A STOCHASTIC ANALYSIS OF INVESTMENT PROSPECTS IN WEST AFRICA: A CASE OF GHANA AND NIGERIA

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A THESIS SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES, UNIVERSITY OF GHANA, LEGON, IN PARTIAL FULFILLMENT FOR THE AWARD OF THE MASTER OF PHILOSOPHY DEGREE IN ACTUARIAL SCIENCE

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DECLARATION

I, Martha Anane-Agyemang, do hereby declare that this submission is my own work towards the acquisition of a master’s degree in Actuarial Science and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the university, except where due and referencing have been made in the text, carried out under the able supervision of Dr. F.O Mettle and Dr. K. Doku-Amponsah.

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ABSTRACT

The main aim of this study is to analyse investment prospect in Ghana and Nigeria using rate of inflation, 90-day Treasury bill rate (interest rate) and local currency per US dollar exchange rate. The dataset span from January, 2003 to June, 2014. Stochastic time series analyses (ARIMA and ARCH-type) models were employed in the forecasting of the macroeconomic variables and simple investment accumulation was also employed in the accumulation of investment returns. The results revealed that forecasting rate of inflation would best be done using the ARIMA(1,1,2)–ARCH(2) model for Ghana and ARIMA(0,1,1)-ARCH(3) model for Nigeria. Treasury bill rate would best be modeled by ARIMA(1,1,2)-ARCH(1) for Ghana and ARIMA(0,1,1)-ARCH(3) for Nigeria. Again, forecasting exchange rate in Ghana was best done using the ARIMA(0,2,4)-ARCH(1) and that of Nigeria was ARIMA(0,1,1)-ARCH(1,1). The results also revealed an upward trend in the forecast of rate of inflation, interest rate and local currency per US dollar exchange rate for both Ghana and Nigeria. Furthermore, the results showed that investment in Ghana is worth more than investment in Nigeria since the accumulated value of investment return in Ghana grew faster than that of Nigeria for both the case of zero inflation and that of varying inflation in the two economies. Also based on the results, it would be prudent to spend return in Nigeria than in Ghana since the rate of inflation in Ghana grows faster than that of Nigeria and hence making cost of living in Ghana relatively higher than that of Nigeria.
DEDICATION

I dedicate this work to the Almighty God for His unconditional favor, mercies and directions. Secondly, to my parents, Mr. George Anane-Agyemang and Mrs. Beatrice Anane-Agyemang and my wonderful brother Samuel Kweku Anane-Agyemang for their support and immense contribution to coming this far in my education. Also to my late grandmother Madam Christiana Adjoa Yeboah whose upbringing has brought me this far.

Finally, this work is dedicated to all individuals in my life who have in one way or the other helped me in reaching this far in my education, I am most grateful.
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CHAPTER ONE: INTRODUCTION

1.1 Background of the study

A decade ago, in a famous cover depicting an image of chaos and war in Sierra Leone, the Economist Magazine scornfully wrote of Africa as ‘The hopeless Continent.’ In doing so, it reflected a view that was pervasive at the time that Africa was backward, condemned inexorably to violence, corruption and failure. “Since the economist regrettably labeled Africa ‘the hopeless continent’ a decade ago, a profound change has taken hold.” As the Economist has been gracious enough to admit, it was wrong. On the contrary, there has been a lot to be enormously hopeful about in Africa. From a peak of 16 wars raging across the Continent in 2002, today we are left with only one or two big conflicts, for example, in Somalia, which the AU is working hard to address. Around two thirds of governments in Africa are democratically elected, compared with just eight in 1991. Recently we have seen a return to democracy in Guinea Conakry, Niger, Cote d’Ivoire and Tunisia, and positive elections in Nigeria, Zambia and Egypt.

The World Bank and other indices show a steady improvement in the quality of governance across Africa. The spread of peace and good governance has unshackled Africa’s entrepreneurs. According to IMF World Economic Outlook figures, region-wide Gross Domestic Product (GDP) growth has averaged 5.5% from 2000 – 2010, more than double the rate in the 80’s and 90’s. During this period, six of the world’s fastest ten growing economies were African, and in eight of the past ten years, Africa has grown faster than East Asia. Moreover, Africa’s growth acceleration was widespread, with 27 of its 30 largest economies expanding more rapidly after the 2000, and also fairly inclusive, with the poorest seeing significant increases in real consumption. Steady progress has also been made in education, health, sanitation, and in
empowering women. Many daunting challenges remain, for example, in tackling poverty, low agricultural productivity and climate change.

The impressive growth momentum that Africa has built up over the last decade is not only expected to continue but also to trend upwards to around 6% in the coming years. The continent will be among the fastest-expanding economic regions in the world, with a long-run growth rate starting to resemble that of the rapid developers of East Asia, and will contain seven out of the world’s ten fastest growing economies over the next five years. But despite all the good news, companies have been slow to enter Africa. As spelt out by Ebrahim Ismail Ebrahim (2012) that some executives are no doubt still stuck in the old mindset.

The Mckinsey Global Institute (2010) set out to investigate into the reasons for the slow entry of companies into Africa and found, to its surprise, that Africa’s growth surge was broadly based and endogenous, with roots extending far beyond the global commodity super-cycle. Its excellent report, "Lions on the move: The progress and potential of African economies," notes that natural resources explain only a part of the African story, directly accounting for just about a quarter of GDP growth from 2000 through 2008, while other industries, particularly manufacturing and services, contributed the rest. In fact, GDP grew at similar rates in both countries with and without resources exports.

The key reasons behind Africa’s growth surge were government reforms that created greater political stability, improved the macroeconomic fundamentals and energized the business environment. Economies became healthier as policymakers reduced inflation, shrank budget deficits, lowered trade barriers, cut taxes, privatized companies, strengthened regulatory and legal systems and liberalized many sectors. Nigeria, for example, privatized more than 116 enterprises between 1999 and 2006. Together these changes helped fuel an African productivity
revolution. As a result, a dynamic African business sector is emerging. The continent now has more than 1,400 publicly listed companies and boasts more than 100 companies with revenue greater than $1 billion.

As Africa’s economies progress and household income climb, substantial new business opportunities are opening in sectors such as retail, construction, telecommunications, banking, infrastructure-related industries, resource-related businesses, and all along the agricultural value chain. Looking ahead, Mckinsey Global Institute (2010) projects that at least four groups of industries on the continent could together generate as much as $2.6 trillion in annual revenue by 2020, or $1 trillion more than today. The biggest business opportunity of the four lies in consumer goods and services, followed by natural resources, agriculture, and infrastructure. Africans spent $860 billion on goods and services in 2008—35% more than the $635 billion that Indians spent. If Africa maintains its current growth trajectory, consumers will buy $1.4 trillion worth of goods and services in 2020, making the prospects for consumer companies exceedingly bright. Africa is also expected to increase the value of its annual agricultural output by $600 billion by 2030, and its annual production of resources to $540 billion by 2020.

Foreign Direct Investment (FDI) inflows to Africa rose by 4 per cent to $57 billion, driven by international and regional market-seeking and infrastructure investments. Expectations for sustained growth of an emerging middle class economy attracted FDI in consumer-oriented industries, including food, IT, tourism, finance and retail (World Investment Report, 2014).

The overall increase was driven by the Eastern and Southern African sub regions, as others saw falling investments. Southern Africa flows almost doubled to $13 billion, mainly due to record-high flows to South Africa and Mozambique. In both countries, infrastructure was the main attraction, with investments in the gas sector in Mozambique also playing a role. In East Africa,
FDI increased by 15 per cent to $6.2 billion as a result of rising flows to Ethiopia and Kenya. Kenya is becoming a favourable business hub, not only for oil and gas exploration but also for manufacturing and transport. Ethiopian industrial strategy may attract Asian capital to develop its manufacturing base. FDI flows to North Africa decreased by 7 per cent to $15 billion.

Central and West Africa saw inflows decline to $8 billion and $14 billion, respectively, in part due to political and security uncertainties.

Intra-African investments are increasing, led by South African, Kenyan, and Nigerian. Between 2009 and 2013, the share of announced cross-border Greenfield investment projects originating from within Africa increased to 18 per cent, from less than 10 per cent in the preceding period. For many smaller, often landlocked or non-oil-exporting countries in Africa, intraregional FDI is a significant source of foreign capital.

Booming West African cities such as Lagos, Abuja and Accra have received a great deal of attention and FDI over the past decade in response to their population growth, economic growth, dynamism and promise. While we expect to see these cities continue as primary investment destinations, domestic and foreign investors alike are always looking for the next big thing (KPMG, 2014).

The above goes to suggest that Africa’s economies gain momentum, democratic governments take hold and the continent’s young population grows wealthier, investing here is becoming both easier and more attractive to international investors. The question now is which region retains the highest prospect for investors? Should investors invest in Ghana or Nigeria?
1.2 Problem statement

Africa’s domestic and foreign direct investment flow is on the increase. This is not surprising as most private capital moves towards countries and regions where there is a higher financial return and perceived safety. Investors detest regions where there are debt problems, governments are unstable, economic reforms are not strong and the risk of capital loss is deemed high. Thus in the quest to attract FDI, certain risk factors need to be considered. Significant among these risk is macroeconomic volatility, these include GDP volatility, inflation volatility, exchange rate volatility and interest rate volatility. These are basic risks that confront any investors’ decision to invest locally or overseas. The volatility of these variables refers to their short run deviations from their long run trends. The volatility and the risk of an investment may affect the expectations of market participants and thus have a feedback effect on the original series of interest. This volatility could be positive or negative but since investors may not be able to predict the direction of the volatility, they will prefer a stable variable. The volatility of these variables has the tendency to increase risk and uncertainty in trade and in effect discourage investment inflows. Theories of financial derivatives posit that risk and uncertainty in investment and trade can be minimized by hedging against these risks through the use of forward and futures contracts. However, such markets are non-existence in the sub region. This makes hedging against such risk impossible.

Numerous empirical researches has gone into assessing the impact of various macroeconomic variables such as exchange rate volatility, interest rate volatility and inflation rate on foreign direct investment in Africa ignoring the role of domestic investment. Tawiri (2010) identify the impact of domestic investment as a determinant of growth in the Libyan economy during the period (1962-2008) and conclude that domestic investment is expected to play an important role
in to stimulate economic growth rates that might be possible if government encourage more domestic private investment projects which should not be neglected at the expense of trend towards FDI.

It is in the light of the above that the study seeks to examine the prospects of investment in Africa (Ghana and Nigeria) by looking at the impact of Macroeconomic Volatility using the stochastic time series models.

1.3 Objectives of the study

There are different objectives to be addressed by this study that is the main and specific objectives.

1.3.1 Main objective

The main objective of this study is to analyse the prospects of investment in West Africa (Ghana and Nigeria) and identify the relative performance of investment in Ghana and Nigeria.

1.3.2 Specific Objectives

The specific objectives would be to:

(i) Fit time series models for each economy using rate of inflation, exchange rate, and interest rate (90-days Treasury bill rate) for Ghana and Nigeria

(ii) Determine future rate of inflation, exchange rate and interest rate in Ghana and Nigeria.

(iii) Determine the accumulated value of investment in Ghana and Nigeria

(iv) Identify the more prudent investing economy
1.4 Significance of the Study
The findings from the study will be of enormous benefit to:

✓ Government and Policy makers:

It is no doubt that this research will benefit government and policy makers very well. If investment prospect is known to impact positively on economic growth, policy makers and government will like to know the effects of macroeconomic volatility relationship since the relationship could serve as a key input in their economic policy formulation.

✓ Investors:

All Investors are much concerned about the factors that could have an adverse impact on their investment. Since uncertainty affects investment, investors would also like to know the regions where the effect of uncertainty is real through empirical studies. This research will basically explore the volatility in macroeconomic variables and how it affects investment in detail and that will be of tremendous benefit to many transnational corporations.

✓ Academicians:

This research will open a new chapter on investment prospects in Africa. It will open further discussions into the relationship between investment and macroeconomic volatility.

1.5 Scope and Methodology

The research is a case study in West Africa specifically Ghana and Nigeria. The data used in this research was secondary monthly dataset obtained from Bank of Ghana and Central Bank of Nigeria spanning from January, 2003 to June, 2014. The total data points used was 126. The macroeconomic variable used were rate of inflation, 90-day Treasury bill rate (interest rate) and local currency per US dollar exchange rate for both Nigeria and Ghana.
The data was analyzed using time series models, specifically ARIMA, ARCH and GARCH models. The maximum likelihood techniques was used in estimation the parameters of the suggested models. The modeling process includes testing for model’s assumption using the Ljung-Box test, model identification using ACF and PACF plot of the series, parameter estimation using the MLE method and finally diagnostic checking using histogram plot and normal Q-Q plot of the selected model’s residual.

The second phase of the data analysis was the optimal investment prospect selection. The simple accumulation method for investment returns with and without the effect of rate of inflation was used.

The statistical software used in this analysis was the R with packages like fBasics, timeSeries, tseries, fGarch, forecast, and MTS.

1.6 Organisation of the Study

Secondary data on rates of inflation, 90 day treasury bill rates and exchange rates will be gathered from the Bank of Ghana, Ghana Statistical Service and the Bank of Nigeria website.
CHAPTER TWO: LITERATURE REVIEW

2.0 Introduction

This chapter reviews relevant theories and concepts associated with investment and related works that has been carried out by other researchers on the topic. The chapter is divided into two main headings namely: Investment theories and Review of related works (foreign Direct Investment prospects, domestic investment prospects, domestic investment verses foreign direct investment and other related literature).

2.1 Empirical Literature

Owusu (2010) used the ARIMA models to model inflation and forecast the monthly inflation on short-term basis. The study used different ARIMA models to model inflation rates from 1990 – 2009. The period under consideration was split into two sub-periods: 1990 – 2000 and 2001 – 2009. The results showed that the best inflation model for the period of 1990 – 2000 was ARIMA (1, 2, 2) whilst that of the period 2001 – 2009 was ARIMA (2, 2, 1). Furthermore, the study concluded that the inflation for the period of January 2001 to December 2009 was less than that of January 1990 to December 2000.

In an attempt to analyse and forecast the macroeconomic impact of oil price fluctuations in Ghana using annual data from 2000 - 2011, Abledu and Agbodah (2012) focused on the feasibility forecast using nested conditional mean (ARIMA) and conditional variance (GARCH, EGARCH and GJR) family of models as the market conditions were volatile. The best model was the ARIMA (1, 1, 0) and it was used to predict the oil prices in Ghana National Petroleum Authority (GNPA) till the end of 2016.
Alnaa and Ahiakpor (2011) also used the ARIMA approach to predict inflation in Ghana. The monthly data from June 2000 to December 2010 was used and it was found that ARIMA(6,1,6) was the best fitted model for forecasting inflation in Ghana. Inflation was predicted highest for the months of March, April and May to be 8.95%, 10.07% and 10.24% respectively. The researchers recommended that the appropriate measures must be put in place to prevent inflation spiral from setting in motion. Since their model suggests that, inflation has a long memory and that once the inflation spiral is set in motion, it will take at least 12 periods (months) to bring it to a stable state.

Amos (2010) examined financial time series with special application to modelling inflation data for South Africa. The data spanned from January 1994 to December 2008. The study considered two families of time series, namely the autoregressive integrated moving averages (ARIMA) with extension to the Seasonal ARIMA (SARIMA) model and the autoregressive conditional Heteroscedastic (ARCH) with extensions to the generalized ARCH (GARCH) model. The study concluded that the SARIMA(1,1,0)x(0,1,1), was the best fitting model from the ARIMA family of models while the GARCH (1, 1) was chosen to be the best fit from the ARCH-GARCH models. Furthermore, a comparison of the two selected models based on the goodness of fit and the forecasting power of the two models was carried out. It was established that the GARCH (1, 1) model was superior to the SARIMA (1, 1, 0) x (0, 1, 1) model according to both criteria as the data was characterized by changing mean and variance.

Awogbemi and Oluwaseyi (2011) described the volatility in the consumer prices of some selected commodities in the Nigerian market. The researchers examined the presence or otherwise of the volatility in their prices using ARCH and GARCH models with monthly Consumer Price Index (CPI) of five selected commodities over a period of 1997 – 2007. The
results showed that ARCH and GARCH models are better models because they give lower values of AIC and BIC as compared to the conventional Box and Jenkins ARMA models. The researchers also observed that since volatility seems to persist in all the commodity items, people who expect a rise in the rate of inflation (the 'bullish crowd') will be highly favoured in the market of the said commodity items.

Lastrapes (1989) observed that ARCH provides a good description of the exchange rate process and that it is broadly consistent with exchange rates behaviour. Bollerslev (1990) however introduces a generalized ARCH (GARCH) process that allows for a more manageable lag structure. According to Bala and Asemota, 2013 the GARCH model has dominated the literature on volatility since the early 1980s. The model allows for persistence in conditional variance by imposing an autoregressive structure on squared errors of the process. The ARCH/GARCH literature had recently focused on analyzing volatility of high–frequency data and their benefits (see Engle, 2002; Andersen 2000).

Mckenzie and Mitchell (2002) state that the GARCH (1,1) model is preferred in case of symmetric reactions to the improvement of market. Likewise, Narayan et al. (2009) conducted a study using EGARCH and the findings showed that foreign exchange rate volatility is positively affected by conditional shock’s evidence.

Brooks and Burke (2003) suggested that the lag order GARCH (1,1) is effective enough to capture all the volatility clustering available in data. To account for asymmetrical and leverage effects, the extension of ARCH models, EGARCH, was introduced by Nelson (1991). Similarly, Akgiray (1989) also supported ARCH and GARCH models and praised the models capability while forecasting volatility in New York Stock Exchange.
Hsieh (1989) used a data of 10 years (1974 – 1983) of daily closing - bid prices, be made up of 2,510 observations, for five countries in comparison of US dollar to estimate the both the ARCH and GARCH models along with the other modified/altered types of ARCH and GARCH. The findings from the study proved that the two understudy models were capable of removing all heteroscedasticity in price changes, and in conclusion the standardized residuals from all the ARCH and GARCH models using the standard normal density were highly leptokurtic, whereas the standard GARCH (1,1) and EGARCH (1,1) were found to be more efficient for removing conditional heteroscedasticity from daily exchange rate movements. Overall the EGARCH proved to fit the data, better than GARCH model, using a variety of diagnostic checks.

Alam and Rahman (2012) explored the application of GARCH type models, for modeling the BDT/USD exchange rate using the daily foreign exchange rate series from July 03, 2006 to April 30, 2012 fixed up by Bangladesh Bank. The study was conducted benchmarking their results with AR and ARMA models. The outcome of the study (Alam and Rahman, 2012) showed that all GARCH type models demonstrate that past volatility of exchange rate significantly influence current volatility. Both the AR and ARMA models were found as the best model as per in-sample statistical performance results, whereas according to out-of-sample, GARCH model is the best model with transaction costs. Moreover, Both the ARMA and AR models are nominated as the best model as per in-sample statistical performance results, whereas according to out-of-sample, the TARCH model is nominated as the best model without transaction costs. The EGARCH and TARCH models outperform all the other models as per in-sample and out-of-sample trading performance outcomes respectively including transaction costs.

Mundaca (1991) modeled the NOK/US Dollar exchange rate through ARCH and GARCH models, the results of which supported that three out of four analyzed data series fitted better
through GARCH than the ARCH model. Further Johnston and Scott (2000) examined the British Pound, Canadian Dollar, German Mark and Japanese Yen against the US Dollar, for the years ranging 1978 to 1992, by applying the GARCH models. Though, the findings of Johnston and Scott (2000) identified that foreign exchange rate time variation were not the only reason of overall volatility but the fact that after removing the GARCH effect, the frequency distributions still showed the existence of independence. This puzzled the authors and forced them to GARCH family models with normality assumption that were unable to provide good description of exchange rate dynamics.

Kazantzis (2001) using information contents and predictive power of implied volatility models against the volatility estimates, based on six - year prices data from the currency options market for six different currencies, deduced that, implied volatility contained more information contents than measures based, where information embedded in past price history.

Hussain and Jalil (2007) applied the parametric and non - parametric techniques on daily exchange rate of Pak Rupee / US Dollar exchange rate and tried to measure the success of intervention in foreign exchange market in Pakistan, which was done either in shape of alteration in the exchange rate level or smoothing the exchange rate fluctuations. The GARCH results, as reported by Hussain and Jalil (2007) proved that intervention was successfully altered, in both direction of exchange rate and smoothed the fluctuations in exchange rate while the event study confirmed that the intervention was successful for level and volatility of the exchange rate.

Khalid (2008) analyzed the capability of existing exchange rate models by using the monthly data of 20 years of Pakistan, India and China and reported that for the developing economies, the model based on macroeconomic fundamentals perform better than the random walk model in both in and out sample.
Kamal et al. (2011) examined the performance of GARCH family models (including symmetric GARCH - M, asymmetric EGARCH and TARCH models) in forecasting the volatility behavior of Pakistani foreign exchange market by using daily foreign exchange rates data, ranging from January, 2001 to December, 2009. The findings of the study showed that the first order autoregressive behavior of the foreign exchange rate is evidenced in GARCH-M and E-GARCH, also proving that the GARCH-M model supports that previous day foreign exchange rate is affected by the current day exchange rate. The outcome also showed that the EGARCH model is the best in explaining the volatility behavior of the data, making all the coefficients of mean and variance equations significant. The TARCH model also supported the time series exchange rate, following the asymmetric behavior and depicts the presence of leverage effect in both the daily and monthly returns.

Bala and Asemota (2013) examined exchange–rate volatility with GARCH models using monthly exchange–rate return series from January 1985 to July 2011 for Naira/US dollar return and from January 2004 to July 2011 for Naira/British Pounds and Naira/Euro returns. The study compared estimates of variants of GARCH models with break in respect of the US dollar rates with exogenously determined break points. The outcome revealed presence of volatility in the three currencies and equally indicated that most of the asymmetric models rejected the existence of a leverage effect except for models with volatility break. Evaluating the models thereafter through standard information criteria, volatility persistence and the log likelihood statistic, showed that results improved with estimation of volatility models with breaks as against those of GARCH models without volatility breaks and that the introduction of volatility breaks reduces the level of persistence in most of the models.
Olowe (2009) used a number of GARCH models to investigate the volatility of Naira/US Dollar exchange rate with monthly data over the period January 1970 to December 2007, the hypothesis of leverage effect was rejected by all asymmetry models, though all the coefficients of the variance equations were significant, the TS-GARCH and APARCH models proved to be the best models. On the other hand, EGARCH model showed that in Nigerian foreign exchange market, with all variances being non-stationary, the volatility is highly persistence.

Appiah and Adetunde (2011) modeled the monthly exchange rate between the Ghana Cedi and the US Dollar and forecast future rates using time series analysis with ARIMA model on the monthly data collected from January, 1994 to December 2010. The result showed that the predicted rates were consistent with the depreciating trend of the observed series. ARIMA (1,1,1) model was found as the most suitable model with least Normalised Bayesian information Criterion (BIC) of 9.111, a forecast for two-year period from January, 2011 to December, 2012 was calculated which showed a depreciating of the Ghana Cedi against the US Dollar.

Bader and Malawi (2014) analyzed the impact of real interest rate on investment level in Jordan over a period (1990-2005). A cointegration analysis with three variables (investment level, real interest rate, and income level) was employed. Two unit root tests (Phillips-Perron test and Augmented DickeyFuller test) were exploited to check the integration order of the variables. The Johansen Cointegration test was mainly used. The study established that for the purpose of supporting the results, the dynamic relationships among the variables were explained through presenting variance decomposition and impulse responses. The results were found to be in line with the economic theory and some other studies in the sense that real interest rate has a negative impact on investment, where it is found that an increase in the real interest rate by 1% reduces the investment level by 44%. On the other hand, the income level has a positive impact.
2.2 Domestic Investment Prospects

Domestic investment is both a driver and an engine of growth in developed and developing countries. It is necessary to sustain growth, create employment and lay the foundation for poverty reduction. Over the past decade there has been a rapid increase in Africa’s needs for resources to finance the development of infrastructure and productive capacity, but domestic investment has not grown fast enough to match these needs. As a result, there is a wide and growing gap between Africa’s investment requirements and domestic resource availability. The decision made by African leaders, during the Summit of the African Union in January 2012, to fast-track the establishment of a continental free trade area with a view to boosting intra-African trade has also made the need for more domestic investment not only urgent but imperative. An expansion of intra-African trade requires investments in infrastructure and in building productive capacity for trade. Therefore, if African governments wish to enhance the likelihood of achieving the objective of boosting intra-African trade, they have to intensify efforts to increase domestic investment.

Over the past decade there has been a significant increase in domestic investment in Africa both in monetary terms and as a percentage of Gross Domestic Product (GDP). In 2010 domestic investment in Africa was about $353 billion compared to $100 billion in 2000. Furthermore, the share of domestic investment in GDP rose from about 17 per cent in 2000 to 21 per cent in 2010. While the increase in domestic investment in Africa is significant, it is worth noting that the share of investment in GDP in Africa is well below the investment share of other developing regions, in particular developing countries in Asia, where the share was about 35 per cent in 2010. In this regard, there is a need for African countries to increase their investment ratios to the
levels observed in rapidly growing emerging developing countries to enhance prospects for sustained economic growth.

Nevertheless, because of their need to narrow the financing gap, most countries in Africa continue to intensify their efforts to attract more FDI. But Africa currently attracts a relatively small share of global FDI and, more importantly, most FDI that goes to the continent is concentrated in a few countries, primarily the large and resource-rich economies (Anyanwu, 2012). Recent data indicate that Africa currently accounts for about 6 per cent of total FDI flows to developing countries (UNCTAD, 2013). Therefore, even when compared with other developing countries, Africa remains a marginal player in attracting global FDI. The attraction of Africa’s natural resources accounts for the bulk of FDI flows and for the uneven distribution of FDI in the continent. This fact also explains why, despite impressive growth trends, African countries have not made very effective use of FDI to support development, as evidenced by the fact that FDI has generated few linkages in African economies, and has not led to significant technology transfer as expected. One of the reasons for the low share in global FDI flows for Africa and the limited impact of FDI in the continent is the approach adopted by African countries in seeking and promoting FDI, which focuses more on providing generous incentives and less on creating a domestic environment conducive to entrepreneurship and business in general. The experience of the past few decades has shown that the most effective way to attract market-seeking or efficiency-seeking FDI is to have a dynamic and growing private sector and a policy environment attractive to both domestic and foreign investors.

2.3 Foreign Direct Investment Prospects

In 2013, FDI flows returned to an upward trend. Global FDI inflows rose by 9 per cent to $1.45 trillion in 2013. FDI inflows increased in all major economic groupings — developed,
developing, and transition economies. Global FDI stock rose by 9 per cent, reaching $25.5 trillion. UNCTAD projects that global FDI flows could rise to $1.6 trillion in 2014, $1.75 trillion in 2015 and $1.85 trillion in 2016. The rise will be mainly driven by investments in developed economies as their economic recovery starts to take hold and spread wider. The fragility in some emerging markets and risks related to policy uncertainty and regional conflict could still derail the expected upturn in FDI flows. As a result of higher expected FDI growth in developed countries, the regional distribution of FDI may tilt back towards the “traditional pattern” of a higher share of developed countries in global inflows.

Nevertheless, FDI flows to developing economies will remain at a high level in the coming years. FDI flows to developing economies reached a new high at $778 billion (table 1), accounting for 54 per cent of global inflows, although the growth rate slowed to 7 per cent, compared with an average growth rate over the past 10 years of 17 per cent. Developing Asia continues to be the region with the highest FDI inflows, significantly above the EU, traditionally the region with the highest share of global FDI. FDI inflows were up also in the other major developing regions, Africa (up 4 per cent) and Latin America and the Caribbean (up 6 per cent, excluding offshore financial centres).

Although FDI to developed economies resumed its recovery after the sharp fall in 2012, it remained at a historically low share of total global FDI flows (39 per cent), and still 57 per cent below its peak in 2007. Thus, developing countries maintained their lead over developed countries by a margin of more than $200 billion for the second year running. Developing countries and transition economies now also constitute half of the top 20 economies ranked by FDI inflows. Mexico moved into tenth place. China recorded its largest ever inflows and maintained its position as the second largest recipient in the world.
FDI by transnational corporations (TNCs) from developing countries reached $454 billion – another record high. Together with transition economies, they accounted for 39 per cent of global FDI outflows, compared with only 12 per cent at the beginning of the 2000s. Six developing and transition economies ranked among the 20 largest investors in the world in 2013. Increasingly, developing-country TNCs are acquiring foreign affiliates of developed-country TNCs in the developing world.

FDI inflows to Africa rose by 4 per cent to $57 billion, driven by international and regional market-seeking and infrastructure investments. Expectations for sustained growth of an emerging middle class attracted FDI in consumer-oriented industries, including food, IT, tourism, finance and retail (World Investment Report, 2014).

The overall increase was driven by the Eastern and Southern African sub regions, as others saw falling investments. In Southern Africa flows almost doubled to $13 billion, mainly due to record-high flows to South Africa and Mozambique. In both countries, infrastructure was the main attraction, with investments in the gas sector in Mozambique also playing a role. In East Africa, FDI increased by 15 per cent to $6.2 billion as a result of rising flows to Ethiopia and Kenya. Kenya is becoming a favoured business hub, not only for oil and gas exploration but also for manufacturing and transport; Ethiopian industrial strategy may attract Asian capital to develop its manufacturing base. FDI flows to North Africa decreased by 7 per cent to $15 billion. Central and West Africa saw inflows decline to $8 billion and $14 billion, respectively, in part due to political and security uncertainties.

Intra-African investments are increasing, led by South African, Kenyan, and Nigerian TNCs. Between 2009 and 2013, the share of announced cross-border greenfield investment projects originating from within Africa increased to 18 per cent, from less than 10 per cent in the
preceding period. For many smaller, often landlocked or non-oil-exporting countries in Africa, intraregional FDI is a significant source of foreign capital. Increasing intra-African FDI is in line with leaders’ efforts towards deeper regional integration. However, for most sub-regional groupings, intra-group FDI represent only a small share of intra-African flows. Only in two Regional Economic Cooperation (REC) initiatives does intra-group FDI make up a significant part of intra-African investments – in EAC (about half) and SADC (more than 90 per cent) – largely due to investments in neighbouring countries of the dominant outward investing economies in these RECs, South Africa and Kenya. RECs have thus so far been less effective for the promotion of intraregional investment than a wider African economic cooperation initiative could be.

Intra-African projects are concentrated in manufacturing and services. Only 3 per cent of the value of announced intraregional greenfield projects is in the extractive industries, compared with 24 per cent for extra-regional greenfield projects (during 2009-2013). Intraregional investment could contribute to the buildup of regional value chains. However, so far, African Global Value Chain (GVC) participation is still mostly limited to downstream incorporation of raw materials in the exports of developed countries.

2.4 Foreign Direct Investment and Domestic Investment

Foreign Direct Investment (FDI) can have a positive effect on domestic investment via spillovers (Borensztein et al., 1998). This is because it ‘pulls in’ other sources of investment. But this positive effect is a result of efficiency gains from FDI rather than higher induced levels of investment. Moreover, FDI “is an important vehicle for the transfer of technology, contributing relatively more to growth than domestic investments” (Borensztein et al., 1998). This is because
FDI is much more efficient than domestic investment (De Gregorio, 1992). Since many developing countries lack the access to international financial markets, the only way for them to achieve growth is through foreign capital inflows. Domestic investment can profit from FDI by seizing the opportunities of technological diffusion and thereby contributing to economic growth. Stagnation of developing countries is caused because those countries failed to seize such opportunities (De Long & Summers, 1991). With this it implies that FDI can have a positive effect on domestic investment.

The diffusion of technologies had a major role in the spillover process from foreign firms with superior knowledge to that of domestic firms (Findlay, 1978) and therefore plays a central role in the technological and economic progress of developing countries (Borensztein et al., 1998). The creation of knowledge is thereby mostly in hands of the multinational firm and this knowledge is spread amongst domestic firms as a result of spillover effects (Jovanovic & Rob, 1989). Here, domestic firms learn by doing and the knowledge base increases as the number of users grows. Differences between countries stem from the fact that such knowledge is unevenly distributed among the accession countries (Spence, 1984). As such knowledge spills over to domestic firms, they are more likely to imitate that knowledge than to invent new knowledge and technologies (Segerstrom, 1990). As imitation has relatively low costs, this implies that economies where imitation dominates over innovation will experience relatively fast growth rates (Barro & Sala-I-Martin, 1997). This is consistent with the convergence effect observed in the European Union (Kutan & Yigit, 2007). Not only will domestic firms in the accession countries profit from the imitation process, but also they can use the knowledge that is obtained for incremental improvements and thereby create superior products (Nelson & Phelps, 1991). Since such knowledge is available via spillovers, domestic firms do not have a disadvantage over the foreign
ones (Aghion & Howit, 1992). All in all, FDI can be beneficial for domestic firms as a tool for knowledge creation and acquisition.

The effect of FDI on domestic investment depends on conditional factors such as the economic policy of the recipient country, the type of FDI and the financial situation of the recipient country (Misum & Tomsik, 2002). Even in situations where FDI competes directly against foreign firms, the effect can be positive, especially in those sectors where production and innovation stagnate (Agosin & Mayer, 2000). As FDI can also cause negative effects, some countries pursued policies to protect their domestic capital, to limit foreign capital inflows and to stimulate domestic investment. But nurturing knowledge and technology effectively is very difficult for domestic governments especially in emerging economies. At the end, such policies resulted in very little domestic investment on the one hand, and in highly inefficient domestic firms who were not able to compete on the global market on the other hand (Wade, 1990). As a result, in many countries governments pursued policies not only to attract new technologies and knowledge from foreign firms, but also to enhance domestic savings by making foreign entry of banks more favourable (Lensink & Murinde, 2006). Domestic firms profit from such entry because they benefit from efficiency gains. Such gains facilitate the domestic savings, which in turn has a positive effect on domestic investments. This effect occurs only in the presence of a large share of foreign banks in total banking activity (Lensink & Murinde, 2006).

The accession countries needed the presence of foreign capital to stimulate their domestic investment.

As FDI enables technological spillovers but also disturbs domestic competition, the effect of FDI on domestic firms depends then on the balance between the technology and competition effect.
(Konings, 2001). Since in the accession countries, after the fall of the Soviet Union, financial markets as well as commercial banking were virtually absent, policy makers started to attract foreign investors in order to stimulate the privatization process. It was expected that foreign firms could stimulate the recipient country’s economy better than domestic firms if a certain threshold of technological spillover is achieved (positive technological effect). This is because foreign firms obtain higher levels of productivity and domestic firms can benefit from these by means of spillovers. As local firms engage in partnerships with foreign affiliates, they broaden the scope of technological and knowledge spillovers. As domestic firms have superior knowledge of local conditions, foreign firms as well as domestic firms benefit since technologies suitable for local markets have the highest effect on the host economy. The more technology that the foreign affiliates bring into the cooperation, the more beneficial spillovers can be observed (Blomstrom & Sjoholm, 1999).

FDI can also have a negative effect on domestic investment. As the accession countries are economically, socially and politically different from each other (Benhabib & Spiegel, 1994), one can expect different effects of FDI on domestic investment. For example, FDI crowded-in investment in Hungary and Czech Republic while it crowded out investment in Poland (Misum & Tomsik, 2002). Here, foreign firms have the ability to disturb the market equilibrium and may increase average costs of domestic firms, thereby pushing them out of the market (Kokko, 1996). In Bulgaria and Romania for example, foreign firms did not perform better than domestic firms. Also, the competition effect dominated over the technological effect, implying negative spillovers. Here, spillovers are not important as foreign investment is only positively related to small enterprises and even negatively related to the production of local firms. On balance, this
means that the net effect of FDI on domestic investment is quite small and slightly negative (Aitken & Harrison, 1999).

For Poland, foreign firms performed better than domestic ones. This result differs from other accession countries because Poland was at that time period in a more advanced economic state (Konings, 2001). So, the effect of FDI on domestic investment involves a trade-off between technological diffusion and competition. As becomes apparent, the positive role of FDI on domestic investment is questionable. Still, the impact of FDI on domestic investment is expected to be positive because of two reasons. First, given the absence of capital and market know-how in the accession countries, FDI is likely to enhance domestic investment in virtually every sector. This is consistent with the reasoning that the relationship between FDI and domestic investment is complementary when such investments are made in underdeveloped sectors of the economy (Agosin & Mayer, 2000). After the fall of the Soviet Union, no sector in the accession countries outperformed that of developed countries. Second, lack of capital, market know-how and managerial skills, made it practically impossible for domestic firms to compete on the global market without foreign interference. Therefore, in the absence of foreign capital, domestic production is unlikely to take place. By the inclusion of foreign firms in domestic markets, local firms learned by watching and doing. Moreover, these firms generated skills, capabilities and technologies which they could not obtain otherwise.

2.5 Other Related Literature

Hymer (1976) stated that a firm will invest in another country if only it can take advantage of those capabilities that firms in the host countries have refused to identify with the hope of
gaining higher returns on investment. The essence of this theory is that it enables firms that invest in overseas to control more markets and increase their firm’s productivity.

Using a panel regression for 22 countries in Sub-Saharan Africa from 1984-2000, Asiedu (2005) emphasized that macroeconomic stability, efficient institutions, political stability and a good regulatory framework have a positive impact on investment. Campa (1993) notes that lack of information in a volatile environment would deter investment. Basically these are macroeconomic issues. In any rational investment, most investors attempt to minimize the level of risk by avoiding investments that are associated with volatile returns. Literature has shown that the macroeconomic environment affects the level of a country’s productivity. In a study of MENA countries, Iqbal (2001) noted that maintaining macroeconomic stability has been an issue of great challenge to the countries in the region, which invariably is affecting the level of investment.

A major macroeconomic variable is the exchange rate. It is the rate at which one currency can be exchanged for another; it is known to be the value of one country’s currency as compared to that of another. Exchange rate could be fixed or floating. A fixed rate is where the state government set a rate and maintain the official exchange rate in the country, thus no currency can be exchanged above or below the official rates. On the other hand, a floating exchange rate is when a country’s exchange rate is determined by the forces of demand and supply. The rate could be higher or less when compared with that of other countries, the devaluation of a country’s currency makes it easier for foreigners to buy goods and services at a reduced price.

According to Barrell and Pain (1996), investors tend to postpone their investment when the currency in the targeted market strengthens. This occurs when they expect to benefit from the fall in the host country’s devaluation, suggesting that there seem to be a relationship between
investment and exchange rate. Erramilli and D’Souza (1995) noted that exchange rate volatility is one of the contributors toward external uncertainty in an economy that have a major effect on investment.

Aizenman and Marion (1996) noted that previous volatility has a tendency to reduce future investment. According to them, the uncertainty that comes from high volatility has a negative effect on economic outcomes of all kind. Economic volatility of all sorts has the tendency of lowering growth in developing countries (Ramsey & Ramsey, 1995).

Inflation, which is a measure of price stability, is also a determinant factor for investors. Price instability affects pricing levels of traders. Thus a host country’s economic instability can be a major deterrent to FDI inflow. Akinboade et al. (2006) noted that “low inflation is taken to be a sign of internal economic stability in the host country. High inflation indicates the inability of the government to balance its budget and the failure of the central bank to conduct appropriate monetary policy.” Studies have commented on the fact that the cost of inflation can have prominent effect on the economy’s growth, a high or low inflation has its own consequence for any economy. Lipsey and Chrystal (2006) define hyperinflation as “Inflation so rapid that money ceases to be useful as a medium of exchange and a store of value.”

However, they also concede that countries with inflation rate higher than 50%, to some 200% plus, have proven to be manageable as the population adjusts in “real term”. It goes to suggest that inflation relatively affects growth and thus the attraction of investors.

Hayami, 2001, Todaro and Smith, 2003 suggested that the contributions that foreign investors brings to the development of a country varies and are widely known to include filling the gap between desired investment and domestically mobilized saving, increasing the tax revenues, and improving management, technology, as well as labour skills in host countries. The effect is that
these could help the country to break the vicious cycle of under development whiles improving the lives of its people

In a recent study by the World Investment Report (2002) on the attraction of FDI, the report noted that most countries seeking to attract investors have diverted or moved away from the first generation idea of investment which involved a country opening up of its economy to investors to the second generation of investment, which on the other hand involves a government marketing its itself through its location by setting up investment promotion agencies to attract investors. According to WIR (2002), investment promotion agencies exist to help investors deal with regulatory and administrative requirements, change the perception of investors about the host countries by participating and organizing investor fairs and seminars.

Aside promotions to attract investors, most countries have resorted to use incentives to win investors into their countries. These Incentives can be described as policies used by various governments to attract internationally mobile investors. These incentives can be grouped into three main groups, namely fiscal Incentives (reduced tax rates, tax holidays and rebates, subsidies, import duty exemptions, etc), Financial Incentives (grants, loans, guarantees, etc) and Rules-based incentives (protection of workers’ right, environmental standards, etc).

Jauch and Endresen (2000) however noted that as to whether incentives are important means of attracting investors is a matter of varied opinions.

Despite these incentives, Africa continues to receive the least amount of FDI as stated earlier on. This misery of Africa and investment can be attributed to a lot of reasons. According to the UNCTAD (1999), the most common reason as to why Africa does not attract much FDI, is the image that Africa is an unfavourable location. Many investors view the continent as one depleted with civil wars, political unrest and unfavourable economic policies. According to United
Nations Conferences and Development (UNCTAD), the bad publicity the African continent gets has played a big role in discouraging foreign investors from investing. CUTS (2002) also noted some variables that hindered investment on the continent. These variables were noted to include market size, lack of policies, lack of profit opportunities, inconsistent setup, negative perceptions, shortage of skills, labour regulations, poor infrastructure and corruption. Other notably variables were extortion, bribery, and the lack of access to global markets.

The delay in getting the requisite permission to get business started is also a determinant of investment in Africa. Since business and returns on investment are time related, the delay in getting business started is important to investors (mostly known as beaucracy). Emery et al (2000) noted the following as factors that affect starting business in time when it comes to Africa. These consist of delays beyond the necessary for approval or signatures, Lack of computerization or lack of capacity in registration or regulatory applications, high costs caused by the requirements for company formation and up-front capital taxes.

For investors who might prefer to inject fund into Africa through the capital market, most financial markets in Africa offer a scanty range of financial products, the financial systems are also not automated or sophisticated. Again most of the markets in African are known to be inefficient and therefore their inability to attract the needed international capital.

According to Ndikumana (2003), most of the stock markets on the continent are not active while those that are a bit active are also small and ill-liquid. Moss (2003) noted that some African stock markets have legal limits on the amount of equity owned by non-resident foreigners. The implication is that there is a cut on the amount that can come in as foreign investment through the capital market.
The banking sector also seems to be a contributing factor in FDI attraction. Bank lending rates are relatively high and mostly short term. In most cases private investors are crowded out when they are found to compete with the government for loans. This hinders business growth and firm expansion. The banking system is still seemed to be a little inefficient with regards to credit allocation and loan repayment which has led to high incidence of non-performing loans. As Gelbard and Leite (1999) noted that some countries have made good progress in improving and modernizing their financial systems over the last decade, particularly with regard to financial liberalization and the adoption of indirect instruments of monetary policy. In many countries, however, the range of financial products remains extremely limited, interest rate spreads are wide, capital adequacy ratios are insufficient, judicial loan recovery is a problem, and the share of non-performing loans is large. Most loans total an average of 20% are recorded as non-performing in a study that comprised over 38 countries in SSA. They further noted that most banks do not issue credit cards while inter-bank lending still remains under-developed. The concerning is that these systems of banking will invariable affect private enterprise borrowing.

Assefa, Bienen and Ciuriak (2013) examined the statistical record and drawn on interviews with companies with experience doing business in Ethiopia to compile a critical investment prospects picture. The study revealed that the macroeconomic overview identifies a number of positive features in Ethiopia’s macroeconomic performance. These include strong growth, based on an increasingly diversified economy, stable non-food price inflation, increasing exports to a diversified range of markets, an improved trade balance, and generally stable economic policies and a solid investor protection framework. The study also revealed that high and volatile
headline inflation rate and negative real interest rate generate diverse macroeconomic management challenges and constrain savings growth needed to fuel investment.

Akanbi (2010) examined the pattern of domestic investment that is consistent with a neoclassical supply-side model in Nigerian economy. The estimations were carried out with time series data from 1970 to 2006 using the Johansen estimation techniques. The study revealed that real output, user cost of capital and the level of financial development are significant determinants of investment. It also revealed that a well-structured and stable socio-economic environment will boost investment over the long run.

Udoh and Egwaikhide (2008) examined the effect of exchange rate volatility and inflation uncertainty on foreign direct investment in Nigeria. The investigation covers the period between 1970 and 2005. Exchange rate volatility and inflation uncertainty were estimated using the GARCH model. Estimation results indicated that exchange rate volatility and inflation uncertainty exerted significant negative effect on foreign direct investment during the period. In addition, the results show that infrastructural development, appropriate size of the government sector and international competitiveness are crucial determinants of FDI inflow to the country.

Brafu-Insaidoo and Beikpe (2011) examined the impact of foreign capital flow on investment volatility in emerging and frontier market economies in Sub-Saharan Africa. The study used the dynamic panel analysis (GMM) to test that increased capital flow increases investment volatility. The study revealed that international capital flows reduce investment volatility. It also revealed that domestic financial markets, inflation volatility and political climate are other determinants of investment volatility.

Razin and Rose (1994) however, argued that the impact of increased capital mobility on investment volatility is also deter- mined by the nature of the underlying productivity shocks. If
shocks are persistent and country-specific, increase capital mobility would heighten investment volatility. Conversely, when shocks are common across countries, the impact of increased capital mobility on investment volatility would be ambiguous. When shocks to productivity are transitory and common across countries, the easing of restrictions on cross-border capital flows would not affect investment spending, because of the resultant changes to international interest rates. Razin and Rose showed that the impact of transitory shocks on investment behavior is marginal, because a non-persistent shock does not lead to a significant change in the expected discounted sum of future profits. In the case of an irreversible investment, a transitory shock may not have any impact on investment.

Soleymani and Akbari (2011) in their paper examined the relationship between exchange rate uncertainty and domestic investment by using the fixed effect approach of panel data model. The results of the theoretical part show that there is nonlinear relationship between these two variables, it means exchange rate uncertainty and investment. The study was based on fifteen countries of the Sub-Saharan African countries and the GARCH (1,1) approach was used to obtain the uncertainty of exchange rate for every country. The results of the estimation show that there is a negative relation between exchange rate uncertainty and investment and the share of investment from growth of GDP in these countries, is very small. In addition, results revealed that the investment in these countries is very sensitive to exchange rate uncertainty not only in current period of time but also about the exchange rate uncertainty in the other periods.

Tawiri (2010) identified the impact of domestic investment as a determinant of growth in the Libyan economy during the period (1962-2008). The study was analyzed using Cobb-Douglas Function to examine the relationship between real per-capita GDP and its most important determinants as described in Cobb-Douglas function. Properties of time series of the model
variables were analyzed by using several tests for determining the integration level of each time series separately. By using Johansen approach, the results showed the significance of the impact of investment on per-capita GDP, the results of tests revealed equilibrium relationship between per-capita GDP and its determinants in the long and short-run. The study concluded that the elasticity of per-capita GDP to changes in domestic investment is greater than the elasticity of labour force which appeared inelastic in the short and long-term.

Another factor that leads to higher investment volatility is macroeconomic instability, usually measured in the context of inflation volatility. Literature, including Agosin & Mayer (2000) & Grenade (2004), indicate that high and volatile inflation increases the uncertainty of investments and heightens risk of long-term investments.

According to Calderon and Schmidt-Hebbel (2008), one of the mechanisms by which agents diversify risk and smooth shocks is accessing credit from the domestic financial market. Deep financial markets make credit available for direct investments and offer investors with funds needed to meet their short and long term needs. Studies, including Denizer et al. (2002) and Easterly et al. (2001), identified deep financial markets to lead to lower macroeconomic volatility. Mileva (2008) also confirm the hypothesis that domestic financial depth increases the rate of investment.

Goldberg (1993) and Darby et al. (1999) found evidence that exchange rate uncertainty can have significant negative long run effects on investment. There are some studies in which has been done by recent theoretical work identifying several channels through which uncertainty can impact on investment, under various assumptions about risk aversion, adjustment costs to investment and other factors (Caballero 1991; Abel & Eberly, 1994). However, some of these effects of uncertainty operate in mutually opposing directions and their magnitude on a variety of
factors identified in the literature. As a result, the sign of this relationship is ambiguous on theoretical grounds. It means, textbook theories of investment under uncertainty present a rather oversimplified rule for a firm deciding whether to invest or not.

Harchaoui et al. (2005) shows that exchange rate can also affect investment through the price of imported investment via adjustment cost. Depreciation causes an increase of investment price, resulting to higher adjustment costs and lower investment. Overall, it is important to note that the global impact of exchange rate on investment is not obvious because it depends on which of these pervious effects prevail and the value of elasticity of demands.

2.6 Summary of Literature Review

Various researchers have dive into modeling rate of inflation, exchange rate and treasury bill rate in Africa and the world at large using stochastic time series models (ARIMA, ARCH and GARCH-type models).

The World Bank and other indices show a steady improvement in the quality of governance across Africa. The spread of peace and good governance has unshackled Africa’s entrepreneurs. According to IMF World Economic Organisation figures, region-wide GDP growth has averaged 5.5% from 2000 – 2010, more than double the rate in the 80’s and 90’s. During this period, six of the world’s fastest ten growing economies were African, and in eight of the past ten years, Africa has grown faster than East Asia. Moreover, Africa's growth acceleration was widespread, with 27 of its 30 largest economies expanding more rapidly after 2000, and also fairly inclusive, with the poorest seeing significant increases in real consumption. Steady progress has also been made in education, health, sanitation, and in empowering women. Many daunting challenges remain, for example, in tackling poverty, low agricultural productivity and climate change.
Booming West African cities such as Lagos, Abuja and Accra have received a great deal of attention and FDI over the past decade in response to their population growth, economic growth, dynamism and promise. While we expect to see these cities continue as primary investment destinations, domestic and foreign investors alike are always looking for the next big thing (KPMG, 2014).

The above goes to suggest that Africa’s economies gain momentum, democratic governments take hold and the continent’s young population grows wealthier, investing here is becoming both easier and more attractive to international investors. The question now is which region retains the highest prospect for investors? Should investors invest in Ghana or Nigeria?
CHAPTER THREE: METHODOLOGY

3.0 Introduction

This chapter entails data collection and sources of data, description of variables, the methods and procedures to be employed in analyzing investment prospects in Africa. The research design will be discussed; the population of the study and sources of data for the research will be looked at. Consequently, this chapter is also concerned with description of time series and the stochastic volatility models that will aid in unveiling the prospect of investment and investment decision making in Africa (Ghana and Nigeria).

3.1 Data collection and Source

Dataset consisting of monthly observations of inflation rate sourced from the Ghana Statistical Service and Nigeria Statistical Service, and government Treasury bill rate and exchange rate (per US dollar) sourced from Bank of Ghana and Central Bank of Nigeria website.

The Government Treasury bill rate was used as interest rate since savers or investors usually invest their savings for higher interest with certainty.

3.2 Description of Variables

A brief description of the variables will be presented in the sub-sections below:

3.2.1 Interest Rates

Interest rate is the monthly effective rate payment on borrowed money. If the person is a creditor, this will be received. It is expressed as the percentage of the borrowed sum. In modern financial theory, interest rates and their determinants are probably the most computationally difficult part.
Although it is hard to compute, the interest rates provide valuable information to economists. The three most important reasons explaining the significance of interest rates include:

- Fixed income market is very sensitive to interest rate changes
- They are used in pricing all other market securities for time discounting
- Most investment decisions are based on some expectations regarding alternative opportunities and the cost of capital; both depend on the interest rates.

### 3.2.2 Inflation Rate

According to Webster (2000), inflation is the persistent and continuous rise in the levels of the consumer prices in an economy. Inflation can also be seen as the persistent decline in the purchasing power of money. That is, inflation means that your money can not buy today as much as what it could have bought yesterday.

The consumer price index is a measure for capturing changes over time (monthly, quarterly, yearly) in the general price level of goods and services. This is determined at the beginning of a period called the base period and according to a fixed pattern of consumption called weight assigned to a representative sampled basket of goods and services. The consumer price index is then used to calculate the inflation rate.

A stable macroeconomic environment reduces cost of investment and leads to the attraction of greater investments. Sneider and Frey (1985) found that high inflation is a disincentive for investment by both local and foreign firms. Monthly inflation rates for both countries will be used.
3.2.3 Exchange Rate

Real exchange rate refers to the exchange rate determined by national authorities or to the rate determined in the legally sanctioned exchange market. It is calculated as an annual average based on monthly averages (local currency units relative to the U.S. dollar). Exchange rate is expected to positively affect investment prospect. The traditional elasticity approach is of the view that a devaluation of any country’s currency will improve trade balances as well as the Marshall Lerner condition is satisfied. The implication of the devaluation is that more Multinational Enterprises will find it attractive to do business in countries with a devalued currency.

3.3 Description of Time Series

Chatfield (2000) defines time series as a series or sequence of data points measured typically at successive times. The data points are commonly spaced in equal times. Time series analysis comprises methods that attempt to understand the underlying generation process of the data points and constructs a mathematical model to represent the process. The constructed model is then used to forecast future events based on known past events. Time series often makes use of the natural one-way ordering of time in that values in a series for a given time will be expressed as being derived from past values rather than future values. A time series model usually reflects on the fact that observations close together in time domain are more correlated as compared to observations further apart. That is, there is ‘volatility clusters’- small (large) shocks are again followed by small (large) shocks.

Original time series data are made up of various patterns that are derived on casual factors which are identified by time series analysis methods. The four patterns that characterize economic and business series are the long-run development known as the trend, cyclical or periodic component, seasonal component and the error or residual component. The trend component deals with the
general and overall pattern of the time series; the cyclical component refers to the variation in the series which arise out of the phenomenon of business cycles. It usually spans within periods of more than one year. The seasonal variations refers to the periodic and repetitive ups and downs that occur in the series within a year and lastly the error term is the component that contains all moments which neither belong to the trend nor to the cycle nor to the seasonal component.

The models for time series data can have many forms and represents different stochastic processes which could be linear or non-linear. Among the linear models include autoregressive (AR) model of order p, Moving Average (MA) of order q, and autoregressive moving average (ARMA) model of order \((p,q)\). A combination of the above models produce the autoregressive integrated moving average (ARIMA) model ordered \((p,d,q)\) with a generalized model known as the autoregressive fractionally integrated moving average (ARFIMA) model.

The non-linear time series model represent or reflect the changes of variance along with time known as heteroscedasticity. With these models, changes in variability are related to and/or predicted by recent past values of the observed series. The wide variety of non-linear models includes the symmetric models such as Autoregressive Conditional Heteroscedastic (ARCH) model with order \((p)\) and Generalized ARCH (GARCH) model with order \((p,q)\). Other asymmetric models are the Power ARCH(PARCH), Threshold GARCH (TGARCH), Exponential GARCH (EGARCH), Integrated GARCH (IGARCH), etc. All these asymmetric models have order \((p,q)\)

The ARIMA, ARCH and GARCH models would be considered in the analysis of the datasets in this thesis.
3.4 Stationarity of Time Series Models

An important fundamental concept of time series analysis is stationarity. A time series \( \{ r_t \} \) is said to be strictly stationary if the joint distribution of \( (r_{t1}, \ldots, r_{tk}) \) is identical to that of \( (r_{t1+t}, \ldots, r_{tk+t}) \) for all \( t \), where \( k \) is an arbitrary positive integer and \( (r_{t1}, \ldots, r_{tk}) \) is a collection of \( k \) positive integers. In other words, strict stationarity requires that the joint distribution of \( (r_{t1}, ..., r_{tk}) \) is invariant under time shift. This is a very strong condition that is hard to verify empirically. A weaker version of stationarity is often assumed. A time series \( \{ r_t \} \) is weakly stationary if both the mean of \( r_t \) and the covariance between \( r_t \) and \( r_{t-l} \) are time-invariant, where \( l \) is an arbitrary integer. More specifically, \( \{ r_t \} \) is weakly stationary if

(a) \( E(r_t) = \mu \), which is a constant, and

(b) \( Cov (r_t, r_{t-l}) = \gamma_l \), which only depends on \( l \).

In practice, suppose we have observed \( T \) data points \( \{ r_t | t = 1, ..., T \} \). The weak stationarity implies that the time plot of the data would show that the \( T \) values fluctuate with constant variation around a fixed level. In applications, weak stationarity enables one to make inferences concerning future observations.

3.5 Autoregressive (AR) model

Most financial time series data has a statistically significant lag-1 autocorrelation which indicates that the lagged return, \( r_{t-1} \) might be useful in predicting return on asset \( (r_t) \).

\[
r_t = \varnothing_0 + \varnothing_1 r_{t-1} + a_t, \tag{3.01}
\]

where \( \{ a_t \} \) is assumed to be a white noise series with mean zero and variance \( \sigma_a^2 \).

Here it suffices to note that an AR(1) model implies that, conditional on the past return \( r_{t-1} \), we have expectation,

\[
E(r_t | r_{t-1}) = \varnothing_0 + \varnothing_1 r_{t-1} \tag{3.02}
\]
and variance

\[ V\text{ar}(r_t|r_{t-1}) = V\text{ar}(a_t) = \sigma_a^2 \]  

(3.03)

That is, given the past return \( r_{t-1} \), the current return is centered around \( \phi_0 + \phi_1 r_{t-1} \) with standard deviation \( \sigma^2 \). This is a Markov property such that conditional on \( r_{t-1} \), the return \( r_t \) is not correlated with \( r_{t-i} \) for \( i > 1 \). Obviously, there are situations in which \( r_{t-1} \) alone cannot determine the conditional expectation of \( r_t \) and a more flexible model must be sought. A straightforward generalization of the AR(1) model is the AR(p) model

\[ r_t = \phi_0 + \phi_1 r_t + \phi_2 r_{t-1} + \cdots + \phi_p r_{t-p} + a_t \]  

(3.04)

### 3.6 Moving Average (MA) Model

A time series is influenced by random shocks in noisy environment. As a result current value of series is affected by the random shocks appeared in previous values. Moving average terms are used to capture the influence of previous random shocks in the future value. First order moving average or MA (1) is a simple time series, given by

\[ Y_t = z + z_t + a_z_t \]  

(3.05)

This equation says, apart from mean \( z \), \( Y_1 \) is a weighed average of \( z_2 \) and \( z_1 \), \( Y_2 \) is the weighted average of \( z_3 \) and \( z_2 \) and so on. The value of \( Y_t \) is defined in terms of random shocks \( z_t \).

### 3.7 Autoregressive Moving Average (ARMA) Model

An Autoregressive Moving Average (ARMA) model combines the ideas of AR and MA models into a compact form so that the number of parameters used is kept small and the concept of ARMA models is highly relevant in volatility modeling.

A time series \( r_t \) follows an ARMA (1,1) model if it satisfies

\[ r_t - \phi_1 r_{t-1} = \phi_0 + a_t - \theta_1 a_{t-1} \]  

(3.06)
where \{a_t\} is a white noise series. The left-hand side of equation 3.08 is the AR component of the model and the right-hand side gives the MA component. The constant term is \(\theta_0\). For this model to be meaningful, we need \(\theta_1 \neq \theta_1\); otherwise, there is a cancellation in the equation and the process reduces to a white noise series.

A general ARMA(p,q) model is in the form

\[
 r_t = \theta_0 + \sum_{i=1}^{p} \theta_i r_{t-i} + a_t - \sum_{i=1}^{q} \theta_i a_{t-i} 
\]

where \{a_t\} is a white noise series and \(p\) and \(q\) are non-negative integers. The AR and MA models are special cases of the ARMA(p,q) model. We require that there are no common factors between the AR and MA polynomials; otherwise the order (p,q) of the model can be reduced.

Like a pure AR model, the AR polynomial introduces the characteristic equation of an ARMA model. If all of the solutions of the characteristic equation are less than 1 in absolute value, then the ARMA model is weakly stationary. In this case, the unconditional mean of the model is

\[
 E(r_t) = \frac{\theta_0}{1-\theta_1-\ldots-\theta_p} 
\]

3.7.1 Order Determination of ARMA Model

Unlike the AR and MA models which use the Autocorrelative factor (ACF) and Partial Autocorrelative Factor (PACF) in determining the order, ARMA does not. Tsay and Tiao (1984) propose a new approach that uses the Extended Auto-Correlation Function (EACF) to specify the order of an ARMA process. The basic idea of EACF is relatively simple. If we can obtain a consistent estimate of the AR component of an ARMA model, then we can derive the MA component. From the derived MA series, we can use the ACF to identify the order of the MA component. Derivation of the EACF is relatively involved. Yet the function is easy to use. The output of the EACF is a two-way table, where the rows correspond to AR order \(p\) and the
columns to MA order q. In general, for an ARMA(p,q) model, the triangle of O’s will have its upper left vertex at the (p,q) position.

### 3.8 Auto Regressive Integrated Moving Average Model (ARIMA Model)

The ARMA model assumes that the time series data is stationary (that is statistical properties of data do not change over time). But the real data are not stationary in nature. Time series data is made stationary by differencing process. The first order differencing process of time series $X_t$ is defined by $X_t^1 = X_t - X_{t-1}$

ARMA time series which is made stationary by differencing process is known as Integrated Autoregressive Moving Average (ARIMA) model. ARIMA model is represented by three parameters: p order of autoregressive model, d order of differencing, and q order of moving average model. ARIMA model takes historical data and decomposes that data into an autoregressive (AR) process which maintains memory of past events, an Integrated (I) process which makes data stationary for easy forecast and a Moving Average (MA) process of forecast errors. It does not suffer from existence of serial correlation between the error residuals and their own lagged values.

An ARIMA (p, d, q) model can be checked if it is a good statistical fit for data or not using Akaike Information Criterion (AIC).

#### 3.8.1 Model Identification

Determine whether the series is stationary or not by considering the graph of ACF. If a graph of ACF of the time series values either cuts off fairly quickly or dies down fairly quickly, then the time series values should be considered stationary. If a graph of ACF dies down extremely slowly, then the time series values should be considered
non-stationary. If the series is not stationary, it can often be converted to a stationary series by differencing. That is, the original series is replaced by a series of differences. An ARMA model is then specified for the differenced series. Differencing is done until a plot of the data indicates the series varies about a fixed level, and the graph of ACF either cuts off fairly quickly or dies down fairly quickly. The theory of time-series analysis has developed a specific language and a set of linear operators. A highly useful operator in time-series theory is the lag or backward linear operator (B) defined by

\[ BY_t = Y_{t-1} \]

Model for non-seasonal series are called Autoregressive integrated moving average model, denoted by ARIMA (p, d, q). Here p indicates the order of the autoregressive part, d indicates the amount of differencing, and q indicates the order of the moving average part. If the original series is stationary, d = 0 and the ARIMA moving average model, denoted by ARIMA (p, d, q). Here p indicates the order of the autoregressive part, d indicates the amount of differencing, and q indicates the order of the moving average part. If the original series is stationary, d = 0 and the ARIMA models reduce to the ARMA models. The difference linear operator (\( \Delta \)) defined by ,

\[ \Delta Y_t = Y_t - Y_{t-1} = Y_t - BY_t = (1 - B)Y_t \]

The stationary series \( W_t \) obtained as the \( dt \)h difference \( (\Delta^d) \) of \( Y_t \)

### 3.9 Testing for ARCH Effect

Let \( a_t = r_t - \mu_t \) be the residuals of the mean equation, where \( \mu_t \) is the mean returns. The squared series \( a_t^2 \) is then used to check for conditional heteroscedasticity, which is also known as the ARCH effects. The test for conditional heteroscedasticity is the Lagrange multiplier test of
Engle (1982). This test is equivalent to the usual F statistic for testing \( a_t = 0 \), \((i = 1, \ldots, p)\) in the linear regression:

\[
a_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_p a_{t-p}^2 + e_t ,
\]

where \( t = p+1, \ldots, T \), \( e_t \) denotes the error term, \( p \) is a specified positive integer and \( T \) is the sample size. Specifically, the null hypothesis is \( H_0: \alpha_1 = \cdots = \alpha_p = 0 \).

Let \( SSR_0 = \sum_{t=p+1}^{T} (a_t^2 - \bar{\omega})^2 \) \((3.10)\)

where \( \bar{\omega} = \frac{1}{T} \sum_{t=1}^{T} a_t^2 \) is the sample mean of \( a_t^2 \), and

\[
SSR_1 = \sum_{t=p+1}^{T} \hat{e}_t^2 \quad (3.11)
\]

where \( \hat{e}_t^2 \) is the least squares residual of the prior linear regression. Then we have

\[
F = \frac{(SSR_0 - SSR_1)/p}{SSR_1/(T-2p-1)} \quad (3.12)
\]

which is asymptotically distributed as a chi-squared distribution with \( m \) degrees of freedom under the null hypothesis. The decision rule is to reject the null hypothesis if \( F > \chi^2_p(\alpha) \), where \( \chi^2_p(\alpha) \) is the upper 100\((1-\alpha)\)th percentile of \( \chi^2_p \), or the \( p \)-value of \( F \) is less than \( \alpha \).

### 3.10 Autoregressive Conditional Heteroskedasticity (ARCH) Model

The first model that provides a systematic framework for volatility modeling is the ARCH model of Engle (1982). The basic idea of ARCH models is that

(a) the shock of an asset return is serially uncorrelated, but dependent, and

(b) the dependence of \( a_t \) can be described by a simple quadratic function of its lagged values.

Specifically, an ARCH\((p)\) model assumes that

\[
a_t = \sigma_t \epsilon_t , \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_p a_{t-p}^2 ,
\]
where \( \{\varepsilon_t\} \) is a sequence of independent and identically distributed (iid) random variables with mean zero and variance 1, \( \alpha_0 > 0 \), and \( \alpha_i \geq 0 \) for \( i > 0 \). The coefficients \( \alpha_i \) must satisfy some regularity conditions to ensure that the unconditional variance of \( a_t \) is finite. In practice, \( \varepsilon_t \) is often assumed to follow the standard normal or a standardized Student-t distribution or a generalized error distribution. From the structure of the model, it is seen that large past squared shocks \( \{a_{t-i}^2\}_{i=1}^{p} \) imply a large conditional variance \( \sigma_t^2 \) for the innovation \( a_t \). Consequently, \( a_t \) tends to assume a large value (in modulus). This means that, under the ARCH framework, large shocks tend to be followed by another large shock.

The conditional expectation of the ARCH(p) model is given as

\[
E(a_t) = E(E(a_t|F_t)) = E(\sigma_t E(\varepsilon_t)) = 0
\]

(3.14)

\[
Var(a_t) = E(a_t^2) = E(E(a_t^2|F_{t-1})) = \alpha_0 + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 ,
\]

(3.15)

where \( F_t \) is the information set at time \( t \).

### 3.10.1 Estimation of the ARCH Model

Three likelihood functions are commonly used in ARCH estimation. Under the normality assumption, the likelihood function of an ARCH(m) model is

\[
f(a_1, ..., a_T | \alpha) = f(a_T | F_{T-1}) f(a_{T-1} | F_{T-2}) ... f(a_{p+1} | F_p) f(a_1, ..., a_p | \alpha)
\]

\[
= \prod_{t=p+1}^{T} \frac{1}{\sqrt{2\pi \sigma_t^2}} \exp \left( -\frac{a_t^2}{2\sigma_t^2} \right) \times f(a_1, ..., a_p | \alpha),
\]

(3.16)

Where \( \alpha = (\alpha_0, \alpha_1, ..., \alpha_p)' \) and \( f(a_1, ..., a_p | \alpha) \) is the joint probability density function of \( a_1, ..., a_p \). Since the exact form of \( f(a_1, ..., a_p | \alpha) \) is complicated, it is commonly dropped from the prior likelihood function, especially when the sample size is sufficiently large. This results in using the conditional likelihood function.
\[
f(a_{p+1}, \ldots, a_T \setminus \alpha, a_1, \ldots, a_p) = \prod_{t=p+1}^{T} \frac{1}{\sqrt{2\pi \sigma_t^2}} \exp \left( -\frac{a_t^2}{2\sigma_t^2} \right) \quad (3.17)
\]

where \(\sigma_t^2\) can be evaluated recursively. We refer to estimates obtained by maximizing the prior likelihood function as the conditional maximum likelihood estimates (MLEs) under normality. Maximizing the conditional likelihood function is equivalent to maximizing its logarithm, which is easier to handle. The conditional log likelihood function is

\[
l(a_{p+1}, \ldots, a_T \setminus \alpha, a_1, \ldots, a_p) = \sum_{t=p+1}^{T} \left( -\frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln(\sigma_t^2) - \frac{1}{2} \frac{a_t^2}{\sigma_t^2} \right) \quad (3.18)
\]

Since the first term \(\ln(2\pi)\) does not involve any parameters, the log likelihood function becomes

\[
l(a_{p+1}, \ldots, a_T \setminus \alpha, a_1, \ldots, a_p) = -\sum_{t=p+1}^{T} \left( \frac{1}{2} \ln(\sigma_t^2) + \frac{1}{2} \frac{a_t^2}{\sigma_t^2} \right) \quad , \quad (3.19)
\]

where \(\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_p a_{t-p}^2\) can be evaluated recursively.

### 3.10.2 Order Determination of ARCH Model

If an ARCH effect is found to be significant, one can use the Partial Autocorrelation Function (PACF) of \(a_t^2\) to determine the ARCH order. Using PACF of \(a_t^2\) to select the ARCH order can be justified as follows. From the ARCH model, we have

\[
\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_p a_{t-p}^2 .
\]

For a given sample, \(a_t^2\) is an unbiased estimate of \(\sigma_t^2\). Therefore, we expect that \(a_t^2\) is linearly related to \(a_{t-1}^2, \ldots, a_{t-p}^2\) in a manner similar to that of an autoregressive model of order \(m\). Note that a single \(a_t^2\) is generally not an efficient estimate of \(\sigma_t^2\), but it can serve as an approximation that could be informative in specifying the order \(m\).

### 3.10.3 Model Adequacy Checking

For a properly specified ARCH model, the standardized residuals

\[
\tilde{a}_t = \frac{a_t}{\sigma_t} \quad (3.20)
\]
form a sequence of independently and identically distributed random variables. Therefore, one can check the adequacy of a fitted ARCH model by examining the series \( \{ \tilde{a}_t \} \). In particular, the Ljung–Box statistics of \( \tilde{a}_t \) can be used to check the adequacy of the mean equation and that of \( \tilde{a}_t^2 \) can be used to test the validity of the volatility equation. The skewness, kurtosis, and quantile-to-quantile plot (i.e., QQ-plot) of \( \{ \tilde{a}_t \} \) can be used to check the validity of the distribution assumption.

### 3.10.4 Forecasting with ARCH(p)

Forecasts of the ARCH model in equation (3.15) can be obtained recursively. Consider an ARCH(p) model. At the forecast origin \( h \), the 1-step ahead forecast of \( \sigma_{h+1}^2 \) is

\[
\sigma_h^2(1) = \alpha_0 + \alpha_1 \tilde{a}_h^2 + \cdots + \alpha_m \tilde{a}_{h+1-m}^2
\]  
(3.21)

The 2-step ahead forecast is

\[
\sigma_h^2(2) = \alpha_0 + \alpha_1 \sigma_h^2(1) + \alpha_2 \tilde{a}_h^2 + \cdots + \alpha_m \tilde{a}_{h+2-m}^2
\]  
(3.22)

and the \( l \)-step ahead forecast for

\[
\sigma_h^2(l) = \alpha_0 + \sum_{i=1}^{m} \alpha_i \sigma_h^2(l - i)
\]  
(3.23)

where \( \sigma_h^2(l - i) = \tilde{a}_{h+l-i}^2, \quad l - i \leq 0 \)

### 3.11 Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model

Although the ARCH model is simple, it often requires many parameters to adequately describe the volatility process of an asset return. Bollerslev (1986) proposes a useful extension known as the generalized ARCH (GARCH) model.

For a log return series \( r_t \), where \( \mu_t \) is the mean returns let

\[
a_t = r_t - \mu_t
\]  
(3.24)

be the innovation at time \( t \). Then \( a_t \) follows a GARCH(p,q) model if
\[ a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \] (3.25)

where again \( \epsilon_t \) is a sequence of iid random variables with mean 0 and variance 1.0, \( \alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0, \) and

\[ \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_j) < 1 \] (3.26)

Here it is understood that \( \alpha_i = 0 \) for \( i > p \) and \( \beta_j = 0 \) for \( j > q \). The latter constraint on \( \alpha_i + \beta_j \) implies that the unconditional variance of \( a_t \) is finite, where as its conditional variance \( \sigma_t^2 \) evolves over time. As before, \( t \) is often assumed to be a standard normal or standardized t distribution or generalized error distribution. Equation (3.23) reduces to a pure ARCH(p) model if \( q = 0 \). The \( \alpha_i \) and \( \beta_j \) denote the ARCH and GARCH parameters, respectively.

### 3.11.1 Estimation of GARCH Model

Once the orders \( p \) and \( q \) have been identified, the parameters of the GARCH (\( p,q \)) model can then be estimated. The maximum likelihood estimation is used to estimate the parameters of the model. The initial values of both the squared returns and past conditional variances are needed in estimating the parameters of the model. Bollerslev (1986) and Tsay (2002) suggest that the unconditional variance or the past sample variance of the returns may be used as initial values. Therefore assuming the orders are known, the conditional log-likelihood is given by

\[ l(a_{p+1}, \ldots, a_T \mid \alpha, a_1, \ldots, a_p) = \sum_{t=p+1}^{T} (-\frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln(\sigma_t^2) - \frac{1}{2} a_t^2) \] (3.27)

It follows that the conditional maximum likelihood estimates are obtained by maximizing the conditional log-likelihood function above.

### 3.11.2 Akaike Information Criterion (AIC)

The Akaike Information Criterion (AIC) was introduced by Hirotogu Akaike in 1973. It was the first model selection criterion to gain widespread acceptance. The AIC was an extension to the
maximum principle and consequently the maximum likelihood principle is applied to estimate
the parameters of the model once the structure of the model has been specified. The AIC is
defined as

\[ \text{AIC} = 2(N) - 2(\text{log likelihood}) \]  

(3.28)

where \( N \) denotes the number of parameters in the model.

Given a family of competing models of various structures, the maximum likelihood estimation is
used to fit the model and the AIC is computed based on each model fit.

The selection of the most appropriate model is then made by considering the model with the
minimum AIC. Akaike’s idea was to combine estimation and structural determination into a
single procedure. The first term of the AIC in equation (3.26) measures the goodness of fit of the
model whereas the second term is called the penalty function of the criterion since it penalizes a
candidate model by the number of parameters used.

### 3.12 Augmented Dickey-Fuller t-test (ADF)

The Augmented Dickey-Fuller t-test is used to test whether a time series requires differencing in
order to be stationary or not. A series with a trend in it and is potentially slow turning around a
trend line would use the test equation:

\[ \Delta r_t = \alpha_0 + \theta r_{t-1} + \gamma_t + \alpha_1 \Delta r_{t-1} + \alpha_2 \Delta r_{t-2} + \cdots + \alpha_p \Delta r_{t-p} + \alpha_t \]  

(3.29)

The number of augmented lags (\( p \)) is determined by minimizing the AIC or lags one dropped
until the last lag is statistically significant. The null hypothesis of the ADF t-test is the data needs
to be differenced to make it stationary whereas the alternative hypothesis is the data is trend
stationary.
3.13 Optimal Investment Portfolio Selection

Let \( a(t) \) be the accumulated value at time \( t \) for an investment of 1 made at time 0, then the effective rate of interest \( R_t \), in year \( t \) is given by:

\[
R_t = \frac{a(t) - a(t-1)}{a(t-1)} \tag{3.30}
\]

The accumulated value \( a(t) \), of an investment at time \( t \) is also given by:

\[
a(t) = \prod_{j=1}^{t} (1 + R_j) \tag{3.31}
\]

To account for the effect of inflation on the interest rate and for that matter future investment returns, the deflated accumulated values would be considered by calculating the true interest. The true interest rate or interest rate \( r_t \), after considering future inflation rate is given by:

\[
r_t = R_t - I_t - R_t I_t \tag{3.32}
\]

The accumulated value of investment for the deflated series at time \( t \) is given by:

\[
a(t) = \prod_{j=1}^{t} (1 + r_j) \tag{3.33}
\]
CHAPTER FOUR: DATA ANALYSIS AND DISCUSSION OF RESULTS

4.1 Modeling Rate of Inflation in Ghana

This section entails modeling and forecasting monthly rate of inflation in Ghana with time series models. It includes the model identification, estimation and twelve month out sample forecast. Monthly rate of inflation data spanning from January, 2003 to June, 2014 was used in the estimation and analysis.

4.1.1 Preliminary Analysis

The plot of monthly rate of inflation for the period January, 2003 to June, 2014 is depicted in Figure 4.1.1 below. From Figure 4.1.1a, it is observed that the monthly rate of inflation series was characterized by a non-constant mean and an unstable variance and hence making the assumption of stationarity in time series modeling questionable. This was confirmed by the Augmented Dickey-Fuller (ADF) test which gave a p-value of 0.03705 leading to the failure to reject the null hypothesis of non stationarity at 1% level of significance.

Figure 4.1.1: Plot of monthly rate of Inflation and its differenced series in Ghana (January, 2003 – June, 2013)
A differencing transformation was carried out on the non-stationery series to bring it to stationarity. The first ordinary differencing which entails subtracting every subsequent 1 step lag value from the original series was used. The ordinary differencing was used because it helps in stabilizing the variance of the time series without exponential growth.

Figure 4.1.1b gives the plot of the first differenced monthly rate of inflation for the period January, 2003 to June, 2014. The plot in Figure 4.1.1b appeared to be stationary; the series fluctuate around a common mean and variance. The Augmented Dickey-Fuller (ADF) test for stationarity of the first differenced series gave a p-value of less than 0.01 which led to the rejection of the null hypothesis that the differenced series is not stationary at 1% level of significance. It was therefore concluded that the differenced series was stationary.

The autocorrelation factor (ACF) and the partial autocorrelation factor (PACF) of the monthly rate of inflation were plotted to test for serial correlation in both series and again to determine the nature of time series model and order of the model. Figure 4.1.2 represents the plot of the ACF and the PACF of the monthly rate of inflation and differenced series in Ghana for the period January, 2003 to December, 2013.

The ACF of the original series from Figure 4.1.2 showed a sine wave pattern which also confirmed that differencing of the series was necessary and hence an autoregression integrated moving average (ARIMA) model might be appropriate for the series. The PACF and the ACF of the differenced series suggested AR(1) and MA(2) respectively. This was confirmed by using the auto arima function in R to fit an appropriate model for the series. The “auto arima” in R software package “forecast” which selected an appropriate order of an arima model by
comparing the Akiake Information Criteria (AIC) of the various arima(p,d,q) models and log-likelihood and selecting the model order with the lowest AIC with highest log-likelihood.

Figure 4.1.2: Plot of the ACF and Partial ACF of the monthly Inflation rate in Ghana and its differenced series (January, 2003 to December, 2013)

4.1.2 Model Estimation and Fitting

The auto arima in R confirmed ARIMA(1,1,2) as the best linear model fit for the differenced monthly rate of inflation in Ghana. The specification of the model is therefore depicted as:

\[ r_t - r_{t-1} = \alpha_1(r_{t-1} - r_{t-2}) - \theta_1 e_{t-1} - \theta_2 e_{t-2} + \epsilon_t \]  

Assuming normality of the residuals of the model, the maximum likelihood estimation (MLE) method was used to estimate the parameters of the model. The fitted ARIMA(1,1,2) model was therefore specified as:
\[ r_t - r_{t-1} = 0.7376(r_{t-1} - r_{t-2})(s.e = 0.2562) + 0.4428e_{t-1}(s.e = 0.2733) + 0.1473e_{t-2}(s.e = 0.1421) \]  
(4.1.1a)

The residuals of the ARIMA (1,1,2) model were examined to determine the presence of ARCH effect. The Lagrange Multiplier test was perform on the squared residuals of the ARIMA (1,1,2) model with some selected lags and results shown in Table 4.1.1 in the appendix. The resultant p-values of the test indicates that there was ARCH effect in the residuals series of the monthly rate of inflation in Ghana and therefore an ARCH model could be used to model monthly rate of inflation in Ghana.

It was therefore necessary to the appropriate order of the ARCH(p) model that could model the differenced series of monthly rate of inflation in Ghana. The statistical software R was used to determine the Log-likelihood and the AIC of some selected ARCH(p) models and the results of the fitting depicted in Table 4.1.2

<table>
<thead>
<tr>
<th>Lag</th>
<th>Log-Likelihood</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH(1)</td>
<td>-203.7153</td>
<td>411.4307</td>
</tr>
<tr>
<td>ARCH(2)</td>
<td>-229.9862</td>
<td>465.9725</td>
</tr>
<tr>
<td>ARCH(3)</td>
<td>-231.3062</td>
<td>470.6124</td>
</tr>
</tbody>
</table>

Results from the table showed that ARCH(1) model was the most appropriate fit for the differenced series. The ARCH(1) model is specified as:

\[ a_t = \sigma_t \epsilon_t \]  
(4.1.2)

\[ \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 \]  
(4.1.2)
Assuming normality of the residuals of the selected model, the parameters of the model were estimated using the maximum likelihood estimation (MLE) method. The fitted ARCH(1) model was therefore specified as:

\[ a_t = \sigma_t \epsilon_t \]  \hspace{2cm} (4.1.2a)

\[ \sigma_t^2 = 0.17312(s.e 0.03866) + 3.5407a_{t-1}^2(s.e 0.40801) \]  \hspace{2cm} (4.1.2a)

Again, the log-likelihood and AIC of some selected ARCH(p) models were computed using the residuals of the ARIMA(1,1,2) model to ascertain an appropriate ARIMA-ARCH model for the series and results shown in Table 4.1.3 in the appendix. It was evident from the results that ARCH(2) model gives appropriate specification with the highest log-likelihood of -232.3061 and lowest AIC of 474.6121.

Assuming normality of the residuals of the ARCH(2) model, the parameters of the model were estimated using the maximum likelihood equation (MLE). The estimated model is therefore:

\[ a_t = \sigma_t \epsilon_t \]  \hspace{2cm} (4.1.3)

\[ \sigma_t^2 = 0.47924(s.e 0.04598) + 1.02221a_{t-1}^2(s.e 0.27231) + 0.29667a_{t-2}^2(s.e 0.13080) \]  \hspace{2cm} (4.1.3)

4.1.3 Diagnostic Checking of the Model

A model check for goodness of fit for ARCH(1) model was carried out using the Ljung Box chi square test and results of the test gave a p-value of 0.3207 which confirmed the failure to reject the null hypothesis of goodness of fit at 5% level of significance. To check for model normality assumption of the model’s residuals, the plot of the residuals with its histogram and normal q-q plot were examined.
Figure 4.1.3: Plot of Residuals of ARCH(1) model with its histogram and Normal Q-Q plot (January, 2009 – December, 2013)

The results of histogram plot of the residuals from figure 4.1.3 did support significantly the normality assumption of the residuals of the ARCH model in estimation.

The Ljung-Box test for goodness of fit gave a p-value of the ARIMA(1,1,2)-ARCH(2) models was 0.8642 signifying goodness of fit. The results from the time plot of the residuals, histogram plot of the residuals and normal q-q plot in Figure 4.1.4 in the appendix also shows compliance with the normality assumption in model parameter estimation.

4.1.4 Forecasting Performance

The forecasting power of the selected ARCH(1) and ARIMA(1,1,2)-ARCH(2) model were examined with various error measure like the Mean error (ME), Random mean square error (RMSE) and Mean absolute error (MAE). The model with the least error was considered to be the best fit for forecasting monthly rate of inflation in Ghana. Table 4.1.4 presents the results of
the comparison of forecasting accuracy of the selected fitted models. Based on the results in Table 4.1.4, it was evident that the ARIMA(1,1,2)-ARCH(2) model is the best fit for the series and it was therefore concluded that the ARIMA(1,1,2)-ARCH(2) would be appropriate model for modeling and forecasting monthly rate of inflation in Ghana.

<table>
<thead>
<tr>
<th></th>
<th>ARCH(1)</th>
<th>ARIMA(1,1,2)-ARCH(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean Error (ME)</strong></td>
<td>0.00016</td>
<td>-2.04429</td>
</tr>
<tr>
<td><strong>Root Mean Squared Error (RMSE)</strong></td>
<td>1.77535</td>
<td>3.79947</td>
</tr>
<tr>
<td><strong>Mean Absolute Error (MAE)</strong></td>
<td>0.9315</td>
<td>2.15223</td>
</tr>
</tbody>
</table>

It was therefore concluded that the most appropriate model for forecasting monthly rate of inflation in Ghana is the ARIMA(1,1,2)-ARCH(2) model and it is depicted as:

\[
r_t - r_{t-1} = 0.7376(r_{t-1} - r_{t-2}) + 0.4428e_{t-1} + 0.1473e_{t-2} + 0.47924 + 1.02221e_{t-1}^2 + 0.29667e_{t-2}^2
\]  

(4.1.4)

4.2 Modeling 90-Day Treasury Bill Rate in Ghana

This section entails modeling and forecasting monthly 90-day Treasury bill rate in Ghana with time series models. It includes the model identification, estimation and twelve month out sample forecast. Monthly Government of Ghana 90-day Treasury bill rate data spanning from January, 2003 to June, 2014 was used in the estimation and analysis.

4.2.1 Preliminary Analysis

The plot of the monthly 90-Day Government of Ghana Treasury bill rate from January, 2003 to June, 2014 is depicted by Figure 4.2.1. It was evident that the plot in figure 4.2.1a was characterized by non-constant mean and unstable variance and hence the series is said to be non-
stationary. This was confirmed by the ADF test which gave a p-value of 0.02646 leading to the rejection of the null hypothesis of stationarity at 1% level of significance.

![Figure 4.2.1a Monthly 90-Day T-bill Rates In Ghana](image)

![Figure 4.2.1b Diff of Monthly T-bill Rates In Ghana](image)

**Figure 4.2.1: Plot of monthly 90-Day Treasury bill rate and its differenced series in Ghana (January, 2003 – June, 2014)**

The ordinary differencing transformation of the series was carried out and results of first ordinary difference of the monthly Treasury bill rates were plotted as shown in Figure 4.2.1b. The ordinary differencing was used because the data does not show any form of exponential growth and seasonal variation.

Figure 4.2.1b shows clearly that the assumption of stationarity is much more reasonable. This was confirmed by the ADF test for stationarity which gave a p-value of less than 0.01 for both trend and level stationarity leading to the rejection of the null hypothesis that the differenced series is not stationary.
The ACF and PACF functions of the series were then plotted to determine the nature of model that would fit the series and possibly the order of the model. Figure 4.2.2 in the appendix shows the plot of the ACF and PACF of the series. The slow decay in the ACF proved that the series requires differencing and therefore an ARIMA(p,d,q) might be appropriate for the series. The PACF and ACF of the differenced series suggest AR(1) and MA(2) respectively and hence it was suggested that ARIMA(1,1,2) will be the order of the model for modeling 90-Day Treasury bill rate in Ghana.

### 4.2.2 Model Estimation and Fitting

The auto arima function in R forecast package was used to confirm the order of the model. The function selected ARIMA(1,1,2) model with lowest AIC of 490.92 and log-likelihood of -241.46 as the appropriate model for the differenced series. The model ARIMA(1,1,2) specification for modeling treasury bill rate in Ghana is given by:

\[
\begin{align*}
    r_t - r_{t-1} &= \alpha_1 (r_{t-1} - r_{t-2}) - \theta_1 e_{t-1} - \theta_2 e_{t-2} + \epsilon_t \\
    \text{(4.2.1)}
\end{align*}
\]

On assumption of normality of the residuals of the model, the MLE method was used to estimate the parameters of the model which led to the specification of the fitted model as:

\[
\begin{align*}
    r_t - r_{t-1} &= 0.6777 (r_{t-1} - r_{t-2})(s.e 0.158) + 0.461 e_{t-1}(s.e 0.1696) - 0.0581 e_{t-2}(s.e 0.1243) \\
    \text{(4.2.1a)}
\end{align*}
\]

The residuals of the ARIMA(1,1,2) were assess to determine the presence of ARCH effect using the LM test. The result of the LM test based on some selected lags is shown in Table 4.2.1 in the appendix. It could be concluded that the there was significant evidence to fail to reject the null hypothesis of no ARCH effect at 5% level of significant. It was there necessary to model the volatility pattern in the series using ARCH model.
The log-likelihood and AIC of some selected ARCH(p) were compared based on monthly differenced series of Treasury Bill Rate in Ghana. It was evident from Table 4.2.2 in the appendix that ARCH(1) had the lowest AIC and highest log-likelihood and therefore conclude that ARCH(1) is the appropriate ARCH fit of the series.

Assuming normality of the residuals of the ARCH(1) model, the parameters of the model was estimated using the MLE method and the fitted model is depicted as:

\[ a_t = \sigma_t \epsilon_t \] (4.2.2)

\[ \sigma_t^2 = 0.3981(e 0.0281) + 1.4144a_{t-1}^2(s.e 0.3175) \] (4.2.2)

Again, the log-likelihood and AIC of selected ARCH(p) models were compared based on the residuals of the ARIMA(1,1,2) model to obtain an ARIMA-ARCH specification for the series. Results from Table 4.2.3 in the appendix shows that ARCH(1) had the lowest AIC of 354.1021 and log-likelihood of -185.4517. It was therefore concluded that the best ARIMA-ARCH fit for the series was ARIMA(1,1,2)-ARCH(1). The fitted model is therefore depicted as:

\[ a_t = \sigma_t \epsilon_t \] (4.2.3)

\[ \sigma_t^2 = 0.31837(e 0.02049) + 1.44113a_{t-1}^2(s.e 0.25838) \] (4.2.3)

4.2.3 Diagnostic Checking of the Models

A check for goodness of fit of the ARCH(1) models revealed that the model fit the series best with a significant Ljung-Box chi square test with p-value of 0.9086 which led to the failure to reject the null hypothesis of goodness of fit at 5% level of confidence. The normality assumption of the model was checked by the histogram plot of the residuals as depicted in Figure 4.2.3 in the appendix. The residual plot exhibit volatility clustering with bell shaped histogram plot. It was therefore concluded that the ARCH(1) model was appropriate model for the series.
The residuals of the ARIMA(1,1,2)-ARCH(1) model was examined to ascertain whether it follows the normality assumption made in estimating the parameters of the model. From Figure 4.2.4 in appendix, it was evident that the histogram plot of the residuals was meet the assumption of normality of the residuals. It was therefore concluded that the ARIMA(1,1,2)-ARCH(1) model for the modeling monthly Treasury bill rate did satisfy all the assumptions of the model with a significant goodness of fit p-value of 0.8515.

4.2.4 Forecasting Performance

The forecasting errors of the ARCH(1) and ARIMA(1,1,2)-ARCH(1) models were compared to ascertain which model can forecast with minimal error. The ME, RSME and MAE of the selected models are shown in table 4.2.4.

<table>
<thead>
<tr>
<th></th>
<th>ARCH(1)</th>
<th>ARIMA(1,1,2)-ARCH(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Error (ME)</td>
<td>-1.419694</td>
<td>-1.267956</td>
</tr>
<tr>
<td>Root Mean Squared Error (RMSE)</td>
<td>2.758372</td>
<td>2.60487</td>
</tr>
<tr>
<td>Mean Absolute Error (MAE)</td>
<td>1.63239</td>
<td>1.473909</td>
</tr>
</tbody>
</table>

The error measures for the ARIMA(1,1,2)-ARCH(1) model was minimal at all error levels as compared to that of the ARCH(1) model. It was therefore concluded that the ARIMA(1,1,2)-ARCH(1) model is the most appropriate model for forecasting monthly 90-day treasury bill rate in Ghana based on data from January, 2003 to June, 2014. The fitted model is depicted as:

\[ r_t - r_{t-1} = 0.6777(r_{t-1} - r_{t-2}) + 0.461e_{t-1} - 0.0581e_{t-2} + 0.51364 + 1.27358e_{t-1}^2 \] (4.2.4)
4.3 Modeling Cedi per US dollar Exchange Rate in Ghana

This section entails modeling and forecasting monthly cedi per US dollar exchange rate in Ghana with time series models. It includes the model identification, estimation and twelve month out sample forecast. Monthly exchange rate data spanning from January, 2003 to June, 2014 was used in the estimation and analysis.

4.3.1 Preliminary Analysis

Figure 4.3.1.1 below shows the plot of monthly exchange rate in Ghana for the period January, 2003 to June, 2014. The plot in figure 4.3.1a is characterized by unstable mean and variance. This was confirmed by the smaller ADF unit root test p-value of 0.972 which led to the failure to reject the null hypothesis of non-stationarity at 1% level of significance.

![Figure 4.3.1.1: Plot of monthly exchange rate and its differenced Logarithm series in Ghana (January, 2003 – June, 2014)](image)

For the smoothing of the geometric growth in the plot of exchange rate series, the natural logarithm differencing approach was used. The first differenced logarithm series was also characterized by non-constant mean and unstable variance. This was confirmed by the ADF test which also gave a p-value (p=0.08935) greater than the level of significance of 1%. The plot of
the second differenced logarithm exchange rate series in figure 4.3.1.1b revealed stationarity of the differenced series. This was confirmed by the significant ADF test for unit root p-value of less than 0.01 which led to the rejection of the null hypothesis of non-stationarity.

The ACF and the PACF were used to determine the order of the model appropriate for forecasting exchange rate in Ghana as shown in figure 4.3.2 in the appendix. The sine wave pattern of the ACF of the original series confirmed that differencing of the series was necessary. The PACF and ACF plot of the second differenced series suggest ARIMA(0,2,4) as the order of the model. This was confirmed by the auto arima function in R on the logarithm series.

4.3.2 Model Estimation and Fitting

The specified ARIMA model with the minimum AIC and maximum log-likelihood based on the auto arima function was ARIMA(0,2,4) with the specification:

$$ r_t = 2r_{t-1} + r_{t-2} - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} - \theta_4 e_{t-4} $$ (4.3.1)

The MLE method based on the assumption of normality of the residuals of the ARIMA(0,2,4) model yielded the fitted model as:

$$ r_t = 2r_{t-1} + r_{t-2} + 0.3176e_{t-1}(s.e 0.0849) + 0.0856e_{t-2}(s.e 0.0875) + 0.3037e_{t-3}(s.e 0.0999) + 0.263e_{t-4}(s.e 0.0984) $$ (4.3.1a)

The squared residuals of the ARIMA(0,2,4) were examined to check for the presence of ARCH effect in the residuals using the LM test. The resultant p-values based on some selected lags were shown in Table 4.3.1 in the appendix. It was evident from Table 4.3.1 in the appendix that there was enough evidence to reject the null hypothesis of no ARCH effect at 5% level of significance for the selected lags.
The log-likelihood and AIC of some selected ARCH(p) models were compared based on the differenced monthly exchange rate data and results shown in Table 4.3.2 in the appendix. From Table 4.3.2, it was evident that ARCH(1) will model the data best with minimal AIC of -789.1702 and log-likelihood of 396.5851. The MLE estimation technique was used to estimate the parameters of the model upon assumption of normality of the model residuals. The fitted model depicted as:

\[ a_t = \sigma_t \epsilon_t \]  
\[ \sigma_t^2 = 0.01969(s.e \ 0.01365) + 0.81927a_{t-1}^2(s.e \ 0.56028) \]  

Again, the AIC and log-likelihood of some selected ARCH(p) models were compared using the residuals of the ARIMA(0,2,4) and results shown in Table 4.3.3 in the appendix. It was evident from Table 4.3.3 that ARCH(1) fit the residuals series best. On assumption of normality of the model’s residuals, the MLE estimation was used to estimate the parameters of the model. The fitted ARCH(1) model is therefore depicted as:

\[ a_t = \sigma_t \epsilon_t \]  
\[ \sigma_t^2 = 0.06685(s.e \ 0.01581) + 0.47052a_{t-1}^2(s.e \ 0.44734) \]

### 4.3.3 Diagnostic Checking of the Model

The residuals of the ARIMA(0,2,4)-ARCH(1) were examined to confirm the assumptions made in the estimation of the parameters of the model. The plot of the residuals and the histogram plot with the normal q-q plot did confirm the normality assumption of the model as shown in figure 4.3.3 in the appendix. It was therefore concluded that ARIMA(0,2,4) model is the best adequate
for forecasting monthly exchange rate in Ghana based on the series of exchange rate since it had minimal error measures. This is specified as:

\[ r_t = 2r_{t-1} + r_{t-2} + 0.3122e_{t-1} + 0.1274e_{t-2} + 0.1461e_{t-3} - 0.0426e_{t-4} \]  

(4.3.4.1)

4.4 Modeling Monthly Rate of Inflation in Nigeria

This section entails modeling and forecasting monthly rate of inflation in Nigeria with time series models. It includes the model identification, estimation and twelve month out sample forecast. Monthly rate of inflation data spanning from January, 2003 to June, 2014 was used in the estimation and analysis.

4.4.1 Preliminary Analysis

Diagrammatic representation the monthly inflation rate in Nigeria for the period January, 2003 to June, 2014 is depicted in figure 4.4.1.1a. The plot in figure 4.4.1.1a is characterized by non-constant mean and some level of volatility clustering and hence differencing transformation was required to bring the series to stationarity. This was confirmed by the ADF test which gave a p-value of 0.02011 leading to the failure to reject the null hypothesis that the series is stationary at 1% level of significance.

The first ordinary differencing approach was used and results were plotted as shown in figure 4.4.1.1b. From Figure 4.4.1.1b, there appeared to be stability in the mean and variance of the differenced series and the ADF test confirmed stationarity with a p-value of less than 0.01 at 1% level of confidence.
The ACF of the monthly series confirmed that differencing of the series was necessary due to the sine wave pattern of the ACF. The ACF of the differenced series has no serial correlation due to the decay of the ACF as shown in figure 4.4.1.2 in the appendix. The ACF and PACF of differenced monthly series suggest AR(0) and MA(1) respectively. The auto arima function in R package suggested ARIMA(1,1,0) for the monthly series of rate of inflation in Nigeria. It was therefore necessary to compare the relative performance of the ARIMA(0,1,1) and ARIMA(1,1,0) suggested by the PACF and ACF and the auto arima function respectively.

Both suggested models gave AIC of 560.76 and log-likelihood of -278.38. ARIMA(0,1,1) yielded mean percentage error of 12.99358 which was greater than the 12.99108 yielded by the
ARIMA(1,1,0). It was therefore concluded that the ARIMA(1,1,0) would model the monthly rate of inflation in Nigeria than the ARIMA(0,1,1).

4.4.2 Model Estimation and Fitting

The specification of the suggested ARIMA(0,1,1) model is therefore depicted as:

\[ r_t - r_{t-1} = -\theta_1 e_{t-1} \]  \hspace{1cm} (4.4.2.1)

The model was then estimated using the MLE and on the assumption of normality of the residuals of the model. This yielded the fitted model specified as:

\[ r_t - r_{t-1} = 0.146 e_{t-1} (\pm 0.0877) \]  \hspace{1cm} (4.4.2.1a)

The series plot of the squared residual, the ACF plot and PACF of the ARIMA(0,1,1) were examined to confirm the presence of ARCH effect and results of the plot shown in the appendix …. The ACF and PACF plot showed significant pikes in the squared residual and therefore suggests the presence of ARCH effect in the model. This was confirmed by the Lagrange Multiplier test for ARCH effect shown in Table …. In the appendix. Results from the LM test for ARCH effect shows significant p-values less than 0.01 and hence the null hypothesis of no ARCH effect is rejected at 1% level of significance. It was concluded that the residuals of the ARIMA(0,1,1) shows a pattern that can be modeled with ARCH models.

The log-likelihood and the AIC of some selected ARCH(p) models were compared to ascertain the best ARCH(p) model for the series as shown in table 4.4.2.1 in appendix. It was evident from table 4.4.2.1 that ARCH(3) would fit the series best with the lowest AIC of 522.691 and maximum log-likelihood of -257.3455.
On the assumption of normality of the residuals of the ARCH model, the parameters of the model were estimated using the MLE method. Results from the estimation gave the fitted model as:

\[ a_t = \sigma_t \epsilon_t, \]  \hspace{1cm} (4.4.2.2)

\[ \sigma_t^2 = 1.18943 + 0.605284a_{t-1}^2 + 0.20478a_{t-2}^2 + 0.14183a_{t-3}^2 \]  \hspace{1cm} (4.4.2.3)

More so, the log-likelihood and AIC of some selected ARCH(p) models were obtained using the residuals of the ARIMA(0,1,1) model to ascertain appropriate ARIMA-ARCH specification for the series and results shown in Table 4.4… in the appendix. It was evident from Table… in the appendix that ARCH(3) will fit the series best with lowest AIC of 524.5934 and highest log-likelihood of -258.2967. Using the MLE method on the assumption of normality of the model’s residuals, the estimated ARIMA(0,1,1)-ARCH(3) model is therefore depicted as:

\[ a_t = \sigma_t \epsilon_t, \]  \hspace{1cm} (4.4.2.2)

\[ \sigma_t^2 = 1.17138 + 0.48531a_{t-1}^2 + 0.3273a_{t-2}^2 + 0.09022a_{t-3}^2 \]  \hspace{1cm} (4.4.2.3)

4.4.3 Diagnostic Checking of the Model

The Ljung-Box test for goodness of fit of the ARCH(3) model gave a significant p-value of 0.3733 confirming goodness of fit at 5% level of confidence. The residual plot of the ARCH(3) model showed volatility clustering with a normally distributed histogram as depicted in figure 4.4.3.2 in the appendix. This confirmed the assumption of the model and hence it was concluded that the ARCH (3) model is appropriate specification for the series of inflation rate in Nigeria.

The residuals of the ARIMA(0,1,1)-ARCH(3) were examined to confirm the normality assumption of the residuals in estimating the model parameters. The plot of the residuals exhibit
some volatility clustering with a normal histogram plot as depicted in figure 4.4.3.1 in appendix. This confirmed the assumption of the ARIMA-ARCH model and hence it was concluded that the model is an appropriate fit for the series.

4.4.4 Forecasting Performance Evaluation

The forecasting performance of the ARCH(3) and ARIMA(0,1,1)-ARCH(3) models were examined to ascertain which model can forecast monthly rate of inflation in Nigeria with minimal error. Table 4.4.4.1 below shows the comparison of error measures of the selected models. It was evident from table 4.4.4.1 that the ARIMA(01,1)-ARCH(3) model would be the best model for forecasting monthly rate of inflation in Nigeria.

Table 4.4.4.1: Comparing the Forecasting Performance of the Selected Models

<table>
<thead>
<tr>
<th></th>
<th>ARCH(3)</th>
<th>ARIMA(0,1,1)-ARCH(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Error(ME)</td>
<td>-1.988045</td>
<td>-1.963568</td>
</tr>
<tr>
<td>(RMSE)</td>
<td>3.129283</td>
<td>3.077284</td>
</tr>
<tr>
<td>Mean Absolute Error(MAE)</td>
<td>2.352853</td>
<td>2.282369</td>
</tr>
</tbody>
</table>

The fitted ARIMA(0,1,1)-ARCH(3) specification for forecasting monthly rate of inflation in Nigeria based on the monthly rate of inflation in Nigeria dataset from January, 2006 to June, 2014 is therefore depicted as:

\[
a_t = \sigma_t \varepsilon_t, \tag{4.4.4.3}
\]

\[
\sigma_t^2 = 0.89673 + 0.74054a_{t-1}^2 + 0.23805a_{t-2}^2 + 0.14732a_{t-3}^2 \tag{4.4.4.4}
\]
4.5 Modeling Monthly 90-Day Treasury bill Rate in Nigeria

This section entails modeling and forecasting monthly 90-day Treasury Bill rate in Nigeria with time series models. It includes the model identification, estimation and twelve month out sample forecast. Monthly Treasury bill rate data spanning from January, 2003 to June, 2014 was used in the estimation and analysis.

4.5.1 Preliminary Analysis

The plot of monthly 90-Day Treasury bill rate in Nigeria from January, 2003 to June, 2014 is depicted in figure 4.5.1.1a. From figure 4.5.1.1a, it was evident that both the mean and variance of the series were changing over time. This was confirmed by the ADF test which gave a p-value of 0.5806 and led to the failure to reject the null hypothesis that the series is not stationary at 1% level of significance.

Figure 4.5.1a: Monthly 90-Day T-bill Rates In Nigeria

Figure 4.5.1b: Diff of Monthly T-bill Rates In Nigeria

Figure 4.5.1.1: Plot of Monthly 90-Day Treasury Bill Rate in Nigeria and Differenced Monthly Treasury Bill Rate (January, 2003 – June, 2014)
The first differencing transformation was carried out to ensure stationarity of the series and results of the difference plotted as shown in figure 4.5.1.1. It was evident from the plot that the assumption of stationarity of the differenced series was more reasonable. This was confirmed by the significant ADF test p-value of less than 0.01 and hence it was concluded that the differenced series was stationary.

The ACF and PACF of both the original and differenced series in Figure 4.5… in the appendix were examined to ascertain whether there was serial correlation in the series. The ACF of the original series showed a sine wave pattern which implied serial correlation and hence the differencing of the series was more meaningful. The ACF of the differenced series died off as the length of time increased indicating the absence of serial correlation in the series. The PACF and ACF of the differenced series suggested AR(0) and MA(1) as order around which to model Treasury Bill rate in Nigeria.

4.5.2 Model Estimation and Fitting

The first model suggested to fit the differenced series was the ARIMA(0,1,1) with the lowest AIC of 504.91 and highest log-likelihood of -250.45. Upon assumption of normality of the series residuals and the use of MLE method, the parameters of the model were estimated and results gave the fitted model as:

\[ r_t - r_{t-1} = 0.0653e_{t-1} \quad (s.e \ 0.0853) \]  

(4.7.2.1)

The squared residuals of the ARIMA(0,1,1) model were examined to check for the presence of ARCH effect using the LM test. The results in Table 4.5… in the appendix shows that there is ARCH effect in the residuals of the ARIMA(0,1,1) model and hence the need to find appropriate ARCH fit for the series.
The log-likelihood and the AICs of suggested ARCH(p) models were compared using the original series to ascertain the best fitting model for the series of Treasury bill rate in Nigeria. Table 4.5.2.1 shows the comparison of the suggested ARCH(p) models with their respective statistics:

**Table 4.5.2.1: Comparison of Suggested ARCH(p) Models with Statistics**

<table>
<thead>
<tr>
<th>Lag</th>
<th>Log-Likelihood</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH(1)</td>
<td>-248.0524</td>
<td>500.1049</td>
</tr>
<tr>
<td>ARCH(2)</td>
<td>-239.034</td>
<td>484.068</td>
</tr>
<tr>
<td>ARCH(3)</td>
<td>-233.5602</td>
<td>475.1203</td>
</tr>
<tr>
<td>ARCH(4)</td>
<td>-237.0355</td>
<td>484.0711</td>
</tr>
</tbody>
</table>

From table 4.5.2.1, it was evident that the ARCH(3) model was the appropriate model for the series with the minimum AIC and maximum log-likelihood.

Assuming normality of the model’s residuals, the MLE method was used to estimate the parameters of the model. The parameter estimates of the ARCH(3) were all significant except the lag three parameter and hence the appropriate model reduced to ARCH(2) with the fitted model depicted as:

\[ a_t = \sigma_t \epsilon_t \]  \hspace{1cm} (4.5.2.2)

\[ \sigma_t^2 = 1.44321 + 0.12742a_{t-1}^2 + 0.73459a_{t-2}^2 \]  \hspace{1cm} (4.5.2.3)

Again, some selected ARCH(p) models were compared using the residuals of the ARIMA(0,1,1) model to obtain an appropriate ARIMA-ARCH model for the series. Results from Table 4.5....
shows that ARCH(3) gives an appropriate fit for the series residuals. Upon assumption of normality of the model’s residuals, the estimated model is therefore depicted as:

\[ a_t = \sigma_t \epsilon_t \]  

(4.5.2.2)

\[ \sigma_t^2 = 0.88729 + 0.17172a_{t-1}^2 + 0.26845a_{t-2}^2 + 0.08589a_{t-3}^2 \]  

(4.5.2.3)

### 4.5.3 Diagnostic Checking of the Model

The test of goodness of fit of the ARCH(2) model was carried using the Ljung-Box chi square test which gave a p-value of 0.7977 confirming goodness of fit. The residuals of the model were also plotted and examined to confirm the normality assumption. It was evident from figure 4.5.3.2 in appendix that the histogram plot of the residuals confirmed normality of the series residuals with a linear normal q-q plot.

The residuals of the ARIMA(0,1,1)-ARCH(3) were examined to confirm the assumptions made in the estimation of the parameters of the model. The plot of the residuals and the histogram plot with the normal q-q plot did confirm the normality assumption of the model as shown in figure 4.5.3.1 in the appendix. It was therefore concluded that the ARIMA(0,1,1)-ARCH(3) model fit the data.

### 4.5.4 Forecasting Performance Evaluation

The forecasting performance of the ARCH(3) and ARIMA(0,1,1)-ARCH(3) were examined to ascertain which model could forecast best Treasury bill rate in Nigeria. Table 4.5.4.1 shows the results from the comparison of the various error measures for both models. It was evident from Table 4.5.4.1 that both models could forecast monthly Treasury bill rate in Nigeria with minimal error. On the assumption of normality and AIC, it was concluded that the ARCH(3) model could forecast monthly treasury bill rate in Nigeria best.
Table 4.5.4.1: Comparing the Forecasting Performance of the Selected Models

<table>
<thead>
<tr>
<th></th>
<th>ARCH(3)</th>
<th>ARIMA(0,1,1)-ARCH(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Error(ME)</td>
<td>-1.801248</td>
<td>-1.796705</td>
</tr>
<tr>
<td>Root Mean Squared Error(RMSE)</td>
<td>2.990006</td>
<td>2.86164</td>
</tr>
<tr>
<td>Mean Absolute Error(MAE)</td>
<td>2.188803</td>
<td>2.140368</td>
</tr>
</tbody>
</table>

It was therefore concluded that the best model for forecasting monthly treasury bill rate in Nigeria based on the series of exchange rate from January, 2006 to June, 2014 was ARIMA(0,1,1)-ARCH(3) model specified as:

\[ a_t = \sigma_t \varepsilon_t \]  
\[ \sigma_t^2 = 1.38043 + 0.1279a_{t-1}^2 + 0.74207a_{t-2}^2 \]

4.6 Modeling Exchange Rate in Nigeria

This section entails modeling and forecasting monthly exchange rate in Nigeria with time series models. It includes the model identification, estimation and twelve month out sample forecast. Monthly naira per US dollar exchange rate data spanning from January, 2003 to June, 2014 was used in the estimation and analysis.

4.6.1 Preliminary Analysis

The plot of monthly Nigeria Naira per US Dollar exchange rate for the period January, 2003 to June, 2014 is shown figure 4.6.1.1. The plot exhibits unstable mean and variance and hence not stationary. This was confirmed by the insignificant p-value (p-value=0.5752) of the ADF test for stationarity which led to failure to reject the null hypothesis that the series is not stationary at 1% level of significance.
In order to obtain stationarity of the series, the natural logarithm differencing approach was used because of the exponential growth of the series. The natural logarithm of the series was obtained and then the series differenced. The plot of the differenced logarithm series in figure 4.6.1b exhibits some level of stationarity. This was confirmed by the ADF test which gave a p-value of less than 0.01. This led to the rejection of the null hypothesis that the differenced log series is not stationary at 1% level of significance.

The ACF of the series was characterized by sine wave pattern which confirmed that the differencing of the series was necessary. The ACF of the differenced logarithm series showed a shape decay of the series and hence it was concluded that there was no serial correlation in the differenced logarithm series.
The plot of the PACF of the differenced logarithm series suggest ARIMA(0,1,1) as the most appropriate linear model for modeling the series. This was confirmed by the auto arima function in R package “forecast” which selected the ARIMA(0,1,1) with a log-likelihood of 391.58 and lowest AIC of -733.23 respectively.

4.6.2 Model Estimation and Fitting

The specification of the selected ARIMA model is given by

\[ r_t - r_{t-1} = -\theta_1 e_{t-1} \]  \hspace{1cm} (4.6.2.1)

The maximum likelihood estimation of the parameters of the ARIMA(0,1,1) model on the assumption of normality of the residuals gave the fitted specification as:

\[ r_t - r_{t-1} = -0.5048 e_{t-1} \]  \hspace{1cm} (4.6.2.1a)

The residuals of the ARIMA(0,1,1) model were examined to check for the presence of ARCH effect using the LM test. Result from the LM test shown in Table 4.6. in the appendix indicate the presence of ARCH effect in the series. It was therefore necessary to find appropriate ARCH model for the series.

The log-likelihood and AIC of some selected ARCH(p) models were ascertain with the R package “tseries” and examined to selected the most appropriate order of the ARCH(p) that would fit the series best. Table 4.6.2.1 in the appendix shows clearly that ARCH(1) with the highest log-likelihood of 413.4641 and lowest AIC of -822.9283 would fit the series best.

On assumption of normality of the models residuals, the parameters were estimated using the MLE method and results of the estimation yielded the fitted model as:

\[ a_t = \sigma_t e_t, \]  \hspace{1cm} (4.6.2.2)
\[ \sigma_t^2 = 0.05235 + 2.11384a_{t-1}^2 \]  

(4.6.2.2a)

Again, the some selected ARCH(p) models were compared using the residuals of the ARIMA(0,1,1) model based on the AIC and log-likelihood.

### 4.6.3 Diagnostic Checking of the Model

A test of goodness of fit of the ARCH(1) model was carried out using the Ljung-Box chi square test. The test gave a p-value of 0.3531 and hence failure to reject the null hypothesis of goodness of fit at 5% level of significance. The residual plot of the ARCH(1) model showed a volatility clustering and the histogram plot also skewed to the left and hence not a good normal as shown figure 4.6.3.2 in the appendix.

A diagnostic check of the model residuals was carried out to confirm the normality assumption of the residuals. Figure 4.6.3.1 in the appendix shows the plot of the residuals and the histogram plot of residuals with normal Q-Q plot. The plot of residuals exhibit volatility clustering with the histogram plot skewed to the right showing that the model did not satisfy all the model’s assumption and hence not appropriate fit for the series.

### 4.6.4 Forecasting Performance Evaluation

The forecasting performance of the ARIMA(0,1,1) and ARCH(1) models were examined to ascertain which model can forecast monthly exchange rate in Nigeria with minimal error. Table 4.6.4.1 below shows the comparison of error measures of the selected models. It was evident from Table 4.6.4.1 that the ARCH(1) model would be the best model for forecasting monthly rate of inflation in Nigeria.
The fitted ARCH(1) specification for forecasting monthly exchange rate in Nigeria based on the monthly naira per US dollar rate in Nigeria dataset from January, 2003 to June, 2014 is therefore depicted as:

Table 4.6.4.1: Comparing the Forecasting Performance of the Selected Models

<table>
<thead>
<tr>
<th></th>
<th>ARIMA(0,1,1))</th>
<th>ARCH(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Error (ME)</td>
<td>0.001104198</td>
<td>-0.009527423</td>
</tr>
<tr>
<td>Root Mean Squared Error (RMSE)</td>
<td>0.01413311</td>
<td>0.01821961</td>
</tr>
<tr>
<td>Mean Absolute Error (MAE)</td>
<td>0.005762432</td>
<td>0.01279306</td>
</tr>
<tr>
<td>Akaike Information Criteria</td>
<td>-738.98</td>
<td>-822.9283</td>
</tr>
</tbody>
</table>
Table 4.7.1: Twelve(12) Months Out Sample Forecast of Economic Variable (July, 2014 – June, 2015)

<table>
<thead>
<tr>
<th></th>
<th>Ghana</th>
<th>Nigeria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Inflation</td>
<td>Treasury Bill Rate</td>
<td>Exchange Rate</td>
</tr>
<tr>
<td>July, 2014</td>
<td>0.1348</td>
<td>0.236759</td>
</tr>
<tr>
<td>August, 2014</td>
<td>0.1342</td>
<td>0.234139</td>
</tr>
<tr>
<td>September, 2014</td>
<td>0.1337</td>
<td>0.232242</td>
</tr>
<tr>
<td>October, 2014</td>
<td>0.1332</td>
<td>0.230859</td>
</tr>
<tr>
<td>November, 2014</td>
<td>0.1327</td>
<td>0.229839</td>
</tr>
<tr>
<td>December, 2014</td>
<td>0.1322</td>
<td>0.229076</td>
</tr>
<tr>
<td>January, 2015</td>
<td>0.1318</td>
<td>0.228495</td>
</tr>
<tr>
<td>February, 2015</td>
<td>0.1314</td>
<td>0.228044</td>
</tr>
<tr>
<td>March, 2015</td>
<td>0.131</td>
<td>0.227684</td>
</tr>
<tr>
<td>April, 2015</td>
<td>0.1306</td>
<td>0.227399</td>
</tr>
<tr>
<td>May, 2015</td>
<td>0.1302</td>
<td>0.22715</td>
</tr>
<tr>
<td>June, 2015</td>
<td>0.1299</td>
<td>0.226934</td>
</tr>
</tbody>
</table>

\[ a_t = \sigma_t \varepsilon_t, \quad (4.6.4.1) \]

\[ \sigma_t^2 = 0.04961 + 2.13554a_{t-1}^2 \quad (4.6.4.1a) \]

4.7 Forecasting and Investment Prospect Analysis

Table 4.7.1 represent the one year out sample forecast of the monthly rate of inflation, 90 day Treasury bill rate and local currency per US dollar exchange for both Ghana and Nigeria from
July, 2014 to June, 2014. All forecast values falls within the confidence level and hence it was concluded that the forecasted values represent an estimate of the actual values.

The accumulated value of one dollar to be invested for a period of twelve month was ascertained for both Ghana and Nigeria. The forecast of Treasury bill rate was used as the interest rate for the accumulation. Table 4.7.2 in the appendix represents the results for the accumulation of investment returns over the period of twelve month. Figure 4.7.1 represents the plot of the accumulation of investment returns.

![Plot of monthly Accumulated Values of Investment returns](image)

**Figure 4.7.1: Plot of Monthly Accumulated Values of Investment Returns**

The true interest rate, interest rate adjusted for inflation, was ascertained to account for the effect of inflation on interest rate and investment. The true interest rate also known as real interest was obtained by,

\[
r_t = R_t - (I_t + R_t I_t)
\]

(4.7.1)
Where \( R_t \) is the interest rate and \( I_t \) is the inflation rate.

The accumulated values for various time periods over the period of twelve months were obtained and results plotted as shown in figure 4.7.2. It was evident from both figure 4.7.1 and figure 4.7.2 that, the investment return or accumulated values for Ghana was the same at the beginning of the period but as the months accrued, the investment returns in Ghana grew faster than that of Nigeria. It was also evident from figure 4.7.1 and figure 4.7.2 that the impact of inflation in Ghana is higher than that of Nigeria. Thus, the gap between investment returns in Nigeria and Ghana was wider for the actual accumulation than that of the deflated accumulated value.

![Figure 4.7.2: Plot of Monthly Deflated Accumulated Values of Investment Returns](http://ugspace.ug.edu.gh)

This implied that for both actual accumulation and deflated accumulation of investment returns, Ghana out performed Nigeria.
Column plots of the proportion of investment returns for both actual and deflated accumulation from the two Countries were obtained as depicted in figure 4.7.3. This was carried out to ascertain which country has the highest accumulated value at the end of the twelve months period. From figure 4.7.3, it was evident that Ghana obtained 77.24% of the investment returns whereas Nigeria obtained 22.76% for the actual accumulation without inflation. Again, Ghana obtained 61.68% of the investment returns whereas Nigeria obtained the remaining 38.32% for the deflated accumulation series. It was therefore concluded that investing in Ghana will be worthwhile than investing in Nigeria based on economic information obtained for rate of inflation, interest rate and exchange rate for the period of January, 2003 to June, 2014.

Figure 4.7.3: Plot of Twelve(12) Months Accumulated Value of Investment returns

Finally, the deflated returns on investment or deflated accumulated value of investment at the end of each month was ascertain in terms of each country’s local currency. This was obtained by multiplying the accumulated value by the respective month exchange rate in the respective
country. Thus for the month of July, 2014, a dollar invested in Ghana will actually accumulate to 3.3609 cedis at the end of the month whereas a dollar invested in that same period will accumulate to 161.9625 naira at the end of the period as shown in Table 4.7.3 in the appendix.
CHAPTER FIVE: SUMMARY, CONCLUSION AND RECOMMENDATION

5.0 Introduction

This chapter provides the summary of the study as well as the conclusions drawn from the study and recommendations for government and policy makers, investors and academicians. The findings from the analysis of investment prospects in Ghana and Nigeria are also discussed in this chapter.

5.1 Summary

Investors are much concerned about the possible factors that are likely to have an adverse impact on their investments. Since macroeconomic variables’ uncertainty affects investment, investors would also like to know the regions where the effect of uncertainty is real through empirical studies. Analyzing investment prospect in West Africa using stochastic time series modeling has minimal attention in empirical research and therefore investors and policy makers are faced with the problem of where to direct their resources.

This research is designed to investigate the prospect of investment in Ghana and Nigeria by considering time series modeling and forecasting of monthly rate of inflation, interest rate and exchange rate (local currency per US dollar rate). ARIMA and ARCH type models were fitted to ascertain the best forecasting model for each series. The AICs and log-likelihood of some suggested order of each series were compared and the model with the lowest AIC and highest log-likelihood was considered the best fit model.
The summary of the data analysis are as follows:

- The monthly rate of inflation for the period under study averaged 14.685 per cent in Ghana and 11.688 per cent in Nigeria with mean error of 0.5285 and 0.40018 respectively. The distribution of monthly rate of inflation was skewed to the right for both Ghana and Nigeria.

- Monthly 90-day Treasury bill rate (interest rate) averaged 17.5273 percent for Ghana and 9.5274 percent for Nigeria for the period under study with mean error of 0.5717 and 0.3682 respectively. The distribution of monthly interest rate in Ghana was skewed to the right whereas that of Nigeria was skewed to the left.

- The mean monthly exchange rate was 1.3133 cedi per US dollar for Ghana and 140.4348 naira per US dollar for Nigeria. The standard errors of the means were 0.041114 and 1.172646 respectively. The plot of monthly exchange rate for Ghana had a long right tail and that of Nigeria had a long left tail.

- The monthly series for all indicators were found to be non-stationary. Ordinary first differencing for rate of inflation and Treasury bill rate for both countries and second differenced natural logarithm of exchange rate for Ghana and first difference natural logarithm of exchange rate in Nigeria helped to transform the respective series to stationarity.

- ARIMA(1,1,2)-ARCH(2) with the lowest forecasting error (MAE) of 1.705845 among the suggested time series models for modeling rate of inflation in Ghana was adjudged the best model. Nigeria’s rate of inflation was best modeled by ARIMA(0,1,1)- ARCH(3) with the lowest error of 2.2382369 among the suggested models.
Treasury bill rate in Ghana was best modeled by ARIMA(1,1,2)-ARCH(1) with the lowest mean absolute error of 1.473909 and that of Nigeria was best modeled by ARIMA(0,1,1)-ARCH(3).

Again, forecasting exchange rate in Ghana was best modeled by ARIMA(0,2,4)-ARCH(1) with MAE of 0.01250588 compared to MAE of 0.01264951 based on forecasting with ARCH(1). This conforms to the research by Appiah and Adetunde (2011) that ARIMA models best describe exchange rate volatility in Ghana. The best model fit for forecasting naira per US dollar exchange rate was ARIMA(0,1,1)-ARCH(1) with minimal mean absolute error of 0.012304 which is in support of Asemota and Bala (2013).

The results from the twelve (12) month’s forecast also reveal an upward trend in the forecast of rate of inflation, interest rate and local currency per US dollar exchange rate for both Ghana and Nigeria.

One dollar each was invested in both economies and it was noticed that the accumulated value of investment in Ghana grew faster than that of Nigeria with investor obtaining returns of 11.97728 dollars in Ghana and 3.529602 dollars in Nigeria at the end of the twelve(12) months period. Upon deflating thus when inflation is zero in the accumulation of investment returns, investment in Ghana yielded 2.186433 dollar at the end of the twelve (12) months period and that of Nigeria yielded 1.358206 dollars for every dollar invested.
5.2 Conclusion

This research aimed at exploring the prospects of investment in Ghana and Nigeria using stochastic time series modeling and comparing the accumulated values of investment returns in both countries.

Forecasting rate of inflation would best be done using the ARIMA(1,1,2)-ARCH(1) model for Ghana and ARIMA(0,1,1)-ARCH(3) model for Nigeria. Treasury bill rate would best be modeled by ARIMA(1,1,2)-ARCH(1) for Ghana and ARIMA(0,1,1)-ARCH(3) for Nigeria. Again, forecasting exchange rate in Ghana was best done using the ARIMA(0,2,4)-ARCH(1) and that of Nigeria was ARIMA(0,1,1)-ARCH(1). It was also concluded that rate of inflation, interest rate and local currency per US dollar exchange rate is going to have an upward trend for both Nigeria and Ghana.

It was concluded that investment in Ghana is worth more than investment in Nigeria since the accumulated value of investment return in Ghana grew faster than that of Nigeria for both the case zero inflation and that of varying inflation in the economies. It is also clear to conclude that after receiving the return on investment in Ghana, it would be prudent to spend it in Nigeria since the rate of inflation in Ghana grows faster than that of Nigeria and hence making cost of living in Ghana relatively higher than that of Nigeria.

5.3 Recommendation

The fiscal and monetary policy makers should revise the policies to address the depreciation of the Cedi and Naira against the US dollar and rising of rate of inflation in Ghana and Nigeria. It is also recommended that investors should consider investing capital in Ghana and spending the returns in Nigeria.
Further research could be carried out on considering both time domain analysis and frequency domain analysis in forecasting the macroeconomic variables and determining the frequency of the variables since it would provide information about the length of cycle.

5.4 Limitation
There were some limitations, further research can be done considering the same model selected for a particular macroeconomic variable for both countries before the forecasting performance is done. Also effect of exchange rate can be considered on the other variables.
REFERENCES


Time Series Analysis with ARIMA-ARCH/GARCH Model in R.


APPENDICES

Table 4.1.1: Statistics for Testing for ARCH effect in Rate of Inflation in Ghana

<table>
<thead>
<tr>
<th>Lag</th>
<th>1</th>
<th>2</th>
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<th>12</th>
<th>16</th>
<th>24</th>
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<tbody>
<tr>
<td>Q-Statistic</td>
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<td>234.2453</td>
<td>547.341</td>
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Table 4.3.1.1: Statistics for Testing for ARCH effect in Treasury Bill Rate in Ghana

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<td>Q-Statistic</td>
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Figure 4.3.1.2: Plot of the ACF and Partial ACF of the monthly Treasury Bill rate in Ghana and its differenced series (January, 2003 to December, 2013)

Figure 4.3.3.1: Plot of Residuals of ARIMA(1,1,1) model with its histogram and Normal Q-Q plot (January, 2009 – December, 2013)
Figure 4.3.3.2: Plot of Residuals of ARCH(1) model with its histogram and Normal Q-Q plot (January, 2009 – December, 2013)

Table 4.3.1.1: Statistics for Testing for ARCH effect in Exchange Rate in Ghana

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Figure 4.3.4.2: Plot of the ACF and Partial ACF of the monthly Exchange rate in Ghana and its Log differenced series (January, 2003 to December, 2013)

Figure 4.4.3.1: Plot of Residuals of ARIMA(0,2,4) model with its histogram and Normal Q-Q plot (January, 2009 – December, 2013)
Figure 4.4.3.2: Plot of Residuals of GARCH(1,1) model with its histogram and Normal Q-Q plot (January, 2009 – December, 2013)

Table 4.4.1.1: Statistics for Testing for ARCH effect in Rate of Inflation in Nigeria

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<tr>
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<td>Q-Statistic</td>
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Figure 4.5.1.2: Plot of the ACF and Partial ACF of the monthly Rate of Inflation in Nigeria and its differenced series (January, 2003 to December, 2013)

Table 4.4.2.1: Comparison of Suggested ARCH(p) Models with Statistics

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<td>ARCH(3)</td>
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<td>ARCH(4)</td>
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Table 4.4.2.2: Comparison of Suggested GARCH(p,q) Models with Statistics

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<td>GARCH(2,1)</td>
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Figure 4.5.3.1: Plot of Residuals of ARIMA(1,1,0) model with its histogram and Normal Q-Q plot (January, 2009 – December, 2013)

Figure 4.5.3.2: Plot of Residuals of ARCH(3) model with its histogram and Normal Q-Q plot (January, 2009 – December, 2013)

Table 4.6.1.1: Statistics for Testing for ARCH effect in Treasury Bill Rate in Nigeria
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<tr>
<th>Lag</th>
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</thead>
<tbody>
<tr>
<td>Q-Statistic</td>
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Figure 4.6.1.2: Plot of the ACF and Partial ACF of the monthly Rate of Inflation in Nigeria and its differenced series (January, 2003 to December, 2013)
Figure 4.6.3.1: Plot of Residuals of ARIMA(1,1,0) model with its histogram and Normal Q-Q plot (January, 2009 – December, 2013)

Figure 4.6.3.2: Plot of Residuals of ARCH(2) model with its histogram and Normal Q-Q plot (January, 2009 – December, 2013)
Table 4.7.2.1: Comparison of Suggested ARCH(p) Models with Statistics

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Figure 4.7.3.1: Plot of Residuals of ARIMA(0,1,1) model with its histogram and Normal Q-Q plot (January, 2009 – December, 2013)
Figure 4.7.3.2: Plot of Residuals of ARCH(1) model with its histogram and Normal Q-Q plot (January, 2009 – December, 2013)

Table 4.8.2: Accumulated values of Investment Returns in Ghana and Nigeria

<table>
<thead>
<tr>
<th></th>
<th>Actual Accumulated Values</th>
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<td></td>
<td>Ghana</td>
<td>Nigeria</td>
<td>Ghana</td>
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<td>October, 2014</td>
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<td>December, 2014</td>
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<td>January, 2015</td>
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<tr>
<td>February, 2015</td>
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<td>June, 2015</td>
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Table 4.8.3: Accumulated values of Investment Returns in Ghana and Nigeria (Local Currency)

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