UNIVERSITY OF GHANA

THE ROLE OF FIXED INCOME IN PENSION SCHEME INVESTMENT IN GHANA

KYEI BAFFOUR AFARI

THIS THESIS IS SUBMITTED TO THE UNIVERSITY OF GHANA, LEGON IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE AWARD OF MPHIL RISK MANAGEMENT & INSURANCE DEGREE

JULY 2014
DECLARATION

I do hereby declare that this work is a result of my own research and has not been presented by anyone for any academic award in this or any other university. All references used in the work have been fully acknowledged.

I bear sole responsibility for any shortcomings.

..........................................................  ..........................................................  
KYEI BAFFOUR AFARI  DATE

(10248297)
CERTIFICATION

We hereby certify that this thesis was supervised in accordance with procedures laid down by the University.

DR. ERIC OFOSU-HENE
(SUPERVISOR)

DATE

PROF. JOSHUA ABOR
(SUPERVISOR)

DATE
DEDICATION

I dedicate this work to my Saviour and redeemer, Jesus Christ and also to my family.
ACKNOWLEDGEMENT

I am indebted to my supervisors, Dr. Eric Ofosu-Hene and Professor Joshua Abor for their timeless dedication during the supervision of this work.

A special appreciation goes to Mr. Paul Ammah of the Computer Engineering Department of the University of Ghana, Legon for his time, contribution and support especially during the modelling phase of my work using a programming language.
ABSTRACT

Pension scheme providers in Ghana adopt different asset allocation (the proportion of pension funds that need to be invested into different assets like equities and bonds) as an investment strategy. For instance, SSNIT seems to adopt a 60% bond allocation (fixed income investment) and 30% equity allocation (non-fixed income investment) as an investment strategy over the last decade. This study investigates the role of fixed income in pension schemes investment in Ghana by specifically looking at the asset allocation and the initial investment required to make the scheme solvent in the future at a specified high probability after matching all liabilities in Ghana.

This thesis examines some basic risk and return characteristics of historical data. The best asset to invest in without matching liabilities as well as the liabilities paid by pension schemes in the future is also investigated. The asset allocation and the minimum initial fund required to make a scheme solvent at a specified probability in the future after matching liabilities using a stochastic asset-liability model under the closed pensioners’ portfolio is also examined. The stochastic asset model (mean-variance model) is adopted in the projection of returns of asset classes as well as the determination and projection on liabilities paid by pension schemes over a 40-year period. The investment strategy is examined using a stochastic asset-liability model. Looking at the assets-only analysis of pension schemes without matching their liabilities, equity appears to be an attractive asset classes to invest in. However, considering asset-liability analysis, bonds (specifically One-year bonds) are the best-matched liabilities since they have good risk-adjusted returns and are less risky. The asset allocation moves from equity towards bonds (specifically One-year bonds) at a higher solvency level and the minimum investment required also increases as the solvency level increases. This study has a significant implication for adopting the appropriate investment strategy by pension fund managers.
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<td>76</td>
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<tr>
<td>ALM</td>
<td>Asset-Liability Management</td>
</tr>
<tr>
<td>CAPM</td>
<td>Capital Asset Pricing Model</td>
</tr>
<tr>
<td>DB</td>
<td>Defined Benefit</td>
</tr>
<tr>
<td>DC</td>
<td>Defined Contributors</td>
</tr>
<tr>
<td>FAS</td>
<td>Financial Accounting Standards</td>
</tr>
<tr>
<td>FRS</td>
<td>Financial Reporting Standards</td>
</tr>
<tr>
<td>GHC</td>
<td>Ghanaian Cedi</td>
</tr>
<tr>
<td>GSE</td>
<td>Ghana Stock Exchange</td>
</tr>
<tr>
<td>IAS</td>
<td>International Accounting Standards</td>
</tr>
<tr>
<td>ILO</td>
<td>International Labour Organization</td>
</tr>
<tr>
<td>IRS</td>
<td>Individual Retirement Account</td>
</tr>
<tr>
<td>IT</td>
<td>Information Technology</td>
</tr>
<tr>
<td>MFR</td>
<td>Minimum Funding Requirement</td>
</tr>
<tr>
<td>MVA</td>
<td>Market Value Adjustment</td>
</tr>
<tr>
<td>PFM</td>
<td>Pension Fund Manager</td>
</tr>
<tr>
<td>PNDC</td>
<td>Provisional National Defence Council</td>
</tr>
<tr>
<td>REIT</td>
<td>Real Estate Investment Trusts</td>
</tr>
<tr>
<td>SEC</td>
<td>Security and Exchange Commission</td>
</tr>
<tr>
<td>SSNIT</td>
<td>Social Security and National Insurance Trust Scheme</td>
</tr>
<tr>
<td>TVAR</td>
<td>Tail Value at Risk</td>
</tr>
<tr>
<td>UK</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>US</td>
<td>United States</td>
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CHAPTER ONE
INTRODUCTION

1.0 Introduction
The introductory section of this thesis describes the historical trend of the investment portfolio of pension schemes in Ghana (using SSNIT) and the structure of the investment incomes derived. The major challenge of the appropriate investment strategy confronting the pension scheme system in Ghana is also addressed.

1.1 Background of the study
Pension scheme investment in Ghana meant government securities, corporate bonds/debts including (REITs, Mortgage and Assets Backed securities and debentures), money market, ordinary shares and open and closed funds (National Pension Act, 2008). However, the investment for pension schemes in this paper has been limited to fixed income investment (that is investment whose returns are known at the time of making the investment like bond and treasury bill) as well as non-fixed income investment like equity from the Ghana Stock Exchange.

Over the years until now, pension scheme providers including SSNIT which is trying to achieve the investment policies which include: implementing an optimal asset allocation policy, maintaining a long-term optimum fund ratio, protecting the corpus of the assets in the scheme and the value of those assets, achieving a real return on the investment of at least +2.25% per annum as well as attracting, training and retaining competent investment talents, are still faced with a challenge as to how to maximize the returns on the investments to meet the benefits and cost of running the scheme. (SSNIT Annual Report, 2012)
1.1.1 Investment portfolio of SSNIT

Now looking critically at one of Ghana’s largest pension scheme providers, SSNIT, a large amount of surplus funds accumulated need to be invested like all funded pension schemes elsewhere. With the SSNIT Scheme under law 247, investment policies are self-sustaining and are therefore expected to be more professional designed and implemented. Two main things are required to achieve the professional attributes that are conceived: a) all restrictions on investments should be removed and, b) responsibility for all investments should be entrusted in the SSNIT Board and largely free of Government interference. Theoretically, investment returns are to be above a minimum acceptable level in aggregate over the long term. Aware of the long term nature of liabilities, investment in long-term projects are to be undertaken as long as short-term requirement are met. Generally, SSNIT’s investment policy is guided by seven principles which are: safety of investment, yield or rate of return, liquidity, maintenance of the fund’s monetary values, diversification, spread of investment by duration and harmonization with national objectives.

With all these guided investment principles, the investment portfolio of SSNIT funds comprised investment in fixed and non-fixed income investment made up as follows: short-term government instruments, government bonds, corporate loans, student loans, equity and property. Table 1.1 shows the relative share of SSNIT investment portfolio between 2005 and 2012. Generally, economically targeted investment made up the least share in the total, hovering around an average of 0.92. Next is followed by real estate, which is on a declining rate. This was declining from 10.6% of total investment in 2005 to 9.4% in 2012. Investment in fixed income which was high as about 58% of SSNIT total investment in 2005 and finally remained at 58% in 2012. Of significance are SSNIT’s equity holdings for both in listed and unlisted companies.
on the Ghana Stock Exchange. The equity portfolio has increased from about 30.0% of total investment in 2005 to 32.6% in 2012. Generally, on the average, SSNIT seems to adopt a 60% bond allocation (fixed income investment) and 30% equity allocation (non-fixed income investment) as an investment strategy. In fact, SSNIT is the single largest holder of shares in the Ghana Stock Exchange. SSNIT is the largest institutional investor in Ghana and is as such badly exposed in the capital market (SSNIT Annual Report, 2012)

Table 1.1 Summary of SSNIT Investment portfolio allocation (Percentage of total)

<table>
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<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity (listed and unlisted)</td>
<td>30.0</td>
<td>29.8</td>
<td>31.5</td>
<td>42.6</td>
<td>46.0</td>
<td>30.0</td>
<td>30.0</td>
<td>32.6</td>
</tr>
<tr>
<td>Fixed Income</td>
<td>58.0</td>
<td>59.9</td>
<td>54.4</td>
<td>46.0</td>
<td>47.0</td>
<td>59.7</td>
<td>60.6</td>
<td>58.0</td>
</tr>
<tr>
<td>Real Estate</td>
<td>10.6</td>
<td>9.5</td>
<td>14.1</td>
<td>11.4</td>
<td>7.0</td>
<td>8.1</td>
<td>8.6</td>
<td>9.4</td>
</tr>
<tr>
<td>Economically Targeted Investment</td>
<td>1.4</td>
<td>0.7</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>1.6</td>
<td>0.8</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Source: Calculated from SSNIT Annual Report

*Calculated from amounts that are net of provisions. Figures may not add up to 100 because of rounding

1.1.2 Structure of investment income

SSNIT has the largest pool of funds that it can manage efficiently to provide an effective social protection for a greater number of Ghanaians. The investment portfolio of SSNIT is about GHC 4.07 billion but still has on-going problems with return, liquidity and asset quality particularly with equity (listed and unlisted) and fixed incomes (SSNIT Annual Report, 2012). As we noted earlier, investments in equity averaged 34.06% of SSNIT’s total investment between 2005 and
2010. However, the dividend incomes to SSNIT from equity investment averaged about 11.18% between 2005 and 2012.

Similarly, investment in fixed income averaged 55.45% of SSNIT’s total investment between 2005 and 2010. The investment income from term deposit and treasury bills as well as government and registered bonds which constitute fixed income investment averaged about 34.09% and 3.82% respectively between 2005 and 2012. Table 1.2 shows the investment incomes from all the assets held by SSNIT between 2005 and 2012.

Table 1.2 Net Investment Income (in percentage)

<table>
<thead>
<tr>
<th></th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government and</td>
<td>0.32</td>
<td>3.42</td>
<td>4.93</td>
<td>3.88</td>
<td>0.05</td>
<td>1.14</td>
<td>4.6</td>
<td>12.2</td>
</tr>
<tr>
<td>Registered bonds</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Term Deposit and</td>
<td>57.12</td>
<td>28.35</td>
<td>49.28</td>
<td>33.44</td>
<td>38.56</td>
<td>33.19</td>
<td>22.12</td>
<td>10.67</td>
</tr>
<tr>
<td>Treasury Bills</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Loan</td>
<td>9.41</td>
<td>1.64</td>
<td>4.11</td>
<td>10.29</td>
<td>14.31</td>
<td>8.06</td>
<td>4.46</td>
<td>6.22</td>
</tr>
<tr>
<td>Corporate Loan</td>
<td>13.24</td>
<td>8.29</td>
<td>15.09</td>
<td>31.08</td>
<td>32.80</td>
<td>31.06</td>
<td>34.95</td>
<td>25.76</td>
</tr>
<tr>
<td>Rent</td>
<td>3.45</td>
<td>2.50</td>
<td>4.92</td>
<td>4.55</td>
<td>3.19</td>
<td>3.86</td>
<td>4.42</td>
<td>2.27</td>
</tr>
<tr>
<td>Dividend</td>
<td>10.22</td>
<td>8.60</td>
<td>12.82</td>
<td>12.74</td>
<td>6.62</td>
<td>12.01</td>
<td>15.83</td>
<td>10.63</td>
</tr>
<tr>
<td>Profit on disposal</td>
<td>-</td>
<td>-</td>
<td>3.77</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>of shares</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>6.24</td>
<td>47.20</td>
<td>5.07</td>
<td>4.01</td>
<td>4.47</td>
<td>10.67</td>
<td>13.6</td>
<td>32.24</td>
</tr>
<tr>
<td>Total</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Source: Calculated from SSNIT Annual Report.
It is recommended that SSNIT should restructure its assets portfolio so as to maximize return and protect the quality of its investments after matching all liabilities to ensure sustainability of the scheme as a social insurance fund.

1.2 Statement of the problem

Pension scheme providers in Ghana adopt different asset allocation (the proportion of pension funds that need to be invested into different assets like equities and bonds) as an investment strategy. For instance, SSNIT seems to adopt a 60% bond allocation (fixed income investment) and 30% equity allocation (non-fixed income investment) as an investment strategy over the last decade. This study intend to investigate the role of fixed income in pension scheme investment in Ghana by specifically looking at the asset allocation and the initial investment required to make the scheme solvent in the future at a specified high probability after matching all liabilities.

In Ghana, many studies have looked generally at the pension system without specifically addressing the role of fixed income in pension scheme investment. Globally, related research have been carried out to investigate the role of fixed income in pension scheme investment in developed economies but this study looks at a developing economy such as Ghana with different economic framework. This study will help pension fund manager to adopt the appropriate investment strategy considering assets and liabilities.

1.3 Research purpose

Undoubtedly, there has been a significant improvement in pension fund investment and returns in Ghana between 2005 and 2012 (SSNIT Annual Report, 2012), but producing the required asset allocation in order to produce good returns on investment into different Ghanaian asset with
equity (listed and unlisted) and fixed incomes remains to be a problem to some pension scheme providers. This study seeks to find out the role of fixed income in pension scheme investment by looking at the asset allocation required as well as the initial amount needed by pension scheme providers to make the scheme solvent in the future at a specified high probability after matching all liabilities

1.4 Objectives of the study

The objectives of the study are:

i. To investigate the historical risk and return characteristics of assets (treasury bills, government bonds and equities) in the Ghanaian market.

ii. To investigate the best asset to invest in the future without matching liabilities.

iii. To investigate the liabilities paid by pension schemes in the future.

iv. To investigate the asset allocation and the minimum initial fund required to make the scheme solvent at a specified probability in the future after matching liabilities using a stochastic asset-liability model.

1.5 Research questions

This study provides possible solutions to the following research questions:

i. What are the risk and return characteristics of assets in the Ghanaian market?

ii. What is the best asset to invest in the future without matching liabilities?

iii. What are the liabilities paid by pension schemes in the future?

v. What is the asset allocation and minimum initial fund required to make the scheme solvent at a specified probability in the future after matching liabilities using a stochastic asset-liability model.
1.6 Significance of the study

This study makes significant contributions in a number of areas. First and foremost, the research will serve as a basis for future research on the role of fixed income in pension scheme investment in Ghana. The findings and recommendations will assist fund managers of pension schemes to know the current risk/return characteristics of asset classes on the Ghanaian market and know the proportion of pension funds that should be invested into these assets like equity, government bond and treasury bill which will produce good returns over time following a stochastic investment model. The findings and recommendations will also help fund managers to know the minimum amount that need to keep together with the asset allocation required to make a scheme solvent in the future at a specified high probability after matching all liabilities. Policy makers will also find it useful. The study will form the basis for the review and evaluation of existing laws and regulations on the subject matter.

1.7 Scope and limitation of the study

It would be an oversight if certain things pertaining to the study which may influence the smooth running of the study are not pointed out. Firstly, literature on the Ghanaian situation was scanty and this made the study a challenge. Due to time and financial constraint, the researcher could not cover situation of all the pension scheme providers in Ghana. There are also limitations in the length of data used due to the limited data available for the Ghanaian market. This dataset is too short to calibrate a model. This will lead to spurious models and parameters estimated (using dataset from 2007 to 2013) which might be dubious and seem unrealistic for future projections. Also the results or outcomes of the research were solely based on information obtained from financial statements, interview, discussion and other relevant materials. The research is therefore
subject to errors, omissions, misstatements and inaccuracies of these financial reports and relevant literature that were obtained in the course of the research. The determination of investment strategy was not optimal because there were limitations in producing the software that will help obtain the optimal solution.

1.8 Summary of results and conclusion

The results show that, considering assets of pension schemes without matching liabilities as well as the risk and return characteristics of asset classes based on the projected average returns, equity appears to be an attractive asset class to invest in. Generally, comparing all the risk and return characteristics (mean, standard deviation, Sharpe ratio and excess kurtosis) of both the historical returns and projected simulated returns, it can be concluded that the historical returns could be used as a good indicator of the future returns without using a stochastic asset model to project future returns on assets since the risk and return characteristics of the historical returns was similar to that of the projected returns as opined by Sweeting (2004).

Under a closed pensioners’ portfolio, total pensions and expenses which constitute the liabilities incurred by scheme total pensions paid to pensioners decreases as the year progress and total expenses made by the scheme also decrease as the years progress.

When liabilities are taken into account, bonds (specifically One-year bonds) are the best-matched liabilities. There is a shift in asset allocation from equity towards bonds (specifically One-year bonds) at a higher solvency level since One-year bonds have good risk-adjusted returns and are less risky. The minimum investment required also increases as the solvency level increases.
1.9 Outline of thesis

The thesis is structured into five main sections.

Chapter 1 describes the historical trend of the investment portfolio of pension schemes in Ghana (using SSNIT) and the structure of the investment incomes derived.

Chapter 2 examines several literature reviewed generally on the pension scheme system in Ghana. This section also tailored to look at literature reviewed on pension fund risk management and asset-liability modelling of pension funds in the global perspective. It finally concludes with papers reviewed on the role of fixed income in pension scheme investment.

Chapter 3 examines the stochastic asset model (mean-variance model) to be adopted in the projection of returns on both fixed and non-fixed income investments. A further look at the determination and projection of liabilities is made. It concludes with a critical look at the stochastic asset-liability model adopted for investment strategy (asset allocation and minimum investment required to make the scheme solvent in the future at a specified high probability after matching all liabilities).

In chapter 4, a detailed analysis on the risk and return characteristics of historical data as well as an asset-only returns investment analysis is made. A further analysis is carried out on liabilities and finally concludes with investment strategy (asset-liability analysis).

Chapter 5 provides a summary of the research. It also draws some conclusions on the investment strategies of pension schemes. Finally recommendations were made to help improve investment strategies adopted by pension fund managers.
CHAPTER TWO
LITERATURE REVIEW

2.0 Introduction
This aspect of the study deals with the review of several literatures. It examines literature reviewed generally on the pension scheme system in Ghana. This section further looks at the asset allocation of pension funds as well as the investment strategy of pension funds in an asset-liability framework. It finally concludes with papers reviewed on the role of fixed income in pension scheme investment.

2.1 Pension scheme system in Ghana
Over the last decade, pension scheme providers like SSNIT seems to adopt a 60% bond allocation (fixed income investment) and 30% equity allocation (non-fixed income investment) as an investment strategy. Although, there has been significant improvement on the returns for pension fund investment in Ghana, the role of fixed income in pension scheme investment is an issue worth investigating for the Ghanaian market.

Most literature reviewed in Ghana looked generally at the pension scheme system in Ghana. A study carried out in Ghana by Dei (2001), looked at the pension fund management in Ghana. He carried out actuarial projections and analysis to ascertain the viability and sustainability of the scheme (SSNIT) into the future. The analysis he carried out entailed the determination and projections of contributors over the next 5 years or more and the determination of the funds inflow expected from contributions and investment returns. He also carried out projections on the number of expected pensioners, invalidity and death cases to arrive at the future funds outflow.
He furthered assessed the financial viability of the social security scheme by looking at the fund ratio. He found out that, there was a rising trend for both pension payments and administrative expenses based on the actuarial projections for the running of the scheme and therefore recommended that management needed to review the procedures and the organization structure to streamline operations aimed at cost cutting and cost reduction. From the results he obtained from the fund ratio to access the financial viability of the social security scheme, the conclusion he drew towards the formulation of investment policies was that, the fund does not have any problem of liquidity in the early years and the long-term minimum returns on investment could be determined. He also concluded that, the fund should aim at longer-term investment which would carry with it, higher returns and capital appreciation to meet the future liabilities of the pension scheme.

Kumado and Gockel (2003) carried out a research on the social security system in Ghana where they conducted a comparative assessment of various social security systems highlighting particularly the best practices. They also investigated the law and practice of social security in Ghana in relation to the best practices elsewhere in order to bring to fore the issues on ownership and control of SSNIT, membership of SSNIT board, impact of the oath of secrecy sworn by workers representatives on the SSNIT board and investment standards of SSNIT. Kumado and Gockel further determined whether there could be additional benefits under the SSNIT scheme and made recommendation for the social security in Ghana which is supported by the legislation, collective bargaining and the trade union policy.

Kumado and Gockel (2003) came up with recommendation that SSNIT should have the force of government legislation behind it as a First Tier National Pension Scheme. However, the heavy presence of government must be removed from its operations and therefore government’s role
must be largely regulatory. The SSNIT Board should be re-engineered: how it is constituted, its responsibilities and associated corporate governance issues must be recast in the light of best practices and SSNIT’s own history. The Board should be reduced in number from fourteen to eleven. SSNIT must be purged of excessive government presence and interference by reducing Government representation on the Board to about four.

He also recommended that SSNIT must be made accountable to Parliament through annual submission of report and appearance in the House to answer questions. The Report which should be accompanied with audited statements should be subject to the scrutiny of the Public Accounts Committee of Parliament to ensure transparency and accountability to its operations and may protect SSNIT from excessive government intervention. The Administrative costs of SSNIT were too high compared with best practices. The organizational structure of SSNIT should be clearly defined, especially as regards to 35 senior staff and junior staff ratios, which was one major source of inefficiency. This would entail staff rationalization as a cost reduction strategy to infuse efficiency into SSNIT operation. It was also recommended that the core business of SSNIT which entails collection of contributions, investing such collected funds and making pension payment could be done using IT solutions as ways of capturing economies of scale and scope so that these services are delivered accurately and efficiently. It was also recommended that stakeholders need not fill their representation on the Board from among their membership. In this connection, qualifications for membership of the Board should be set, taking into account the core business of SSNIT and its allied responsibilities to guide the stakeholders in choosing their representatives. Since the Board is made up of representatives, they must have the legal authority to report back to their constituency. This will suggest that the Oath of Secrecy required presently to be taken by Board members should be abolished because it may have a chilling
effect on members and therefore if need be, confidentiality requirements which protect SSNIT’s trade secrets and competitiveness may be introduced.

The Chilean example was overstretched and cannot replace SSNIT. The Chilean example was recommended as second tier private retirement scheme. CAP 30 should be preserved only for the following security service personnel who are already under the scheme: Armed Forces, Police Service, Prisons Service, and Fire Service. This conformed to best practices worldwide where these security services are under unfunded schemes. This does not however preclude them from taking part in other schemes in so far as they have not exceeded the allowable tax incentives.

It was also recommended that Government facilitates the establishment of second tier long term savings plan, probably designated as Private Retirement Plan. The proposed Private Retirement Plans must be employer-sponsored and must be tax deductible. The Funds of the Plans must be managed by approved private fund managers regulated by the Securities and Exchange Commission (SEC) under the Securities Industry Law. Fund managers must be required to offer an Individual Retirement Plan as a way of reaching out to persons in the informal sector.

It is recommended that fund managers should offer Home Ownership Plans and Education Plans. A third tier private scheme was also recommended to meet the needs of merit goods such as housing and education, along the lines of the Singaporean example. Alternatively, the Second tier Private Retirement Plan could be made not entirely forced saving for old age but to meet other social things that workers desire to acquire before retirement. It was also recommended that any restructuring of the SSNIT Scheme should enable it to cover all the 9 products listed in ILO Convention 102.

Boon (2007) also carried out a study on the knowledge system and social security in Africa where he carried out a case study on Ghana. He looked at the formal and informal social security
systems in Ghana and indicated that, an optimal combination of the formal and informal social security was the best option for improving the delivery of social security services to the majority of people in Ghana. He concluded that social security scheme in Ghana had undergone a series of transformations. The traditional in-kind social security scheme serviced by the extended family system provided an appropriate environment through which tradition and culture acted as a safe net for children, the aged and the vulnerable and protected them from total deprivation. He also pointed out that, the aged and the children were left to their own fate as the youth moved to the urban centres. Boon (2007) further indicated that, an accelerated process of social and economic disintegration was gradually replacing the traditional non-cash social security scheme provided by the extended family arrangement. He also mentioned that, serious policy and institutional problems had limited the scope of various formal national social security schemes introduced in the country. A major result was that, only 11% of workers in Ghana were covered by the formal social security scheme and the majority of the workforce which were in the informal sector did not benefit from the scheme. Boon (2007) also indicate that, the NHIS was introduced to redress the shortcomings of the previous social security scheme and prevent poverty from denying any citizen and resident of the country the right to good health services. He addressed a number of fundamental challenges that required urgent redress in order to help achieve this objective. He also mentioned that, effective implementation of the strategies proposed will lead to a significant improvement of the social security scheme in Ghana. Concerning problems associated with the low coverage of the informal sector, he recommended an extension of coverage in the informal sector. The difficulty of collecting contributions from contributors was addressed by encouraging government and employers to pay a realistic living wages in order to help them save and prepare for old age. Concerning the challenge of delays in
processing benefits received by pensioners, he recommended that payment of occupational pensions should be made mandatory for all employers in order to help make pensions transferable. The issue of low benefits was addressed by improving the coverage of the social security scheme through organization of effective and comprehensive educational and awareness raising programmes. The low investment returns by SSNIT was also addressed by allocating adequate resources into good business ventures. The high administrative cost was resolved by restricting the administrative structure of SSNIT. He also indicated that excessive government control and interference could be resolved by restricting SSNIT strategically as proposed by Kumado and Gockel (2003). Boon (2007) indicated that, one of the surest ways of improving and extending social security services to the poor and the deprived in Ghana was to introduce an innovative combination of the traditional in-kind social security scheme which was serviced by the extended family arrangement with the formal social security scheme.

Kpessa (2011) looked at the politics of retirement income security policy in Ghana where he analyzed the development and transformation of retirement income policy in Ghana. He concluded that, multiple policy objectives that often subordinate provision of retirement income security to the pursuit of nation building, socio-economic development and political mobilization had been the motivating drive for the development and transformation of income security policies in Ghana. He also indicated that, moving from the parallel public pension programs (SSNIT and CAP-30) to a three-tier model designed to incorporate both defined benefit and defined contribution in Ghana was an attempt to restructure the nation’s pension system on the basis of existing old age income support ideas and institutions in a manner that addressed the problem of institutional fragmentation. Kpessa (2011) also concluded that, Ghanaian policy makers argued that, the three-tier pension framework was a reflection of a synthesized version of
the preferences expressed by stakeholders within the context of what is known about social security and retirement income in Ghana according to a personal interview conducted in Accra during March, 2008. The views expressed by these Ghanaian policy makers indicated that, the content and institutional arrangement of the new scheme were determined by actors’ understanding of the policy challenges in a collective problem-solving domain and within the context of domestic politics and policy legacies. However, Kpessa (2011) indicated that, the shift to three-tier pension plan continued to reflect the legacies of a minimalist approach to formal old income protection, reinforcing interest of urban working class as it happened in the previous reforms. He further indicated that, most policy makers are of the view that, informal sector workers could take advantage of the voluntary third tier of the current arrangement to save towards their old age income security needs. However, this arrangement raised several questions. Due to the lack of knowledge of the operation capital market and an understanding of investment by most informal sector workers, it was unlikely that competitive private old age income security plan based on defined contributions would address their needs. Secondly, Kpessa (2011) indicated that, the current system was troubled with inequality against informal sector workers, in the sense that the problem of myopia, which was addressed through a mandatory second tier for formal sector workers, had not addressed for informal sector workers. Both the mandatory first and second tiers were designed to prevent that possibility for this category of workers since it was assumed that, that employees in the formal sector may undervalue the future old age income security needs. Kpessa (2011) further indicated that, the third tier which policymakers claim was designed for informal sector workers was a voluntary savings scheme, and being voluntary meant the questions of myopia for informal sector workers was ignored. Kpessa (2011) indicated that, this point was important for several reasons which included the fact that, Ghana
was a signatory to international conventions that recognized the need to ensure income security of all the aged and secondly, the country’s own constitution was clearly specified that the “state shall provide social assistance to the aged such as will enable them to maintain a decent standard of living” according to article 37, section 6b of the 1992 Constitution of the Republic of Ghana. Kpessa (2011) indicated that, Ghanaian policy makers were of the view that, it was wise to grant privileges to groups whose cooperation in the socio-economic and political transformation of the state was necessary considering the financial constraints. According to Kpessa (2011), it was possible that, such exclusive rights could become the basis for transforming such privileges into citizenship rights. He finally concluded that, retirement income policy in Ghana had mostly been used strategically for the purposes of economic development and reforms across time and not on retirement only.

Missing from the literature on pension scheme system in Ghana is specifically the role of fixed income in pension scheme investment which specifically looks at the asset allocation and the initial investment required to make the scheme solvent in the future at a specified high probability after matching all liabilities.

### 2.2 Asset allocation of pension funds.

Globally, many interesting surveys on the pension fund asset allocation have been carried out especially in the U.S. and U.K markets.

Papke (1991) looked at the asset allocation of private pension funds in the U.S market considering both defined benefits and defined contribution plans. He came out with some interesting finding for both single-employer and multi-employer defined benefit and defined contribution plans. Papke (1991) reported that, both single-employer and multi-employed defined benefits plans shown a large investment of pension funds into fixed income investment.
With single-employer and multi-employer defined contribution plans, a similar conclusion was drawn with large portion of pension funds being invested into fixed income securities as well.

However, Heaveley and Rozenov (2004) also studied asset allocation of defined pension funds in the U.S market. They considered the 200 largest defined pension funds in the U.S. They came out with interesting results indicating that, the equity allocation increased in shares from 48 percent in 1991 to 57 percent in 2001. They also discovered that, other assets such as alternative investment, real estate, enhance indexed equities and bonds also enjoyed an increasing portion of pension fund asset allocations.

On the other hand, Blake et al. (1998) looked at asset allocation and performance of pension funds in U.K. He assessed as many as 300 U.K pension funds and came out interesting results. They found out that allocation practices of pension funds had remained rather steady from 1986 to 1994. One of the notable observations was the high allocation of pension funds to equities which was hovering around 78 percent with only 14 percent into fixed income investments. However, Blake et al. (1998) study concentrated on the performance rather than asset allocation of pension funds and therefore, it remains somewhat unclear why U.K pension funds invest so much in equities than their U.S counterparts.

However, Blake (2001), looked at the pension fund management in the U.K. market. He argued that, fixed income investments should be encouraged by regulators simply because the discount rate used by actuaries and accountants was based on bond yields. This meant that, pension fund should invest heavily in bonds in order to avoid the short-term match between assets and liabilities. However, in Europe, many pension funds are encouraged to invest heavily in government bonds in order to help governments finance their national debt. Some financial
economists such as Bodie (1988, 1995, 1998, 1999) and radical actuaries such as Exley, Mehta and Smith (1997), Gold (2001), Bader and Gold (2003) as well as Alestabo and Puttonen (2005) would argue that pension funds should be entirely invested in bonds on the grounds that pension funds should not take risks with the sponsoring company’s shareholders’ funds and that, pension payments are bond-like in nature.

Over the years, there has been an interesting debate over asset allocation of pension funds and this has created two extreme views in the world today. Some researchers are of the view that, investing in bonds is the only way to match assets with liabilities whiles the other contradicting view recommends investments in equities as a best assets to match liabilities. In view of this, this study which looks at the role of fixed income in pension scheme investments in Ghana will also contribute to this debate.

According to Bodie et al. (1999), pension funds in the U.S. have a special tax treatment and this gives them incentives to create asset mix with large spread between pre-tax and after-tax returns and therefore, tax reasons drives pension funds to invest more in bonds than in equities. A paper by Bodie (1988) recommended investment only in taxable fixed income securities. Bodie (1988) carried out a study to investigate the investment policy of pension funds in the U.S. market. He considered the defined contribution and defined benefit plans. He argued that, the investment policy of pension funds depended heavily on the type of plan. He indicated that, the investment policy for the defined contribution plan was not much different than it was for an individual who was deciding on how to invest the money in an Individual Retirement Account (IRA). The guiding principle for the study was the efficient diversification, which was, achieving the maximum expected returns for any given level of risk exposure. He further reported that, one special feature was the fact that, investment earnings were not taxed as long as the money was
held in a pension fund and this would cause investors to tilt the asset mix of pension funds
towards least tax advantaged securities such as corporate bonds. Considering a defined benefit
plan, he urged on immunization strategies to hedge benefits owed to retired employees and
portfolio insurance strategies to hedge benefits accruing to active employers. He further
indicated that, most academic research into the theory of optimal funding and asset allocation
rules for corporate defined benefit plan concluded that, if there is shareholder wealth
maximization, then these plans should pursue extreme policies. He concluded that, considering a
health plan, full funding and investment in taxable fixed income securities was the optimum
investment policy. However, the optimum investment policy for much unfunded plans was
minimum full funding and investment in the riskiest assets. He further argued that, empirical
research so far had failed to decisively confirm or reject the prediction of this theory of corporate
pension policy. The Financial Accounting Standards adopted rule changes regarding corporate
reporting of defined benefit plan assets and liabilities which led to significant shift of pension
fund asset allocation into fixed income securities. He also indicated that, the introduction of
price-level indexed securities in the U.S. financial markets led to notable changes in pension
fund asset allocation. He finally concluded that, by giving plan sponsors a simple way to hedge
inflation risk, these securities made it possible to offer plan offer participant inflation protection
for both before and after retirement.

Now, considering the paper by Papke (1991) on asset allocation of the U.S private pension fund
in detail, he used the Form 5500 data from 1981 to 1987 and summarized the Form 5500 data on
the private pension fund investment. Using the Form 5500 data, the asset allocation of single-
employer and multi-employer defined benefit and defined contribution plans were reported. The
asset mix of the defined contribution plan which was categorized by plan funding ratio, sole and
multiple defined contribution plan savings or thrift, money purchase and 401(k) defined contribution plan was also reported. Papke (1991) found out that, the average single-employer defined benefit plan held about 50 percent of pension funds in fixed income investment, 20 percent in equities and 20 percent in pooled funds. Also, large single-employer defined benefit plan held 60 percent of pension funds in fixed income investments, 30 percent in equities and 2 percent in pooled funds on the average. He further reported that, few portfolios are extreme eventhough portfolios for these defined benefit and defined contribution plans predicts extreme investments policies. He further concluded that, multi-employer defined benefits plans held 63 percent of pension funds in fixed income securities, 19 percent in equities and 8 percent in pooled funds. However, considering the single-employer and multi-employer defined contribution plan, he concluded that, single-employer defined contribution plan invested 41 percent of pension funds in fixed income investment, 30 percent in equities and 20 percent in pooled funds. Large single-employer defined contribution had 49 percent of pension funds invested in fixed income investment, 38 percent in equities and 2 percent in pooled funds. Also, multi-employer defined contribution plans invested more heavily in fixed income securities with 73 percent in fixed income securities, 5 percent in equities and 8 percent in pooled funds.

Exley, Mehta and Smith (1997) looked at the financial theory of defined benefits pension schemes. They first analyzed the corporate pension provision from the financial theory perspective. The results of the analysis were reconciled with the contradictory message from the traditional actuarial valuation approach and the option of the market-based valuation paradigm was introduced. Exley, Mehta and Smith (1997) also suggested a pattern for this market-to-market valuation subject and considered its practicality to pension scheme. They declared that, adoption of the market based approach appeared to be important in many sector of actuarial
advice of defined benefit corporate pension provision. They further declared that, the principles can be used to create more efficient and transparent methodologies in sectors which had depended on subjective methods. They also gave the hope that, the insight gain on financial theory could be applied to level the playing field between defined benefit and defined contribution arrangements from both corporate and members perspective.

Exley, Mehta and Smith (1997) first concluded that, there was confusion over the valuation terminology between actuaries and economist and the actuarial fund valuation had a different purpose from an economist’s value. Also there was lack of clarity not only as to when one method or another should be used but also as to the theoretical basis for calculation carried out. In their analysis of corporate pension provision from the financial theory perspective, this difference was brought into focus. One of the main conclusions was that, the way asset of funds are arranged between equities and bonds didn’t have an influence on economic cost of liabilities. Exley, Mehta and Smith (1997) further examined their findings and concluded that, equity related discounting of pension liabilities using funding valuation techniques are flawed as a means of calculating economic value. According to the actuary performing the valuation, funding levels and assessed values of assets varied greatly. Even if actuaries assume the same long-term rate of return, values assessed by one actuary could be above current market values whereas another actuary could also assessed values below current market values. They furthered concluded that some inconsistencies could be created if actuarial values were held as economic values. They therefore set out a pattern for a market-based approach using the conventions adopted by banks. They concluded that, financial theory offered no good reason for the historical distinction drawn between asset and liability. Also, they concluded that, corporate management of pension schemes as well as member’s understanding of this distinction could also be improved
by the adoption of these principles. They also indicated that, there was also a tax-advantage associated with shareholders being able to invest in bonds using pension funds. Therefore, it made sense from the tax perspective to hold equities in private portfolios but invest 100% in bonds if only shareholder value should be maximized.

However, Exley, Mehta and Smith (1997) gave some practical applications to their study. First, many of the practical difficulties related with pricing more complicated liabilities could be worked around using standard techniques and guiding principles from modern finance even though they based their initial analysis on simple examples. Comparison of these results with traditional actuarial approaches casted more light on some consistencies in the standard actuarial evaluation theory particularly the supposed link between equity returns and salary growth which they considered false. Secondly, the issue of subjectivity remains but to a lesser degree under the market approaches. Bond-like liabilities which were not exactly replicated by traded bonds could be priced using standard term structures. They furthered indicated that, these values produced could be interpreted by both shareholders and members and have meaning. Thirdly, the positive gains for shareholders and members arising from material issues ignored by the conventional actuarial approaches were shown by the application of market principle to setting both contribution rates and investment policy. Finally, Exley, Mehta and Smith (1997) suggested that the current method of costing pension provisions overshadowed a number of important issues and therefore they advanced an alternative framework for measuring employment cost based on commodity pricing principles. They indicated that, defined benefit scheme would once again be able to compete with defined contribution scheme if desirable adjustment either to transfer value bases or rate of returns and a better understanding of cost structure were given.
Blake (2001) looked at the how to value the assets and liabilities of a defined benefit pension fund when the assets are liquid and subject to market value fluctuations, while the liabilities are less liquid and potentially less volatile. He also investigated on how to ensure that there are always sufficient cash flows from the assets to meet the promised pension payments when they fall due by investing in fixed income securities. He considered the UK market. These were analysed from the actuary, accounting and economic perspective.

From the actuary point of view, Blake found out that assets were measured at market value, while the discount rate for valuing liabilities was based on the actuaries’ assessment of long-run returns on the assets in the pension fund. The liabilities were measured using the current unit method (which takes into account accrued service but not future pay rises) and then rescaled by various Market Value Adjustments (MVAs) to reflect current market conditions. For young active members (and for pensioners in large schemes on payments over 12 years), the relevant MVA was the equity MVA; for older active members (within 10 years of the MFR pension age), the relevant MVA was a mixture of the equity and gilt MVAs; while for pensioners, the gilt MVA was used. The equity MVA is the ratio of the long-run dividend yield (currently set at 3.25%) to the current dividend yield on the FT-SE Actuaries All-Share Index. The gilt MVA was equal to the fair price of a notional 15-year gilt with an annual coupon of 8%.

However, from the accounting perspective, assets and liabilities were valued by reference to current market conditions. FRS17 valued liabilities on a completely different basis from the MFR, using the projected unit method (which takes into account anticipated pay rises up to the retirement date) and a discount rate equal to the market yield on AA corporate bonds, the same
yield used in the corresponding US and international accounting standards FAS87 and IAS19.

Considering the economist, Blake (2001) found out that assets should be valued at market prices and that liabilities should be valued consistently using the market returns on appropriate assets. The optimal asset allocation would be determined using ‘horizon matching’. This uses bonds with their reliable cash flows to meet current and near-maturing pension obligations (using a strategy called cash flow matching) and equity and property with their growth potential to match long-maturing liabilities that grow in line with earnings (using a strategy called surplus management).

Blake (2001) concluded that few people would now justify valuing assets on anything other than a market basis. Yet there are currently three official valuation bases for pension liabilities in the UK: statutory, MFR and FRS17. He also concluded that moves should be made to develop a single valuation basis for pension liabilities. Also, the discount rates that are being currently used or proposed by actuaries and accountants were based on bond yields and therefore likely to push pension fund asset allocations towards bonds in an attempt to lower the short-term volatility mismatch between assets and liabilities at the cost of lower should be made to ensure that the valuation basis for pension liabilities does not distort pension fund asset allocations.

Gold and Bader (2003) also looked at the case against stocks in public pension funds where he encouraged that, pension funds should be entirely invested in bonds (all–bond strategy). Gold and Bader (2003) indicated that, most actuarial critics argued that, government had no shareholders and also paid no federal taxes. The further argued that, Government Accounting Standards Board (GASB) was not rushing towards a transparent economic accounting model. These critics also mentioned that, taxpayer may escape
a troubled local pension plan by moving whereas corporate shareholders find buyers for their shares. Again, the indefinite lifetime of government plan suggested the possibility of intergenerational risk sharing that could deliver the equity risk premium without commensurate risk. After carrying out some critical analysis on risk-adjusted cost and tax effect on pension funds, Gold and Bader (2003) ascertained that, shifting government pension funds from equities to bonds added value to local taxpayers (a Federal tax arbitrage gain) in a transparent financial environment. Gold and Bader (2003) indicated that, they had ignored issues of risk in their paper although government pension funds had default risk. They also observed that, equity investment by government plans involved many other risks besides the market risk. Some of these risk included the intergenerational taxpayers conflicts as well as undercharges to employees’ compensation packages for the value of pensions, employee claims on pension surplus and higher governmental borrowing cost. Gold and Bader (2003) also observed that, there were greater practical obstacles to an all-bond strategy in public pension plans than in corporate settings. However, economic analysis suggested that, avoiding equities helped local taxpayers in much the same way that corporate all-bond strategies helped shareholders. They also looked at how public plans sponsors and their actuaries should prepare for changes that would help support the idea of greater transparency. Gold and Bader (2003) advised that, public pension plan actuaries should master the anti-equity reasoning irrespective of the fact that, they or their clients are convinced. They also concluded that, poor decision-making of pension plans stemmed from their inability to understand the risk nature of equity investments hence pension funds should be entirely invested in bonds (all-bond strategy) on the grounds that, pension funds should not take risk.
However, a paper carried out by Sweeting (2004) also looked at the role of fixed income in pension scheme investment. He expanded on the work carried out by ABN Amro team by and extending the sample period and using the arithmetic mean instead of geometric mean as a more appropriate measure of mean-variance analysis. He concentrated on the US market data only. There are several analyses that he carried out. The first was to compare the historical risk and return characteristics of US high yield corporate debt/bond, investment grade corporate debt/bond, treasury bonds and equities. In addition to the mean and variance of asset returns as well as correlation between asset classes, he measured the Sharpe ratio, skewness and excess kurtosis. The Sharpe ratio was used to calculate the risk-adjusted returns and found out that treasury bond and investment grade corporate debt which had greater Sharpe ratio gave a better risk adjusted return than high yield corporate debt and equity. Because investors are interested in the one-sided measure of risk such as the expected shortfall, the shape of the return distribution that is skewness and excess kurtosis was analyzed. He found out that the high yield corporate debt had a high excess kurtosis (fat–tail) and was less normal than the other assets.

However, Sweeting (2004) restricted his calculation to historical data. Although he acknowledged that it would be possible to carry out risk and return analysis using stochastic asset-liability models, he failed to address it because he did not believe that these would give any additional information, since they would have been calibrated using past data. He believe this to be especially true since the dataset that he was using (1984-2002) contains a good range of different economic scenarios, although the steady reduction in long-term interest rates over the period had resulted in higher bond returns than could reasonably be expected in future.

Looking at the asset–liability aspect of his study, he considered both the open and closed pensioners portfolio. Although relative to other debt asset classes, high-yield corporate debt
appeared to be the attractive asset classes in asset-only income analysis, treasury bond and investment grade corporate debt performed fare better when liabilities were taken into account. He assumed initial liabilities of $10,000, and initial asset of $8,000, £10,000 and $12,000. He found out that the net cashflow shown that, allowing for the early and recent poor performance of high yield corporate debt, still provided a consistently high level of net income than investment grade corporate debt or treasury bond. It was worth knowing that investment grade corporate debt outperformed treasury bond in this respect. Unlike the funding level calculation, there was no real difference in the result for the different initial funding levels. The net cashflow for the different initial funding levels for both the open and closed pensioners portfolio were similar with few differences. First, the net cashflow were smoother for the closed portfolio than the open ones. Also, the initial funding level had a significant difference on the relative attractiveness of the different bond asset classes; as the initial funding level increases, corporate bonds becomes more attractive than treasury bonds.

Alestabo and Puttonen (2005) also examined the strategic asset allocation and asset-liability issues in the Finnish defined benefit pension funds. They looked at data set consisting of 44 pension funds at the end of 2002. These data were collected from the Finnish Centre for Pensions and the Insurance Supervisory Authority. Asset allocation figures, liability structure information and solvency margin limits were that main parameter of pension funds which were used in this study. The liability structure of each pension fund was studied with an age structure of employers in the sponsoring company as a proxy. Alestabo and Puttonen (2005) reported from their results that, there was a relationship between the liability structure and asset allocation of pension funds. They concluded that, pension funds with younger participant had more equity investments whiles mature pension funds had more fixed income investments. They also concluded that, there was a
wide dispersion in asset allocation found between the funds with one fund holding its entire portfolio in fixed income investments and the other fund holding none or few of its portfolio in fixed income investments. Interestingly, they also found out that, investment into equities also varied, ranging from 0 percent to over 70 percent of the asset allocation. Investment in sponsors, real estate and money market also experienced this dramatic variation of the asset allocation. They finally concluded that, a portion of this asset allocation was explained by the liability structure whiles another part remained unexplained.

However, most of these empirical studies tended to focus mainly on developed economies. The question is, do these studies hold in a developing economy such as Ghana with different economic framework? This question among others is what this study seeks to also provide answers to.

2.3 Investment strategy of pension funds in an asset-liability framework

Since this study seeks to use an asset-liability model to investigate the role of fixed income in pensions scheme investment, literatures on asset-liability modelling is worth reviewing. Globally, several literatures have looked at asset and liability modeling. Considering a multi-period framework, Teper (1976) looked at a dynamic stochastic programming model which was used for deducing optimal funding and investment strategy. Sundaresan and Zapareto (1987) looked at how to relate asset allocation of pension funds to the marginal productivity of workers in an organization. They looked at both risk and riskless assets and the constant investment opportunities in these different types of assets. They looked at the defined benefit plan. Sundaresan and Zapareto provided a framework that connected pension plan formula and the valuation and asset allocation policies of defined benefit plan with the marginal productivity schedule of workers. They also looked at the retirement policies that are expressed by the
primitives of the model and the value of pension obligation. The model examined by Sundaresan and Zapareto (1987) also gave a precise and clear valuation formula for the stylized defined benefit plan. They also indicated that, the optimal asset allocation was made up of the replicating portfolio of the pension liabilities and the growth optimum portfolio independent of the pension liabilities. Sundaresan and Zapareto (1987) shown that, workers will go on retirement when the ratio of pension benefits to current wages reached a critical value which depended on the parameters of the pension plan and the discount rate. They also used the numerical technique to examine the feedback effect of retirement policies on the valuation of plans and on asset allocation.

Later in the subsequent years, Leibowitz (1987) and Sharpe and Tint (1990) introduced surplus optimization in the presence of pension liabilities in a single-period framework. Sharpe and Tint (1990) came out with interesting conclusion when they introduced surplus optimization in the presence of liabilities. They concluded that, the wide direction within domestic bond market acted as a important bridge towards a new allocation procedure directed towards surplus management. They also concluded that, asset class percentages were set a the macro level and the composition of each asset were determined by a manager using or by the nature of the index selected as a core fund when considering allocation. Sharpe and Tint (1990) also indicated that, this process led to duration for the bond component and for the total portfolio that had been selected for all sorts of reasons but with less concern for the control of surplus risk. Considering the new surplus context, Sharpe and Tint (1990) concluded that, a more efficient portfolio would result through closer integration of the macro and micro decision especially with respect to bond component. They further concluded that, the bond duration could be derived by an interaction process with the macro decision that sets the percentage weightings of all asset
classes. The total risk incurred a greater equity ratio and the tendency towards an even greater duration gap could be counter balanced by setting higher duration targets for fixed income portfolios using this interactive approach. They furthered indicated that, a deeper problem for effective surplus management was the tendency for sponsors to view the surplus value itself as being of a strictly short-term nature. Sharpe and Tint (1990) further concluded that, in reality, the surplus function truly links long and short term considerations. The long-term interpretation of surplus could be classified by viewing it as an earning rate cushion. Through the purchase of annuities or the construction of an immunizing bond portfolio at current market rate, a fund with zero surplus should be in a position to fulfil its associated liabilities exactly. They indicated that, a fund with a positive surplus should have some room to fulfil its liabilities, even if the long term earnings rate should fall below current annuity rate. The fund then has a cushion that allows it to take on market risk in search for excess returns. Thus the short-term measure of surplus status clearly has an important implication in terms of the long-term earnings rate need to achieve fulfilment of the fund’s liabilities.

Hilli et al. (2007) looked at the stochastic programming model for asset-liability management of the Finnish pension company. They indicated that the model had some unique features which stemmed from the statutory restrictions for Finnish pension insurance companies. They paid particular attention to modelling the stochastic factors, numerical solution of the resulting optimization problem and evaluation of the solution.

The modelling was done in two phases. First and foremost, they modelled the decision problem which included the specification of the decision variables, stochastic factors, objective and constraints. Secondly, they modelled the stochastic factors where they used the model developed in Koivu et al. (2004). This resulted in an infinite-dimensional stochastic optimization problem
which was solved in two steps. Firstly, there were issues of discretization which resulted in a finite dimensional optimization problem where the uncertainty was approximated by a scenario tree. Secondly, the numerical solution of the discretized model was done using an algebraic modelling language and an interior point solver for nonlinear convex optimization.

Numerical results indicated that the model is robust and superior to more traditional asset-liability management approaches. Out-of-sample tests clearly favour the strategies suggested by their model over static fixed-mix and dynamic portfolio insurance strategies.

Pension plan investment in an asset-liability framework has also been considered by Leibowitz, Kogelman and Bader (1994), Peskin (1997) as well as Binsbergen and Brandt (2014).

Binsbergen and Brandt (2014) looked at the optimal asset allocation in asset liability management. They examined the dynamic Asset Liability Management (ALM) problem of a defined benefits pension plan that faced a time varying investment opportunity set and was subject to various regulatory constraints such as a Value-at-Risk (VAR) constraint, and mandatory contributions by the plan sponsor when the plan had less assets than the reported liabilities. Binsgergen and Brandt (2014) indicated that, risk management and financial reporting could have first order effect on funds’ optimal investment policies. For instance, the current requirement to discount liabilities at a rolling average of yields as opposed to at current yields, encouraged risk taking by the plan and increased the portfolio weight of short-term debt instruments that do not hedge against liability risk. Binsbergen and Brandt (2014) further examined the influence of ex ante (preventive) and ex post (punitive) risk constraints on the gains to dynamic (strategic) as opposed to myopic (tactical) decision making. They found out that, Value-at-Risk (VAR) constraint which was preventive measure, led to a decrease in gains of dynamic investment. On the other hand, they concluded that, punitive constraints such as
mandatory additional contributions from the sponsor when the plan was underfunded, led to large utility gains from solving the dynamic program.

2.4 Chapter Summary
Extensive literatures were reviewed on the pension scheme system in Ghana and the role of fixed income in pension scheme investment from the global perspective. Some literatures were also reviewed on how asset-liability modelling had been used to investigate the investment strategy of pension funds from the global perspective.

Concerning literatures on the pension system in Ghana, Dei (2001) looked at the pension fund management in Ghana. Later, Kumado and Gockel (2003) looked at the social security system in Ghana. Boon (2007) also addressed the knowledge system and social security in Africa where he carried out a case study on Ghana’s formal and informal social security scheme. Finally, Kpessa (2011) also looked at the politics of retirement income security policy in Ghana. None of these literatures reviewed in Ghana looked at the role of fixed income in pension scheme investment which specifically looks at the asset allocation and the initial investment required to make the scheme solvent in the future at a specified high probability after matching all liabilities hence the need to carry out this study.

Concerning some literatures on asset allocation of pension funds, interesting debate over asset allocation of pension funds had created two extreme views in the world today. Globally, some researchers like Heaveley and Rozenov (2004) were of the view that equity was the best asset that matched liabilities. Others like Papke (1991), Bodie (1988, 1995, 1998, 1999) and radical actuaries such as Exley, Mehta and Smith (1997), Gold (2001), Bader and Gold (2003) as well as Alestabo and Puttonen (2005) were also of the view that bonds are the best asset to matched
liabilities. This study also contributed to this debate by looking at role of fixed income in pension scheme investment in Ghana.

Since this study examined the role of fixed income in pension scheme investment in Ghana using a stochastic asset-liability modelling, literatures on investment strategy of pension funds in an asset-liability framework were worth reviewing. Tepper (1976) looked at a dynamic stochastic programming model which was used for deducing optimal funding and investment strategy. Leibowitz (1987) and Sharpe and Tint (1990) introduced surplus optimization in the presence of pension liabilities in a single-period framework. Sundaresan and Zapareto (1987) looked at how to relate asset allocation of pension funds to the marginal productivity of workers in an organization. Hilli et al. (2007) looked at the stochastic programming model for asset-liability management of the Finnish pension company. Pension plan investment in an asset-liability framework has also been considered by Leibowitz, Kogelman and Bader (1994), Peskin (1997) as well as Binsbergen and Brandt (2014).
CHAPTER THREE

RESEARCH METHODOLOGY

3.0. Introduction

This section of the study seeks to examine the stochastic asset model (mean-variance model) adopted in the projection of returns on both fixed and non-fixed income investment. A further look at the determination and projection of liabilities is made. It concludes with a critical look at the stochastic asset-liability model adopted for investment strategy (asset allocation and minimum investment required to make the scheme solvent in the future at a specified high probability after matching all liabilities).

3.1 Area of study

The study area for the research is the Ghanaian market.

3.2 Data collection and sample size

The data for the study are secondary data which are gathered from published and unpublished records of treasury bills (91-days), One-year, and Two-year bonds from the Bank of Ghana (BOG). All share-index from the Ghana Stock Exchange was also gathered. All data were gathered from 2007 to 2013 since that was the longest whole-year period for which yields on bonds were available eventhough data on GSE All-Share index existed from 1990 to 2013. The chosen period from 2007 to 2013 for all dataset created a consistency in the dataset.

3.3 The Model. (Asset only)

The Wilkie (1995, 1999) model is a widely used model in actuarial literature especially in the UK market. The autoregressive, conditional heteroskedastic (ARCH) model as proposed by
Wilkie was developed from historical data using the version from Box and Jenkins’ (1970) methodology. Wilkie’s model was found to be inconsistent by Huber and Verall (1999). Kemp (1996) found that, the Wilkie model was quite difficult to comprehend because it was built on several parameters. Kemp (1996) furthered argued that, the features of the Wilkie model doesn’t fit with intuition. For example, he found out the expected returns on property are more than that on equities when considering a long term period.

Smith (1996), on the other hand, came up with the jump equilibrium model. This model was designed based on financial economic theories like the efficient market hypothesis. Smith gave more less attention to statistical fitness of the model and paid less attention to theoretical considerations. Huber (1998), however, challenged the theoretical framework surrounding the jump model. He specifically looked at the heroic assumptions implied by the CAPM model. For instance, looking at the CAPM model, it is assumed that borrowing and lending by agents is done at the same risk-free rate and short selling is also allowed. Merton (1976) used the jump-diffusion processes to generate returns assuming that random large moves occur in returns.

The VAR approach is a widely adopted model in most economic literature. Optimal portfolio strategy was examined by Campbell and Viciera (2005). Kouwenberg (2001) and Hoovernaars (2007) used the VAR model to analyze the optimal portfolio strategy using historical data for Dutch pension funds. The VAR model was used to analyze the optimal contribution policy and investment strategy for a German pension fund as carried out by Maurer, Mitchell and Rogalla (2009).

The regime-switching model has also been used by other researchers. Ang and Bekaert (2002) evaluated the benefits of international diversification using the regime-switching model. They concluded that the existence of high volatility does not negate the benefits of international
diversification and techniques, allowing for returns to be randomly drawn from regimes, each with its own distribution,

However, in spite of all these models, I choose to use the mean-variance model to simulate asset returns for assets of pension funds. The mean-variance model has been found be the most fundamental model in financial economics. The model requires not only knowledge of the expected returns and the standard deviation of the returns on each asset, but also the correlation of returns for each and every pair of assets which helps to uncover large risk reduction opportunities through diversification.

The mean-variance model is developed consisting of four asset classes. Each individual asset class is modeled as a mean-variance time series in which the parameters are estimated using historical data, taking the future economic outlook into account. The asset classes are basically grouped into two which are equity and bond asset classes (One-year bond, Two-year bond and treasury bill).

Generally, the mean-variance model is specified as:

\[ R_{it} = \mu_i + \sigma_i Z_{it} \]  \hspace{1cm} (1)

Where \( \mu_i \) = mean of the return of asset \( i \)

\( \sigma_i \) = standard deviation of the return of asset \( i \)

\( Z_{it} \) = randomly generated random numbers for asset \( i \) over a time \( t \) period. \( Z \sim N(0,1) \)

\( R_{it} \) = returns produced on asset \( i \) over time period \( t \).
3.3.1 Data set, parameters and valuation bases for asset

3.3.1.1 Equities.

We generate the equity returns from the GSE All share index. The returns on equity ($R_e$) are computed as follows:

$$R_e = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$  \hspace{1cm} (2)

Where $Y_t$ denote the current GSE is share index and $Y_{t-1}$ denote the previous GSE share index.

We model the returns as a simple random walk using the mean return and the volatility of the returns computed from historical data for the whole period (2007-2013) and projecting the equity returns forward over 40-year period and simulating 10,000 scenarios of the equity returns.

If $R_{et}$ is the expected return on equity, then we model the expected returns as:

$$R_{et} = \mu_e + \sigma_e Z_{et}$$  \hspace{1cm} (3)

Where $Z_{et} \sim N(0,1)$, $\mu_e$ is the mean return for the whole period, 2007-2013 and $\sigma_e$ denotes the volatility of the return for the whole period, 2007-2013.

It is worth noting that the future equity returns changes randomly and are independent of each other

3.3.1.2 Bonds

Bond returns $R_t$ are calculated from the yields. The assumption made is that, an annual par fixed coupon bond is bought within a given time period and its held for one year, and then rolled into a new bond with a given parameter $m$.

$$R_t = Y_t + \frac{Y_t (1 - \frac{1}{(1 + Y_{t+1})^m})}{Y_{t+1}} + \frac{1}{(1 + Y_{t+1})^m - 1}$$  \hspace{1cm} (4)

Where $m$ denotes the duration of the bond.
For instance, Equation (4) is used to derive the returns from a bond if an annual par coupon bond of duration $m$ is purchased at a par value of 1 at time $t$, is held for one year and it then rolled over into a new bond with the same duration. The income (coupon payments) at time period $t$ which is denoted by the current yield $Y_t$ at that time and the capital gain (gain from change in price) from the bond in year $t$ are estimated. The bond price at end of period $t$ is the sum of discounted future coupons, assuming a fixed coupon bond based on the yield at time period $t$ and the discounted face (par) value of 1 at maturity discounted by the yield $Y_{t+1}$ at time period $t + 1$. Here, the coupons and the face value are discounted using the yield at the next time period $t + 1$.

Hence, the return from the bond $R_t$ is given as the income $Y_t$ and price change $Y_t (1 - (1/(1 + Y)^{m-1})/Y_{t+1} + 1/(1 + Y_{t+1})^{m-1} - 1$ divided by the initial purchase (par) value 1 of the bond.

We model the bond returns as a simple random walk using the mean return and volatility of returns computed from historical data for the whole period (2007-2013) and projecting the bond returns forward over 40-years period and simulating 10,000 scenarios of the bond returns.

If $R_{bt}$ is the expected return on bonds, then we model the expected returns as follows:

$$ R_{bt} = \mu_b + \sigma_b Z_{bt} \quad (5) $$

Here $Z_{bt} \sim N(0,1)$ where $\mu_b$ is the mean of bond return for the whole period, 2007-2013 and $\sigma_b$ denotes the volatility of the return for the whole period, 2007-2013.

It is worth noting that the future bond returns changes randomly and are independent of each other.
When the random numbers are projected, it produces uncorrelated random numbers and this make the projected simulated returns uncorrelated. In order to make the projected simulated returns correlated, cholesky decomposition on these uncorrelated random numbers is performed. The cholesky decomposition is carried out by multiplying the uncorrelated random numbers (error terms) by the lower or upper triangular cholesky decomposition of the correlation matrix all assets.

Now let $L = l_{ij}$ be the lower triangular cholesky decomposition of the correlation matrix $A$ (that is $l_{ij} = 0$ for all $j > i$ and $A = LL^T$), therefore projected simulated returns for each asset (equity and bonds) will be given by:

$$R_{it} = \mu_i + \sigma_i(Z_{it} * L)$$

(6)

Where $\mu_i$ = mean of the return of asset $i$

$\sigma_i$ = standard deviation of the return of asset $i$

$Z_{it}$ = randomly generated random numbers for each asset $i$ over a time period $t$. $Z \sim N(0,1)$

$R_{it}$ = returns produced on asset $i$ over time period $t$.

$L$ = Lower triangular cholesky decomposition of the correlation matrix $A$

### 3.4. Liability determination and projection

This section explains the approach adopted to determine liabilities. The liabilities are projected forward over time across ages.

Based on the available data and the projection made, the number of expected pensioners, average pensions and total pensions to be paid to pensioners were computed. The projected contributors, total contributions and hence the total expense of the scheme was also computed. Combining the
projected total pensions and the projected total expenses resulted in the total liabilities incurred by the scheme.

Some important assumptions made in the analysis are that, the chosen age for the members who could start contributing to the scheme to await their pensions paid to them later during their retirement age was 20 years and the age for retirement was 60. More so, all pensioners are assumed to die at age 100.

Again, one of the principal assumptions made is that, the pensioners portfolio is a closed portfolio where there are no additional contributors added to the scheme as the years go by. In view of this, the number of pensioners will run-off by 40 years time and therefore there will be no cash inflow from any other sources than the investments. However, in open pensioners’ portfolio, additional contributors are added to the scheme as the years progress.

3.4.1 Data set, parameters and valuation bases for liabilities

3.4.1.1 Contributors

3.4.1.1.1 Projected Survivors for contributors.

The number of contributors for a particular age who survived in next year is given as follow:

\[ l_{x+1} = l_x \times p_x \quad (7) \]

Where \( l_{x+1} \) denotes the number of persons (contributors) at age \( X \) who will live to age \( X + 1 \) in the following year.

\( l_x \) denotes the number of persons (contributors) at age \( X \).

\( p_x \) denotes the probability that a person (contributor) age \( X \) will live in one year.

The number of contributors for a particular age (say age 30) who survived in the next year (say to age 31) is computed by multiplying the number of contributors (at age 30) to the survival probability at that age.
3.4.1.1.2 Projected deaths for contributors

The number of death recorded as the contributors at age \( X \) move to age \( X + 1 \) in the following year is given as:

\[
d_x = l_x - l_{x+1} \quad (8)
\]

Where \( d_x \) denotes the number of persons (contributors) who die between age \( X \) and \( X + 1 \) in the following year

\( l_{x+1} \) denotes the number of persons (contributors) at age \( X \) who will live to age \( X + 1 \) in the following year.

\( l_x \) denotes the number of persons (contributors) at age \( X \).

The projected deaths for contributors for a particular age is calculated by subtracting the number of contributors who survived (say to age 21) from their membership when they were (say at age 20) in the previous year.

3.4.1.1.3. Projected contributors

The projected contributors that moved from age \( X \) to age \( X + 1 \) in the following year is given as:

\[
l_{x+1} = l_x - d_x \quad (9)
\]

Where \( l_{x+1} \) denotes the number of projected contributors at age \( X \) who move to age \( X + 1 \) in the following year.

\( l_x \) denotes the number of persons (contributors) at age \( X \).

\( d_x \) denotes the number of persons (contributors) who died between age \( X \) and \( X + 1 \) in the following year.

The projected contributors for a particular age (say age 20) is derived by subtracting the number of death projected for contributors at this age (say age 20) from the number of contributors who survived in the past year (when the same contributors were aged 19).
3.4.1.1.4 Projected total salary

The projected total salary on which contributions were paid is given as:

\[ AS_{(x+1,t+1)} = AS_{(x,t)} \times (1 + g) \quad (10) \]

Where \( AS_{(x+1,t+1)} \) denotes the projected total salary received by contributors in the following year \( t + 1 \) and age \( X + 1 \)

\( AS_{(x,t)} \) denotes the average salary received by contributors at current time \( t \) and age \( X \).

\( g \) denotes a fixed indexation rate

The projected total salary for a particular age in the following year is derived by multiplying the indexed total salary in the previous year by one plus the index value.

One important economic assumption made is that, average salary of contributors was indexed annually by a fixed indexation rate.

3.4.1.1.5 Projected total contributions

The projected total contributions for a particular age are given as:

\[ TC_{(x+1,t+1)} = AS_{(x+1,t+1)} \times m \quad (11) \]

Where \( TC_{(x+1,t+1)} \) denotes projected total contribution paid by contributors in the following year at time \( t + 1 \) and age \( X + 1 \)

\( AS_{(x+1,t+1)} \) denotes the projected average salary received by contributors in the following year \( t + 1 \) at age \( X + 1 \)

\( m \) denotes a fixed contribution rate.

The projected total contributions for a particular age are derived by multiplying the projected total salary for that age by a fixed contribution rate.
3.4.1.2 Projected total expenses

The total expenses made by the scheme are given as:

\[ TE_{(x+1,t+1)} = TC_{(x+1,t+1)} \times n \]  \hspace{1cm} (12)

Where \( TE_{(x+1,t+1)} \) denotes the projected total expenses made by scheme for the following year \( t + 1 \) and age \( X + 1 \).

\( TC_{(x+1,t+1)} \) denote projected total contribution paid by contributors in the following year at time \( t + 1 \) and age \( X + 1 \).

\( n \) denotes a fixed expense rate.

The total expenses made for a particular age are therefore derived by multiplying the projected total contributions for that age by a fixed expense rate.

3.4.1.3 Pensioners

3.4.1.3.1 Projected Survivors for pensioners.

The number of pensioners for a particular age who survived in the following year is given as follow:

\[ l_{x+1} = l_x \times p_x \]  \hspace{1cm} (13)

Where \( l_{x+1} \) denote the number of pensioners at age \( X \) who will live to age \( X + 1 \) in the following year.

\( l_x \) denote the number of pensioners at age \( X \).

\( p_x \) denotes the probability that a person (pensioner) age \( X \) will live in one year.

The number of pensioners for a particular age (say age 65) who survived in the next year (say to age 66) is computed by multiplying the number of pensioners (at age 65) to the survival probability at that age.
3.4.1.3.2 Projected deaths for pensioners

The number of death recorded as pensioners at age $X$ move to age $X+1$ in the following year is given as:

$$d_x = l_x - l_{x+1} \quad (14)$$

Where $d_x$ denotes the number of pensioners who died between age $X$ and $X + 1$ in the following year.

$l_{x+1}$ denote the number of pensioners at age $X$ who will live to age $X + 1$ in the following year.

$l_x$ denote the number of pensioners at age $X$.

The projected deaths of pensioners for a particular age are calculated by subtracting the number of pensioners who survived (say to age 66) from their membership when they were (say at age 65) in the previous year.

3.4.1.3.3 Projected pensioners

The projected pensioners that moved from age $X$ to age $X + 1$ in the following year is given as:

$$l_{x+1} = l_x - d_x \quad (15)$$

Where $l_{x+1}$ denote the number of projected pensioners at age $X$ who move to age $X + 1$ in the following year.

$l_x$ denote the number of pensioners at age $X$.

$d_x$ denotes the number of pensioners who died between age $X$ and $X + 1$ in the following year.

The projected pensioners for a particular age (say age 66) was derived by subtracting the number of death projected for pensioners at this age (age 66) from the number pensioners who survived in the past year (when the same pensioners were aged 65).
3.4.1.3.4 Projected average pensions.

The projected average pensions paid to pensioners in the following year $t + 1$ and age $X + 1$ is given as:

$$AP_{(x+1,t+1)} = AP_{(x,t)} \times (1 + r)$$  \hspace{1cm} (16)

Where $AP_{(x+1,t+1)}$ denote the projected average pension paid to pensioners in the following year $t + 1$ and age $X + 1$.

$AP_{(x,t)}$ denote the average pension paid to pensioners at current time $t$ and age $X$.

$r$ denote a fixed indexation rate.

The projected average pension for a particular age in the following year is derived by multiplying the indexed average pension in the previous year by one plus the index value.

3.4.1.4 Projected total pensions.

The total pensions paid to pensioners for a particular age are derived as follows:

$$TP_{(x+1,t+1)} = AP_{(x+1,t+1)} \times l_{x+1}$$  \hspace{1cm} (17)

Where $TP_{(x+1,t+1)}$ denote the total pensions paid to a pensioner in the following years as time $t + 1$ and age $X + 1$

$l_{x+1}$ denote the number of projected pensioners at age $X$ who move to age $X + 1$ in the following year.

$AP_{(x+1,t+1)}$ denote the projected average pension paid to pensioners in the following year $t + 1$ and age $X + 1$.

The projected total pension is therefore derived by multiplying the projected average pensions received by pensioners for a particular age by the projected pensioners for the same age.
3.5 Approach to determine investment strategy (Asset-liability analysis)

This section explains the approach adopted to determine investment strategy. Based on the economic scenarios, the assets and liabilities of the pension schemes are projected forward. This step is repeated many times, each time based on a fresh simulation of a projected economic scenario. In particular, assuming a start date of 31 December 2007, 10,000 40-year scenarios projecting forward the assets and liabilities of the pension schemes are simulated from that date until run-off. Asset and liability optimization modeling is then carried out to determine the investment strategy.

3.5.1 Asset and Liability Management

Asset and liability management is a risk management technique which takes into account the assets, liabilities and interactions of policies which may be adopted by the board of trustees of a pension fund. In the early 2000s, taking pension funds into consideration, the traditional asset-only investment strategy which focused on outperforming a market index failed. This followed the perfect storm of the equity bubbles and low interest rate which led to large deficits in pension funds. The required investment strategy that ensures that the solvency of a fund is enough to pay off all liabilities is determined by the pension fund trustee. The solvency of the fund in the long run may be measured over a specified solvency probability (that is the probability that all liabilities are covered in the long run).

The sponsors of the fund adjust the contributions to compensate for the shortfalls when the fund is in deficit. Otherwise surpluses may be redistributed to sponsors or used to improve benefit levels in some circumstances. The changes in assets and liabilities of the scheme cause changes in solvency level over time. Practically, the conflicting interest of stakeholders influences investment strategy decisions. The stochastic influences from the market and economic and
actuarial risks intensify the influence on investment strategy decision. In the next section, the asset and liability model incorporating these stochastic influences is described. The last two sections looked at the optimization model and determination of the investment strategy.

3.5.2 Asset and Liability Modelling

The asset and liability model considered a closed pensioners portfolio using the SSNIT 2005 male pensioner mortality, shown in Table A1 of the appendix. Even though, pension scheme can operate under the closed and open pensioners’ portfolio, only the situation for the closed pensioners portfolio was considered in this study. Based on the pension plan design and actuarial assumption made, the liabilities are calculated. 10,000 40-year scenarios of the pension liabilities are simulated and projected forward.

The changes in the characteristics of the pension plan participants and demographics as well as risk factors such as interest risk, longevity risk and ageing affect the pension liabilities. The scope of this research does not cover these risks. The exposure of longevity risk on pensions will cause pension payment to be made for a longer period as far as the recipient (pensioner) lives longer. This may directly affect the funding status of the fund. Blake, Cairns and Dowd (2006), looked at the longevity risk into details and discussed the various ways to manage this risk exposure. Once the assets and liabilities have been calculated, the solvency of the fund is obtained at the run-off horizon. A scheme is solvent if it is able to pay all liabilities in the long run. The solvency at any point in time is measured by the difference in the market value of assets and liabilities. Once the asset and liabilities at time zero are known, the fund values are projected based on a recurring relation as follows:

$$F_{t+1} = F_t (1 + \bar{R}) - M_t \quad (8)$$
where $\bar{R}$ denotes the stochastic investment (expected) return obtained from the mean-variance model specified, assuming liabilities $M_t$ are paid at the end of the year, where $M_t$ denotes the sum of all liabilities in the portfolio. The procedure is projected into the future until the liabilities are paid off in 40 years time when all pensioners are assumed to be dead at age 100. The step above is repeated for 10,000 simulations of the assets and liabilities.

The most common and fundamental framework that is used by pension fund managers as a financial measure to assess a particular strategy before identifying the investment strategy is the efficient frontier concept as introduced by Markowitz (1952). Looking at the Markowitz framework, a PFM must choose a return measure (e.g. expected surplus) and a risk measure (e.g. value-at-risk, see Blake et al. 2001), or a worst conditional mean as a coherent risk measure (see Artzner et al. 1999). Then the measured risk and return of each strategy is plotted in a risk-return space. When there is a no other strategy with a low risk at the same return level, or a higher return at the same level of risk, then a strategy is said to be efficient. The Markowitz framework was extended to an optimal investment strategy indifference map of efficient frontiers by Maurer, Mitchell and Rogalla (2009). The TVAR cost and volatility of contribution against expected return of each strategy was a measure of risk in the model by Maurer, Mitchell and Rogalla (2009).

Efficient frontiers have some shortcomings although it is a good means of communicating asset-liability analysis. For instance, Cumberworth et al. (1999) indicated that a typical efficient frontier uses risk measures that mix systematic risk (non-diversifiable by shareholders) and non-systematic risk, which blurs the shareholder value perspective. Moreover, efficient frontiers may give misleading information if they are used to make investment decisions once the systematic risk has been factored into the risk measure.
As indicated in Cumberworth et al. (1999), the often disregarded and unappreciated aspect of dynamic financial modeling is the interpretation of outputs. For instance, an efficient frontier still leaves a variety of equally desirable strategies.

### 3.5.3 Portfolio Optimisation and Investment Strategy

Over a specified time period, an actuarial approach is used to determine the probability of solvency for a minimum investment required and then to determine the investment strategy. The investment strategy is determined after all liabilities are paid in 40 years time when all pensioners are assumed to be dead at age 100.

At time zero, the investment strategy is determined for the starting fund such that the proportion of scenarios before the assets are run off by the liabilities is, say, \((1 - \beta)\%\). Then it can be said that the scheme is solvent at that level of confidence.

In this model, this translates into determining an investment strategy that will ensure that a minimum amount of assets is kept now at an agreed confidence level. In particular, the strategic asset allocations are obtained for which the amount of assets kept now is minimized to ensure that the probability of the pension fund being ruined at the run-off horizon is at most \(\beta\%\), where \(\beta\) denotes a very small probability. Mathematically, the following optimization problem is solved:

\[
\begin{align*}
\text{Min}_{w_i}(F_0) \\
\text{subject to:} \\
P\{(F_T - M_T) \geq 0\} \geq (1 - \beta), \quad (19) \\
\sum_{i}^{n} s_i = 1,
\end{align*}
\]
\[ s_i \geq 0 \forall i \]

where \( s_i \) denotes the weights in asset \( i \), \( F_0 \) denotes the amount of asset kept at time zero, and \( P\{ (F_T - M_T) \geq 0 \} \) denotes the probability of solvency. In this model there is no allowance for short-selling, hence \( s_i \geq 0 \) for all \( i \). This approach serves as a solvency testing tool and also provides a whole probability distribution of surplus in the long run. One of the rationales underlying asset-liability management (ALM) is the minimization of ruin probability in a DB scheme. Schneiper (1997, 1999) and Leibowitz, (1986) further explained the solvency tool using dynamic financial analysis.
CHAPTER FOUR

ANALYSIS AND DISCUSSION OF FINDINGS

4.0 Introduction

This section of the study seeks to address some basic risk and return characteristics of historical data. An analysis on investment return of the asset classes as well as the analysis on liabilities paid by pension schemes over a projection of 40 years was carried out. Finally, the analysis on investment strategy (that is the asset allocation and the minimum initial investment that need to be kept in order to make the fund solvent at a specified probability in the future) under closed pensioners portfolio is also analysed.

4.1 Some Basic Risk and Return Measures

The data used covered the period 2007 to 2013 since that was the longest whole-year periods for which yields on bonds are available. I generally used annual data in my analysis and rolling two-year periods (1 January of the starting years to 31 December of the ending year), as well as analysis of the whole period.

Figure 4.1 shows the annual returns for each asset class over the period 2007 to 2013. There are quite a number of interesting points to note about the Figure. One of the first points relates to the volatility of the asset classes. The returns on equity are clearly more volatile than those on other asset classes. It is interesting to note that equity have performed well in the last year even though its performance generally from 2009 to 2012 was quite poor. However, treasury bill and bonds (both One-year and Two-year bonds) have both performed well over the same periods.
In order to get a clearer idea of the relative attractiveness of asset classes, it is needful to calculate some more statistics.

Looking at the mean monthly returns and the standard deviations of the monthly returns in Figure 4.2, it is quite clear that equities have the highest average returns but with the greatest volatility. This pattern of high returns for higher risk also holds for 2-year bonds but for treasury bills and One-year bonds, the volatility was small for a high return. Now looking at the annual returns, it can be seen that the recent good performance by One-year bond in the last three years (2011-2013) is as big a factor in explaining the high mean return and low volatility of One-year bond as compared to the Two-year bond with high mean return and high volatility.
This can be demonstrated and further explained by looking at the rolling two-year mean returns in Figure 4.3. These show that it is only in the last two two-year periods (last three single years from 2011-2013) that two-year returns on Two-year bond have fallen to a level comparable below that of One-year bond and even treasury bills.

Figure 4.2: Mean and standard deviation of monthly returns, 2007-2013

![Mean and standard deviation of monthly returns, 2007 - 2013](image)

Source: Author’s construct
Figure 4.3: Mean monthly returns, rolling two-year periods, 2007-2013

![Mean monthly returns, rolling two-year periods, 2007-2013](image)

Source: Authors construct

Figure 4.4: Standard deviation of monthly returns, rolling two-year periods, 2007-2013

![Standard deviation of monthly returns, rolling two-year periods, 2007-2013](image)

Source: Author’s construct
The picture of the standard deviation shown in Figure 4.4 is more consistent, with equity still at highest risk, One-year bond and treasury bill at a lower risk and the Two-year bond in the middle. Standard deviation of returns was also unsteady over time for all assets until the last two years when the volatility for equity and Two-year bond increased again.

Figure 4.5: Mean – variance analysis, rolling two-year periods, 2007-2013

Source: Author’s construct

*The black dots at the tip of each of the lines represent the last Two-year periods.

The mean and standard deviation can be combined to give the traditional risk and return chart as shown in Figure 4.5, with risk measured as standard deviation of monthly returns. As shown clearly in the Figure, the bond asset classes (treasury bill, One-year bond and Two-year bond) move in the broadly similar ways which is very different from equity. Now looking more closely, the risk showing between the Two-year bond and the other bond asset classes (treasury bills and One-year bond) stay reasonable constant over the various periods. However the return
advantage of the Two-year bond over the other bond asset classes dropped suddenly in the last rolling period. On the other hand, the returns of equities rose suddenly in the last rolling period.

**Figure 4.6 Mean-variance analysis, 2007-2013**

Source: Author’s construct

Figure 4.6 shows the mean variance analysis for the whole period (2007-2013). It shows that Two-year bond has the highest risk and also do provide the highest returns as compared to that of the other bond asset classes (treasury bill and One-year bond).

One way of demonstrating the risk-adjusted returns on the various asset classes is to look at the Sharpe ratio. This is calculated as the mean of the excess returns over the risk-free asset divided by the standard deviation of the same asset returns. For the risk-free asset class, I used treasury bills, so the risk free rate is the treasury bill rate. The Sharpe ratio over the successive periods for the four asset classes is given as Figure 4.7.

Figure 4.7 shows that, for the period under investigation, on a risk-adjusted basis, there is not much to choose between the various asset classes (that is the relative attractiveness of each asset
changes over the two-year rolling periods) although equities have fared particularly well over recent times.

Now looking at the results for the whole period in Figure 4.8, the asset classes appear to all fall into two distinct group. Treasury bills, Two-year bonds and One-year bonds all give a good risk-adjusted return (Sharpe ratio greater than 1 indicating a good risk-adjusted returns, a value of 1 is good, a value of 2 is great and 3 is exceptional) but equities was considered to have a poor risk-adjusted returns as compared to the bond asset classes.

Figure 4.7: Sharpe ratios, rolling two-year periods

Source: Author’s construct
4.2 Skew and Kurtosis

Although the mean-variance analysis gives an indication of the risk-return trade off, it does not always give the whole picture. For instance, investors interested in a one-sided measure of risk such as expected shortfalls should consider the shape of the return distribution that is skewness and excess kurtosis.

The skew of a distribution measures how lop-sided the distribution is (a positive skew indicates that the right tail of the distribution is longer than the left and the mean is greater than the median, which is greater than the mode) whiles negative skew indicates the opposite. The normal distribution is symmetric and therefore the mean, median and mode are equal.

If skewness is less than -1 or greater than +1, the distribution is highly skewed (negative if less than 0 and positive if greater than 0). If skewness is between -1 and -1/2 or between +1/2 and +1, the distribution is moderately skewed (negative if less than 0 and positive if greater than 0). If
skewness is between -1/2 and +1/2, the distribution is approximately symmetric (negative if less than 0 and positive if greater than 0).

For an investor, a negatively skewed distribution means frequent small gains and few extreme losses indicating a greater chance of negative outcomes and a positively skewed distribution means frequent small losses and few extreme gains indicating lesser chance of negative outcomes. A nonsymmetrical distribution (exactly 0 mark) are described as being either negatively skewed (greater chance of negative outcomes) or positively skewed (lesser chance of negative outcomes).

Figure 4.9: Skew of monthly returns, 2007-2013

Source: Author’s construct

Excess kurtosis also measures the fatness of the tail of the distribution. The fatter the tails, the greater the chances of an extreme results relative to the probability implied by the normal distribution. Excess kurtosis is calculated as the kurtosis minus 3 (that is excess kurtosis = kurtosis -3) since the normal distribution is the reference standard which has a kurtosis of 3.
A distribution with kurtosis ≈ 3 (excess kurtosis exactly 0) is called mesokurtic.

A distribution with kurtosis < 3 (excess kurtosis < 0) is called platykurtic.

A distribution with kurtosis > 3 (excess kurtosis > 0) is called leptokurtic.

It is important to note that the skewness and excess kurtosis can move a great deal depending on the sample period (that is adding an additional year of data can have a large impact on the skewness and excess kurtosis of the return distribution.

The skewness for the whole period in Figure 4.10 indicates that equity is highly skewed (positive) whiles treasury bills, Two-year bond and One-year bonds are approximately symmetric but with positive, positive and negative skewness respectively.

This indicates that equity is likely to produce frequent small losses and few extreme gains hence lesser chances of negative outcomes as compared to treasury bills and Two-year bond. On the other hand, a One-year bond has greater chance of producing negative outcomes since it is negatively skewed.

Figure 4.10: Excess kurtosis of monthly returns, 2007-2013

Source: Author’s construct.
However, the excess kurtosis for the full period in Figure 4.10 marks out bond asset classes (that is treasury bill, Two-year bond and One-year bond) as being less normal because their excess kurtosis are less than 3 whiles equities is more normal as compared to the bond asset classes.

4.3 Investment returns analysis of asset classes

This section of the work concentrates on the returns of the assets only. First to look at the returns produced by the assets and the level, stability and development of the projected returns over time (projection over 40 year period).

The returns calculated from GSE indices and bond yields (2007 to 2013) are presented in Table 4.1 below. In this analysis, it was preferable to calculate annual returns for all asset classes.

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>Treasury bill</th>
<th>Two-year bond</th>
<th>One-year bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>0.29</td>
<td>0.10</td>
<td>0.18</td>
<td>0.12</td>
</tr>
<tr>
<td>2008</td>
<td>0.61</td>
<td>0.19</td>
<td>0.08</td>
<td>0.20</td>
</tr>
<tr>
<td>2009</td>
<td>-0.49</td>
<td>0.27</td>
<td>0.18</td>
<td>0.21</td>
</tr>
<tr>
<td>2010</td>
<td>-0.81</td>
<td>0.15</td>
<td>0.46</td>
<td>0.13</td>
</tr>
<tr>
<td>2011</td>
<td>-0.07</td>
<td>0.11</td>
<td>0.14</td>
<td>0.11</td>
</tr>
<tr>
<td>2012</td>
<td>0.15</td>
<td>0.19</td>
<td>0.07</td>
<td>0.23</td>
</tr>
<tr>
<td>2013</td>
<td>3.34</td>
<td>0.12</td>
<td>0.30</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Source: Author’s calculation.
4.3.1. Long-Term Results

The returns on the different asset classes (equities, treasury bills, One-year bonds and Two-year bonds) over a long term period (say 40 years) can be calculated and analyzed. First is to consider the mean and standard deviations of the returns of the various assets classes for the whole period (2007 to 2013). The calculation of returns on assets is straightforward as shown in Equations (2) and (4). The projections of simulated correlated returns on both equity and bond assets are also shown in Equation (3) and (5) respectively.

The simulated projected correlated returns for the different asset classes over 40 year period are shown in Figure 4.11, 4.12, 4.13 and 4.14

Figure 4.11: Projected average equity returns

![Projected average equity returns](image)

Source: Author’s construct
Figure 4.12: Projected average treasury bills returns

![Projected average treasury bills returns](image1)

Source: Author’s construct

Figure 4.13: Projected average Two-year bond returns

![Projected average Two-year bond returns](image2)

Source: Author’s construct
As can be seen in the figures on the projected average returns of the different asset classes, the returns produced by the portfolio of equity is higher than that produced by bond asset classes (treasury bill, Two-year bond and One-year bond).

From the graphs and table describing the risk and return characteristics of the projected returns on asset classes, it is quite interesting to note that projected average returns and volatility of the returns for equity is the highest among all asset classes followed by Two-year bond with higher projected average return and higher volatility as well. This pattern of high returns for higher risk in the projected years is consistent with the same pattern for equity and Two-year bond in the historical data.

However, treasury bill and One-year bond also exhibit a pattern of higher returns for low risk in the projected years which is also consistent with the same pattern in the historical data.

Now looking at other measures of risk and return characteristic of the projected average returns such as Sharpe ratio, all the asset classes appear to have good risk-adjusted returns.
Considering the skewness of the projected returns, equities and treasury bills show positively skewed distribution indicating frequent small losses and few extreme gains hence lesser chance of negative outcomes. However, Two-year bonds and One-year bonds show negatively skewed distribution indicating frequent small gains and few extreme losses hence greater chance of producing negative outcomes. Equity appear to be more normal as compared to the bond asset classes by virtue of its value in measuring excess kurtosis.

Table 4.2: Summary characteristics of simulated returns over the entire projection (40 years)

<table>
<thead>
<tr>
<th></th>
<th>Equities</th>
<th>Treasury bills</th>
<th>Two-year bond</th>
<th>One-year bond</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.484785248</td>
<td>0.16135145</td>
<td>0.196178705</td>
<td>0.181454535</td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td>0.243382526</td>
<td>0.008526696</td>
<td>0.019531433</td>
<td>0.005812108</td>
</tr>
<tr>
<td><strong>Sharpe ratio</strong></td>
<td>1.92605685</td>
<td>17.04467723</td>
<td>42.36764812</td>
<td>28.46434884</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>2.0607486</td>
<td>0.428271481</td>
<td>-0.305544272</td>
<td>-0.402889096</td>
</tr>
<tr>
<td><strong>Excess Kurtosis</strong></td>
<td>4.918962426</td>
<td>-3.396230865</td>
<td>-2.684256723</td>
<td>-2.68920158</td>
</tr>
</tbody>
</table>

Source: Author’s calculation

Table 4.3: Summary characteristics of returns on historical data (2007-2013)

<table>
<thead>
<tr>
<th></th>
<th>Equities</th>
<th>Treasury bills</th>
<th>Two-year bond</th>
<th>One-year bond</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.43</td>
<td>0.16</td>
<td>0.20</td>
<td>0.18</td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td>1.266179028</td>
<td>0.054766519</td>
<td>0.125591525</td>
<td>0.047656625</td>
</tr>
<tr>
<td><strong>Sharpe ratio</strong></td>
<td>0.328521</td>
<td>2.884301355</td>
<td>1.447426247</td>
<td>3.704470557</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>1.93767</td>
<td>0.040858</td>
<td>0.122994</td>
<td>-0.01122</td>
</tr>
<tr>
<td><strong>Excess kurtosis</strong></td>
<td>1.536190475</td>
<td>-2.434702003</td>
<td>-1.703488815</td>
<td>-5.534605102</td>
</tr>
</tbody>
</table>

Source: Author’s calculation
Therefore considering assets-only analysis of pension schemes without matching liabilities, pension fund manager can now decide which best asset to invest in looking at various risk and return characteristics (mean, standard deviation, Sharpe ratio, skewness and excess kurtosis) of projected average returns in the future (over 40 year period). Here equity appears to be an attractive asset classes to invest in.

Comparing all the risk and return characteristics (mean, standard deviation, Sharpe ratio, skewness and excess kurtosis) of both the historical returns and projected simulated returns, it can be concluded that the risk and return characteristics of the historical returns are similar to that of the projected simulated returns except in the case of skewness for Two-year bond which was negatively skewed for the projected simulated returns and positively skewed for historical returns and secondly, equity which had poor risk-adjusted return for the historical returns but good risk-adjusted returns for the projected simulated returns. On the whole, we could conclude that the historical returns could be taken as a good indicator of future returns without using a stochastic asset model to project future returns on assets.

The subsequent works will look at which best asset to invest in, considering liabilities over time.

4.4 Liability analysis.

This section of the analysis concentrates solely on the liabilities. First to calculate the liabilities that are paid by social security scheme and the projection made into the future in order to ensure the viability and sustainability of the scheme. The analysis entails the determination and projection of contributors, total salaries and hence the projected total contributions which indicate the funds inflow expected from contributions in the future (over the next 40 years).

The projected total contributions and the investment returns makes up to the total asset of the scheme.
Based on the available data and the projection made, the number of expected pensioners, average pensions and total pensions are computed. The projected total expenses of the scheme are also computed. Combining the projected total pension and the projected total expenses results in the total liabilities incurred by the scheme.

Some important assumptions made in the analysis are that, all pensioners will die by age 100. Also, the chosen age for the members who could start contributing to the scheme to await their pension payments during their retirement age was 20 years and the age for retirement was 60.

It is worth noting that the principal assumption made in our analysis is that the pensioners portfolio was a closed portfolio where there are no additional contributors added to the scheme hence the number of pensioners will run-off by 40 years time and therefore there will be no cash inflow from any other sources than the investments.

It is also important to note that the number of projected years of 40 was chosen because per the projected pensioners analysis made, all pensioners will die by 40 years (that is pensioners at age 60 at the start of scheme will die by age 100) and this will be the only time when the scheme can determine that it has paid off all its liabilities (especially total pensions) and can then determine the sustainability of the scheme.

4.4 Contributors

4.4.1 Projected survivors for contributors.

The falling trend of the number of contributors for a particular age who survived to the next age in the following year can be expected. The graph indicated that by forty year time, all the contributors age 20 who survived to age 59 will be approximately 10409. Now at age 60, these surviving contributors (10409) will become pensioners by then. In view of this, considering a closed pensioners portfolio where no additional contributors are added to the scheme as the years
progress, there will be no contributors in the scheme which is indicated by the zero mark at time 40. The projected survivors for contributors are computed from Equation (7)

Figure 4.15: Projected survivors for contributors

![Projected survivors for contributors](image)

Source: Author’s construct

The straight line nature of the graph also tells that, the number of survivors decreases proportionally as the years progress. First and foremost, this falling trend occurs because it is expected that some contributors will die as the years progress and no additional contributors are added to the scheme as the years progress (that is considering a closed pensioners portfolio).

### 4.4.1.2. Projected deaths for contributors

Looking at Figure 4.16, the death projected for contributors for a particular age who moved to the next year was also decreasing quite proportionally as the years progressed. The projected death for contributors are computed from Equation (8)

This falling trend occurs because the number of contributors who survive as the years go by reduces and therefore the number of people who will die out of these surviving contributors will
turn to decrease as the years progress. It is, therefore, quite interesting to notice from Figure 4.17 that the number of death recorded for contributors at age 20 (at start of scheme) who move to age 59 will reduce to 281 deaths.

Figure 4.16: Projected deaths for contributors

Source: Author’s construct

The surviving contributors at age 59 will become pensioners in the fourth year (age 60) hence they were no more contributors in the scheme at that time which is indicated by zero mark at time 40.

4.4.1.3 Projected contributors

The graph of the projected contributors shows a falling trend as well. Figure 4.17 show that, in forty years time, the projected contributors that move from age 20 to age 59 will be approximately 10128. In the fourth year, these projected contributors (10128) will become pensioners by then so there will be no projected contributors in the scheme considering a closed pensioners portfolio where no additional contributors are added to the scheme as the years progress. This is indicated by the zero mark at time 40.
The straight line nature of the graph also tells that the number of projected contributors decreases proportionally as the years progress. From Equation (9), it can be seen that the projected contributors depends on both the projected survivors and the projected deaths for contributors. Hence since the latter parameters show a decreasing trend, the former will also show a falling trend.

4.4.1.4 Projected total salary

Figure 4.18 shows the falling trend in the projected total salary on which contributions were paid. It can be explained from the graph that contributors at age 20 who move to age 59 will be receiving a total salary of GHC16449.01.

The concavity of the graph gives an indication that the projected total salary received by contributors decreases slowly as the years progress. This falling trend occurs because the number of projected contributors reduces as the years progress hence the total salary received by these contributors.
projected contributors reduce as the years go by. The projected total salary is calculated using
Equation (10).

Figure 4.18: Projected total salary

![Projected total salary graph]

Source: Author’s construct

4.4.1.5 Projected total contributions.

The picture of the projected total contributions shown in graph 4.19 is more consistent with that of projected total salary, still showing a falling trend since the former depends on the latter.

The projected total contributions decrease slowly as the years progress because the total salary on which total contributions are made also decreases as the years go by. The relationship is clearly shown in Equation (11).
4.4.2 Projected total expenses.

The graph in Figure 4.20 also shows a falling trend of expenses based on the actuarial projections for the running of the scheme. This pattern seems to occur because of the closed pensioners portfolio assumption made.

The trend of this graph is further confirmed by that of projected total contribution since the projected total expenses depends largely on the projected total contributions hence the concave nature of both graphs indicating a slow decrease in total expenses and total contributions respectively as the years progress. The projected total expenses reduce because the total contributions paid by contributors to the scheme reduce as the years progress which is computed from Equation (12).
4.4.3 Pensioners

4.2.3.1. Projected survivors for pensioners.

There seem to be a falling trend for the number of pensioners of a particular age who survive to the next age in the following year. The graph shows that all the pensioners at age 60 who survive to age 99 will be approximately 24. Interestingly, at age 100, it is assumed that there will be no survivor in the scheme which is indicated by the zero mark at time 40. The projected survivors for pensioners is computed from Equation (13).

The convex nature of the graph also tells that the number of survivors for pensioners decreases fast as the years progress. This occurs because some pensioners are expected to die as the years progress and no additional pensioners are also added to the scheme as the years progress (that is considering a closed pensioners portfolio).
4.4.3.2 Projected deaths for pensioners

Figure 4.22 shows that the deaths projected for pensioners of a particular age who move to the next year decreases fast as the years progress due to the convex nature of the graph shown.

It is worth noting from Figure 4.22 that, the number of death that would be recorded as the number of pensioners at age 60 move to age 99 would be approximately 14 deaths. In the fourth year, it is assume that all pensioners will die which is indicated by the zero mark at time 40. The projected deaths for pensioners are calculated from Equation (14).

This falling trend occurs because the number of pensioners who survive as the years go by reduces and therefore the number of people who will die out of these surviving pensioners will turn to decrease as the years progress.
4.4.3.3 Projected pensioners

The falling trend of the graph for projected pensioners of a particular age who move to the next age in the following year can be expected.

Figure 4.23 show that, in forty years time, the projected pensioners that move from age 60 to age 100 (where they all die) will be 0. The convex nature of the graph also tells that the number of projected pensioners decrease rapidly as the years progress. From Equation (15), it can be seen that the projected pensioners depend on both the projected survivors and the projected deaths for pensioners. Hence since the latter parameters show a decreasing trend, the former also shows a falling trend.
4.4.3.4 Projected average pension

The graph in Figure 4.24 shows the falling trend in the projected average pensions that are paid to pensioners. It can be explained from the graph that, pensioners at 60 who moves to age 99 will be receiving an average pension of GHC1965.16.

The concavity of the graph gives an indication that the average pensions paid to pensioners as the years progress decreases slowly. This falling trend occurs because the projected pensioners who will receive these average pensions decreases as the years progress hence the projected average pensions also decreases as time progresses. The projected average pension is computed using Equation (16)
4.4.4 Projected total pensions

The graph shows a falling trend of total pension payments. The falling trend of benefit paid to pensioners (total pensions) can be expected since in a closed pensioner portfolio since the projected pensioners as well as the projected average pension decreases as the years progress.

It is quite interesting to note that the total pensions to be paid to pensioners at age 100 will be 0 since it was assumed that all pensioners will die by age 100.

The convexity of the graph tells that the pension payments made by the scheme to pensioners decreases fast as the year progress. This is more consistent with the graph on projected average pensioners and projected average since the former (projected total pensions) depends on the two latter variables (projected average pensions and pensioners).

The projected total pension is one of the two main liabilities which are incurred by the scheme with the other liability being the total expenses.
4.5. Investment strategy

This section provides an analysis of investment strategy (that is the asset allocation and the minimum initial investment that need to be kept in order to make the fund solvent at a specified probability in the future). It is worth being reminded that the closed pensioners’ portfolio was considered in this case.

4.5.1. Investment strategy and solvent probabilities.

Table 4.4 summarizes the solution reached when the basic problem in Equation (19) is solved. The minimum investment required as well as the sensitivity of the asset allocation to changing solvency probabilities for a 40-year horizon is also shown. A horizon of this length (40 years) is sufficient to examine risk and return characteristics of a selected portfolio because it is at this period that the scheme would have paid off all its liabilities and can determine the solvency of the scheme.
Table 4.4: Investment strategy under varying solvent probabilities

<table>
<thead>
<tr>
<th>Solvency probability</th>
<th>Minimum investment required</th>
<th>Asset allocations (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Equity</td>
</tr>
<tr>
<td>97.5%</td>
<td>700,000,000</td>
<td>0%</td>
</tr>
<tr>
<td>92.5%</td>
<td>510,000,000</td>
<td>70%</td>
</tr>
<tr>
<td>90%</td>
<td>495,000,000</td>
<td>70%</td>
</tr>
</tbody>
</table>

Source: Author’s calculation

Tables 4.4 depict the investment strategy. The table shows the asset allocation and the minimum investment required at different solvency levels.

The maximum risk portfolios at 90% and 92.5% solvency levels have quite similar asset allocations. At 90% solvency level, the asset allocation consist of a 70% equity allocation and 30% bond allocation with equal proportions of 10% under treasury bills, Two-year bonds and One-year bonds. Interestingly, at 92.5% solvency level, there is a 70% equity allocation and 30% bond allocation with 20% and 10% under Two-year bonds and One-year bonds respectively.

However, the minimum risk portfolio at 97.5% solvency level consists of 100% bond allocation, specifically One-year bonds. This can be explained by the fact that One-year bonds have very good risk-adjusted returns and low risk. There is a general trend of asset allocation shifting from equities to bonds (specifically One-year bonds) at higher solvency levels.

Having considered the general trend in asset allocation, the general trend in minimum investment required is considered next. From table 4.3, there is a direct relationship between solvency...
probability and minimum investment required. The minimum investment required increases as the risk tolerance is reduced (approaching higher solvency levels).

**4.6 Chapter Summary**

Considering the risk and return characteristics of asset classes based on historical data, equities have the highest average returns with the greatest risk. Two-year bonds have higher risk and returns as compared to that of the other bond asset classes (treasury bill and One-year bond). Treasury bills, Two-year bonds and One-year bonds all give good risk-adjusted return whiles equities have poor risk-adjusted returns as compared to the bond asset classes. Equities have lesser chances of producing negative outcomes as compared to treasury bills and Two-year bonds whiles One-year bonds have greater chances of producing negative outcomes. Bond asset classes (that is treasury bill, Two-year bond and One-year bond) are less normal whiles equities are more normal as compared to the bond asset classes.

Looking at the risk and return characteristics of asset classes based on the projected average returns and considering assets of pension schemes without matching liabilities, equity appears to be an attractive asset class to invest in. Generally, comparing all the risk and return characteristics (mean, standard deviation, Sharpe ratio and excess kurtosis) of both the historical returns and projected simulated returns, it can be concluded that the historical returns could be used as a good indicator of the future returns without using a stochastic asset model to project future returns on assets since the risk and return characteristics of the historical returns was similar to that of the projected returns as opined by Sweeting (2004).

Under a closed pensioners’ portfolio, total pensions and expenses which constitute the liabilities incurred by scheme total pensions paid to pensioners decreases as the year progress and total expenses made by the scheme also decrease as the years progress.
When liabilities are taken into account, bonds (specifically One-year bonds) are the best-matched liabilities. There is a shift in asset allocation from equity towards bonds (specifically One-year bonds) at a higher solvency level and the minimum investment required also increases as the solvency level increases.
CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Introduction

This chapter gives a summary of the research. It also draws some conclusions on the investment strategies of pension schemes. Finally recommendations were made to help improve investment strategies adopted by pension fund managers.

5.1 Summary

Pension scheme providers in Ghana adopt different asset allocation as investment strategy in order to produce good returns on investment. These pension scheme providers invest most of their assets into fixed income investments like bonds and treasury bills and non-fixed income investments like equities. For instance, SSNIT, one of the largest pension scheme providers in Ghana, adopt a 60% bond allocation and 30% equity allocation. Even though there has been significant improvement in pension fund investment and returns over the last decade, the asset allocation required to produce good returns on investment continues to be a challenge to pension scheme providers in Ghana. This study tend to investigate the role of fixed income in pension scheme investment by looking at the asset allocation and the the initial amount needed by pension scheme providers to make the scheme solvent in the future at a specified high probability after matching all liabilities.

A stochastic asset model (specifically mean-variance model) was used to project historical returns on equities and bonds forward over 40-year period and 10,000 scenarios of equity and bond returns were simulated to derive the projected returns on equities and bonds. The risk and return characteristics of asset classes based on the projected average returns were also computed
to know the attractive asset class to invest in the future without matching liabilities. The number of expected pensioners as well as the average pensions and total pensions to be paid to pensioners were computed. The projected contributors, total contributions and hence the total expense of the scheme was also computed. Looking at the projected returns on assets (equities, treasury bills and bonds), investment returns on assets as well as the projected total contributions constitute the total assets of pension schemes. On the other hand, projected total pensions and the projected total expenses which constitute the total liabilities incurred by the scheme were projected over 40-year period across ages. The minimum investment required and the asset allocation to changing solvent probabilities for a 40-year horizon after matching liabilities was derived using a stochastic asset-liability model as described in Chapter 3.

5.2 Conclusions

Conclusions drawn from the study will help pension fund manager know the best asset to invest in, without matching liabilities of pension schemes as well as the asset allocation and minimum investment required to make the scheme solvent in the future at a specified high probability after matching all the liabilities of pension schemes.

Analysis of the risk and return characteristics of asset classes based on the projected average returns show that equity appears to be an attractive asset class to invest in considering assets of pension schemes without matching liabilities.

Generally, comparing all the risk and return characteristics (mean, standard deviation, Sharpe ratio and excess kurtosis) of both the historical returns and projected simulated returns, it can be concluded that the historical returns could be used as a good indicator of the future returns without using a stochastic asset model to project future returns on assets since the risk and return
characteristics of the historical returns was similar to that of the projected returns as opined by Sweeting (2004).

Concerning total pensions and expenses which constitute the liabilities incurred by scheme under a closed pensioners’ portfolio, total pensions paid to pensioners decreases as the year progress and total expenses made by the scheme also decrease as the years progress.

When liabilities are taken into account, the picture changes and bonds (specifically One-year bonds) is the best-matched liabilities since they have good risk-adjusted returns and are less risky. The asset allocation moves from equity towards bonds (specifically One-year bonds) at a higher solvency level and the minimum investment required also increases as the solvency level increases.

5.3 Recommendations

The recommendations provided in this section of the study are to help pension fund managers to know the best asset class to invest in without matching liabilities and also how to manage their liabilities. Recommendations regarding the best asset that matches liabilities as well as the minimum investment required to make the scheme solvent in the future at a specified high probability is also addressed.

Pension fund manager are advised to invest more in equities if they should consider asset-only without matching liabilities.

In the case of the falling trends of expenses and pension payments under the closed pensioners portfolio, management of pension schemes are advised to maintain the procedures and the organization structure that streamline operations aimed at cost cutting and cost reduction of expenses to keep it within manageable limits. Actuaries who manage the pension computations of pension scheme in Ghana are also advised to maintain the formula for pension computation to
ensure moderate pension payments in order to ensure the sustainability of the scheme in the future.

Pension scheme providers are traditionally regarded as long term investors in the financial market since they have liabilities to match in the future. It is therefore advisable that, pension scheme know the best asset that matches liabilities. In this regard, pension fund managers are further advised to look at bond asset classes (that is treasury bills, One-year bonds and Two-year bonds) especially One-year bonds as the most attractive asset to match liabilities since they have good risk-adjusted returns and are less risky. The minimum initial fund recommended in this study will help to make pension schemes solvent in the future hence pension fund manager are advised to adopt these investment strategies.
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APPENDIX
Table A1: Mortality Table.

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*Source: Author’s calculation.*

*The SSNIT 2005 Males Mortality Table, Actuarial Review of the State Pension Scheme in Ghana, ILO Report.*