MODELLING RADIONUCLIDES TRANSPORT AND DOSE ASSESSMENT IN A GROUNDWATER SYSTEM IN SOMITATAPARKO GOLDMINE IN BURKINA FASO

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DECLARATION

This thesis is the outcome of research work undertaken by KABORE KARIM in the Department of Medical Physics, School of Nuclear and Allied Sciences, University of Ghana, under the supervision of Prof E. O. Darko and Rev. Dr. S. Akoto-Bamford

Kabore Karim (Student) Date

Prof E. O. Darko (Supervisor) Date

Rev. Dr. S. Akoto-Bamford (Supervisor) Date
DEDICATION

This work is dedicated to my parents, my mother Madam Fati Kabore, my father who is no longer in this world and all my family.
ACKNOWLEDGEMENTS

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<tbody>
<tr>
<td>1-D</td>
<td>One-Dimension</td>
</tr>
<tr>
<td>2-D</td>
<td>Two-Dimension</td>
</tr>
<tr>
<td>3-D</td>
<td>Three-Dimension</td>
</tr>
<tr>
<td>ADE</td>
<td>Advection-Diffusion Equation</td>
</tr>
<tr>
<td>BEIR</td>
<td>Biological Effects of Ionizing Radiation</td>
</tr>
<tr>
<td>ALARA</td>
<td>As low as reasonably achievable</td>
</tr>
<tr>
<td>Bq/L (BqL-1)</td>
<td>Becquerel per litre</td>
</tr>
<tr>
<td>Bq/m (Bqm⁻¹)</td>
<td>Becquerel per cubic meter</td>
</tr>
<tr>
<td>DCF</td>
<td>Dose conversion factor</td>
</tr>
<tr>
<td>EC</td>
<td>European Commission</td>
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<tr>
<td>GAEC</td>
<td>Atomic Energy Commission</td>
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<td>GHARR-1</td>
<td>Ghana Research Reactor One</td>
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<tr>
<td>BSS</td>
<td>Basic Safety Standards</td>
</tr>
<tr>
<td>BTCS</td>
<td>Backward in Time, Centre in Space</td>
</tr>
<tr>
<td>CEA</td>
<td>Commissariat à l’Energie Atomique</td>
</tr>
<tr>
<td>CEIA</td>
<td>Centre for Environmental Impact Assessment</td>
</tr>
<tr>
<td>CT</td>
<td>Computed Tomography</td>
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<tr>
<td>DEM</td>
<td>Discontinuous Enrichment Method</td>
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<td>DNA</td>
<td>Deoxyribonucleic Acid</td>
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<td>ECL</td>
<td>Exempt Concentration Levels</td>
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<td>EMS</td>
<td>Environmental Management Systems</td>
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<tr>
<td>Abbreviation</td>
<td>Full Form</td>
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<tr>
<td>EPA</td>
<td>Environmental Protection Authority</td>
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<tr>
<td>FTBS</td>
<td>Forward in Time, Backward in Space</td>
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<tr>
<td>FTCS</td>
<td>Forward in Time, Centre in Space</td>
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<tr>
<td>GDP</td>
<td>Gross Domestic Product</td>
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<tr>
<td>GIS</td>
<td>Geographic Information System</td>
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<tr>
<td>HDM</td>
<td>Hydrological Dispersion Model</td>
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<tr>
<td>HEC</td>
<td>Hydrologic Engineering Center</td>
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<tr>
<td>HPS</td>
<td>Health Physics Society</td>
</tr>
<tr>
<td>IAEA</td>
<td>International Atomic Energy Agency</td>
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<tr>
<td>ICPRB</td>
<td>Interstate Commission on the Potomac River Basin</td>
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<tr>
<td>ICRP</td>
<td>International Commission on Radiological Protection</td>
</tr>
<tr>
<td>IFC</td>
<td>International Finance Co-operation</td>
</tr>
<tr>
<td>IG</td>
<td>Iduapriem Goldfields</td>
</tr>
<tr>
<td>ISEs</td>
<td>Intermediate Scale Experiments</td>
</tr>
<tr>
<td>K</td>
<td>Potassium</td>
</tr>
<tr>
<td>K_d</td>
<td>Distribution Coefficient</td>
</tr>
<tr>
<td>LET</td>
<td>Linear Energy Transfer</td>
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<tr>
<td>LLRW</td>
<td>Low Level Radioactive Waste</td>
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<tr>
<td>MARS</td>
<td>Merrick Advance Remote Sensing</td>
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<tr>
<td>NaCN</td>
<td>Sodium cyanide solution</td>
</tr>
<tr>
<td>NAS</td>
<td>National Academy of Sciences</td>
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<tr>
<td>NORM</td>
<td>Naturally Occurring Radioactive Material</td>
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<tr>
<td>Abbreviation</td>
<td>Full Form</td>
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<td>--------------</td>
<td>-----------------------------------------------</td>
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<tr>
<td>NRC</td>
<td>National Research Council</td>
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<tr>
<td>PCBs</td>
<td>Polychlorinated Biphenyls</td>
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<tr>
<td>PDE</td>
<td>Partial Differential Equation</td>
</tr>
<tr>
<td>Ra</td>
<td>Radium</td>
</tr>
<tr>
<td>RA</td>
<td>Regulatory Authority</td>
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<tr>
<td>Rn</td>
<td>Radon</td>
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<tr>
<td>NRPANS</td>
<td>National Radiation Protection Authority and Nuclear Safety</td>
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<tr>
<td>NWOR</td>
<td>National Water Office and Rehabilitation</td>
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<tr>
<td>TAM</td>
<td>Tidal Anacostia Model</td>
</tr>
<tr>
<td>Th</td>
<td>Thorium</td>
</tr>
<tr>
<td>U</td>
<td>Uranium</td>
</tr>
<tr>
<td>UN</td>
<td>United Nations</td>
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<tr>
<td>UNSCEAR</td>
<td>United Nations Scientific Committee on the Effects of Atomic Radiation</td>
</tr>
<tr>
<td>USACE</td>
<td>U.S. Army Corps of Engineers</td>
</tr>
<tr>
<td>USNRC</td>
<td>United States Nuclear Regulatory Commission</td>
</tr>
<tr>
<td>UV</td>
<td>Ultraviolet</td>
</tr>
<tr>
<td>WASP</td>
<td>Water quality Simulation Program</td>
</tr>
<tr>
<td>WHO</td>
<td>World Health Organization</td>
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<tr>
<td>WNA</td>
<td>World Nuclear Association</td>
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ABSTRACT

Radionuclides transport and dose assessment to the public in a groundwater system are described using theoretical and experimental methods. It is known that radionuclides could accumulate during mining and mineral ore processing and reach into water bodies, and thus contribute to the radiation dose received by the public who consume this water. As groundwater is inaccessible, a mathematical model with a numerical method is used to describe the relevant physical processes during radionuclide transport in the groundwater system. A computer programme has been written in MatLab to implement the numerical solution. The activity concentration and annual committed effective dose to the public from natural radionuclides (NORM) are determined for both theoretical and experimental methods. The NORM elements of interest are $^{238}$U, $^{232}$Th and $^{40}$K. The results obtained from the study revealed that the concentration of NORM in Taparko groundwater system increases with time at the point of release and becomes constant at a certain time. The concentration then decreases rapidly as far as from the release point. The average activity concentrations for theoretical method were 0.3 Bq/L, and 0.05 Bq/L for $^{238}$U and $^{232}$Th respectively at 600 m from the discharged point where the water from boreholes and wells is used by the public. For experimental analysis the mean activity concentrations were 5.57 Bq/L, 0.58Bq/L and 4.78 Bq/L for $^{238}$U, $^{232}$Th and $^{40}$K respectively. The corresponding average annual committed effective doses estimated were 0.13 mSv/y, 0.28 mSv/y, for theoretical and experimental methods respectively. The theoretical and experimental results indicate insignificant exposure of the public to Natural Occurring Radioactive Materials (NORM) from the activities of the Goldmine.
CHAPTER ONE

INTRODUCTION

This section provides background information on groundwater system, the statement of the problem, the target objectives, relevance and justification, scope and limitation and the structure of the Thesis.

1.1 Background

Groundwater is water that occurs in pores and fractures in soil and rocks below the watertable. Formally, the watertable (sometimes referred to as the phreatic surface) is defined as the level at which the water pressure equals the atmospheric pressure.

One of the most important pathway for transport of radioactive contaminations in the soil is groundwater. Unfortunately, this type of contamination is difficult to sample and monitor and it requires great dependence on models to predict the transport and fate as well as the variation of concentration along this pathway.

Groundwater contamination can be considered as a potential exposure pathway if the radionuclide concentration in the groundwater exceeds the legal levels. if the concentrations of radionuclides in the groundwater downflow from a source, or in leachate at the source, exceed these values, the groundwater is used as a source of drinking water.

Fortunately, groundwater modeling can be used as a support tool to predict an eventual decision of site remediation. It allows to identify the type and to estimate the amount of radionuclide present. This will not only determine the potential offsite impact, but it will also help to identify the magnitude of the risks to potentially exposed receptors, the radionuclides mobility and the time period over which the radionuclides may be hazardous.
The types of radionuclides will also determine whether radioactive decay and the ingrowth of radioactive daughters are important parameters that will need to be modeled (Dinis and Fiuza, 2005).

On the other hand, the measurement of natural radioactivity in our physical environment allows the determination and assessment of population exposure to radiation. Considering the presence of natural radionuclides in the groundwater system and the radiotoxicity due to human, consumption of groundwater with high amounts of natural radionuclides may give rise to internal exposure caused by the decay of natural radionuclides taken into body through ingestion. During the decay process, they release several alpha and beta particles which are also responsible for a generally small fraction of the total radiation dose received from natural radioactivity as well as artificial radioactivity. Gross alpha in water is usually presented by $^{238}$U series while gross beta is presented by non-series of $^{40}$K as well as $^{232}$Th series (Nguelem, et al, 2013).

The increase in the level of radioactivity is as a result of possible migration of radionuclides to groundwater from anthropogenic activities: seepage of pollutants into the groundwater bodies, excessive fertilization of agricultural land, abandoned industrial sites, thermonuclear testing or nuclear power plants, post-mining areas, waste dump areas, could also contribute to the radioactivity increase. It is possible for groundwater to naturally regenerate itself but, sometimes, takes many years as it proceeds very slowly.

Importance of water quality assessment and monitoring in the context of radioactivity cannot be overemphasized (Nwankwo, 2013).

The protection of the public and workers from the harmful effects of ionizing radiation has been a subject of interest and concern of radiation protection professionals since the
early part of the 20th century when harmful effects were first observed. Thus, a range of models useful for predicting radionuclide transport in water in general and in groundwater system in particular have been developed (NORM-MMPG, Western Australia, 2010).

The International Basic Safety Standards for Protection against Ionizing Radiation and for the Safety of Radiation Sources (BSS) established basic and detailed requirements for protection against the risks associated with exposure to radiation and for the safety of radiation sources that may deliver such exposure (IAEA -BSS, 2001). The standards are based primarily on the 2007 Recommendations of the International Commission on Radiological Protection (ICRP) and other International Atomic Energy Agency (IAEA) Safety Series publications (ICRP. 2007).

The development of mechanisms to implement effective management of wastes and residues, including those containing Naturally Occurring Radioactive Materials (NORM) should be taken into account by companies, research institutions, and the general public (A/CONF.151/26, UN, New York, 1992).

The NORM elements of interest are $^{238}$U and $^{232}$Th decay chains as well as $^{40}$K. These radionuclides as well as some progenies such as $^{226}$Ra are long-lived. The distribution of these radionuclides in the geosphere depends on the distribution of the geological media from which they are derived and the processes which concentrate them at a specific location in specific media. (IAEA, 2003).

In many developing countries including Burkina Faso, mining which has the potential to generate NORM has not been duly investigated and as a result no radiological regulatory
controls are applied. Therefore, there is general lack of awareness and knowledge of the radiological hazards and exposure levels by legislators, regulators and operators.

The aim of this work is to model the transport of natural radionuclides in groundwater system from SOMITA-TAPARKO goldmine and use experimental data in order to assess the magnitude of the impact on the public and the environment. The results that obtained from the study will be used to determine the contamination level of NORM elements in the mine site and the vicinity.

1.2 Radionuclide Transport Modeling Approaches

The transport and behavior of radionuclides in groundwater flow can be predicted using various sophisticated methods, ranging from a simple algebraic mass-balance approach to a multi-dimensional numerical solution of the problems. The basis of all the computer models is on the law of mass conservation of any contaminant. It can be expressed in terms of the advection-diffusion equation in Cartesian coordinates (IAEA, 2001).

There are two basic types of models used to estimate radionuclide transport in groundwater system.

a. Numerical models usually transform basic equations describing radionuclide dispersion/diffusion into finite difference or finite element forms.

b. Analytical models that solve the basic radionuclide transport equations. Simplifying assumptions are made regarding water body geometry, flow conditions and dispersion processes in order to obtain analytical solutions to the governing equations (IAEA, 2001)
1.3 Statement of the Problem

Water is vital and, concurrently, one of the most important natural resources. About 70% of the Earth’s surface is covered with water, which is estimated at a volume of approximately 1.4 billion km$^3$. However, most of it is salty, and only around 2.5% of the global water resources (about 35 million km$^3$) consists of freshwater (Nwankwo, 2013). Groundwater is the most important and highly used resource; however, its quality can be endangered. Because of the increasing interest in radioactivity and its applications, there is the need for an assessment of human exposure to radiation. Therefore, it is necessary to examine naturally occurring radioactivity in the environment, especially the occurrence of natural radioactivity in groundwater (Nwankwo, 2013). Harnessing the natural resources of a country is an unavoidable venture in national development.
Nevertheless, related activities often expose groundwater to radioactive contamination. Very little work has been done on groundwater. No such studies have been carried out regarding the impact of gold mining on groundwater quality in Burkina Faso.

1.4 Objectives

The main objective of the study is to contribute to the assessment of the impact of gold activities on groundwater quality in Burkina Faso. This will be achieved by determining the concentration of certain radionuclides such as U-238, Th-232, and K-40 in the groundwater system.

Specific objectives are to:

- Evaluate the level of radioactivity in the groundwater system within the site and the vicinity of the mine;
- Assess the Annual Committed Effective Dose due to water ingestion;
- Assess the risk to the consuming public;
- Predict radionuclide concentration in groundwater at various distances from the site using a mathematical model.

1.5 Relevance and Justification

Radiation exposure can result in stochastic health effect (IAEA BSS, 1996). The greater incidence of cancer in population exposed to ionizing radiation indicates that the probability or risk of developing cancer is proportional to the level of radiation at low and low dose rate of exposure (BEIR, 2006).
The results from this work could help to know the exposure of the public from consumption of groundwater around the mining site; and therefore the dose received by the public who are consuming the water from wells and boreholes in the vicinities, and the cancer risk associated with the exposure.

1.6 Scope and Limitation

The sampling covers some water resource points in the site and most of the boreholes in the vicinity of SOMITA-TAPARKO Goldmine Company in the center north of Burkina Faso. It is the first time this kind of studies is being carried out in the country. However, the study is limited to only one mine. It is important to extend the study to other areas especially in the old abandoned mine site.

1.7 Structure of the thesis

The thesis has five chapters, including introduction and conclusion. Chapter one consists of a summary of the basic concept of the study. Chapter two includes the literature review, discussing some previous works done on the field of study. Chapter three explains the methodology and the techniques used to attain the results. Chapter four concerns analysis and discussions of the results. Chapter five outlines conclusions and recommendations based on the outcomes.
CHAPTER TWO

LITERATURE REVIEW

This section provides an overview on natural radionuclides, radioactivity effects, radioactivity in drinking water, and groundwater models are discussed.

2.1 Mining industry of Burkina Faso

Burkina Faso is geologically rich. There has been substantial interest in mining, particularly gold mining, in Burkina Faso over the last 40 years, with the Poura region producing over 25t of gold. Some of the world's most prolific mines are on greenstone belts. These belts cover about 3 000 000 km$^2$ of West Africa, making the area's exploration potential substantial (fig.2.1). Burkina Faso currently accounts for 21% of West Africa's greenstone belt exposure (Mbendi, 2014). In 2012, there were 6 gold mines in Burkina Faso. It is the 3rd biggest site for gold exploration in Africa and 4th biggest gold producer in Africa. West African gold is not as high grade as gold from South Africa (www.theglobeandmail.com).

Gold Mining plays a significant role in Burkina Faso’s economy. Production of mineral commodities is limited to cement, dolomite, gold, granite, marble, phosphate rock, pumice, other volcanic materials, and salt (U.S. Geological Survey, 2007). Gold mining in Burkina Faso continues to grow rapidly, with more than fifteen major discoveries made since 2006. This could contribute to increase the amounts of natural activity and more explorers continue to flock in the region.
A recent report from publication AllAfrica revealed that the recent boom in the country’s gold mining industry in the last three years has made the country one of Africa’s leading producers and also asserted that it was luring a whole generation into the sector for work (AllAfrica, 2012).

![Simplified map of activity and mining potentials of Burkina Faso:](image)

Figure 2.1 Map showing mining potentials of Burkina Faso, (Ministry of Mines and Energy, 2014)

### 2.2 Contribution of gold mining in Burkina Faso’s economy

According to the Ministry of Finance, gold has become the top export commodity. In 2011, it earned Burkina Faso 127 billion CFA (US$247 million). Between 2007 and 2011, the country earned 440 billion CFA, accounting for 64.7 percent of all exports and 8 percent
of GDP. Production rose from 23 tons in 2010 to 32 tons in 2011. Gold mines are spread across the country’s northern, western, southwestern and central regions (AllAfrica, 2012). So the mine resources contribute to the development of the country but the exploitation could introduce some amounts of natural radioactivity in groundwater system.

2.3 Natural Radioactivity in the Environment

The naturally occurring radionuclides of terrestrial origin are present in various levels in all media in the environment, including living tissues (UNSCEAR, 2010). Irradiation of the human body from external sources is mainly by gamma radiation from radionuclides in the $^{238}$U and $^{325}$Th series as well as $^{40}$K. These are referred to as primordial NORMs and they include two categories, namely: series and non-series.

- **Non-series primordial** radionuclides are nuclei with very long half-lives that do not produce radioactive daughter nuclei, or have only two or three stages in their decay chain. The most common example to be found on earth is potassium-40.

- **Series NORMs** form long, complex decay chains, with their daughter nuclei decaying into further daughter nuclei, emitting gamma radiation (UNSCEAR, 2000). The primary examples on earth are the actinium chain, the uranium chain, and the thorium chain, and they are particularly important in understanding the relative quantities of each particular type of NORM. There is a fourth long decay chain known as the neptunium series that begins with neptunium-237, but due to the relatively short half life of neptunium and its relatively low abundance on earth, it is considered a negligible factor in the earth’s background radiation levels.
Radionuclides are present in the body and irradiate various organs with alpha, beta particles as well as gamma rays (UNSCEAR, 2000). Other terrestrial radionuclides, such as those of $^{235}$U series, $^{138}$La, $^{176}$Lu, $^{87}$Rb and $^{147}$Sm exist in nature but at low levels such that their dose contributions to humans are very small.

External exposure to gamma radiation outdoor comes from terrestrial radionuclides naturally occurring at trace levels at ground formations. Higher radiation levels are known to be associated with igneous rocks, such as granite, and low levels with sedimentary rocks. However, some phosphate and shale rocks are also known to have relatively high content of radionuclides (UNSCEAR, 2008; Florou and Kritidis, 1992). Thus, natural background radiation exposure depends on geological and geographical conditions (Florou and Kritidis, 1992).

The earth is continuously receiving cosmic radiation from the outer space. The radiation from the sun and the secondary radiation generated by the entry of charged particles to the earth’s atmosphere are at high speed and energy. Cosmic radiation is a mixture of photons, alpha particles, electrons and other high energy particles. Cosmogenic radionuclides are formed as a result of interactions between primary and secondary cosmic ray particles with atomic nuclei. Best known of these Cosmogenic radionuclides produced by interaction of galactic cosmic rays with earth’s atmosphere are $^3$H, $^{14}$C, $^{10}$Be, $^{53}$Mn, $^{41}$Ca, $^{26}$Al. These galactic cosmic rays incident on the top of the atmosphere consist of nucleonic component, which accounts for 98% of the total, and electrons which accounts for the rest of the 2%. The nucleonic component which is 88% is primarily protons, 11% of alpha particles and the remaining 1% percentage been heavier nuclei (UNSCEAR, 2008).
2.4 Radioactivity in Drinking Water

According to the World Health Organization, the recommended reference dose level (RDL) of committed effective dose is 100 μSv from 1 year’s consumption of drinking water, and for practical purposes, the recommended screening level for drinking water below which no further action is required are 0.1 Bq L⁻¹ for gross alpha activity and 1 Bq L⁻¹ for gross beta activity (WHO, 1998).

Also in the third edition of WHO, the recommended RDL and the recommended screening level for gross beta activity remained the same as those in the second edition, but the recommended screening level for gross alpha activity was revised from 0.1 to 0.5BqL⁻¹. Therefore, in drinking water the gross alpha, gross beta and tritium guideline values are 0.1, 1 and 50 Bq.L⁻¹ respectively with the exception of the gross alpha for samples originating from regions with high natural radioactive background (WHO, 2004).

The results of environmental monitoring programme have shown, over the years that the concentrations of the artificial radionuclides (¹³⁷Cs, ⁹⁰Sr and ³H) are, in general, very low and usually below the minimum detectable activity (0.3×10⁻³, 0.5×10⁻³ and 0.8BqL⁻¹ for ¹³⁷Cs, ⁹⁰Sr and ³H, respectively) (Madruga et al., 2007).

The dose contribution of drinking water is largely due to naturally occurring radionuclides in the uranium and thorium decay series, such as ²³⁸U, ²³⁴U, ²³⁴Th, ²³⁰Th, ²²⁶Ra, ²²²Rn, ²¹⁰Pb, ²¹⁰Po, and ²³²Th, ²²⁸Th, ²²⁸Ra and ²²⁴Ra (Jia and Torri, 2007) and so on. It is obvious that the dose contribution from α-emitters is the most important fraction from the drinking water intake. Due to the fact that the dose coefficients (DC) of different α-emitters can vary by several orders of magnitude, for instance
$D_{C_{U-238}} = 4.5 \times 10^{-8} \text{SvBq}^{-1}$, $D_{C_{Ra-226}} = 2.8 \times 10^{-7} \text{SvBq}^{-1}$, $D_{C_{Po-210}} = 1.2 \times 10^{-6} \text{SvBq}^{-1}$, taking a value of $0.5 \text{Bq L}^{-1}$ gross alpha activity as the screening level (UNSCEAR, 2000). This can lead to a possible underestimation of the committed effective dose and increase the exposure risk to the public if it is mainly composed of radium and/or polonium.

Although the activity concentrations from gross alpha and gross beta monitoring in drinking water can guide the competent authorities in determining whether the water is of appropriate quality for human consumption (EML, 1990). For accurate dose assessment, it is necessary to determine the activity concentrations of the specific radionuclides in drinking water. There are a variety of techniques for the specific radionuclide identification and determination, such as: α-spectrometry, liquid scintillation counting (LSC), gamma-ray spectrometry, etc.

### 2.5 Hazards and Risk associated with Exposure to NORM in the Mining Industry

According to the National Research Council (NRC) of USA, Risk is defined as the characterization of the potential adverse health effect of human exposures to environmental hazards (NRC, 1999). Due to the stochastic nature of the adverse effects of the exposure, together with their extremely low probability of occurrence, risk assessments/estimates have always been based on studies on large population groups using mathematical models. The most current of such studies is that by the U.S. National Academy of Sciences Committee on the Biological Effects of Ionizing Radiation (BEIR Committee) known as BEIR V Report and the latest version of BEIR VII: Health Effects of Exposure to low levels of Ionizing Radiation (NAS, 1990). This assessment was based on a review of new scientific information from several different studies including:
- Epidemiological studies of Japanese survivors of nuclear bombing during World War II;
- Radiation accidents;
- Patients who had been exposed to radiation during the course of their medical treatments;
- Laboratory studies on chemistry, physics and biology of ionizing radiation (Cember and Johnson, 2009).

Studies by American Health Physics Society have recommended quantitative estimation of radiation health risk below an individual dose of 50 mSv per year, additional to background radiation. The reason is that, there is no conclusive evidence of health risks for low dose rate up to 50 mSv/year (HPS, 1996).

It should however be noted that interaction of NORM with the cell can cause breaks in the strands of the Deoxyribonucleic Acid (DNA) molecule. This process can cause changes or damage to the DNA structure of the body. Biological effects of NORM are concentrated mainly on the effects of low doses of ionizing radiations. Low doses of ionizing radiation exposure has the probability of inducing cancer proportional to the dose received which show up after a latency period (IAEA, 2003).

Direct estimate of the risk associated with NORM from a combination of epidemiological and radiobiological studies at the molecular and cellular levels is a useful tool for elucidating the consequences of low doses of ionizing radiations.
The main effects that manifest in later life due to changes or damage to the DNA structure include.

- Changes in the genetic code resulting in the death of the cell progeny.

- Cancer induction due to damage to single cells. Cancer initiation involves a loss of regulation of growth or reproduction and development in somatic stem cells, i.e. loss of control over cell production cycle and differentiation process. Point mutation and Chromosomal abrasion or damage play important roles in the initiation of cancer formation (neoplasia).

- Hereditary effects which occur due to changes in genetic codes being transmitted.

This may become manifested as hereditary disorders in the descendants of the exposed individuals (IAEA. (2003).

2.6. Models of Radionuclide Transport in groundwater

2.6.1 Groundwater models

In general, models are conceptual descriptions or approximations that describe physical systems using mathematical equations – they are not exact descriptions of physical systems or processes. (MDEQ, 2014)

Groundwater models describe groundwater-flow and fate-and-transport processes using mathematical equations that are based on certain simplifying assumptions. These assumptions typically involve the direction of flow, geometry of the aquifer, the heterogeneity or anisotropy of sediments or bedrock comprising the aquifer, the
contaminant transport mechanisms, and chemical reactions. Because of the simplifying assumptions embedded in the mathematical equations and the many uncertainties in the values of data required by the model, a model provides predictions as an approximation and not an exact duplication of field conditions (MDEQ, 2014).

### 2.6.1.1 Groundwater-Flow Models

Groundwater-flow models are used to calculate the rate and direction of movement of groundwater through aquifers and confining units in the subsurface, and the exchange of groundwater between aquifers and sources and sinks, where groundwater is added or removed from the aquifer. These calculations are referred to as “simulations. The simulation of groundwater-flow depends upon a thorough understanding of the hydrogeologic characteristics of the facility and the surrounding area (MDEQ, 2014).

A groundwater-flow model simulates the following processes:

- Movement of groundwater through aquifers and confining layers,

- Addition of groundwater by sources such as precipitation, leakage from surface water bodies, injection wells, infiltration galleries, etc.

- Removal of groundwater by sinks such as pumping wells, drains, surface water bodies, interceptor trenches, etc., or

- The change in hydraulic-head and hydraulic gradients as a result of the addition or removal of groundwater by sources and sinks.
The outputs from groundwater-flow model simulations are the hydraulic-heads and groundwater-flow rates that are in equilibrium with the hydrogeologic conditions (hydrogeologic framework, hydrologic boundaries, initial and transient conditions, hydraulic properties, and sources or sinks) defined for the modeled area (MDEQ, 2014).

2.6.1.2 Fate-and-Transport Models

Fate-and-transport models simulate the migration and chemical alteration of contaminants as they move with groundwater through the subsurface. Fate-and-transport models rely on the development of a calibrated groundwater-flow model or, at a minimum, an accurate determination of the velocity and direction of groundwater-flow, which has been based on field data (MDEQ, 2014).

A fate-and-transport model may simulate the following processes:

- Movement of contaminants by advection and diffusion,

- Spread and dilution of contaminants by dispersion,

- Removal, or release, of contaminants by sorption, or desorption, of contaminants onto, or from, subsurface sediment or rock,

- Addition or removal of contaminants by contaminant sources or sinks, and

- Chemical alteration of the contaminant by chemical reactions which may be controlled by biological processes or physical-chemical reactions.
The outputs from the model simulations are the contaminant concentrations, which are in equilibrium with the groundwater-flow system, and the geochemical conditions (described above) that have been defined for the modeled area (MDEQ, 2014).

2.6.2. Types of Models

The equations that describe the groundwater-flow and fate-and-transport processes may be solved using different types of models;

2.6.2.1 Analytical models.

Analytical models are exact solution of a specific, greatly simplified, groundwater-flow or transport equation. The equation is a simplification of more complex three-dimensional groundwater-flow or solute-transport equations (MDEQ, 2014). Specifically, these simplifications resulted in reducing the groundwater-flow to one dimension and the solute-transport equation to one or two dimensions. This resulted in changes to the model equations that include one-dimensional uniform groundwater-flow, simple uniform aquifer geometry, homogeneous and isotropic aquifers, uniform hydraulic and chemical reaction properties, and simple flow or chemical reaction boundaries (MDEQ, 2014).

Examples of common analytical model codes that might be used for groundwater-flow or fate-and-transport simulations are: any of the well-hydraulics models (e.g. Theis equation), the Domenico model, BIOSCREEN, or BIOCHLOR.

2.6.2.2 Analytic Element Method Models

Analytic Element Method (AEM) models are computer codes that do not require the model domain to be discretized into network of grid cells or elements. The only discretization
involves representing surface-water features as arcs or polygons. The discharge potential for each of these arcs or polygons is represented by analytical solutions (elements) describing groundwater-flow or transport processes. AEM’s superpose the exact solutions for each element resulting in a solution to a more complex groundwater-flow or solute-transport problem (MDEQ, 2014).

Examples of analytic element model codes that might be used for groundwater-flow or fate-and-transport simulations are SLAEM, MLAEM, GFLOW, WinFlow, WhAEM2000, MODAEM, and CZAEM. (Haitjema, 1995; Strack, 1989).

2.6.2.3 Numerical Models

Numerical models are capable of solving the more complex equations that describe groundwater flow and solute transport. These equations generally describe multi-dimensional groundwater flow, solute transport, and chemical reactions, although there are one-dimensional numerical models. Numerical models use approximations (e.g. finite differences, or finite elements) to solve the differential equations describing groundwater flow or solute transport. The approximations require that the model domain and time be discretized. In this discretization process, the model domain is represented by a network of grid cells or elements, and the duration of the simulation is represented by a series of time (MDEQ, 2014).
Figure 2.2: A model domain represented by a network of grid cells or elements
(Introduction to Groundwater Modelling, C.P. Kumar, 2009)

Examples of some of the more common numerical model codes are MODFLOW,
BIOPLUME II, BIOPLUME III, MOC, SUTRA, and FEFLOW (MDEQ, 2014).
5.3 Dimensional Models

Table 2.1: Examples of different model dimensions (Barnett, 2012).

<table>
<thead>
<tr>
<th>Model dimension</th>
<th>Description</th>
<th>Range of application</th>
<th>Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1D)</td>
<td>Radial flow</td>
<td>Predicting responses to pumping.</td>
<td>1D models generally provide a simplified groundwater flow domain and geometry. They are useful to assess groundwater behaviour where such simplifications can be justified or where complex calculations are not required.</td>
</tr>
<tr>
<td></td>
<td>Horizontal flow models</td>
<td>Applications of Darcy’s Law.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Solute movement along a single flow</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Vertical slice or vertical section</td>
<td>Where vertical flow is important.</td>
<td>These models are ideally suited to assess vertical flow processes and are commonly used to model density-dependent solute transport.</td>
</tr>
<tr>
<td>model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Density-dependent solute transport</td>
<td>Because flow to a pumping well is usually radial in direction, a 2D vertical slice of unit width does not provide an appropriate geometry with which to model the convergence or divergence of flow to or from pumping or injection wells. Accordingly, the implementation of groundwater extraction and injection must be approached with caution. It is recommended that such models be avoided if the flow to and from extraction and injection wells is an important feature of the model. In this case a</td>
<td>models for sea water intrusion.</td>
</tr>
<tr>
<td></td>
<td>Aquifer geometry that allows the</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>definition of fluxes per unit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2D)</td>
<td>length of aquifer.</td>
<td>2D radial flow model (as described below) should be adopted.</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>------------------</td>
<td>-------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Areal flow model</td>
<td>Where vertical flow is not important.</td>
<td>These models assume that the aquifer is a 2D planar feature where groundwater flow is predominantly in the horizontal plane. This assumption is usually valid for aquifers that have a horizontal extent that is much larger than the aquifer thickness, and have such a high vertical hydraulic conductivity that vertical head gradients within the aquifer are negligible.</td>
<td></td>
</tr>
<tr>
<td>Radial flow</td>
<td>Flow to extraction and injection wells with negligible regional groundwater flow.</td>
<td>In this type of model the spatial domain is defined as a radial slice or wedge of defined angle with a source or sink at the centre. Such models are ideally suited to model flow towards or away from extraction or injection wells.</td>
<td></td>
</tr>
<tr>
<td>(2D)</td>
<td>Where flows occurs in all directions.</td>
<td>3D groundwater flow models are needed to simulate groundwater movement in both the horizontal and vertical planes and are required when there are several overlying hydrogeological units where horizontal flow in individual units and flow between adjoining units are important.</td>
<td></td>
</tr>
</tbody>
</table>
2.7 Some study approaches to solving the Advection Diffusion Equation

Analytical methods basically provide solutions to governing equations of groundwater flow and contaminant transport with simplified boundary conditions and hydrogeological and chemical properties. Limitations for analytical solutions include: assumptions of a uniform thickness of aquifer, a uniform hydraulic conductivity and specific yield, a single inland boundary condition, and a single-phase homogeneous fluid, among others (Ataie-Ashtiani et al. 1999).

It is extremely difficult or even impossible to derive analytical solutions to solute transport when a more complex system is considered. Thus, Numerical modeling has been given more and more attention recently only with the aid of computers. It is more powerful than analytical analysis of a complex system. Unfortunately, even though numerical simulation has the potential to undertake the tasks, a comprehensive simulation model that can account for all the factors has not yet been constructed. Still, numerous numerical models have been developed dealing with one or more facets concerning groundwater flow and contaminant transport in groundwater system. The major topics that are most frequently considered and intensively studied in numerical modeling include variable density flow and contaminant transport, tidally influenced water table fluctuation or periodic boundary conditions (Kolditz et al., 1998; Li et al., 1997).

One of the most successful models is the saturated-unsaturated transport finite-element ground-water simulation model (SUTRA), developed by Voss which can be used to simulate variable-density flow with chemically reactive single species contaminant transport in saturated-unsaturated formations (Voss, C.I., 1984). Later, Ataie-Ashtiani
modified SUTRA such that it can account for periodic boundary conditions and sloping beach faces, in addition to variable density and variably saturated flow (Ataie-Ashtiani et al., 1999).

The widely used models for groundwater flow and solute transport, such as MODFLOW and SUTRA, or the others developed by various authors, do not account for tidal fluctuations. SUTRA modified by Ataie-Ashtiani et al., (1999) can incorporate tidal fluctuations, it is based on a boundary condition describing contaminant concentration (Li et al. 1997).

The laboratory experiments, known as Intermediate Scale Experiments (ISEs) have been able to investigate contaminant advection, diffusion, dispersion, and fluid flow in saturated and partially saturated systems, flow and transport in homogeneous and heterogeneous media, transport of contaminants under uniform- and variable-density flow fields, multiphase transport, chemical reactions, particle transport, and microbial interactions, as summarized by Silliman (Silliman, 1998). The few studies include those done by Volker and Zhang (Li et al., 1997; Zhang et al., 2002).

2.7.1 Numerical Approaches for Solving Partial Differential Equations

2.7.1.1 Classical Random Walk

Advection –dispersion equation can be approached using either Lagrangian or Eulerian point of view (Schumer et al., 2009).
2.7.1.2 The Lagrangian Approach to Advection –Diffusion Equation

Due to the difficulty in solving the advection- diffusion equation analytically, it is necessary to resort to numerical methods such as finite difference or finite element methods which are able to incorporate data only as boundary conditions or as calibration parameters (Tandon, 2000).

Based on experimental and theoretical results, the three-dimensional advection-diffusion equation has been solved by the Lagrange approach where the pollutant is represented by a predetermined large number of tracer particles. At any desired time interval, pollutant concentrations are calculated using a local mass balance (Lorin, 1999; Tsanis and Valeo, 1994). Despite other numerical methods that simulate the contaminant spreading by calculating concentration in a mesh nodes, the particle tracer model tracks several individual particles and then calculates the concentration. The main advantage of the Lagrangian technique over the other widely used numerical methods is the elimination of numerical diffusion but only when a high resolution scheme is used to solve the equations (García and Rodríguez, 1998). It should be noted that the Lagrangian approach does not solve the advective -diffusive transport equation directly.

2.7.1.3 Eulerian Approach to Advection-Diffusion Equation

The Eulerian approach to Advection-Diffusion Equation is well adapted to transport problems. It preserves the performance of characteristic methods and treats general boundary conditions naturally in their formulations. The approach chooses a particle in a specific location in space and describes particle motion through that location with time. A conservative particle mass equation that relates the rate of mass change at that location with
the difference between the mass of particles entering and leaving the location is employed (Schumer, 2009).

\[ n \frac{\partial C}{\partial t}(x,t) = - \frac{\partial F(x,t)}{\partial x} \]  

(2.1)

Where, effective porosity \( n = 1 \) for transport at the surface, particle concentration \( C \) is mass per unit volume, and flux \( F \) is the mass per unit area per unit time. If particle flux is assumed by advection and Fickian dispersion,

\[ F = vnC - nD \frac{\partial C}{\partial x} - \lambda C \]  

(2.2)

Where, \( v \) is average particle velocity, \( \lambda \) is the decay constant of the radionuclide and D is the dispersion coefficient. From (2.1) and (2.2) the advection diffusion in one dimension can be written as

\[ \frac{\partial C}{\partial t} = -v(x,t) \frac{\partial C}{\partial x} + D(x,t) \frac{\partial^2 C}{\partial x^2} - \lambda C \]  

(2.3)

2.7.1.4 Coupling geological and numerical models to simulate groundwater flow and contaminant transport in fractured media

Transport processes in fractured rocks include molecular diffusion, mechanical dispersion and advection. In the rock matrix, molecular diffusion usually dominates over advection, such that the porous rock mass may attenuate and retard the advective propagation of contaminants along fractures (Tang et al., 1981; Therrien and Sudicky, 1996).

The 3D geometrical representation of subsurface geological structures, including fractures, is called the geological model, or the Geomodel (Mallet, 2002). A mesh generation phase,
representing the connection between the geological and the numerical models, is necessary to discretize the Geomodel.

2.8 Mathematical modelling

The numerical code selected is HydroGeoSphere (Therrien et al., 2007).

HydroGeoSphere is a numerical simulator specifically developed for supporting water resource and engineering projects pertaining to hydrologic systems with surface and subsurface flow and contaminant transport components. It has been developed by Therrien et al. (2007), who extended the FRAC3DVS code to accommodate surface water flow and contaminant transport.

The numerical implementation is presented in Therrien et al. (2007) and Therrien and Sudicky (1996). The governing equation for 3D fully saturated groundwater flow in the porous matrix, without sources or sinks, is

\[
\frac{\partial}{\partial x_i} \left( \frac{K_{ij}}{\partial t} \right) = \frac{\partial S_j}{\partial t}, \quad i, j = 1, 2, 3
\]  

(2.4)

Contaminant transport in 3D porous media is described by

\[
\frac{\partial C}{\partial t} + \frac{v_i}{R} \frac{\partial C}{\partial x_i} - \frac{\partial C}{\partial x_i} \left( \frac{D_{ij}}{R} \frac{\partial C}{\partial x_j} \right) + \lambda C = 0, \quad i, j = 1, 2, 3
\]

(2.5)

The standard Galerkin technique is used to discretize the above equations (Therrien et al., 2007; Therrien and Sudicky, 1996). Using an approximation of the time derivative by a
finite difference representation and a lumped mass approach to treat the storage terms, the
discretized porous medium flow equation becomes

\[ \sum_{\eta_i} \left( h_i^{t+1} - h_i^t \right) \frac{V_i}{\partial \tau} = \sum_{\gamma_{ij}} \left( h_j^{t+1} - h_j^t \right), \quad i, j = 1, 2, 3 \]  

(2.6)

Where, parameter \( \eta_i \) is defined as being the set of nodes connected to node \( i \) and the term \( \gamma_{ij} \) contains the integral of the standard finite element basis functions that depend on the
element type. The term \( \gamma_{ij} \) is sometimes referred to as the transmissibility (Letniowski and Forsyth, 1991; Gable et al., 1996b), and Eq. (2.6) indicates that a negative transmissibility value will cause fluid flow from nodes with lower hydraulic heads towards nodes with higher hydraulic heads, which is physically unrealistic.

2.8.1 CVFE method and the fluid conductance matrix

The Control-Volume Finite Element method applied to numerical modeling in
hydrogeology is discussed in Letniowski and Forsyth (1991). In this method, a finite
volume subgrid is constructed as a complement to the finite element grid (Geiger et al.,
2004). The CVFE method combines the flexibility of a finite element method with a local
conservation property, which is typical of finite volume schemes. To evaluate the fluid
conductance matrix, the influence coefficient technique proposed by Huyakorn et al.
(1984) is used. The technique was first developed for linear rectangular elements and then
applied to 3D blocks and prism elements (Huyakorn et al., 1986; Huyakorn et al., 1987;
It provides a rapid valuation of fluid conductance matrix coefficients, without requiring numerical integration phases and therefore reducing computation effort Huyakorn et al., 1984). The integral of the basic functions, represented by \( \gamma_{ij} \) in Eq. (2.6), is given by

\[
\gamma_{ij} = \int \nabla N_i K \nabla N_j \, dv
\]  

(2.7)

Simplex tetrahedra have been chosen here. Simplex elements have linear sides and linear polynomials as interpolation function. For a tetrahedron whose vertices are i, j, k and l, the following known shape function is used (Allaire, 1985):

\[
\{N\} = \frac{1}{6V} \{1, x, y, z\} \begin{bmatrix}
  a_i & a_j & a_k & a_l \\
  b_i & b_j & b_k & b_l \\
  c_i & c_j & c_k & c_l \\
  d_i & d_j & d_k & d_l 
\end{bmatrix}
\]  

(2.8)

where \( V \) is the tetrahedron volume, calculated from the four nodes coordinates. After evaluating the Jacobian matrix, or element derivative matrix, the elemental stiffness matrix can be calculated as

\[
[A]^{(e)} = K_{xx}[A_{xx}] + K_{yy}[A_{yy}] + K_{zz}[A_{zz}]
\]  

(2.9)

Matrix A is called the stiffness matrix. This matrix may be an M-matrix, which is a real, square, nonsingular matrix A, whose off-diagonal elements \( ij \) are either zero or negative and whose diagonal elements are strictly positive. The diagonal element \( \gamma_{ii} \) in the control-volume discretization is the sum of the absolute values of the other entries in row i, causing
the matrix to be diagonally dominant. M-matrix properties can be summarized as follows (Kosik et al., 2000):

\[
\begin{cases}
\gamma_{ij} \leq 0 \\
\gamma_{ij} \geq -\sum_j \gamma_{ij}, \quad \forall i \neq j \\
A^{-1} \geq 0
\end{cases}
\] (2.10)

Coefficients \(\gamma_{ij}\) of the fluid conductance matrix \(A\) are called transmissibilities (Letniowski and Forsyth, 1991; Letniowski, 1992). Every \(\gamma_{ij}\) constitutes the entry \((i,j)\) in the fluid conductance matrix \(A\) and corresponds to the tetrahedral edge of extremities \(i\) and \(j\). Therefore, the flux between nodes \(i\) and \(j\) is given by

\[
Q_{ij} = \gamma_{ij}(h_j - h_i)
\] (2.11)

The Laplace operator of the partial differential equation is often discretized using the Galerkin method, but in the case of a 3D tetrahedral mesh it does not lead to an M-matrix. Therefore, it is interesting to analyze the orthogonal subdomain collocation (OSC) method proposed by Putti and Cordes (1998, 2000). The elemental stiffness coefficient can be obtained as the negative ratio between the area of the Voronoi cell face \(F_{ij}\) of the tetrahedron and the length of the corresponding element edge \(r_{ij}\) (Putti and Cordes, 1998)

\[
\gamma_{ij}^{osc} = -\frac{F_{ij}}{r_{ij}}
\] (2.12)

The expression for the first element \(ij\) would be (Cordes and Putti, 2001; 1998)
\[ \gamma_{ij}^c = \frac{1}{48V} \left[ 2(r_{ik}r_{jk})(r_{ii}r_{jj}) + A_x A_y \left( \frac{(r_{il}r_{lk})^2}{A_x A_y} + \frac{(r_{jl}r_{jk})^2}{A_x A_y} \right) \right] \]  

(2.13)

Transmissibilities are evaluated with Eq. (13) and an M-matrix is obtained.

### 2.9 Model verification

#### 2.9.1 Test case 1: Groundwater flow and solute transport in a horizontal fracture

In this example, the propagation of uranium, in its isotope form U\(^{234}\), is simulated along a horizontal fracture embedded in a porous rock matrix. The horizontal fracture has been created in GOCAD and then imported into LaGriT (Therrien and Sudicky, 1996)

![Figure 2.3: Model design for test case 1: transport along a horizontal fracture](image)

Figure 2.3: Model design for test case 1: transport along a horizontal fracture
2.9.2. Test cases 2: Transport in a single inclined fracture

The refined mesh (Figure 2.5) has 3609 nodes and 14,279 tetrahedra. The block-based and tetrahedral mesh simulations both consider a constant concentration of contaminant equal to 1.0 on the top of the domain (Figure 2.6). All other boundary areas are signed zero dispersive flux for transport. An inclined fracture crosses the whole domain. Specified head conditions are imposed at x = 0 and x = 12, with a hydraulic head difference equal to 0.5m.
Figure 2.5: Tetrahedral mesh with refinement around an inclined fracture for case 2

Figure 2.6: Model design for test cases 2: transport along a single inclined fracture.

The surrounding rock matrix is considered impermeable, such that the solute propagation along the fracture could be evaluated with the simplified Ogata–Banks analytical solution applicable when the observation point is far from the source of solute, and which expresses concentration as
The velocity along the fracture is 0.00093 m/s. Simulation results are shown in Figure 2.7

\[ C(x,t) = \frac{C_0}{2} \text{erfc}\left(\frac{x-vt}{2\sqrt{D}t}\right) \]  

Figure 2.7: Results test case 2: tetrahedral mesh, block-based mesh and analytical solution.

The solute break through curves computed at the observation point show that the tetrahedral mesh gives a very good approximation of the Ogata–Banks analytical solution, even better than the Blocks_Internal_Faces solution. The Blocks Stairway configuration is obviously the worst one, as the travel distance is lengthened compared to a planar fracture. Thus, the mesh generation process adopted here leads to a better representation of inclined fractures.
CHAPTER THREE

MATERIALS AND METHODS

This section explains the location of the study area; the sampling procedure; samples preparation and the analysis used in the study. Experimental method and theoretical method were used to determine the concentrations of NORMs at a distance from the mine representing the location of boreholes and wells. The experimental method describes the sampling, sample preparation procedure and analysis of water samples from boreholes in the vicinity of the mine. The theoretical model is formulated in order to assess the NORM discharges at the TAPARKA goldmine. A computer program is written in MathLAB to implement the solution.

3.1 Description of the Study Area

3.1.1 Location and Size

The Taparko mine is located in the Namantenga province of Burkina Faso in the Centre-North Region, approximately 200km by road of Ouagadougou, the Capital City of Burkina Faso at a latitude of 13° 27'55.28'' N and a longitude of 0° 19'27.99'' W. The mine is operated by SOMITA SA, owned by High River Gold, a Canadian company. Taparko is in the Department of Yalgo which belongs to Namantenga province. The population of Yalgo is about 31,641 (INSD, 2006) with an area of 576 km². The estimated population of the Region is 1,202,025 (INSD, 2006) with an area of 19677 km².

The figure 3.1 is done by using the software « Arcview 3.2 » with the Samples location coordinates (APPENDIX G). It shows the sampling points around the Taparko- goldmine.
3.1.2 Climate & Vegetation

The tropical climate is characterized by alternating dry period approximately 8 months and a wet period for 4 months, commissioned by the monsoon (PRB.MED, 2005). Temperatures, depending on the latitude, are always high, between 17 and more than 40°C. There are three periods:

- Dry and cool, 17-30° C, as the Harmattan blows from the northeast, from October to February,

- Hot and dry from March to June, temperatures reach 40°C or more

- And the rainy season, called "wintering" from June to September.
In the rainy season, rainfall is very strong and important (average 600 to 750 mm of water in the Center-North Region) [PRB.MED, 2005]. They feed surface water, rivers, reservoirs and reserves underground. They also sometimes cause floods and still significant soil erosion.

In the dry season, evaporation is intense because of the heat and the creeks dry up.

Three vegetation standards: forest steppe, shrub land and savanna and there are 4 classified forests, relict forests and departmental forests. There is a strong biological diversity (PRB.MED, 2005).

3.1.3 Geology and Soil

The base of Namentinga belongs to two geological eras:

- Inferior precambrian formations (Anttebirimian) represented by compound gneisses with biotites and amphiboles and compound gneisses and granites.
- those of mean Precambrian (Birimian) represented by the plutonic rocks and the volcanogenic rocks. The plutonic rocks consist of granites with biotites. You can see in some places some outcrops of rocks in the forms of "back of whale". The volcanogenic rocks are made up of metavolcanites neutral to basic, the tuffs, the lava and sediments associated. To these geological formations are added the argillaceous alluvia of the great rivers and the bafonds (MPS-NP, 2001).

The area of the Northern Center counts 6 principal types of soil: lithosols on armour, brown soil eutrophic on basic rocks, washed tropical ferruginous soil, hydromorphic soil, not very advanced soil of gravillonnaires erosion, hydromorphic sodic soil. (PRB.MED, 2005).
Figure 3. 2: Geology of Burkina Faso showing Taparko

Figure 3. 3: Local Geology of Taparko area (Nordgold, Nov 2013)
3.1.4 Hydrography & Drainage

The hydrographic network of the area is organized around two principal basins: That of Nakambé and the under-basin of Niger which collect principal water of the area and drain them towards the principal rivers. The area shelters 98 reserves of water. About potentialities of the subsoil water resources, the area of the Northern Center is one of most underprivileged (PRB.MED, 2005).

3.2 Methods

Two methods are used in this study: experimental method and theoretical method.

3.2.1 Experimental method

3.2.1.1 Sampling

The groundwater samples were collected from water reserves inside the mine site and from the boreholes in Taparko village around the site. The sampling bottles (one and half liter polyethylene container) were rinsed three (3) times with the water to be sampled, followed by filling the bottle with water to the brim (Donkor et al., 2006; Serfo-Armah et al., 2004). Each water sample was acidified with two (2) drops of concentrated 65% HNO₃ just after the collection of the water samples. This was to prevent adsorption of the radionuclides on the walls of the container. Then water samples were tightly sealed to avoid spill during transport from sampling points in Burkina Faso to the GAEC laboratory in Ghana for preparation and analysis.

The Global Positioning System (ashtech, MobileMapper 10.) was used to take the reading for the sampling points and recorded in the field after collection of the samples.
3.2.1.2 Sample preparation

Each of the sampled borehole water is used to fill one liter of a Marinelli beaker and tightly sealed to cut off the background radiation. The water samples were analysed for $^{238}$U, $^{232}$Th and $^{40}$K activity concentration using a Gamma Spectrometry with High Purity Germanium detector.

Figure 3. 4: Sampling of water from borehole

Figure 3. 5: Sample preparation for analysis
3.2.1.3 Description of the Gamma Ray Spectrometer System

The gamma ray spectrometry system used for this study consists of a High Purity Germanium (HPGe) detector with the following characteristics (Canberra detector model GX4020, cryostat model 7500SL and preamplifier model 2002CSL). It has a diameter of 60.5 mm, length of 61.5 mm. The resolution of the detector is 2.0 keV and relative efficiency of 40% for 1.33 MeV gamma energy of $^{60}$Co. The output from the detector is connected to a desk top computer provided with “Gennie 2000”configuration software for spectrum acquisition and evaluation. The detector is surrounded by a lead shield (100 mm) on all sides to reduce the background radiation level of the system, and lined inside with copper, cadmium and plexiglass (3 mm each) sheets to minimize the X-rays emitted due to interaction of cosmic radiation with lead.

The detector is cooled in liquid nitrogen at a temperature of -196 °C (77 k). In order to determine the background distribution in the environment around the detector, 10 empty Marinelli beakers were thoroughly cleaned and filled with distilled water and counted for 36,000 s in the same geometry as the samples. The background spectra were used to correct the net peak area of gamma rays of measured isotopes. The background spectra were also used to determine the minimum detectable activities of $^{238}$U, $^{232}$Th and $^{40}$K of the detector.

3.2.1.4 Energy Calibration

Energy calibration was done by matching the energies of major gamma-rays in the spectrum of the standard reference material (Appendix B) to the channel number of the spectrometer. This was done by a computer with installed “Gennie 2000”software. The
relationship relating the energy and channel number is given by Gilmore et al (1995),

\[ E = a_1 X + a_0 \]  

(3.1)

Where, \( E \) is the energy of the radionuclide used for the calibration in keV, \( a_0 \) and \( a_1 \) are the calibration constants for the selected geometry and \( X \) is the channel number for the given radionuclide.

The energy calibration curve is given in figure 3.6

![Energy Calibration Curve](image-url)

**Figure 3.6**: Energy Calibration Curve
3.2.1.5 Efficiency Calibration

The efficiency calibration was carried out by acquiring a spectrum of the calibration standard (Appendix B) until the count rate at the peak of total absorption can be calculated with statistical uncertainty of less than 1% at a confidence level of 95%. The net count rate was determined at the photo peaks for all the energies to be used for efficiency calibration standard at the time of measurement. The efficiency of each energy peak was plotted as a function of the peak energy and extrapolated to determine the efficiencies at other peak energies for the measurement geometry used. The expression used to determine efficiency is given in relation (3.1) (Vimalnath et al., 2005).

\[
EFF = \frac{\text{Net.Area}}{A_{st} \times p_{\gamma} \times T_{st}}
\]  

(3.2)

Where, \(EFF\) is the efficiency of the detector, \(A_{st}\) is the activity (Bq) of the radionuclide in the calibration standard at the time of calibration, \(p_{\gamma}\) is gamma emission probability for energy \(E\) and \(T_{st}\) is the counting time of the standard time. The efficiency is related to the energy by the expression,

\[
\ln(EFF) = a_0 + a_1 \ln E + a_2 \ln E^2
\]  

(3.3)

Where, \(a_0, a_1\) and \(a_2\) are calibration constants for a given geometry. The efficiency calibration curve is shown in figure 3.7
3.2.1.6 Determination of Activity Concentration of Radionuclides

From the spectrum analysis, count rates for each detected photo peak and activity per unit volume (specific activity) for each of the detected nuclides are calculated. The specific activity (in $\text{Bq.L}^{-1}$), $A_{Sk}$, of a nuclide $k$, and for a peak at energy $E$, is given by:

$$A_{Sk} = \frac{(\text{NetArea})_{Ek}}{\varepsilon_{Ek} \times p \times T} \quad (3.4)$$

$$y = -0.0257x^4 + 0.744x^3 - 7.954x^2 + 36.472x - 63.559$$

$R^2 = 0.9968$
Where, \( \varepsilon_{Ek} \) is the detection efficiency at energy \( E \), \( T \) is the counting live time, \( P_\gamma \) is the number of gamma-rays per disintegration of this radionuclide for a transition at energy \( E \) of the measured sample of water (Darko et al., 2003). If there is more than one peak in the energy analysis range for a nuclide, then an attempt is made to average the activities for the peak. The result is then the weighted radionuclide’s average activity concentration. The activity concentrations of \(^{238}\text{U}\) and \(^{232}\text{Th}\) in samples collected were determined using the measured \( \gamma \)-ray photo peaks, emitted by specific radionuclides in their decay series whereas the activity concentrations of \(^{40}\text{K}\) is calculated from the measured \( \gamma \)-ray photo peaks directly. In other words, the activity concentration of \(^{238}\text{U}\) was calculated from the average of 609.31 keV of \(^{214}\text{Bi}\) and 1764.5 keV of \(^{214}\text{Bi}\), \(^{232}\text{Th}\) from the average of 2614.53 keV peak of \(^{208}\text{Tl}\) and 911.2 keV of \(^{228}\text{Ac}\), and \(^{40}\text{K}\) was determined from 1460.0 keV.

### 3.2.1.7 Calculation of Annual Committed Effective Dose due to Drinking Water

The committed effective doses (E) were estimated from the activity concentrations of each individual radionuclide by applying the annual water consumption rate for adults of 730 L/year (2 L/day multiplied by 365 days) and the dose conversion factors of \(^{238}\text{U}\), \(^{232}\text{Th}\) and \(^{40}\text{K}\) taken from the BSS and UNSCEAR report, (IAEA, 1996; UNSCEAR, 2000) using the equation below. The total annual effective dose \( D \) (Sv y\(^{-1}\)) to an individual was established by summing contributions from all radionuclides present in the water samples,

\[
E_{ing} = \sum A_{sp} \times I_W \times C_f
\]  

(3.5)
Where, $A_{sp}$ is the activity concentration of the radionuclides in the water samples in Bq/L, $I_w$ is the annual intake of water in liters per year, and $C_f$ is the ingestion dose coefficient in Sv/Bq (UNSCEAR, 2000).

### 3.2.2 Theoretical Method

#### 3.2.2.1 Conceptual Model of the Research Problem

The level of radioactivity in groundwater system may increase as a result of possible migration of radionuclides to groundwater due to the processing activities of the mines. As the groundwater flows, the contaminant is carried out from boreholes and wells because most of the time as the groundwater serves as the major source of water for drinking, household purposes and irrigation.

Figure 3.8: Descriptive representation of groundwater system incorporating interpretation of geological & hydrological conditions (Groundwater Modeling-1, Groundwater Hydraulics, Daene C. McKinney, 2010)
At most sites, the migration rates and contaminant concentrations are influenced by several parameters and flow and transport processes that typically were not fully characterized in the early phase of the investigation. The parameters include recharge, hydraulic conductivity, effective porosity, hydraulic gradient, and distribution coefficients, thicknesses of the aquifer and confining unit, and source concentrations. During this early phase, questions pertaining to flow and transport processes typically are limited to general considerations, such as whether flow and transport are controlled by porous media or fractures, and whether the wastes are undergoing transformation from one phase to another (e.g., liquid to gas). In this study, the focus was on the influence of porosity and hydraulic conductivity.

### 3.2.2.2 Assumptions made in the Model

In this model, equation for transport of radionuclides in groundwater was used (Lovanh, 2000). The following assumptions were made:

- The contaminant concentration at initial time is neglected (i.e., $C(x, 0) = 0$);
- The aquifer is homogenous and isotropic, i.e., $K$ is constant in all directions;
- The groundwater flow characteristics (e.g. seepage velocity, groundwater depth, groundwater width, etc.) do not change significantly with distance and time;
- Porosity and dispersion coefficients are constant;
- Adsorption is a reversible process at equilibrium, represented by a linear isotherm.
3.2.2.3 Mathematical Model Formulation of Groundwater Flow and Fate Transport

a. Derivation of the governing equation

In general, the flow and contaminant transport equations are developed from the fundamental principle, namely; the conservation of fluid and dissolved mass. The general conservation equation can be expressed as (GWM-SRA, 1990),

\[ \text{Rate of mass input} - \text{rate of mass output} + \text{rate of mass production/consumption} = \text{rate of mass accumulation} \]

The differential equations describing the transport of contaminant in groundwater are also developed from conservation statement. Let the flux of a particular dissolved constituent into and out of a volume element of porous medium be represented by \( J \). The continuity equation has the following form (GWM-SRA, 1990).

\[
\frac{\partial}{\partial x} (J_x) + \frac{\partial}{\partial y} (J_y) + \frac{\partial}{\partial z} (J_z) \pm r = \frac{d(\varepsilon C)}{dt} \quad (3.6)
\]

Where

\( J_k \): flux of a particular dissolved constituent in a volume element of a porous medium in the direction (\( k=x,y,z \))

\( \varepsilon \): Porosity

\( r \): source term within the volume

\( C \): mass concentration

Mathematically the advective flux of a contaminant is described by the following equation:

$$ J_x = V_x \varepsilon C $$

(3.7)

Where, $V_x$ is the mean groundwater velocity or seepage velocity in the x direction.

The dispersion component is described by Fick’s Law represented by the equation (3.8) below

$$ J_x = -\varepsilon D_x \frac{\partial C}{\partial x} $$

(3.8)

Where, $D_x$ is the dispersion coefficient in the x direction.

The combined effects of advection and dispersion can be accounted for simply by adding (3.7) and (3.8) to give

$$
\begin{align*}
J_x &= V_x \varepsilon C - \varepsilon D_x \frac{\partial C}{\partial x} \\
J_y &= V_y \varepsilon C - \varepsilon D_y \frac{\partial C}{\partial y} \\
J_z &= V_z \varepsilon C - \varepsilon D_z \frac{\partial C}{\partial z}
\end{align*}
$$

(3.9)

Substituting (3.9) into (3.6) in gives:

$$ - \left[ \frac{\partial}{\partial x} \left( V_x \varepsilon C - \varepsilon D_x \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( V_y \varepsilon C - \varepsilon D_y \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left( V_z \varepsilon C - \varepsilon D_z \frac{\partial C}{\partial z} \right) \right] \pm r = \frac{\partial}{\partial t} (\varepsilon C) $$

(3.10)
Taking account of the assumptions made and for 1-D in x-direction, the following is obtained

\[
\frac{\partial C}{\partial t} = -V_x \frac{\partial C}{\partial x} + D_x \frac{\partial^2 C}{\partial x^2} \pm \frac{r}{\varepsilon}
\]  

(3.11)

For radionuclide decay and the mass transport accompanied by sorption that is described in terms of a simple linear isotherm, the source term is given respectively by:

\[
r = \frac{d}{dt(xC)} = -\lambda \delta C
\]

and

\[
-r = \rho \frac{\partial S}{\partial t} = \rho K_d \frac{\partial C}{\partial t}
\]

(3.12)

Where,

\( \lambda \) is the radionuclide decay constant related to the half-life for decay

\( \rho \) is the bulk density of the medium

\( S \) is the quantity of mass sorbed on the surface, and

\( K_d \) is the distribution coefficient.

Substituting (3.12) into (3.11) and rearranging gives a resulting constituent transport equation in 1-D for a continuous release as:

\[
R \frac{\partial C}{\partial t} + V_x \frac{\partial C}{\partial x} = D_x \frac{\partial^2 C}{\partial x^2} - \lambda RC
\]

(3.13)

Where, \( C \) - Radionuclide concentration (Bq/m³)

\( V_x \) - mean groundwater velocity or seepage velocity in x direction (m/s)
\( x \) - Longitudinal direction (m),

\( D_x \) - dispersion coefficients in \( x \) directions (m\(^2\)/s)

\( \lambda \) - Radionuclide decay constant (s\(^{-1}\))

\( t \) - time (s)

\( R \) - Retardation factor is given by

\[
R = 1 + \frac{\rho K_d}{\varepsilon} 
\]

(3.14)

The elements of the formula (3.14) are defined before.

Equation (3.13) is solved using the following initial and boundary conditions:

**b. Initial Condition and boundary conditions**

**Initial conditions**

The initial conditions considered are

\[
C(x, t)_{t=0} = 0 \quad 0 \leq x \leq L 
\]

(3.15)

**Boundary Conditions**

In general the boundary conditions used in the contaminant-transport equation are three types. They can be described as follows (Yuan Ding and Yong Peng, 2009).

1-The first-type (Dirichlet) boundary condition specifies the value of the concentration along a cross-section of the flow and transport boundary as

\[
C(x, t)_{x=\text{boundary}} = y(t) \quad x = 0 \quad 0 \leq t \leq T 
\]

(3.16)

Where, \( y(t) \) is the measured concentration in the effluent water as a function of time.

2-The second-type (Neumann) boundary condition specifies the gradient in solute concentration along a cross-section of the boundary as
\[
\frac{\partial C}{\partial x}_{x=\text{boundary}} = s(t) \quad x = L \quad 0 \leq t \leq T \quad (3.17)
\]

Where, \(s(t)\) is the measured concentration gradient in the effluent water as a function of time.

3- The third-type (Cauchy) boundary condition specifies the flux of solute across the boundary as

\[
(V_x C - D_x \frac{\partial C}{\partial x})_{x=\text{boundary}} = V_x g(t) \quad x = L \quad 0 \leq t \leq T \quad (3.18)
\]

Where, \(g(t)\) is the measured concentration in the flow as a function of time (Yuan Ding and Yong Peng, 2009). These three types of boundary conditions are used to describe the conditions at the effluent ends of a flow system. In this study, the effluent boundary is generally assigned with the second-type boundary condition to specify a constant or variant effluent of contaminant source.

Application of the first-type boundary condition presumes that the concentration gradient across the boundary equals zero as soon as flow begins, which may lead to overestimation of the mass of contaminant in the system at early times. The second and third-type boundary conditions allow for contaminant concentration at the effluent boundary to be lower than the effluent concentration initially, and then to increase as more contaminant enters the system. Over time, the concentration gradient across the boundary decreases as the concentration at the effluent boundary approaches the effluent concentration, which eventually reaches the case of the first type boundary condition (Yuan Ding and Yong Peng, 2009).
For this study the boundary conditions used are:

\[ C(0,t) = C_0 \quad 0 < t_0 \leq t \]

\[ C(0,t) = 0 \quad t > t_0 \] \hspace{1cm} (3.19)

\[ C(L,t) = 0 \]

c. Contaminant Input

The first or the second-type boundary condition is generally used to specify a constant input of contaminant source. For example, a continuous contaminant input at the effluent boundary at a constant rate may be simply represented by the equation below, assuming it is well-mixed instantly over the entire vertical section,

\[ C(0,t) = C_0 \] \hspace{1cm} (3.20)

And, a pulse-type input can be represented by

\[ C(0,t) = \begin{cases} 
C_0, & 0 \leq t \leq t_0 \\
0, & t \geq t_0 
\end{cases} \] \hspace{1cm} (3.21)

Where, \( t_0 \) is the time span of the contaminant release (assuming release starts at time zero).

d. Discretization of the differential equation

The partial differential equation in (3.13) above was solved by employing finite difference method to obtain a solution. The finite difference method was used because it allows to approximate the derivatives appearing in the equation by a set of values of the function at
a selected number of points. The time and distance were discretized using Euler’s forward in time and centered difference in space, a matrix was then generated to determine the concentration of natural radionuclides in the Taparko groundwater system.

**e. Euler’s explicit Method**

Equation (3.13) can be solved by employing a finite difference mesh and discretizing the distance and time axes into intervals of $\Delta x$, and $\Delta t$ respectively. A mesh of uniformly spaced grid-lines were introduced as:

$$
\begin{align*}
  x_i &= i\Delta x \\
  t^n &= n\Delta t \\
  i &= 1,2,3...I \\
  n &= 1,2,3...N
\end{align*}
$$

(3.22)

Where, $I$ denote the total number of spatial grid-spacing and $N$ denotes the total number of temporal grid-spacing.

![Model of domain discretization](image-url)
Now, approximating the derivatives using Euler’s forward difference in time and centered difference in space at point O, the following is obtained

\[
\frac{\partial C}{\partial t} = \frac{C_{i}^{n+1} - C_{i}^{n}}{\Delta t} \quad (3.23)
\]

\[
\frac{\partial C}{\partial x} = \frac{C_{i+1}^{n} - C_{i-1}^{n}}{\Delta x} \quad (3.24)
\]

\[
\frac{\partial^2 C}{\partial x^2} = \frac{C_{i+1}^{n} - 2C_{i}^{n} + C_{i-1}^{n}}{\Delta x^2} \quad (3.25)
\]

Inserting (3.20), (3.21) and (3.22) into (3.13) gives

\[
R \frac{C_{i}^{n+1} - C_{i}^{n}}{\Delta t} + V_{x} \frac{C_{i+1}^{n} - C_{i-1}^{n}}{\Delta x} = D_{x} \frac{C_{i+1}^{n} - 2C_{i}^{n} + C_{i-1}^{n}}{\Delta x^2} - R \lambda C_{i}^{n} \quad (3.26)
\]

Multiplying through by \(\Delta t\), dividing by \(R\) and rearranging gives,

\[
C_{i}^{n+1} = (1 - \lambda \Delta t) \frac{2D_{x} \Delta t}{R \Delta x^2} C_{i}^{n} + \left(\frac{V_{x} \Delta t}{R \Delta x} + \frac{D_{x} \Delta t}{R \Delta x^2}\right) C_{i+1}^{n} + \left(\frac{V_{x} \Delta t}{R \Delta x} + \frac{D_{x} \Delta t}{R \Delta x^2}\right) C_{i-1}^{n} \quad (3.27)
\]

Equation (3.24) can be rewritten simply:

\[
C_{i}^{n+1} = AC_{i-1}^{n} + BC_{i}^{n} + DC_{i+1}^{n} \quad (3.28)
\]

Where,

\[
A = \frac{V_{x} \Delta t}{R \Delta x} + \frac{D_{x} \Delta t}{R \Delta x^2} \quad ; \quad B = 1 - \lambda \Delta t - \frac{2D_{x} \Delta t}{R \Delta x^2} \quad \text{And,} \quad D = \frac{V_{x} \Delta t}{R \Delta x} + \frac{D_{x} \Delta t}{R \Delta x^2}
\]

Applying the initial and boundary conditions, the following matrix form is obtained
\[
\begin{pmatrix}
C^{n+1}_1 \\
C^{n+1}_2 \\
\vdots \\
C^{n+1}_{i-1} \\
C^{n+1}_i \\
\end{pmatrix}
= 
\begin{bmatrix}
B & D & 0 & 0 & \cdots & 0 & 0 \\
A & B & D & 0 & \cdots & 0 & 0 \\
0 & A & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & D & 0 & \vdots \\
0 & \cdots & \cdots & \cdots & 0 & A & B \\
\end{bmatrix}
\begin{pmatrix}
C^n_1 \\
C^n_2 \\
\vdots \\
C^n_{i-1} \\
C^n_i \\
\end{pmatrix}
+ 
\begin{bmatrix}
A \ast C^n_0 \\
0 \\
\vdots \\
0 \\
0 \\
D \ast C^n_{i-1} \\
\end{bmatrix}
\]

(3.29)

Or simply

\[ C^{n+1} = MC^n + V \]  

(3.30)

This matrix system is implemented in MatLab code (APPENDIX H) to obtain the activity concentration of radionuclides of interest.

### 3.2.3 Risk assessment from consumption of water

The risk incurred by the population is estimated by assuming a linear dose-effect relationship with no threshold as per ICRP practice. For low doses ICRP fatal cancer risk factor is 0.05 Sv\(^{-1}\) (IAEA, 2004). The risk factor states that the probability of a person dying of cancer increases by 5% for a total dose of 1 Sv received during his lifetime.

The average annual committed effective dose \(D\) (Sv/y) for the measured water samples in this study, is used to estimate cancer risk for an adult person using the following relationship:

\[
\text{Risk} = \text{Dose(Sv)} \times \text{risk factor (Sv}^{-1})
\]  

(3.31)

Where,

Dose(Sv) is equals to mean annual committed effective dose \(D\) (Sv/y) x life –time (years).

Risk factor (Sv\(^{-1}\))= 0.05 (IAEA-ICRP, 2004) for low dose.
CHAPTER FOUR
RESULTS AND DISCUSSIONS

This Chapter discusses the results from water samples collected around Taparko gold mine in Burkina Faso and the results obtained from modelling radionuclides transport in groundwater system of the same area using MathLab code. It focuses on activity concentrations of the NORM, effective dose, and risk assessment. Comparison of the results is also made with other works done elsewhere.

4.1 Experimental results

4.1.1 Activity Concentration of Radionuclide in water Sample

The data from water samples were categorized into two groups. Samples collected within the site ranged from WSTi-001 to WSTi-004 and those from the area around the site ranged from WSTO-001 to WSTO-013. The activity concentrations of \( \text{U}^{238} \), \( \text{Th}^{322} \) and \( \text{K}^{40} \) from each of these samples are plotted in figure 4.1. The corresponding values of the activity concentrations are given in APPENDIX A1.
The mean activity concentrations of $^{238}\text{U}$, $^{232}\text{Th}$ and $^{40}\text{K}$ in the water samples are shown in figure 4.1. The mean values for $^{238}\text{U}$, $^{232}\text{Th}$ and $^{40}\text{K}$ are $5.57 \pm 0.85$ Bq.L$^{-1}$ in a range of $0.36 - 9.31$ Bq.L$^{-1}$, $0.58 \pm 0.09$ in a range of $0.43 - 0.79$ Bq.L$^{-1}$ and $4.78 \pm 0.67$ Bq.L$^{-1}$ in a range of $3.62 - 6.30$ Bq.L$^{-1}$ respectively. These values are all below the standards recommended for drinking water. The activity concentrations $^{238}\text{U}$ and $^{232}\text{Th}$ of all the samples were lower than the recommended guideline values of 10.0 and 1.0 Bq/L respectively.

Figure 4.1 A graph of activity concentration of natural radionuclides ($^{238}\text{U}$, $^{232}\text{Th}$ and $^{40}\text{K}$) from water sample of Taparko gold mine
4.1.2. Annual Committed Effective Dose from Water samples

The annual committed effective dose (ACED) from water sample concentration is shown in figure 4.2. The figure also shows the contribution of each radionuclide to the total ACED. The corresponding values of ACED are given in APPENDIX A2.

The largest contribution to the overall dose for all the samples came mainly from $^{238}\text{U}$ with the lowest value of 0.013 mSv and the highest being 0.29 mSv compared to the dose contribution from $^{232}\text{Th}$ and $^{40}\text{K}$ with ranges of (0.069-0.12) mSv and (0.016-0.028) respectively. The mean Annual Committed Effective Dose is calculated to be $0.28 \pm 0.06$ mSv in a range of 0.13 – 0.41 mSv which is above the recommended reference dose level.
(RDL) of committed effective dose of 0.1 mSv (100 μSv) from 1 year’s consumption of drinking water. Generally, the mean Annual Committed Effective Doses of all the water sources investigated in the various communities had values above the WHO recommended value of 0.10 mSv/year. (Faanu, 2011).

4.1.3 Risk assessment

The Annual Committed Effective Dose due to adults water ingestion was estimated to be 0.28 mSv. Using this value and a life-time of 56 years in Burkina Faso, the exposure was evaluated as 15.68 mSv. The risk from this exposure is estimated to be \(7.8 \times 10^{-4}\) due to water consumption, using a risk factor \(0.05 \text{ Sv}^{-1}\) (IAEA, 2004). This value is less than ten times the total risk \(6.0 \times 10^{-3}\) from all natural radiation sources based on global average annual radiation dose of 2.4 mSv yr\(^{-1}\) to man (UNSCEAR, 2000). The estimated values are far less than the ICRP cancer risk of \(2.5 \times 10^{-3}\) based on annual dose limit of 1 mSv for the general public, which gives annual death probability of \(10^{-5}\), i.e. 1 in 100,000 (ICRP, 1990).
4.2 Theoretical results (Simulation)

Results of the simulation of long-lived radioisotopes with half-lives of several years show an accumulation of the radionuclide as the discharge commences and gradually decreases in concentration with distance. Even though these radionuclides have longer half-lives, the reduction in the concentration may be attributed not only to decay but due to dispersion of the radionuclide as well as adsorption sediments, etc. However, some few amount of the radioisotope is present at the receptor location. This is illustrated by Uranium-238 and Thorium-232.

4.2.1 Simulation of Uranium-238

Figures 4.3 and 4.4 show the results from the simulation of Uranium-238 with respect to time and distance.

![3D Plot of Variation of the concentration of Uranium-238 with time and distance from discharge point](image)

Figure 4.3: 3D Plot of Variation of the concentration of Uranium-238 with time and distance from discharge point
The results indicate that the concentration of Uranium-238 is accumulated at the discharge point and it decreases when the distance increases (Fig.4.3). The concentration becomes very low at 600 m beyond the release point. Also, at various distances from the release point, the concentration increases rapidly with time after the beginning of the release and becomes constant after a certain time. (fig.4.3). It reaches a maximum of 4.2 Bq/L at 200 m and a minimum value of 0.3 Bq/L at 600 m from the release point. This means that the wells and boreholes which supplying people as drinking water should be beyond 600 m from the mining site to avoid contamination of people with Uranium-238 that has been concentrated due to the mining activities through drinking water.

Figure 4. 4: 2D Plot of Variation of the concentration of Uranium-238 with time and at various distances from discharge point

The results indicate that the concentration of Uranium-238 is accumulated at the discharge point and it decreases when the distance increases (Fig.4.3). The concentration becomes very low at 600 m beyond the release point. Also, at various distances from the release point, the concentration increases rapidly with time after the beginning of the release and becomes constant after a certain time. (fig.4.3). It reaches a maximum of 4.2 Bq/L at 200 m and a minimum value of 0.3 Bq/L at 600 m from the release point. This means that the wells and boreholes which supplying people as drinking water should be beyond 600 m from the mining site to avoid contamination of people with Uranium-238 that has been concentrated due to the mining activities through drinking water.
4.2.2 Simulation of Thorium-232

The variation of Thorium-232 concentration with time and distance from the discharge point is illustrated in Figures 4.5 and 4.6, which are 3D et 2D plots respectively.

Figure 4.5: 3D Plot of Variation of the concentration of Thorium-232 with time and distances from discharge point
The results from Thorium-232 simulation are similar to the case of Uranium-238 such, the concentration is accumulated at the discharge point and decreases when the distance increases (Fig.4.5). However, the concentration becomes very low at 500 m beyond the release point compared to Uranium-238 which is very low at 600 m beyond the release point. Also, at various distances from the release point, the concentration increases rapidly with time after the beginning of release and becomes constant after a certain time (Fig.4.6). For Thorium-232, the concentration reaches to a maximum of 0.7 Bq/L at 200 m, 0.2 Bq/L at 500 m from the release point. So, for Thorium-232, wells and boreholes which supplying

Figure 4.6: 2D Plot of Variation of the concentration of Thorium-232 with time and at various distances from discharge point

The results from Thorium-232 simulation are similar to the case of Uranium-238 such, the concentration is accumulated at the discharge point and decreases when the distance increases (Fig.4.5). However, the concentration becomes very low at 500 m beyond the release point compared to Uranium-238 which is very low at 600 m beyond the release point. Also, at various distances from the release point, the concentration increases rapidly with time after the beginning of release and becomes constant after a certain time (Fig.4.6). For Thorium-232, the concentration reaches to a maximum of 0.7 Bq/L at 200 m, 0.2 Bq/L at 500 m from the release point. So, for Thorium-232, wells and boreholes which supplying
people drinking water should be beyond 500 m from the mining site to protect people from ingestion of Thorium-238 through drinking water.

4.2.3 Activity Concentration and Annual Committed Effective Dose from theoretical results.

In this part of the study, the average activity concentrations were 2.25 Bq/L, 0.37 Bq/L with corresponding committed effective doses of 0.072 mSv/y and 0.06 mSv/y, for $^{238}\text{U}$ and $^{232}\text{Th}$ respectively at a distance of 600 m from the discharged point. The $^{238}\text{U}$ activity concentration and committed effective dose are greater than $^{232}\text{Th}$. The same results were obtained with the experimental method. This could be explain by the fact that Uranium is more soluble than Thorium.

4.2.4 Comparison of theoretical and experimental results

A comparison of theoretical and experimental results is showed in table 4.3

Table 4.1: Comparison of theoretical and experimental results

<table>
<thead>
<tr>
<th>Radionuclide</th>
<th>Theoretical Activity Concentration (Bq/L)</th>
<th>Experimental Activity Concentration (Bq/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U-238</strong></td>
<td>2.25±0.11</td>
<td>5.57±0.85</td>
</tr>
<tr>
<td><strong>Th-232</strong></td>
<td>0.37±0.02</td>
<td>0.58±0.009</td>
</tr>
<tr>
<td><strong>K-40</strong></td>
<td>-</td>
<td>4.78±0.67</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Committed effective dose (mSv/y)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Theoretical</strong></td>
<td>0.13</td>
</tr>
<tr>
<td><strong>Experimental</strong></td>
<td>0.28</td>
</tr>
</tbody>
</table>
The results from the experimental method are almost two times those from theoretical methods. This difference could be due to the unavailability of some of the input parameters and real field data for the simulation. Some data were taken from the literature.

In spite of this, the differences between the theoretical and experimental results are not so much, therefore the theoretical compares well with the experimental results within the uncertainty limits.

4.3.1 Reported values of Natural Radionuclide concentrations (Bq.L\(^{-1}\)) and annual committed effective Dose (mSv.y\(^{-1}\)) in drinking water compared to results from this study.

The results obtained from this study were compared with the results of the radiological studies carried out in some other publications.

Table 4.2: Comparison of results from current study with data from other Publications

<table>
<thead>
<tr>
<th>Activity concentration (Bq/L)</th>
<th>Annual Committed Effective Dose (mSv/y)</th>
<th>Reference</th>
<th>source</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-238</td>
<td>Th-232</td>
<td>K-40</td>
<td></td>
</tr>
<tr>
<td>05.57</td>
<td>0.58</td>
<td>4.78</td>
<td></td>
</tr>
<tr>
<td>2.25</td>
<td>0.37</td>
<td>0.28, Experimental</td>
<td>borehole</td>
</tr>
<tr>
<td>0.54</td>
<td>0.41</td>
<td>7.76</td>
<td>0.13, Theoretical</td>
</tr>
<tr>
<td>1.59</td>
<td>4.44</td>
<td>14.39</td>
<td>1.08, 0.73</td>
</tr>
<tr>
<td>4.0</td>
<td>2.0</td>
<td>67</td>
<td>-</td>
</tr>
<tr>
<td>University of Ghana <a href="http://ugspace.ug.edu.gh">http://ugspace.ug.edu.gh</a></td>
<td>University of Ghana <a href="http://ugspace.ug.edu.gh">http://ugspace.ug.edu.gh</a></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The results of the investigations indicate that the average specific activity concentration in drinking water (wells and borehole) have consistent values compared to those reported in other publications. However, the concentration of $^{238}\text{U}$ is high for the current study compared to other studies.

Additionally, the current model has been compared with similar work done by Blessent (2009). He used coupling geological and numerical models to simulate groundwater flow and contaminant transport in fractured media.

Preliminary results revealed that the concentration of pollutant increases with time at a point of release and becomes constant at a certain time. The concentration decreases rapidly as the distance increases from the point of release (Blessent, 2009).
CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

This chapter provides the main conclusion based on the results of the study and relevant recommendations made to inform decision makers about how radiological health and safety policies of gold mining companies in Burkina Faso can be improved.

5.1 Conclusions

The release of radioactive materials into the environment due to human activities may lead to adverse health effects.

The present study describes the use of a mathematical model to simulate transport of Natural Radionuclides in a groundwater system and experimental data to investigate groundwater contamination. The simulation allows the determination of the concentration of the radionuclides from the point of contamination to the receptor location. At the receptor locations, the average activity concentrations were 0.3 Bq/L and 0.05 Bq/L for $^{238}\text{U}$ and $^{232}\text{Th}$ respectively at 600 m from the discharged point where the water from boreholes and wells is used by the public. For the experimental results, the mean activity concentrations were 5.57 Bq/L, 0.58 Bq/L and 4.78 Bq/L for $^{238}\text{U}$, $^{232}\text{Th}$ and $^{40}\text{K}$ respectively. The average annual committed effective doses estimated were 0.13 mSv/y and 0.28 mSv/y for theoretical and experimental methods respectively.

The activity concentrations measured for both theoretical and experimental methods are far below the ICRP recommended level of 1000 Bq/L for which remedial action is needed.
The annual committed effective doses are also lower than the 1 mSv per year dose limit recommended by the ICRP for public radiation exposure control (WHO, 2004; ICRP, 1990; 2007). The results from this work indicate insignificant levels of the natural radionuclides, implying that the mining activities do not pose any significant radiological hazard due to NORMS to the communities who are drinking the water from boreholes and wells in this area.

Even though the activity concentrations and the annual committed effective doses are far below recommended levels, studies have shown that long term accumulation of NORM may call for health concern (Von et. al., 2012).

5.2 Recommendations

Recommendations made in this study have been put into three categories:

i- Recommendations for future study

ii- Recommendations to Gold mining Companies and

iii- Recommendations to the National Radiation Protection Authority and Nuclear Safety (NRPANS) and the Environmental Protection Authority (EPA).

5.2.1 Recommendations for future study

i- It is recommended that simulation and experimental analysis of radionuclides continue in other mining sites.

ii- It is also recommended that hydrologic studies are needed in order to obtain some input data from the field.
5.2.2 Recommendations to Gold mining Companies

5.2.2.1 Water Management Planning

Site-specific groundwater management plan should be developed and implemented by mining companies. The plan should include:

i- The identification of the mine property subwatersheds, including those for mine waste areas, drainage flow paths, and receiving water bodies;

ii- Analysis of the local groundwater regime, including flow direction and rates, recharge and discharge areas, and relationship with the local surface water regime;

iii- Descriptions of measures to be implemented to manage water;

iv- Indication of the locations of mine water and seepage sampling stations and mine waste areas;

v- Monitoring of water quality and level in retention facilities, such as tailings management facilities, sedimentation ponds and polishing ponds.

5.2.2.2 Water Use and Recycling

Ore processing facilities should be designed to:

1. Minimize the volume of fresh water that is used for ore processing by:
   i. Using ore processing methods that require less water; and
   ii. Maximizing the recycling of water to reduce requirements for freshwater intake; and

2. Avoid or minimize the use of reagents that require treatment prior to effluent discharge.
5.2.3 Recommendation to the National Radiation Protection Authority and Nuclear Safety (NRPANS) and the Environmental Protection Authority (EPA)

5.2.3.1 National Water Office and Rehabilitation

Under the provision of the National Water Office and Rehabilitation (NWOR), the EPA should issue drinking water standards for NORM. These standards for the protection of both groundwater and surface water will prohibit the disposal of NORM that could jeopardize drinking water quality.

The NRPANS should also regulate the discharge of radioactive materials into surface water.

5.2.3.2 Environmental Auditing

Periodic environmental audits should be conducted to determine

a- Whether the site is operating in compliance with applicable regulatory requirements and appropriate non-regulatory and corporate requirements. For authorization by licensing, specific conditions will be required for:
   (i) Discharge limits
   (ii) Effluent and environmental monitoring
   (iii) Reporting of monitoring to the regulatory body (RPB); and

b- Whether the Environmental Management Systems (EMS) and other environmental plans have been properly implemented and maintained.
REFERENCES


Dinis, M.D.L and Fiùza, A. (2005). Modelling and Assessment of Radionuclides Differential Transport in Groundwater, Mining Department, Engineering Faculty, University of Porto


http://bigstory.ap.org/article/child-trafficking-victims-freed-burkina-faso


IAEA, (2003). Extent of environmental contamination by naturally occurring radioactive material (norm) and technological options for mitigation. IAEA technical reports series no. 419.


Mbendi and its associated information providers, 1995-2014.


### Appendix A

#### Table A1: Activity concentration of $\text{U}^{238}$, $\text{Th}^{232}$ and $\text{K}^{40}$ from water samples

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>Activity concentration (B/L)</th>
<th>U</th>
<th>Th</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>WSTO-001</td>
<td>0.42 ± 0.06</td>
<td>0.60 ± 0.04</td>
<td>3.85 ± 0.58</td>
<td></td>
</tr>
<tr>
<td>WSTO-002</td>
<td>0.56 ± 0.01</td>
<td>0.52 ± 0.03</td>
<td>4.12 ± 0.56</td>
<td></td>
</tr>
<tr>
<td>WSTO-003</td>
<td>0.14 ± 0.93</td>
<td>0.50 ± 0.04</td>
<td>3.62 ± 0.54</td>
<td></td>
</tr>
<tr>
<td>WSTO-004</td>
<td>0.84 ± 0.91</td>
<td>0.53 ± 0.05</td>
<td>5.23 ± 0.71</td>
<td></td>
</tr>
<tr>
<td>WSTO-005</td>
<td>0.14 ± 0.95</td>
<td>0.60 ± 0.08</td>
<td>3.85 ± 0.62</td>
<td></td>
</tr>
<tr>
<td>WSTO-006</td>
<td>0.65 ± 0.84</td>
<td>0.59 ± 0.13</td>
<td>6.30 ± 0.85</td>
<td></td>
</tr>
<tr>
<td>WSTO-007</td>
<td>0.42 ± 0.01</td>
<td>0.60 ± 0.18</td>
<td>4.80 ± 0.69</td>
<td></td>
</tr>
<tr>
<td>WSTO-008</td>
<td>0.91 ± 1.83</td>
<td>0.59 ± 0.17</td>
<td>4.33 ± 0.32</td>
<td></td>
</tr>
<tr>
<td>WSTO-009</td>
<td>0.16 ± 0.01</td>
<td>0.62 ± 0.06</td>
<td>3.95 ± 0.59</td>
<td></td>
</tr>
<tr>
<td>WSTO-010</td>
<td>0.89 ± 0.01</td>
<td>0.56 ± 0.07</td>
<td>4.73 ± 0.68</td>
<td></td>
</tr>
<tr>
<td>WSTO-011</td>
<td>0.40 ± 0.83</td>
<td>0.62 ± 0.20</td>
<td>4.29 ± 0.67</td>
<td></td>
</tr>
<tr>
<td>WSTO-012</td>
<td>0.89 ± 0.96</td>
<td>0.79 ± 0.12</td>
<td>4.92 ± 0.71</td>
<td></td>
</tr>
<tr>
<td>WSTO-013</td>
<td>0.46 ± 0.10</td>
<td>0.58 ± 0.07</td>
<td>5.16 ± 0.73</td>
<td></td>
</tr>
<tr>
<td>WSTI-001</td>
<td>0.68 ± 0.92</td>
<td>0.43 ± 0.002</td>
<td>5.62 ± 0.80</td>
<td></td>
</tr>
<tr>
<td>WSTI-002</td>
<td>0.36 ± 0.06</td>
<td>0.76 ± 0.24</td>
<td>5.45 ± 0.77</td>
<td></td>
</tr>
<tr>
<td>WSTI-003</td>
<td>0.75 ± 0.01</td>
<td>0.46 ± 0.06</td>
<td>5.30 ± 0.76</td>
<td></td>
</tr>
<tr>
<td>WSTI-004</td>
<td>0.72 ± 0.01</td>
<td>0.50 ± 0.06</td>
<td>5.68 ± 0.82</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>0.36 ± 0.06</td>
<td>0.93 ± 1.83</td>
<td>5.57 ± 0.85</td>
</tr>
<tr>
<td>Th</td>
<td>0.43 ± 0.002</td>
<td>0.79 ± 0.12</td>
<td>0.58 ± 0.09</td>
</tr>
<tr>
<td>K</td>
<td>3.62 ± 0.54</td>
<td>6.30 ± 0.85</td>
<td>4.78 ± 0.67</td>
</tr>
</tbody>
</table>
## APPENDIX A2

Table A2: Annual committed effective dose due to water ingestion

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>Dose contribution (µSv/y)</th>
<th>Average Committed Annual Effective Dose (mSv/y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U</td>
<td>Th</td>
</tr>
<tr>
<td>WSTO-001</td>
<td>13.58</td>
<td>96.94</td>
</tr>
<tr>
<td>WSTO-002</td>
<td>210.84</td>
<td>82.98</td>
</tr>
<tr>
<td>WSTO-003</td>
<td>165.00</td>
<td>81.58</td>
</tr>
<tr>
<td>WSTO-004</td>
<td>155.43</td>
<td>85.82</td>
</tr>
<tr>
<td>WSTO-005</td>
<td>210.87</td>
<td>82.101</td>
</tr>
<tr>
<td>WSTO-006</td>
<td>181.36</td>
<td>95.14</td>
</tr>
<tr>
<td>WSTO-007</td>
<td>206.15</td>
<td>97.07</td>
</tr>
<tr>
<td>WSTO-008</td>
<td>299.13</td>
<td>96.13</td>
</tr>
<tr>
<td>WSTO-009</td>
<td>165.79</td>
<td>99.30</td>
</tr>
<tr>
<td>WSTO-010</td>
<td>189.13</td>
<td>89.26</td>
</tr>
<tr>
<td>WSTO-011</td>
<td>135.08</td>
<td>99.25</td>
</tr>
<tr>
<td>WSTO-012</td>
<td>157.11</td>
<td>127.49</td>
</tr>
<tr>
<td>WSTI-003</td>
<td>216.86</td>
<td>73.09</td>
</tr>
<tr>
<td>WSTI-004</td>
<td>248.11</td>
<td>81.34</td>
</tr>
<tr>
<td>Min</td>
<td>11.72</td>
<td>82.115</td>
</tr>
<tr>
<td>Max</td>
<td>299.13</td>
<td>82.116</td>
</tr>
<tr>
<td>Mean</td>
<td>171.98</td>
<td>92.49</td>
</tr>
</tbody>
</table>
# APPENDIX B

## Table B: Standard Certificate

![Certificate Image]

<table>
<thead>
<tr>
<th>Radionuclide</th>
<th>Half life days</th>
<th>Activity kBq</th>
<th>Combined standard uncertainty, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Am-241</td>
<td>157800</td>
<td>4.694</td>
<td>1.1</td>
</tr>
<tr>
<td>Cd-109</td>
<td>462.6</td>
<td>14.54</td>
<td>1.4</td>
</tr>
<tr>
<td>Ce-139</td>
<td>337.5</td>
<td>1.355</td>
<td>1.1</td>
</tr>
<tr>
<td>Co-57</td>
<td>271.26</td>
<td>1.156</td>
<td>1.1</td>
</tr>
<tr>
<td>Co-60</td>
<td>1925.4</td>
<td>2.697</td>
<td>1.1</td>
</tr>
<tr>
<td>Ca-137</td>
<td>11019</td>
<td>2.689</td>
<td>1.3</td>
</tr>
<tr>
<td>Sr-90</td>
<td>28.5</td>
<td>4.800</td>
<td>2.2</td>
</tr>
<tr>
<td>Y-88</td>
<td>580.6</td>
<td>4.570</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Mass: 980.0 g  
Density: 0.98 g/cm³  
Volume: 1000 cm³  

Radionuclide impurities: gamma < 0.1 %  
Reference date: 23.3.2014  
Homeogeneity better than: 1 %

Description:  
Radioactive material is homogeneously dispersed in silicone resin. Composition of the matrix: C - 0.204  
H - 0.816  
O - 0.218  
Si - 0.379 (mass ratio).  

Measuring method:  
Preparation issues from standard SR solutions whose activities were determined by suitable absolute method. Final control is based on gamma spectrometry on HPGe detector.  
Note:  
As the criterion of homogeneity standard deviation of the activity value of 1 cm³/3 elements was chosen  
σ<sub>0</sub> = 0.7. The volume is calculated from the mass and the density.

Date of the certificate issue: 25.2.2014  
Validity: 3 years

Customer:  
CANBERRA-PACKARD CENTRAL EUROPE  
Wienersiedlung 6  
A-2432 Schwadorf  
Austria

Control: RNDr. Richard Bludovský, CSc., RNDr. Pavel Dryák, CSc.

Tel.: +420 266 020 497  
Fax: +420 266 020 466
APPENDIX C

Table C: Detector characteristics

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detector Model</td>
<td>G4020</td>
</tr>
<tr>
<td>Cryostat Model</td>
<td>300GSL</td>
</tr>
<tr>
<td>Pressure/Heater Model</td>
<td>2003C03L</td>
</tr>
<tr>
<td>Specifications</td>
<td></td>
</tr>
<tr>
<td>Diameter</td>
<td>90.1 mm</td>
</tr>
<tr>
<td>Active volume</td>
<td>90.1 mm</td>
</tr>
<tr>
<td>Length</td>
<td>91.2 mm</td>
</tr>
<tr>
<td>Distance from window (nominal)</td>
<td>6 mm</td>
</tr>
<tr>
<td>Physical Characteristics</td>
<td></td>
</tr>
<tr>
<td>Resolution</td>
<td>3.0 keV (FWHM) at 1.33 MeV</td>
</tr>
<tr>
<td></td>
<td>5.34 keV (FWHM) at 5.25 MeV</td>
</tr>
<tr>
<td></td>
<td>10.22 keV (FWHM) at 122.4 keV</td>
</tr>
<tr>
<td>Peak/Compton</td>
<td>56.1</td>
</tr>
<tr>
<td>Crystal well diameter</td>
<td></td>
</tr>
<tr>
<td>Well depth</td>
<td>mm</td>
</tr>
<tr>
<td>Crystal absorption or Drawing number (if special)</td>
<td>7.00GSL</td>
</tr>
<tr>
<td>Electrical Characteristics</td>
<td></td>
</tr>
<tr>
<td>Deposition voltage</td>
<td>5000 Vdc</td>
</tr>
<tr>
<td>Recommended bias voltage Vdc</td>
<td>(+4500 Vdc</td>
</tr>
<tr>
<td>Leakage current at recommended bias</td>
<td>0.01 mA</td>
</tr>
<tr>
<td>Pre-amplifier test point voltage at recommended voltage</td>
<td>-0.8 Vdc</td>
</tr>
<tr>
<td>Resolution and Efficiency</td>
<td></td>
</tr>
<tr>
<td>With angle-time constant of</td>
<td>4 ps</td>
</tr>
<tr>
<td>Isotope</td>
<td>58-Co</td>
</tr>
<tr>
<td>Energy (keV)</td>
<td>122</td>
</tr>
<tr>
<td>FWHM (keV)</td>
<td>878</td>
</tr>
<tr>
<td>FWHM (keV)</td>
<td>2.31</td>
</tr>
<tr>
<td>Peak/Compton</td>
<td>63.4</td>
</tr>
<tr>
<td>Rel. Efficiency</td>
<td>44.3%</td>
</tr>
<tr>
<td>Tests are performed following IEEE standard test ANS/IEEE std323-1996</td>
<td></td>
</tr>
<tr>
<td>Standard Canberra electronics used - See Germanium detector manual Section 7</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX D

Table D: Input parameters of simulation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulic conductivity $K$ (m/s)</td>
<td>$5.5 \times 10^{-6}$</td>
<td>Roose et al., 1978</td>
</tr>
<tr>
<td>Effective Porosity ($\theta$)</td>
<td>0.24</td>
<td></td>
</tr>
<tr>
<td>Bulk density ($\rho$)</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>Longitudinal dispersivity $\alpha_L$ (m)</td>
<td>5</td>
<td>EPA, 1996</td>
</tr>
<tr>
<td>Distribution coefficient $K_d$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Hydraulic gradient $i$</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

Darcy’s velocity is given: $V = \frac{K \times i}{\theta}$,  
Groundwater seepage velocity: $V_x = \frac{V}{R}$

R: retardation factor given by: $R = 1 + \frac{\rho \times K_d}{\theta}$

Dispersion coefficient: $D_x = \alpha_L \times V_x$

After calculation: $V_x = 5.5 \times 10^{-8} m/s$; $D_x = 2.75 \times 10^{-7} m^2/s$

- Initial concentrations

$C_0=9.31$Bq/L for $^{238}$U, $C_0=0.79$Bq/L for $^{232}$Th, they are the maximum activity concentration values obtained from the experimental method.
APPENDIX E

Table E: Default values of intake per person for various critical groups in the world (Adults)

<table>
<thead>
<tr>
<th></th>
<th>Far East</th>
<th>Near East</th>
<th>Africa</th>
<th>South America</th>
<th>Central America</th>
<th>North America</th>
<th>Europe</th>
<th>Oceania</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water (m³/a)</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Milk (L/a)</td>
<td>65</td>
<td>140</td>
<td>80</td>
<td>135</td>
<td>135</td>
<td>325</td>
<td>250</td>
<td>410</td>
</tr>
<tr>
<td>Meat (kg/a)</td>
<td>40</td>
<td>55</td>
<td>35</td>
<td>90</td>
<td>75</td>
<td>205</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>Grain, root crops, vegetables and fruits (kg/a)</td>
<td>510</td>
<td>600</td>
<td>380</td>
<td>470</td>
<td>445</td>
<td>535</td>
<td>410</td>
<td>500</td>
</tr>
<tr>
<td>Freshwater fish (kg/a)</td>
<td>35</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>25</td>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>Marine fish (kg/a)</td>
<td>60</td>
<td>20</td>
<td>30</td>
<td>35</td>
<td>45</td>
<td>40</td>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td>Shellfish (kg/a)</td>
<td>20</td>
<td>5</td>
<td>10</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>10</td>
</tr>
</tbody>
</table>
### APPENDIX F

**Table F: Chemical symbols and Characteristics of Uranium-238 Series, Thorium-232 Series and K-40 Decay Series**

<table>
<thead>
<tr>
<th>NORM Symbol</th>
<th>Half-Life</th>
<th>Major Emissions</th>
<th>NORM Symbol</th>
<th>Half-Life</th>
<th>Major Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uranium 238</td>
<td>$^{238}$U</td>
<td>$4.5 \times 10^9$ y</td>
<td>α</td>
<td>Thorium 232</td>
<td>$^{232}$Th</td>
</tr>
<tr>
<td>Thorium 234</td>
<td>$^{234}$Th</td>
<td>24.0 d</td>
<td>β, γ</td>
<td>Radium 228</td>
<td>$^{228}$Ra</td>
</tr>
<tr>
<td>Protactinium 234m</td>
<td>$^{234m}$Pa</td>
<td>1.2 m</td>
<td>β, γ</td>
<td>Actinium 228</td>
<td>$^{228}$Ac</td>
</tr>
<tr>
<td>Uranium 234</td>
<td>$^{234}$U</td>
<td>$2.5 \times 10^5$ y</td>
<td>α, γ</td>
<td>Thorium 228</td>
<td>$^{228}$Th</td>
</tr>
<tr>
<td>Thorium 230</td>
<td>$^{230}$Th</td>
<td>$7.7 \times 10^4$ y</td>
<td>α, γ</td>
<td>Radium 224</td>
<td>$^{224}$Ra</td>
</tr>
<tr>
<td>Radium 226</td>
<td>$^{226}$Ra</td>
<td>$1.6 \times 10^3$ y</td>
<td>α, γ</td>
<td>Radon 220</td>
<td>$^{220}$Rn</td>
</tr>
<tr>
<td>Radon 222</td>
<td>$^{222}$Rn</td>
<td>3.83 d</td>
<td>α</td>
<td>Polonium 216</td>
<td>$^{216}$Po</td>
</tr>
<tr>
<td>Polonium 218</td>
<td>$^{218}$Po</td>
<td>3.1 m</td>
<td>α</td>
<td>Lead 212</td>
<td>$^{212}$Pb</td>
</tr>
<tr>
<td>Lead 214</td>
<td>$^{214}$Pb</td>
<td>27 m</td>
<td>β, γ</td>
<td>Bismuth 212</td>
<td>$^{212}$Bi</td>
</tr>
<tr>
<td>Decay Chain</td>
<td>Radioactive Isotope</td>
<td>Half-Life</td>
<td>Decay Type</td>
<td>Stable End Product</td>
<td></td>
</tr>
<tr>
<td>-------------</td>
<td>---------------------</td>
<td>-----------</td>
<td>-------------</td>
<td>-------------------</td>
<td></td>
</tr>
<tr>
<td>Bismuth 214</td>
<td>$^{214}$Bi</td>
<td>20 m</td>
<td>$\beta, \gamma$</td>
<td>Polonium 212</td>
<td>$^{212}$Po</td>
</tr>
<tr>
<td>Polonium 214</td>
<td>$^{214}$Po</td>
<td>$1.6 \times 10^4$ s</td>
<td>$\alpha, \gamma$</td>
<td>Thallium 208</td>
<td>$^{208}$Tl</td>
</tr>
<tr>
<td>Lead 210</td>
<td>$^{210}$Pb</td>
<td>22.3 y</td>
<td>$\beta, \gamma$</td>
<td>Lead 208</td>
<td>$^{208}$Pb</td>
</tr>
<tr>
<td>Bismuth 210</td>
<td>$^{210}$Bi</td>
<td>5.01 d</td>
<td>$\beta$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polonium 210</td>
<td>$^{210}$Po</td>
<td>138 d</td>
<td>$\alpha$</td>
<td>Potassium-40</td>
<td></td>
</tr>
<tr>
<td>Lead 206</td>
<td>$^{206}$Pb</td>
<td>Stable</td>
<td>none</td>
<td>Potassium 40</td>
<td>$^{40}$K</td>
</tr>
</tbody>
</table>
APPENDIX G

Table G: Samples location coordinates

<table>
<thead>
<tr>
<th>Sample Code</th>
<th>Samples location coordinates</th>
<th>Height above sea level</th>
</tr>
</thead>
<tbody>
<tr>
<td>WSTI-001</td>
<td>N 13° 31' 40.50&quot; W 00° 20' 37.41&quot;</td>
<td>164</td>
</tr>
<tr>
<td>WSTI-002</td>
<td>N 13° 30' 19.46&quot; W 00° 19' 51.70&quot;</td>
<td>263</td>
</tr>
<tr>
<td>WSTI-003</td>
<td>N 13° 29' 52.13&quot; W 00° 19' 45.22&quot;</td>
<td>234</td>
</tr>
<tr>
<td>WSTI-004</td>
<td>N 13° 31' 29.87&quot; W 00° 20' 56.95&quot;</td>
<td>238</td>
</tr>
<tr>
<td>WSTO-001</td>
<td>N 13° 29' 25.78&quot; W 00° 19' 22.89&quot;</td>
<td>322</td>
</tr>
<tr>
<td>WSTO-002</td>
<td>N 13° 29' 36.54&quot; W 00° 19' 33.52&quot;</td>
<td>330</td>
</tr>
<tr>
<td>WSTO-003</td>
<td>N 13° 29' 57.82&quot; W 00° 18' 59.46&quot;</td>
<td>328</td>
</tr>
<tr>
<td>WSTO-004</td>
<td>N 13° 29' 39.19&quot; W 00° 19' 08.41&quot;</td>
<td>326</td>
</tr>
<tr>
<td>WSTO-005</td>
<td>N 13° 29' 09.97&quot; W 00° 19' 01.26&quot;</td>
<td>318</td>
</tr>
<tr>
<td>WSTO-006</td>
<td>N 13° 29' 11.97&quot; W 00° 19' 93.00&quot;</td>
<td>322</td>
</tr>
<tr>
<td>WSTO-007</td>
<td>N 13° 28' 50.88&quot; W 00° 19' 36.22&quot;</td>
<td>309</td>
</tr>
<tr>
<td>WSTO-008</td>
<td>N 13° 28' 55.94&quot; W 00° 19' 44.48&quot;</td>
<td>313</td>
</tr>
<tr>
<td>WSTO-009</td>
<td>N 13° 29' 02.36&quot; W 00° 19' 47.84&quot;</td>
<td>315</td>
</tr>
<tr>
<td>WSTO-010</td>
<td>N 13° 27' 44.52&quot; W 00° 19' 46.58&quot;</td>
<td>299</td>
</tr>
<tr>
<td>WSTO-011</td>
<td>N 13° 27' 39.49&quot; W 00° 19' 33.15&quot;</td>
<td>311</td>
</tr>
<tr>
<td>WSTO-012</td>
<td>N 13° 28' 16.63&quot; W 00° 19' 26.74&quot;</td>
<td>307</td>
</tr>
<tr>
<td>WSTO-013</td>
<td>N 13° 28' 30.64&quot; W 00° 19' 36.67&quot;</td>
<td>310</td>
</tr>
</tbody>
</table>
APPENDIX H

Table H: MatLab code

%Contaminants transport equation numerical solution;

clear all
clc

%Constants
%k, decay constant;
%Dx ,Dispersivity along x direction(m2/s)
%Vx; seepage velocity (m/s)
R %Retardation factor
T; %element period
Day =3600*24; %# day in second
year=365.25*jour; %# year in second
t=1000; % time

%numerical parameters
lamda=(0.69)/(T*an);
dx=10 % should be less than 2*D/U for stability
dt = 200 % should be less than (dx^2)/(2*D+k*dx^2)
Cin = c0; %initial concentration
%first node and last node are fictitious
Tn=ceil(L/dx)+3;
%Total nodes ;
Tt=ceil(Tf/dt);
%total time periods in the simulation
\[ A = \text{zeros}(Tn, Tn); \]
\[ b = \text{zeros}(Tn, 1); \]
\[ x = \text{zeros}(Tt, Tn); \]
\[ V = \text{zeros}(Tn, 1); \]
\%flops(0);
\[ \text{val1} = \frac{D * \text{dt}}{(R * \text{dx}^2)} + \left(\frac{U * \text{dt}}{R * \text{dx}}\right); \]
\[ \text{val2} = 1 - \left(\frac{2 * D * \text{dt}}{(R * \text{dx}^2)}\right) - \text{lamda} * \text{dt}; \]
\[ \text{val3} = \left(\frac{D * \text{dt}}{(R * \text{dx}^2)}\right) - \left(\frac{U * \text{dt}}{R * \text{dx}}\right); \]
\%first equation is from applying the left boundary condition
\[ A(1,1) = 1; \]
\[ A(1,2) = 2 * \text{dx} * \frac{U}{D}; \]
\[ A(1,3) = -1; \]
\[ b(1,1) = 2 * \text{dx} * \frac{U * \text{Cin}}{D}; \]
\[ V(1,1) = c0; \]
\%now for nodes 2 (first actual node!) to Last actual node
\%which corresponds here to node=Tn-1;
\%in the AX=b equation for each time will place


%this in time stepping loop later.

end

%the last equation is from applying the right boundary condition

A(Tn,Tn-1)=1;
A(Tn,Tn)=-1;

for timeindex=2:Tt
  for i=2:Tn-1
    b(i,1)=x(timeindex-1,i);
    end %rhs that changes every time step
  end %now solve the Ax=b to get the solution
  x(timeindex,:) = (A*b)+V;
  %you can use LU to make this step efficient
  end %of time loop

save -ASCII -TABS x.xls x

%flops

figure(1)

plot([1:dt:Tf]',1/c0*x(:,20),[1:dt:Tf]',
    1/c0*x(:,30),[1:dt:Tf]',1/c0*x(:,40),[1:dt:Tf]',1/c0*x(:,50),[1:dt:Tf]',1/c0*x(:,60))
xlabel('time (s)')
legend('x=200','x=400','x=500','x=600');
ylabel('Radionuclide Concentration ')
grid on

figure(2)

X=[0:dx:L];

T=[1:dt:Tf];

mesh(T,X,x(:,2:Tn-1)')