border trade; threatening economic markets ' integrity through large-scale money laundering and threatening businesses operations.

Although insurance companies join other bodies in launching a significant crackdown on fraudulent insurance proposals and claims such as fake robbery reports, staged accidents, private profit burning claims, numerous claims on the same property and material falsehood at the proposal point. Needless to say, even the most honest claims may sometimes have been denied their compensation due to the trouble of saying if the claim was real.

2.9 Premium Adjustment Mechanism

Premium Adjustment Mechanisms (alternately referred to as Insurance Experience Adjustments, Premium Equalization, Claims Experience Discounts, and Profit Sharing Discounts) are given as processes in group insurance contracts maintained by members ' superannuation funds, to return a part of premiums to employees where the experience of claims arises more favourably than expected in pricing.The mechanisms may, in some conditions, be two-way payments and may allow for enhanced future premiums reflecting bad knowledge of underlying claims.

Premium adjustment mechanisms have existed since the earliest agreements for group life (which were used in employer agreements). As more data about the experience of claims becomes known, they are typically evaluated annually over a number of years. Premium adjustment mechanisms are separate from volume bonuses. They are not commissions, but a mechanism for adjusting premiums over time to align more tightly with the contract's real claims experience. For superannuation insurance, any payments arising from Premium Adjustment Mechanisms are usually payable to the fund for the benefit of members (usually in the insurance reserves of the fund) and are not paid to the trustee company, its directors, fund managers, sponsoring employers or unions.

Members are beneficiaries of these Premium Adjustment Mechanisms where cumulative modifications are used to decrease future membership premiums or to provide members with extended insurance services such as extra assistance for claimants, enhanced insurance management, online insurance instruments and insurance benefits enhancements. These adjustment mechanisms can be paid in two ways. They can be either an insurer's payment to the fund or the insurer's fund.

2.10 Benefits of Premium Adjustment Mechanisms

Premium Adjustment Mechanisms are a way to address the uncertainty associated with group insurance policy setting premium rates. They are intended to generate higher alignment between superannuation funds. It helps guarantee that insurers are not making excessive profits at the cost of members of the superannuation fund. Premium Adjustment Mechanisms are a way to address the uncertainty associated with group insurance policy setting premium rates. Adjustment Mechanisms, the insurer will set premiums based on its anticipated future level of claims expenses and will seek to include a contingency margin in the premium prices set to represent uncertain future experiences. By using Premium Adjustment Mechanisms, for the benefit of members, excess premiums can be returned to the superannuation fund where the future level of claims cost is less than expected. This enables insurers and funds to smooth members' premium results over time and prevent using conservative margins of contingency in setting premium prices. As a consequence, insurers must hold less capital against the company, thereby lowering premium expenses. These mechanisms have been especially crucial in recent years where the incidence of claims has increased, including an rise in late reported claims. It was hard to predict in this setting when the growing number of claims would stabilize.

Premium adjustment mechanisms provide the insurer with a level of security in that the premiums paid over time will align with the price of the experience of claims. This also helps to make superannuation funds more competitive in the insurance industry where insurers otherwise may not be involved in contract tenders. At the same moment, rates paid by employees in funds with Premium Adjustment Mechanism agreements are often more stable compared to those without them (especially when coupled with lower price guarantees) as the processes enable more gradual adjustments to reflect experience rather than important step changes at the end of a longer guarantee period. Dependent claims continue to impact the insurance company's reserve. The business must, however, take certain steps to prevent payment of various claims.

2.11 Appropriate and Inappropriate Use Of Premium Adjustment Mechanism Payments.

Industry rules should govern the correct use of payments from the Premium Adjustment Mechanism and identify forbidden or inappropriate uses. Premium adjustment mechanisms must be used in accordance with the ' sole purpose test ' in the SIS Act only for reasons. Without Premium Adjustment Mechanisms, the effect of accepting lower (or greater) volumes of claims will be postponed until the next reset of the premium prices. However, when a Premium Adjustment Mechanism is in place, lower (or greater) claim volumes lead to a more instant shift in insurance rates for employees – this does not conflict with the obligation of the trustee to do everything to pursue a fair claim.

2.12 Mandatory Motor Insurance

Compulsory motor insurance requires all those operating a motor vehicle purchase insurance to ensure or guarantee some compensation to those injured in an automobile accident. In the United States, compulsory insurance was introduced in Massachusetts in 1927 known then as Compulsory Automobile Liability Insurance Law (Cohon et al., 2004). The Road Tra_c Act 1930 was enacted to commence compulsory insurance in the United Kingdom. Compulsory motor insurance observed in many countries is backed by some legislature with a minimum of third party insurance. This is known as Third Party Personal Injury Insurance (EU, 2015).

He proposed that mandatory third-party insurance would require adequate public oversight to produce the economic growth required in the motor insurance industry. This rise may lead from an rise in the total premium received by insurance companies, but it can also boost the general claim made on insurance companies. However, according to Lawrence (2001), in order to match the increased competition and understand the possibilities that liberalization will bring, there is a need to inject some liberalization in phases.

Hoffer et al. (1991) proposed alternative funding scheme for uninsured motorist coverage in order to tackle the danger associated with car use in public space. In their studies, they suggested that premiums for uninsured motor vehicle coverage should be gathered by a tiny universal motor fuel surcharge and therefore distributed to insurers based on the amount of cars they insure in that jurisdiction. Despite the benefit of reduced costs for the average motorist using this strategy over the different other funding system for uninsured motorists, Hoffer et al. (1991) created a lot of assumptions about the fuel consumption pattern of cars, the driver's driving abilities and experience as well as the hazards connected with the highway mostly used by the driver. However, these are very critical parameters which determine the likelihood of the claim being ever present and ignore them in the price of the motor insurance premium eliminates the actuarial precision of the estimate.

2.13 Pricing In Insurance

Several scientists have suggested different methods to correctly calculate the claims insurance companies need to pay the insured when the danger happens. The actuarial precision of these different methods depends on the accessible information and on the admissible assumptions.

Edward et al. (2008) used detailed micro-level records that included experience at the individual level of the vehicle, the type of insurance claim and the corresponding amount of claims and used a hierarchical model for the three components. These elements include claim frequency, claim form, and claim seriousness. Edward et al. (2008) used a negative binomial regression model to evaluate the frequency of claims. The model used the gender, age, no claim discount of the owner as well as the age and type of the car to forecast a claim occurrence. As proposed by other scientists, these factors showed a powerful connection to the likelihood of a claim. The next model used to predict the type of insurance claim that is whether third party injury, third party property damage, insured own damage or some combination is the multinomial logit model. This model also considered year, vehicle age and vehicle type as important predictors for this model.The third model which this research is about is the severity component. Edward et al., (2008) proposed the use of a generalized beta of the second kind of long-tailed distribution for claim amounts and also incorporate predictor variables. This model also found year and vehicle age and insurers age to be important predictors of this model. They found in their research a significant dependence among the different claim types and concluded that the three component models proficient prediction of automobile claims compared with the traditional methods. This however cannot be said for countries where data integrity cannot be confirmed. Despite the actuarial accuracy in computing motor insurance premium using this approach, a lot of the information needed is provided by the insured and some audit or double checking is required, which means more time and cost. A more detailed personal information approach was suggested by Vlad et al.,(2006) in estimating motor insurance premium.

2.14 Previous works on Probability of Ruins and Time to Ruin

Rudź (2015), studies the exact ruin probabilities in discrete time models. To determine the precise formulae for ruin probabilities, he applies an integral operator produced by the discrete time risk process. The methodology is based on discovering an operator's fixed point and checking if it is identical to the probability of ruin. He derived the exact probabilities of ruin for both an absolute continuous distribution of claims and a discrete amount.

Livshits (1999), examines the likelihood of a Poisson model insurance company's ruin. This is because there are issues in estimating an insurance company's probability of ruin over an infinite interval and the associated conditional average time before the ruin. He discovered that an insurance company's likelihood of ruin over an infinite interval and the conditional average time before the insurance company's ruin for insurance premium and payment flows from Poisson.

Kasumo., Kasozi and Kuznetsov (2018), in their study entitled “On minimizing the ultimate ruin probability of an insurer by reinsurance,” consider an insurance company whose dynamic reserves follow a diffusion-disturbed model of risk. They discovered that the business chose reinsurance using proportional or excessive reinsurance to decrease its risk.

They used the Hamilton-Jacobi-Bellman (HJB) strategy to achieve a second-order Volterra integrodifferential equation (VIDE) and use the block-by-block technique for ideal reinsurance strategy to minimize the ultimate probability of ruin for the parameters selected. They also found that proportional reinsurance increases the company's survival for the Cramér-Lundberg and diffusion-perturbed models in both light- and heavy-tailed distributions.

Tovstik (2014), examined the probabilities of ruin for an insurance business based on certain stochastic risk models. With autonomous random claims and premiums, he concentrated on stochastic risk model. Recurrence formulas for an insurance company's ruin probabilities when paying claims are obtained and assumed that both the random premiums and the insurance damages are independent and distributed identically. The number of claims and premiums are independent processes of Poisson, both independent of the size of premiums and claims. He also presumed the exponential distribution of the random premiums and insurance damages. They were able to calculate the probabilities of ruin on infinite and finite time periods based on the probabilities acquired.

In his studies, Charpentier (2010), considers ideal reinsurance from the point of perspective of an insurer. This is because insurers may want to discover the ideal risk transfer system given a (small) likelihood of ruin. He derives an effective algorithm from Monte Carlo to connect the probability of ruin with the parameter of reinsurance. In the event of nonproportional reinsurance, he also obtained a methodology estimating the ruin likelihood target.

Panjer (2015), examines an insurance company's excess generally modelled on the classical Poisson compound risk model and the risk model of Sparre-Andersen. The claim quantities and inter-claim times under these models were presumed to be distributed separately, which is not always suitable in practice. Risk models relaxing the hypothesis of autonomy have attracted extensive study attention in latest years. He integrated small fluctuations in the underlying excess mechanism, the dependent insurance risk is further generalized to a disturbed version. Together with apps and examples, explicit solutions for the Gerber-Shiu function were deduced. Finally, he brought the disturbed structure of reliance into the dual risk model and study the issue of ruin time. Exact solutions are acquired for the transformation of the Laplace and the first moment of ruin with an arbitrary gain-size distribution. He also gave numerical examples to demonstrate the effect of the structure of reliance and the disturbance.

Yang, (2015), discusses how it is possible to relax this often-unwarranted hypothesis in a straightforward manner while integrating rating variables into the model. His strategy consisted of fitting generalized linear models to the marginal frequency and the conditional effect elements of the complete claim price; reliance is caused by treating the amount of claims as a covariate in the average claim size model. In addition to being simple to enforce, this modelling approach has the benefit that when Poisson counts are assumed for the conditional severity model together with a log-link, the resulting pure premium is the result of a marginal mean frequency, a modified marginal mean magnitude, and an readily interpreted correction term that represents the reliance.

Wang, (2013), for insurance claims and dimensions, presented a joint copula-based model. To accommodate the reliance between these amounts, she utilizes bivariate copula. Without the restrictive premise of independence, she derived the overall distribution of the political loss. She demonstrated that this distribution tends to be skewed and multi-modal, and that an assumption of independence can lead to significant bias in policy loss assessment. Furthermore, by merging marginal generalized linear models with a copula, she extended the structure to regression models. She demonstrated that this strategy leads to a flexible model class and that maximum likelihood can be used to estimate the parameters effectively.

To select the ideal copula family, she suggested a test operation. A simulation research and an assessment of car insurance policies illustrated the usefulness of her strategy.

Yang, J (2015), explored how the heterogeneity between probabilities of incidence and severity of claim impacts aggregate claim figures and aggregate claim quantity for a portfolio of insurance. He demonstrated that greater heterogeneity (and reliance) between probabilities of occurrence outcomes in both lower aggregate claim amounts and aggregate claim quantity in the mean remaining lifetime order. He also demonstrated that as the heterogeneity among the claims rises, the aggregate claim quantity rises in the sense of the usual stochastic order when the probability of occurrence vector remains weakly stochastic arrangement. These theoretical results were applied to characteristics of binomial random variables convolutions, providing upper limits for the mean residual lifetime features of aggregate claim figures and amounts, and comparing stop-loss premiums and risk capital of various insurance portfolios.

2.15 Previous works on adjustment of premiums

Fred Jawadi (2009) studied the adjustment dynamics of the non- life insurance premium (NLIP) and its dependence on the financial markets. He also examined the relationship between the NLIP, the interest rate, and the stock price using the recent developments of nonlinear econometrics. He therefore suggested a strong evidence of significant linkages between insurance and financial markets. He found that the NLIP adjustment toward equilibrium is time varying, with a convergence speed that varies according to the insurance disequilibrium size. Therefore adjusting premiums becomes very necessary in the insurance business to hedge against negative cashflow and prolonging the ruin time of an insurance companies.

Wynand P.M.M (1994) explained there exists a consumer information surplus that may result in adverse selection. He indicated that insurers can greatly reduce this surplus by risk-adjusting the premium. He therefore concluded that there need not be any substantial unavoidable consumer information surplus if consumers can choose whether to take a deductible for a one- or two-year health insurance contract with otherwise identical benefits. Therefore, adverse selection should not be a problem in a competitive insurance market with risk-adjusted premiums.

According to Boikov (2003), the adjusted premium process is a linear function of time in the classic Cramer--Lundberg model. The premium process is stochastic and it is also independent of a risk process. The nonruin probability is chosen as a measure of the payment ability. Integral equations and exponential bounds are obtained similarly with the classic Cramer--Lundberg model. He also proposed that adjusted premiums should be a function of the numder of claims and the claim size.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

The methodology of the study is presented in this chapter. It includes data source and structure, methods of data analysis and the models adopted for the study.

3.2 Data Source and Structure

Data that was used for the study was secondary and was extracted from Annaly reports of National Insurance Commission (NIC), from 2010 to 2017. The researcher randomly selected four general insurance companies. The data obtained includes annual claims paid, premiums received and the number of claims reported.

3.3 Method of Data Analysis

To ensure accuracy in the data processing, data editing and clearing of the data were done before analysing the data. The data were coded in order to make possible inputting into the data processor. The codes were transformed into units to facilitate their description and analyses. Diagrammatic presentation by means of tables and graphs were done. Although there will be several electronic means of analysing data R were used for the analysis. The study employed Pollaczek-Khinchine formula, which was used to estimate probability of ruin and time to ruin, copulas and Pearson correlation was used to determine claims dependence.

3.3.1 Copula Test of Independent

Copulas are mathematical objects that fully capture the dependence structure among random variables and hence, offer a great flexibility in building multivariate stochastic models. In statistics, a copula is used as a general way of formulating a multivariate distribution in such a way that various general types of dependence can be represented. The Frank Copula Test of dependence is presented as

$$c_\theta(x,y) = -\frac{1}{\theta}\ln\left[1+\frac{\left(e^{-\theta x}-1\right)\left(e^{-\theta y}-1\right)}{\left(e^{-\theta}-1\right)}\right] \quad (3.1)$$

where x is the number of claims and y is the amounts of claims,

$\theta \in [1,\infty)$ control the degree of dependence between x and y.

3.3.2 Estimating Probability of Ruin and Time to Ruin

In classical risk theory, the company's surplus process is given as

$$U(t) = \mu_0 + \Pi(t) - \sum_{k=1}^{N(t)} X_k \quad (3.2)$$

where the aggregate claims process $\sum_{k=1}^{N(t)} X_k$ consists of the sum of individual claim amounts assumed to be independent and identically distributed random variables, together with a random number of claims $\{N(t)\}$ also assumed to be independent of the claim sizes.

If the number of claims and claim amount assumed to be dependent, the company surplus process is given as

$$U(t) = \mu_0 + \Pi(t) - \sum_{k=1}^{N(t)} Y_k \quad (3.3)$$

where the aggregate claims process $\sum_{k=1}^{N(t)} Y_k$ consists of the sum of individual dependent claims.

Time to ruin

The time to ruin is defined to be the first time that the $\mu(t)$ becomes negative

$$\gamma = inf\{t > 0 : x(t) < 0\} \qquad (3.4)$$

Premium Process

The premium process typically has the form

$$\Pi(t) = c_t \qquad (3.5)$$

where *c*, the rate of premium per unit of time, is expressed as

$$c = E(N)E(X)(1 + \theta) \qquad (3.6)$$

with θ denoting the relative security loading.

3.3.3 Adjustment of Premiums

From equation 3.3, the surplus fuction with the assumption of cliam dependent is given as

$$U(t) = \mu_0 + \Pi(t) - \sum_{k=1}^{N(t)} Y_k$$

making $\Pi(t)$ the subject

$$\Pi(t) = U(t) - \mu_0 + \sum_{k=1}^{N(t)} Y_k$$
$$= U(t) - \mu_0 + S(t)$$
$$E[\Pi(t)] = E[U(t)] - \mu_0 + E[S(t)]$$

Now to prevent ruin for occurring it is expexted that the positive surplus process

$E[U(t)|U(t) > 0]$ should be added to the premiums (Trufin & Loisel, 2009)

$$Adj\Pi(t) = E[\Pi(t)] + E[U(t)|U(t) > 0] \qquad (3.7)$$

3.3.4 Probability of Ruin

We employed Pollaczek-Khinchine Formula to compute for probability of ruin. This is because when data are tailed, Pollaczek-Khinchine precisely compute the stationary waiting time distribution in $M\ /\ G1$ queue.

$$\psi(x) = \left(1 - \frac{\lambda u}{c}\right) \sum_{n=0}^{\infty} \left(\frac{\lambda u}{c}\right)^n \left(1 - F_l(x)\right) \qquad (3.8)$$

where $F_l(x) = \frac{1}{\mu}\int_0^x (1 - F(u))\, du$.

3.3.5 Maximum Likelihood Estimator (MLE)

A value $\hat{\theta}_n$ {where $\hat{\theta}_n$ is a function of the observations $x_1, x_2, \ldots x_n$; say $\hat{\theta}_n = t(x_1, x_2, \ldots x_n)$ which maximizes the likelihood function $L(\theta;\ x_1, x_2, \ldots x_n)$ over all $\theta \in \Theta$ is called a maximum likelihood estimate (MLE) for θ.

Hence, for all $\theta \in \Theta$;

$$L[t(x_1, x_2, \ldots x_n); x_1, x_2, \ldots x_n] \geq L(\theta;\ x_1, x_2, \ldots x_n) \qquad (3.9)$$

The Random Variable $T_n = t(x_1, x_2, \ldots x_n)$ is called a Maximum Likelihood Estimator for θ.

Procedure:

1. Find (obtain) the likelihood function $L(\theta; \underline{x})$
2. Find the natural log of L, i.e.

$$l((\theta; \underline{x}) = \ln\{L(\theta; \underline{x})\}$$

3. Obtain the score function $S(\theta; \underline{x})$

$$S(\theta; \underline{x}) = \frac{\delta l}{\delta \theta}$$

4. Obtain the information function $I(\theta; \underline{x})$

$$I(\theta;\underline{x}) = \frac{\delta^2 l}{\delta\theta^2}$$

Let $\hat{\theta}$ be the MLE for θ, then $\hat{\theta}$ is a solution to the equation

$$S(\hat{\theta};\underline{x}) = 0$$

i.e. Solve θ in the $S(\theta;\underline{x}) = 0$

5. Finally check that $I(\hat{\theta},\underline{x}) \geq 0$

MLE for Exponential

Let $x_1, x_2 \dots, x_n$ be identical and independent random variables that use exponentially distributed as $X \sim \exp(\lambda)$

$$f(x) = \theta e^{-\theta x} \qquad \theta > 0\,, x > 0$$

$$L(P(x)) = \prod_{i=1}^{n} \theta e^{-\theta x}$$

$$L(P(x)) = \theta^n e^{-\theta \sum x_i}$$

$$= log\theta^n + loge^{-\theta \sum x_i}$$

$$= nlog\theta - \theta \sum x_i$$

$$\frac{l(Px)}{d\theta} = \frac{n}{\theta} - \sum x_i = 0$$

$$\frac{n}{\theta} = \sum x_i$$

$$\hat{\theta} = \frac{n}{\sum x_i}$$

$$\hat{\theta} = \frac{1}{\bar{x}}$$

Where $\bar{x}$ is the mean $f\{x_i\}$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

By the invariant property of the MLE, the MLE estimator $\hat{\mu}$ is simply $\hat{\mu} = \bar{x}$

MLE Poisson

Let $x_{1,}\ x_{2,\dots}x_n$ identically independent random variable, the likelihood function is defined as

$$L(\lambda) = \prod f(x_i\,|\lambda)$$

$$L(\lambda) = \frac{\lambda^{\sum x_i} e^{-nx}}{\prod x!}$$

$$L(\lambda) = log\lambda^{\sum x_i} + loge^{-n\lambda} - log\prod x_i$$

$$= \sum x_i\, log\lambda - n\lambda - log\prod x!$$

$$l'(\lambda) = \frac{1}{\lambda}\sum x_i - n = 0$$

$$\sum x_i = n\hat{\lambda}$$

$$\hat{\lambda} = \frac{\sum x_i}{n}$$

$$\hat{\lambda} = \bar{x}$$

3.3.6 Chi Square Goodness of Fit Test

Assumptions

- The data available for analysis consist of a random sample n independentobservations.
- The measurement scale be norminal.

- The observation can be classified into r nonoverlapping categoriesthat exhaust all classification possibilities. The number of observations falling into a given category is called the observed frequency of the category.

Hypothesis

H_0: $The\ sample\ has\ been\ drawn\ from\ a\ population\ that\ follows\ a\ specified\ distribution.$

H_1 : $The\ sample\ has\ not\ been\ drawn\ from\ a\ population\ that\ follows\ the\ specified\ distribution$

Test statistics

The test statistic Chi Square Goodness of Fit test

$$X^2 = \sum_{i=1}^{r} \frac{(\theta_i - E_i)^2}{E_i} \qquad (3.10)$$

where θ_i is the observed frequency and E_i is the expected frequency.

3.3.7 Wilcoxon matched-pairs signed-ranks test

The Wilcoxon signed-ranks test is a non-parametric equivalent of the paired t-test. It is most commonly used to test for a difference in the mean (or median) of paired observations - whether measurements on pairs of units or before and after measurements on the same unit (MacFarland & Yates, 2016).

Procedure

- Determine the sign of the difference $\left(D_i\right)$ between each pair of observations. Examine the distribution of the differences.
- Rank the differences in order of absolute size with a rank of 1 assigned to the smallest difference. Differences of zero are (usually) dropped from the analysis. The rank assigned to tied ranks is the mean of the ranks that would have been given if the observations had not been tied.
- Reassign the signs of the differences to their respective ranks $\left(R_i\right)$.

Test Statistics

Then calculate the appropriate test statistic.

Calculate S^+ and S^- which are the sums of positive and negative ranks respectively.

The test statistic is given as

$$S = \min(S^-, S^+) \qquad (3.11)$$

.

3.3.7 Pearson Correaltion

Peasrson correlation is a method used to study the strength of a relationship between two numerical variables. The correlation analysis was used to established the relationship between claims amount and claims count in the study.

$$r\left(x, y\right) = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \sqrt{n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2}} \qquad (3.12)$$

where x is the number of claims and y is the amounts of claims.

CHAPTER FOUR

DATA ANALYSIS AND DISCUSSION

4.1 Introduction

This chapter presents the analysis and discussion pertaining to effects of dependent claims on the probability of ruin, the time to ruin given ruin occurs. The analysis includes modelling the amount of claims, modelling number of claims and estimating probability of ruins, and time to ruin.

4.2 Descriptive Analysis

The descriptive statistics such as mean, standard deviation, skewness, kurtosis of the premiums, claims paid and number of claims for four selected insurance companies. The results were presented in Table 4.1.

Table 4. 1Descriptive Analysis of Premiums, Claims Paid, Number of Claims

	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
Premiums						
A	10017757.00	69660874.00	45062120.38	22523078.63	-0.19	-1.32
B	64042765.00	161928878.00	117461352.88	35145129.99	-0.14	-0.94
C	28168710.00	129593555.00	74252986.38	36866587.64	0.40	-1.09
D	11559429.00	35671068.00	19101680.75	9152316.98	1.22	-0.01
Claims Paid						
A	2823494.08	19633842.73	12700710.37	12700710.37	1.19	-1.32
B	12565341.00	47818099.00	31099659.13	31297659.82	1.11	-1.39
C	2560791.82	25505000.00	10587097.95	10588079.42	0.98	-0.77
D	2311885.80	10176799.33	4408686.43	4402286.33	1.56	1.62
Number of claims						
A	3589.00	7092.00	5749.50	75.41	-0.71	-1.30
B	4507.00	8000.00	6856.25	82.55	-1.27	-0.03
C	2911.00	6404.00	5050.50	71.01	-0.71	-1.28
D	1004.00	4497.00	3158.00	56.38	-0.74	-1.16

From Table 4.1, on the average company A received a premium of 45,062,120.38 annually with a standard deviation of 22,523,078.63. The premiums ranges from 10,017,757.00 to

69,660,874.00. With regards to claim paid by Company A, on the average company A paid claims amounted to 12,700,710.37 annually. Also, Company B on the average received a premium of 117,461,352.88, with claims paid annually as 31,099,659.13 and the number of claims recorded annually was 6856.25 and ranges from 4,507 to 8,000. With regards to Company C the mean premiums received annually was 74,252,986.38, the amount of claims paid annually was 10587097.95 and mean the number of claims recorded annually 5050.50. In Company D, it was found that the average premium was 19,101,680.75 and the number of claims recorded was 3158.00.

4.3 Modelling Amount of Claims

From the descriptive statistics it was found that the mean and the standard deviation of the amount of claims paid were almost similar. This implies that the suggested model for the amount of claims paid is exponential distribution. Hence a chi-square of goodness of fit test was done to determine how well the data was fit to exponential distribution.

Table 4.2: Chi-square goodness of fit of Exponential on Amount of Claims

Company	Chi-Square	DF	P-Value
A	8.458	6	0.2064
B	5.652	6	0.4632
C	7.067	6	0.5781
D	6.7154	6	0.6522
Overall	27.243	23	0.7542

From Table 4.2, it can be seen that all the p-value of the chi-square test is greater than zero. This implies that the null hypothesis that the found that the amount of claims follows exponential was failed to be rejected. Hence the data follows an exponential distribution.

Since the amount of claims follows both exponential, Maximum Likelihood Estimator (MLE) was used to estimates the parameters of the distributions. The results can be seen in Table 4.3.

Table 4.3: Modelling Amount of Claims

Company	Rate ($\hat{\vartheta}$)	Stand. Error
A	0.000000079	0.000000028
B	0.000000032	0.000000011
C	0.000000087	0.000000033
D	0.000000243	0.000000020
Overall	0.000000363	0.000000080

From Table 4.3, it can be found in Company A, the amounts of claims follow exponential distribution with parameter 0.000000079 with a standard error of 0.000000028, whiles Company B follows an exponential distribution with parameter 0.000000032 with a standard error of 0.000000011. With respect to Company C and D the amounts of claims follow exponential distribution with parameters 0.000000087 and 0.000000243 respectively. Putting all the companies together, it was found that the amounts of claims follows distribution with parameter 0.000000363 with a standard error of 0.000000080.

4.4 Modelling Number of Claims

From the descriptive, it was found that average number of claims, variance (square of standard deviation) of the number of claims are similar. Hence, it is likely to follow a Poisson distribution. Tovstik (2014), in his study also found out that the number of claims observed follows Poisson distributions. Therefore, chi-square goodness of fit test was done to determine whether the number of claims recorded in the insurance companies follows Poisson distribution.

Table 4.4: Model Adequacy for Number of claims

Company	Chi-Square	DF	P-Value
A	10.245	6	0.1147
B	9.452	6	0.1497
C	8.845	6	0.0576
D	7.452	6	0.2811
Overall	29.425	23	0.1667

From the chi-square test of goodness fit, since the p-value of the test statistic is greater than zero. It implies that the number of claims follows a Poisson distribution. As a result of that, MLE was used to estimate the parameter of the Poisson distribution. The results of MLE were presented in Table 4.5.

Table 4.5: Modelling the number of claims

Company	Rate ($\hat{\lambda}$)	Stand. Error
A	5739.500	26.785
B	6647.500	28.826
C	5051.500	25.128
D	3144.500	19.826
Overall	5739.500	26.785

From Table 4.5, it can be seen that the number of claims modelled for Company A, B, C and D follows Poisson distribution with parameter 5739.500, 6647.500, 5051.500 and 3144.500 respectively.

4.5 Estimating Dependency between Claims Amount and Number of Claims

The dependency between claims paid and number of claims recorded, was estimated using Pearson correlation and Frank Copula test of dependence.

Table 4.6: Estimating Dependency between Claims Amount and Number of Claims

Company	Pearson Correlation r	Test of dependence η
A	0.5377329	0.3622671
B	0.5703038	0.3296962
C	0.5913531	0.3086469
D	0.604668	0.195332
Overall	0.7119262	0.0880738

From Table 4.6, it was found that there is a positive correlation, between claims amount and number of claims. This implies that as the number of claims increases, amounts of claims also increases. From the Copulas test of Independent it was also found that the level of independency between claims amount and number of claims was low. Company D, recorded the highest dependency among the four selected companies ($r = 0.60$; $\eta = 0.1956$). The overall results imply that there is a higher dependency between claims amount and number of claims in Ghana companies ($r = 0.7$; $\eta = 0.1956$).

4.6 Probability of Ruins

The probability of rule was estimated for two cases, firstly when the dependency between number of claims and amount of claims is considered in the model and when the dependency is not considered. Pollaczek-Khinchine formula was used to estimate the ruin probabilities given there is no initial surplus and the results can be seen in Table 4.7.

Table 4.7: Probability of Ruins

Company	Probability of Ruin	
	Assumptions of Dependence	Assumptions of Independence
A	0.288	0.0808
B	0.382	0.0935
C	0.423	0.1285
D	0.509	0.1737
Overall	0.597	0.1836

From Table 4.7, it was revealed in all the companies that when the dependency between number of claims and amounts of claims is considered, the ruin probability is higher than when the dependency is not considered. This implies the insurance companies can underestimate their ruin probabilities when the dependencies are not considered.

4.7 Time to Ruin

The time it takes for the insurance company to ruin was estimated using equation 3.4 and the reults were presented in Table 4.8.

Table 4.8: Time to Ruin

Company	Time to Ruin (days)	
	Assumptions of Dependence	Assumptions of Independence
A	30.29	92.94
B	23.67	84.12
C	21.60	55.11
D	19.28	42.13
Overall	16.03	39.77

From Table 4.8, it can be found that the time to ruin for all companies when dependency between number of claims and amount of claims is taken into consideration are lower than when the dependency are ignored. This implies that it will take shorter time for ruin to occur when dependency between number of claims and amount of claims is considered than when it is ignored. Therefore, a company may have shorter time to ruin than the actual time estimated when the dependency is ignored.

4.8 Relationship between level of independency and Time to Ruin

The level of correlation was plot against time to ruin, in order to display the relationship between the relationship between level of independency and Time to Ruin.

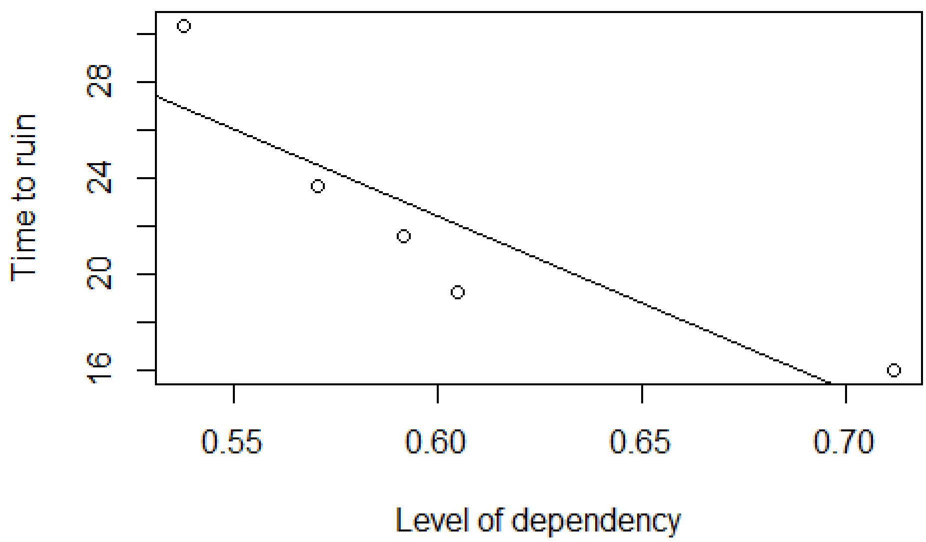

Figure 4.1: Relationship between level of Dependency and Time to Ruin

Figure 4.1, it can be seen that there is a negative relationship between dependency and time to ruin. This implies that the higher the dependency the lower the time to ruin. Therefore, when the dependency between number of claims and amount of claims are higher, the insurance company ruin faster.

4.9 Relationship between level of dependency and Probability of Ruin

A line plot was done to examine the relationship between the relationship between level of dependency and probability of ruin. The level of dependency from correlation was plot against probability of ruin and results were displayed in Figure 4.2.

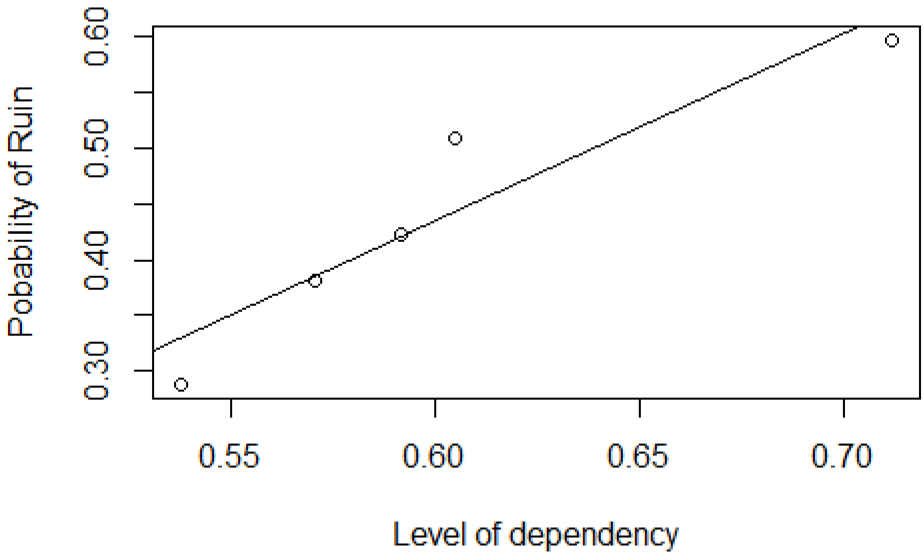

Figure 4.2: Relationship between level of independency and Probability of ruin

Figure 4.2, it can be seen, there is a positive relationship between probability of ruin and level of dependency. This implies that the higher level of dependency, the higher probability of ruin and vice versa. Hence given the same initial surplus, insurance companies that exhibit higher relationship between amount of claims paid and number of claims are more likely for ruin to occur than companies with relatively lower dependency.

4.10 Premiums Adjustment to meet Claims dependency

From Table 4.7, it was found that the probability of ruin based on assumption of claim dependency is significantly higher than the assumption of claim independency. This is because with the assumption of claim dependency, claims amount are higher. Hence to ensure solvency, premiums must be adjusted. This section, present results of how the premiums are adjusted to to prevent ruin occuring in case claim dependency. The adjustment is based on adding the expected positive surplus process to the initial premiums as shown in equation 3.7.The results of the adjusted premiums can be seen in Table 4.9.

Table 4.9: Adjusted Premiums

Company	Premiums	$E\left[U(t)\mid U(t)>0\right]$	Adjusted Premiums
A	45,062,120.38	2,577,499.28	47,639,619.66
B	117,461,352.88	5,155,099.96	122,616,452.84
C	74,252,986.38	1,112,070.20	75,365,056.58
D	19,101,680.75	110,513.96	19,212,194.71

The premiums are adjusted by the proposed formula in equation 3.7. For examples the adjusted premiums for company A was computed as $45062120.38+2577499.28=47639619.66$.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 Introduction

This chapter presents the summary, conclusions and recommendations on effects of dependent claims on the probability of ruin, the time to ruin given ruin occurs.

5.2 Summary

Probability of ruin and time to ruin are mostly estimated by the insurance companies without considering the dependent claims. In practice, the amount of claims paid are dependent on number of claims recorded. Therefore, this study determines how dependent claims, affect the ruin probability companies in Ghana. Also, how fast a given insurance company goes to ruin when the assumption of dependency of claims hold.

Secondary data was used and was extracted from Annaly reports of National Insurance Commission (NIC), from 2010 to 2017. The researcher randomly selected four general insurance companies. The data includes annual claims paid, premiums received and the number of claims reported.

The amount of claims was modelled using exponential distribution and maximum likelihood estimator (MLE) was used to estimates the parameters. The study found out that amount of claims are well fit using exponential distribution. Also, with regards to number of claims, the data follows Poisson distribution. The study also found out that there is a positive correlation, between claims amount and number of claims and from the Copulas test of Independent it was also found that the level of independency between claims amount and number of claims was low.

With regards to the relationship between the level of independency and time to ruin and probability of ruin. The study found out there is a positive relationship between level of independency and time to ruin and a negative relationship between level of independency and probability of ruin. This means that as the level of dependency between number of claims and amount of claims is high, ruin occurs faster with higher probability of ruin.

5.3 Conclusions

The following conclusions were drawn from the study:

Findings from the study indicate that when there is claim dependency it plays an important role in the insolvency insurance companies as it significantly increases the likelihood of ruin of insurance companies.

Based on the results, it can be concluded that when the assumption of dependency of claims hold, time left for a given insurance company to go to ruin decreases.

From the study it can be concluded that using the proposed adjusted premium, the impact of claim dependencies on ruin occurring can be neutralised.

5.4 Recommendations

Based on the findings from this study, it is recommended that insurance companies with high claim dependencies should adjust their premiums in order to enhance their chances of survival.

Computation of time to ruin without the assumptions of claims dependency may be misleading results and this will lead to wrong decision making. Hence, any policies on the solvency of the insurance companies should be supported by the level of claim dependencies.

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APPENDIX: R codes

```
datac=read.delim("clipboard") ##loading the data

names(datac) ### Variables names

library(MASS)


claimfit<- dexp(datac$Glico_C, rate = 1, log = FALSE)

AIC(claimfit)

#### number of Claims ######

#### Poisson #####

172.600000/12

AADC<-fitdistr(datac$Claim_Amount, "weibull")

AIC(AADC)

BIC(AADC)

AADC

AB<-fitdistr(datac$SIC_nc, "Poisson")

AIC(AB)

BIC(AB)

AC<-fitdistr(datac$Star_nc, "Poisson")

AIC(AC)

BIC(AC)

AC

AD<-fitdistr(datac$Donewell_nc, "Poisson")

AIC(AD)

BIC(AD)
```

```
AD

OV<-c(datac$Glico_nc,datac$SIC_nc,datac$Star_nc,datac$Donewell_nc)

OV1<-fitdistr(datac$Glico_nc, "Poisson")
AIC(OV1)
BIC(OV1)
OV1
OV2<-fitdistr(datac$SIC_nc, "negative binomial")
AIC(OV2)
BIC(OV2)
OV2
com.fit(OV2)
AC<-fitdistr(datac$Star_nc, "Poisson")
AIC(AC)
BIC(AC)
AC
AD<-fitdistr(datac$Donewell_nc, "Poisson")
AIC(AD)
BIC(AD)
AD

#### negative binomial#####
```

```
BA<-fitdistr(datac$Glico_nc, "negative binomial")

AIC(BA)

BIC(BA)

BA

BB<-fitdistr(datac$SIC_nc, "negative binomial")

AIC(BB)

BIC(BB)

BB

BC<-fitdistr(datac$Star_nc, "negative binomial")

AIC(BC)

BIC(BC)

BC

BD<-fitdistr(datac$Donewell_nc, "negative binomial")

AIC(BD)

BIC(BD)

BD


CA<-fitdistr(datac$Glico_nc, "compo")

AIC(CA)

BIC(CA)

CA

library(compoisson)

MA<-matrix(datac$SIC_nc, nrow = 1, ncol =8 )
```

```
DDC<-com.fit(MA)

DDC


MAM<-matrix(OV, nrow = 1, ncol =32 )

DDC<-com.fit(MAM)

DDC


CC<-as.numeric(DDC$lambda)

AIC(CC)

com.fit

?fitdistr

compoisson

compoisson::compoisson::

  Glico

WA1<-fitdistr(datac$Glico_C, "exponential")

AIC(WA1)

BIC(WA1)

WA1


WA2<-fitdistr(datac$SIC_C, "exponential")

AIC(WA2)

BIC(WA2)

WA2
```

```
WA3<-fitdistr(datac$Star_C, "exponential")

AIC(WA3)

BIC(WA3)

WA3



WA4<-fitdistr(datac$Donewell_C, "exponential")

AIC(WA4)

BIC(WA4)

WA4


OVC<-c(datac$Glico_C,datac$SIC_C,datac$Star_C,datac$Donewell_C)

GA<-fitdistr(OVC, "exponential")

AIC(GA)

BIC(GA)

GA



dpois(x, lambda, log = FALSE)

mean(datac$Glico_nc)

lambda=5749.5, log = FALSE)

dpois(datac$Glico_nc, lambda=5749.5, log = FALSE)

rpois(1000,lambda=5749.5)
```

```
as.matrix(datac$Glico_nc)

com.fit(dglico)


datac$Glico_nc

dglico<-matrix(datac$Glico_nc, nrow = 1, ncol = 8)

dsic<-matrix(datac$SIC_nc, nrow = 1, ncol = 8)

dstar<-matrix(datac$Star_nc, nrow = 1, ncol = 8)

ddonwell<-matrix(datac$Donewell_nc, nrow = 1, ncol = 8)

aa<-com.fit(dglico)

ab<-com.fit(dsic)

ac<-com.fit(dstar)

ad<-com.fit(ddonwell)

## fitdistr()included in package MASS

fitdistr(datac$Glico_C, "gamma")



x <- rgamma(100, shape = 5, rate = 0.1)

fitdistr(x, "gamma")

set.seed(123)

x3 <- rweibull(100, shape = 4, scale = 100)

fitdistr(x3, "weibull")


set.seed(123)

x4 <- rnegbin(500, mu = 5, theta = 4)
```

```
fitdistr(x4, "Negative Binomial")

options(op)

mean(x4)

poisson(datac$Glico_nc)


####

library(copula)

library(IndepTest)


cor.test(OVC, OV)

indepTest {copula}


indepTestSim(n, p, m = p, N = 1000, verbose = interactive())

indepTest(datac$Donewell_C, dp, alpha=0.05)

dependogram(test, pvalues = FALSE, print = FALSE)

ggraph-tools {copula}

pairwiseIndepTest(datac$Donewell_C,datac$Donewell_nc)

pairwiseIndepTest


###  copulas



## Case with an explicit formula: exponential claims and exponential

## interarrival times.
```

```
psi <- ruin(claims = "e", par.claims = list(rate = 5),
            wait   = "e", par.wait   = list(rate = 3))
psi
psi(0:10)
plot(psi, from = 0, to = 10)

ruin(claims = c("exponential", "Erlang", "phase-type"), par.claims,
     wait = c("exponential", "Erlang", "phase-type"), par.wait,
     premium.rate = 1, tol = sqrt(.Machine$double.eps),
     maxit = 200L, echo = FALSE)

# S3 method for ruin

plot(x, from = NULL, to = NULL, add = FALSE,
     xlab = "u", ylab = expression(psi(u)),
     main = "Probability of Ruin", xlim = NULL, …)
psi <- ruin(claims = "exponential",
            par.claims = list(rate = c(3, 7),weights = 0.5),
            wait = "exponential",
            par.wait = list(rate = 3),
            premium.rate = 1)

ruin::ruin_probability(claims = "exponential",
                       par.claims = list(rate = c(3, 7),weights = 0.5),
```

```
                wait = "exponential",
                par.wait = list(rate = 3),
                premium.rate = 1)

ruin_probability {ruin}
### INDEPENDENCY
model <- CramerLundberg(initial_capital = 0,
                premium_rate = 63969535.09375,
                claim_poisson_arrival_rate =,
                claim_size_generator = rexp,
                claim_size_parameters = list(rate = 0.000000223))
ruin_probability(model = model,
            time_horizon = 10,
            simulation_number = 4,
            return_paths = TRUE,
            parallel = FALSE)

pchisq(27.245, df=40, lower.tail=FALSE)
pchisq(25.4253, df=40, lower.tail=FALSE)
pchisq(11.245, df=10, lower.tail=FALSE)
pchisq(14.452, df=10, lower.tail=FALSE)
pchisq(12.243, df=10, lower.tail=FALSE)
pchisq(13.845, df=10, lower.tail=FALSE)
pchisq(11.452, df=10, lower.tail=FALSE)
```

```
pchisq(12.458, df=10, lower.tail=FALSE)

pchisq(12.652, df=10, lower.tail=FALSE)

pchisq(14.06714,df=7)
```